A Road to local quantum criticality

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Spin coupled to bosons

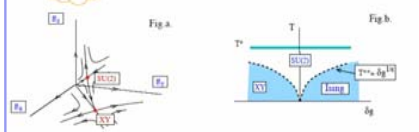
From Zaránd & Demler, PRB 2002

$$H_B^{int} = \sum_\alpha \Lambda^{\epsilon/2} \gamma_\alpha S^\alpha \phi^\alpha$$

ε measures deviations from ohmic bosons

« Sub-ohmic case »

$$\langle T\phi_\alpha(\tau)\phi_\beta(0)\rangle = cst.\ \frac{\delta_{\alpha\beta}}{\tau^{2-\epsilon}}$$



Large N+NCA: Vojta, Buragohain & Sachdev, PRB 2000

Including the fermions SU(2) summetry

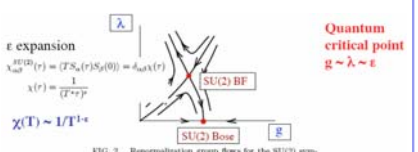
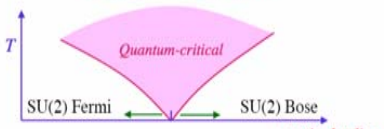


FIG. 2. Renormalization group flows for the SU(2) symmetrical Bose-Fermi Kondo model. "BF" denotes the SU(2) symmetrical Bose-Fermi fixed point. Here the spin is partially screened and both the Fermi and Bose fields couple to it. At the Bose fixed point the fermionic degrees of freedom fully decouple from the spin.

λ denotes the Kondo coupling with electrons

Road to local quantum criticality …

Quantum phase transition: ground states on either side of g_c have distinct "order"



Smith & Q.Si, EuroPhys.Lett. 1999
Sengupta, PRB 2000

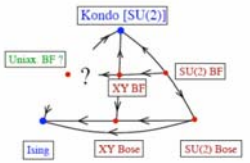
On the Bose-Fermi Kondo model: Q.Si et al., Nature 2001

Our proposal for noisy qubits

Take into account different anisotropies

Applicability to noisy qubits

K. Le Hur, PRL 2004



Ohmic case, ε=0
- exact mapping onto the Caldeira-Leggett model
- endeavors on numerical renormalization group applied to bosonic systems coupled to spin

M.-R. Li, K. Le Hur & W. Hofstretter, cond-matt/0501755

The starting point

$$H = hS_z + H_{sf} + H_{sb}, \quad (1)$$

$$H_{sf} = \frac{J_\perp}{2}\left(\Psi_\downarrow^\dagger(0)\Psi_\uparrow(0)S_+ + \text{h.c.}\right) + v_f \sum_{\sigma=\uparrow,\downarrow}\int_{-\infty}^{\infty} dx\ \Psi_\sigma^\dagger(x) i\partial_x \Psi_\sigma(x), \quad (2)$$

$$H_{sb} = -\frac{v_b \partial_x \Phi(0)}{\sqrt{2K_b}} S_z + \frac{v_b}{4\pi}\int_{-\infty}^{\infty} dx [\partial_x \Phi(x)]^2, \quad (3)$$

φ: bosons with ohmic spectrum

ψ: fermions with Dirac spectrum

A J_z fermionic coupling will slightly affect the mapping

Caldeira-Leggett mapping

First: Bosonization of the fermionic bath

$$H_{sf} = \frac{\Delta}{2}\left[e^{i\sqrt{2}\varphi(0)}S_+ + \text{h.c.}\right] + \frac{v_f}{4\pi}\int_{-\infty}^{\infty} dx [\partial_x \varphi(x)]^2$$

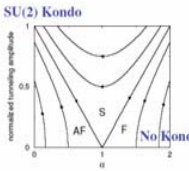
$$\Delta = (J_\perp \omega_c / 2\pi v_f)$$

Now: Spin coupled to two bosonic baths

(this point was previously noted by Grempel and Q. Si, PRL 2003)

Eventually, for the ohmic case, we can recombine exactly those two environments → Caldeira-Leggett physics

Brief reminder



S: Strong tunneling
AF: Kondo realm
Bethe-Ansatz region
F: ferromagnetic Kondo
Unscreened spin

Analogy with the anisotropic Kondo model:

This diagram has been investigated by Anderson, Yuval, Hamman

Bose-Fermi Kondo model: α in the vicinity of the quantum phase trans.

Applicability of Ohmic case & J_z =0: Noisy mesoscopic qubits

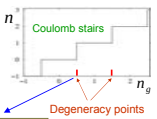
Large $E_c = e^2/2C$

C : capacitance decreases with size

Gate voltage controls electron number

$E_{ch} = E_c(n - n_g)^2$

$n_g = C_g V_g / e$

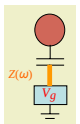


Two-state system
Artificial spin S =1/2
$n = 1$ state
$n = 0$ state
$E_{ch} = hS_z$ $h = 2E_c(1/2 - n_g)$

Resonant level

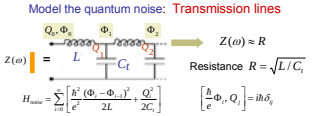
K. Le Hur, PRL 2004

Gate voltage fluctuations quantum noise



Fluctuations in electromagnetic medium …

Model the quantum noise: Transmission lines



$H_{noise} = \sum_{i=0}^{n}\left[\frac{\hbar^2}{e^2}\frac{(\Phi_i - \Phi_{i-1})^2}{2L} + \frac{Q_i^2}{2C_t}\right]$ $\left[\frac{\hbar}{e}\Phi_i, Q_j\right] = i\hbar\delta_{ij}$

(coupled) quantum harmonic oscillators (bosonic bath)

Gate voltage fluctuation $\delta V_{gi} = Q_{0i}/C_{gi}$

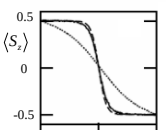
Spin-boson coupling $e(\delta V_{gL} - \delta V_{gR})\, S_z$

Metallic dot +Fermi liquid leads

Dense energy spectrum: $H_t = t c_{lead}^\dagger c_{dot} S^- + h.c.$ Matveev, JETP 1991

Exotic Kondo effect

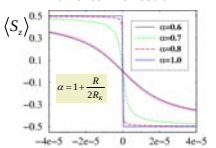
Experimentally measurable



Lehnert et al, PRL 1999

Noise tuned quantum phase transition

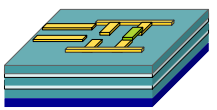
Numerical RG result



$R_c / R_K \approx |\epsilon|/2\pi v_f << 1$ v_f Fermi velocity

Li, Le Hur, Hofstetter, cond-mat/0501755

What we propose to experimentalists



GaAs : $Al_x Ga_{1-x} As$

First GaAs layer: quantum dot + lead

Second GaAs layer: *tunable* R through tuning electron density

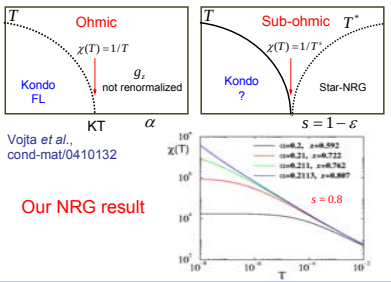
$R \uparrow$

Dissipative Josephson junction

Rimberg *et al.*, PRL1991

Kycia *et al.*, PRL 2001

Beyond ohmic: Sub-ohmic case



Vojta *et al.*, cond-mat/0410132

Our NRG result

Conclusion

For the ohmic situation and Ising coupling with bosons:

Hidden Caldeira-Leggett model --- Single boson bath

Kosterlitz-Thouless quantum phase transition

Observable in a noisy mesoscopic qubit

Subohmic case: A way to perform and ε expansion from the ohmic case?

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