Quantum phase transitions and disorder: Rare regions, Griffiths effects, and smearing

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- Phase transitions and quantum phase transitions
- Quenched disorder and critical behavior: the common lore
- Rare regions and Griffiths effects
- Smeared phase transitions
- An attempt of a classification

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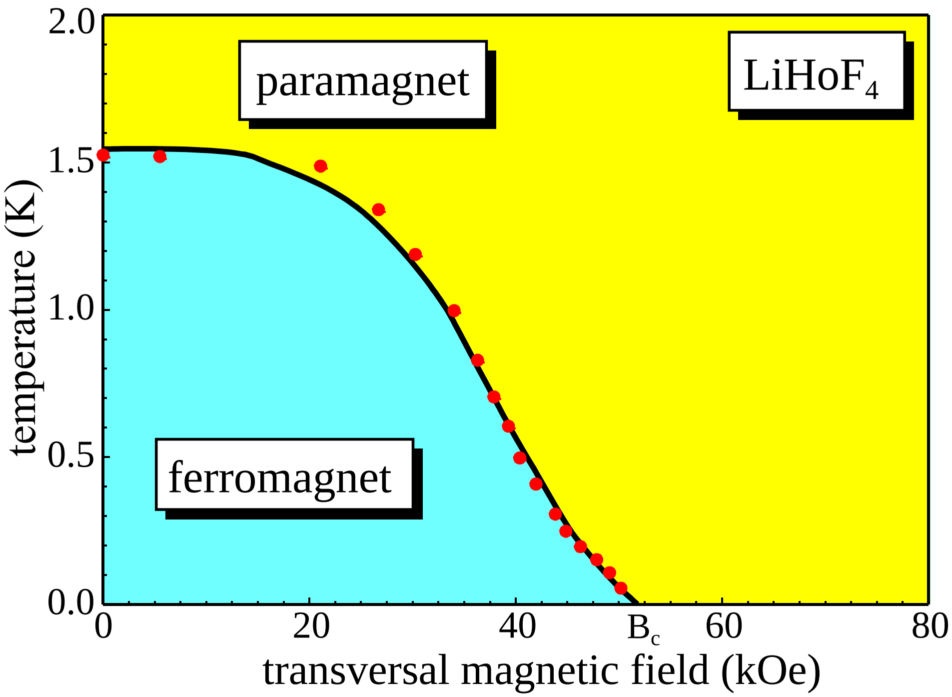
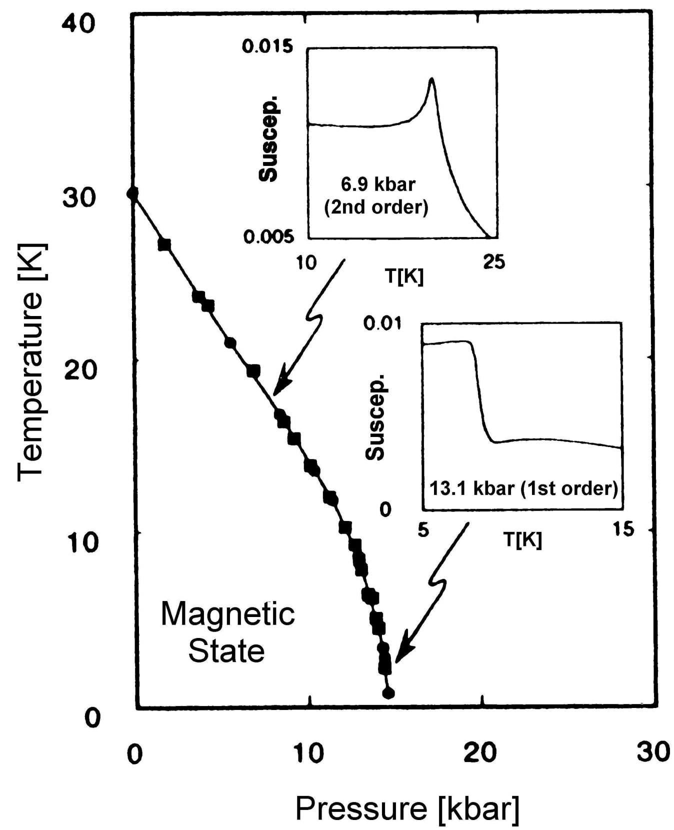
Classical and quantum phase transitions

classical phase transitions: at non-zero temperature, asymptotic critical behavior dominated by classical physics (thermal fluctuations)

quantum phase transitions: at zero temperature as function of pressure, magnetic field, chemical composition, ..., driven by quantum fluctuations





phase diagrams of $LiHoF_4$ (Bitko et al. 96) and MnSi (Pfleiderer et al. 97)

Imaginary time and quantum to classical mapping

Classical partition function: statics and dynamics decouple

$$Z = \int dpdq\ e^{-\beta H(p,q)} = \int dp\ e^{-\beta T(p)} \int dq\ e^{-\beta U(q)} \sim \int dq\ e^{-\beta U(q)}$$

Quantum partition function:

$$Z = \mathrm{Tr} e^{-\beta \hat{H}} = \lim_{N\to\infty} (e^{-\beta \hat{T}/N} e^{-\beta \hat{U}/N})^N = \int D[q(\tau)]\ e^{S[q(\tau)]}$$

imaginary time τ acts as additional dimension
at $T = 0$, the extension in this direction becomes infinite

Caveats:

- mapping holds for thermodynamics only
- resulting classical system can be unusual and anisotropic ($z \neq 1$)
- extra complications with no classical counterpart may arise, e.g., Berry phases

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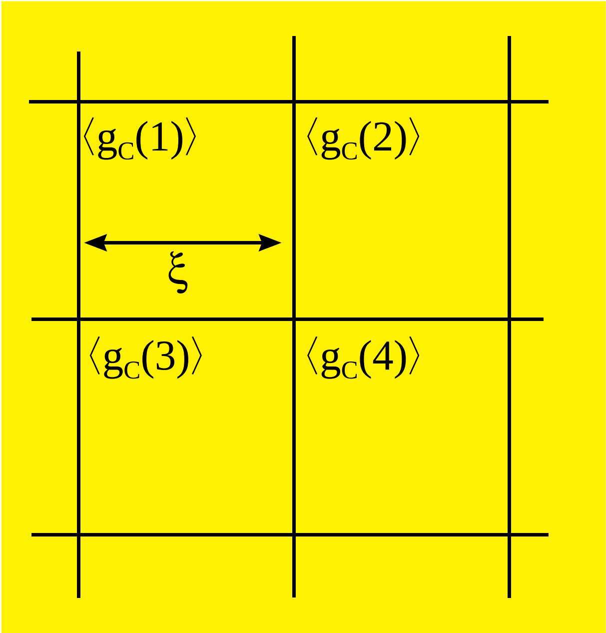
Weak disorder and Harris criterion

weak disorder: impurities lead to spatial variation of coupling strength $g(x)$

Theory: random mass disorder **Experiment:** "good disorder" ???

Harris criterion: variation of average local $g_c(x)$ in correlation volume must be smaller than distance from global g_c



variation of average g_c in volume ξ^d

$\Delta\langle g_c(x)\rangle \sim \xi^{-d/2}$

distance from global critical point $t \sim \xi^{-1/\nu}$

$\Delta\langle g_c(x)\rangle < t \quad \Rightarrow \quad d\nu > 2$

- if clean critical point fulfills Harris criterion $\Rightarrow$ stable against disorder
- inhomogeneities vanish at large length scales
- macroscopic observables are self-averaging
- example: 3D classical Heisenberg magnet: $\nu = 0.698$

Finite-disorder critical points

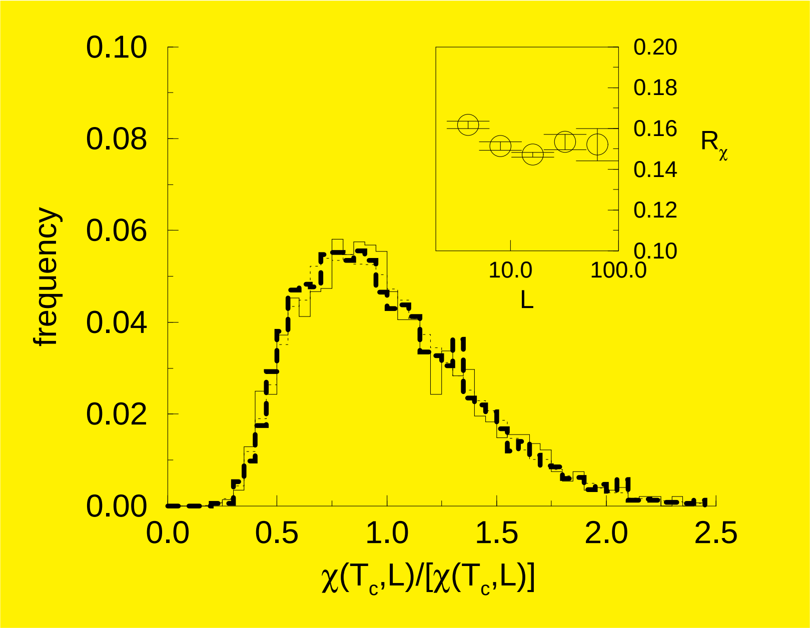
if critical point violates Harris criterion ⇒ unstable against disorder

Common lore:

- system goes to new different critical point which fulfills $d\nu > 2$
- inhomogeneities remain finite at all length scales ("finite disorder")
- macroscopic observables are not self-averaging
- example: 3D classical Ising magnet: clean $\nu = 0.627 \quad \Rightarrow \quad$ dirty $\nu = 0.684$

Distribution of critical susceptibilities of 3D dilute Ising model

(Wiseman + Domany 98)

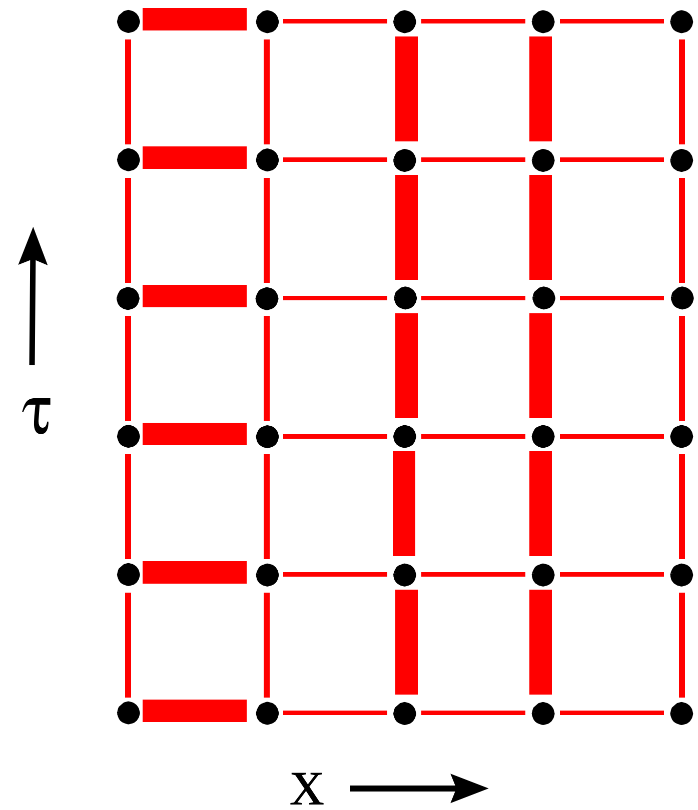


Disorder and quantum phase transitions

Disorder is quenched:

- impurities are time-independent
- disorder is perfectly correlated in imaginary time direction

⇒ correlations increase the effects of disorder ("it is harder to average out fluctuations")



Disorder generically has stronger effects on quantum phase transitions than on classical transitions

Random quantum Ising model

$$H = -\sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^x$$

nearest neighbor interactions J_{ij} and transverse fields h_i both random

Exact solution in 1+1 dimensions:

Ma-Dasgupta-Hu-Fisher real space renormalization group

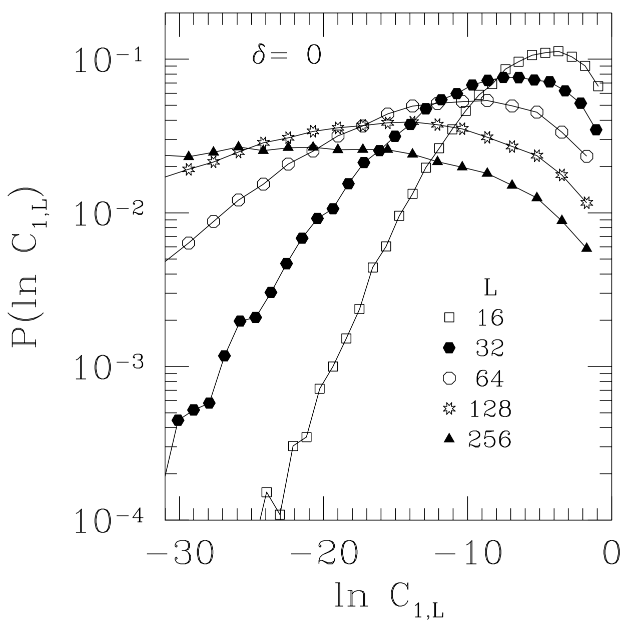
- in each step, integrate out largest energy among all J_{ij} and h_i
- cluster aggregation/annihilation procedure
- becomes exact in the limit of large disorder

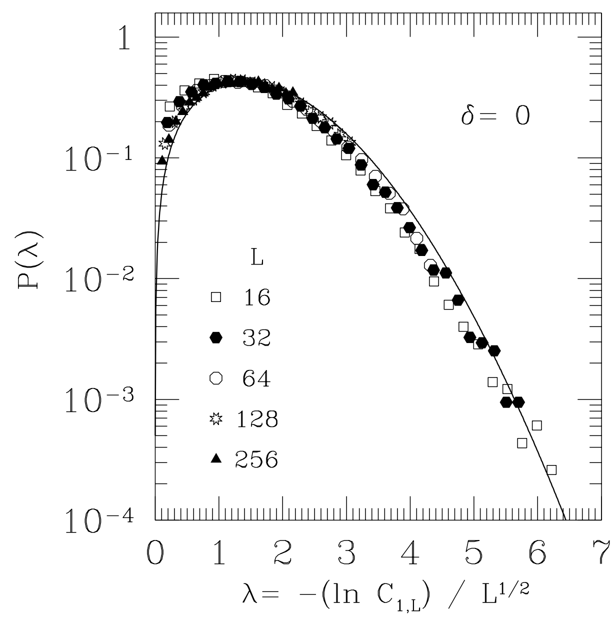
Infinite-disorder critical point:

- under renormalization the disorder increases without limit
- relative width of the distributions of J_{ij}, h_i diverges

Infinite-disorder critical point

- extremely slow dynamics $\log \xi_\tau \sim \xi^\mu$ (activated scaling)
- distributions of macroscopic observables become infinitely broad
- average and typical values can be drastically different
  correlations: $-\log G_{typ} \sim r^\psi \qquad G_{av} \sim r^{-\eta}$
- averages are dominated by rare events





Probability distribution of the end-to-end correlations in a random quantum Ising chain (Fisher + Young 98)

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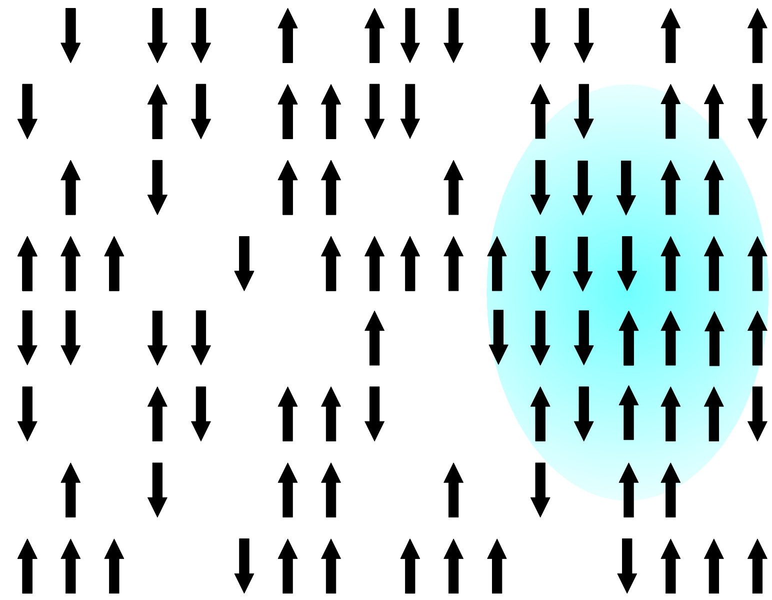
Griffiths effects in a classical dilute ferromagnet

- critical temperature T_c is reduced compared to clean value T_{c0}
- for $T_c < T < T_{c0}$:
  no global order but local order on rare regions devoid of impurities
- probability: $w(L) \sim e^{-cL^d}$:



rare regions have slow dynamics
$\Rightarrow$ singular free energy everywhere in the Griffiths region $(T_c < T < T_{c0})$

Classical Griffiths effects are generically weak and essentially unobservable

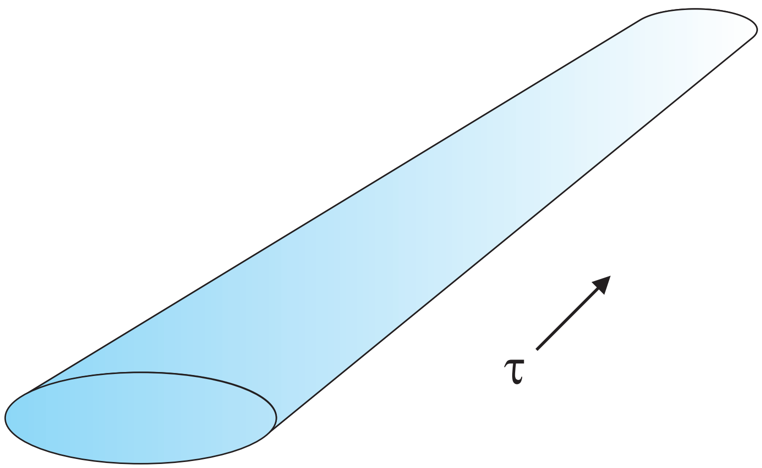
contribution to susceptibility: $\chi_{RR} \sim \int dL\ e^{-cL^d} L^{\gamma/\nu} =$ finite

Quantum Griffiths effects

rare regions at a QPT are finite in space but infinite in imaginary time

fluctuations of the rare regions are even slower than in classical case $\Rightarrow$

Griffiths singularities are enhanced



rare region at a quantum phase transition

Random quantum Ising systems

local susceptibility (inverse energy gap) of rare region: $\chi_{loc} \sim \Delta^{-1} \sim e^{aL^d}$

$\chi_{RR} \sim \int dL\, e^{-cL^d} e^{aL^d}$ can diverge inside Griffiths region

finite temperatures:

$\chi_{RR} \sim T^{d/z'-1}$ (z' is continuously varying exponent)

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Rare regions at quantum phase transitions with overdamped dynamics

itinerant Ising quantum antiferromagnet

magnetic fluctuations are damped due to coupling to electrons

$$\Gamma(\mathbf{q}, \omega_n) = t + \mathbf{q}^2 + |\omega_n|$$

in imaginary time: long-range power-law interaction $\sim 1/(\tau - \tau')^2$

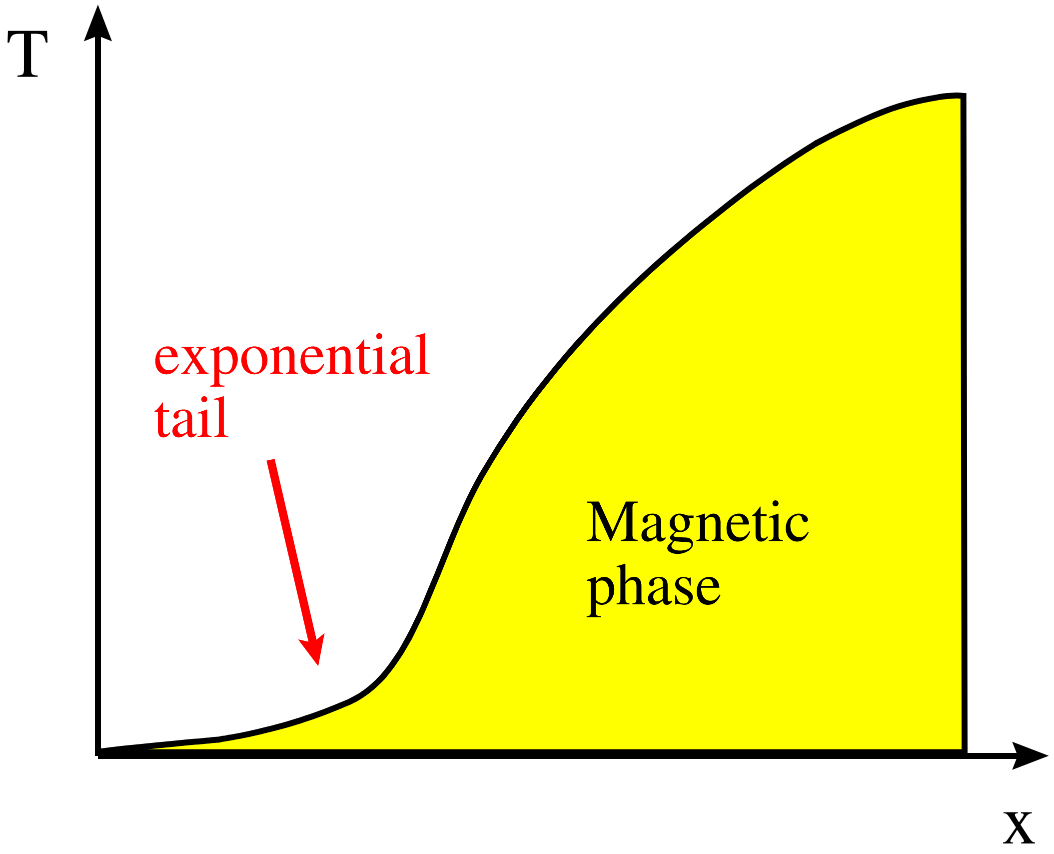
1D Ising model with $1/r^2$ interaction is known to have an ordered phase

$\Rightarrow$ isolated rare region can develop a static magnetization, i.e., **large islands do not tunnel** (c.f. Millis, Morr, Schmalian and Castro-Neto)

$\Rightarrow$ conventional quantum Griffiths behavior does **not** exist
magnetization develops **gradually** on **independent** rare regions

quantum phase transition is smeared by disorder

Phase diagram at a smeared transition



possible realization: ferromagnetic quantum phase transition in Ni_xPd_{1-x}

Universality of the smearing scenario

Condition for disorder-induced smearing:

isolated rare region can develop a static order parameter
$\Rightarrow$ rare region has to be above lower critical dimension

Examples:

- quantum phase transitions of itinerant electrons
  (disorder correlations in imaginary time + long-range interaction $1/\tau^2$)
- classical Ising magnets with planar defects
  (disorder correlations in 2 dimensions)
- classical non-equilibrium phase transitions in the directed percolation universality class with extended defects
  (disorder correlations in at least one dimension)

Disorder-induced smearing of a phase transition is a ubiquitous phenomenon

Isolated islands – Lifshitz tail arguments

probability to find rare region of size L devoid of defects: $w \sim e^{-cL^d}$

region has transition at distance $t_c(L) < 0$ from the clean critical point
finite size scaling: $|t_c(L)| \sim L^{-\phi}$ $\quad$ ($\phi =$ clean shift exponent)

Consequently:
probability to find a region which becomes critical at t_c:

$$w(t_c) \sim \exp(-B\ |t_c|^{-d/\phi})$$

total magnetization at coupling t is given by the sum over all rare regions having $t_c > t$:

$$m(t) \sim \exp(-B\ |t|^{-d/\phi}) \qquad (t \to 0-)$$

Computer simulation of a model system

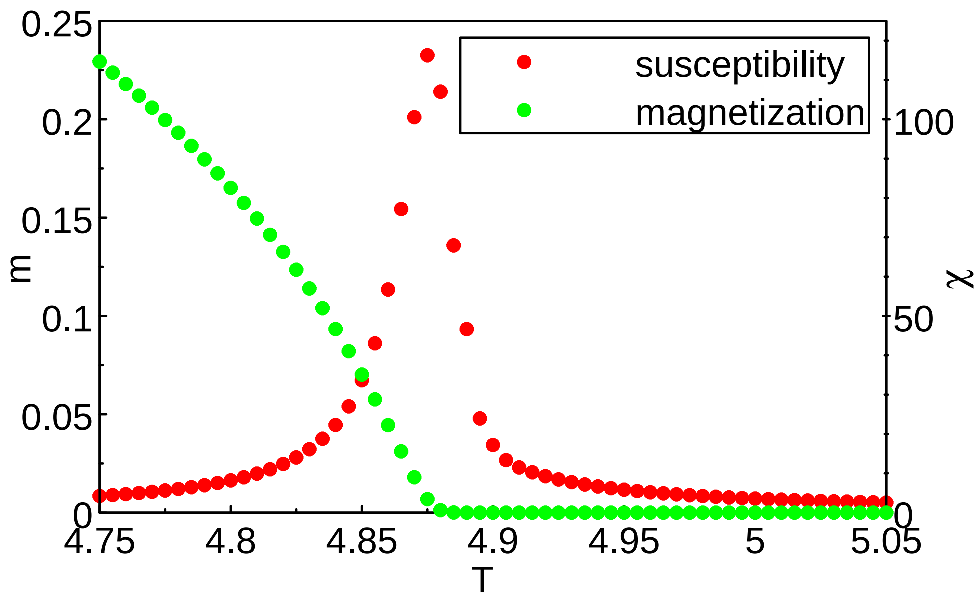
Classical Ising model in 2+1 dimensions

$$H = -\frac{1}{L_\tau} \sum_{\langle \mathbf{x},\mathbf{y}\rangle,\tau,\tau'} S_{\mathbf{x},\tau} S_{\mathbf{y},\tau'} - \frac{1}{L_\tau} \sum_{\mathbf{x},\tau,\tau'} J_{\mathbf{x}} S_{\mathbf{x},\tau} S_{\mathbf{x},\tau'}$$

$J_{\mathbf{x}}$: binary random variable, $P(J) = (1-c)\,\delta(J-1) + c\,\delta(J)$
totally correlated in the time-like direction

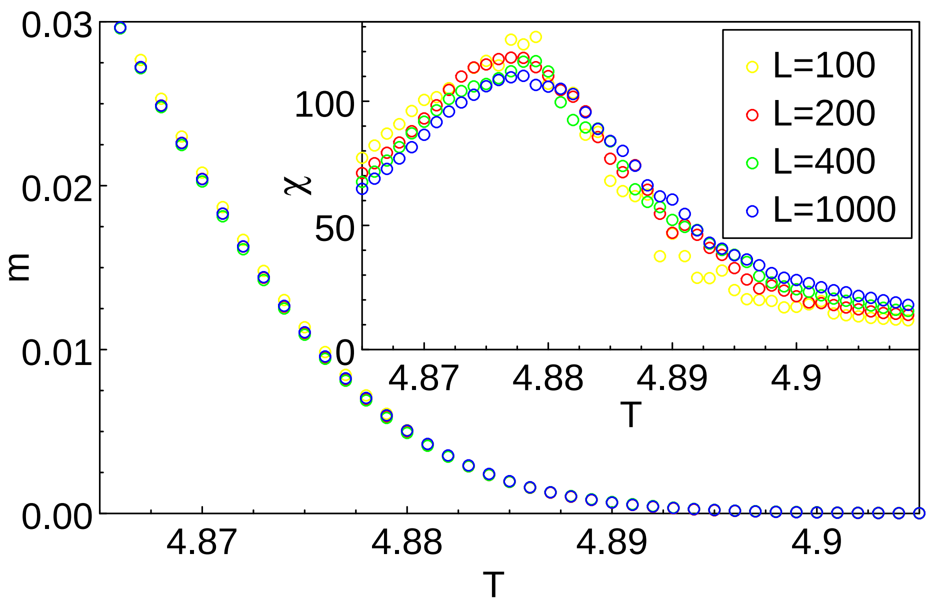
- short-range interactions in the two space-like directions
- infinite-range interaction in the time-like direction
  (static magnetization on the rare regions is retained, but time direction can be treated exactly, permitting large sizes)

Smeared transition in the infinite-range model



Magnetization + susceptibility of the infinite-range model

seeming transition close to $T = 4.88$

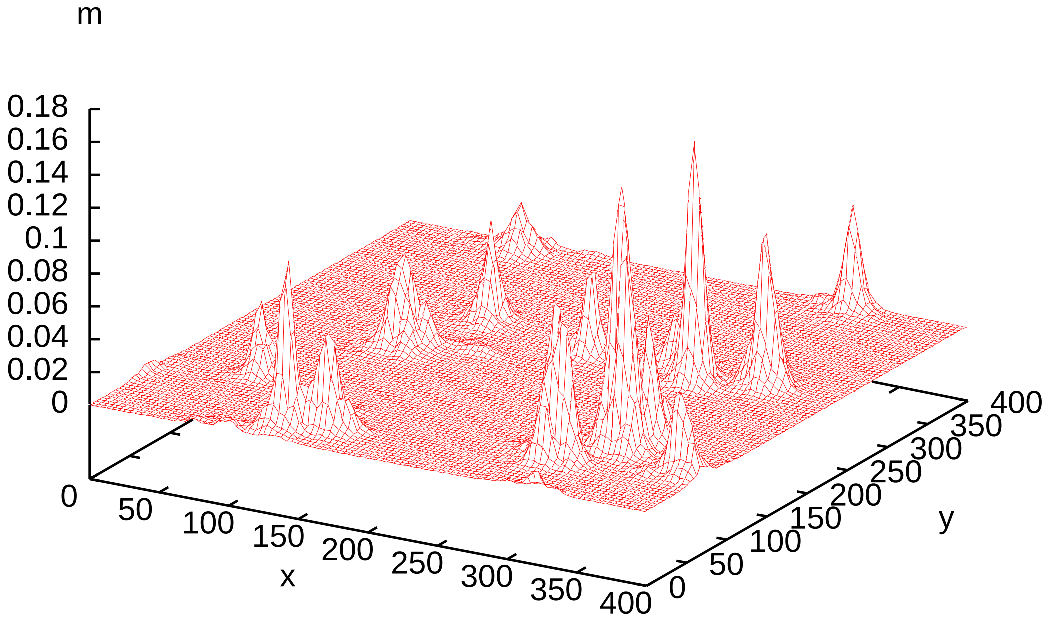


phase transition is smeared

(m and χ are independent of L)

Lifshitz magnetization tail towards disordered phase $\log(m) \sim -1/(T_{c0} - T)$

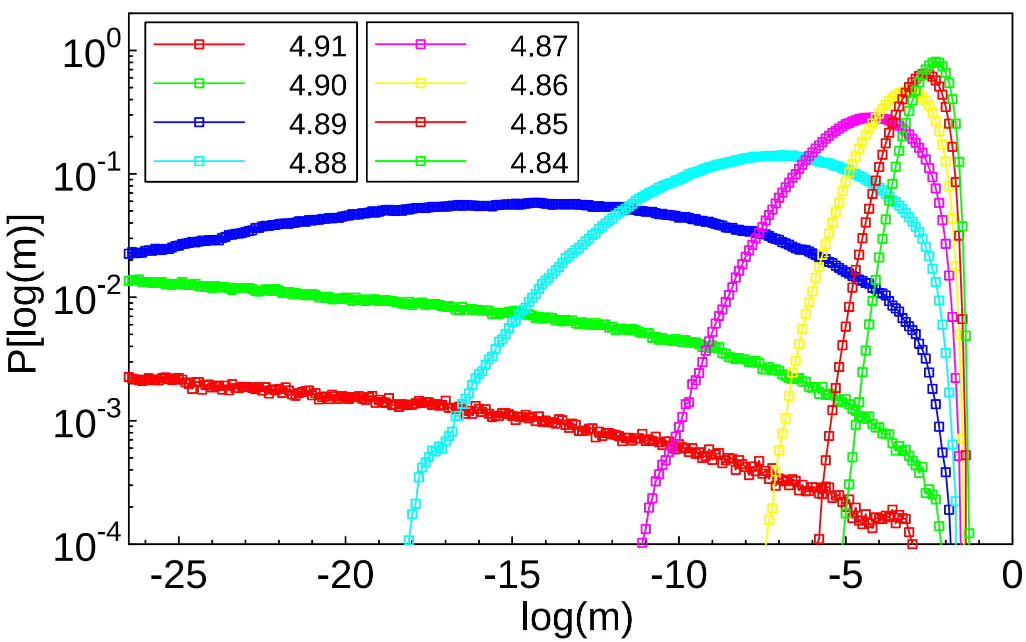
Local magnetization distribution



Local magnetization in the tail region ($T = 4.8875$)

global magnetization starts to form on isolated islands

very inhomogeneous system



Distribution of the local magnetization values

very broad, even on logarithmic scale

$\ln(m_{typ}) \sim \langle m \rangle^{-1/2}$

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Classification of dirty phase transitions according to importance of rare regions

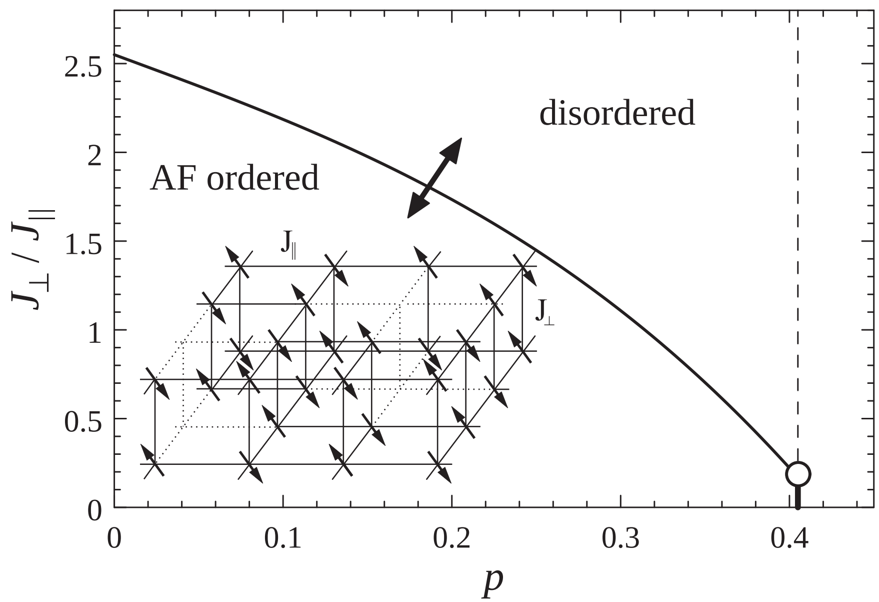
Dimensionality of rare regions	Griffiths effects	Dirty critical point	Examples (classical PT, QPT)
$d_{RR} < d_c^-$	weak exponential	conv. finite disorder	class. magnet with point defects dilute bilayer Heisenberg model
$d_{RR} = d_c^-$	strong power-law	infinite randomness	Ising model with linear defects random quantum Ising model itin. quantum Heisenberg magnet?
$d_{RR} > d_c^-$	RR become static	smeared transition	Ising model with planar defects itinerant quantum Ising magnet

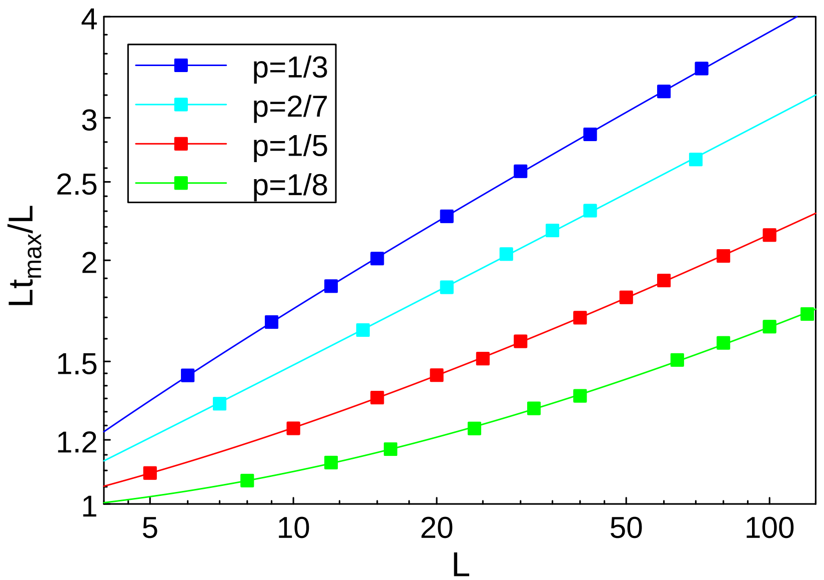
Dimer-diluted 2d Heisenberg quantum antiferromagnet

$$H = J_{\|} \sum_{\substack{\langle i,j \rangle \\ a=1,2}} \epsilon_i \epsilon_j \hat{\mathbf{S}}_{i,a} \cdot \hat{\mathbf{S}}_{j,a} + J_{\perp} \sum_i \epsilon_i \hat{\mathbf{S}}_{i,1} \cdot \hat{\mathbf{S}}_{i,2},$$

Large scale Monte-Carlo simulations:

conventional finite-disorder critical point with power-law scaling
critical exponents are universal, dynamical exponent $z = 1.31$
(after accounting for corrections to scaling)





Conclusions

- even weak disorder can have surprisingly strong effects on a quantum phase transition
- **rare regions** play a much bigger role quantum phase transitions than a classical transitions
- **effective dimensionality** of rare regions determines **overall phenomenology** of phase transitions in disordered systems
- Ising systems with overdamped dynamics: sharp phase transition is destroyed by **smearing** because static order forms on rare spatial regions
- at a smeared transition, system is extremely **inhomogeneous**, even on a logarithmic scale

Griffiths effects at quantum phase transitions leads to a rich variety of new and exotic phenomena