Singular Corrections to the Fermi-liquid Behavior: 1D Physics in Higher Dimensions

Dmitrii L. Maslov
University of Florida

Suhas Gangadharaih — *U of Florida*

Andrey V. Chubukov — *U of Maryland*

Leonid I. Glazman — *U of Minnesota*

Andy Millis — *Columbia*

KITP, 20 January. 2004

- Main subleading corrections to the Fermi liquid behavior are singular (non-analytic) functions of temperature, spatial scale

  Finite |Q|-singularity in the spin susceptibility changes the nature of the FM phase transition (→to integrate out or not …)

- Singularities result from 1D scattering processes embedded into D>1 space: link between D=1 and D>1.
  Precursors of the 1D—Luttinger-liquid-- behavior in D>1

- Naïve perturbation theory breaks down in 2D
  Re-summed perturbation theory:
  Fermi liquid survives but
  Non-Fermi-liquid features remain
  (non-Lorentzian spectral function)

Lowest energies:

Fermi liquid=Fermi gas with renormalized parameters

Fermi gas

$$C/T=\gamma$$
$$\chi_s(T=0,Q=0)=\chi^0{}_s$$

Fermi liquid

$$C/T=\gamma^*$$
$$\chi_s(T=0,Q=0)=\chi^*{}_s$$

What about not so low energies?
(next-order term in T/E_F)

$$C/T=\gamma\left(1-T^2/E^2{}_F\right)$$
$$\chi_s(T,Q)=\chi^0{}_s\left(1-\max\{T^2,Q^2\}/E^2{}_F\right)$$

$$C=\gamma^*T+???$$
$$\chi_s=\chi^*{}_s+???$$

Specific heat and spin susceptibility are *singular* beyond the leading order

3D

$$C(T)/T=\gamma-\Gamma T^2\ln\left(E_F/T\right)$$
$$\chi_s(Q)=\chi_0+\beta Q^2\ln\left(k_F/|Q|\right)$$

Eliashberg 63,
Doniach & Englesberg 66
Amit, Kane, Wagner 68 …

Belitz, Kirkpatrick, Vojta 97

2D

$$C(T)/T=\gamma-\Gamma T$$
$$\chi_s(Q)=\chi_0+\beta|Q|$$

Coffey & Bedell 93, Chubukov & DLM 03,
Das Sarma et al. 03

Belitz, Kirpatrick, Vojta '97
Chitov & Millis 01
Chubukov & DLM 03

1D

$$C(T)/T\propto\ln T$$
$$\chi_s(Q)\propto\ln|Q|$$

Dzyaloshinskii & Larkin 72
Japaridze & Nersesyan 83

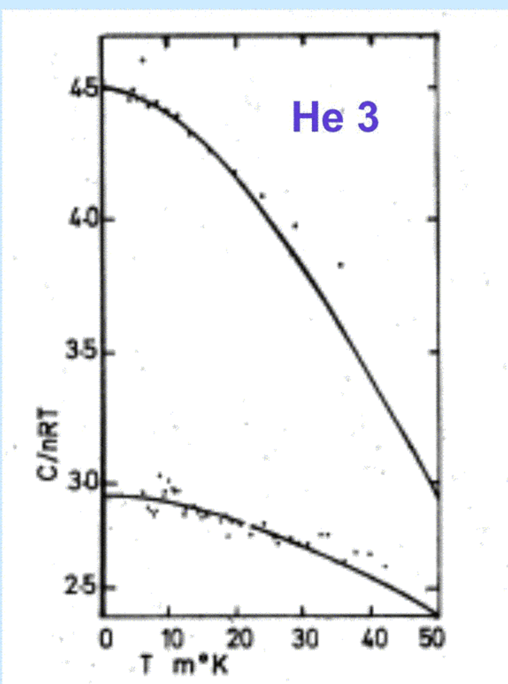
Non-analytic term near the FM quantum critical point

Chubukov, Pepin, Reich 04

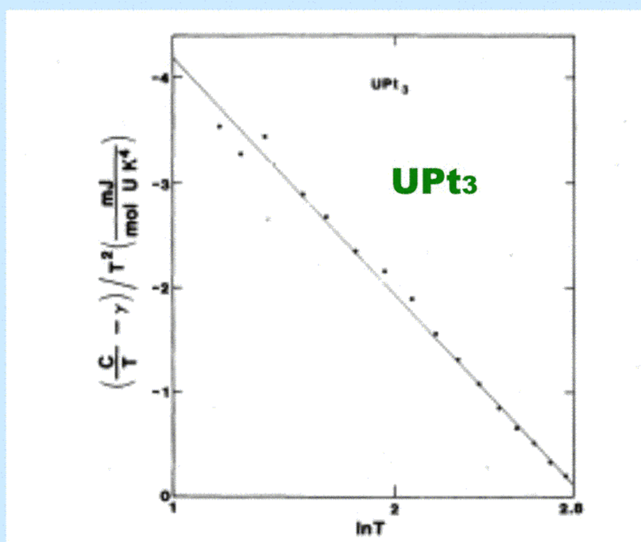
Spin- Fermion model, 2D

$$\chi_s(q) = \frac{\chi_0}{q^2 - 0.17|q|^{3/2} p_F^{1/2}}.$$

3D: $C(T)/T = \gamma - \alpha T^2 \ln(W/T)$



Abel et al. 66



Stewart 84

"Paramagnon anomaly"

2D $C/T = \gamma - \Gamma_{2D} T$

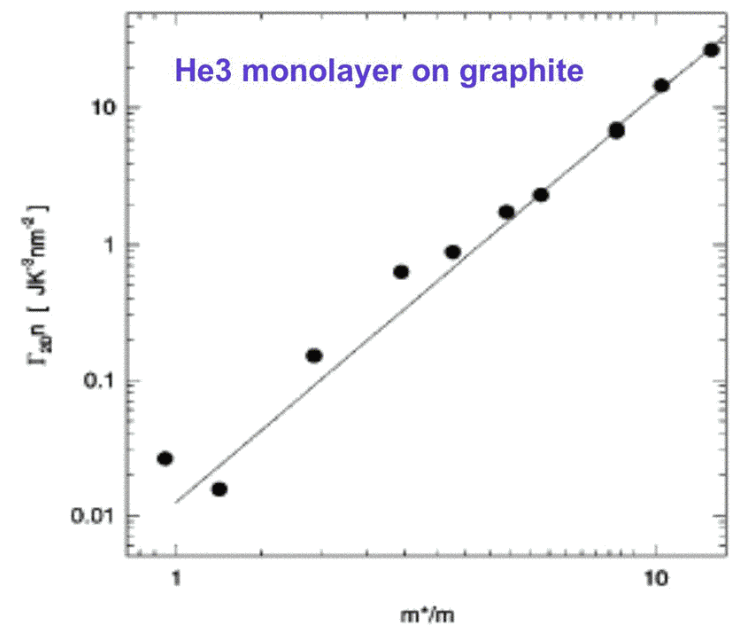
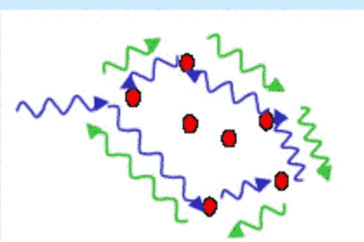


FIG. 4. $\Gamma_{2D} n$ vs m^*/m, where Γ_{2D} is a coefficient of the T^2 term and n is the number of ^{3}He atoms per unit area.

Casey et al. 03

Singularities in transport properties of disordered metals

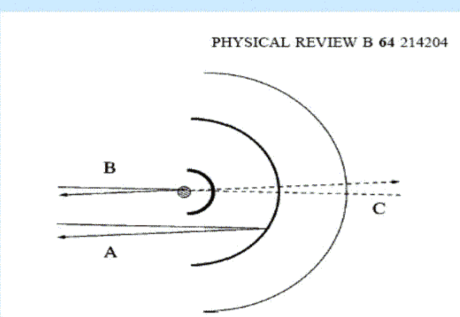
Weak localization



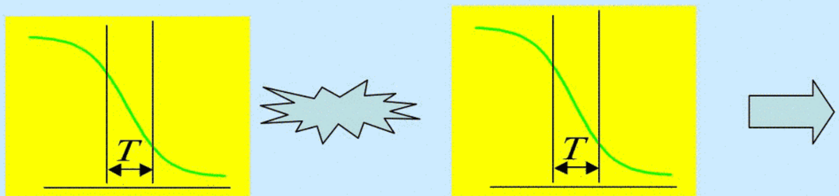
$$\delta\sigma_{\mathrm{WL}} \propto |\omega|^{1-D/2}$$

Gor'kov, Larkin, Khmel'nitskii 79

Altshuler-Aronov effect:
interaction corrections to tunneling density of states and conductivity



Altshuler & Aronov 79
…
Matveev, Glazman, Yue 94
Aleiner, Glazman, Ruzin 97
Zala, Narozhny, Aleiner 01



Fermi Liquid: $\mathrm{Im}\Sigma \propto (\pi T)^2 + \omega^2$

NOT an expansion in ω^2

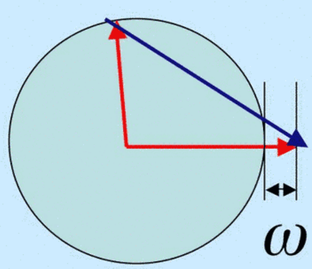
$$3D: \mathrm{Im}\Sigma(\omega) \propto \omega^2 + \underline{|\omega|^3} + \ldots$$

$$2D: \mathrm{Im}\Sigma(\omega) \propto \omega^2 + \underline{\omega^2 \ln|\omega|}$$

$$1D: \mathrm{Im}\Sigma(\omega) \propto \omega^2 + \underline{|\omega|}$$

Analytic, "Fermi liquid" (ω^2) and singular terms come from *different* processes

$\mathrm{Im}\Sigma \propto \omega^2$:

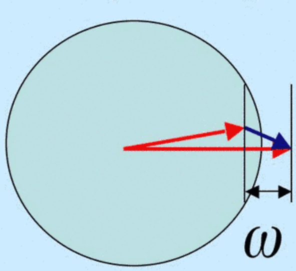


$Q \sim \Lambda$

$\Omega \sim \omega$

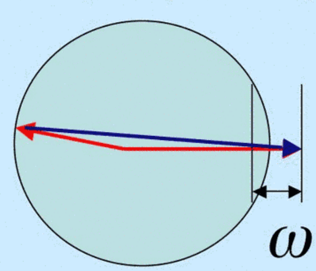
Non-analytic part of $\mathrm{Im}\Sigma$

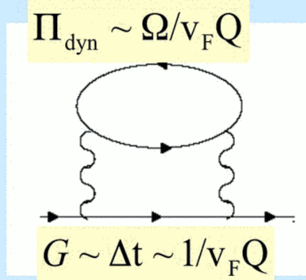
$Q \sim |\omega| / v_F$



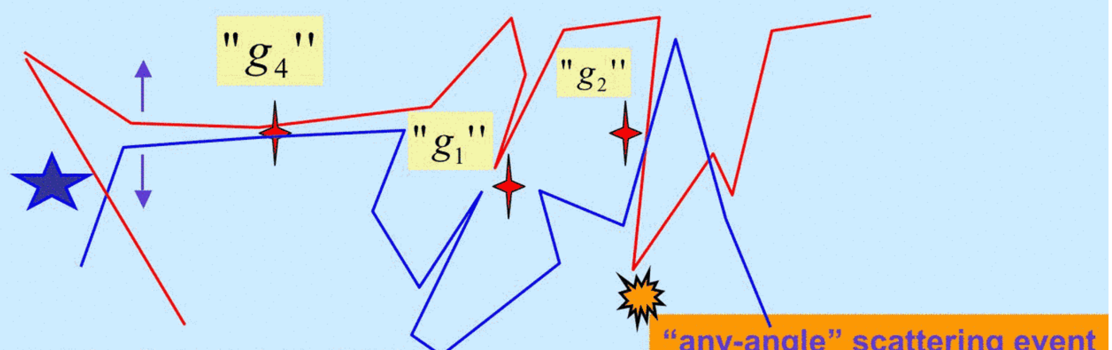
$\Omega \sim \omega$

$|Q - 2k_F| \sim |\omega| / v_F$





$$\mathrm{Im}\Sigma_{\text{sing}}(\omega) \propto \int_0^{\omega} d\Omega \int_{\Omega/v_F} dQ Q^{D-1} \frac{\Omega}{Q^2} = \omega^2 \omega^{D-2}$$

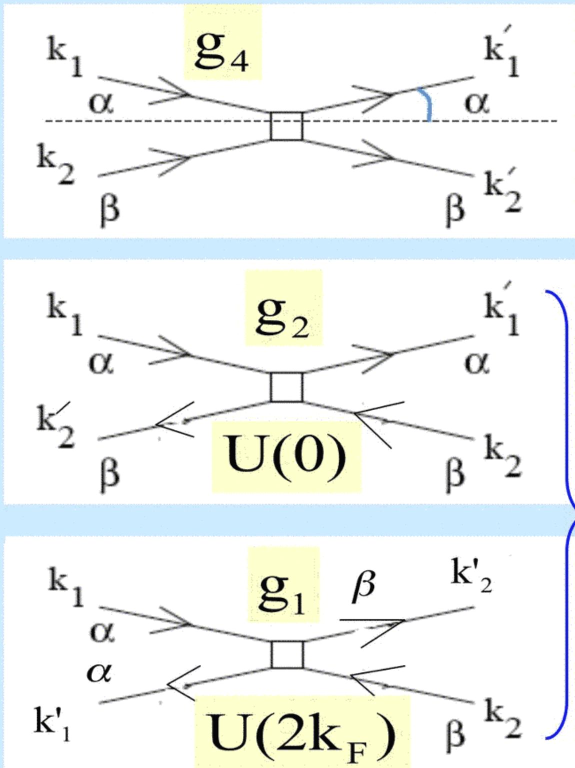


★ dynamic forward scattering

✦ 1D dynamic forward or backscattering Q or $|Q\text{-}2k_F| \propto |\omega|$

type	3D	2D
★	Yes (``old" paramagnon term) `	No
✦	Yes (``new" paramagnon contribution)	Yes

1D processes embedded into D>1 space



$\alpha \sim |\omega| / E_F$

``Infrared catastrophe'' in D=2
Breakdown of the naïve perturbation theory.
Features in single-particle properties (spectal function)

- Momentum transfer is either near 0 or 2kF
- Total momentum is near 0

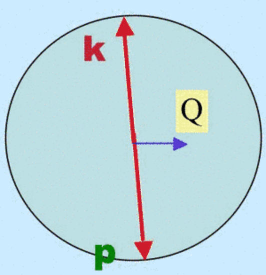
Singular terms in thermodynamics

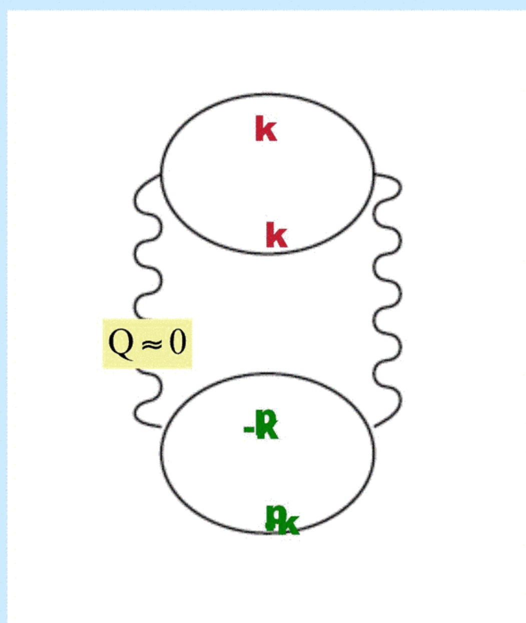
Q=0 scattering

Regular part: k and p are not correlated

$\Pi(Q,\Omega) \propto \frac{\Omega}{Q}$ comes from $k \perp Q$

Likewise, $p \perp Q$

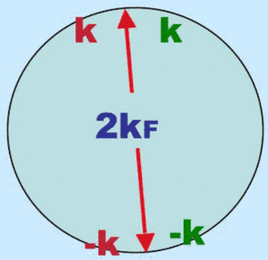


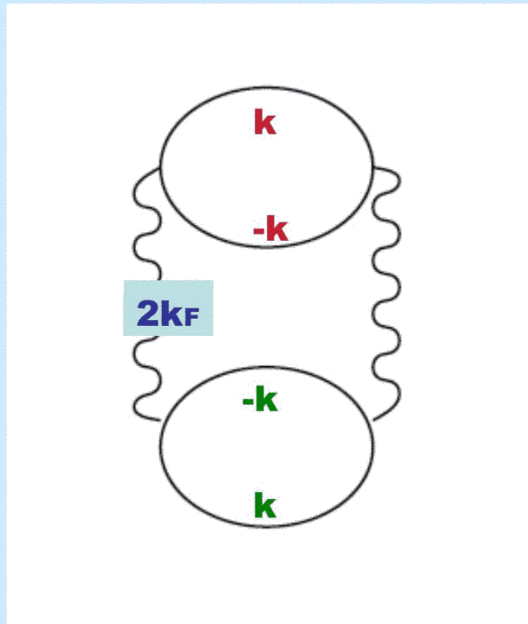


Singular part: k and p are either parallel or anti-parallel

Parallel k and p do not contribute

$2k_F$ scattering





2D: beyond the perturbation theory

Landau interaction function $\hat{F}(\theta)$

Scattering amplitude

$\hat{A}(\theta)$

$$C(T)/T = \gamma\left(1+F_1^s\right) - a\left[A_s^{\,2}(\pi)+3A_a^{\,2}(\pi)\right]T$$

Landau function averaged over the Fermi surface

Scattering amplitudes NOT averaged over the Fermi surface

$$A_s(\pi) = \sum_n (-)^n \frac{F_n^s}{1+F_n^s} \quad F_n^s \gg 1 \ \text{or} \ < 1 \ \text{or} \ \gg 1$$

(annotations: $\gg 1$, < 1, $\gg 1$)

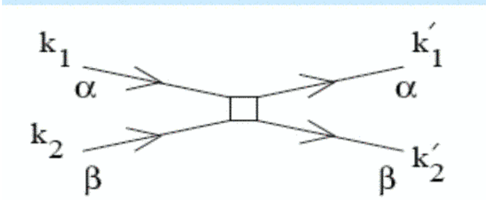
$$\chi_s(Q,T) = \frac{\chi_0(1+F_1^s)}{1+F_0^a} + bA_a^{\,2}(\pi)\max\left\{v_F|Q|,T\right\}$$

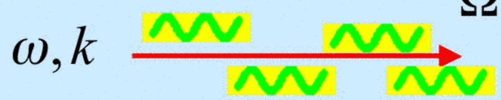
Infrared catastrophe: 1D

Bychkov, Gor'kov & Dzyaloshinskii 66

1D +linearized spectrum →

on-shell fermion can emit an infinite number of soft bosons: charge and spin density fluctuations





$$\Omega = v_F Q$$

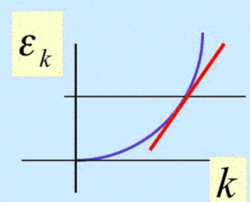
$$\Rightarrow \quad \Sigma \sim \frac{\omega^2}{\omega - v_F\left(k-k_F\right)+i\delta}$$

On-shell:
energy and momentum conservations are the same

Does not renormalize the wave function but leads to spin-charge separation

Infrared catastrophe: 2D

$$\mathrm{Im}\,\Sigma\left(\omega, k=k_F\right) \sim g^2\omega^2 \ln|\omega|$$



Linearized dispersion: $\varepsilon_k = v_F(k-k_F)$

Mass-shell singularity

$$\mathrm{Im}\,\Sigma(\omega,k) \sim g^2\left[\omega^2 \ln|\omega-\varepsilon_k| + \omega^2 \ln|\omega+\varepsilon_k|\right]$$

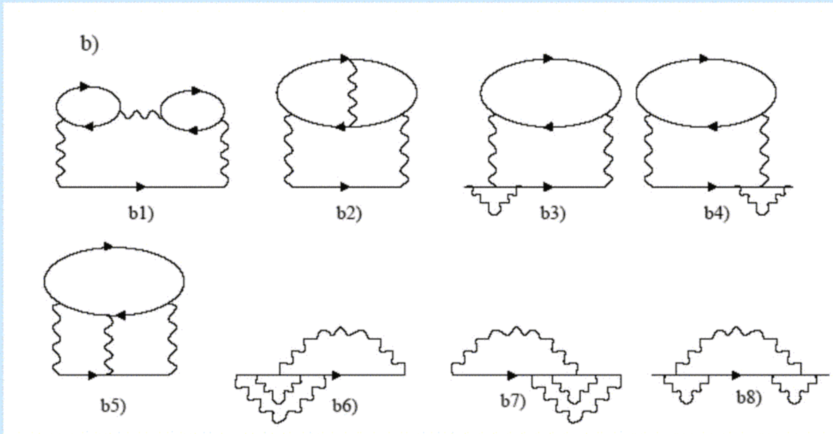
Castellani, di Castro, Metzner 94
Metzner 98
Chubukov and DLM 03

Finite curvature of the spectrum →finite mass

Singularity is reguralized at $|\omega-\varepsilon_k| \sim \omega^2/E_F$

$$\mathrm{Im}\,\Sigma \sim g^2\omega^2 \ln|\omega|$$

3^{rd} order and beyond …



$$\mathrm{Im}\Sigma_n \sim u^n \omega^2 \left(\frac{1}{\omega - \varepsilon_k}\right)^{n/2-1}$$

Finite curvature does not help: series diverges in the infrared

$$\omega < u^2 E_F$$

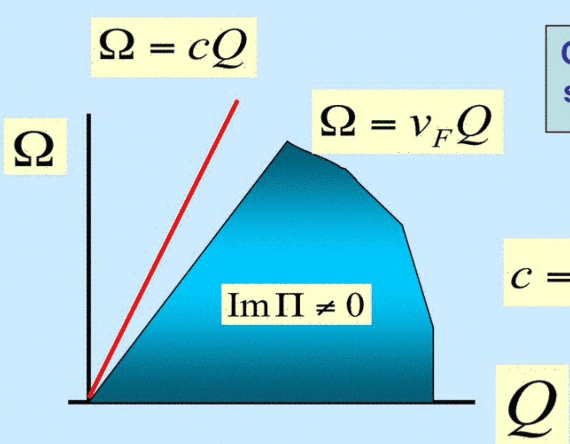
Perturbation theory must be re-summed even for $u \to 0$

Re-summed perturbation theory: (asymptotically) exact

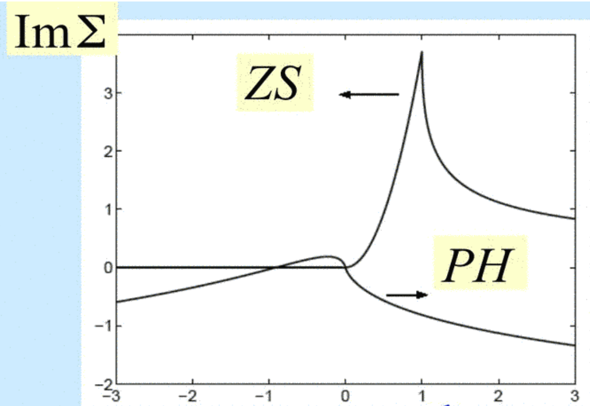
$$\Sigma_F(p) = \frac{1}{2}\int_q G(p-q) \times \left[4U - 2U^2\Pi(q) + \frac{U}{1-U\Pi(q)} - \frac{3U}{1+U\Pi(q)}\right]$$

Charge zero-sound mode

Spin zero-sound mode (damped)



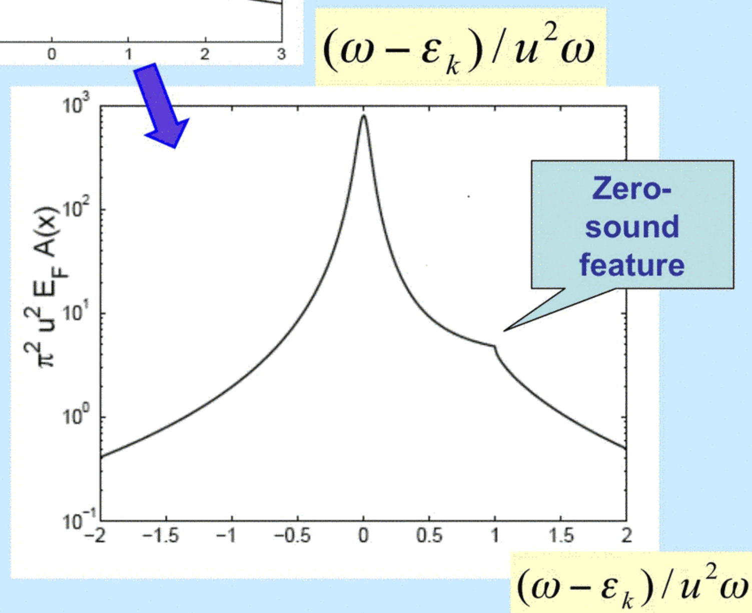
$$c = v_F(1 + U^2/2 + \ldots)$$



Spectral function

$-(1/\pi)\,\mathrm{Im}\,G(\omega,k)$

ARPES
Momentum-resolved tunneling between layers of 2D gas

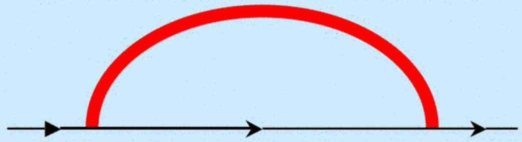


- **Main subleading corrections to the Fermi liquid behavior are singular (non-analytic) functions of temperature, spatial scale …**
- **Singularities result from 1D scattering processes embedded into D>1 space: link between D=1 and D>1.**
  Precursors of the 1D—Luttinger-liquid-- behavior in D>1
- **Naïve perturbation theory breaks down in 2D**
  Re-summed perturbation theory:
  Fermi liquid survives but
  Non-Fermi-liquid features remain
  (non-Lorentzian spectral function)

A. V. Chubukov and DLM, PRB 68, 155113 (2003), *ibid*. 69, 121102 (2004)
A. V. Chubukov, DLM, S. Gangadharaiah cond-mat/0412283, 0501013

Long-range dynamic interaction

$$V_{\text{eff}}(\Omega,Q)\sim g^2\Pi(Q,\Omega)$$



Singularity near Q=0 →long-range dynamic interaction

$$V_{\text{eff}}(\Omega,r)\sim\frac{\Omega}{r^{D-1}}$$

Singularity near Q=2kF → dynamic Friedel oscillation

$$V^{2k_F}{}_{\text{eff}}(\Omega,r)\sim\frac{\Omega}{r^{D-1}}\sin 2k_F r$$

Non-analytic term near the quantum critical point

Chubukov, Pepin, Reich 04

Spin-Fermion model

$$\mathbf{H}=\sum_{p,\alpha}v_F(p-p_F)c^\dagger_{p,\alpha}c_{p,\alpha}+\sum_q\chi^{-1}_{s,0}(q)\mathbf{S}_q\mathbf{S}_{-q}+g\sum_{p,q}c^\dagger_{p+q,\alpha}\boldsymbol{\sigma}_{\alpha,\beta}c_{p,\beta}\mathbf{S}_q.$$

→

$$\chi_s(q)=\frac{\chi_0}{q^2-(0.17)|q|^{3/2}p_F^{1/2}}.$$

$$q<(0.17)^2p_F$$

$$q\propto\omega^{1/3}\sim T^{1/3}$$

$$T<(0.17)^6E_F$$

$$Sr_2RuO_4 : \chi_s(T) = \chi_s(0) + AT$$ *Maeno 97*

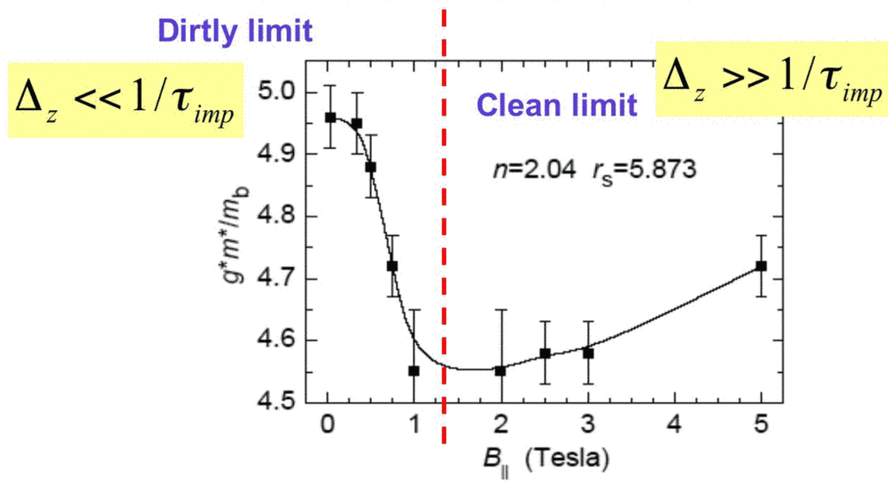


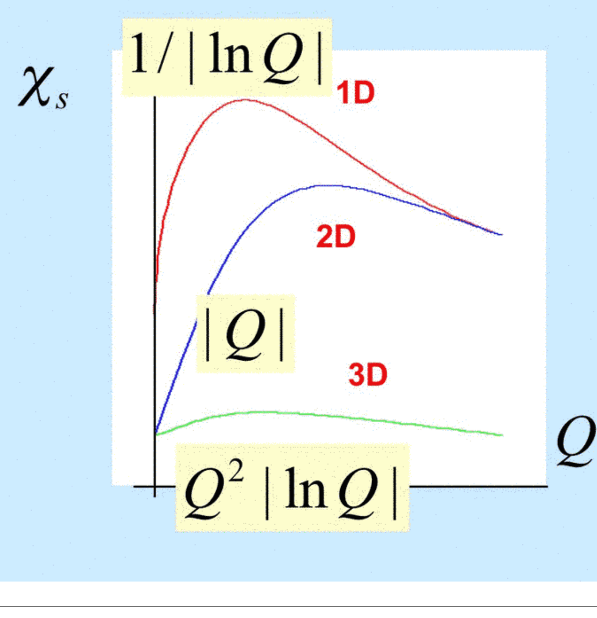
Figure 1.5. Typical dependence of the spin susceptibility on the in-plane magnetic field, measured for n-Si-MOS sample at $T \approx 0.15$K. Density is given in units of 10^{11}cm^{-2}.

Data: Gershenson, Kojima & Pudalov 04

in ALL dimensions, $\chi_s(Q)$

a) non-analytic in Q

b) peaked at finite Q

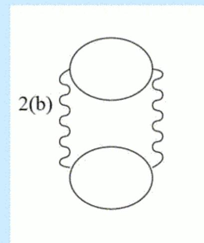


In 3D

$$\delta C(T) \propto T^3 \log T, \quad \ddot{a} \div (T) \propto T^2, \ \ddot{a} \div (Q) \propto Q^2 \log Q$$

This is not the same log as in paramagnon theory

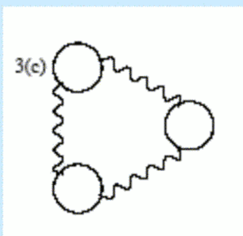
Our story



$$\Omega \propto T \sum_\omega \int d^3 q\, \Pi^2 (q,\omega)$$

$$\Pi \propto \frac{\omega}{q} + \left(\frac{\omega}{q}\right)^2, \quad \Omega \propto T \sum_\omega \omega^3 \log \omega$$

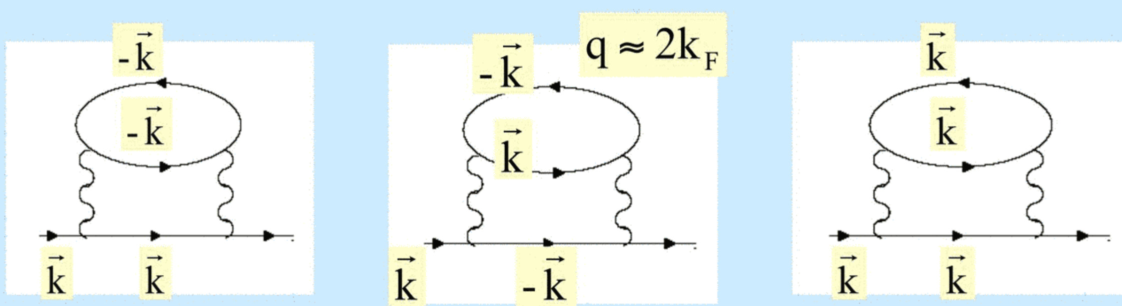
Paramagnon story

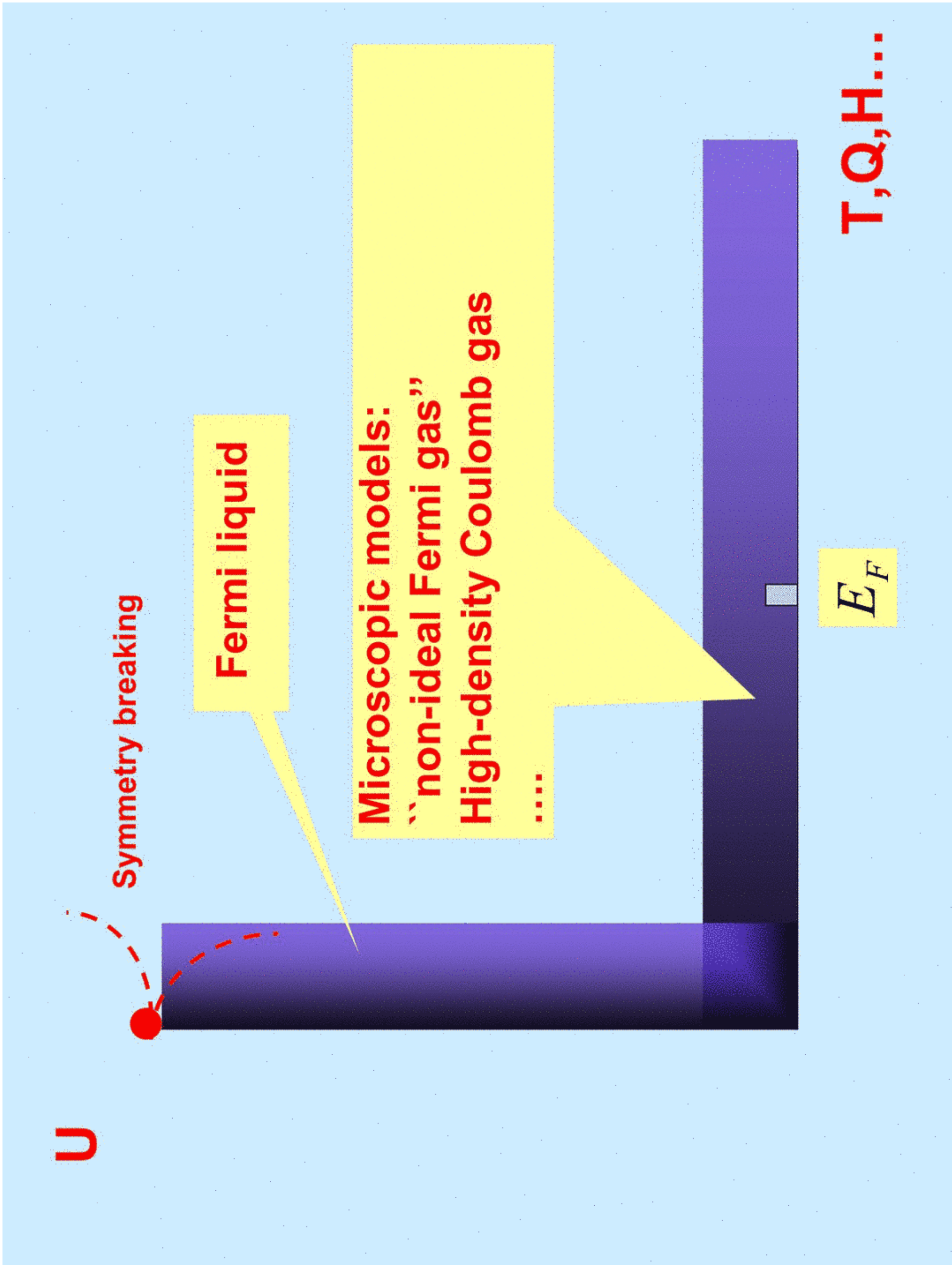


$$\Omega \propto T \sum_\omega \int d^3 q\, \Pi^3 (q,\omega)$$

$$\Pi \propto \frac{\omega}{q}, \quad \Omega \propto T \sum_\omega \omega^3 \log \omega$$

Doniach et al,
Larkin et al, ...,




U
Symmetry breaking
Fermi liquid
Microscopic models:
``non-ideal Fermi gas''
High-density Coulomb gas
....
E_F
T,Q,H…