Ferromagnetic quantum criticality: an overview

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Outline

- The nature of the effective action:
  - Paramagnons and Hertz-Millis…
  - Analyticity in m and/or q.
  - The tricritical point and metamagnetism.
  - Coupling to other degrees of freedom.
- Correlated states near the critical point:
  - Superconductivity and beyond.

The old wisdom:

- Paramagnon theory (1960s onward):
  - metals on the border of ferromagnetism.
  - Pd, Ni_3Al, Ni_3Ga, YNi_3, $ZrZn_2$
  - Puzzles: neither Stoner nor Heisenberg-like
    - Stoner: $B=a\,M + b\,M^3$ with $a » a_0-\lambda-(T/T_F)^2$
    - T_c small, χ Curie-Weiss like, yet μ_{eff}/μ_0 large
  - Resolution: include spin-fluctuations
    - Berk, Schrieffer, Doniach, Engelsberg, Rice, Moriya, Yamada, Beal-Monod, Misawa, Lonzarich … and more
    - $B=(a+ b\,\text{h}m^2\text{i})\,M + b\,M^3$

Renormalization group approach

- Hertz 1976, Millis 1993, …
- Integrate out the electrons to give an Ornstein-Zernike form for the effective action:

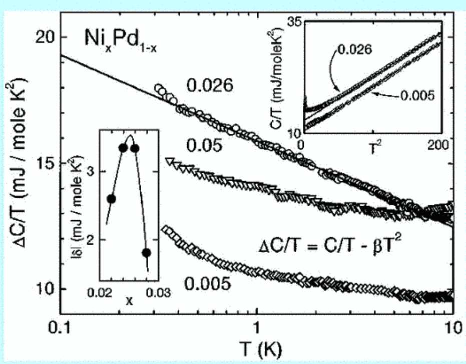
$$S = \int d^D q \, d\omega \left[\delta + q^2 + \frac{|\omega|}{vq} \right] \phi^2_{q,\omega} + u\phi^4 + v\phi^6 - h\phi$$

← damping vanishes as $q \to 0$

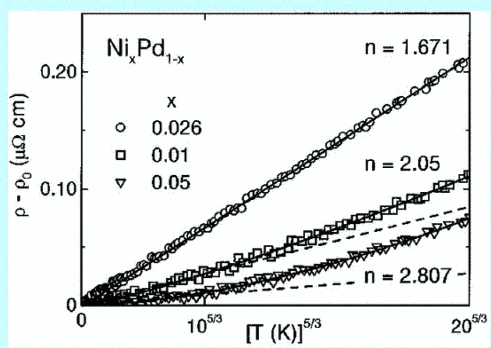
- RPA/Lindhard function determines above form.
- Do momentum shell RG.
  - $D_{uc} = D\text{-}z$ so above upper critical dimension.

QC properties	Specific Heat	Resistivity exp.	Tuning
D=2	$T^{2/3}$	$T^{4/3}$	T_c » $(p_c\text{-}p)$
D=3	$-T \ln T$	$T^{5/3}$	$T_c^{4/3}$ » $(p_c\text{-}p)$

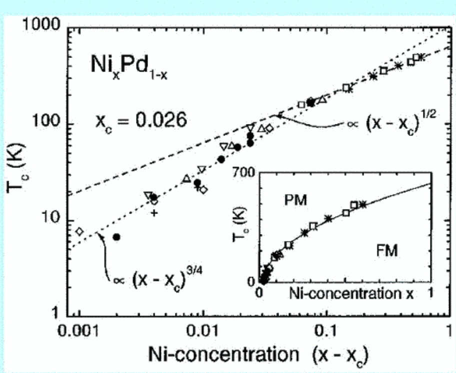
Initially theory compared well to experiment



C» T ln T



$\Delta \rho$ » $T^{5/3}$



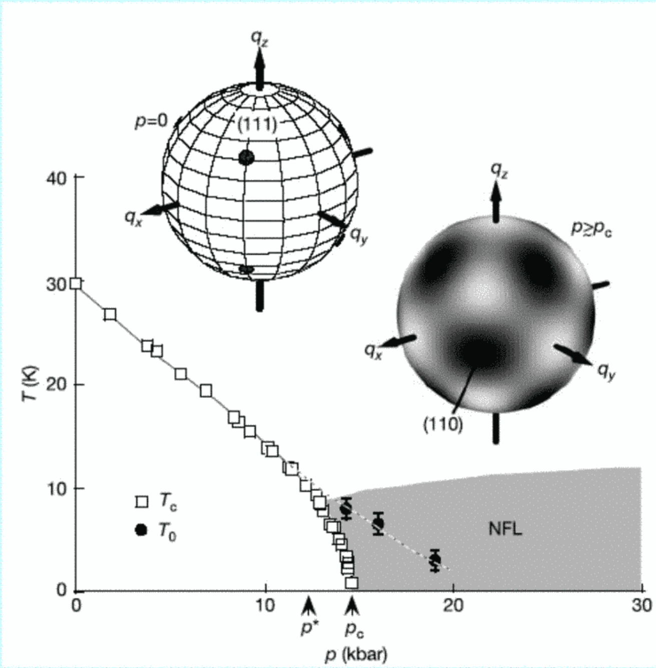
T_c » $(x\text{-}x_c)^{4/3}$

Data from $Ni_x Pd_{1-x}$: with x_c=0.026

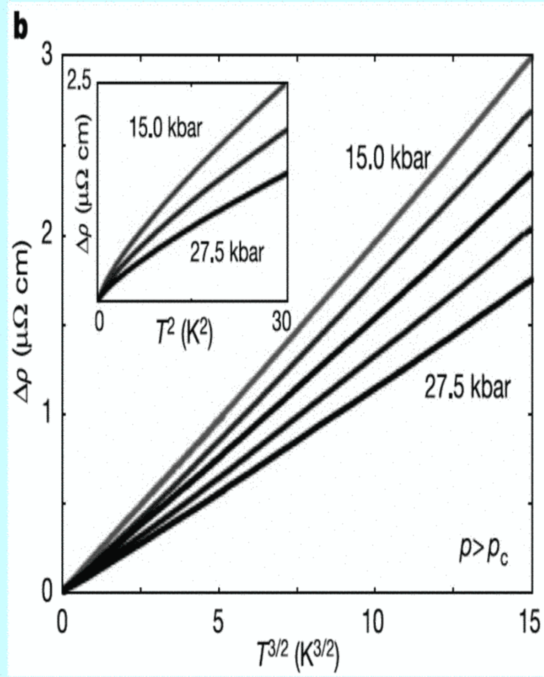
M. Nicklas, M. Brando, G. Knebel, F. Mayr, W. Trinkl and A. Loidl, Phys. Rev. Lett. 82, 4268 (1999).

But cleaner systems showed the quantum critical point was never reached – transition went first order.

Aside: MnSi – helimagnet a special case?

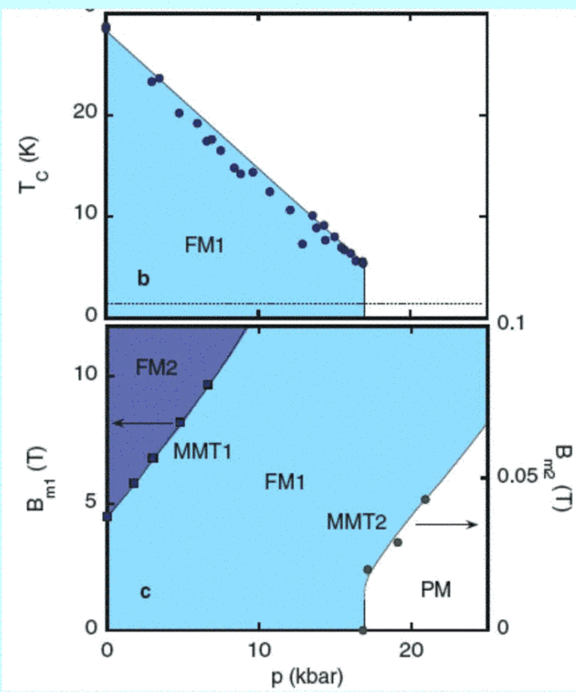


C. Pfleiderer, D. Reznik, L. Pintschovius, H. v. Lohneysen, M. Garst and A. Rosch, Nature **427**, 227 (2004).



N. Doiron-Leyraud, I. R. Walker, L. Taillefer, M. J. Steiner, S. R. Julian & G. G. Lonzarich, Nature **425**, 595 (2004).

$ZrZn_2$: the latest casualty



M. Uhlarz, C. Pfleiderer and S. M. Hayden, Phys. Rev. Lett. **93**, 256404 (2004).

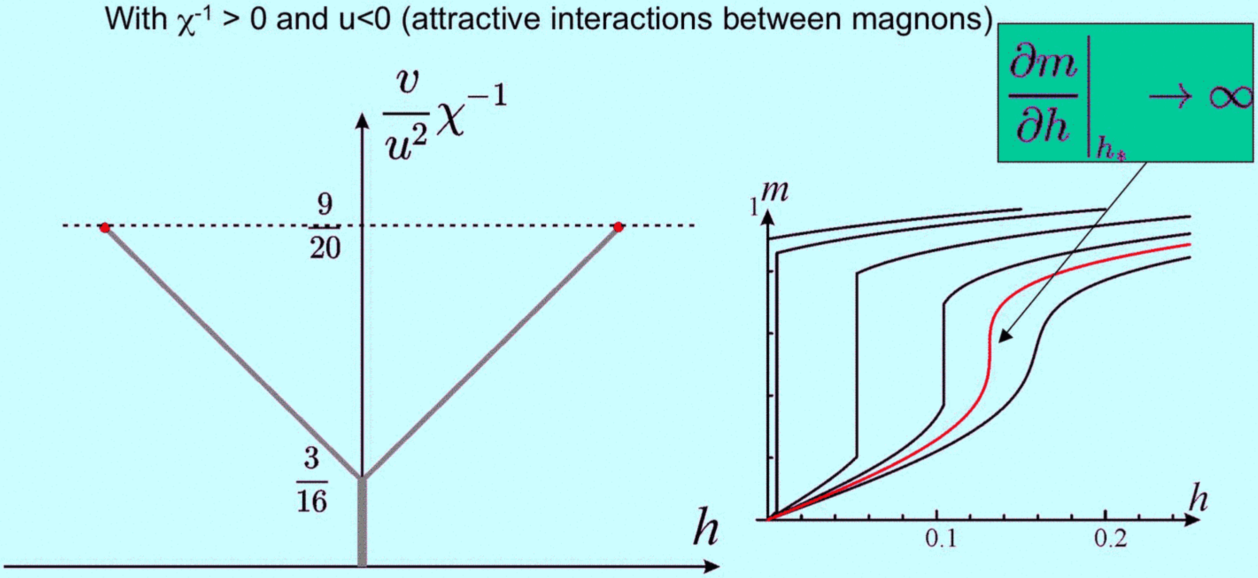
- Transition goes first order at some low temperature.
- There is a tricritical point.
- Follow the critical end-point by introducing a magnetic field axis.

Simplest Landau theory of such a phase diagram

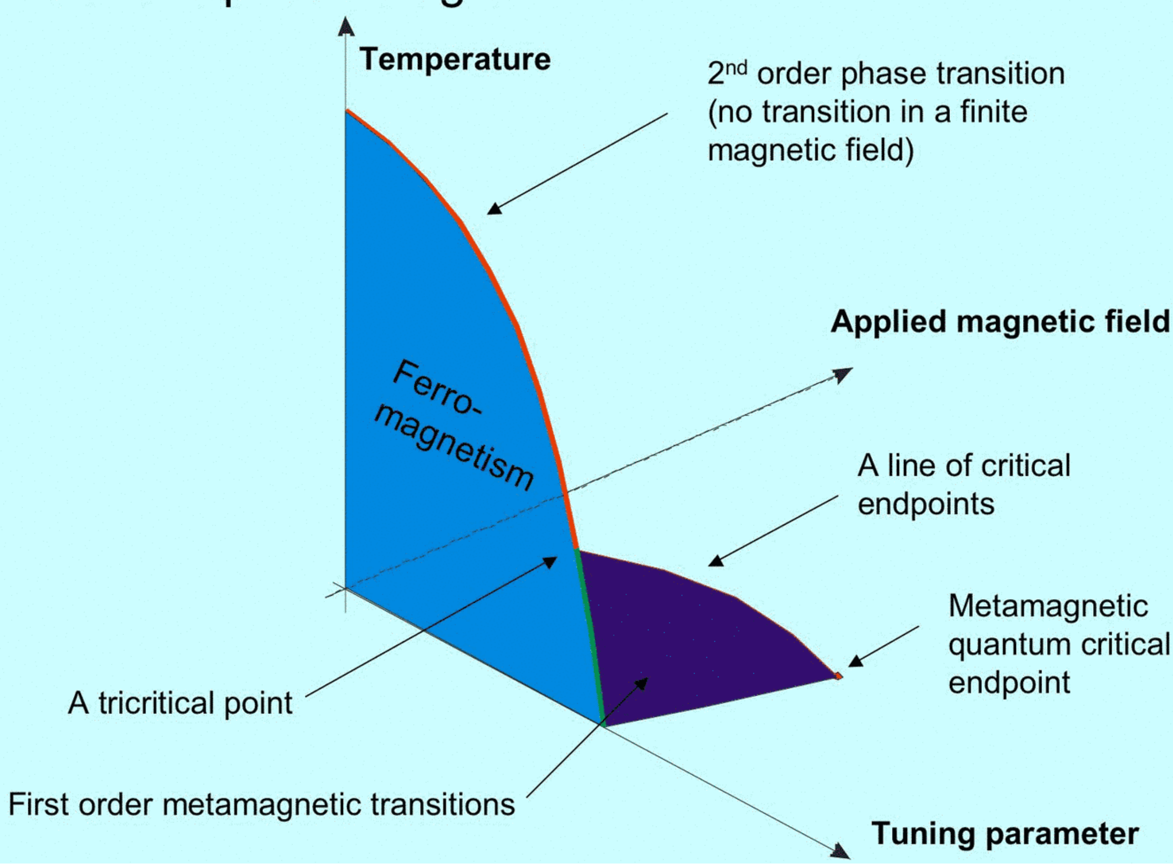
E.P. Wohlfarth & P. Rhodes, Phil. Mag B, **6**, 1817 (1962)

$$\mathcal{F} = \frac{1}{2}\chi^{-1}m^2 + \frac{1}{4}um^4 + \frac{1}{6}vm^6 - hm$$

With $\chi^{-1} > 0$ and $u<0$ (attractive interactions between magnons)



Generic phase diagram

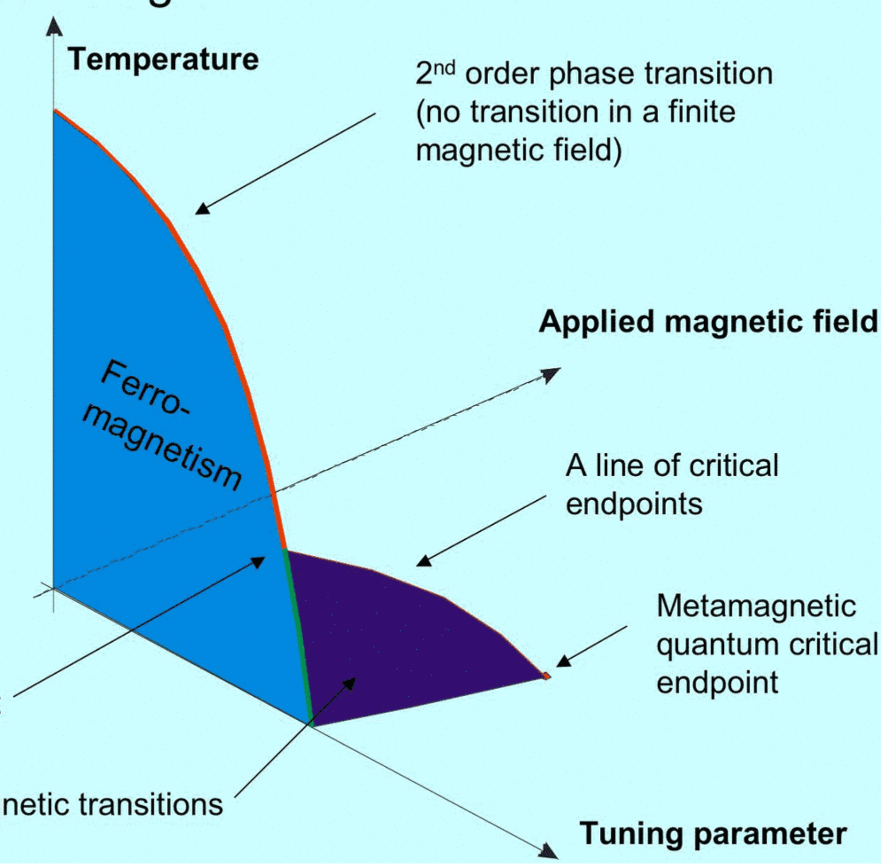


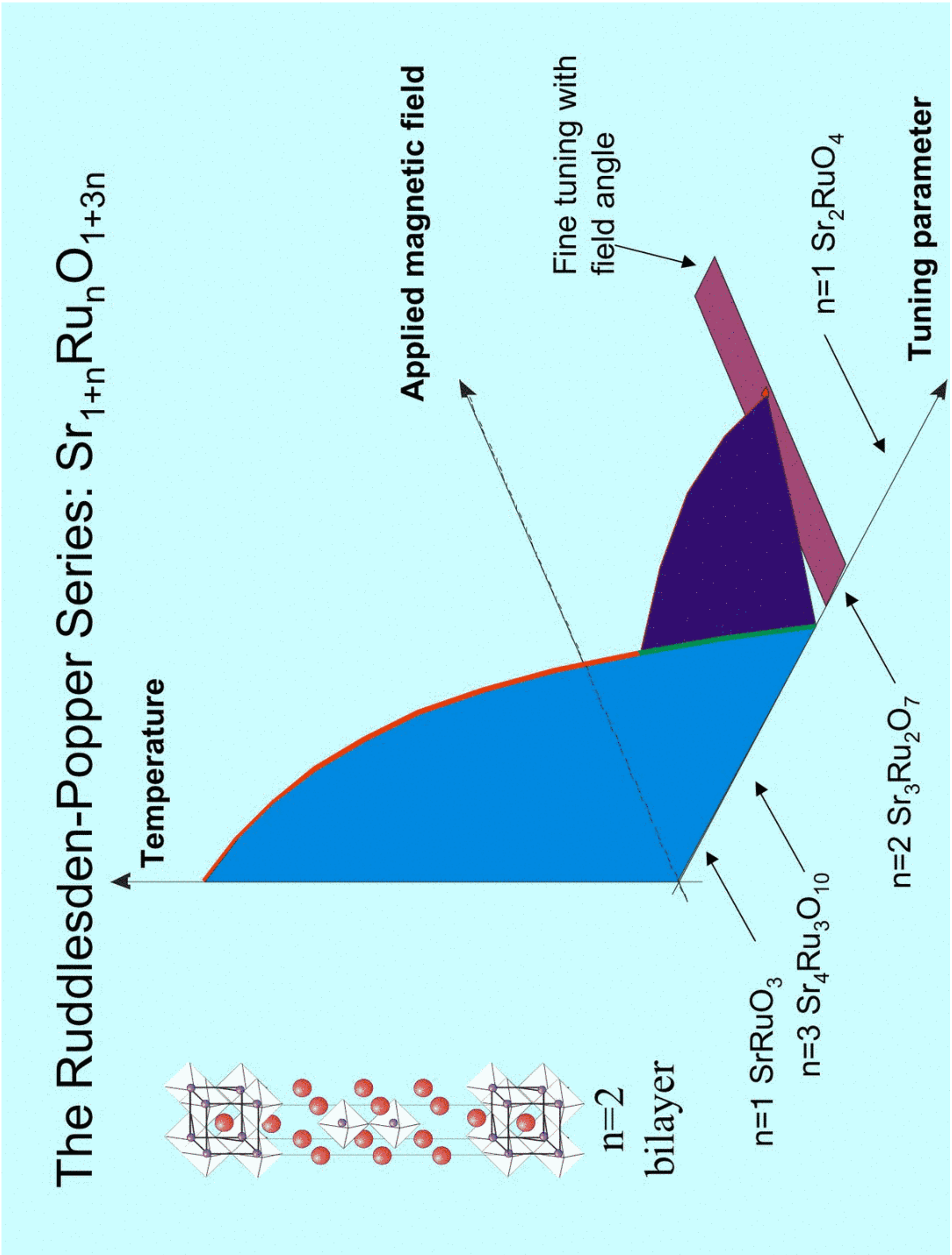
Why attractive interactions?

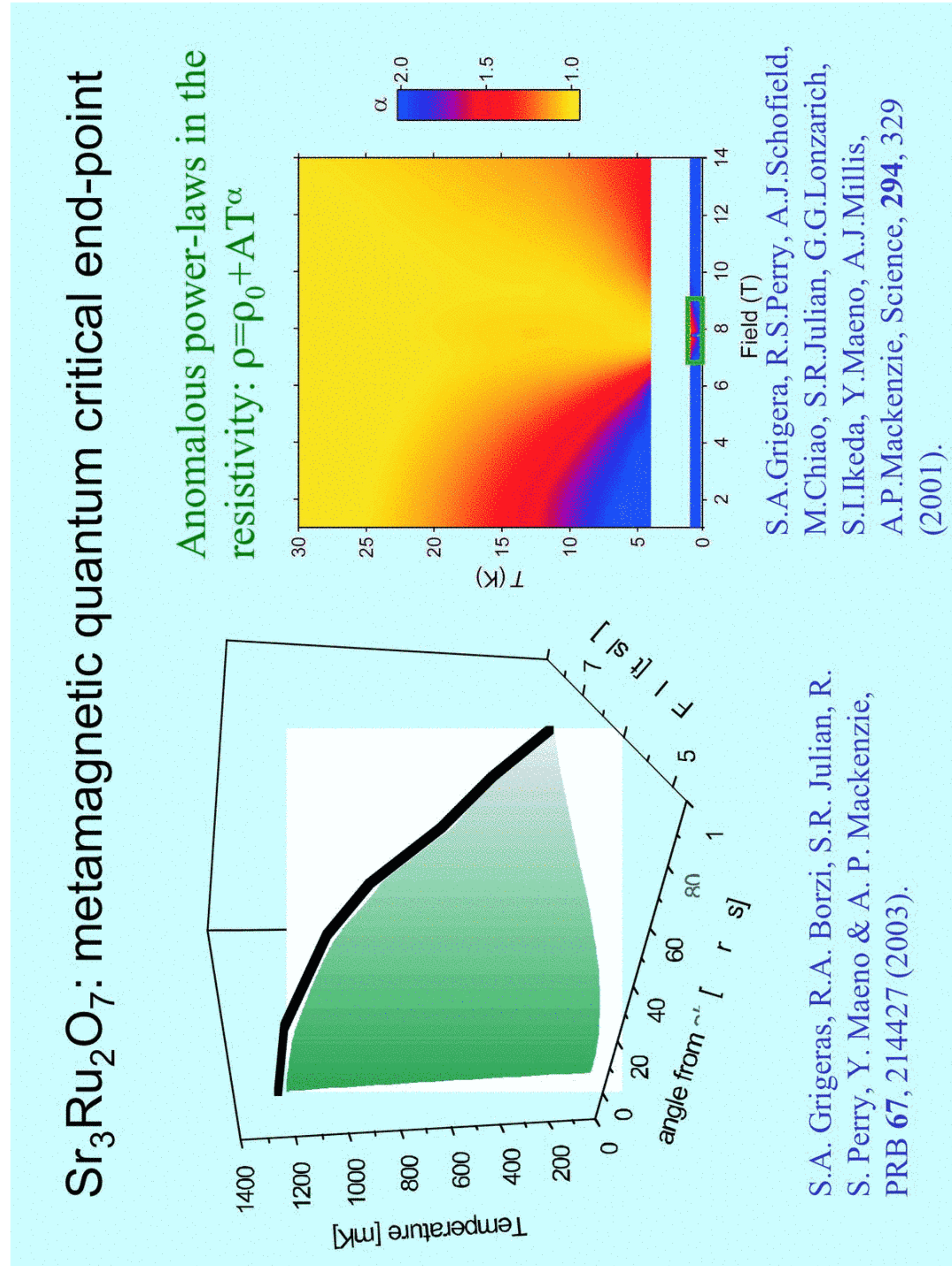
- Negative curvature of density of states
  - Chain features in UGe_2:
    K. Sandeman, G. Lonzarich, A. J. Schofield, PRL 90, 167005 (2003).
  - Van-Hove point in ruthenates.
    B. Binz and M. Sigrist, *Europhys. Lett.*, **65,** 816 (2004).
- Coupling to soft modes: particle-hole excitations in the spin channel:
  - F ! $F_{conventional} + w\, m^4 \ln (m^2/m_0^2 + T^2/T_0^2)$
    D. Belitz, T.R. Kirkpatrick, Jorg Röllbühler, cond-mat/0410344 and references therein. Reminiscent of other fluctuation induced first order transitions.
  - Non-analytic action in q: *e.g.* D=2 ! $-|q|^{3/2}\, m_q^2$ at QCP. |q| near QCP, D=3 ! $q^2 \ln q$.
    A. V. Chubukov, C. Pépin, J. Rech, PRL **92**, 147003 (2004). D. Belitz, T. R. Kirkpatrick, and T. Vojta, Phys. Rev. B **55**, 9452 (1997).
  - Over counting? No – observe corrections in Fermi liquid theory.
    G.Y.Chitov and A. J. Millis, PRB 64, 054414 (2001).

Generic phase diagram

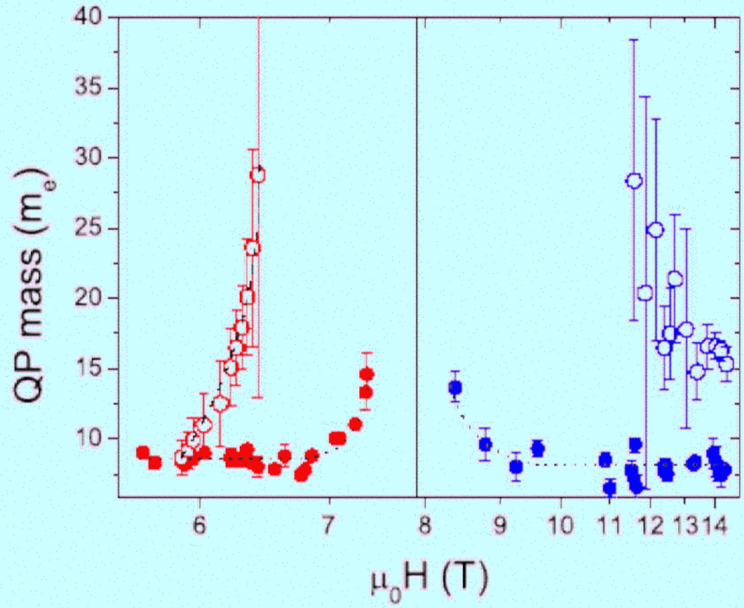
No one material has all features of this phase diagram accessible. Nearest so far – the Ruddlesden-Popper series of ruthenates.




The Ruddlesden-Popper Series: $Sr_{1+n}Ru_nO_{1+3n}$
Temperature
Applied magnetic field
Fine tuning with field angle
n=1 Sr_2RuO_4
Tuning parameter
n=2 $Sr_3Ru_2O_7$
n=3 $Sr_4Ru_3O_{10}$
n=1 $SrRuO_3$
n=2
bilayer


$Sr_3Ru_2O_7$: metamagnetic quantum critical end-point
Temperature [mK]
1400
1200
1000
800
600
400
200
0
0
20
40
60
80
1
5
S.A. Grigeras, R.A. Borzi, S.R. Julian, R. S. Perry, Y. Maeno & A. P. Mackenzie, PRB **67**, 214427 (2003).
Anomalous power-laws in the resistivity: $\rho=\rho_0+AT^\alpha$
α
2.0
1.5
1.0
T (K)
30
25
20
15
10
5
0
2
4
6
8
10
12
14
Field (T)
S.A.Grigera, R.S.Perry, A.J.Schofield, M.Chiao, S.R.Julian, G.G.Lonzarich, S.I.Ikeda, Y.Maeno, A.J.Millis, A.P.Mackenzie, Science, **294**, 329 (2001).

How good is an itinerant picture for Sr327?



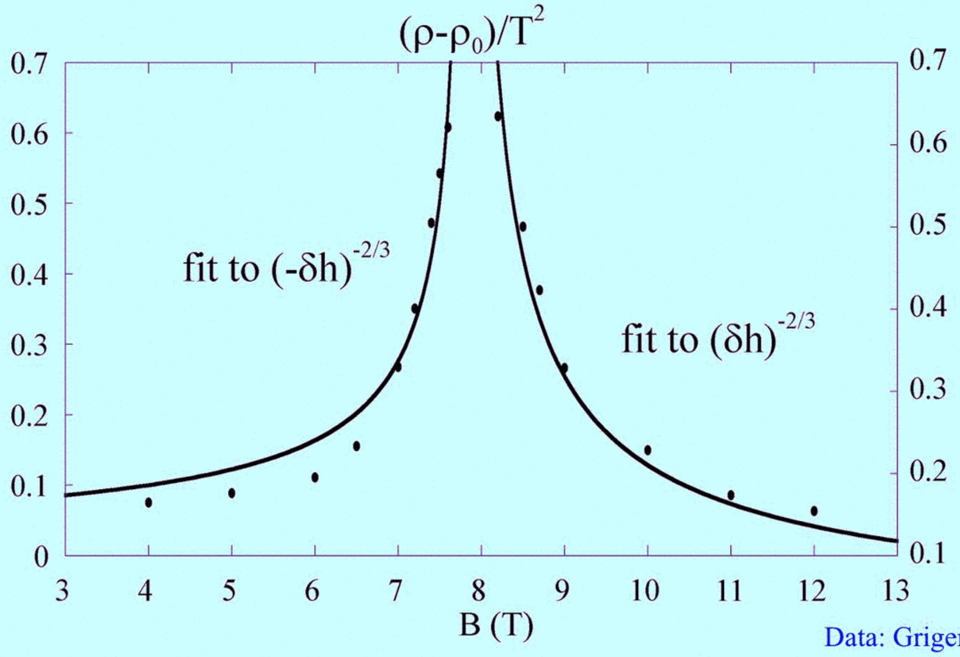
Also, small changes in Fermi surface volume through B_c.

R.A. Borzi, S.A. Grigera, R.S. Perry, N. Kikugawa, Y. Maeno and A. P. Mackenzie, PRL **92**, 216403 (2004).

The theory of the metamagnetic quantum critical endpoint:

- SCR approach:
  H. Yamada, PRB **47**, 11211 (1993).
- Hertz RG approach:
  A.J.Millis, A.J.Schofield, G.G.Lonzarich and S.A.Grigera PRL **88**, 217204 (2002). Springer Lecture Notes in Physics, **603**, 271 (2002).
  - Action at the critical endpoint is that of original Hertz-Millis theory [no non–analyticities: D. Belitz, T.R. Kirkpatrick, T. Vojta, Phys Rev B, **65** 165112 (2002).]
  - Field is the tuning parameter: δ » $|B-B_c|^{2/3}$
  - Theory of the quantum critical liquid-gas transition:
    - C.f. valence fluctuations: Miyake et al. [see A. T. Holmes, D. Jaccard, K. Miyake PRB **69**, 024508 (2004) and references therein.]

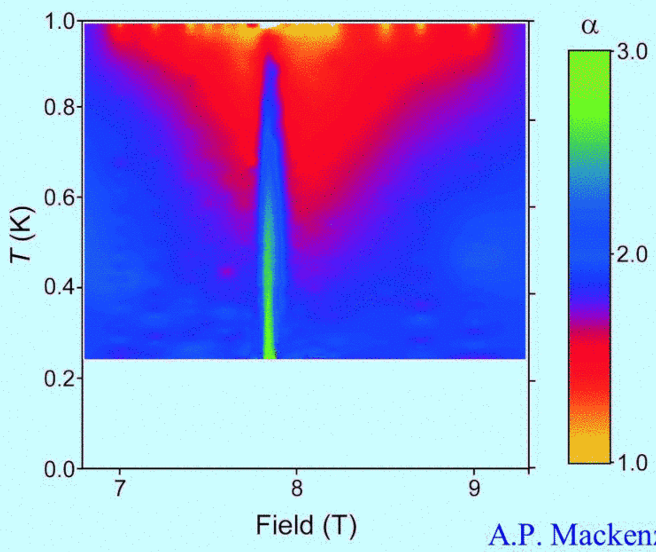
So – how well does this theory work?



Data: Grigera *et al.*

Just when you are getting close to the quantum critical endpoint $|\delta h| < 0.1$T something else happens.

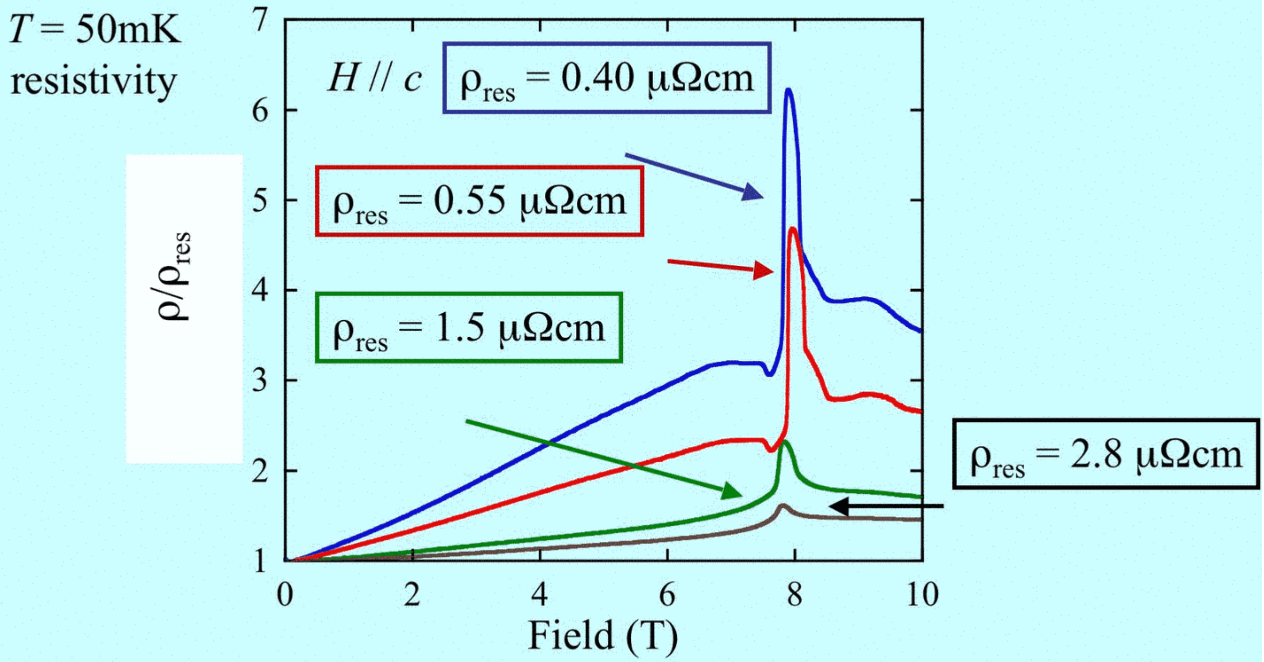
First indications of the change of behavior:



A.P. Mackenzie (unpublished 2002)

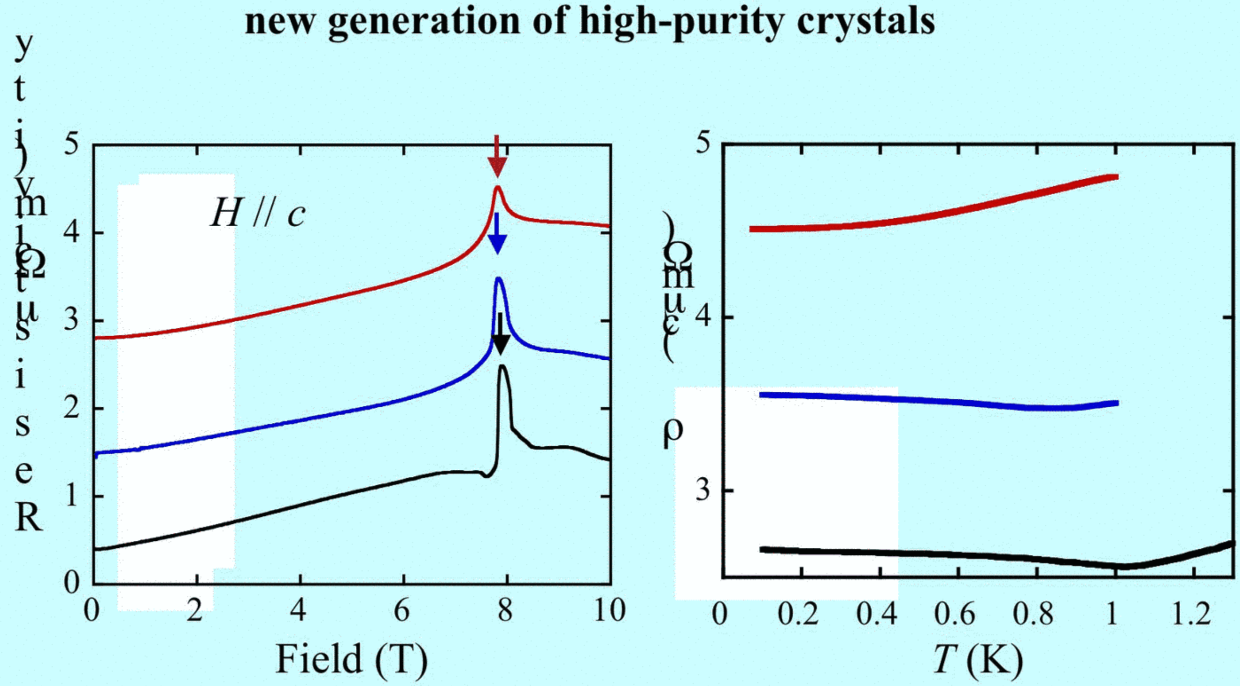
With a $\rho_0(B=0)=2.8\mu\Omega$cm sample:
ρ » T^3 for $T<0.5$K and $B \sim B^* +/- 50$mT
(outside the conventional quantum critical end-point scenario).

Anomalous transport effects *grow* with increasing purity



R.S. Perry**, K. Kitagawa, S.A. Grigera, R.A. Borzi, A.P. Mackenzie, K. Ishida and Y. Maeno, Phys. Rev. Lett.* ***92*, 166602 (2004).*

Flat T^3 seen in old crystals becomes a distinct *minimum* in the new generation of high-purity crystals



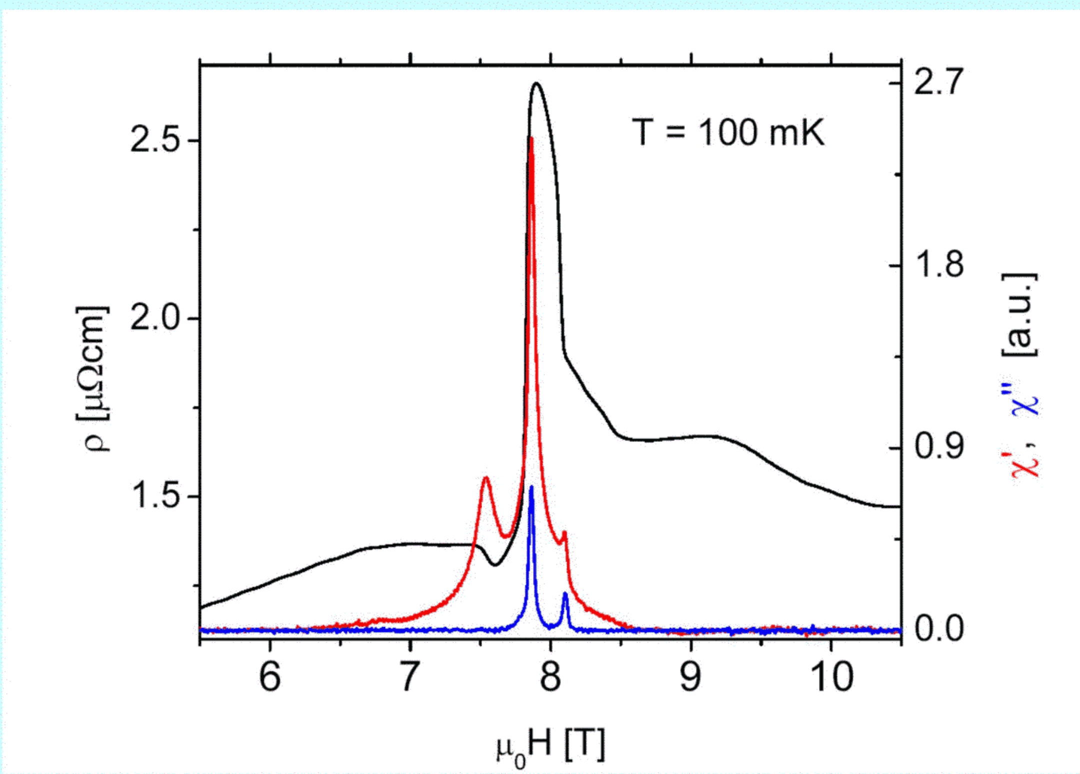
Also minimum moves to higher T with increasing sample purity!

What happens is very sensitivity to weak disorder

- What is the role of disorder at the quantum critical endpoint?
  - "Only Fermi liquids are metals",
    - C. M. Varma, PRL **79**, 1535 (1997).
  - Residual resistivity enhanced:
    - K. Miyake and O. Narikiyo, J. Phys. Soc (Japan), **71**, 867 (2002).
  - Altshuler-Aronov corrections to resistivity near the QCEP:
    - Y. B. Kim and A. J. Millis, Phys. Rev. B 67, 085102 (2003).
  - See Indranil Paul this afternoon

- In $Sr_3Ru_2O_7$ it looks to be more than this…

'Sidewalls' in ρ delineate a region bounded by *first-order transitions*

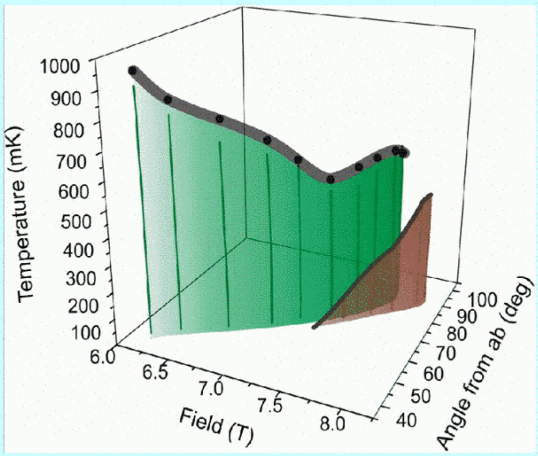


S. A. Grigera, P. Gegenwart, R. A. Borzi, F. Weickert, A. J. Schofield, R. S. Perry, T. Tayama, T. Sakakibara, Y. Maeno, A. G. Green, A. P. Mackenzie, Science 306 1154 (2004).

Multiple first order transitions near the QCEP

A. G. Green, S. A. Grigera, R. A. Borzi, A. P. Mackenzie, R. S. Perry, and B. D. Simons, cond-mat/0410470

Harmonic fluctuations of another degree of freedom (eg strain) can further drive the critical endpoint first order leading to two transitions:



$$\phi = m - M_c$$

$$\beta F[\phi] = \frac{r}{2}\phi^2 + \frac{s}{3}\phi^3 + \frac{u}{4}\phi^4 + \frac{1}{6}\phi^6 - h\phi$$

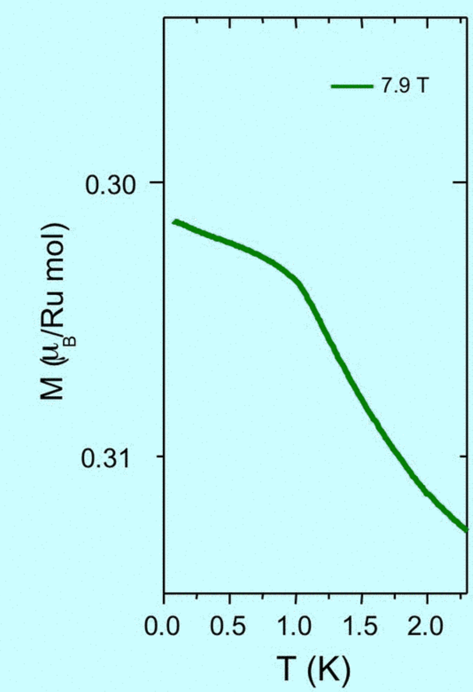
$$\beta F'[\phi;\psi] = \gamma (M_c + \phi)^2 \psi + \beta F'[\psi]$$

Integrate out the other field, ψ. Find that

$$u \to u - 4\gamma \langle \psi^2 \rangle$$

So could change sign of u. If so we get another line of first order transitions

Divergence of magnetisation cut off as a function of *T*



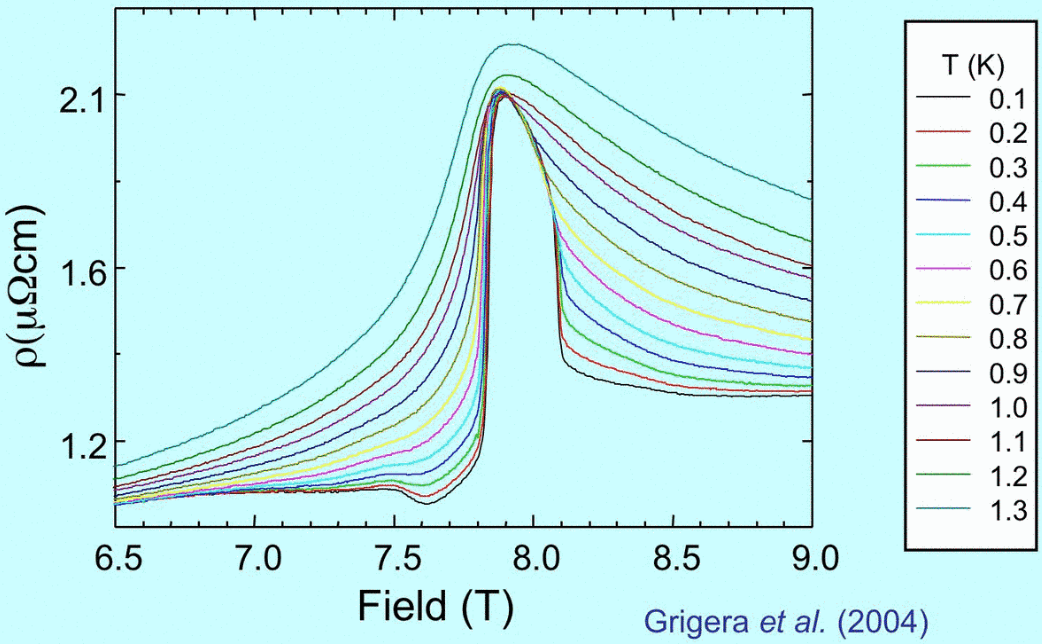
Thermodynamic hint at entry into a new phase.

Divergences as underlying QCP is approached are cut off in both field and temperature.

Grigera *et al.* (2004)

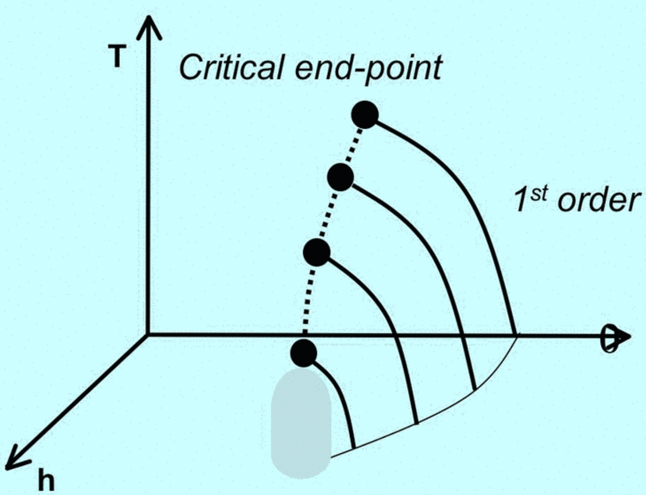
Characteristics of this unusual region:

“Saturation” of the temperature dependence of ρ inside the peak

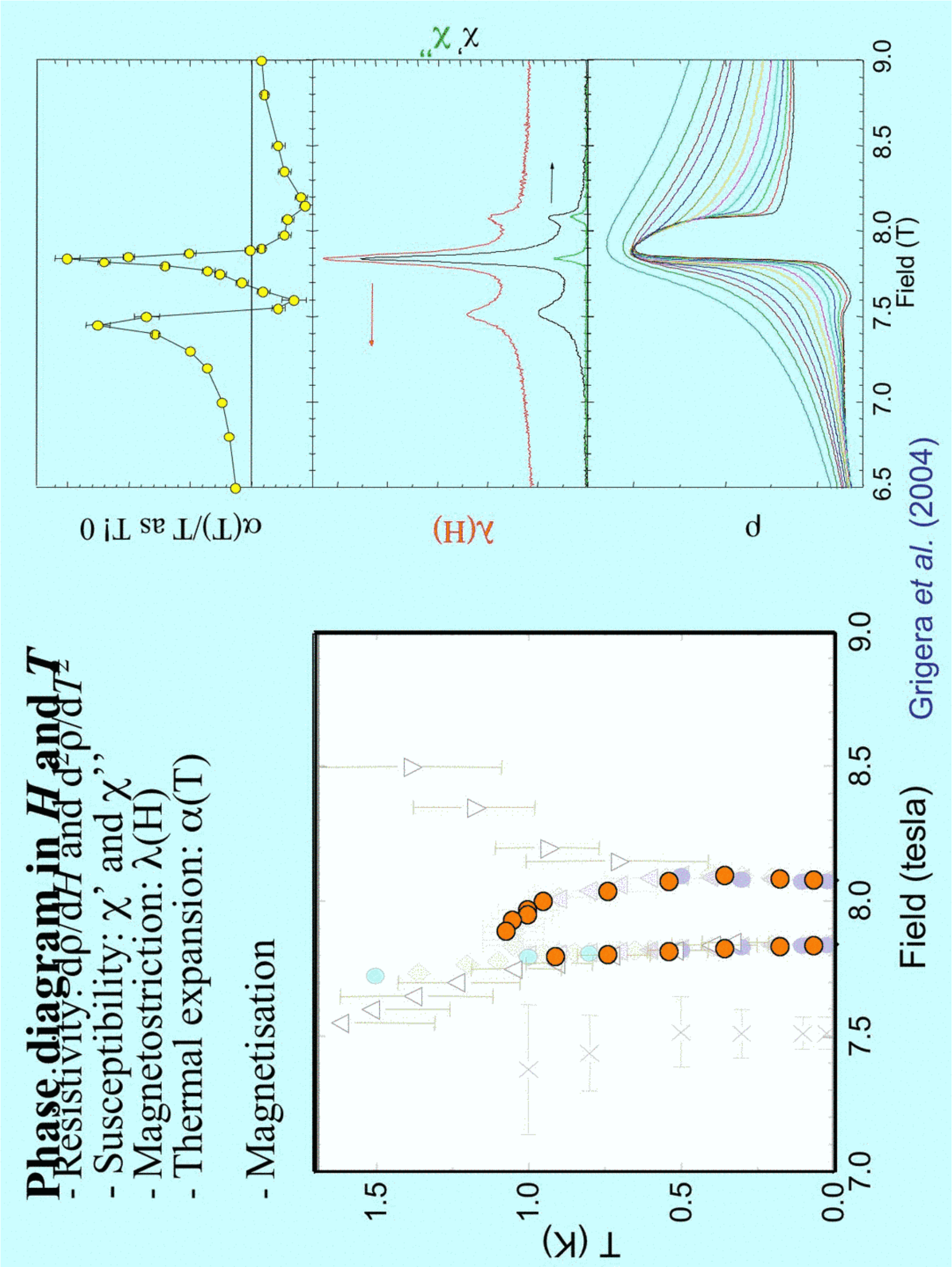


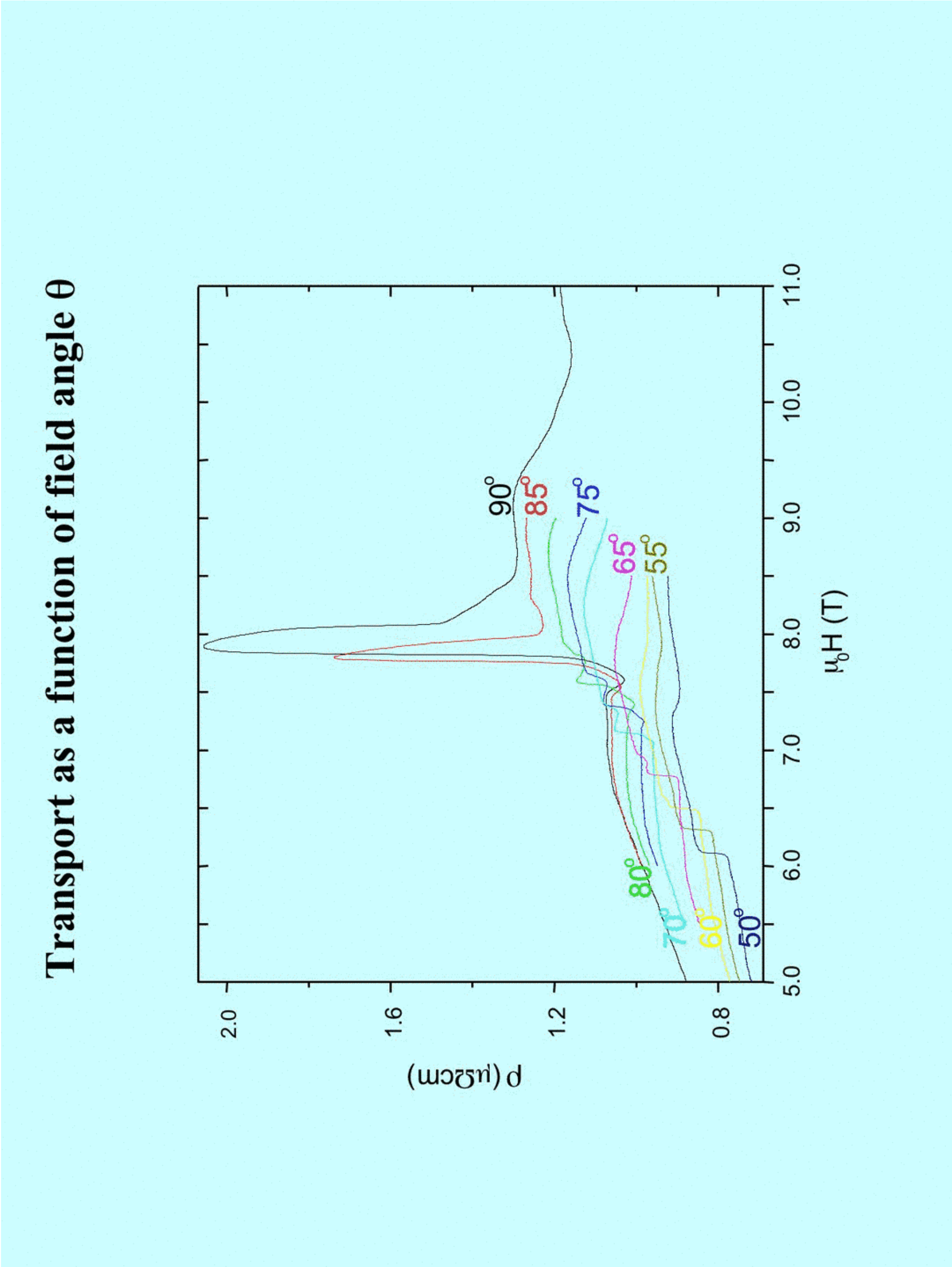
Grigera *et al.* (2004)

Suggestion: another phase at the quantum critical endpoint.



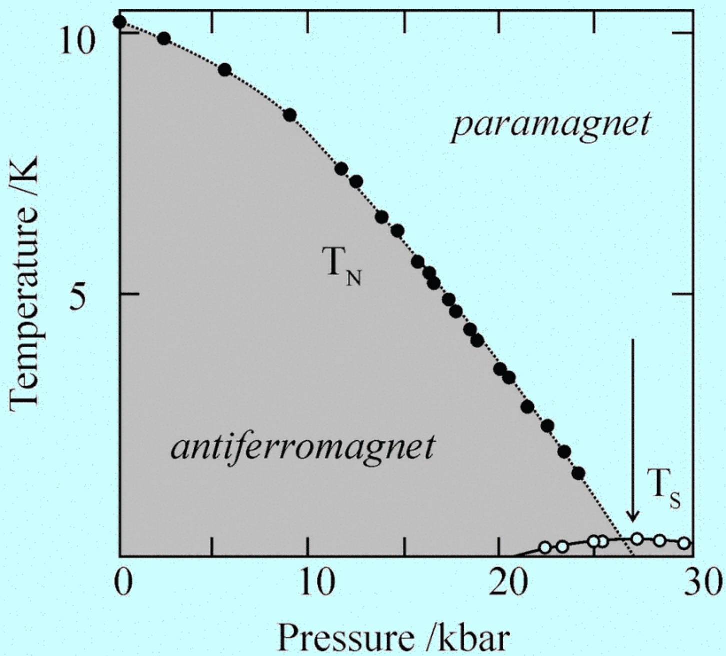
Expectation – a phase would be well-defined in temperature, field and field angle.

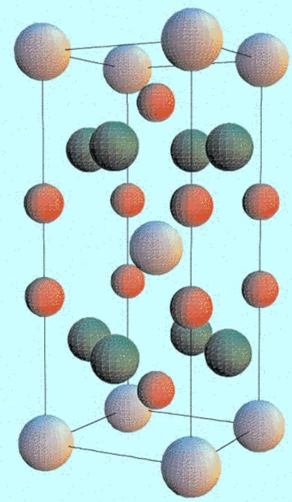

Phase diagram in H and T
- Resistivity: dρ/dH and d²ρ/dT²
- Susceptibility: χ' and χ''
- Magnetostriction: λ(H)
- Thermal expansion: α(T)
- Magnetisation
T (K)
Field (tesla)
α(T)/T as T→0
λ(H)
ρ
χ' χ''
Field (T)
Grigera et al. (2004)


Transport as a function of field angle θ
ρ (μΩcm)
μ0H (T)
90°
85°
80°
75°
70°
65°
60°
55°
50°

Phases at quantum critical points: superconductivity

e.g. $CePd_2Si_2$ under pressure

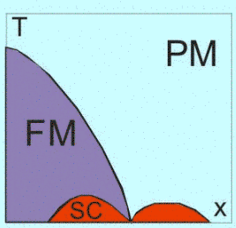




S. R. Julian *et al.* J. Phys: Condens. Matt., **8**, 9675 (1996)

Aside: superconductivity at a FM QCP: triplet superconductivity

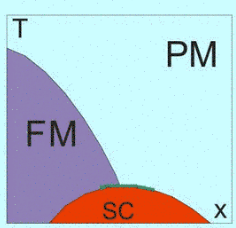
- Lots of theoretical work:



T_c=0 at the QCP: D. Fay and J. Appel, Phys. Rev. B **22**, 3173 (1980): Ph. Monthoux and G.G. Lonzarich, Phys. Rev. B **59**, 14598 (1999).

T_c a minimum at the QCP: R. Roussev and A. J. Millis, PRB 63, 140504 (2001).

T_c higher in FM phase than PM: T. R. Kirkpatrick, D. Belitz, Thomas Vojta and R. Narayanan, PRL 87, 127003 (2001).

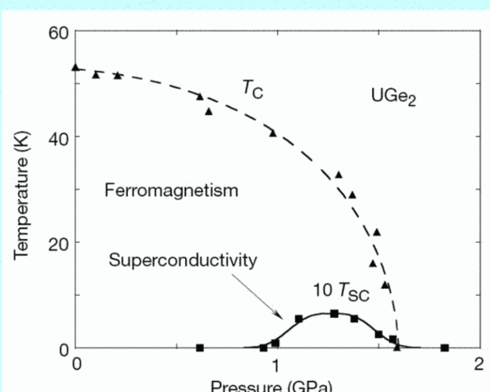


Transition goes first order near the QCP: A. V. Chubukov, A. M. Finkel'stein, R. Haslinger and D. K. Morr, PRL 90, 077002 (2003).

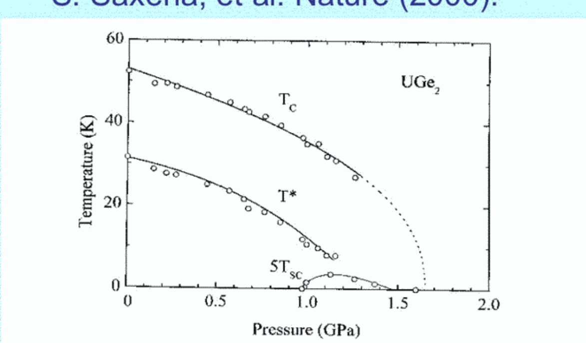
Aside: Experimental examples?

- Arguably Sr_2RuO_4: p-wave superconductor
  K. D. Nelson, Z. Q. Mao, Y. Maeno, Y. Liu , Science **306**, 1151 (2004).
- URhGe: superconducting ferromagnet
  Dai Aoki, Andrew Huxley, Eric Ressouche, Daniel Braithwaite, Jacques Flouquet, Jean-Pascal Brison, Elsa Lhotel and Carley Paulsen, Nature **413**, 613 (2001).
- UGe_2: pressure tuned ferromagnetic superconductor
  S. S. Saxena, P. Agarwal, K. Ahilan, F. M. Grosche, R. K. W. Haselwimmer, M. J. Steiner, E. Pugh, I. R. Walker, S. R. Julian, P. Monthoux, G. G. Lonzarich, A. Huxley, I. Sheikin, D. Braithwaite and J. Flouquet, Nature **406**, 587 (2000).

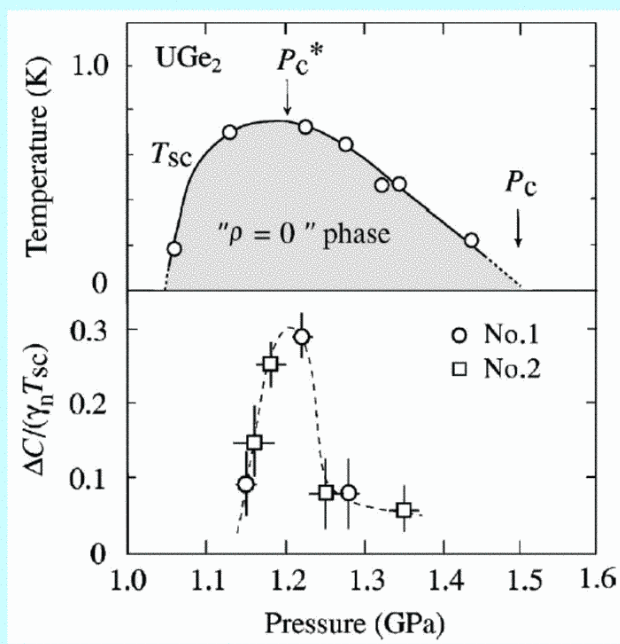
UGe_2: is it really anything to do with a FM "QCP"



S. Saxena, et al. Nature (2000).



Tateiwa *et al*. J Phys Condens Matter (2001)



N. Tateiwa *et al*., PRB 69 180513(R) (2004).

Looks like T* is responsible for pairing

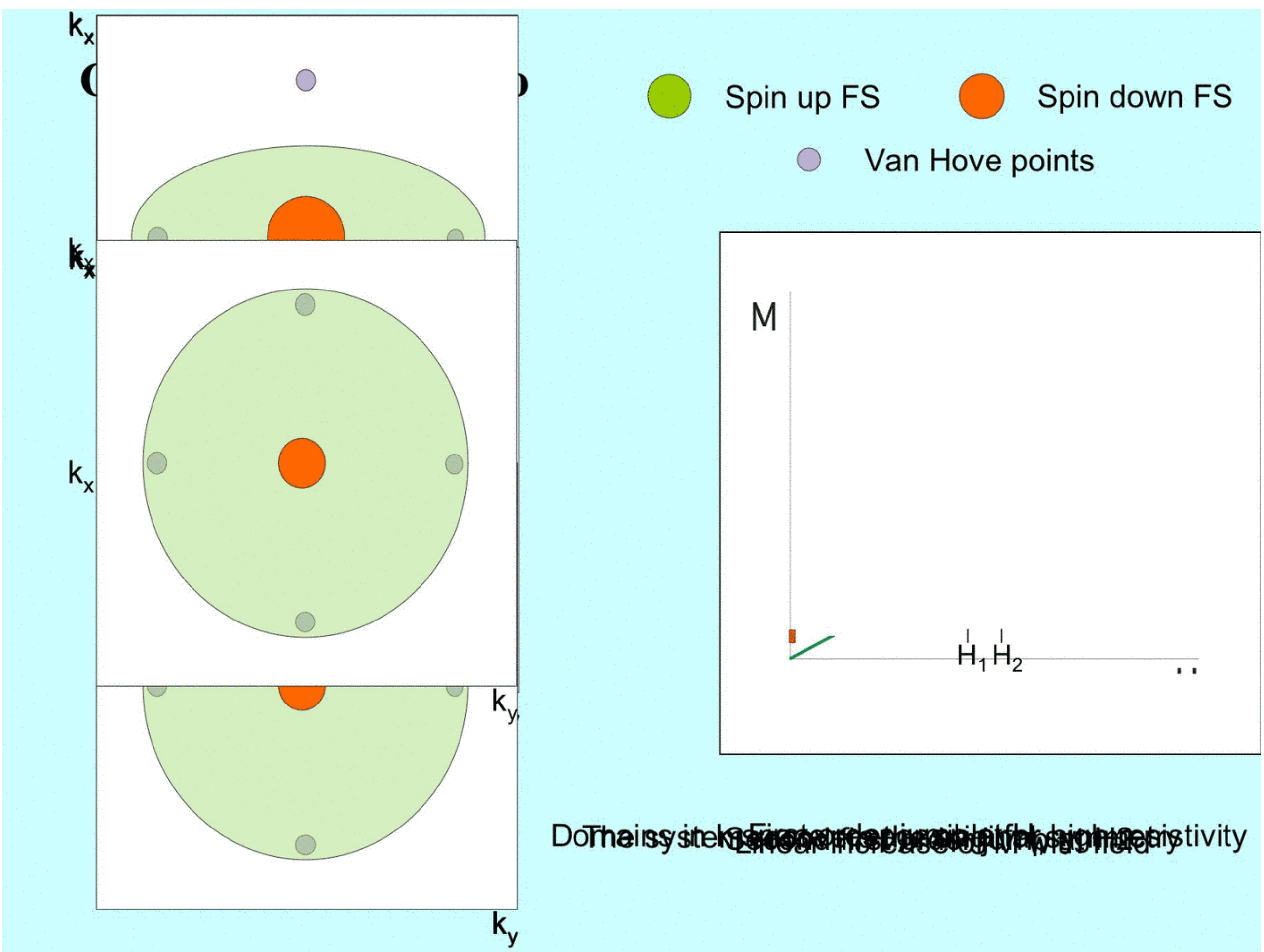
What about the anomalous region in $Sr_3Ru_2O_7$?

1. A theory based on metal physics: dHvA oscillations are observed both above and below the metamagnetic transition.
R.A. Borzi, S.A. Grigera, R.S. Perry, N. Kikugawa, K. Kitagawa, Y. Maeno and A.P. Mackenzie, Phys. Rev. Lett ***92****, 216403 (2004).*

2. Field-dependent transitions which only increase the moment.

3. A temperature dependent transition that does not give a sudden change of moment.

4. Something leading to the formation of domains, our only plausible explanation for the behaviour of the resistivity in the anomalous phase.



Conclusions – open questions

- Where is the ferromagnetic quantum critical point?
- What is the connection between various approaches to the non-analytic terms in the effective action?
- Are other (non-superconducting) phases induced by proximity to a quantum critical endpoint?