Quantum Criticality in String/M Theory

KITP Conference on Quantum Phase Transitions

January 21, 2005

Various dualities have revealed the existence of a large number of scale invariant (actually, conformally invariant) quantum field theories, especially in 2, 3, 4, and 6 spacetime dimensions. Many of these are strongly coupled, and many have no known Lagrangian description.

I will focus on the most general and powerful example, AdS/CFT duality:

conformally invariant field theory in d spacetime dimensions = string or M theory in (d+1)-dimensional anti-de Sitter space (times a compact space to give total dimension 10 or 11).

Outline:

- $\mathcal{N} = 4$, $d = 4$ supersymmetric gauge theory = IIB string theory in $AdS_5 \times S^5$ (the most-studied example).
- Other examples and extensions.
- Some remarks on emergent gravity.

One side of the duality: $\mathcal{N} = 4$, $d = 4$ supersymmetric gauge theory:

- $SU(N_c)$ gauge theory with 4 2-component fermions ψ and 6 real scalars ϕ, all in the $N_c \times N_c$ adjoint representation.
- Specific $\psi\psi\phi$ and $\phi\phi\phi\phi$ couplings governed by a single coupling constant, the gauge coupling g.
- The coupling g doesn't run, $\beta(g) = 0$, so there is a line of fixed points $0 \le g \le \infty$.
- For $g^2 N_c << 1$ ordinary perturbation theory gives the anomalous dimensions and correlation functions.

Other side of the duality, when $g^2 N_c$ and N_c are both large: IIB in theory in $AdS_5 \times S^5$. The metric of AdS_5 is

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$$

Five coordinates: the usual $x^{0,1,2,3}$ plus z, where $0 < z < \infty$. $z = 0$ is a *boundary*; $z = \infty$ is a *horizon.*

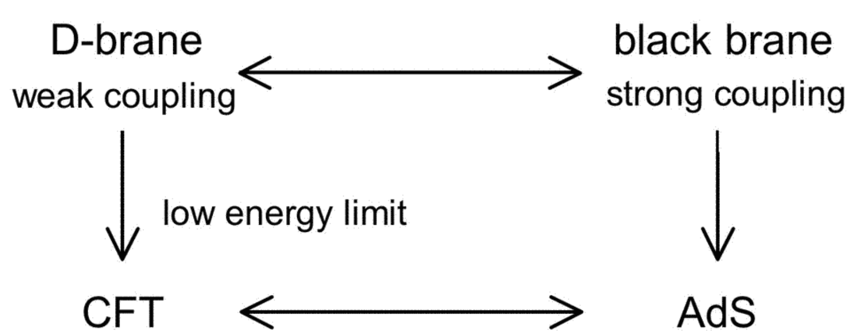
$$R = (g^2 N_c)^{1/4}, \;\; g_{\text{string}} = g^2.$$

Duality is a complete equivalence, the same theory in different variables, like Sine-Gordon/Thirring and high-T Ising/low-T Ising.

Unlike those examples, we do not have an explicit transformation between the two descriptions.

`Derivation' (Maldacena):

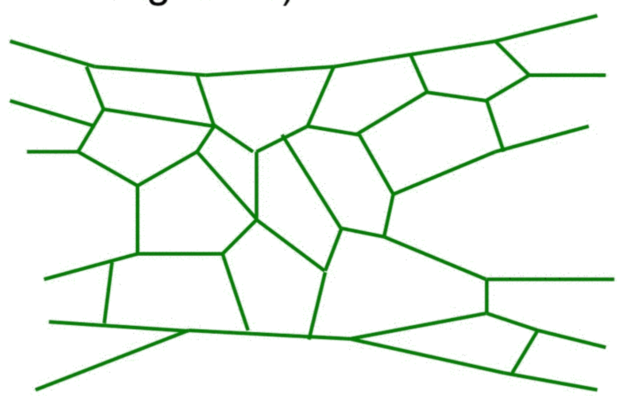
Studies of black hole quantum mechanics in string theory are largely based on the adiabatic continuation



This explained surprising agreements between seemingly very different calculations.

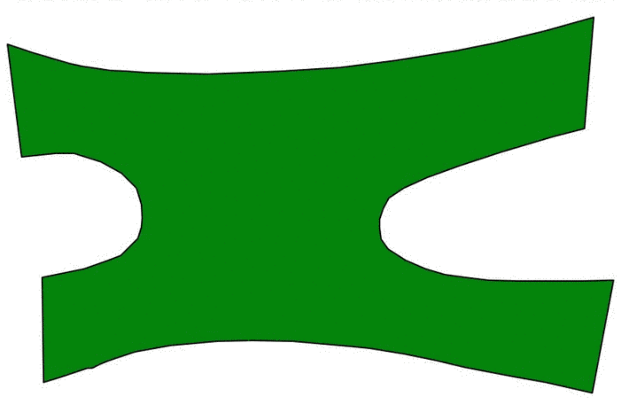
Another motivation ('t Hooft, 1974):

In the large N_c limit gauge theory reduces to planar graphs (to leading order):



Another motivation ('t Hooft, 1974):

At strong coupling it is plausible that the vertices become dense and form a continuous sheet:



Thus, the strongly coupled dynamics would be given by a string theory. AdS/CFT realizes this in a precise but unexpected way.

Dictionary for observables:

In the field theory take a complete set of local operators $O_i(x)$ of dimension Δ_i. For each such operator there is a field $\phi_i(x,z)$ in the AdS space, with $m_i^2 R^2 = \Delta_i(\Delta_i - 4)$, and

$$\mathcal{O}_i(x) = \lim_{z \to 0} z^{-\Delta_i} \phi_i(x, z)$$

Thus the dimensions and correlators in the strongly coupled gauge theory can be calculated on the AdS side. The leading order in $1/N_c$ is given by tree graphs in AdS, with curved spacetime propagators.

How can 5 = 4? I.e. where does z come from?

- The AdS description is useful only when N_c is large, so there are *many* degrees of freedom. Remarkably, they organize themselves in a way that is local in a new dimensions.
- The metric $ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu + dz^2)$ is invariant under

$$x^\mu \to \lambda x^\mu \ , \quad z \to \lambda z$$

Thus, the scale invariance is a *geometric* symmetry of the AdS space, and z seems to be associated with the *scale.*

Relation between z and scale:

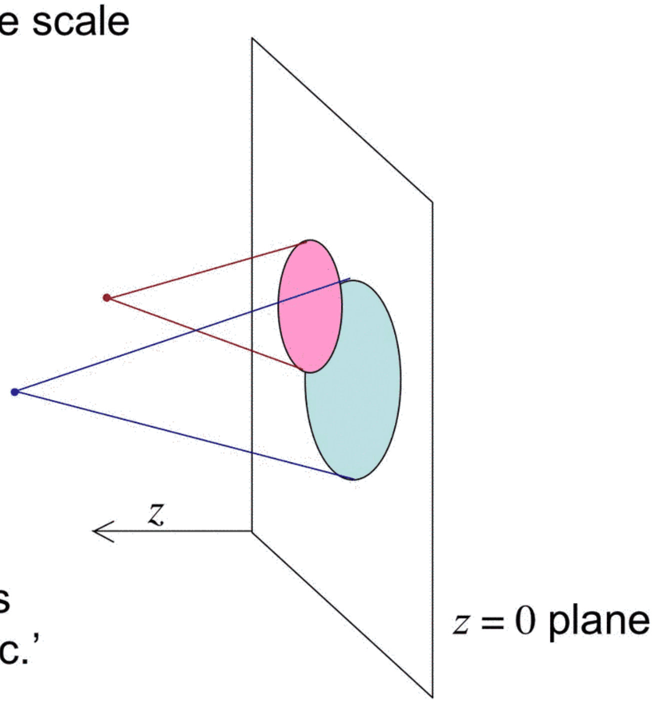
The AdS metric $ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$ is *warped*: its normalization depends on the position in the z-direction. There is a gravitational redshift,

$$E_{4-\text{dimensional}} = \frac{R}{z} E_{10-\text{dimensional}}$$

The AdS theory has a characteristic energy/momentum scale $E_{\text{10-dimensional}} = 1/R$.

Thus, $E_{\text{4-dimensional}} \sim z$: the z coordinate of a typical excitation is its energy/momentum scale (inverse length scale) in the gauge theory.

Thus z comes from the scale of the field theory:



The 5 to 4 mapping is nonlocal, `holographic.'

Another point: suppose that we heat the system up. On the gauge side, get a plasma of gauge bosons etc. On the AdS side get a `black brane' in place of the AdS horizon. Gauge theory entropy = Hawking entropy.

Transport properties are obtained from classical field equations on the AdS/CFT side.

Curiously, this gives the best estimate for the viscosity as seen at RHIC.

Further examples:

The same gauge theory reduced to $d = 3$ flows to strong coupling at low energy, but a massless sector survives. The dual theory is M theory on $AdS_4 \times S^7$.

Any string solution of the form $AdS_{d+1} \times M$ should have a CFT dual, through the dictionary

$$\mathcal{O}_i(x) = \lim_{z \to 0} z^{-\Delta_i} \phi_i(x, z)$$

The number of such solutions is probably very large, considering the possible topologies for M. We have little to no understanding of the dual CFTs.

Recent `string landscape' ideas suggest $10^{O(100)}$ `sporadic' solutions (cf. 5 sporadic Lie algebras and 26 sporadic finite groups).

This duality is not limited to conformal theories. One can add relevant perturbations O_i in the gauge theory, and the dictionary $\mathcal{O}_i(x) = \lim_{z\to 0} z^{-\Delta_i}\phi_i(x,z)$ implies that in the AdS theory one adds a boundary action, or (equivalently) modifies the boundary conditions.

If the field theory has a mass gap m, the infrared range of the coordinate z is cut off at $z_{\rm max} \sim m$.

In this way one can get theories that are qualitatively similar to QCD (mass gap, confinement, chiral symmetry breaking).

Does every CFT define a solution to string theory? This is harder to say. Klebanov and Polyakov have conjectured a dual description of the Wilson-Fisher fixed point, but it is rather degenerate (zero string tension limit) and poorly understood.

Fermi liquids?

Emergent Gravity

- Why it is difficult
- How it can arise

Emergent gauge symmetry is now a familiar idea
(D'Adda, DiVecchia, Luscher; Witten; Forster, Nielsen, Ninomiya; many QHE and high-T_c models)

Important points:

- The gauge symmetry is invisible in the underlying theory, it acts only on emergent quantities:

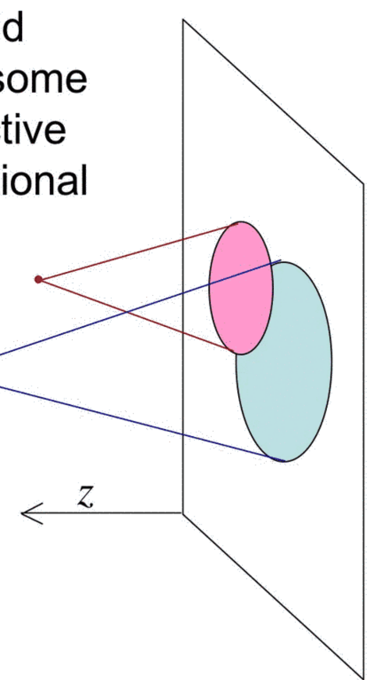
$$e(x) = f(x)b^\dagger(x) \; ;$$
$$f(x) \rightarrow e^{i\lambda(x)} f(x) \; , \quad b(x) \rightarrow e^{i\lambda(x)} b(x) \; .$$

- A global $U(1)$ symmetry is not demoted to a gauge symmetry, the gauge symmetry emerges `from nothing.'

In general relativity, the time and space coordinates are gauge-dependent. They are therefore not the same as the time and space coordinates of the underlying theory, which are physically observable. *The emergence of gravity requires the emergence of spacetime itself.*

In general relativity, there are no gauge invariant local observables. To define $\phi(x)$ requires a system of rods and clocks to define the coordinates, and these depend nonlocally on the dynamical metric. An underlying nongravitational theory would have local observables, and so this theory must be related *nonlocally* to the emergent spacetime (cf. Weinberg-Witten).

Nevertheless, these conditions can be satisfied: AdS/CFT is an example. Starting with a nongravitational field theory, the CFT, one finds that in some parts of parameter space the effective description is in terms of a gravitational theory, in adifferent (higher dimensional) spacetime. The relation is indeed nonlocal:



Puzzles:

• More complete understanding of the emergence of gravity, without the special AdS boundary.

• The emergence of *time …*