Open Quantum Systems: from the transmission of correlations to the measurement problem.

Antonella De Pasquale

KITP, UC Santa Barbara

SCUOLA
NORMALE
SUPERIORE

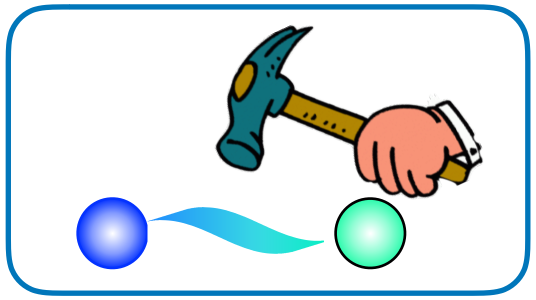
UNIVERSITÀ
DEGLI STUDI
FIRENZE

INFN
Istituto Nazionale di Fisica Nucleare
SEZIONE DI FIRENZE

Outline

Main topic: open quantum systems

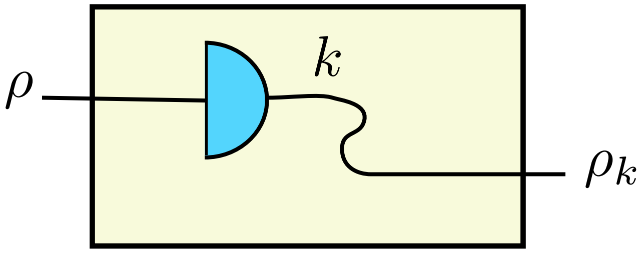
environment as NOISE



transmission of correlations

Phys. Rev. A **86**, 052302 (2012); Phys. Rev. A **96**, 022322 (2017)
Phys. Rev. A **96**, 012314 (2017); Phys. Rev. A **98**, 042301 (2018)

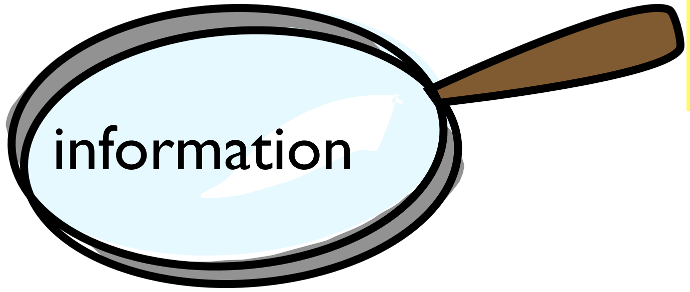
EB maps



quantum measurements

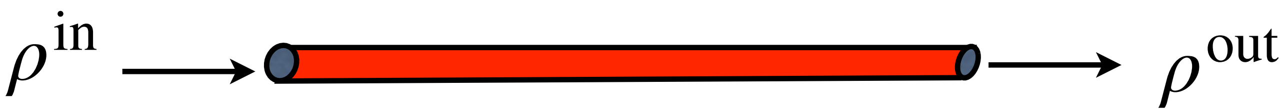
arXiv:1902.03628v2

environment ENCODES information



what
we read

our framework: quantum maps



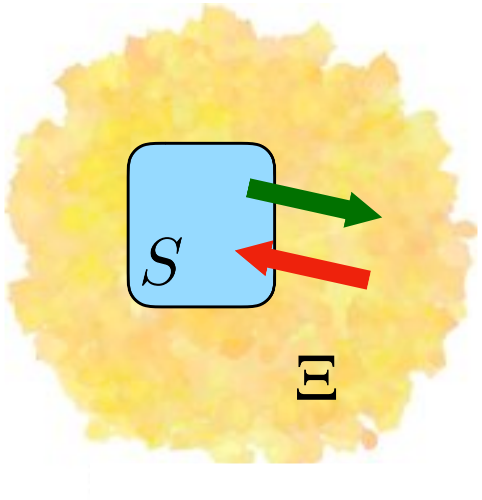
coherent evolution:
(isolated systems)

$$\frac{d\rho}{dt} = -i[H,\rho] \xrightarrow[\text{integrated}]{} \rho^{\text{out}} = \mathcal{U}[\rho^{\text{in}}]$$ unitary CHANNEL

noisy evolution:
(open systems)

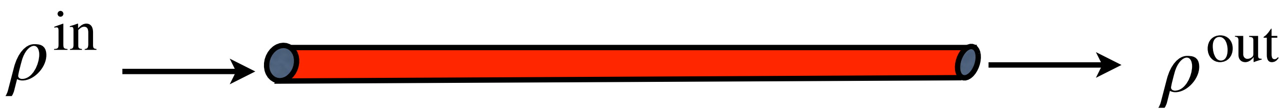
$\Phi[\rho^{\text{in}}] = \rho^{\text{out}} \cdots$ <u>not necessarily local in time</u>

generic CHANNEL



$\rho(t)$ depends on all the previous history,
and not only on $\rho(t+dt)$

our framework: quantum maps



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unitary CHANNEL

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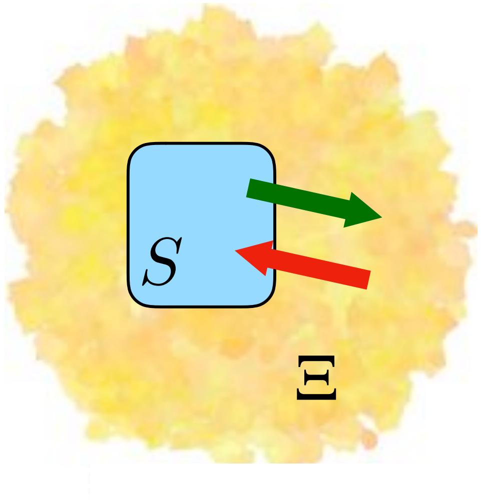
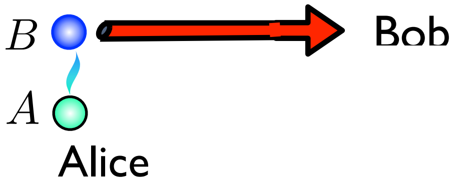
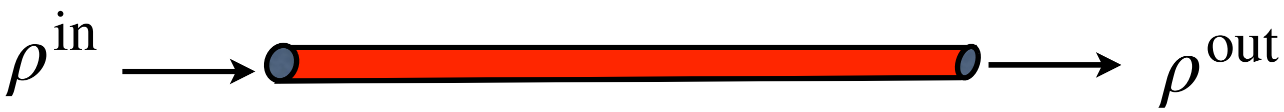


Φ MUST BE

- **linear**: superposition
- **trace preserving**: states $\longrightarrow$ state
- **completely positive**: local operations $\Phi \otimes \mathrm{I}$



our framework: quantum maps



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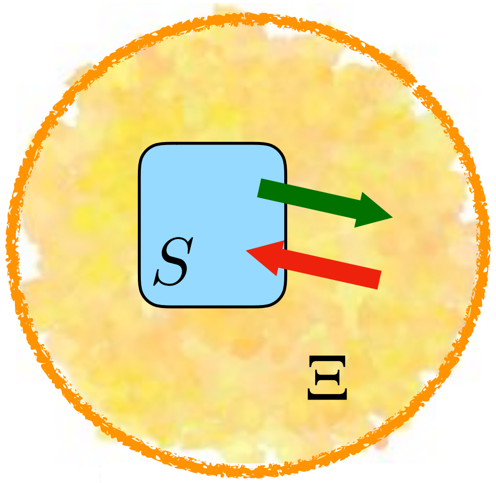
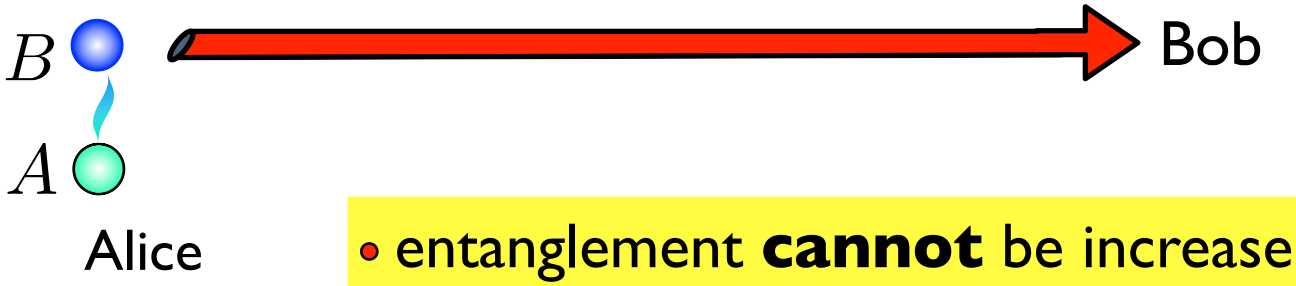


Φ: Stinespring representation

$$\rho^{\rm out} = \Phi[\rho^{\rm in}] = {\rm Tr}_\Xi\left[U_{S\Xi}\left(\rho^{\rm in}\otimes|D\rangle_\Xi\langle D|\right)U_{S\Xi}^\dagger\right]$$

Part I: correlations trasmission

Entanglement distribution



- entanglement **cannot** be increased by:
  - LOCAL OPERATIONS on A or B,
  - letting Alice and Bob share CLASSICAL INFO

quantum information & technologies:

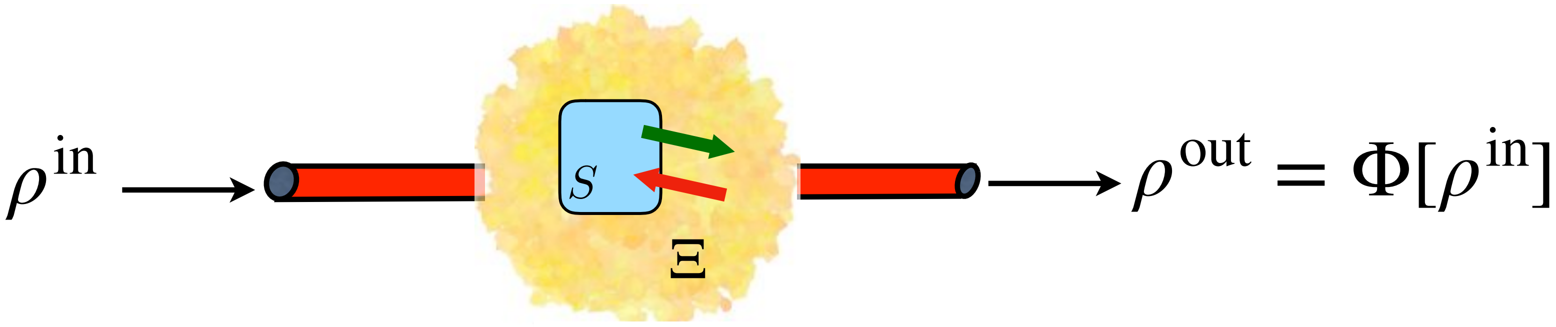
q. ***computation*** & **teleportation** (Bennett, Jozsa, Peres, Wootters, Zeillinger, Polzik, Di Vincenzo)

dense coding (Bennett and Wiesner)

q. ***cryptography*** (Bennett, Brassard, Deutsch, Ekert, Popescu, Gisin..)

q. ***metrology*** (Giovannetti, Lloyd, Adesso, Braunstein, Caves, ..)

Evolution of quantum systems



$$\rho^{\rm out} = \Phi[\rho^{\rm in}]$$

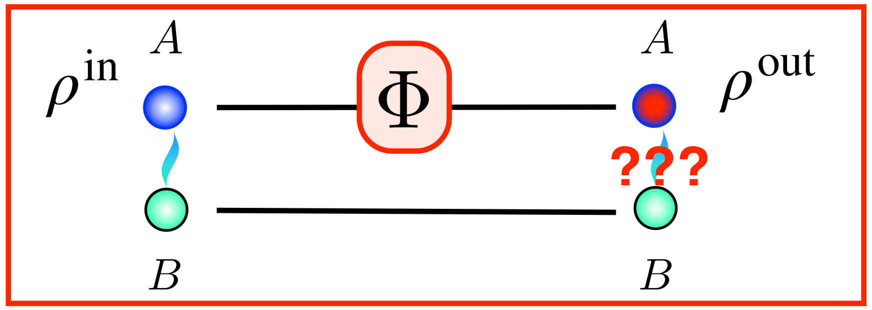
What is the “quality” of such evolution?

- **fidelity:** $\mathcal{F}(\rho^{\rm in}, \Phi(\rho^{\rm in})) \xrightarrow[\text{convexity}]{} \mathcal{F}(\Phi) = \min_{\rho^{\rm in}=|\psi\rangle\langle\psi|} (|\psi\rangle, \Phi[|\psi\rangle\langle\psi|])$

- **capacity:** optimal communication rate in parallel on multiple copies of S

⋮

- **entaglement transmission:**



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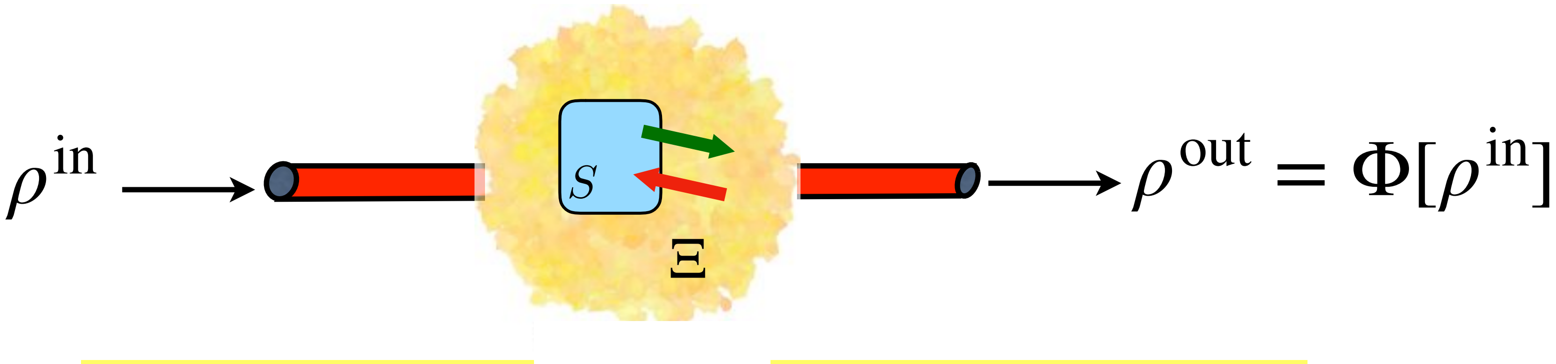

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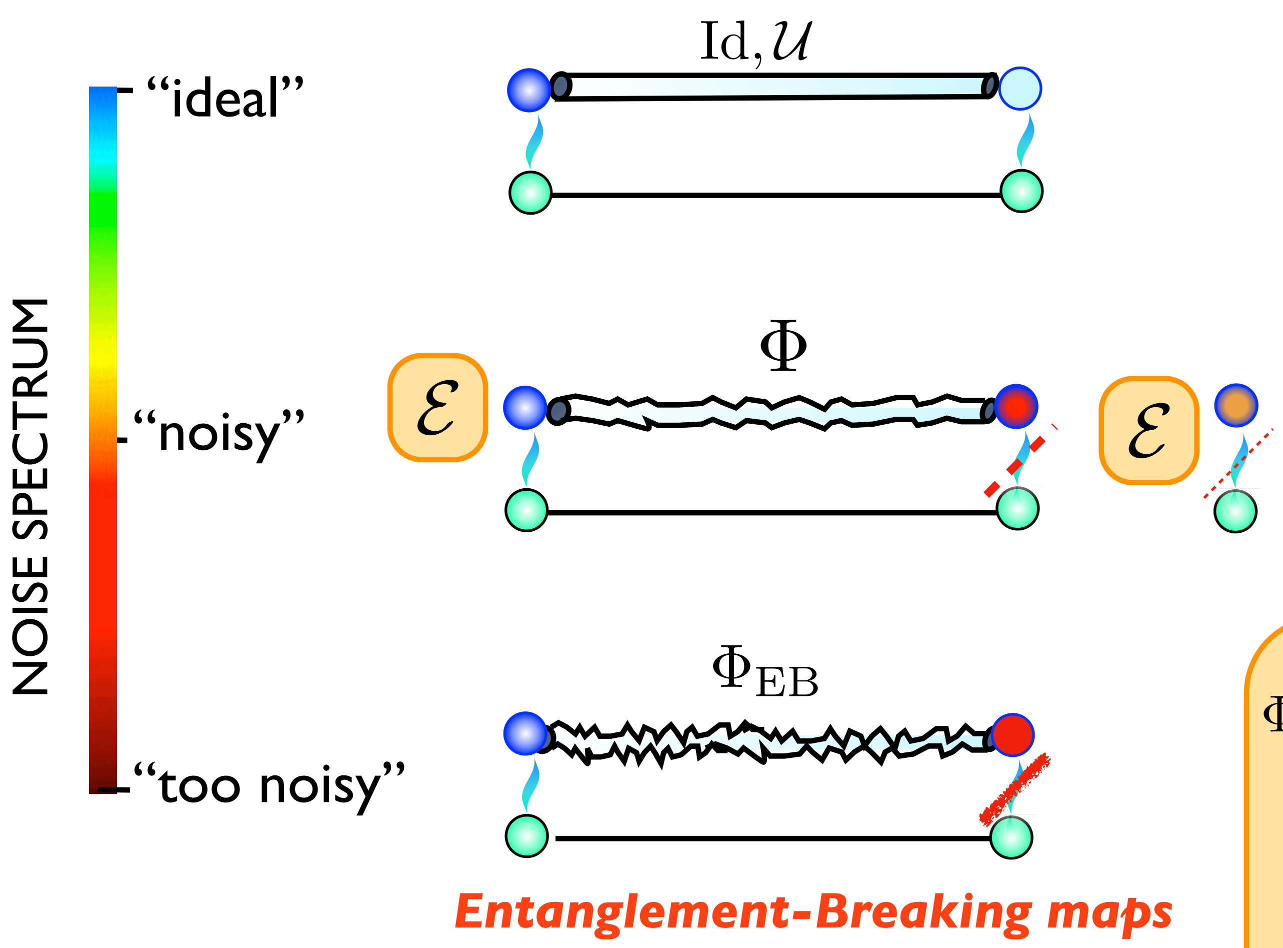
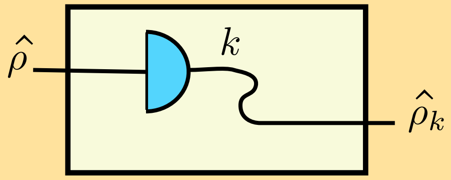


Entanglement-Breaking maps

error - correction codes

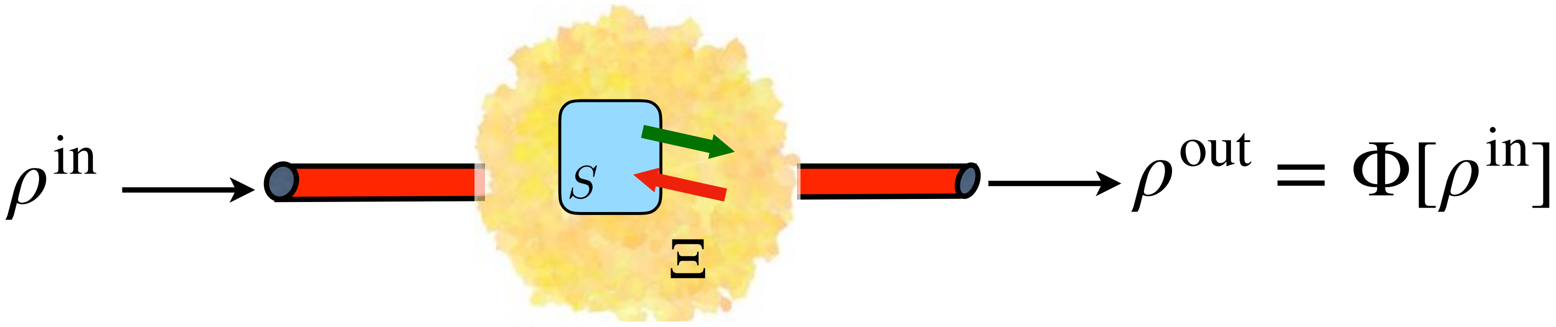
$$\Phi_{EB}[\hat{\rho}] = \sum_k \hat{\rho}_k \mathrm{Tr}[\hat{F}_k \hat{\rho}], \quad \{\hat{F}_k\}$$

POVM



A. S. Holevo, Russian. Math. Surveys **53**, 1295 (1999)

Evolution of quantum systems



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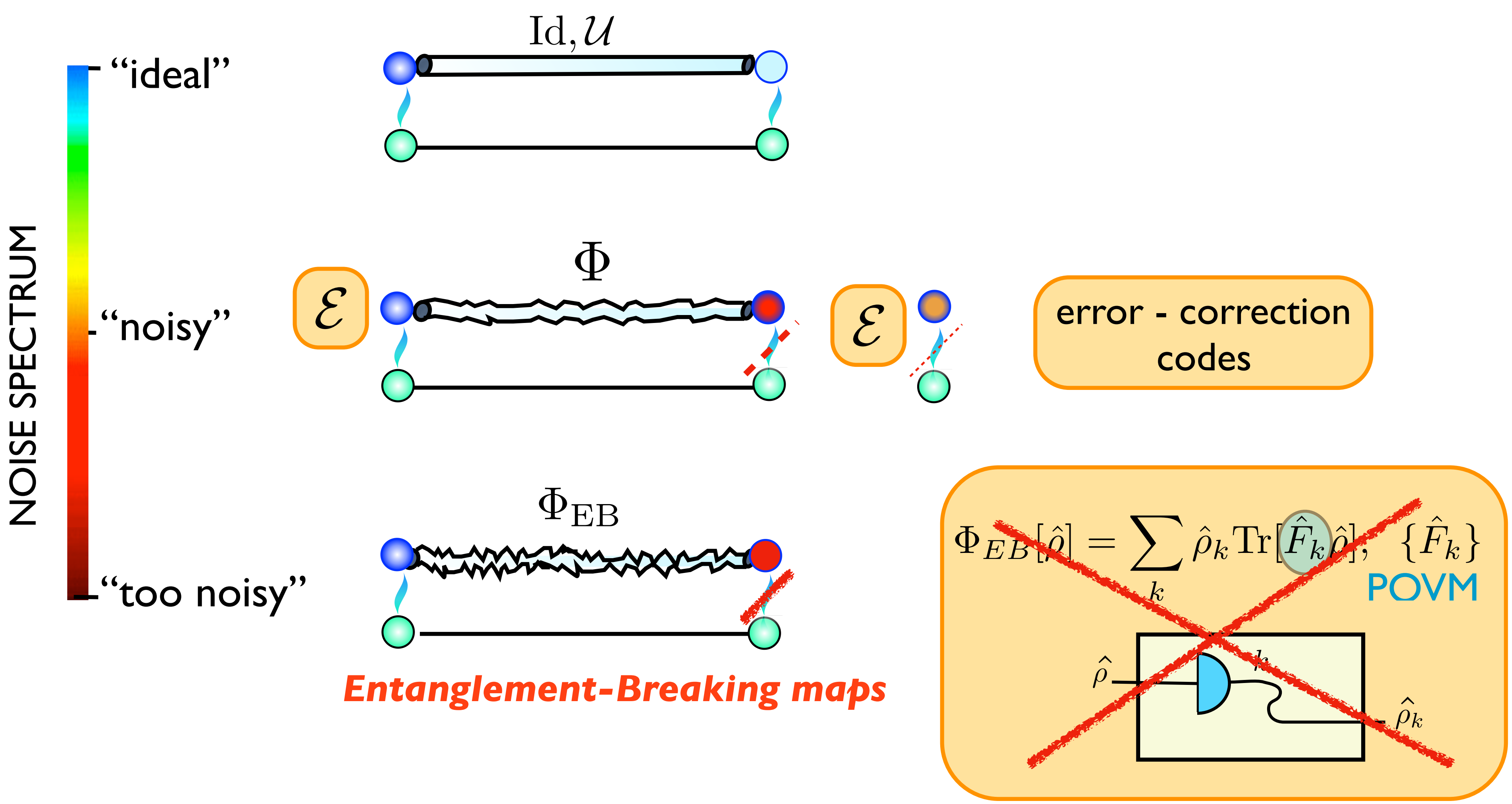

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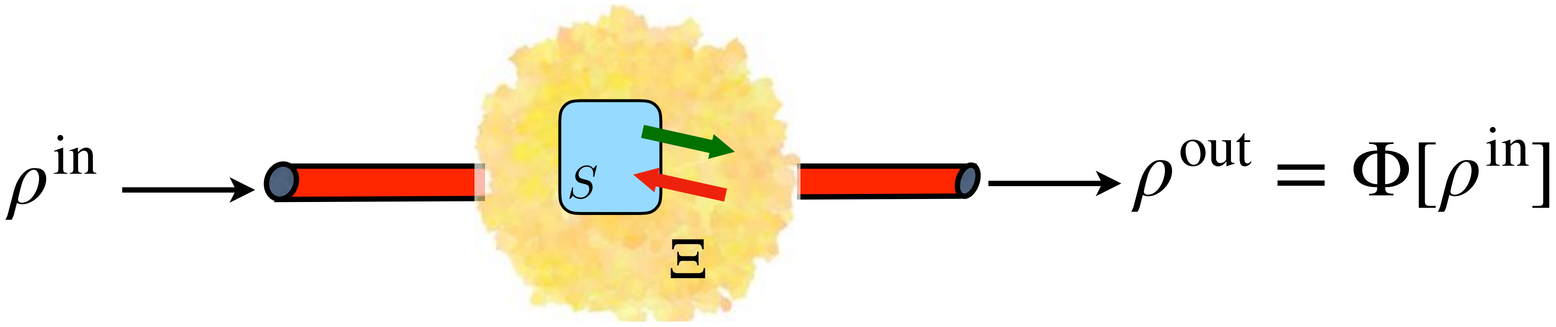

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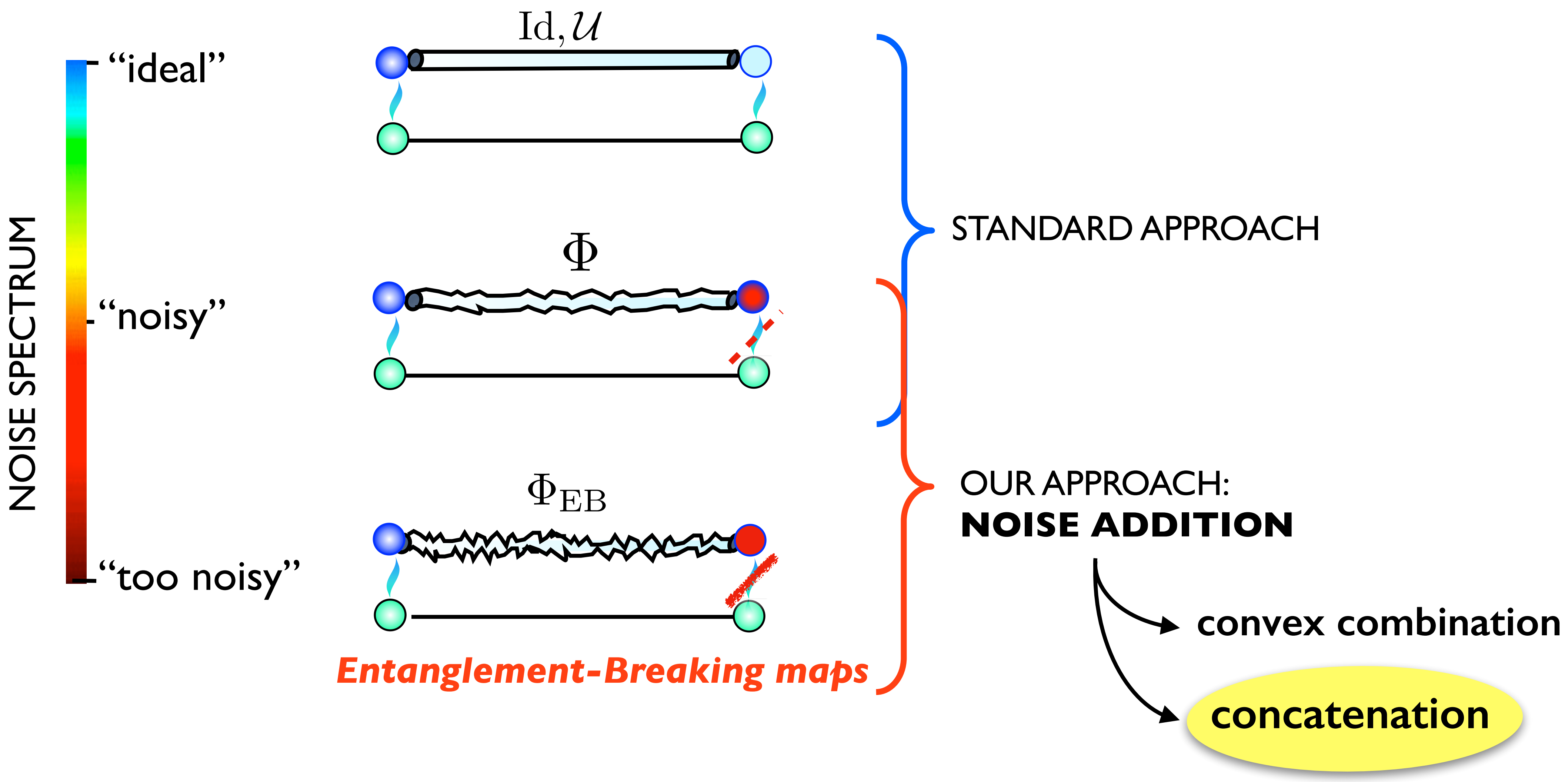


Entanglement-Breaking maps

STANDARD APPROACH

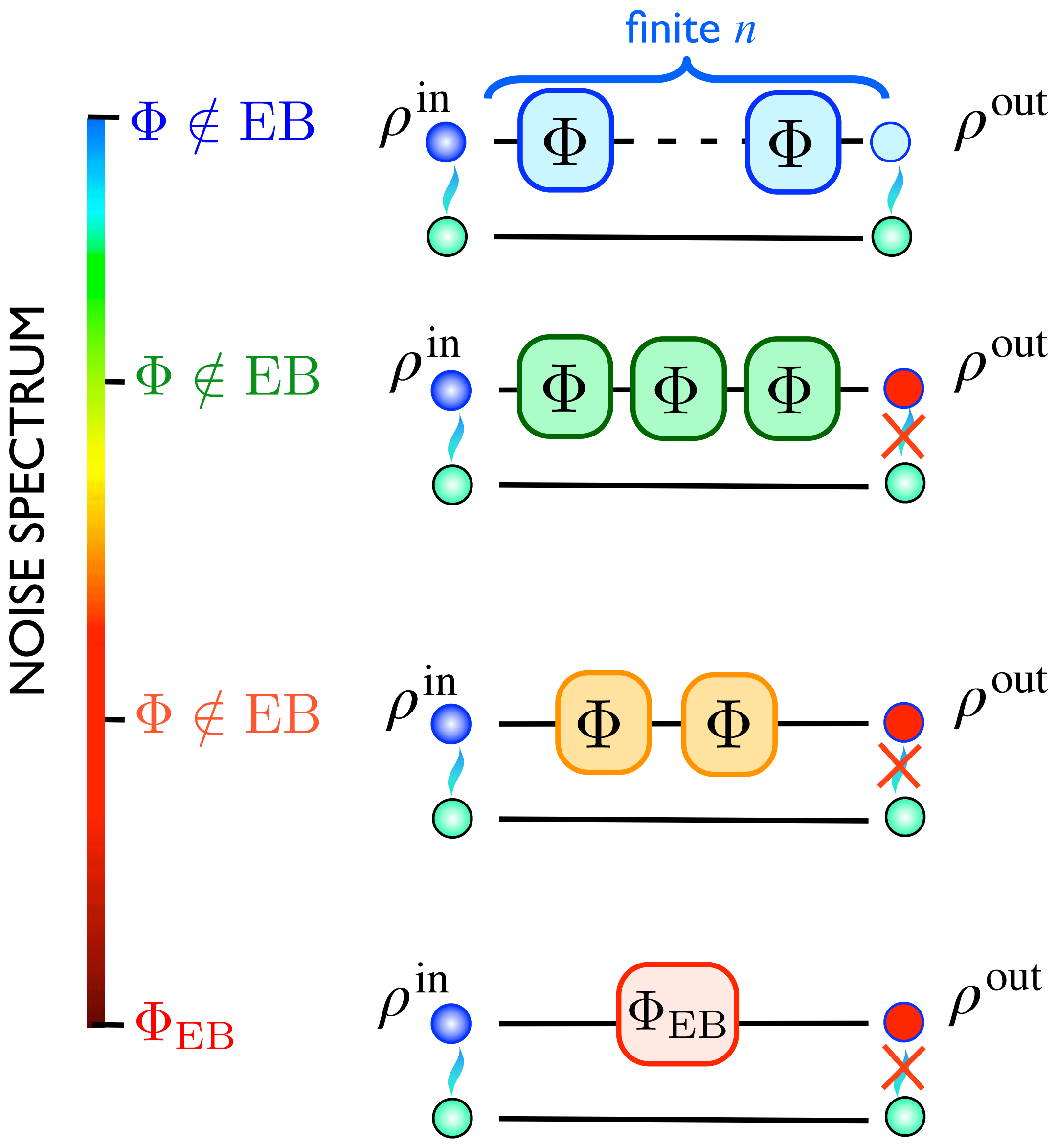
OUR APPROACH:
NOISE ADDITION

convex combination

concatenation

A. D.P. and V. Giovannetti, Phys. Rev. A **86**, 052302 (2012)

Noise addition: concatenation



$$\Phi \circ \Phi \circ \ldots \circ \Phi \notin \mathrm{EB} \overset{\text{def.}}{\iff} \Phi \in \mathrm{EB}^{\infty}$$

Ent.-Break. maps of order infinite

$$\Phi \circ \Phi \circ \Phi \in \mathrm{EB} \overset{\text{def.}}{\iff} \Phi \in \mathrm{EB}^{3}$$

Ent.-Break. maps of order 3

$$\Phi \circ \Phi \in \mathrm{EB} \overset{\text{def.}}{\iff} \Phi \in \mathrm{EB}^{2}$$

Ent.-Break. maps of order 2

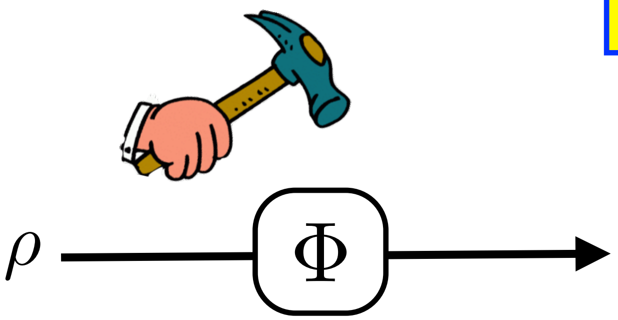
Entanglement-Breaking maps:
WORST CASE SCENARIO

A. De Pasquale and V. Giovannetti, Phys. Rev. A **86**, 052302 (2012)

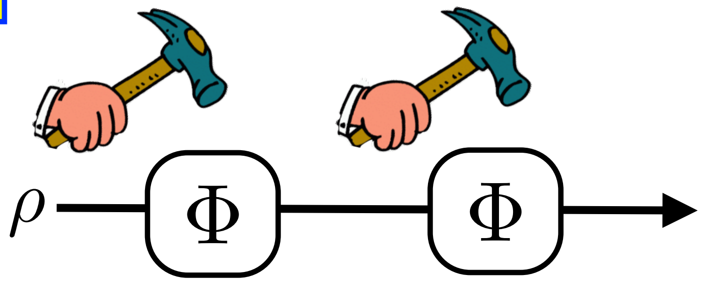
Amending EB evolutions: the discrete case

A. De Pasquale and V. Giovannetti, Phys. Rev. A **86**, 052302 (2012).

ent.-break. of order 2



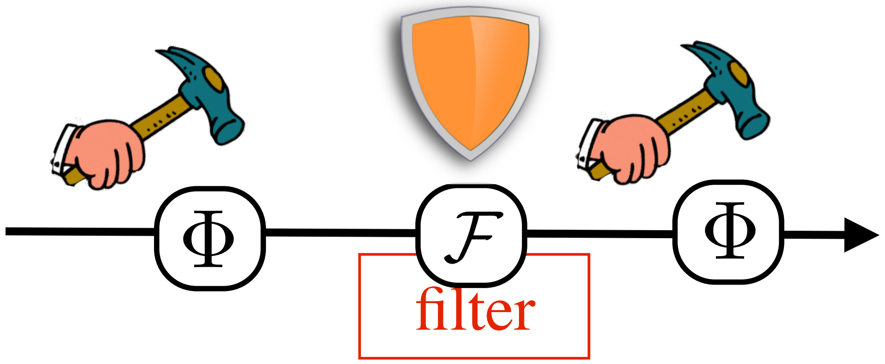
$\Phi \notin \mathrm{EB}$ but



$\Phi \circ \Phi \in \mathrm{EB}$

$\hookrightarrow \Phi \in \mathrm{EB}^2$

amendable maps



$\Phi \circ \mathcal{F} \circ \Phi \notin \mathrm{EB}$

change the direction along which the system is hit

amendable maps of order m

detrimental effect
delayed of m-2 steps

$$\underbrace{(\Phi \circ \mathcal{F}) \circ (\Phi \circ \mathcal{F}) \circ \ldots \circ (\Phi \circ \mathcal{F})}_{m' < m} \notin \mathrm{EB}$$

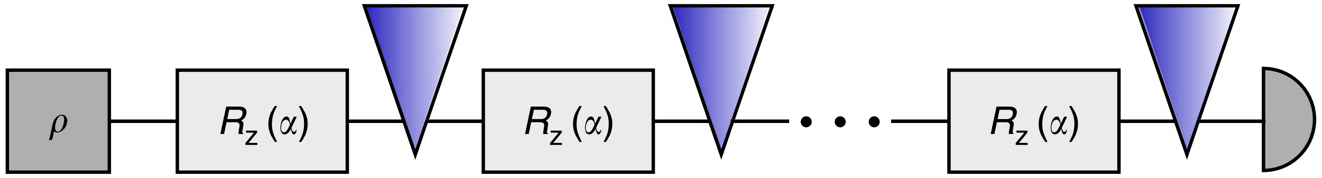
eg. Generalized Amplitude damping maps,
Gaussian attenuation/amplification maps

Amending EB evolutions: the discrete case

A. De Pasquale and V. Giovannetti, Phys. Rev. A **86**, 052302 (2012).

…somehow in the same spirit of Daniel's talk..

$$H_{\mathrm{KT}}(t)=\alpha J_z+\frac{k}{2J}J_y^2\sum_{n=-\infty}^{\infty}\tau\delta(t-n\tau)$$



$$\underbrace{(\Phi\circ\mathcal{F})\circ(\Phi\circ\mathcal{F})\circ\ldots\circ(\Phi\circ\mathcal{F})}_{m'<m}\notin \mathrm{EB}$$

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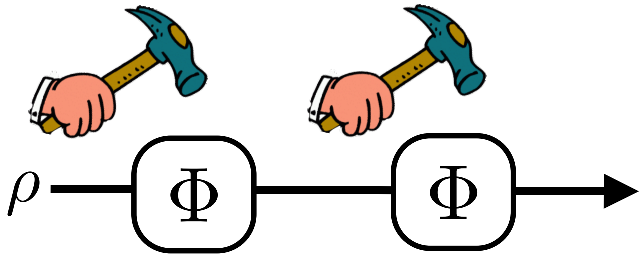
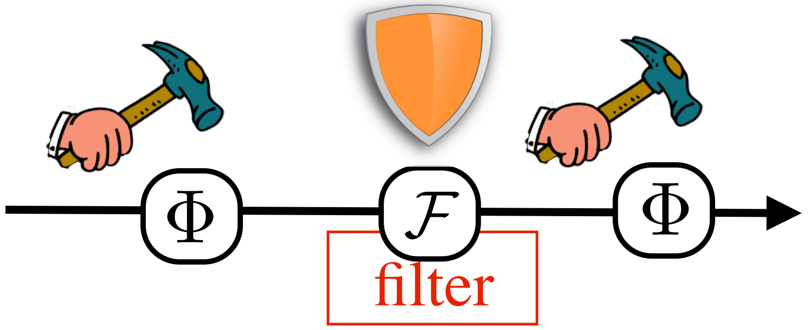
Amending EB evolutions: the discrete case

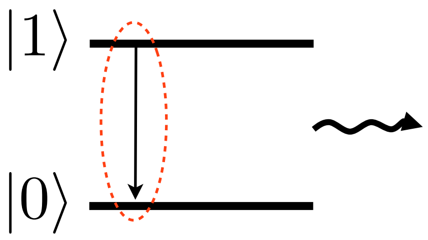


$\Phi \in \mathrm{EB}^2$ but

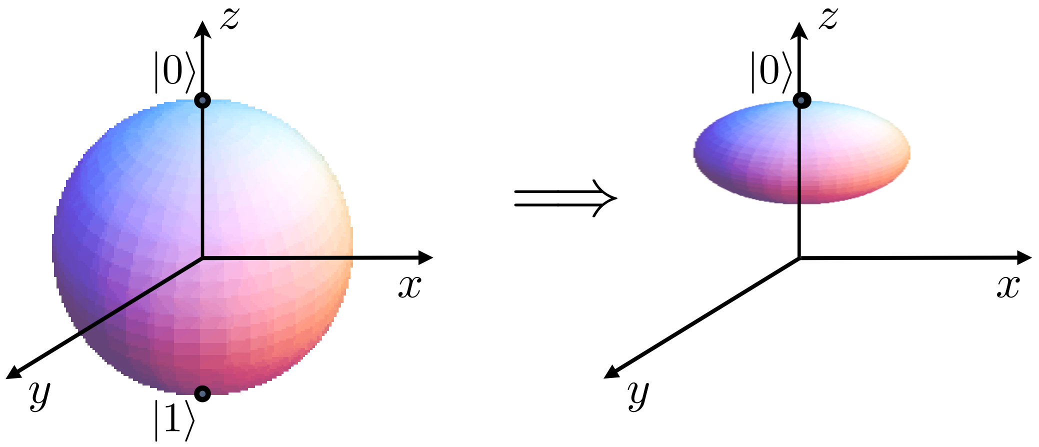


$\Phi \circ \mathcal{F} \circ \Phi \notin \mathrm{EB}$

ENERGY DISSIPATION
(amplitude damping channel)



eg. spontaneous emission
high temp. equilibrium

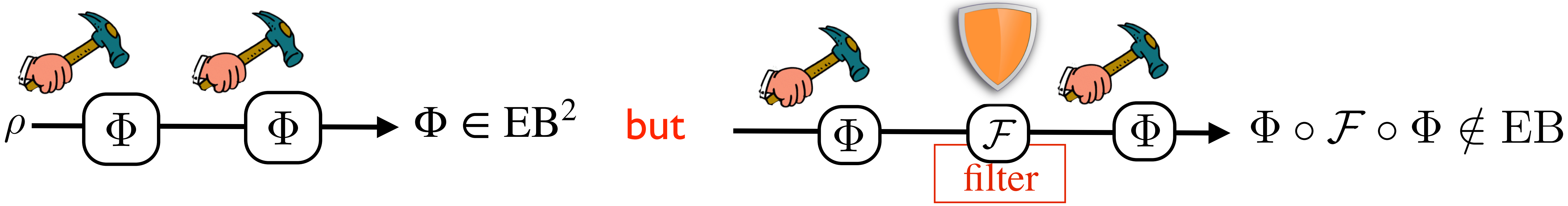


$$\Sigma_\eta[\rho] = E_1 \rho E_1^\dagger + E_2 \rho E_2^\dagger$$

$$E_1 \equiv \left[\begin{array}{cc} 1 & 0 \\ 0 & \sqrt{\eta} \end{array}\right], \; E_2 \equiv \left[\begin{array}{cc} 0 & \sqrt{1-\eta} \\ 0 & 0 \end{array}\right]$$

transmission coefficient: $\eta \in [0,1]$

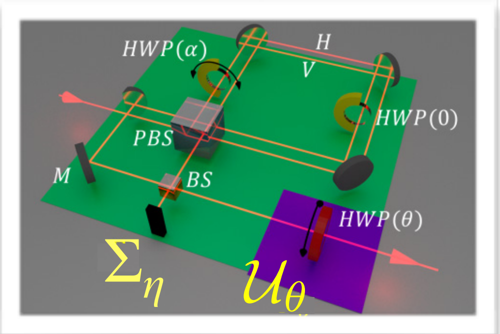
Amending EB evolutions: the discrete case

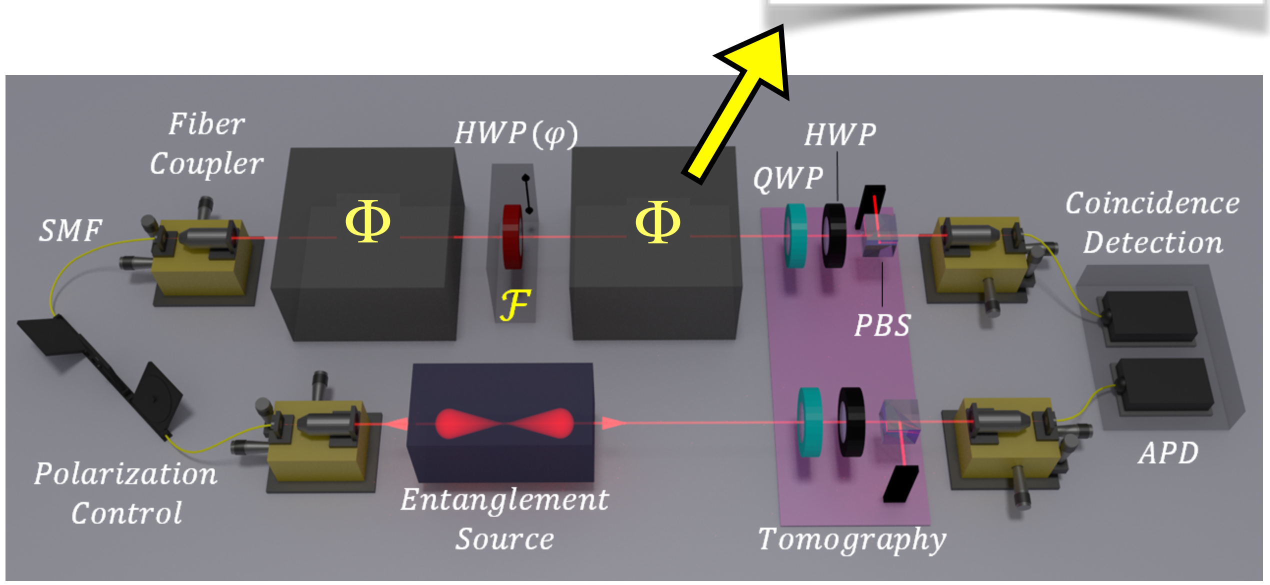


$\Phi \in \mathrm{EB}^2$ but $\Phi \circ \mathcal{F} \circ \Phi \notin \mathrm{EB}$

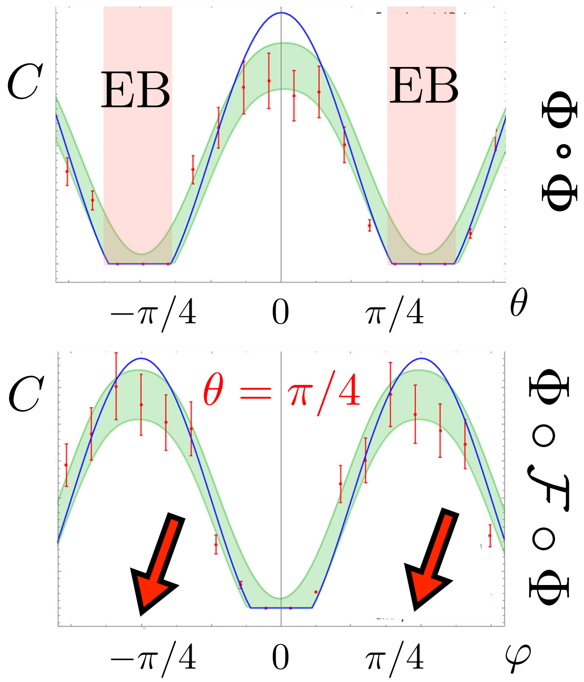
ENERGY DISSIPATION
(amplitude damping channel)

$$\Phi = \mathcal{U}_\theta \circ \Sigma_\eta, \mathcal{F} = \mathcal{U}_\varphi$$





$\eta = 0.66 \pm 0.017$



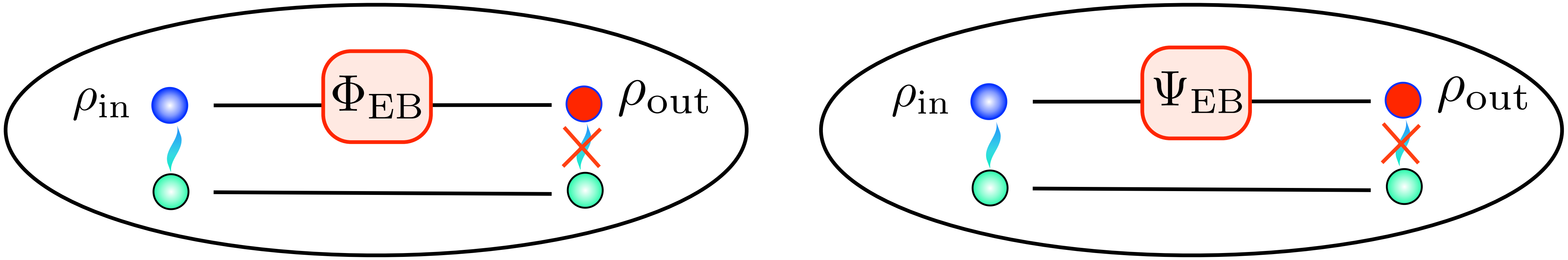
Entanglement restored!

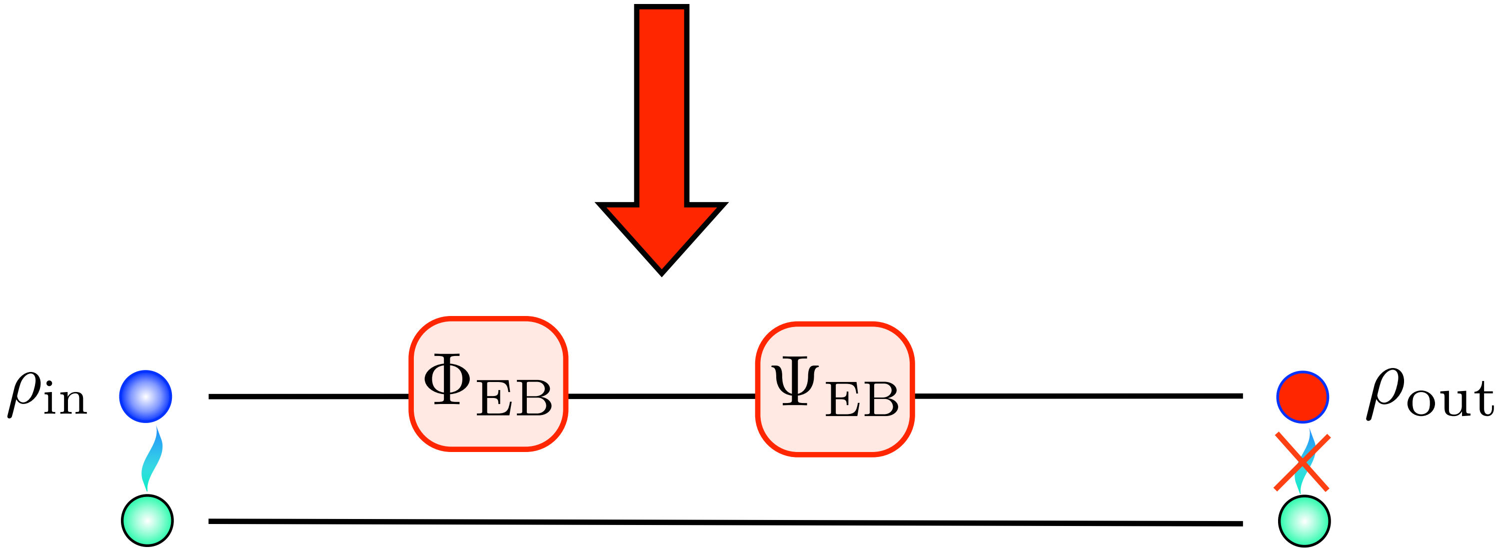
SCUOLA
NORMALE
SUPERIORE

Á. Cuevas, A. De Pasquale, et al, Phys. Rev. A 96, 022322 (2017)

cut-and-paste protocol

What happens if we have at disposal **only** EB channels ?





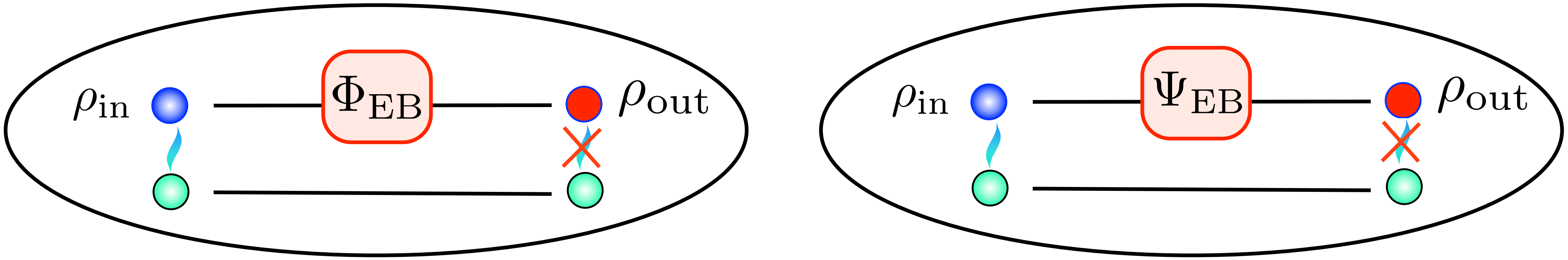
NO!!!

$$\Psi_{\rm EB} \circ \Phi_{\rm EB} \in {\rm EB}$$

once entanglement is destroyed, it is **not** possible to create it again with local operations

cut-and-paste protocol

What happens if we have at disposal **only** EB channels ?

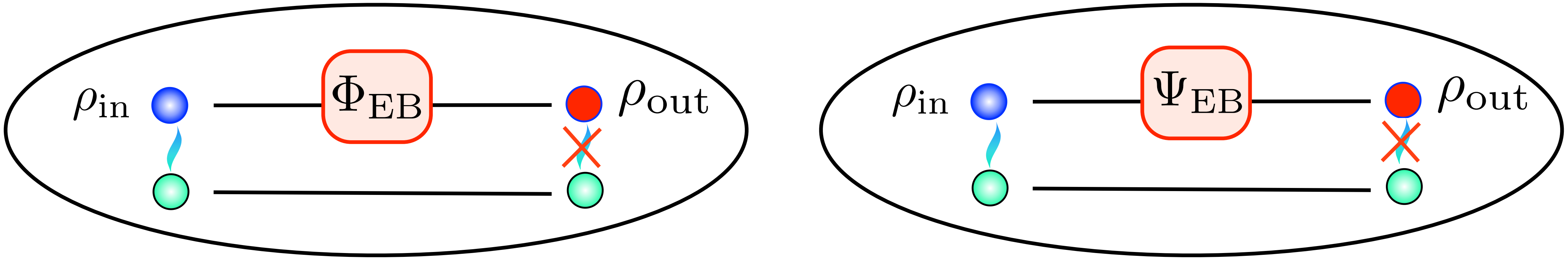


IF

$$\Phi_{\text{EB}} = \Phi \circ \Phi \qquad \Psi_{\text{EB}} = \Psi \circ \Psi$$

cut-and-paste protocol

What happens if we have at disposal **only** EB channels ?



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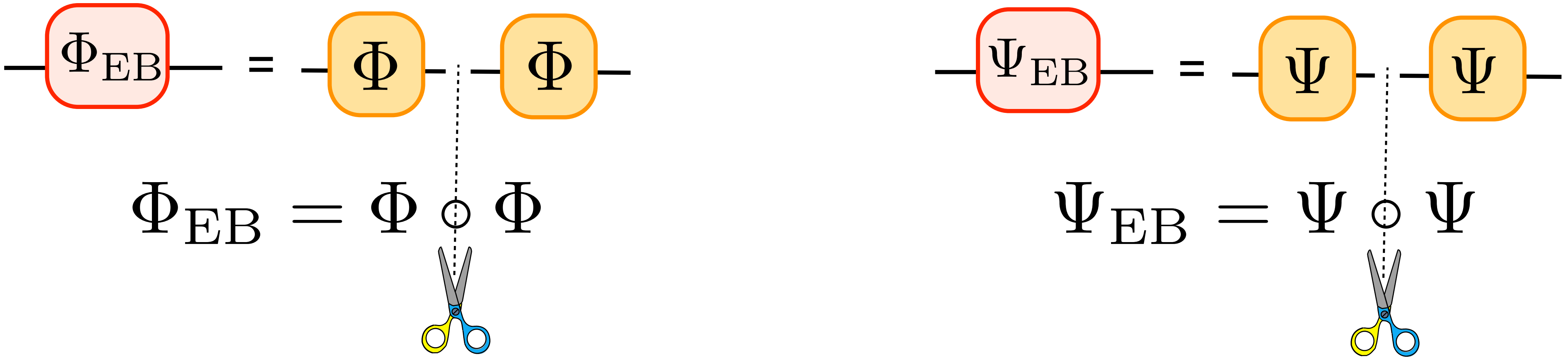
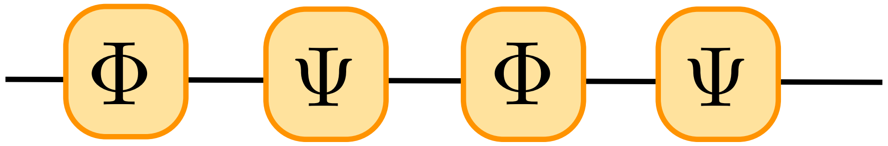


$$\Phi_{\mathrm{EB}} = \Phi \circ \Phi$$

$$\Psi_{\mathrm{EB}} = \Psi \circ \Psi$$

it may happen that ...



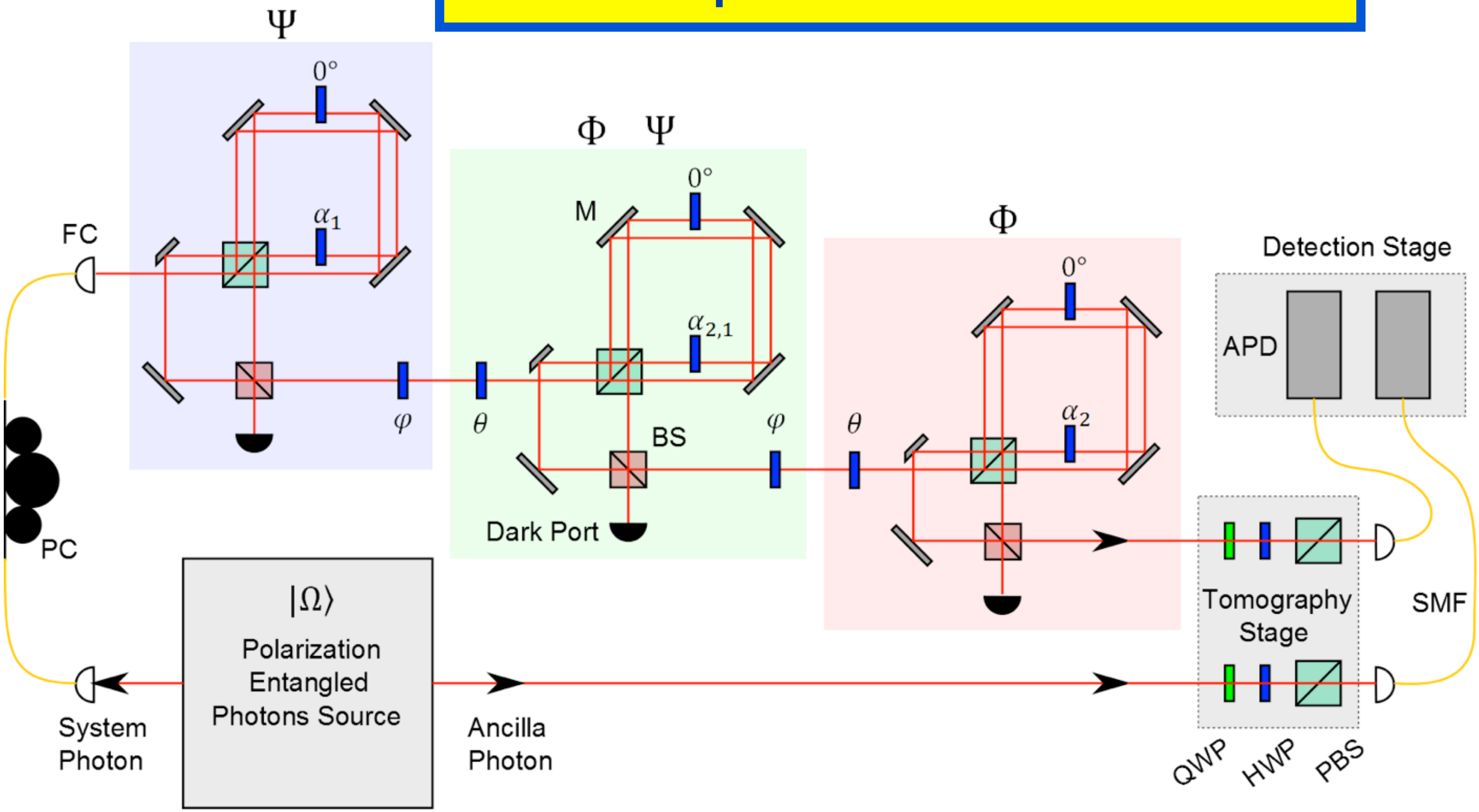
$\notin \mathrm{EB}$

IT IS POSSIBLE TO TRANSMIT ENTANGLEMENT HAVING AT DISPOSAL **ONLY** EB MAPS!!!

Á. Cuevas, A. Mari, A.D. P., A. Orieux, S. Duranti, M. Massaro, F. Sciarrino, P. Mataloni, and V. Giovannetti, Phys. Rev. A 96, 012314 (2017)

Experimental test

$$\Psi = \mathcal{U}^\dagger \circ \Sigma_\eta$$
$$\Phi = \Sigma_\eta \circ \mathcal{U}$$



Avalanche Photodiods

Single Mode Fibers

$$\frac{1}{\sqrt{2}}\left(|H\rangle_S|V\rangle_A + e^{i\phi}|V\rangle_S|H\rangle_A\right)$$

high-brilliance, high-purity polarization entangled source **more than 98% fidelity**

- The generated pairs of photon at 810nm by type-II parametric down conversion. The generated pairs (**more than 50000 detected coincidences/sec**) have a **coherence length** of Lcoh = 1.02mm and spectral bandwidth Δλ = 0.43nm.

nonlinear PPKTP crystal pumped by a single mode laser at 405nm and 2.75mW of power within a Sagnac interferometer

Let's check (experimentally)!

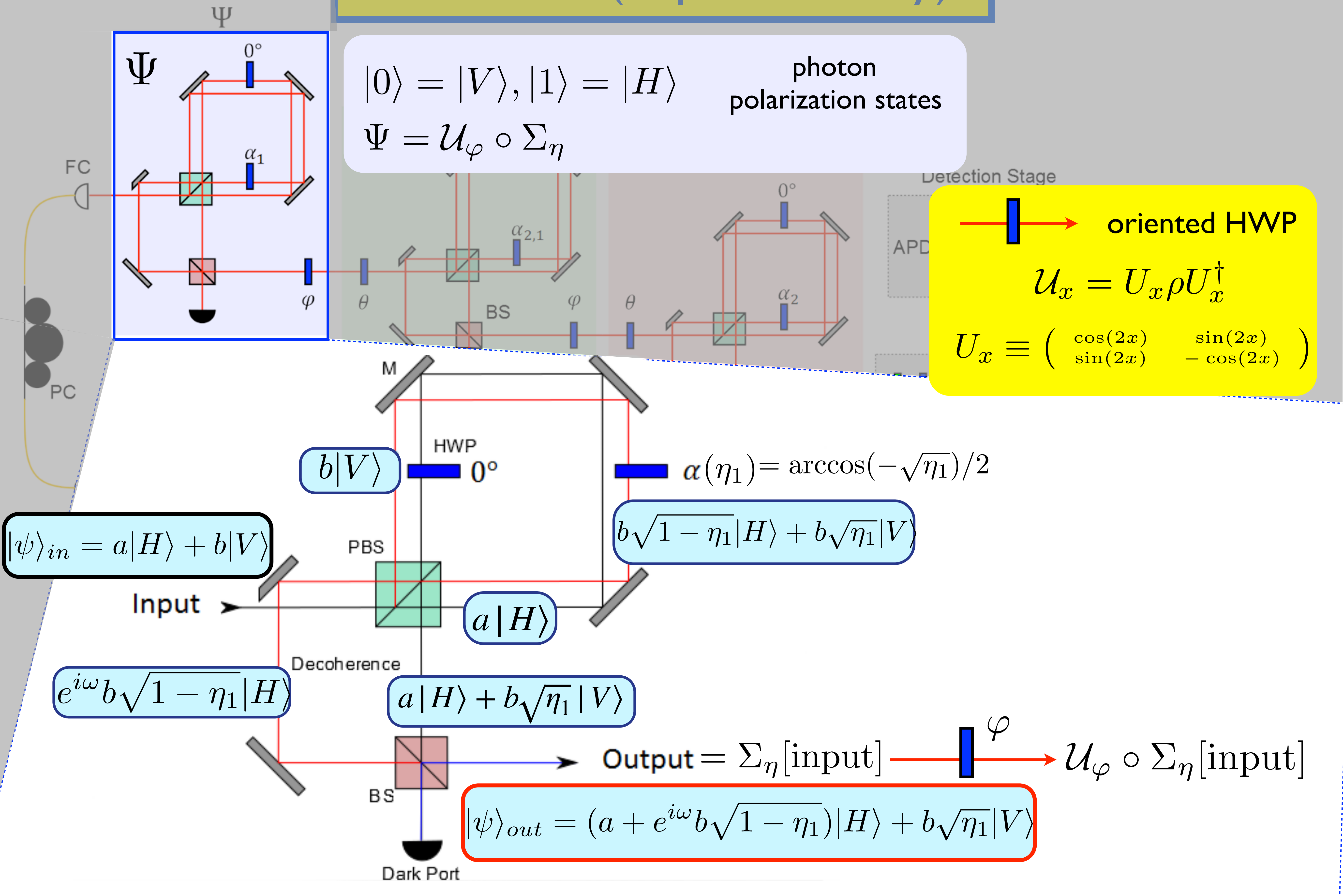
$|0\rangle = |V\rangle, |1\rangle = |H\rangle$ photon polarization states

$\Psi = \mathcal{U}_\varphi \circ \Sigma_\eta$

oriented HWP

$$\mathcal{U}_x = U_x \rho U_x^\dagger$$

$$U_x \equiv \begin{pmatrix} \cos(2x) & \sin(2x) \\ \sin(2x) & -\cos(2x) \end{pmatrix}$$



$b|V\rangle$

$\alpha(\eta_1) = \arccos(-\sqrt{\eta_1})/2$

$b\sqrt{1-\eta_1}|H\rangle + b\sqrt{\eta_1}|V\rangle$

$|\psi\rangle_{in} = a|H\rangle + b|V\rangle$

$a|H\rangle$

$e^{i\omega}b\sqrt{1-\eta_1}|H\rangle$

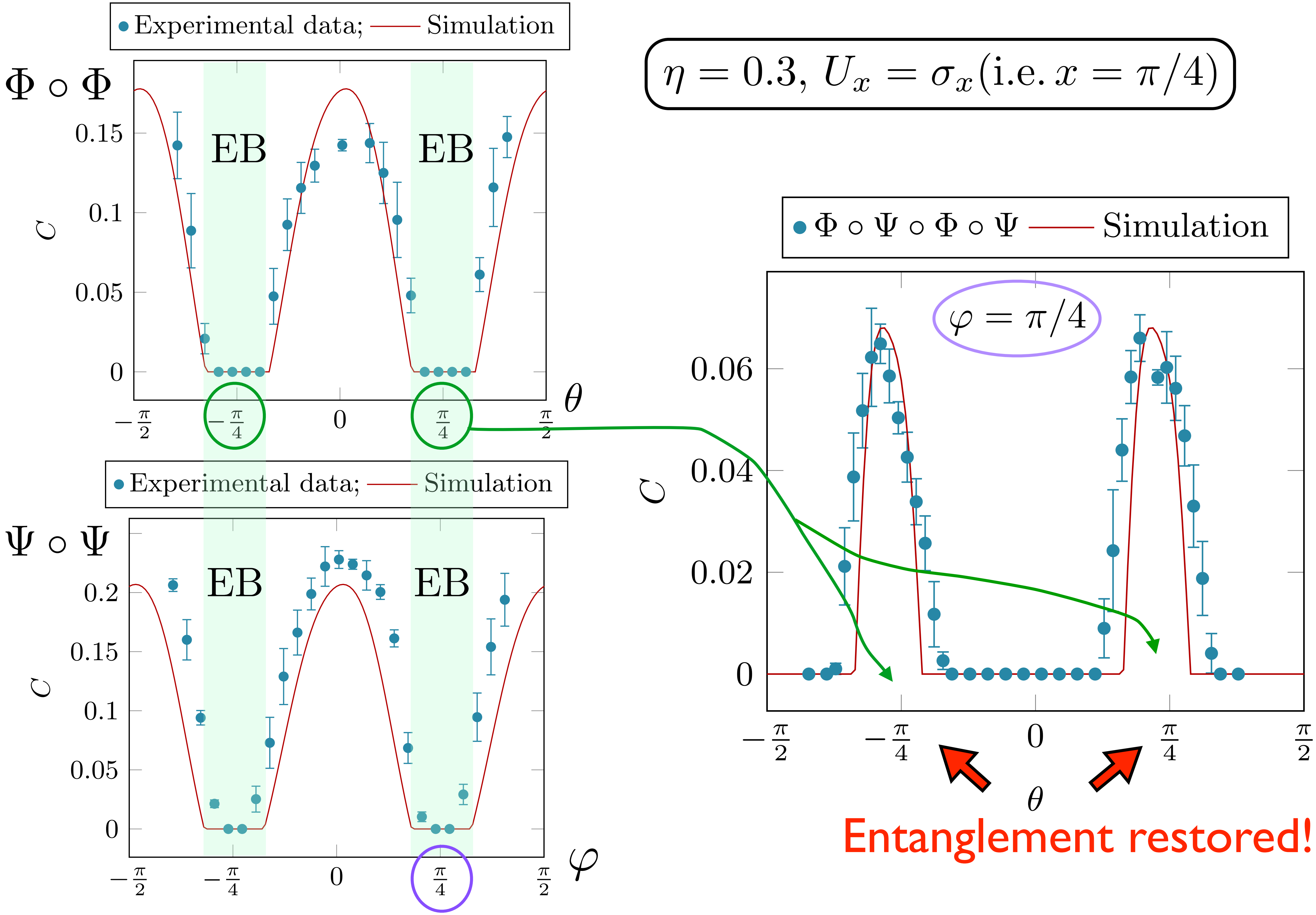
$a|H\rangle + b\sqrt{\eta_1}|V\rangle$

Output $= \Sigma_\eta[\text{input}] \xrightarrow{\varphi} \mathcal{U}_\varphi \circ \Sigma_\eta[\text{input}]$

$|\psi\rangle_{out} = (a + e^{i\omega}b\sqrt{1-\eta_1})|H\rangle + b\sqrt{\eta_1}|V\rangle$

Experimental data

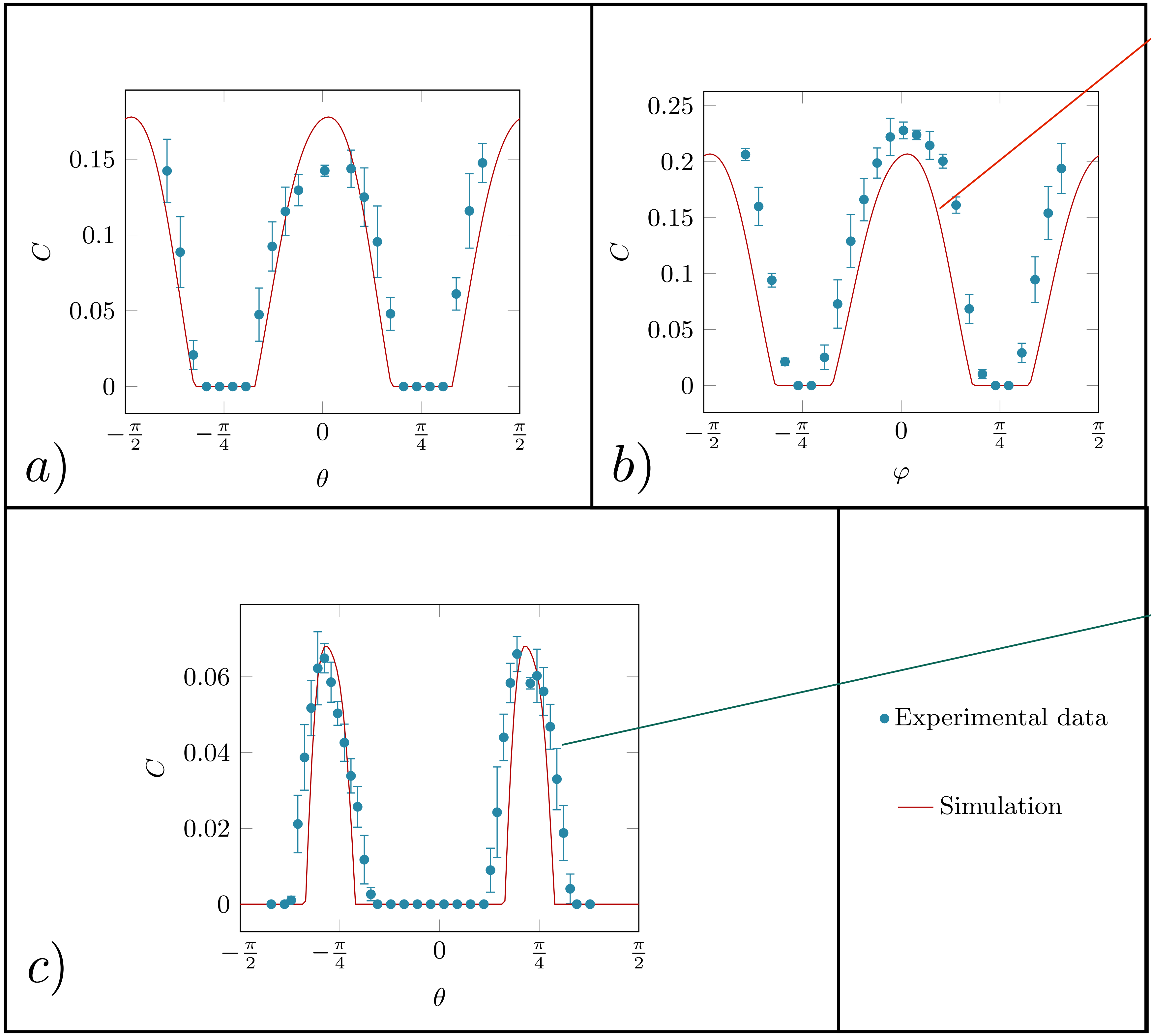
$$\eta = 0.3,\ U_x = \sigma_x (\text{i.e. } x = \pi/4)$$



Entanglement restored!

Á. Cuevas, A. Mari, A. De Pasquale, A. Orieux, S. Duranti, M. Massaro, F. Sciarrino, P. Mataloni, and V. Giovannetti, Phys. Rev. A 96, 012314 (2017)

Experimental data



theoretical prediction obtained with real optical elements

1) entanglement degradation on each dual interferometric set up > 1.3%, therefore maximum concurrence at end of the entire sequence of channels decreases from 98% to 94%:

2) losses L of the beam splitters:
T, R ⟶ T/(1-L), R/(1-L)

Each point and the associated statistical error was taken from a set of N measurements ($3 \leq N \leq 11$), under equivalent mode coupling conditions.

Á. Cuevas, A. Mari, A. De Pasquale, A. Orieux, S. Duranti, M. Massaro, F. Sciarrino, P. Mataloni, and V. Giovannetti, Phys. Rev. A 96, 012314 (2017)

Continuous time
evolution of quantum systems

…related to Daniel's presentation..

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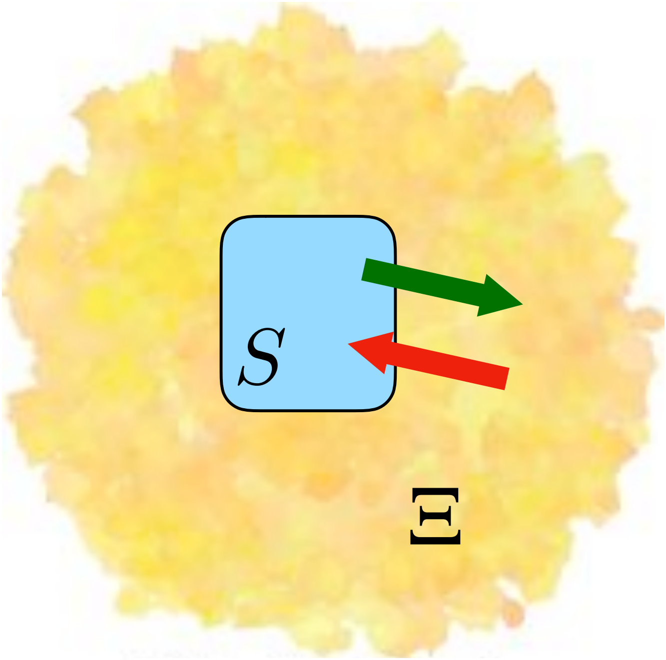


$\rho(t)$ depends on all the previous history, and not only on $\rho(t+dt)$

⇨ **Markovian evolution:** D. Gatto, A. De Pasquale, and V. Giovannetti, Phys. Rev. A **99**, 032307 (2019)

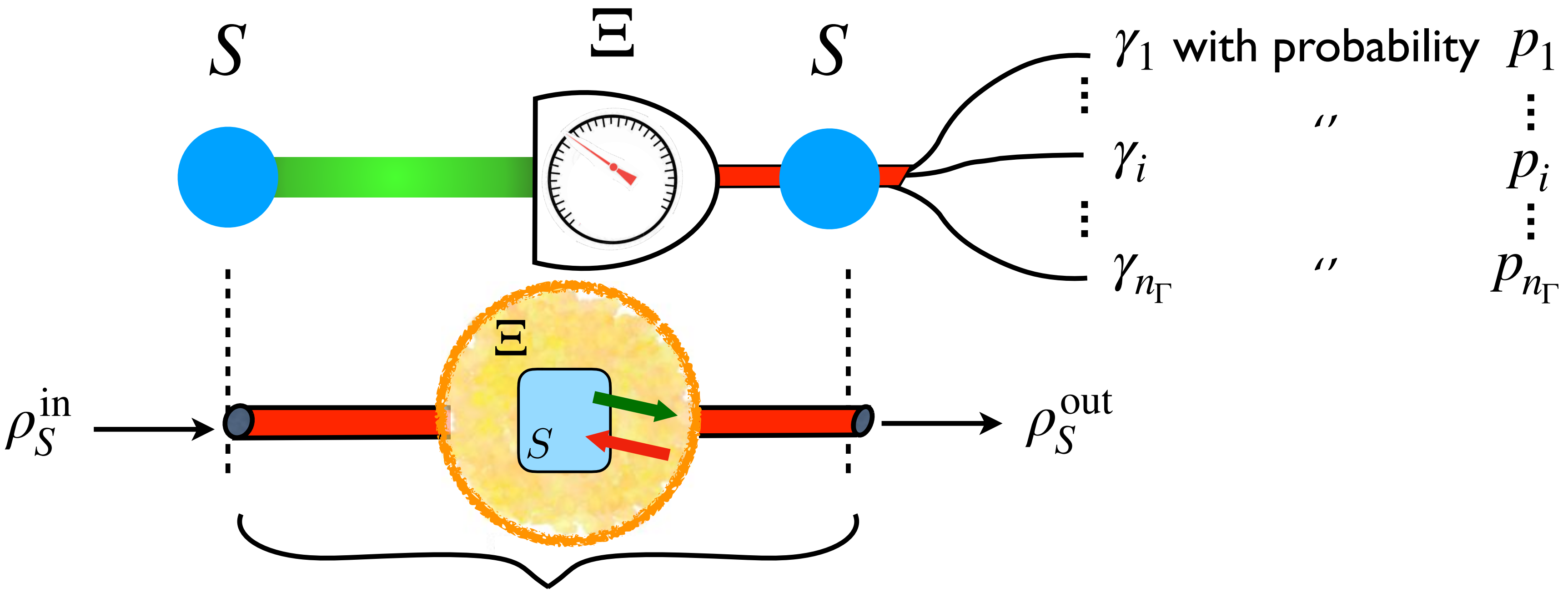
⇨ **non-Markovian evolution:**

T. Bullock, F. Cosco, M. Haddara, S. H. Raja, O. Kerppo, L. Leppäjärvi, O. Siltanen, N.W. Talarico, A.D.P., S. Maniscalco and V. Giovannetti, Phys. Rev. A **98**, 042301 (2018)

Part II: the measurement problem

A.D.P., C. Foti, A. Cuccoli, V. Giovannetti and P. Verrucchi, **arXiv:1902.03628v2**

Quantum measurements



γ_1 with probability p_1

$\vdots$ $\vdots$

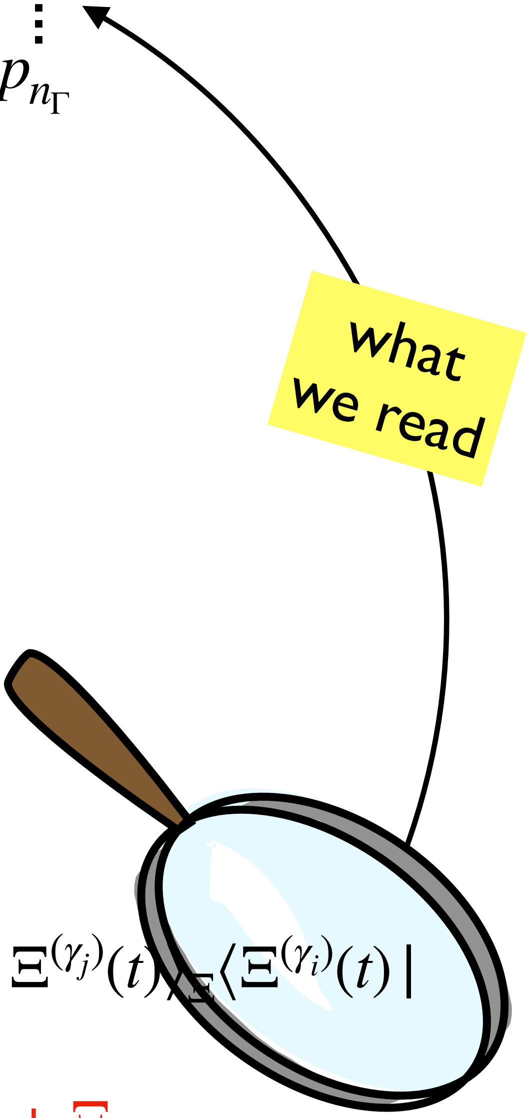
γ_i '' p_i

$\vdots$ $\vdots$

γ_{n_Γ} '' p_{n_Γ}

Is it possible to establish a dynamical description?

we read the info. on Ξ $\Longrightarrow$ we need to consider the evolution of $S+\Xi$



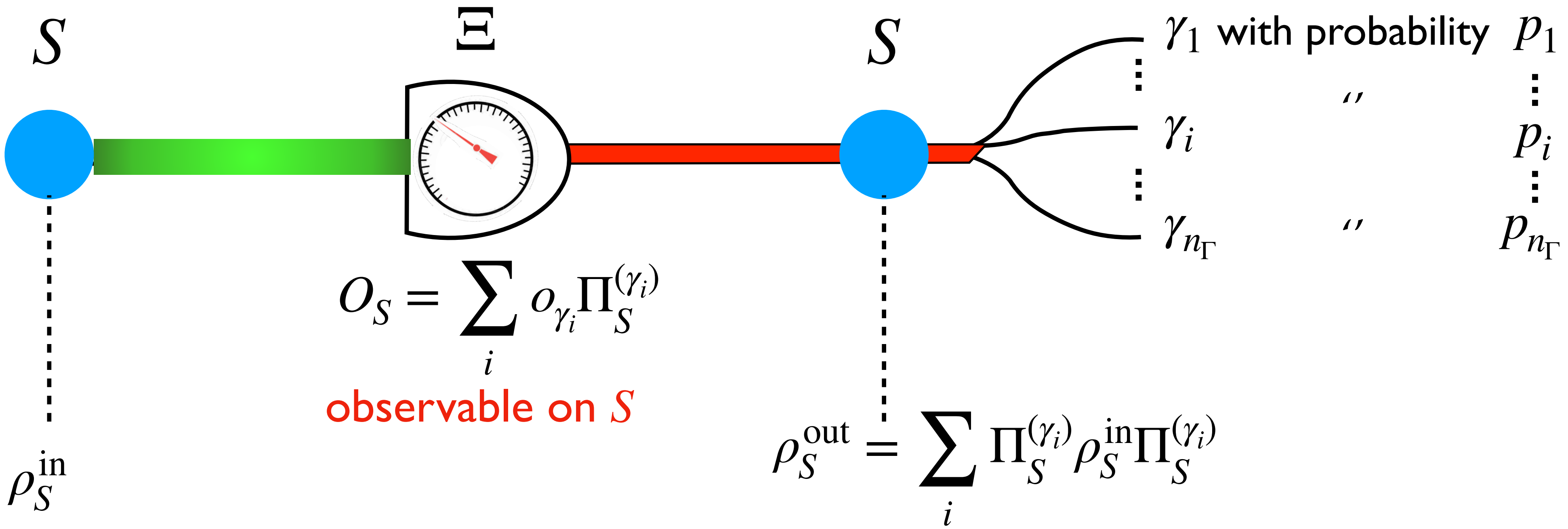
$$\rho_{S\Xi}(0) = \rho_S^{\rm in} \otimes |D\rangle_\Xi\langle D| \xrightarrow[U_{S\Xi}(t)]{\qquad\qquad} \rho_{S\Xi}(t) = \sum_{i,j} \left[\rho_S^{\rm out}\right]_{ji} \otimes |\Xi^{(\gamma_j)}(t)\rangle_\Xi\langle\Xi^{(\gamma_i)}(t)|$$

initial state of $S+\Xi$

final state of $S+\Xi$

Projective q. measures (PVM)

von Neumann (1927)
Ozawa (1984)



γ_1 with probability p_1

$\vdots$

γ_i '' $p_i = \mathrm{Tr}[\Pi_S^{(\gamma_i)} \rho_S^{\mathrm{in}} \Pi_S^{(\gamma_i)}]$

$\vdots$

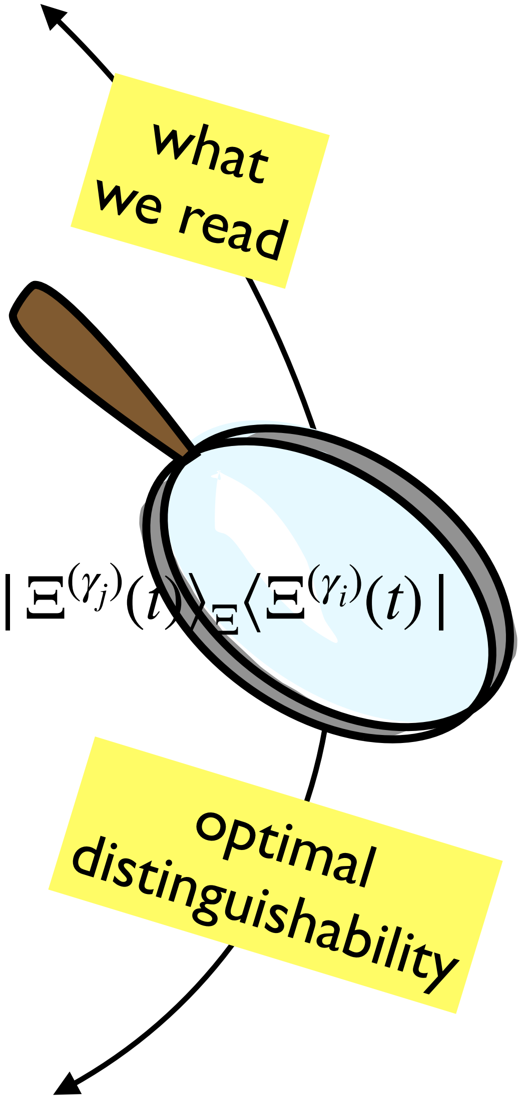
γ_{n_Γ} '' p_{n_Γ}

$$O_S = \sum_i o_{\gamma_i} \Pi_S^{(\gamma_i)}$$

observable on S

ρ_S^{in}

$$\rho_S^{\mathrm{out}} = \sum_i \Pi_S^{(\gamma_i)} \rho_S^{\mathrm{in}} \Pi_S^{(\gamma_i)}$$



what we read

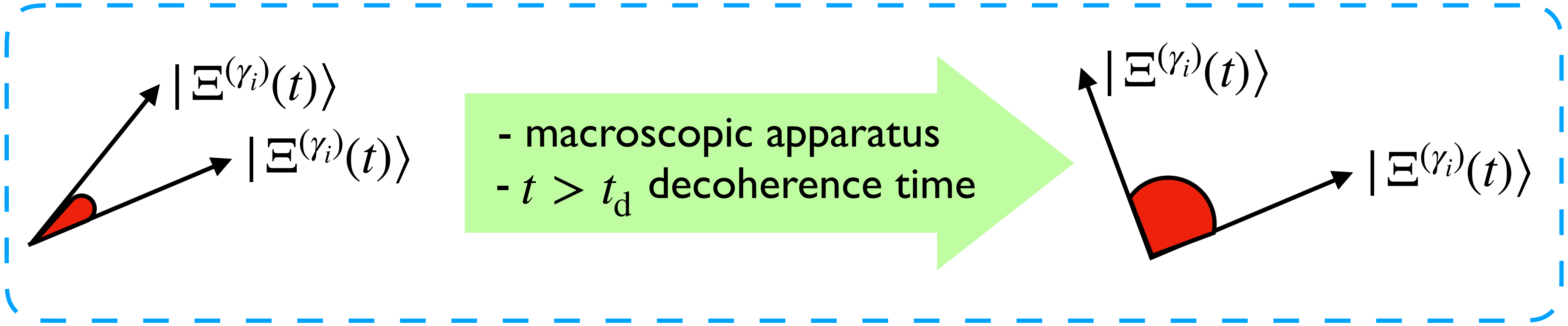
optimal distinguishability

$$\rho_{S\Xi}(0) = \rho_S^{\mathrm{in}} \otimes |D\rangle_\Xi\langle D| \longrightarrow \rho_{S\Xi}(t) = \sum_{i,j} \Pi_S^{(\gamma_j)} \rho_S^{\mathrm{in}} \Pi_S^{(\gamma_i)} \otimes |\Xi^{(\gamma_j)}(t)\rangle_\Xi\langle \Xi^{(\gamma_i)}(t)|$$

initial state of $S + \Xi$

$$U_{S\Xi}(t) = e^{-itO_S \otimes O_\Xi}$$

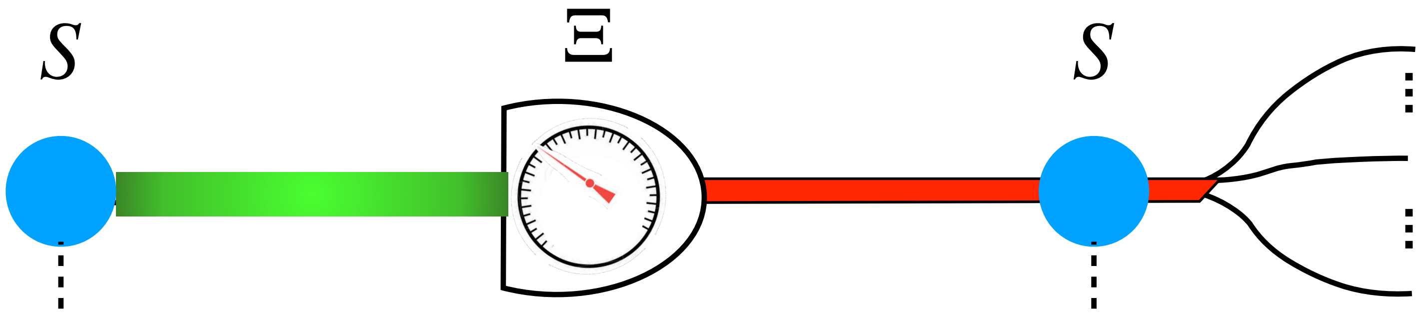
final state of $S + \Xi$



- macroscopic apparatus
- $t > t_{\mathrm{d}}$ decoherence time

Projective q. measures (PVM)

von Neumann (1927)
Ozawa (1984)



γ_1 with probability p_1

γ_i ″ $p_i = \mathrm{Tr}[\Pi_S^{(\gamma_i)}\rho_S^{\mathrm{in}}\Pi_S^{(\gamma_i)}]$

γ_{n_Γ} ″ p_{n_Γ}

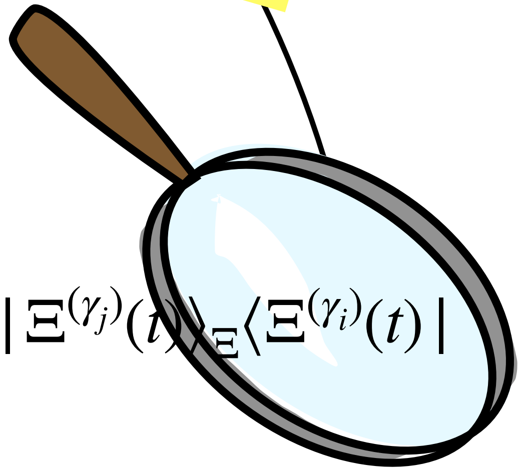
what we read

ρ_S^{in}

$$O_S = \sum_i o_{\gamma_i}\Pi_S^{(\gamma_i)}$$

observable on S

$$\rho_S^{\mathrm{out}} = \sum_i \Pi_S^{(\gamma_i)}\rho_S^{\mathrm{in}}\Pi_S^{(\gamma_i)}$$



$$\rho_{S\Xi}(0) = \rho_S^{\mathrm{in}} \otimes |D\rangle_\Xi\langle D| \longrightarrow \rho_{S\Xi}(t) = \sum_{i,j} \Pi_S^{(\gamma_j)}\rho_S^{\mathrm{in}}\Pi_S^{(\gamma_i)} \otimes |\Xi^{(\gamma_j)}(t)\rangle_\Xi\langle\Xi^{(\gamma_i)}(t)|$$

initial state of $S + \Xi$

$$U_{S\Xi}(t) = e^{-itO_S\otimes O_\Xi}$$

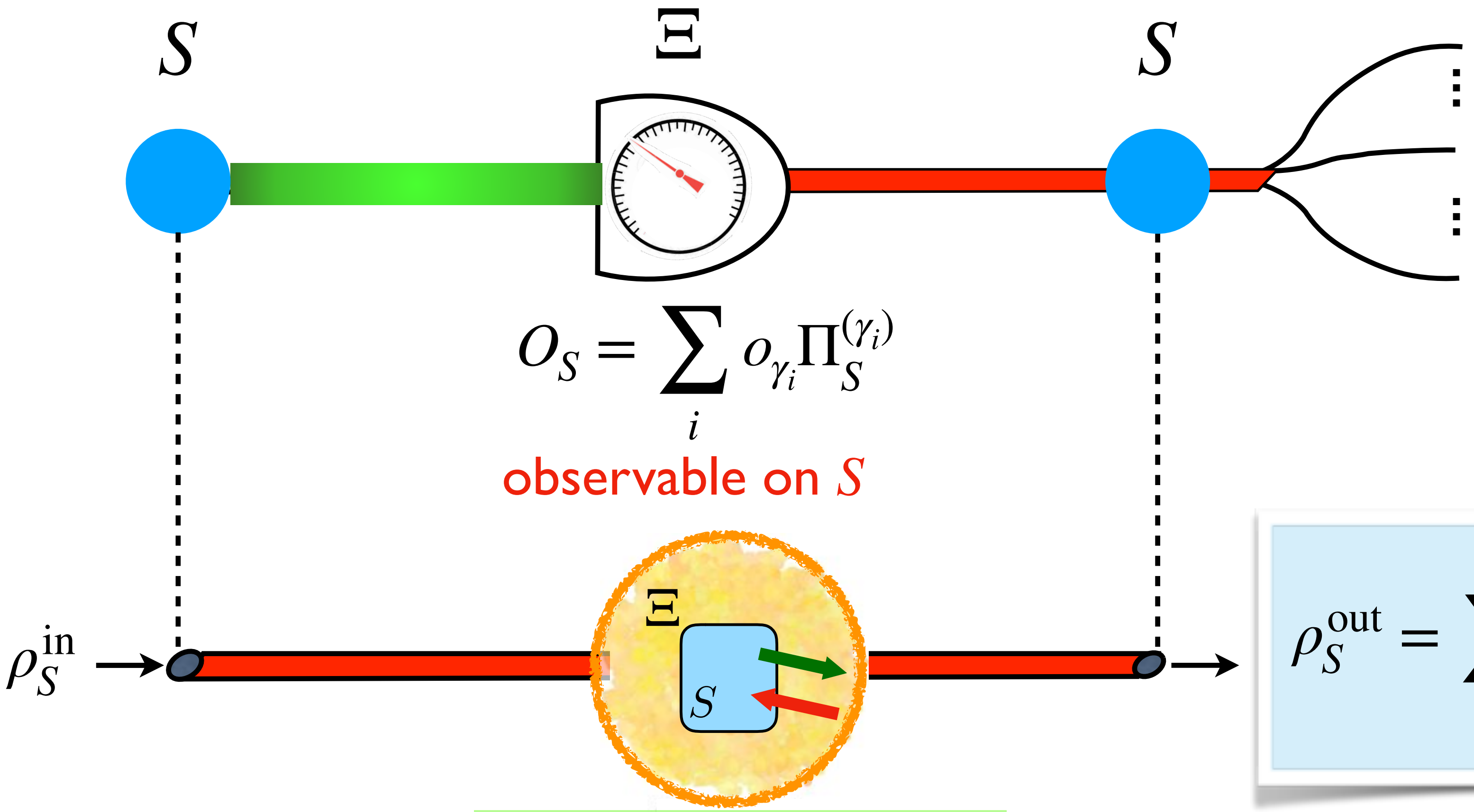
final state of $S + \Xi$

$\mathrm{Tr}_\Xi[\ldots]$

$$\rho_S^{\mathrm{out}} = \sum_i \Pi_S^{(\gamma_i)}\rho_S^{\mathrm{in}}\Pi_S^{(\gamma_i)} = \Phi[\rho_S^{\mathrm{in}}]$$

Projective q. measures (PVM)

von Neumann (1927)
Ozawa (1984)



γ_1 with probability p_1

γ_i $''$ $p_i = \mathrm{Tr}[\Pi_S^{(\gamma_i)} \rho_S^{\text{in}} \Pi_S^{(\gamma_i)}]$

γ_{n_Γ} $''$ p_{n_Γ}

$$O_S = \sum_i o_{\gamma_i} \Pi_S^{(\gamma_i)}$$

observable on S

$$\rho_S^{\text{out}} = \sum_i \Pi_S^{(\gamma_i)} \rho_S^{\text{in}} \Pi_S^{(\gamma_i)} = \Phi[\rho_S^{\text{in}}]$$

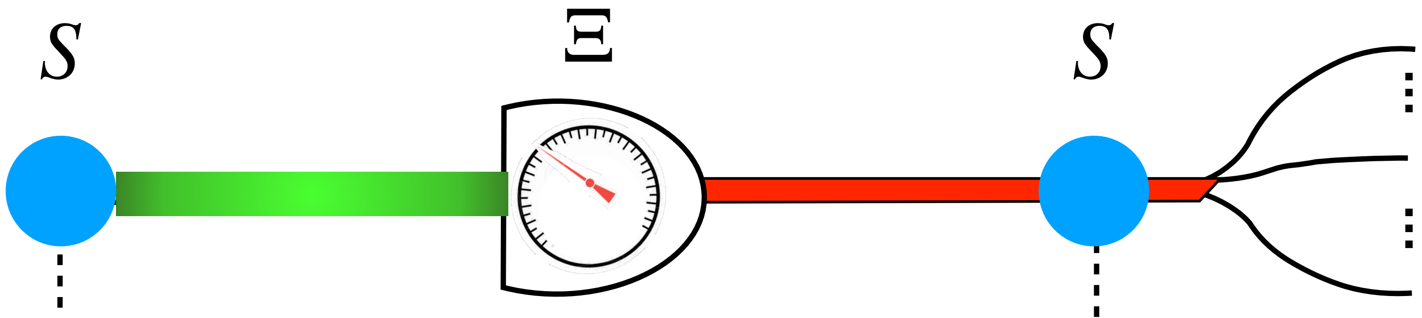
$$U_{S\Xi} = e^{-itO_S \otimes O_\Xi}$$

$t > t_{\text{d}}$

$$\rho_S^{\text{out}} = \Phi[\rho_S^{\text{in}}] = \mathrm{Tr}_\Xi \left[U_{S\Xi} \left(\rho_S^{\text{in}} \otimes |D\rangle_\Xi \langle D| \right) U_{S\Xi}^\dagger \right]$$

Stinespring representation of the quantum map

Positive Operator-Valued Measures (POVMs)



γ_1 with probability p_1

$\vdots \qquad \qquad \vdots$

$\gamma_i \qquad ” \qquad p_i = \mathrm{Tr}[M_S^{(\gamma_i)} \rho_S^{\mathrm{in}} M_S^{(\gamma_i)^\dagger}]$

$\vdots \qquad \qquad \vdots$

$\gamma_{n_\Gamma} \qquad ” \qquad p_{n_\Gamma}$

$$\{E_S^{(\gamma_i)} = M_S^{(\gamma_i)^\dagger} M_S^{(\gamma_i)} \geq 0\}$$

$$\sum_i E_S^{(\gamma_i)} = \mathbb{I}_S$$

$$\xcancel{E_S^{(\gamma_i)} E_S^{(\gamma_j)} = \delta_{ij} E_S^{(\gamma_i)}}$$

ρ_S^{in}

$$\rho_S^{\mathrm{out}} = \sum_i \underbrace{M_S^{(\gamma_i)} \rho_S^{\mathrm{in}} M_S^{(\gamma_i)^\dagger}}$$

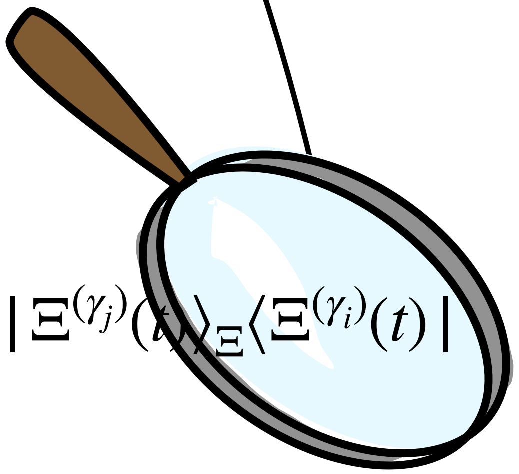
what we read

due to the lack of orthogonality this transformation would not be physically implementable (i.e. CPT)

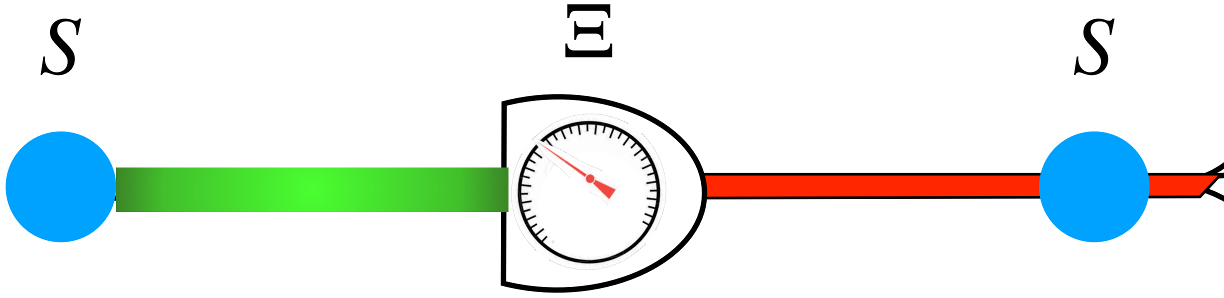
$$\rho_{S\Xi}(0) = \rho_S^{\mathrm{in}} \otimes |D\rangle_\Xi \langle D| \xrightarrow{\xcancel{?}} \rho_{S\Xi}(t) = \sum_{i,j} \overbrace{M_S^{(\gamma_j)} \rho_S^{\mathrm{in}} M_S^{(\gamma_i)^\dagger}} \otimes |\Xi^{(\gamma_j)}(t)\rangle_\Xi \langle \Xi^{(\gamma_i)}(t)|$$

initial state of $S + \Xi$

final state of $S + \Xi$



Positive Operator-Valued Measures (POVMs)



γ_1 with probability p_1

$\vdots$ $\vdots$

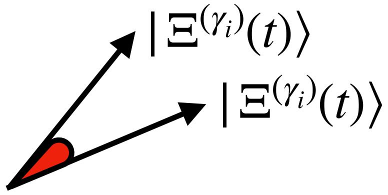
γ_i '' $p_i = \mathrm{Tr}[M_S^{(\gamma_i)}\rho_S^{\text{in}}M_S^{(\gamma_i)\dagger}]$

$\vdots$ $\vdots$

γ_{n_Γ} p_{n_Γ}

projective measures

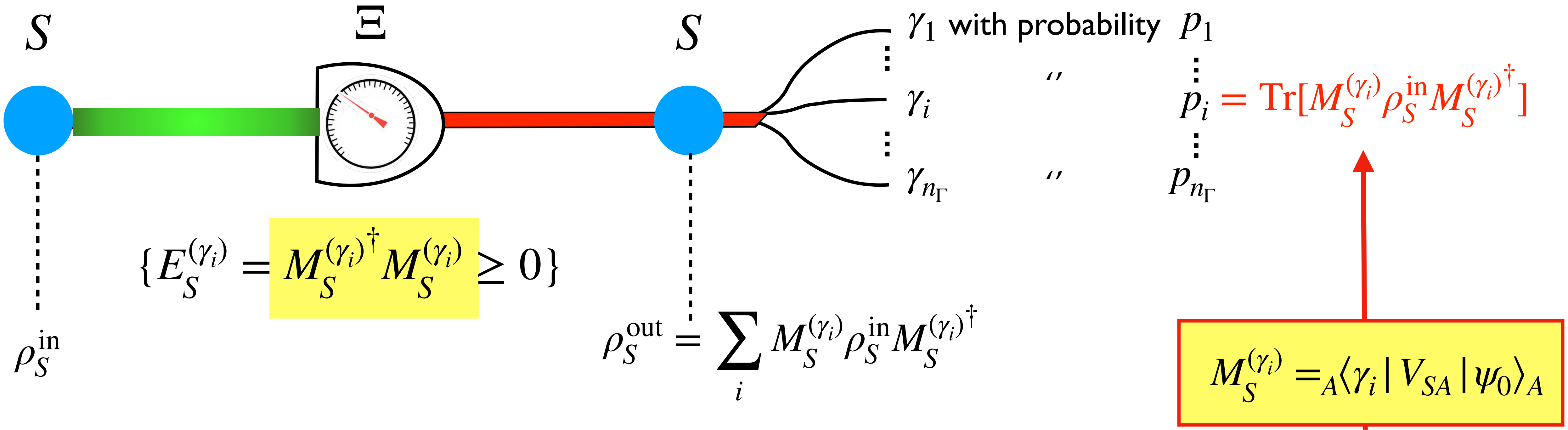
$$\rho_{S\Xi}(0) = \rho_S^{\text{in}} \otimes |D\rangle_\Xi\langle D| \xrightarrow{\checkmark} \rho_{S\Xi}(t) = \sum_{i,j} \underbrace{\Pi_S^{(\gamma_j)}\rho_S^{\text{in}}\Pi_S^{(\gamma_i)}} \otimes |\Xi^{(\gamma_j)}(t)\rangle_\Xi\langle\Xi^{(\gamma_i)}(t)|$$



$$\rho_{S\Xi}(0) = \rho_S^{\text{in}} \otimes |D\rangle_\Xi\langle D| \xrightarrow{\cancel{?}} \rho_{S\Xi}(t) = \sum_{i,j} \overbrace{M_S^{(\gamma_j)}\rho_S^{\text{in}}M_S^{(\gamma_i)\dagger}} \otimes |\Xi^{(\gamma_j)}(t)\rangle_\Xi\langle\Xi^{(\gamma_i)}(t)|$$

POVMs

Positive Operator-Valued Measures (POVMs)



γ_1 with probability p_1

γ_i " $p_i = \mathrm{Tr}[M_S^{(\gamma_i)} \rho_S^{\text{in}} M_S^{(\gamma_i)\dagger}]$

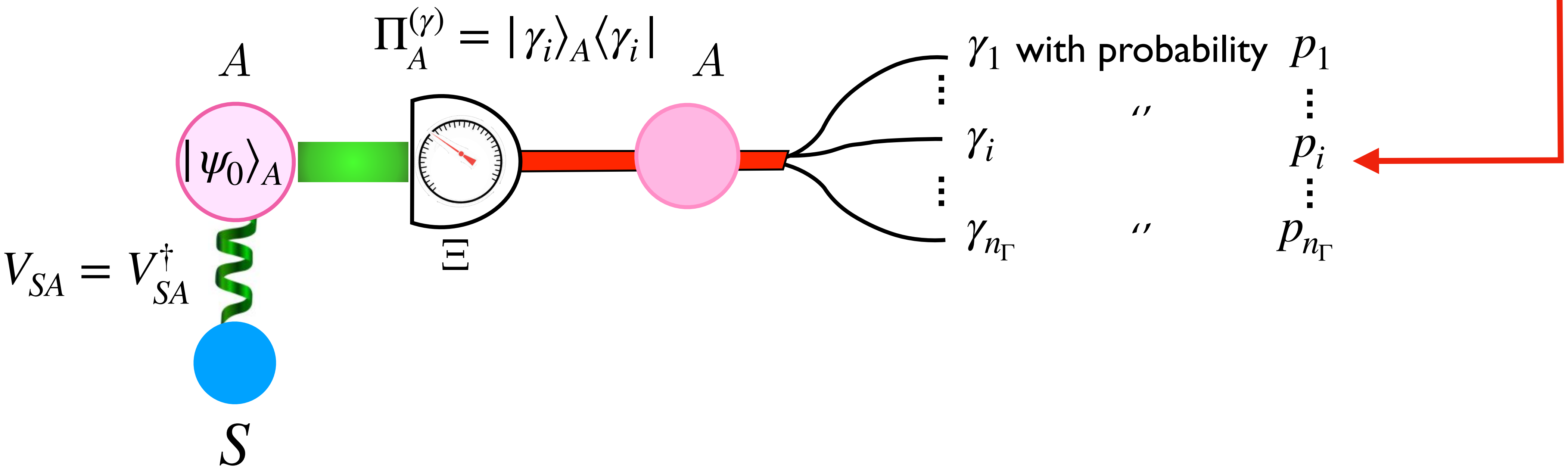
γ_{n_Γ} " p_{n_Γ}

$$\{E_S^{(\gamma_i)} = M_S^{(\gamma_i)\dagger} M_S^{(\gamma_i)} \geq 0\}$$

$$\rho_S^{\text{out}} = \sum_i M_S^{(\gamma_i)} \rho_S^{\text{in}} M_S^{(\gamma_i)\dagger}$$

$$M_S^{(\gamma_i)} = {}_A\langle \gamma_i | V_{SA} | \psi_0 \rangle_A$$

Naimark Theorem *: two-step procedure*



$\Pi_A^{(\gamma)} = |\gamma_i\rangle_A \langle \gamma_i|$

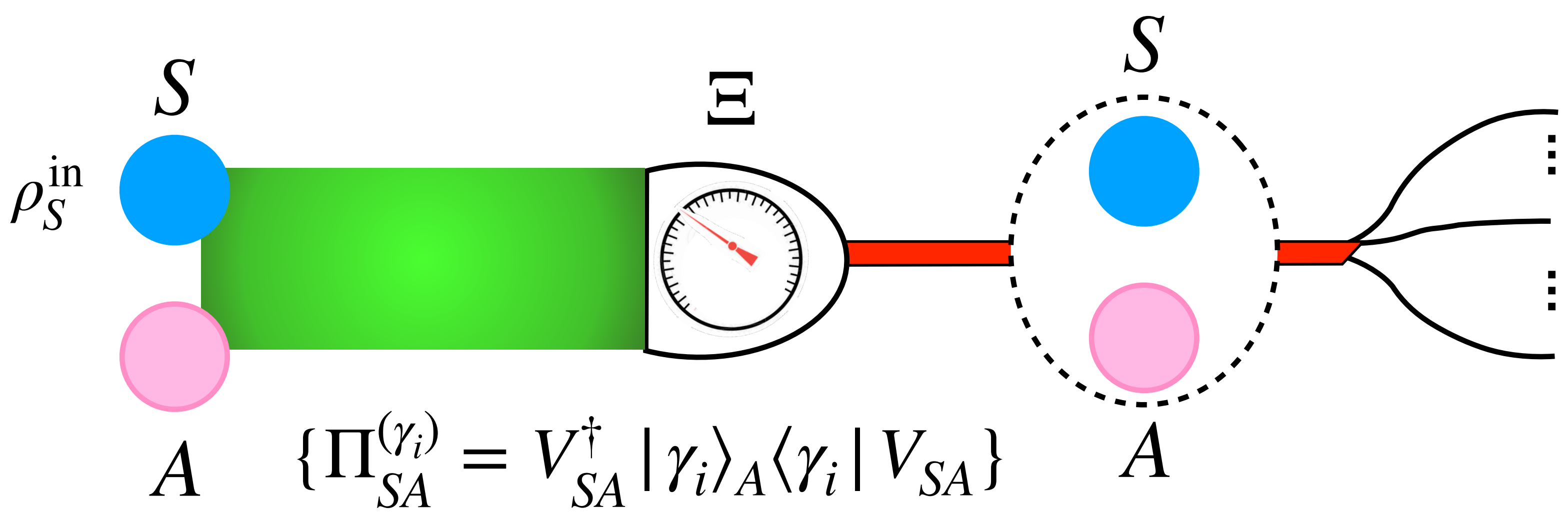
$V_{SA} = V_{SA}^\dagger$

γ_1 with probability p_1

γ_i " p_i

γ_{n_Γ} " p_{n_Γ}

Positive Operator-Valued Measures (POVMs)



$\{\Pi_{SA}^{(\gamma_i)} = V_{SA}^\dagger \, |\gamma_i\rangle_A\langle\gamma_i| \, V_{SA}\}$

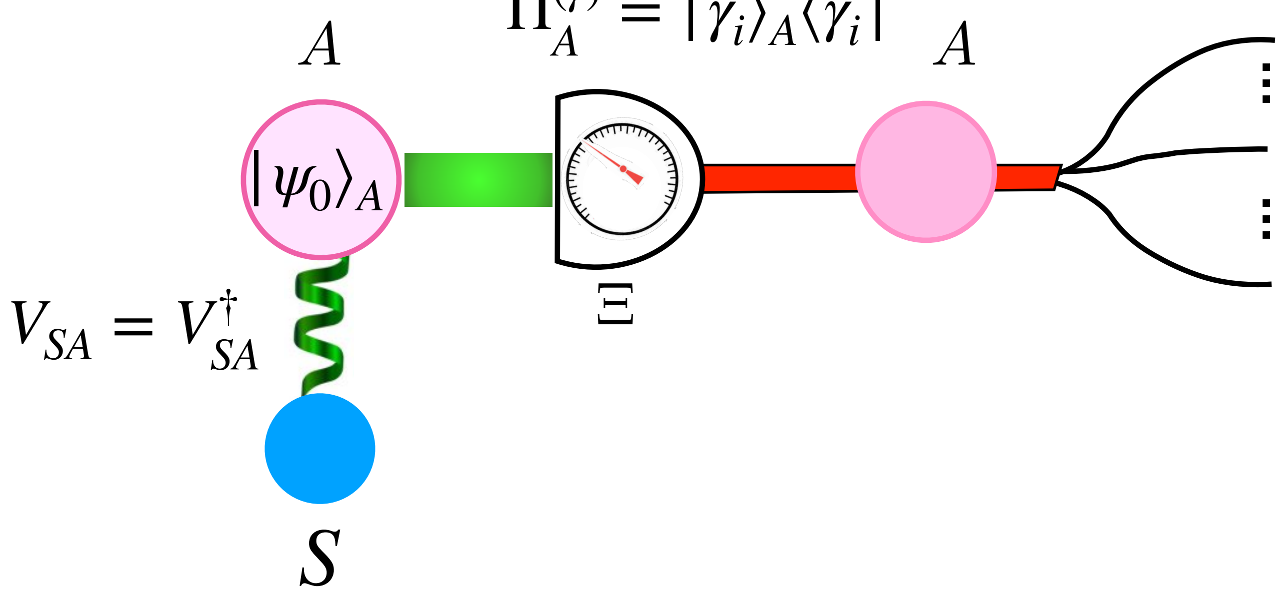
orthogonal projectors

γ_1 with probability p_1
⋮
γ_i ″ $p_i = \mathrm{Tr}[M_S^{(\gamma_i)} \rho_S^{\rm in} M_S^{(\gamma_i)\dagger}]$
⋮
γ_{n_Γ} ″ p_{n_Γ}

$M_S^{(\gamma_i)} = {}_A\langle\gamma_i \,|\, V_{SA} \,|\, \psi_0\rangle_A$

Naimark Theorem *: two-step procedure*

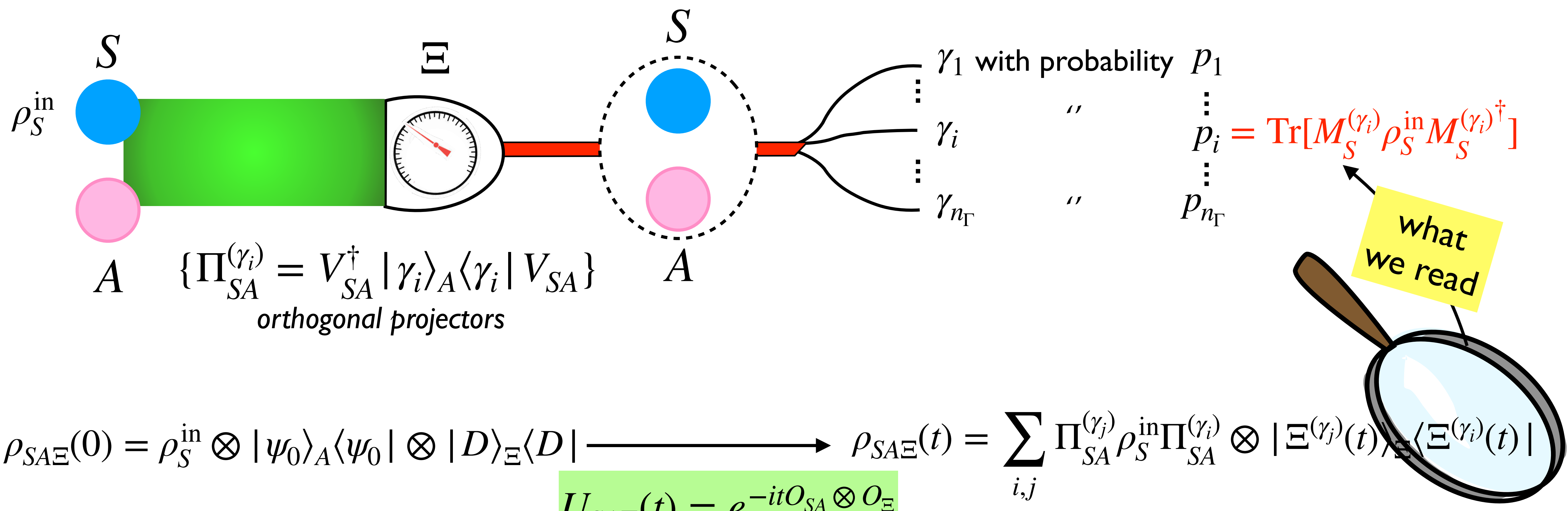
$\Pi_A^{(\gamma)} = |\gamma_i\rangle_A\langle\gamma_i|$



$V_{SA} = V_{SA}^\dagger$

γ_1 with probability p_1
⋮
γ_i ″ $p_i = \mathrm{Tr}[\rho_S^{\rm in} E_S^{(\gamma_i)}]$
⋮
γ_{n_Γ} ″ p_{n_Γ}

Positive Operator-Valued Measures (POVMs)



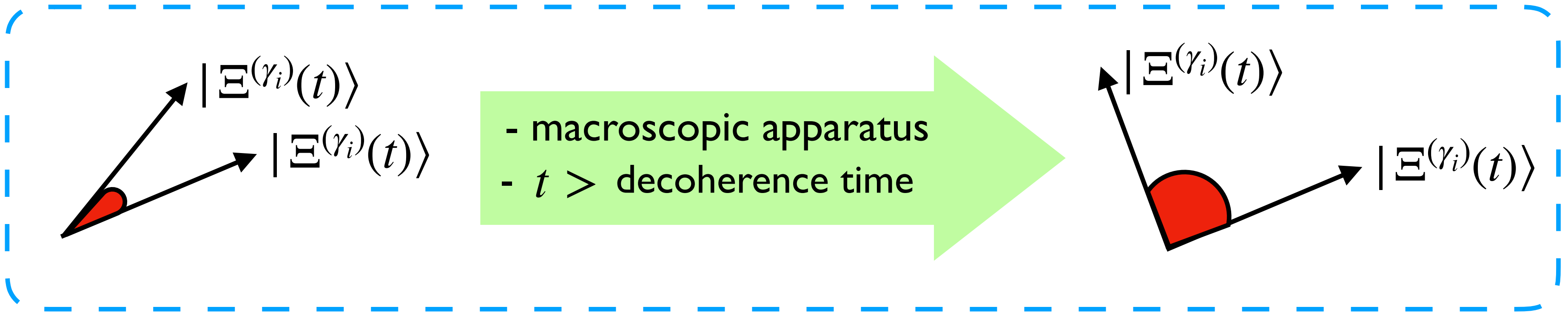
$\{\Pi_{SA}^{(\gamma_i)} = V_{SA}^\dagger \,|\,\gamma_i\rangle_A\langle\gamma_i\,|\,V_{SA}\}$

orthogonal projectors

$p_i = \mathrm{Tr}[M_S^{(\gamma_i)}\rho_S^{\text{in}}M_S^{(\gamma_i)\dagger}]$

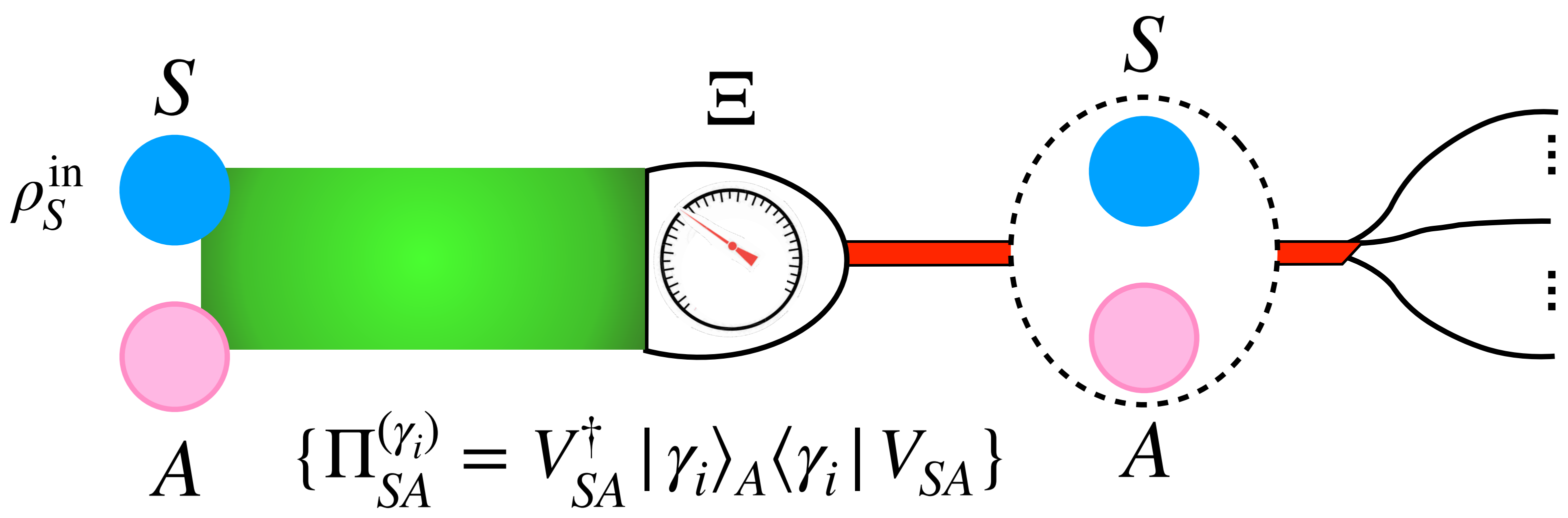
$$\rho_{SA\Xi}(0) = \rho_S^{\text{in}} \otimes |\,\psi_0\rangle_A\langle\psi_0\,|\otimes|\,D\rangle_\Xi\langle D\,| \xrightarrow{\hspace{3cm}} \rho_{SA\Xi}(t) = \sum_{i,j}\Pi_{SA}^{(\gamma_j)}\rho_S^{\text{in}}\Pi_{SA}^{(\gamma_i)}\otimes|\,\Xi^{(\gamma_j)}(t)\rangle_\Xi\langle\Xi^{(\gamma_i)}(t)\,|$$

$$U_{SA\Xi}(t) = e^{-itO_{SA}\otimes O_\Xi}$$



- macroscopic apparatus
- $t >$ decoherence time

Positive Operator-Valued Measures (POVMs)



$\{\Pi_{SA}^{(\gamma_i)} = V_{SA}^\dagger\,|\gamma_i\rangle_A\langle\gamma_i|\,V_{SA}\}$

orthogonal projectors

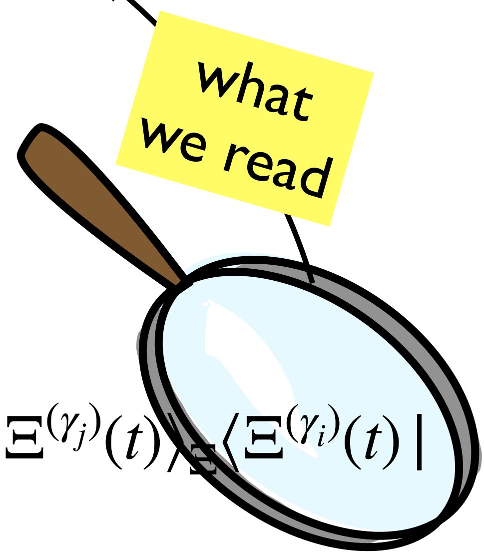
γ_1 with probability p_1

$\vdots$ ″ $\vdots$

γ_i ″ $p_i = \mathrm{Tr}[M_S^{(\gamma_i)}\rho_S^{\text{in}}M_S^{(\gamma_i)\dagger}]$

$\vdots$ $\vdots$

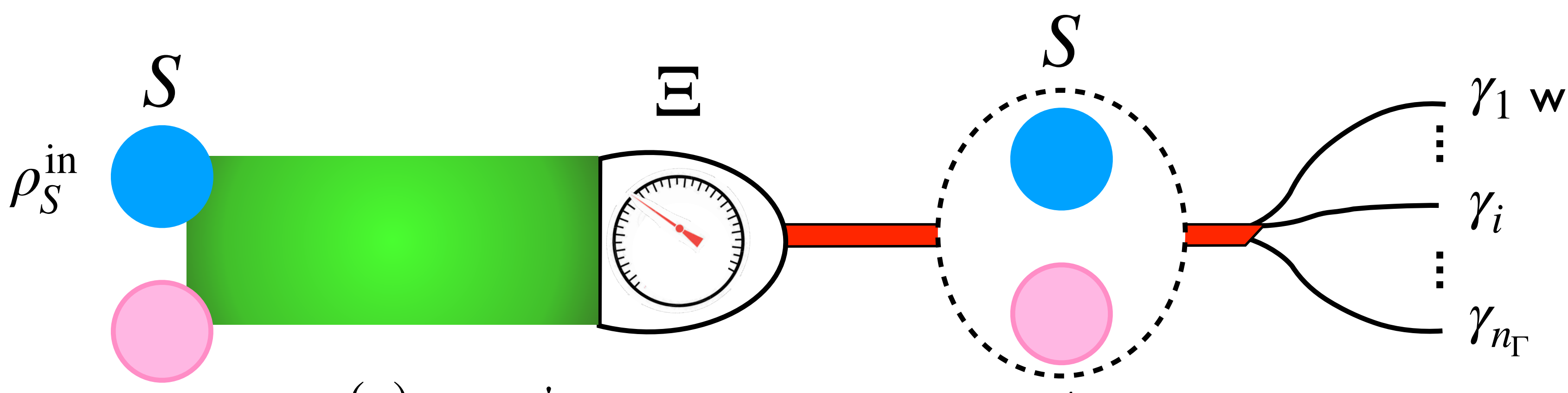
γ_{n_Γ} ″ p_{n_Γ}



$$\rho_{SA\Xi}(0) = \rho_S^{\text{in}} \otimes |\psi_0\rangle_A\langle\psi_0| \otimes |D\rangle_\Xi\langle D| \xrightarrow[U_{SA\Xi}(t) = e^{-itO_{SA}\otimes O_\Xi}]{} \rho_{SA\Xi}(t) = \sum_{i,j} \Pi_{SA}^{(\gamma_j)}\rho_S^{\text{in}}\Pi_{SA}^{(\gamma_i)} \otimes |\Xi^{(\gamma_j)}(t)\rangle_\Xi\langle\Xi^{(\gamma_i)}(t)|$$

what happens at the level of the principal system?

Positive Operator-Valued Measures (POVMs)



ρ_S^{in}

S

A

Ξ

S

A

γ_1 with probability p_1

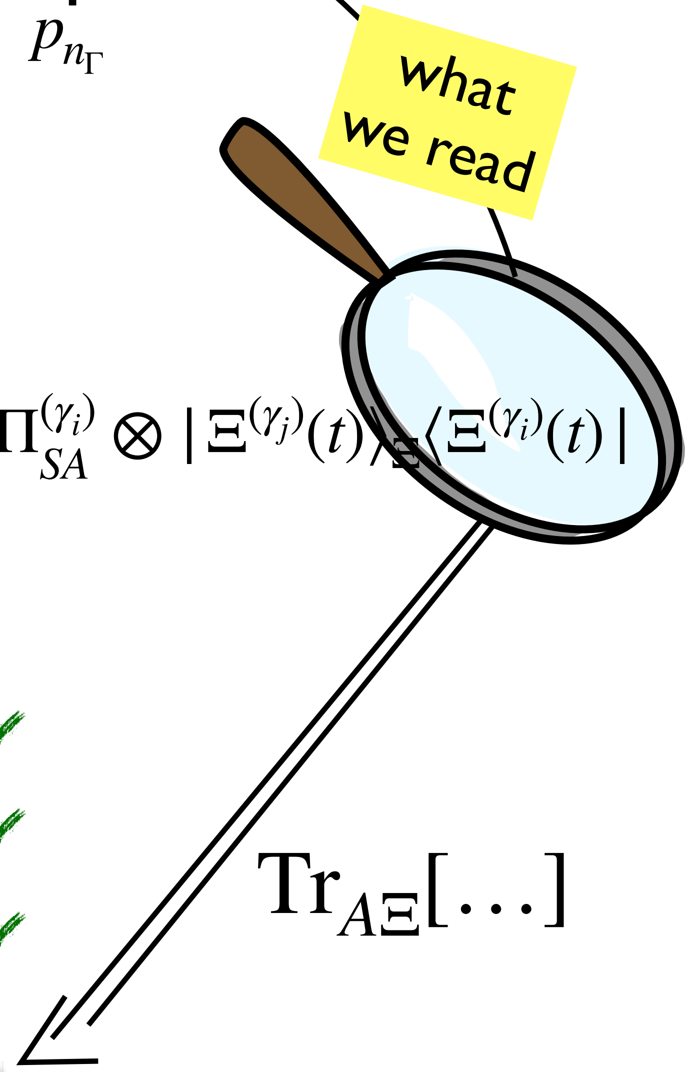
γ_i ″ $p_i = \mathrm{Tr}[M_S^{(\gamma_i)}\rho_S^{\text{in}}M_S^{(\gamma_i)\dagger}]$

γ_{n_Γ} ″ p_{n_Γ}

$$\{\Pi_{SA}^{(\gamma_i)} = V_{SA}^{\dagger}\,|\gamma_i\rangle_A\langle\gamma_i|\,V_{SA}\}$$

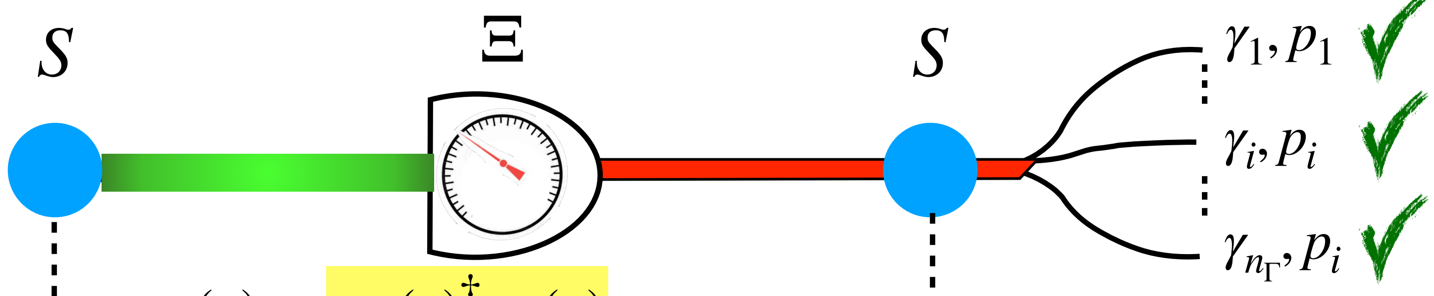
orthogonal projectors

what we read



$$\rho_{SA\Xi}(0) = \rho_S^{\text{in}} \otimes |\psi_0\rangle_A\langle\psi_0| \otimes |D\rangle_\Xi\langle D| \xrightarrow[U_{SA\Xi}(t) = e^{-itO_{SA}\otimes O_\Xi}]{} \rho_{SA\Xi}(t) = \sum_{i,j}\Pi_{SA}^{(\gamma_j)}\rho_S^{\text{in}}\Pi_{SA}^{(\gamma_i)} \otimes |\Xi^{(\gamma_j)}(t)\rangle_\Xi\langle\Xi^{(\gamma_i)}(t)|$$

$\mathrm{Tr}_{A\Xi}[\ldots]$



S

Ξ

S

γ_1, p_1 ✓

γ_i, p_i ✓

γ_{n_Γ}, p_i ✓

$$\{E_S^{(\gamma_i)} = M_S^{(\gamma_i)\dagger}M_S^{(\gamma_i)} \geq 0\}$$

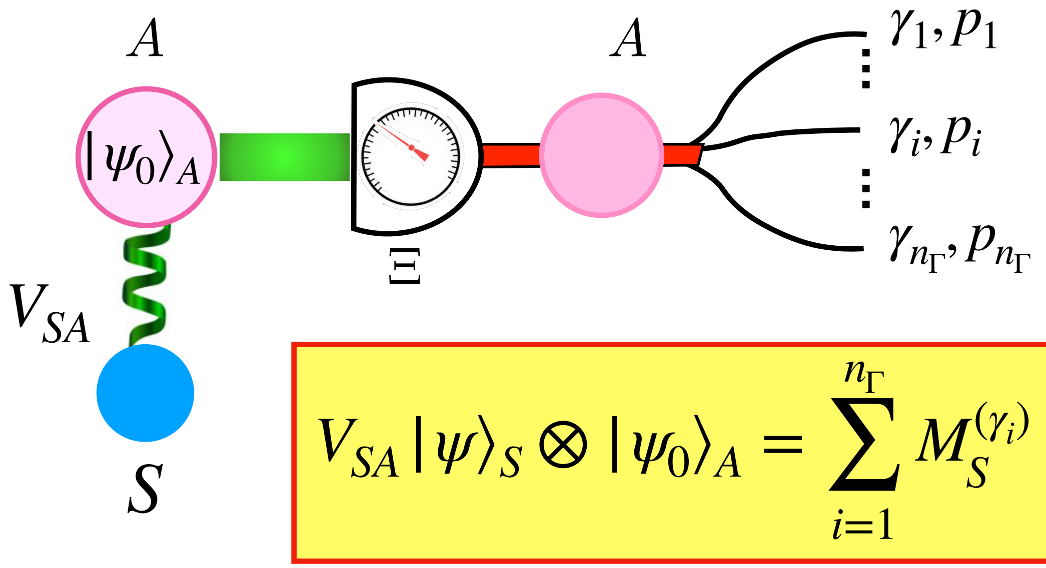
ρ_S^{in}

$$\rho_S^{\text{out}} = \sum_i M_S^{(\gamma_j)}\rho_S^{\text{in}}M_S^{(\gamma_i)\dagger}$$ ✗

Positive Operator-Valued Measures (POVMs)

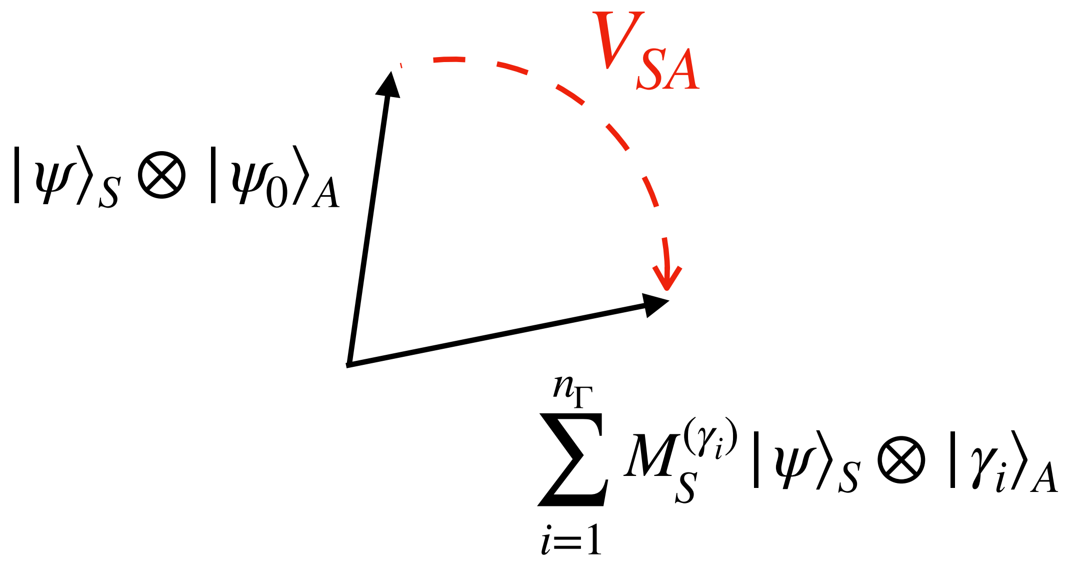
new strategy **:**

1. ***one time-independent Hamiltonian***
2. ***beyond standard decoherence***



$$V_{SA}\,|\psi\rangle_S \otimes |\psi_0\rangle_A = \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)}\,|\psi\rangle_S \otimes |\gamma_i\rangle_A$$

$$U_{SA}(t) := e^{-iH_{SA}t} \dashrightarrow V_{SA}$$



Positive Operator-Valued Measures (POVMs)

new strategy **:**
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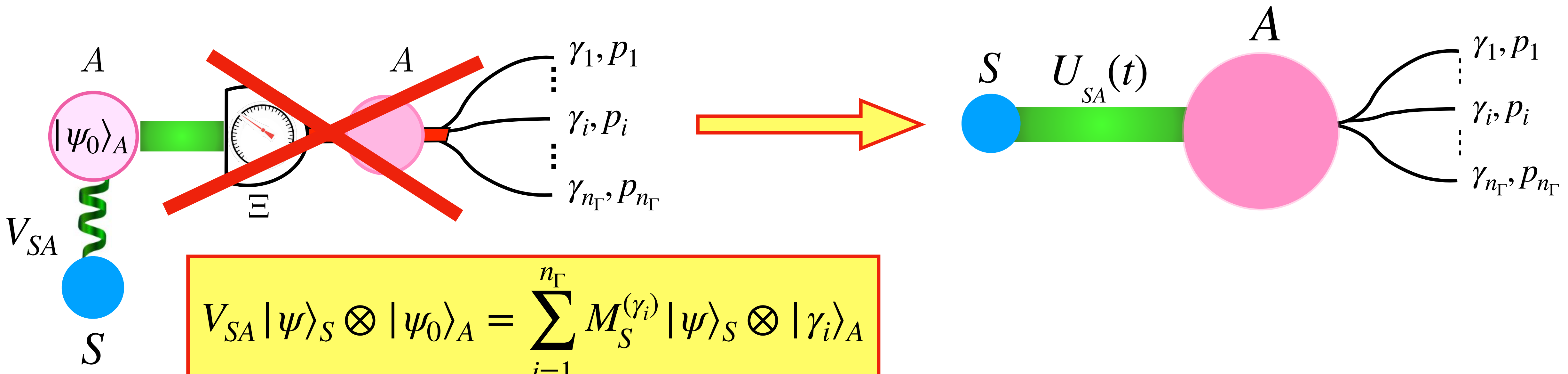
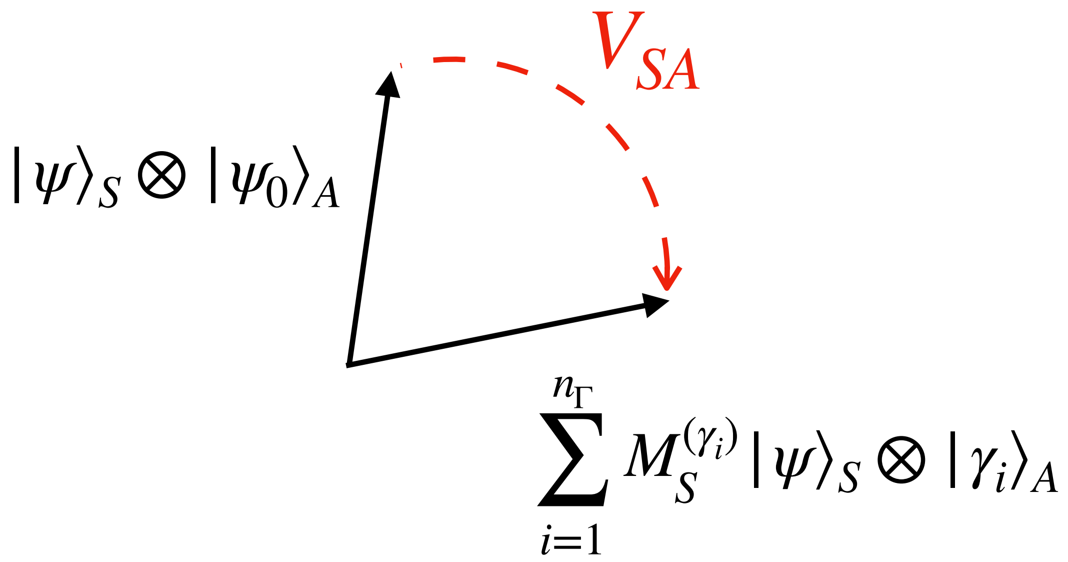


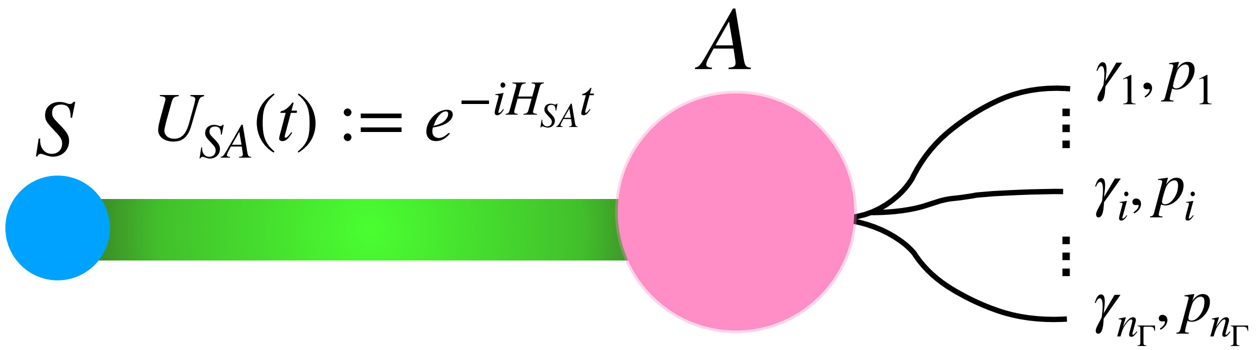
$$V_{SA}\,|\psi\rangle_S \otimes |\psi_0\rangle_A = \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)}\,|\psi\rangle_S \otimes |\gamma_i\rangle_A$$

$$U_{SA}(t) := e^{-iH_{SA}t} \dashrightarrow V_{SA}$$



Positive Operator-Valued Measures (POVMs)

new strategy : *1. one time-independent Hamiltonian; 2. beyond standard decoherence*



$$|\psi\rangle_S \otimes |\psi_0\rangle_A \xrightarrow{V_{SA}} \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)} |\psi\rangle_S \otimes |\gamma_i\rangle_A$$

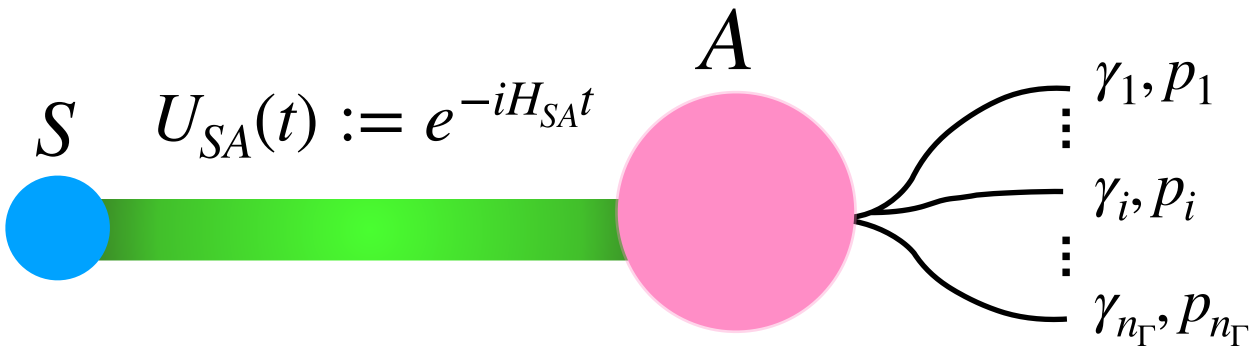
$$H_{SA} = \omega \sum_{j=1}^{n_S} \underbrace{|\xi_j^{(0)}\rangle_{SA}\langle\xi_j^{(1)}| + |\xi_j^{(1)}\rangle_{SA}\langle\xi_j^{(0)}|}_{\text{Pauli matrix } \sigma_{SA}^{(j)}}$$

$\{|j\rangle_S\}$ basis of $\mathscr{H}_S$

$$\begin{cases} |\xi_j^{(0)}\rangle_{SA} = |j\rangle_S \otimes |\psi_0\rangle_A \\ |\xi_j^{(1)}\rangle_{SA} := \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)} |j\rangle_S \otimes |\gamma_i\rangle_A \end{cases}$$

Positive Operator-Valued Measures (POVMs)

new strategy : ***1. one time-independent Hamiltonian; 2. beyond standard decoherence***



S

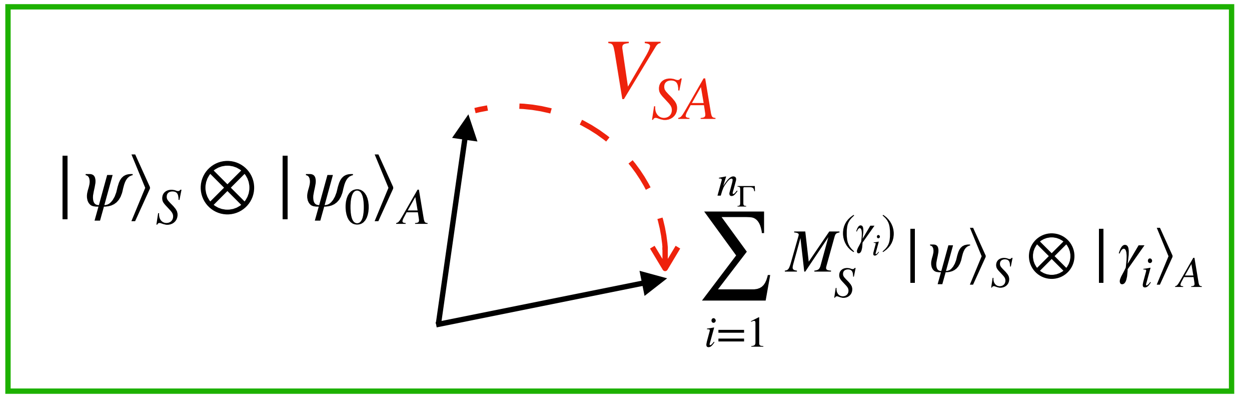
$U_{SA}(t) := e^{-iH_{SA}t}$

A

γ_1, p_1

γ_i, p_i

$\gamma_{n_\Gamma}, p_{n_\Gamma}$



V_{SA}

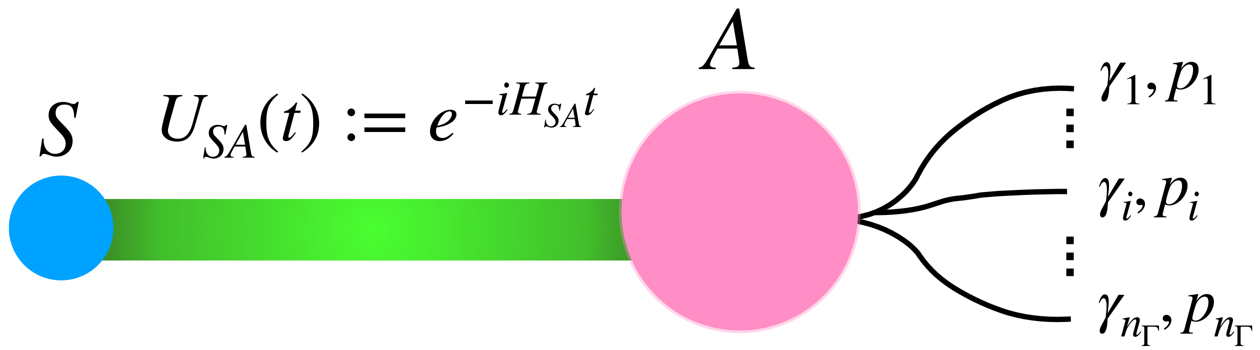
$|\psi\rangle_S \otimes |\psi_0\rangle_A$

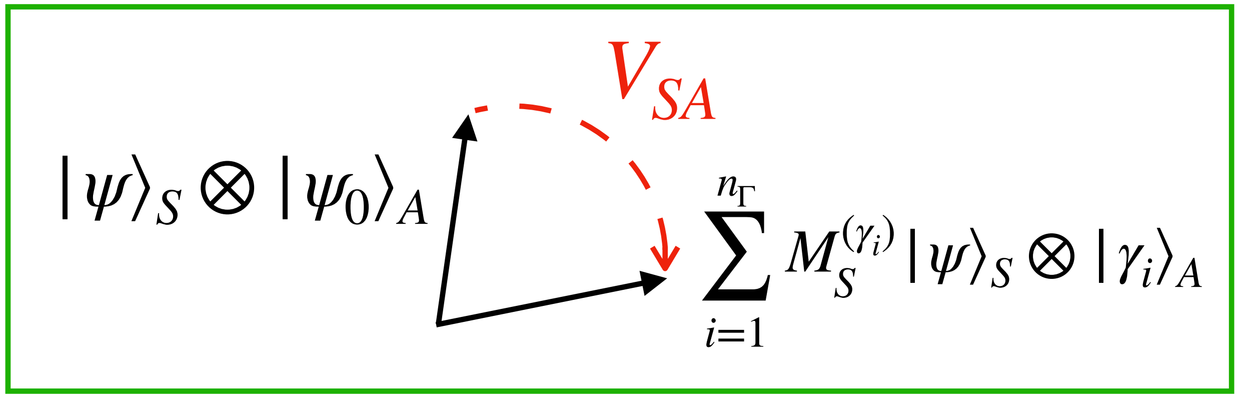
$\sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)} |\psi\rangle_S \otimes |\gamma_i\rangle_A$

$$|\Psi(t)\rangle_{SA} = \cos(\omega t)\,|\psi\rangle_S \otimes |\psi_0\rangle_A - i\sin(\omega t) \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)} |\psi\rangle_S \;\otimes\; |\gamma_i\rangle_A$$

Positive Operator-Valued Measures (POVMs)

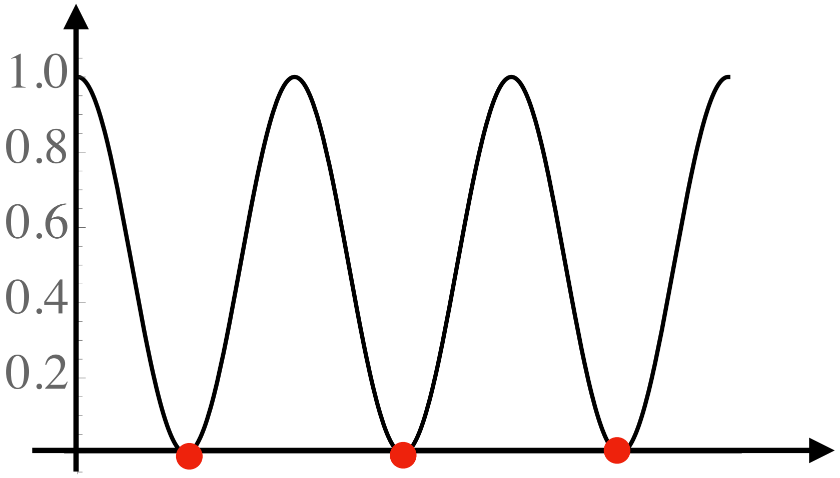
new strategy : *1. one time-independent Hamiltonian; 2. beyond standard decoherence*





$$|\Psi(t)\rangle_{SA} = \xcancel{\cos(\omega t)|\psi\rangle_S} \otimes |\psi_0\rangle_A - \underset{\downarrow \atop \pm i}{\boxed{i\sin(\omega t)}} \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)} |\psi\rangle_S \otimes |\gamma_i\rangle_A \propto V_{SA}|\psi\rangle_S \otimes |\psi_0\rangle_A$$

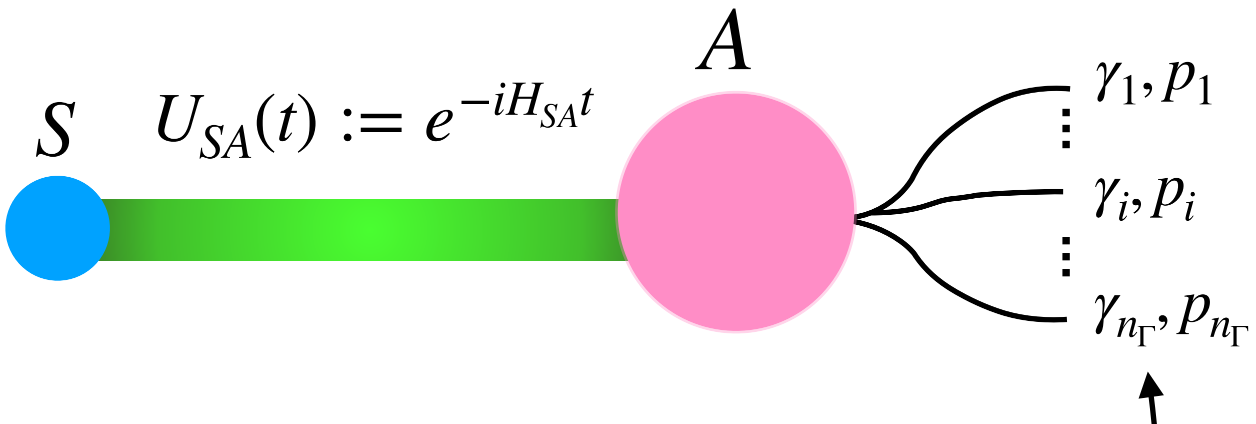
$$t^* = \frac{\pi}{2\omega} + m\pi$$



$t^* = \pi/2\omega$

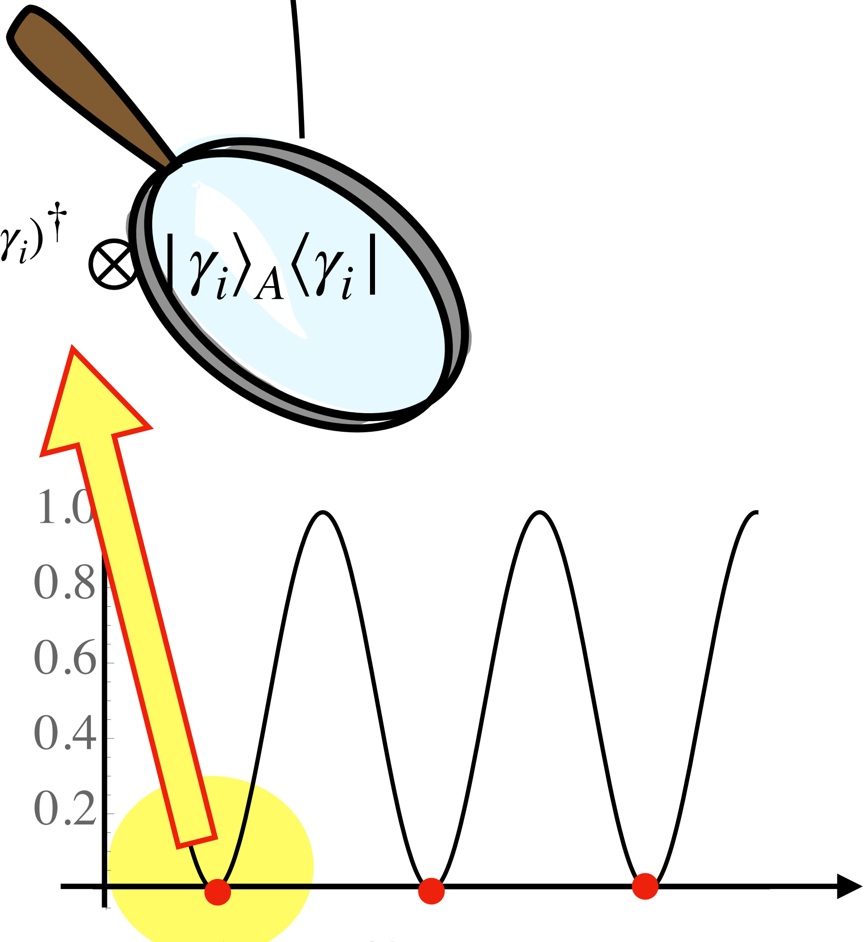
Positive Operator-Valued Measures (POVMs)

new strategy : *1. one time-independent Hamiltonian; 2. beyond standard decoherence*



$$|\psi\rangle_S \otimes |\psi_0\rangle_A \xrightarrow{V_{SA}} \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)} |\psi\rangle_S \otimes |\gamma_i\rangle_A$$

$$|\Psi\rangle_{SA}\langle\Psi| = \sum_{i,j} M_S^{(\gamma_i)} |\psi\rangle_S\langle\psi| M_S^{(\gamma_i)\dagger} \otimes |\gamma_i\rangle_A\langle\gamma_i|$$



$t^* = \pi/2\omega$

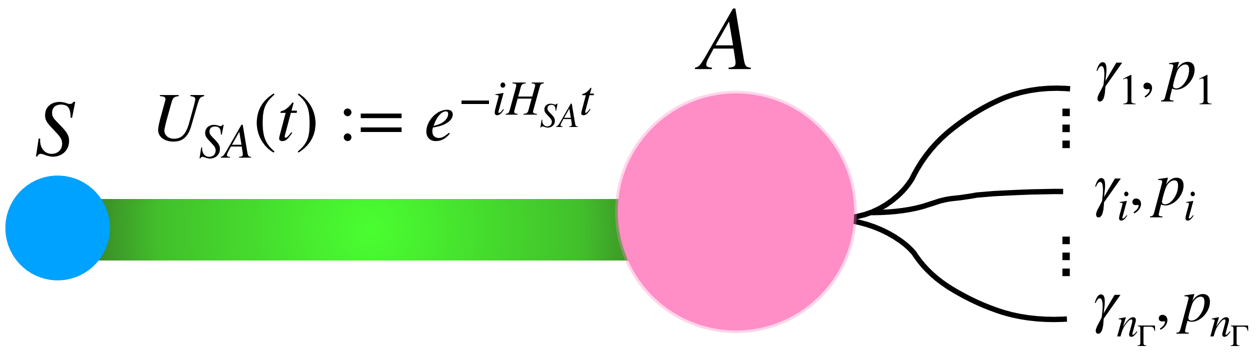
however it is JUST a PRECISE instant in which we should read the ANCILLA..

⇝

*we need to restore "**macroscopicity**"*

Positive Operator-Valued Measures (POVMs)

new strategy : *1. one time-independent Hamiltonian; 2. beyond standard decoherence*



$$|\psi\rangle_S \otimes |\psi_0\rangle_A \xrightarrow{V_{SA}} \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)} |\psi\rangle_S \otimes |\gamma_i\rangle_A$$

"macroscopicity"

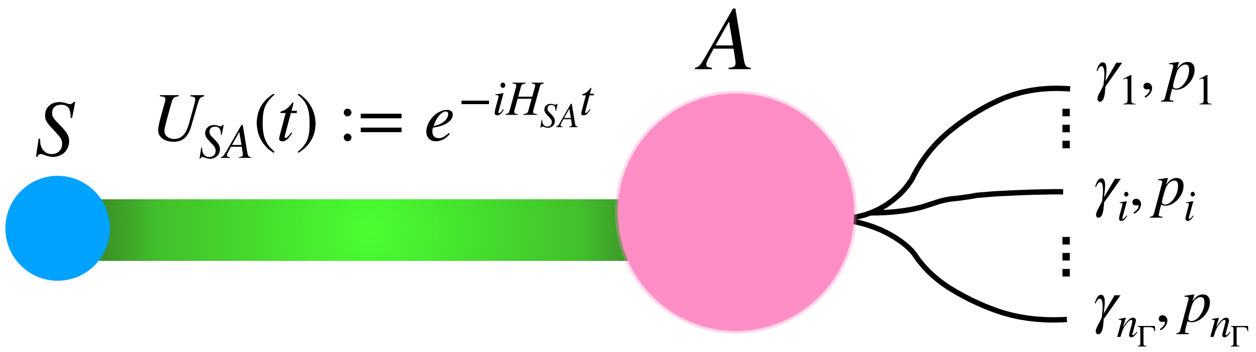
$$H_{SA} = \sum_{j=1}^{n_S} \sum_{\ell=0}^{n_L-1} \omega_\ell \Big(\underbrace{|\xi_j^{(\ell)}\rangle_{SA}\langle\xi_j^{(\ell+1)}| + |\xi_j^{(\ell+1)}\rangle_{SA}\langle\xi_j^{(\ell)}|}_{\sigma_{SA}^{(j,\ell)}} \Big)$$

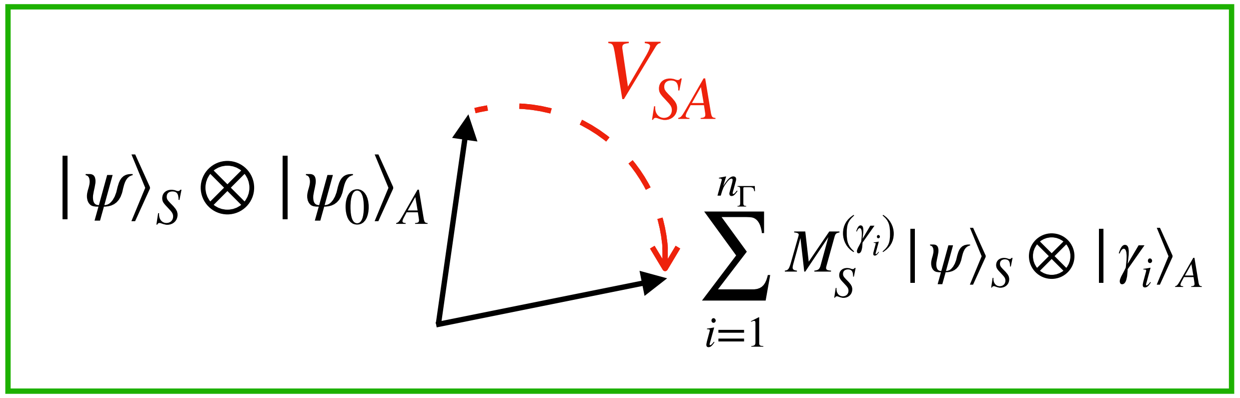
basis of $\mathscr{H}_S$ $\{|j\rangle_S\}$

$$\begin{cases} |\xi_j^{(0)}\rangle_{SA} = |j\rangle_S \otimes |\psi_0\rangle_A \\ |\xi_j^{(\ell)}\rangle_{SA} := \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)} |j\rangle_S \otimes |\gamma_i, \ell\rangle_A, \ \ell \geq 1 \end{cases}$$

Positive Operator-Valued Measures (POVMs)

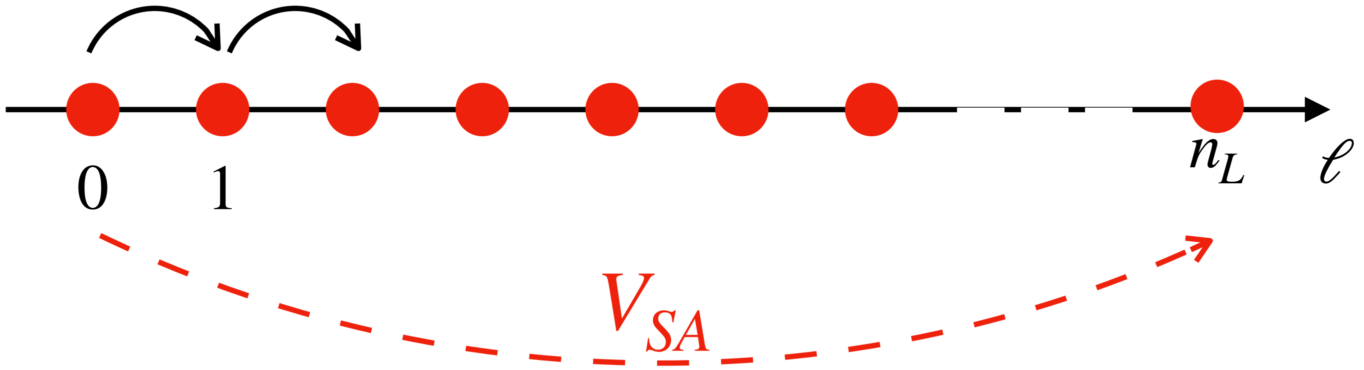
new strategy : ***1. one time-independent Hamiltonian; 2. beyond standard decoherence***





$$H_{SA} = \sum_{j=1}^{n_S} \sum_{\ell=0}^{n_L-1} \omega_\ell \left(\underbrace{|\xi_j^{(\ell)}\rangle_{SA}\langle\xi_j^{(\ell+1)}| + |\xi_j^{(\ell+1)}\rangle_{SA}\langle\xi_j^{(\ell)}|}_{\sigma_{SA}^{(j,\ell)}} \right)$$

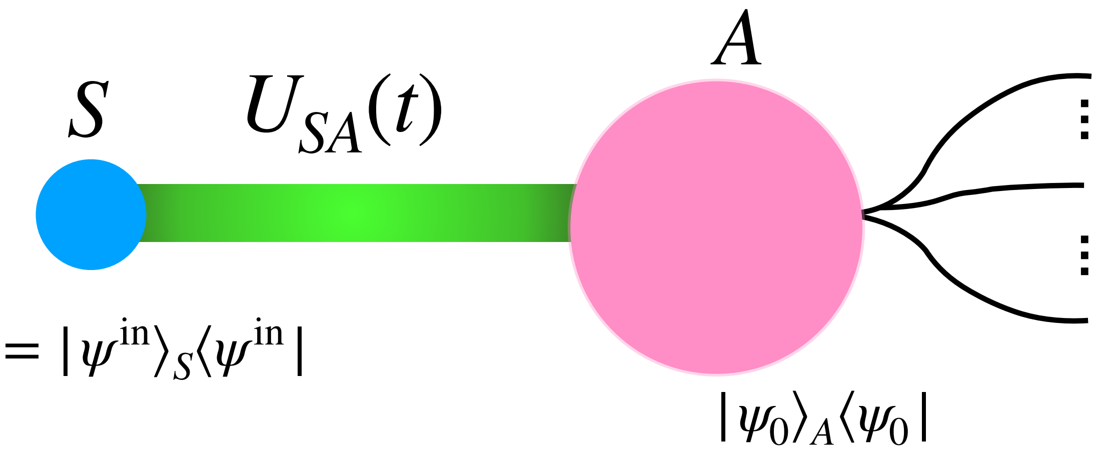
$\sim$ 1 excitation sector of a spin chain with first neighboring hopping terms



STATE TRANSFER PROBLEM

Christandl, et al, PRL (2004)

Positive Operator-Valued Measures (POVMs)

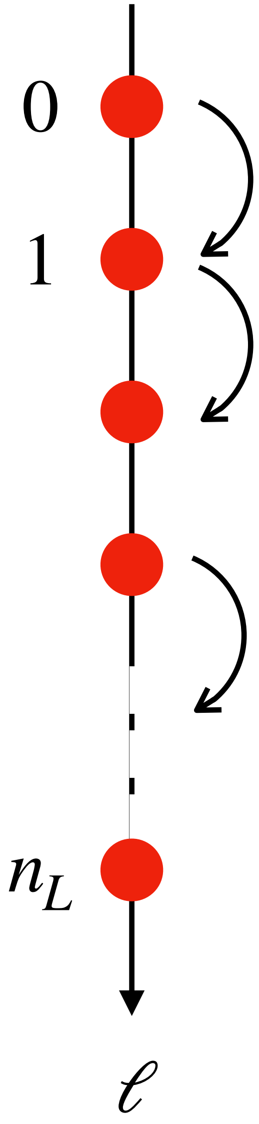


γ_1 with probability p_1

γ_i " $p_i = \mathrm{Tr}[M_S^{(\gamma_i)}\rho_S^{\mathrm{in}}M_S^{(\gamma_i)^\dagger}]$

γ_{n_Γ} " p_{n_Γ}

hp $\rho_S^{\mathrm{in}} = |\psi^{\mathrm{in}}\rangle_S\langle\psi^{\mathrm{in}}|$



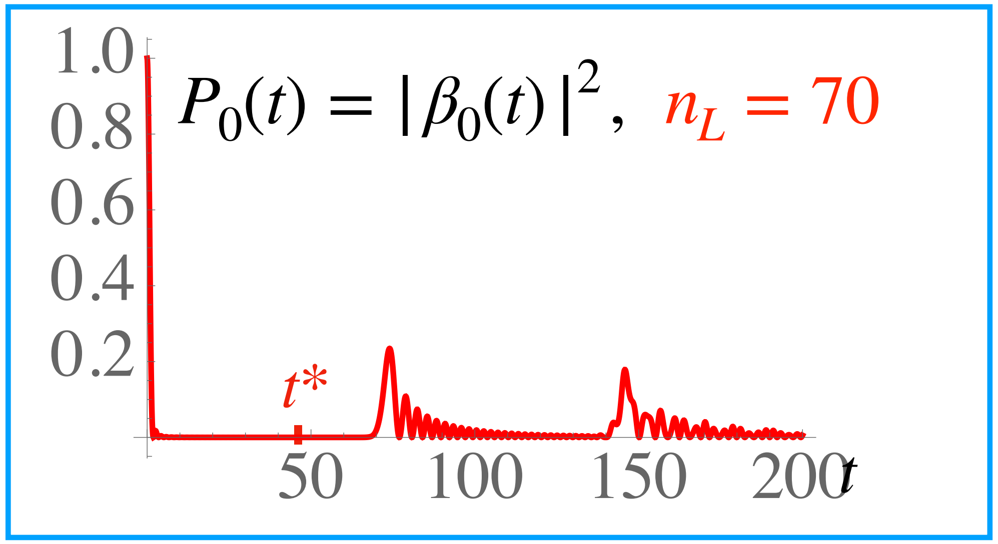
$$|\Psi(0)\rangle_{SA} = |\psi^{\mathrm{in}}\rangle_S \otimes |\psi_0\rangle_A$$

$$\downarrow$$

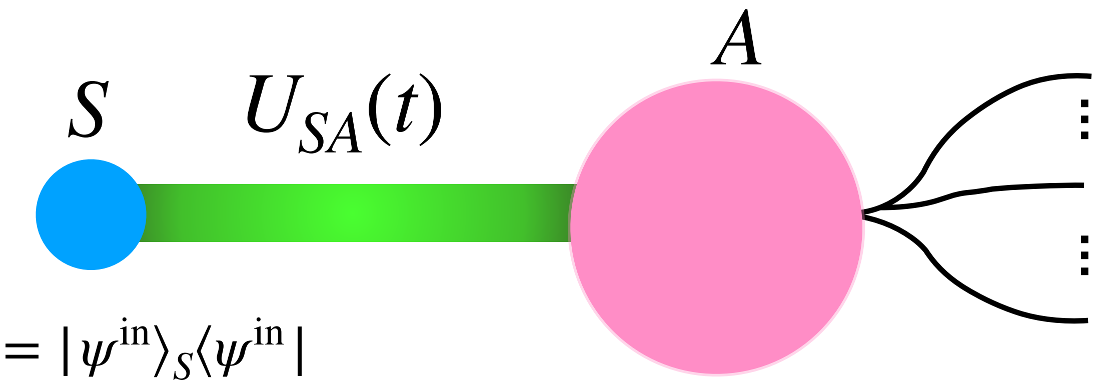
$$|\Psi(t)\rangle_{SA} = \beta_0(t)|\psi^{\mathrm{in}}\rangle_S \otimes |\psi_0\rangle_A + \sqrt{1-|\beta_0(t)|^2}\sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)}|\psi\rangle_S \otimes |A^{\gamma_i}(t)\rangle_A$$

$$\downarrow$$

$$|\Psi(t^*)\rangle_{SA} \simeq \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)}|\psi^{\mathrm{in}}\rangle_S \otimes |A^{\gamma_i}(t^*)\rangle_A$$



Positive Operator-Valued Measures (POVMs)



γ_1 with probability p_1

$\vdots$ $\vdots$

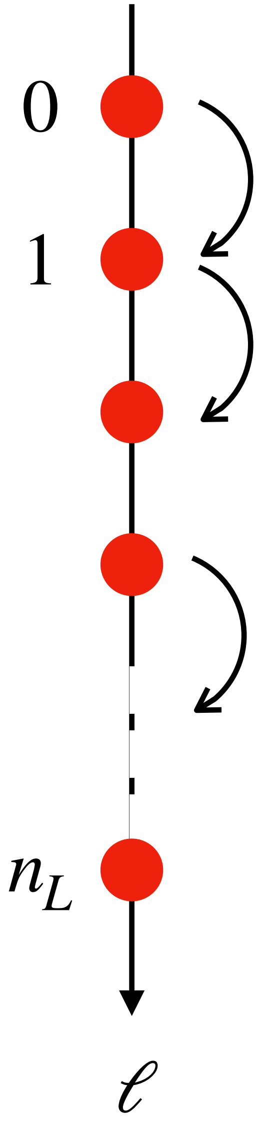
γ_i ″ $p_i = \mathrm{Tr}[M_S^{(\gamma_i)}\rho_S^{in}M_S^{(\gamma_i)^\dagger}]$

$\vdots$ $\vdots$

γ_{n_Γ} ″ p_{n_Γ}

hp $\rho_S^{in} = |\psi^{in}\rangle_S\langle\psi^{in}|$

$|\psi_0\rangle_A\langle\psi_0|$

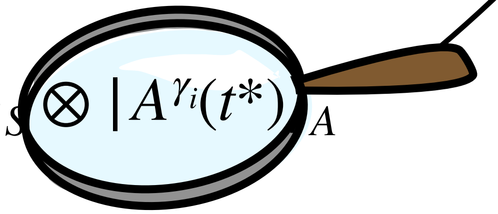


$$|\Psi(0)\rangle_{SA} = |\psi^{in}\rangle_S \otimes |\psi_0\rangle_A$$

$$|\Psi(t)\rangle_{SA} = \beta_0(t)|\psi^{in}\rangle_S \otimes |\psi_0\rangle_A$$

$$+\sqrt{1-|\beta_0(t)|^2}\sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)}|\psi\rangle_S \otimes |A^{\gamma_i}(t)\rangle_A$$

$$|\Psi(t^*)\rangle_{SA} \simeq \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)}|\psi^{in}\rangle_S \otimes |A^{\gamma_i}(t^*)\rangle_A$$

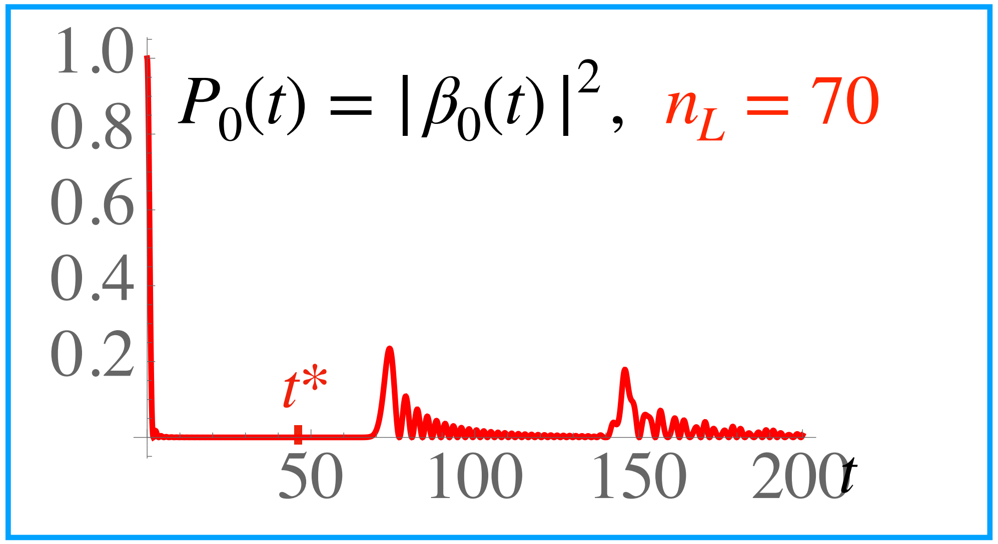


what we read

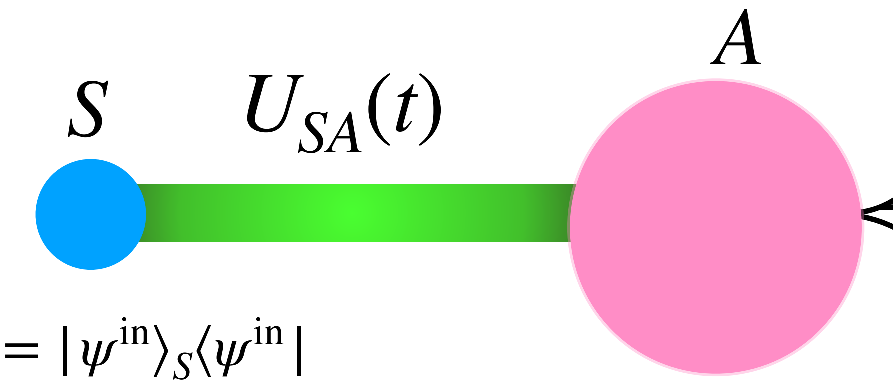
$${}_A\langle A^{\gamma_i}(t)|A^{\gamma_j}(t)\rangle_A = \delta_{i,j}$$

$${}_A\langle\psi_0|A^{\gamma_i}(t)\rangle_A = 0$$

$$\forall t$$



Positive Operator-Valued Measures (POVMs)



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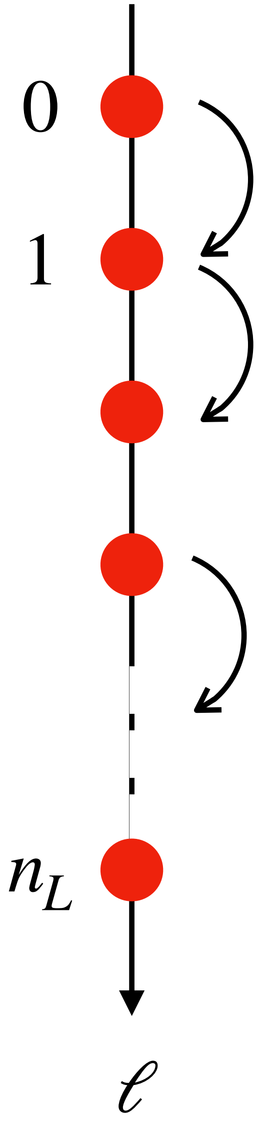
γ_{n_Γ} ″ p_{n_Γ}

hp $\rho_S^{\mathrm{in}} = |\psi^{\mathrm{in}}\rangle_S \langle \psi^{\mathrm{in}}|$

$|\psi_0\rangle_A \langle \psi_0|$



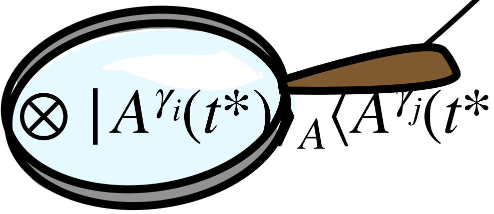
$$\rho_S^{\mathrm{in}} \otimes |\psi_0\rangle_A \langle \psi_0|$$

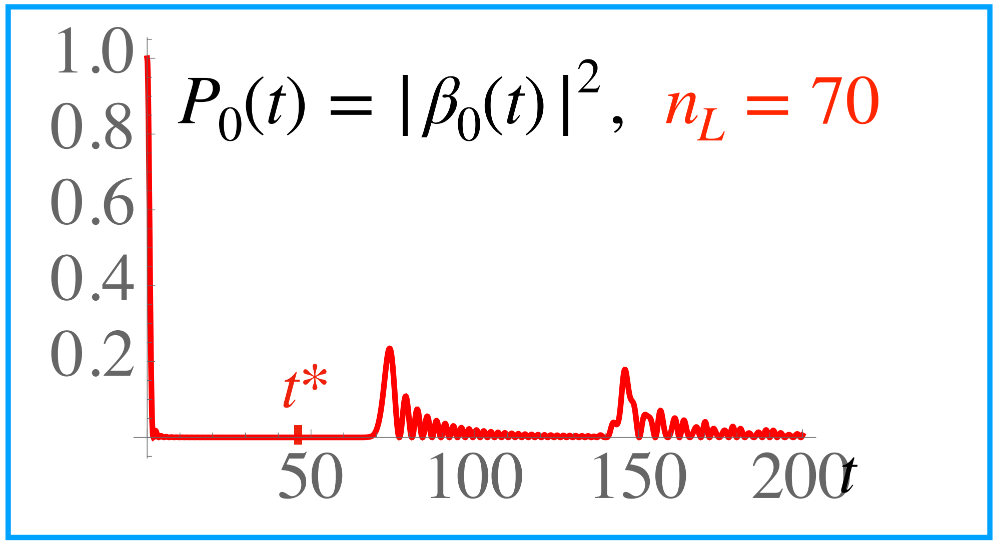
$$\rho(t)_{SA} = |\beta_0(t)|^2 \rho_S^{\mathrm{in}} \otimes |\psi_0\rangle_A \langle \psi_0| + |\beta_0(t)| \Delta_{SA}(t)$$
$$+ (1 - |\beta_0(t)|^2) \sum_{i,j=1}^{n_\Gamma} M_S^{(\gamma_i)} \rho_S^{\mathrm{in}} M_S^{(\gamma_j)^\dagger} \otimes |A^{(\gamma_i)}(t)\rangle_A \langle A^{(\gamma_i)}(t)|$$

what we read

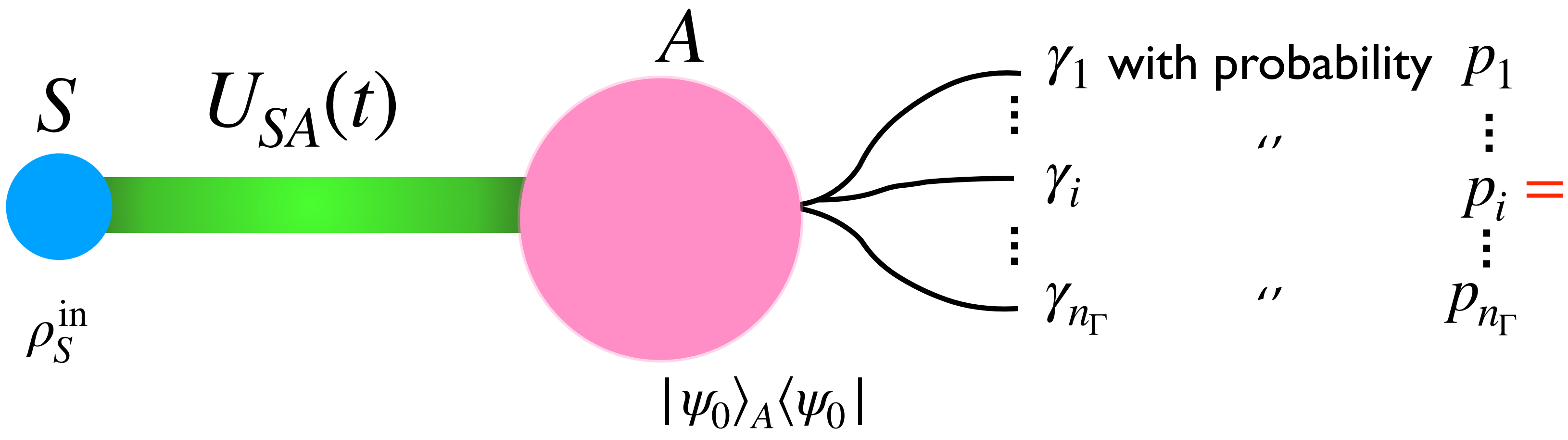
$${}_A\langle A^{\gamma_i}(t) | A^{\gamma_j}(t)\rangle_A = \delta_{i,j}$$
$${}_A\langle \psi_0 | A^{\gamma_i}(t)\rangle_A = 0$$
$$\forall t$$

$$\rho_{SA}(t^*) \simeq \sum_{i=1,j}^{n_\Gamma} M_S^{(\gamma_i)} \rho_S^{\mathrm{in}} M_S^{(\gamma_j)^\dagger} \otimes |A^{\gamma_i}(t^*)\rangle_A \langle A^{\gamma_j}(t^*)|$$





Positive Operator-Valued Measures (POVMs)



$$p_i = \mathrm{Tr}[M_S^{(\gamma_i)}\rho_S^{\rm in}M_S^{(\gamma_i)\dagger}]$$

$$\rho_S^{\rm in}\otimes|\psi_0\rangle_A\langle\psi_0| \xrightarrow[{}_A\langle A^{(\gamma_i)}(t)|A^{(\gamma_j)}(t)\rangle_A=\delta_{i,j}]{\textbf{CPT}} \rho_{SA}(t)\simeq\sum_{i=1,j}^{n_\Gamma} M_S^{(\gamma_i)}\rho_S^{\rm in}M_S^{(\gamma_j)\dagger}\otimes|A^{\gamma_i}(t)\rangle_A\langle A^{\gamma_j}(t)|$$

standard decoherence

$$\cancel{\textbf{CPT}}$$

$${}_A\langle\Xi^{(\gamma_i)}(t)|\Xi^{(\gamma_j)}(t)\rangle_A\neq\delta_{i,j}$$

$$\downarrow$$

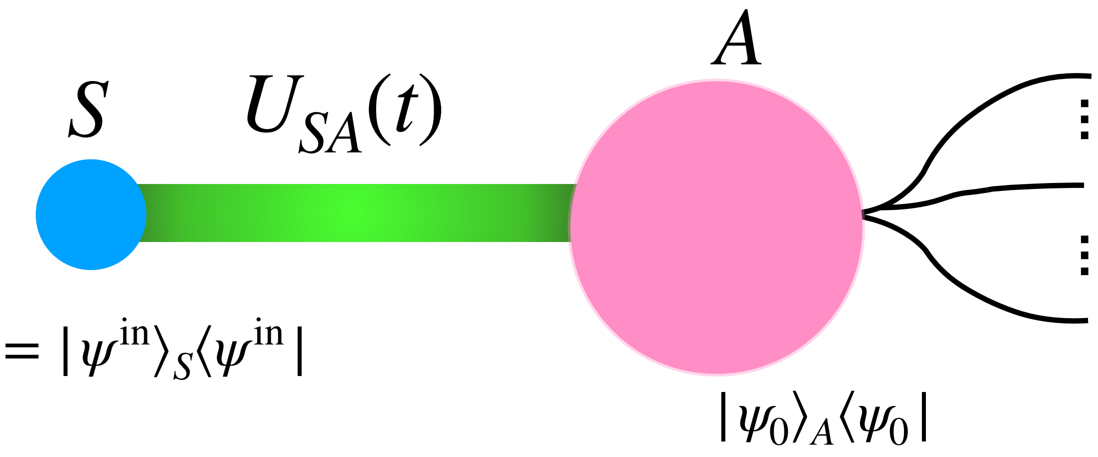
$$\rho_{S\Xi}(t)=\sum_{i,j}M_S^{(\gamma_j)}\rho_S^{\rm in}M_S^{(\gamma_i)\dagger}\otimes|\Xi^{(\gamma_j)}(t)\rangle_\Xi\langle\Xi^{(\gamma_i)}(t)|$$

$$\{E_S^{(\gamma_i)}=M_S^{(\gamma_i)\dagger}M_S^{(\gamma_i)}\geq 0\}$$

$$\sum_i E_S^{(\gamma_i)}=\mathbb{I}_S$$

arXiv:1902.03628v2

Positive Operator-Valued Measures (POVMs)



γ_1 with probability p_1

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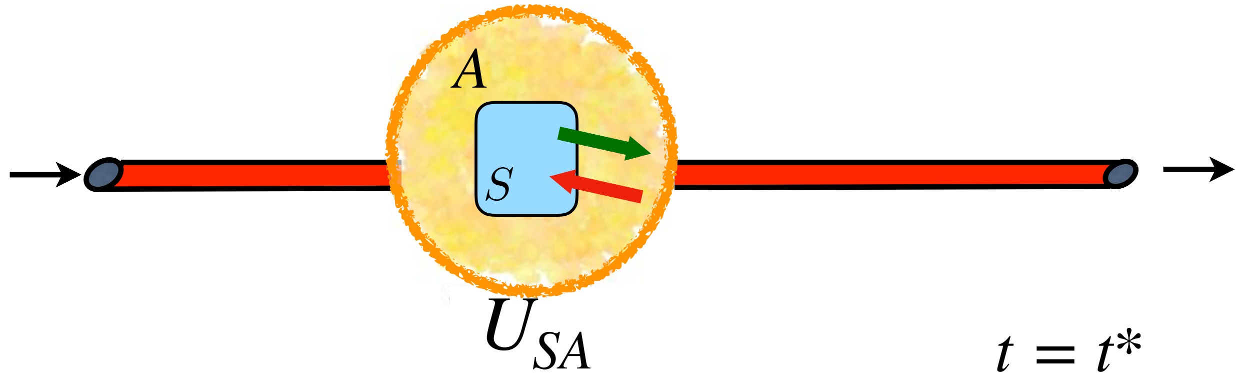
$\vdots$

γ_{n_Γ} ″ p_{n_Γ}

hp $\rho_S^{\mathrm{in}} = |\psi^{\mathrm{in}}\rangle_S\langle\psi^{\mathrm{in}}|$

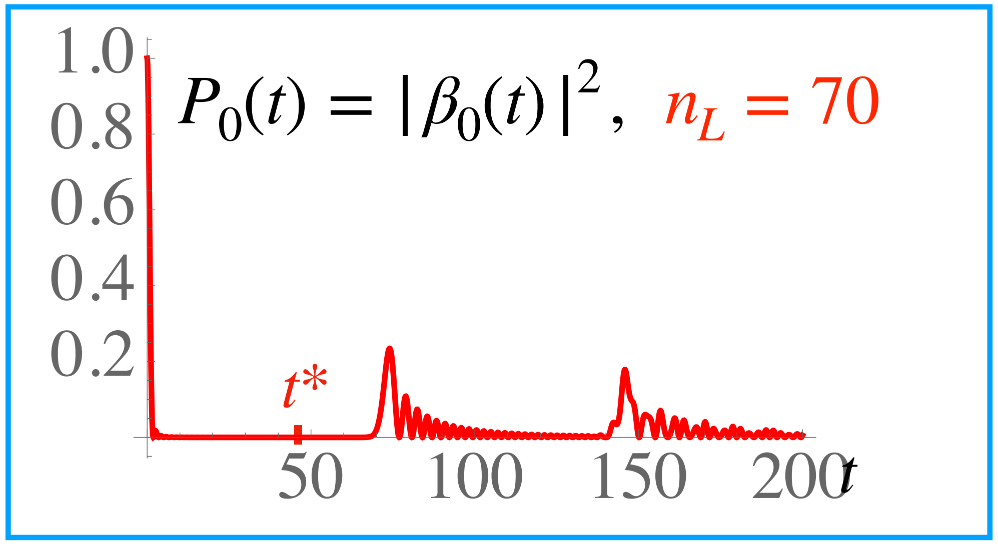
$\rho_S^{\mathrm{in}} \rightarrow$



$$\rightarrow \rho_S^{\mathrm{out}} = \sum_{i=1}^{n_\Gamma} M_S^{(\gamma_i)} \rho_S^{\mathrm{in}} M_S^{(\gamma_i)\dagger} = \Phi[\rho_S^{\mathrm{in}}]$$

$$\Phi[\rho_S^{\mathrm{in}}] = \mathrm{Tr}_A\left[U_{SA}\left(\rho_S^{\mathrm{in}} \otimes |\psi_0\rangle_A\langle\psi_0|\right) U_{SA}^\dagger\right]$$

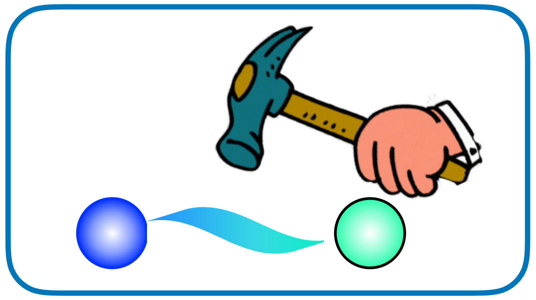
Stinespring representation of the quantum map



Conclusions

transmission of correlations

environment as NOISE



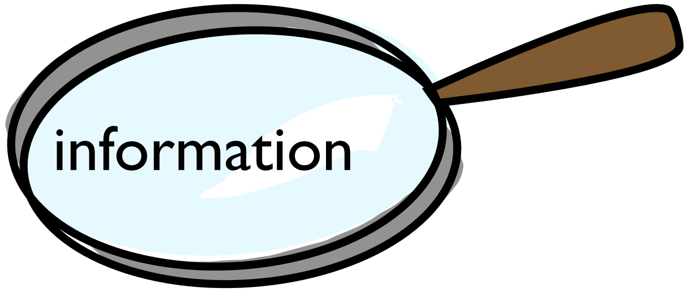
Phys. Rev. A **86**, 052302 (2012)
Phys. Rev. A **96**, 022322 (2017)
Phys. Rev. A **96**, 012314 (2017)
Phys. Rev. A **98**, 042301 (2018)
Phys. Rev. A **99**, 032307 (2019)

- we have introduced some protocols to **amend** the corrupting role of the environment on entanglement

quantum measurements

arXiv:1902.03628v2

environment ENCODES information



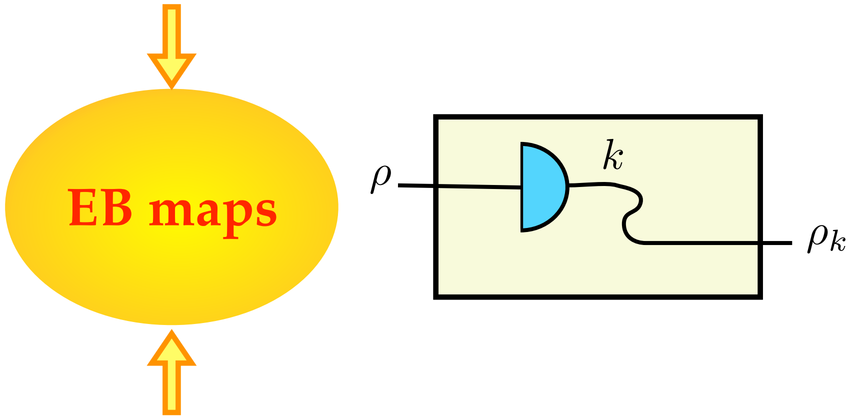
- in order to write a dynamical model for arbitrary **POVMs** we need to go **beyond** standard decoherence

Open questions

Paola Verrucchi
at the PROGRAM!

- optimal control techniques to limit/delay the destructive effects of environment on q. correlations?

- transmission of correlations



quantum measurements

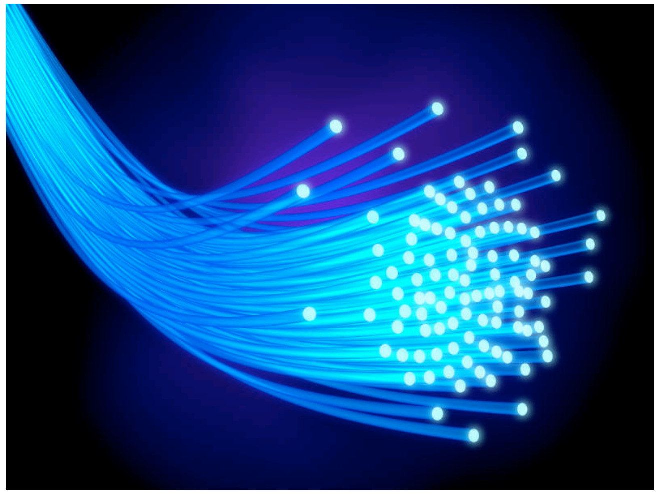
is it possible to determine a simple dynamical description for arbitrary EB maps?

- is it possible to translate the incompatibility of quantum observables at a dynamical level?

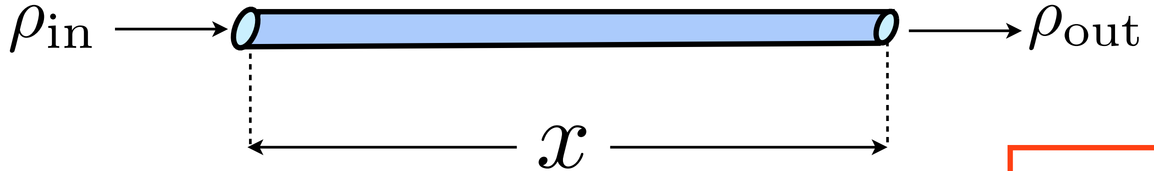
Thank you for your attention!

propagation of polarization qubits in optical fibers

Phys. Rev. A 96, 012314 (2017)



polarization qubits in optical fibers



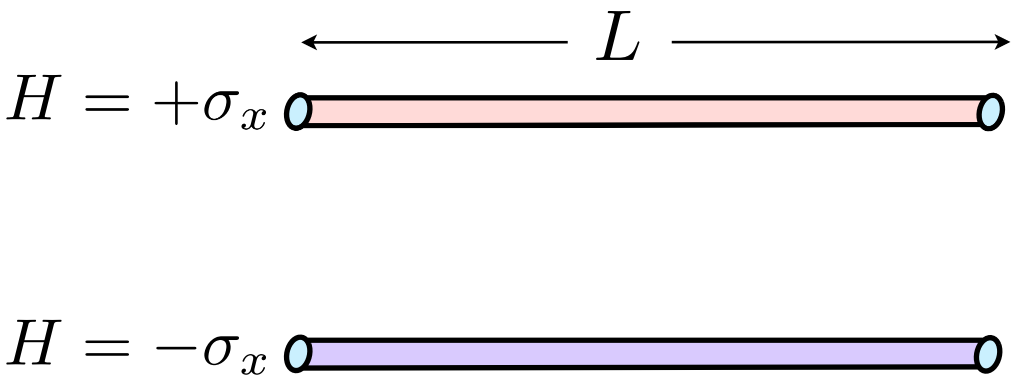
$$\rho_{\rm out} = \boxed{e^{\mathcal{L}x}}[\rho_{\rm in}]$$

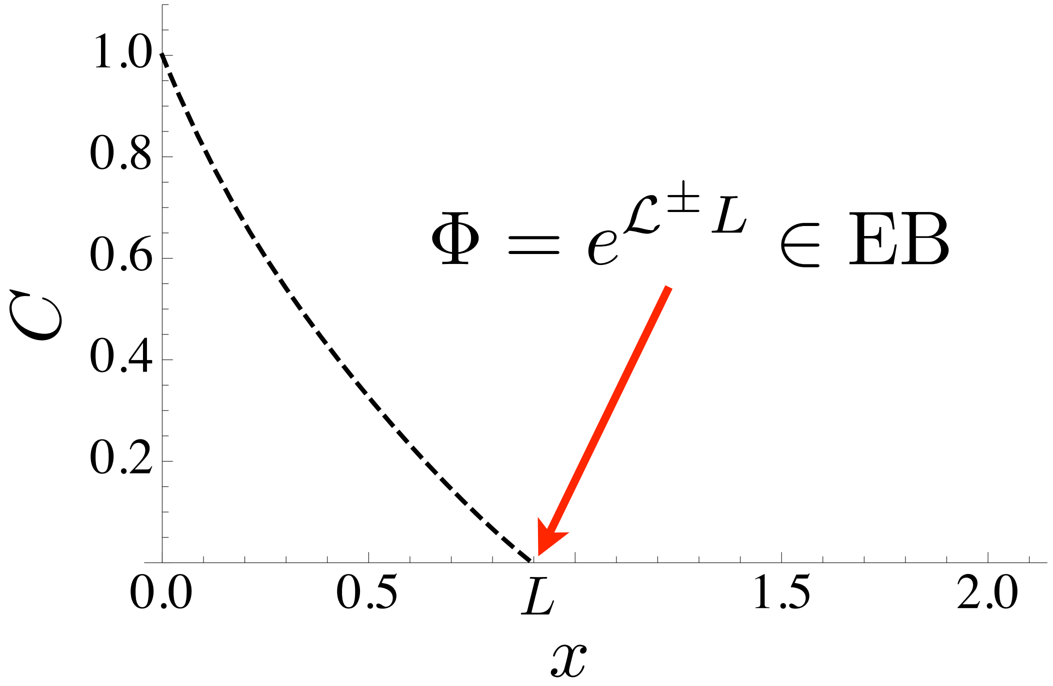
$$\updownarrow$$

$$\boxed{\Phi}$$

Markovian approx.

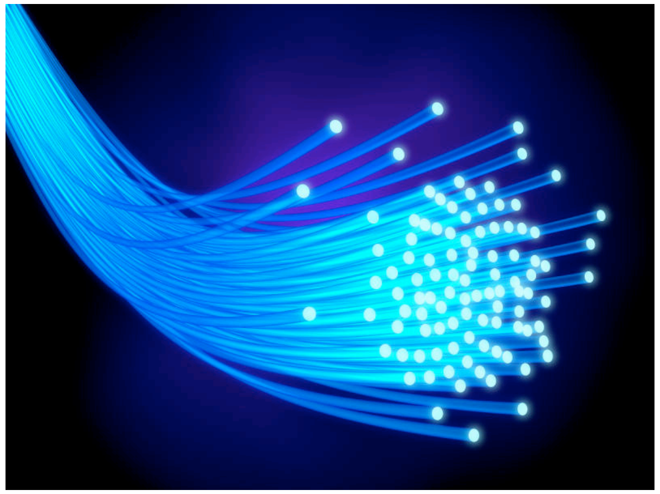
- $\dfrac{d\rho}{dx} = \mathcal{L}(\rho)$
- $\mathcal{L}(\rho) = -i[H,\rho] + \overbrace{\dfrac{\epsilon}{2}[\sigma_z,[\sigma_z,\rho]]}^{\text{dephasing noise}}$



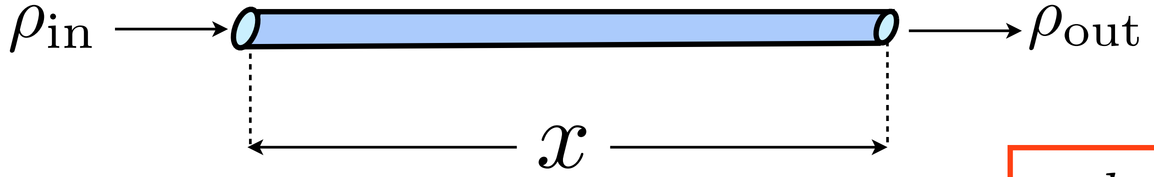


propagation of polarization qubits in optical fibers

Phys. Rev. A 96, 012314 (2017)



polarization qubits in optical fibers



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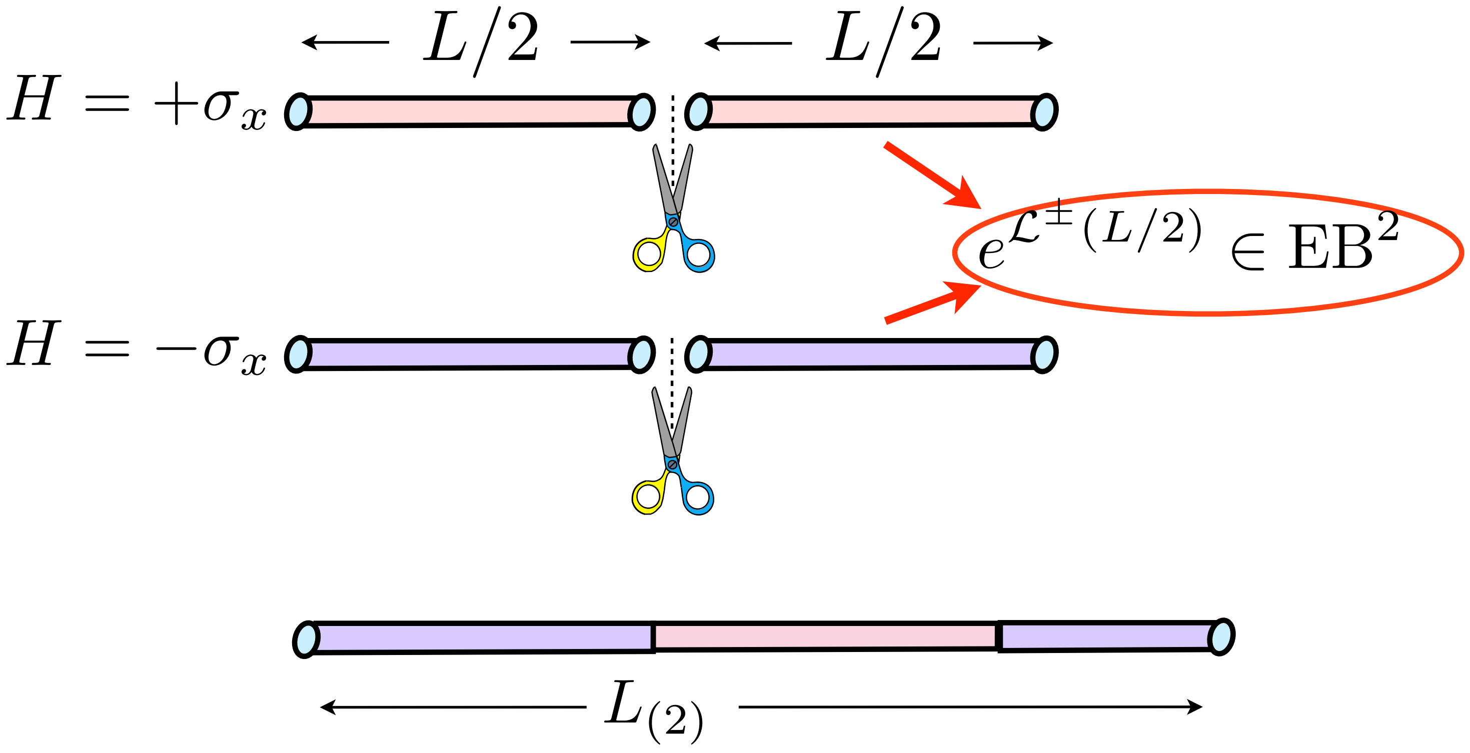
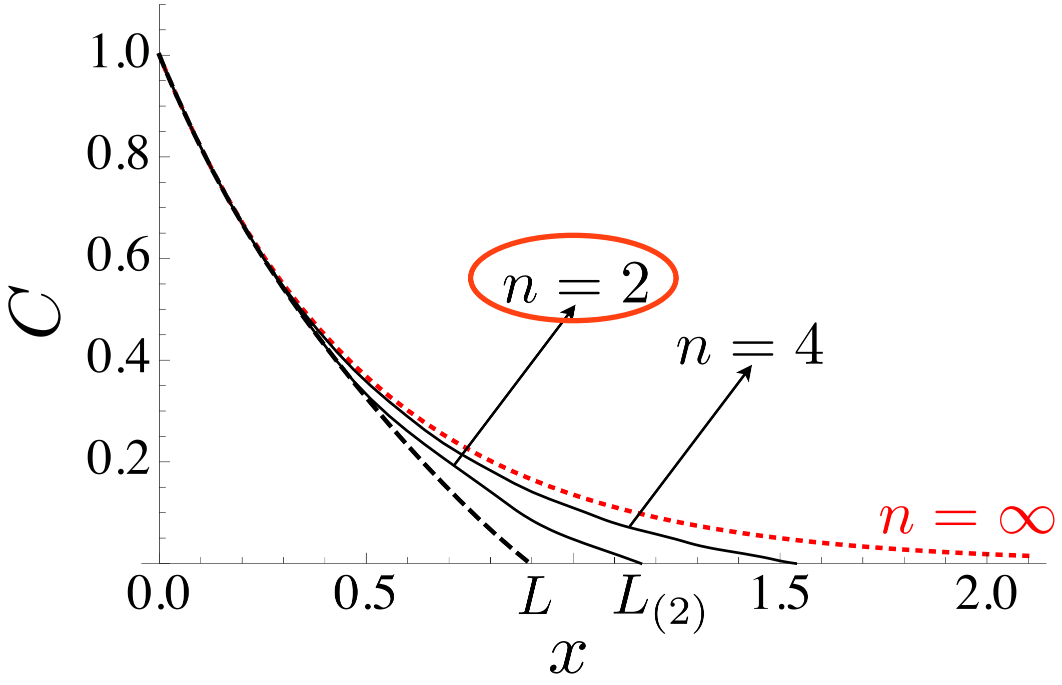
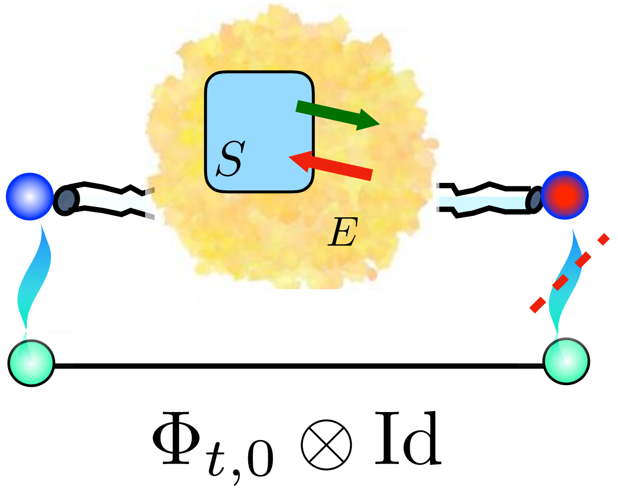




Continuous evolution of quantum systems



$\Phi_{t,0} \otimes \mathrm{Id}$

memory effects: non-Markovian regime

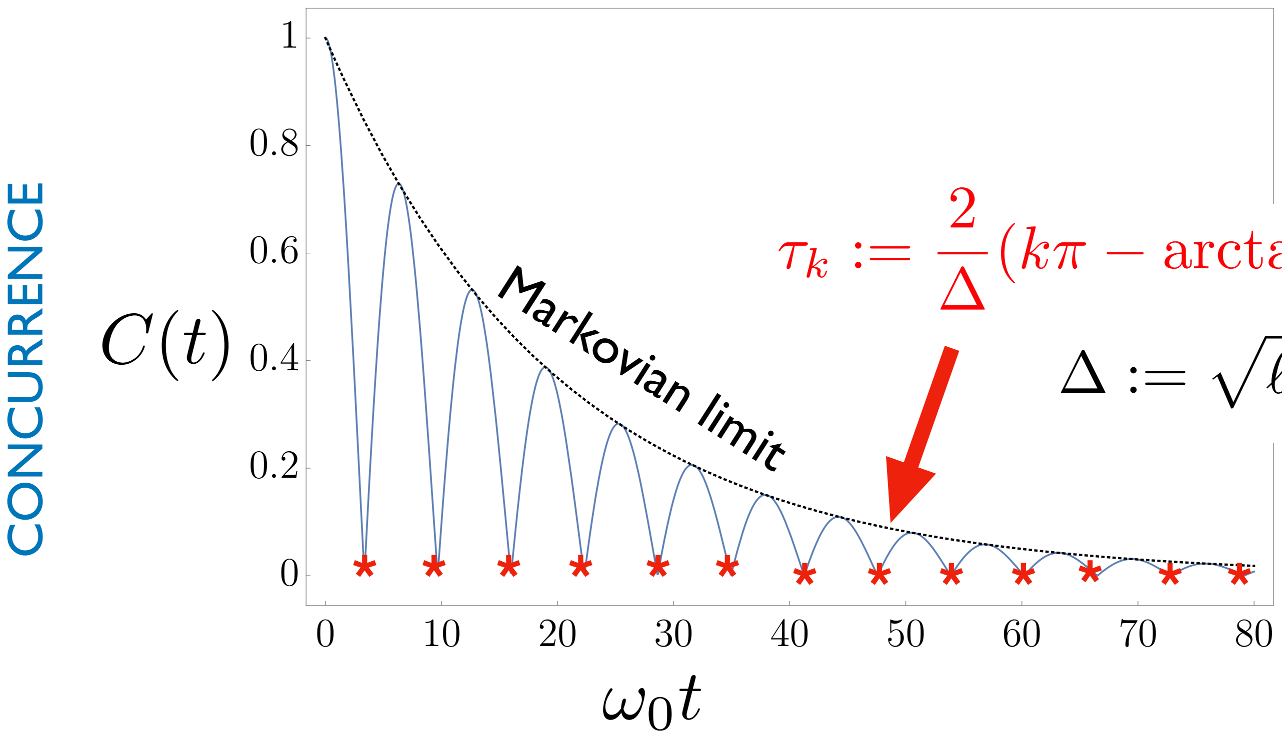
possible non-monotonic behavior of quantum correlations

A. Rivas, S. F. Huelga, and M. B. Plenio, Rep. Prog. Phys. 77, 094001 (2014).

L. Mazzola, S. Maniscalco, J. Piilo, *et al* Phys. Rev. A 79, 042302 (2009).

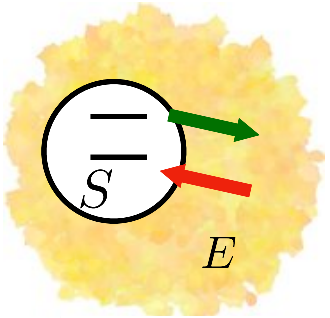
Time-local amplitude damping channels

$$\frac{d\varrho(t)}{dt} = \gamma(t)\left(\sigma_-\varrho(t)\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \varrho(t)\}\right)$$



$$\tau_k := \frac{2}{\Delta}(k\pi - \arctan(\Delta/\ell))$$

$$\Delta := \sqrt{\ell(2\alpha - \ell)}$$



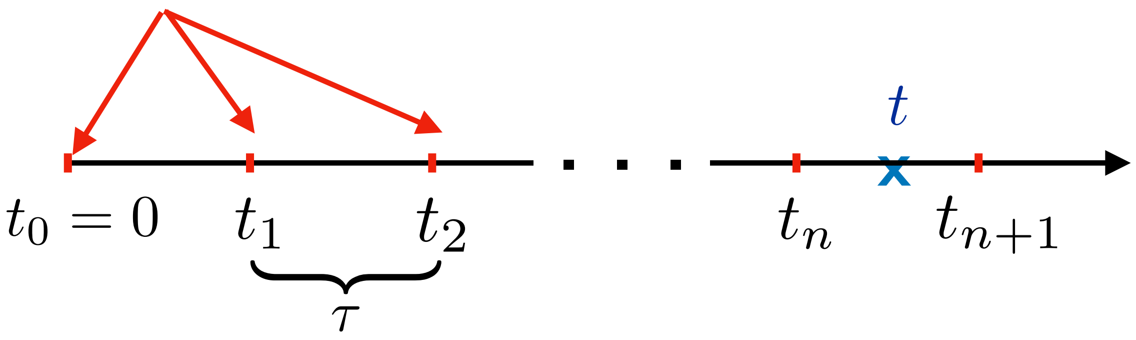
bosonic reservoir with Lorentian spectral density

$$J(\omega) = \frac{1}{2\pi}\frac{\alpha\ell^2}{(\omega_0 - \omega)^2 + \ell^2}$$

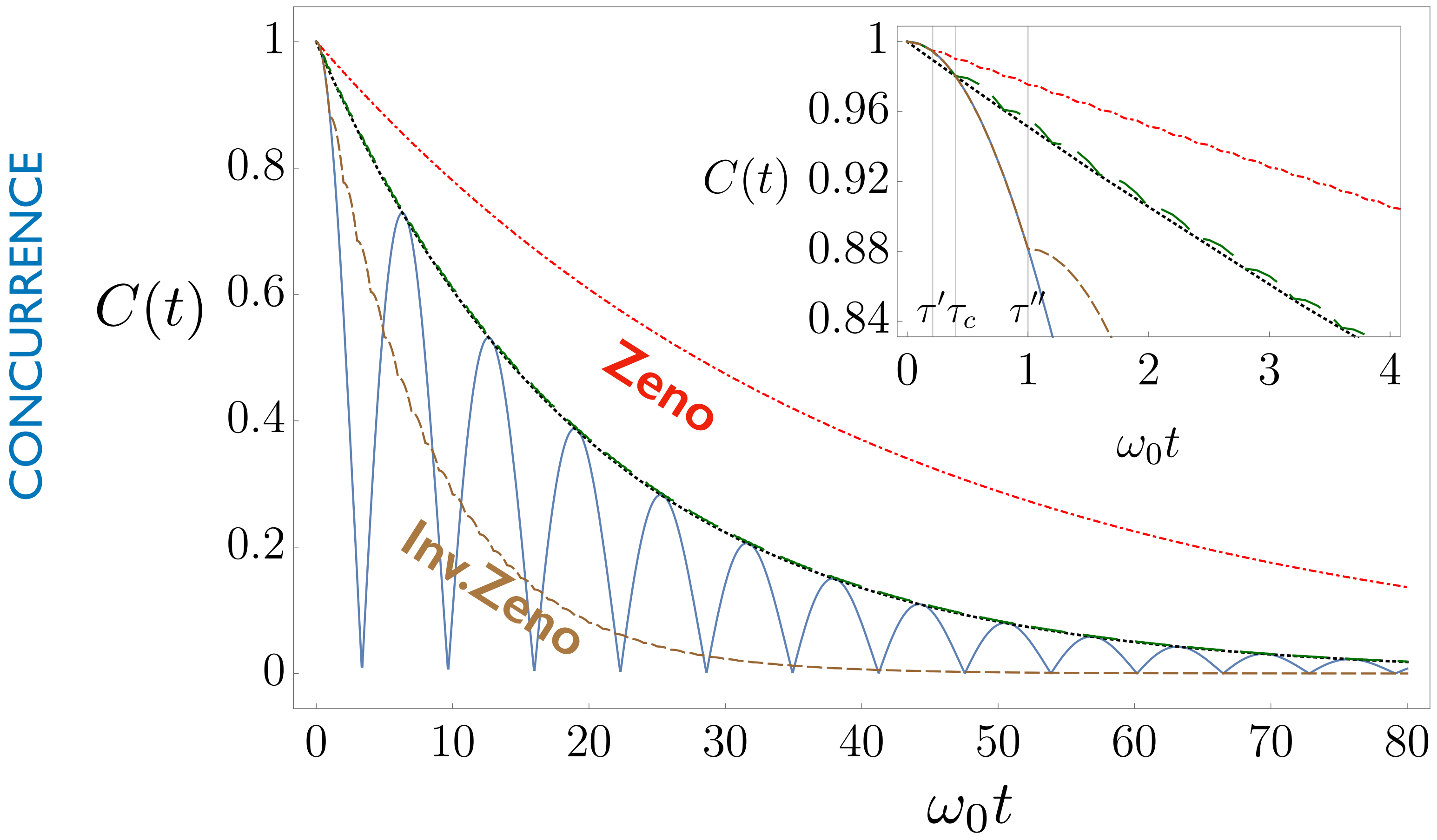
T. Bullock, F. Cosco, M. Haddara, S. H. Raja, O. Kerppo, L. Leppäjärvi, O. Siltanen, N. W. Talarico, A.D.P., S. Maniscalco and V. Giovannetti, Phys. Rev. A **98**, 042301 (2018)

Continuous evolution of quantum systems

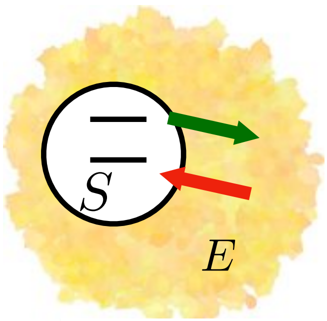
Assume to instantaneously reset the environment at the end of each interval τ at the input state it had at the beginning



$$\tilde{\Phi}^{(\tau)}_{0\to t} := \Phi_{0\to t-n\tau} \circ \overbrace{(\Phi_{0\to\tau}) \circ \cdots \circ (\Phi_{0\to\tau})}^{n \text{ times}}$$



$\tau_c = 1/\ell$



bosonic reservoir with Lorentian spectral density

$$J(\omega) = \frac{1}{2\pi} \frac{\alpha \ell^2}{(\omega_0 - \omega)^2 + \ell^2}$$

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