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Negative Refraction in Metamaterials

David R. Smith
David Schurig
Sheldon Schultz
Marshall Rosenbluth
Norman Kroll

Physics Department, University of California, San Diego

John Pendry
Anantha Ramakrishnan

Imperial College, London

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Collaborators & Funding

- Prof. Sia Nemat-Nasser (UCSD)
- Prof. Xiang Zhang (UCLA/MURI)
- Dr. Minas Tanelian (Boeing)
- Prof. Dimitri Basov (UCSD)
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- DARPA
- ONR
- AFOSR
- NSF
- DOE

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Topics

- **Negative Refraction**
- **Material Characterization**
- **Experiments on Negative Refraction**
- **The Perfect Lens – Calculations**
- **Modulated Beams**

Materials with negative ε/μ are dispersive

$$\frac{\partial(\omega\varepsilon_r)}{\partial\omega} > 1 \quad \varepsilon_r(\omega\to\infty)\to +1$$

$$\frac{\partial(\omega\mu_r)}{\partial\omega} > 1 \quad \mu_r(\omega\to\infty)\to +1$$

$$\frac{\partial(\omega n)}{\partial\omega} > 1 \quad n(\omega\to\infty)\to +1$$

- Large bandwidth possible
- Excessive losses not implied

Refractive Index in Dispersive Media

Reasonable causal forms for ε and μ:

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega^2} \qquad \mu(\omega)=1-\frac{F\omega_0{}^2}{\omega^2-\omega_0^2-i\omega\Gamma}$$

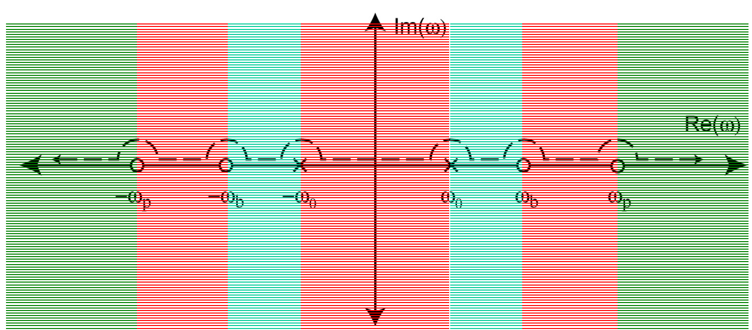
Lead to a dispersive index:

$$n(\omega)=\sqrt{\varepsilon}\sqrt{\mu}=\pm\frac{1}{\omega}\sqrt{\frac{(\omega^2-\omega_b^2)(\omega^2-\omega_p^2)}{(\omega^2-\omega_0^2)}}$$

Refractive Index in Dispersive Media

$$n(\omega)=\sqrt{\varepsilon}\sqrt{\mu}=\pm\frac{1}{\omega}\sqrt{\frac{(\omega^2-\omega_b^2)(\omega^2-\omega_p^2)}{(\omega^2-\omega_0^2)}}$$

Re(n)>0
evanescent
Re(n)<0



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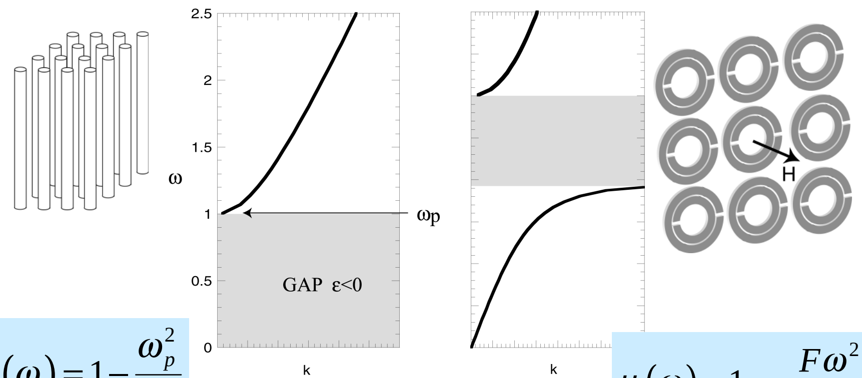
Building Blocks for Negative Index Media



$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}$$

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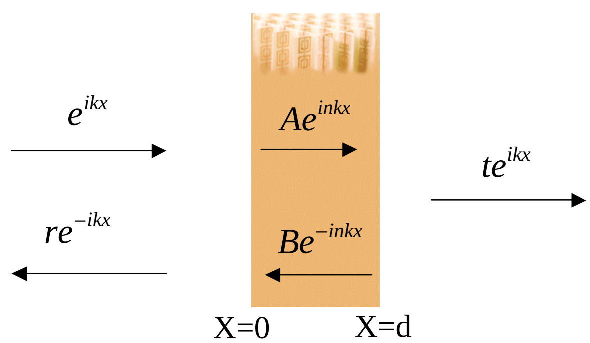
Characterization by S-parameter Analysis



Difficulty: S-parameters for metamaterials more complex!

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Inverting t and r to find n and z

A continuous material can be characterized by two complex variables t and r, or n and z.

$$t^{-1} = \left[\cos(nkd) - \frac{i}{2}\left(z + \frac{1}{z}\right)\sin(nkd)\right]e^{ikd}$$

$$\frac{r}{t} = -\frac{e^{ikd}}{2}\, i\left(z - \frac{1}{z}\right)\sin(nkd)$$

Inversion yields…

$$n = \frac{1}{kd}\cos^{-1}\left(\frac{1}{2t'}\left[1 - \left(r^2 - t'^2\right)\right]\right) + \frac{2\pi m}{kd}$$

$$z = \pm\sqrt{\frac{(1+r)^2 - t'^2}{(1-r)^2 - t'^2}}$$

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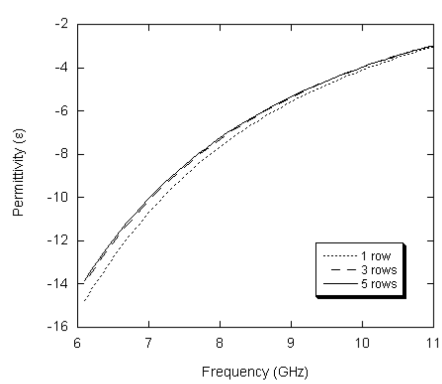
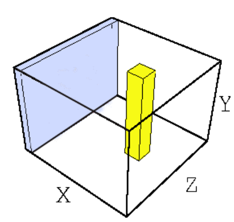
Wire Data from ISU Transfer-Matrix Calculations





$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

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Free Space Apparatus
Anechoic Chamber
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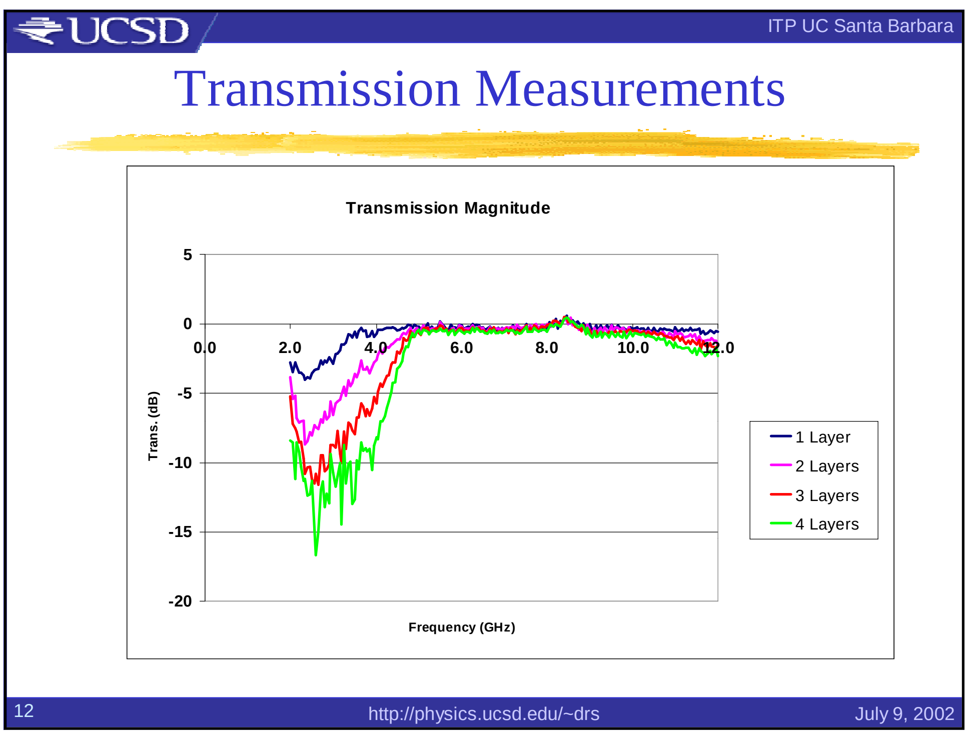
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Transmission Measurements
Transmission Magnitude
5
0
-5
-10
-15
-20
0.0
2.0
4.0
6.0
8.0
10.0
12.0
Trans. (dB)
Frequency (GHz)
1 Layer
2 Layers
3 Layers
4 Layers
12
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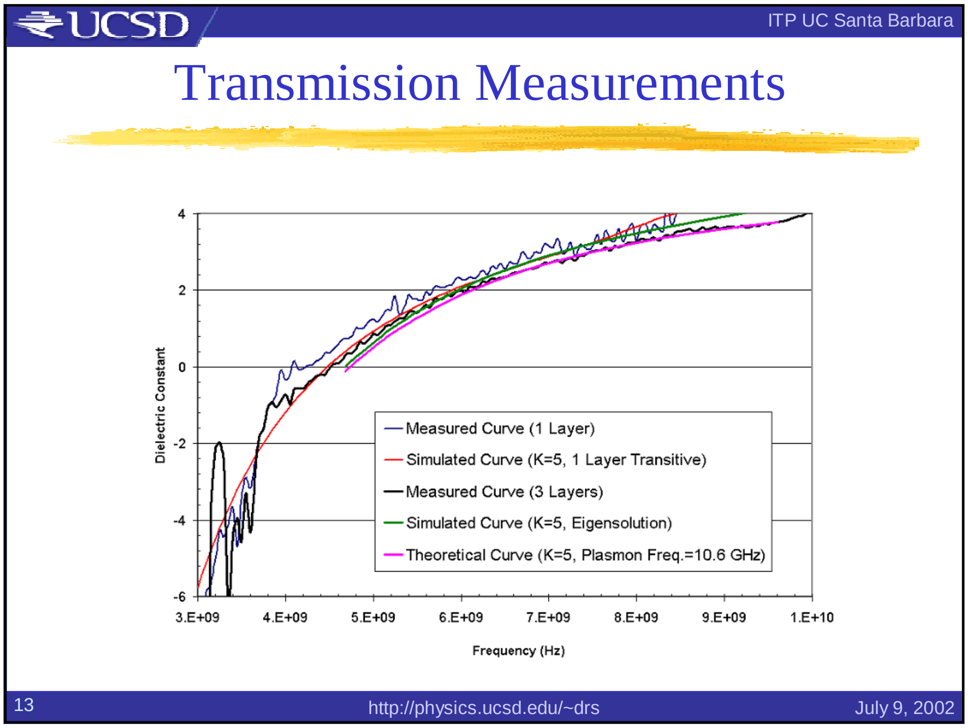
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Transmission Measurements
Dielectric Constant
Frequency (Hz)
Measured Curve (1 Layer)
Simulated Curve (K=5, 1 Layer Transitive)
Measured Curve (3 Layers)
Simulated Curve (K=5, Eigensolution)
Theoretical Curve (K=5, Plasmon Freq.=10.6 GHz)
3.E+09 4.E+09 5.E+09 6.E+09 7.E+09 8.E+09 9.E+09 1.E+10
4 2 0 -2 -4 -6
13
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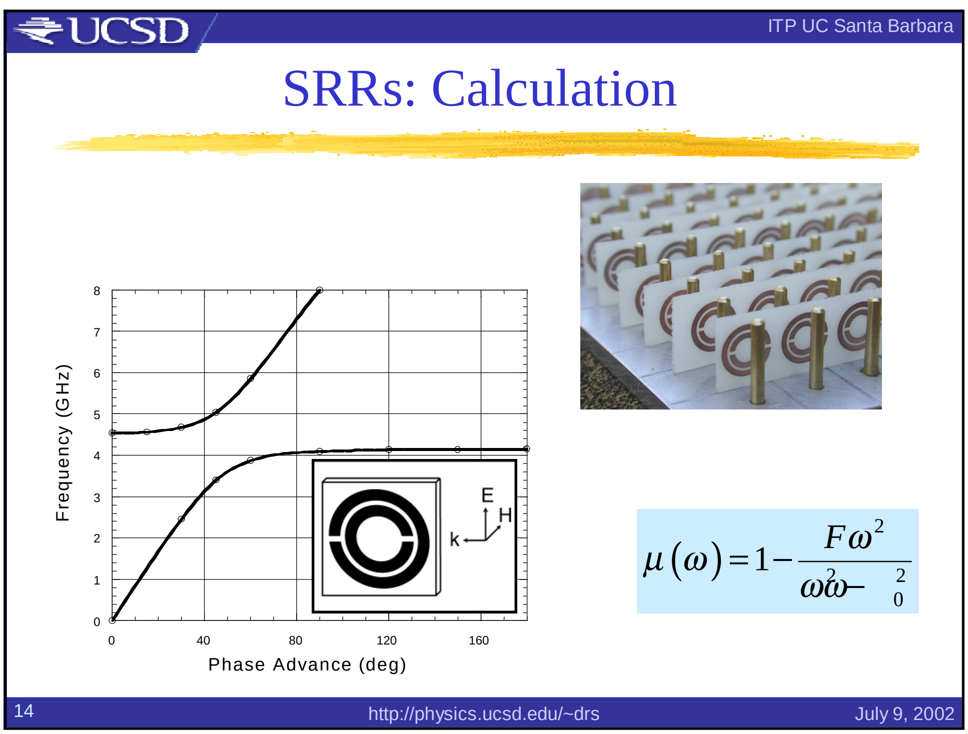
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SRRs: Calculation
Frequency (GHz)
Phase Advance (deg)
0 40 80 120 160
0 1 2 3 4 5 6 7 8
E
H
k
$\mu(\omega)=1-\frac{F\omega^2}{\omega^2-\omega_0^2}$
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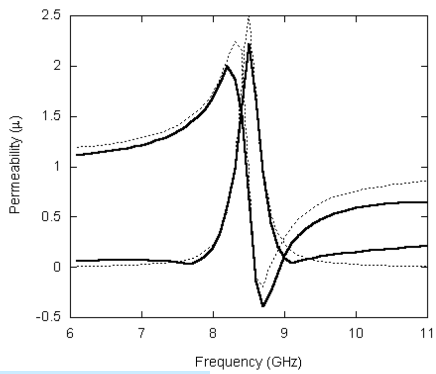
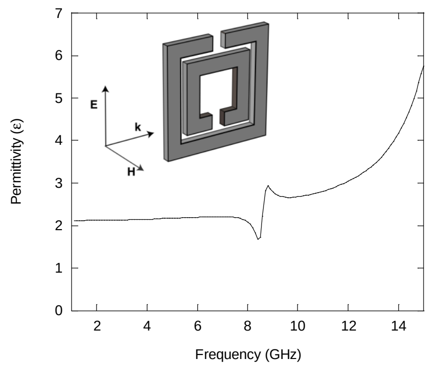
SRR μ and ε (from ISU Transfer-Matrix Calculations)





$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}$$

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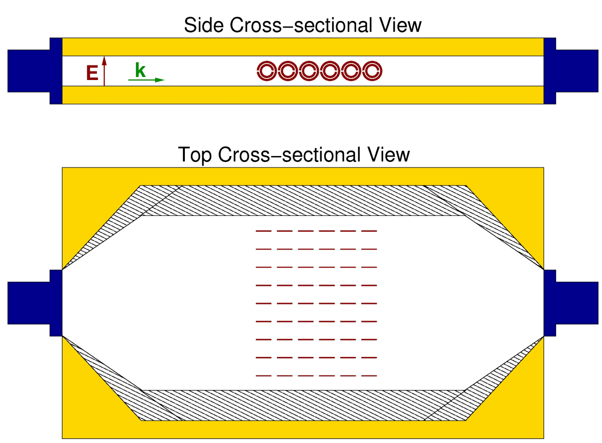
Depiction of the UCSD Experimental Apparatus



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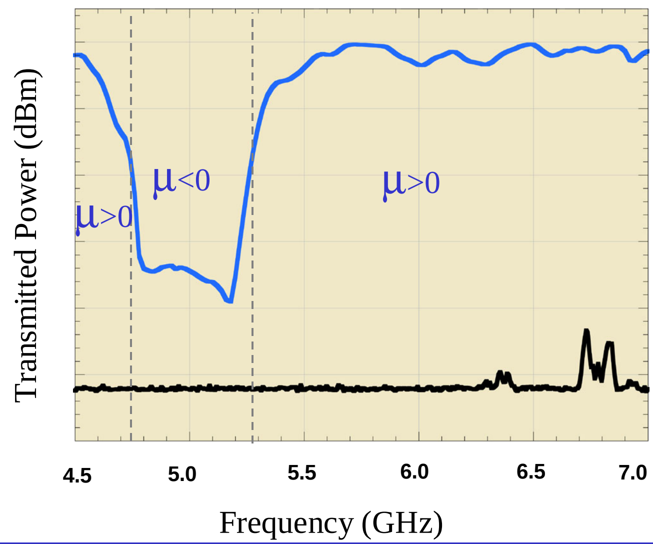
Transmission Measurements



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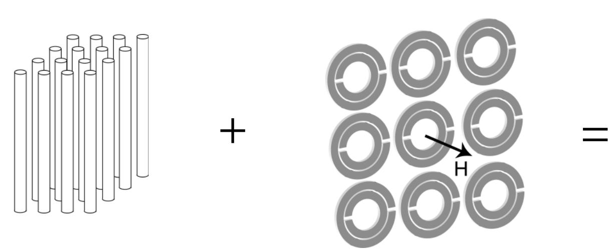
Prediction



Negative Refractive Index Material

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}$$

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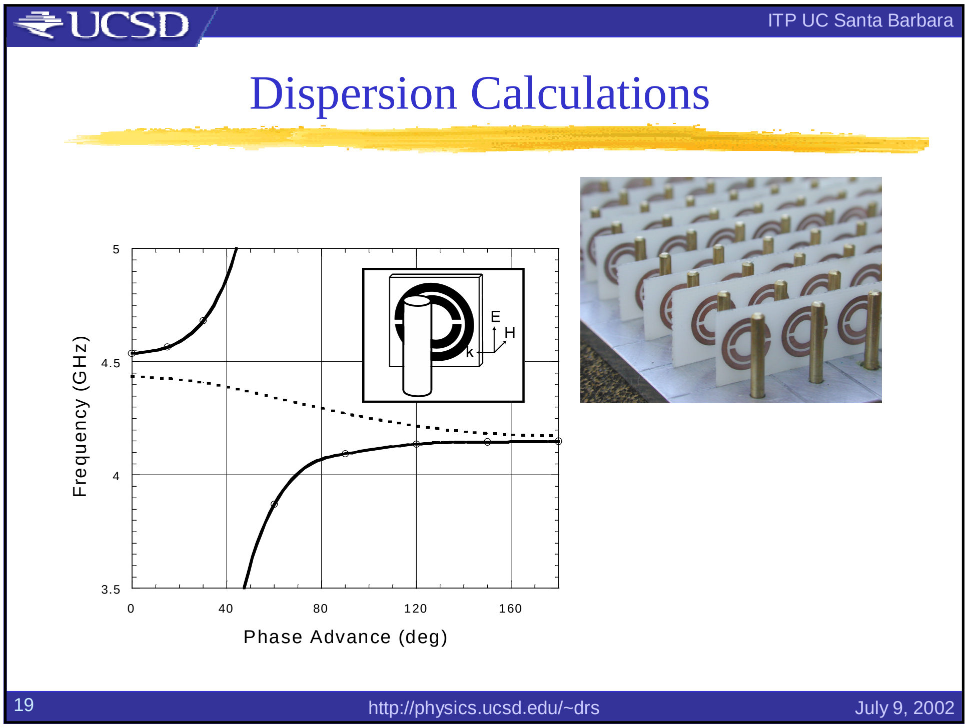
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Dispersion Calculations
Frequency (GHz)
5
4.5
4
3.5
0
40
80
120
160
Phase Advance (deg)
E
H
k
19
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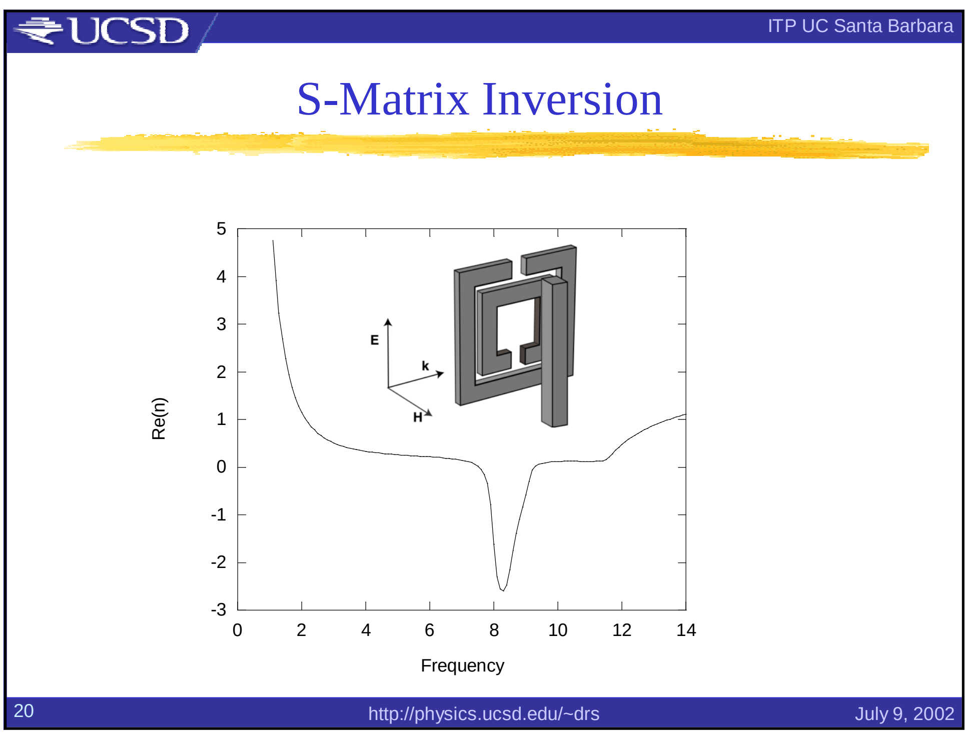
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S-Matrix Inversion
Re(n)
5
4
3
2
1
0
-1
-2
-3
0
2
4
6
8
10
12
14
Frequency
E
k
H
20
http://physics.ucsd.edu/~drs
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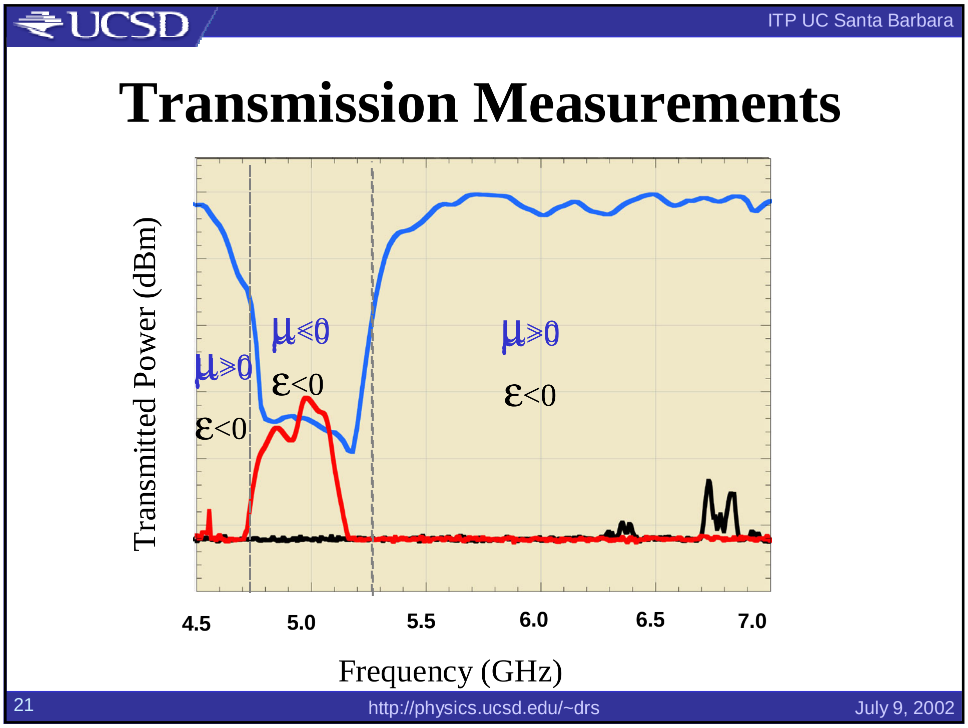
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Transmission Measurements
Transmitted Power (dBm)
μ>0
ε<0
μ<0
ε<0
μ>0
ε<0
4.5
5.0
5.5
6.0
6.5
7.0
Frequency (GHz)
21
http://physics.ucsd.edu/~drs
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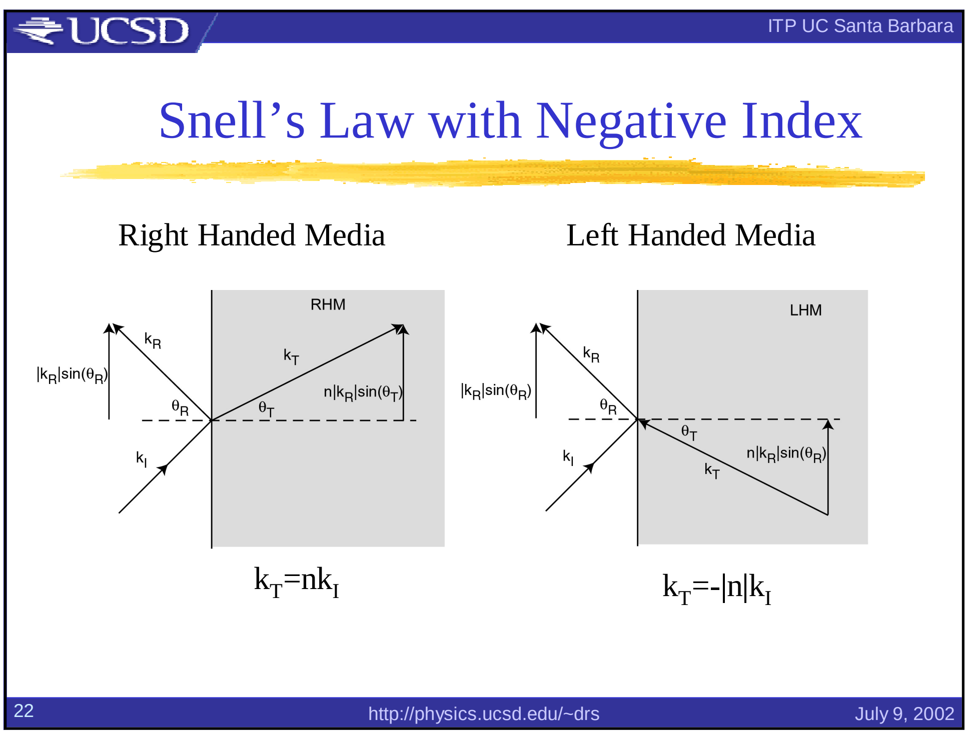
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Snell's Law with Negative Index
Right Handed Media
Left Handed Media
RHM
k_R
k_T
$|k_R|\sin(\theta_R)$
$n|k_R|\sin(\theta_T)$
θ_R
θ_T
k_I
LHM
k_R
$|k_R|\sin(\theta_R)$
θ_R
θ_T
$n|k_R|\sin(\theta_R)$
k_I
k_T
$k_T=nk_I$
$k_T=-|n|k_I$
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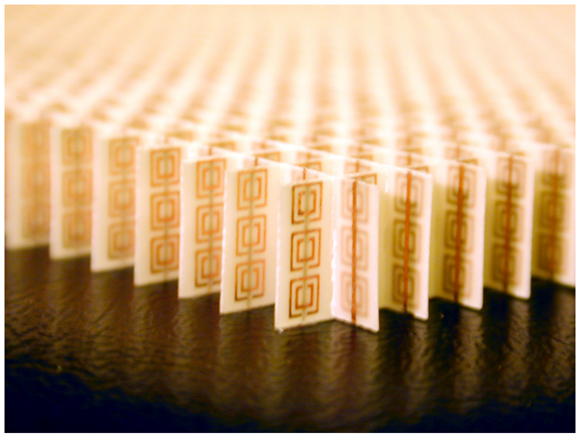
A 2-D Isotropic Structure



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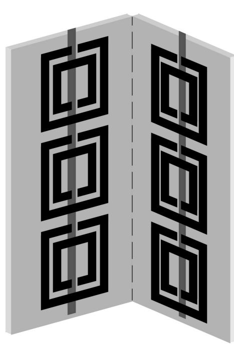
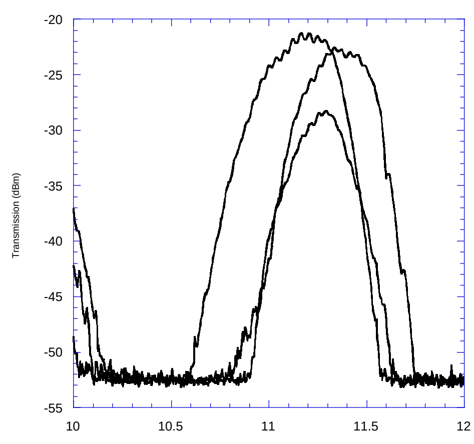
A 2-D Isotropic Structure





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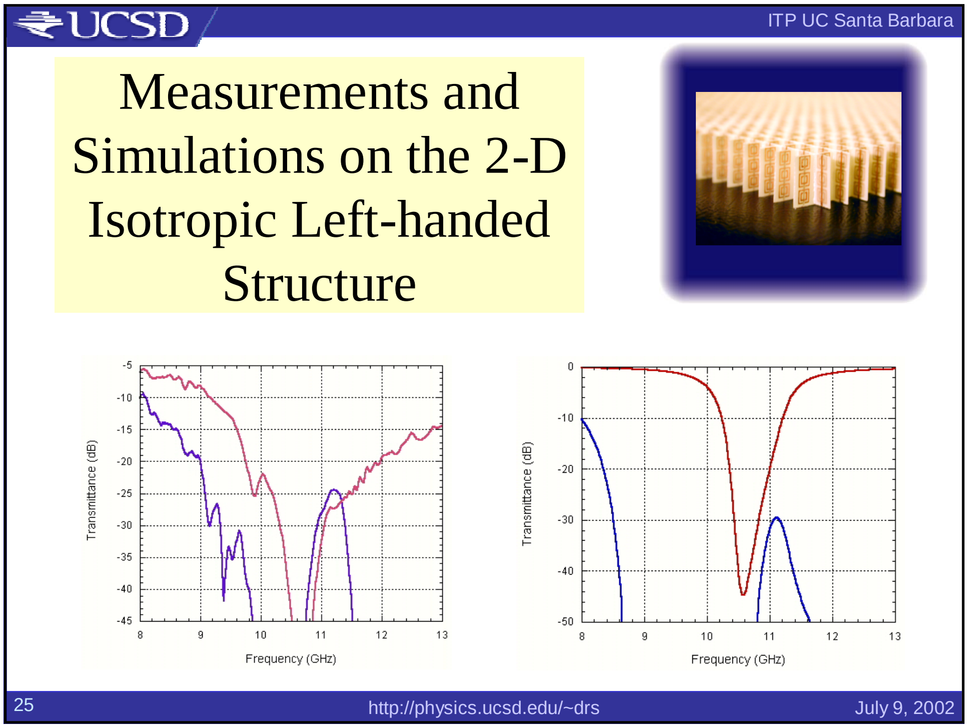
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Measurements and Simulations on the 2-D Isotropic Left-handed Structure
Transmittance (dB)
Frequency (GHz)
Transmittance (dB)
Frequency (GHz)
25
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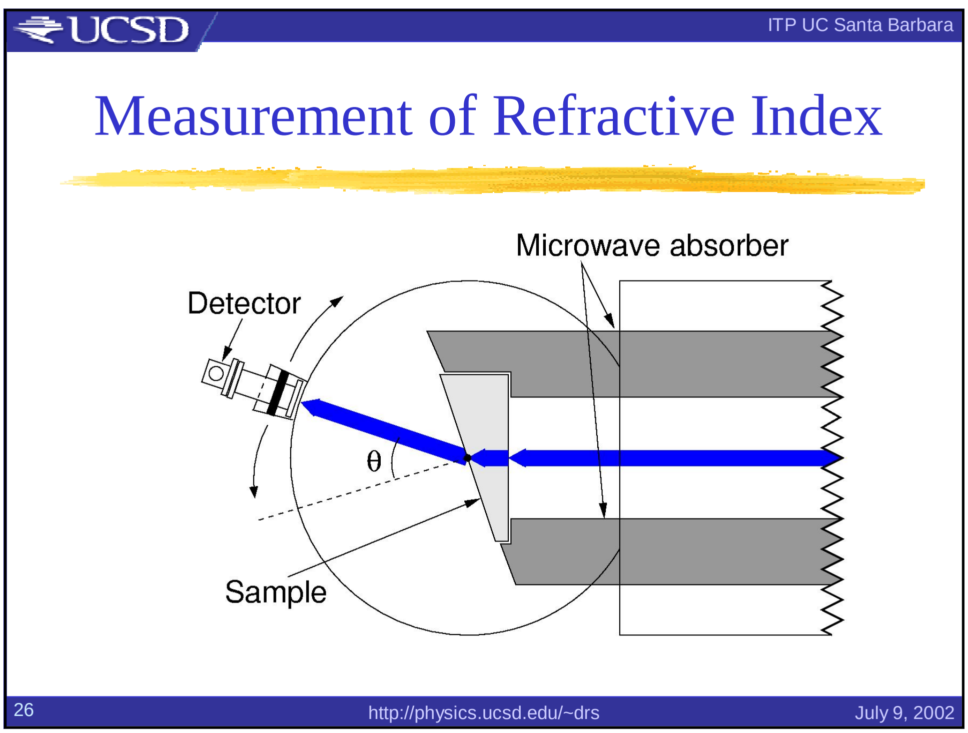
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Measurement of Refractive Index
Microwave absorber
Detector
θ
Sample
26
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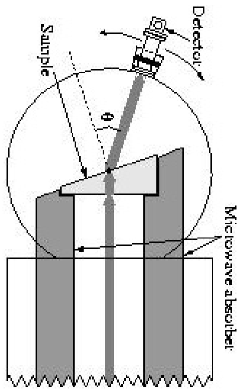
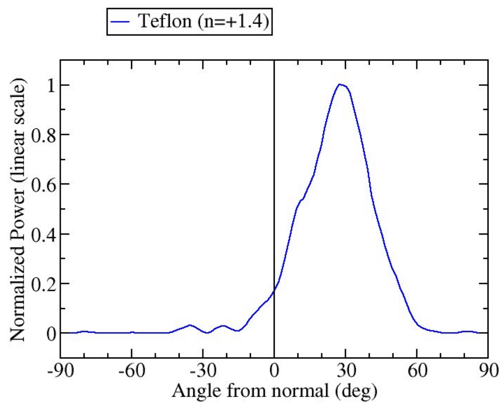
Measurement of Refractive Index





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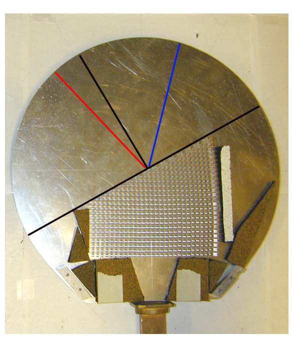
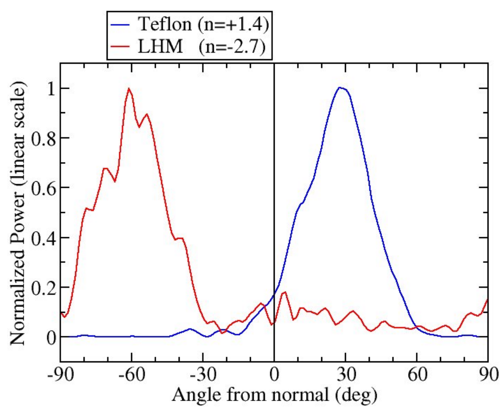
Measurement of Refractive Index





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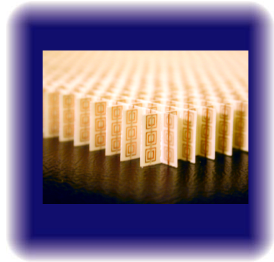
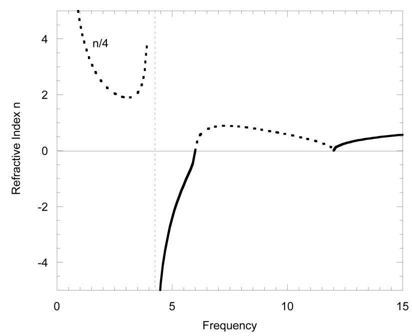
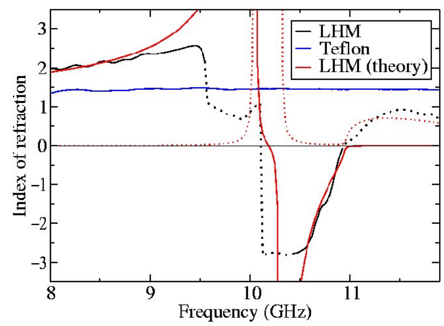
Frequency Dispersion Consistent with Theory







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Conclusion

- All data is consistent with negative index.
- For metamaterials, n<0 is physically meaningful
- How can negative index materials be useful?

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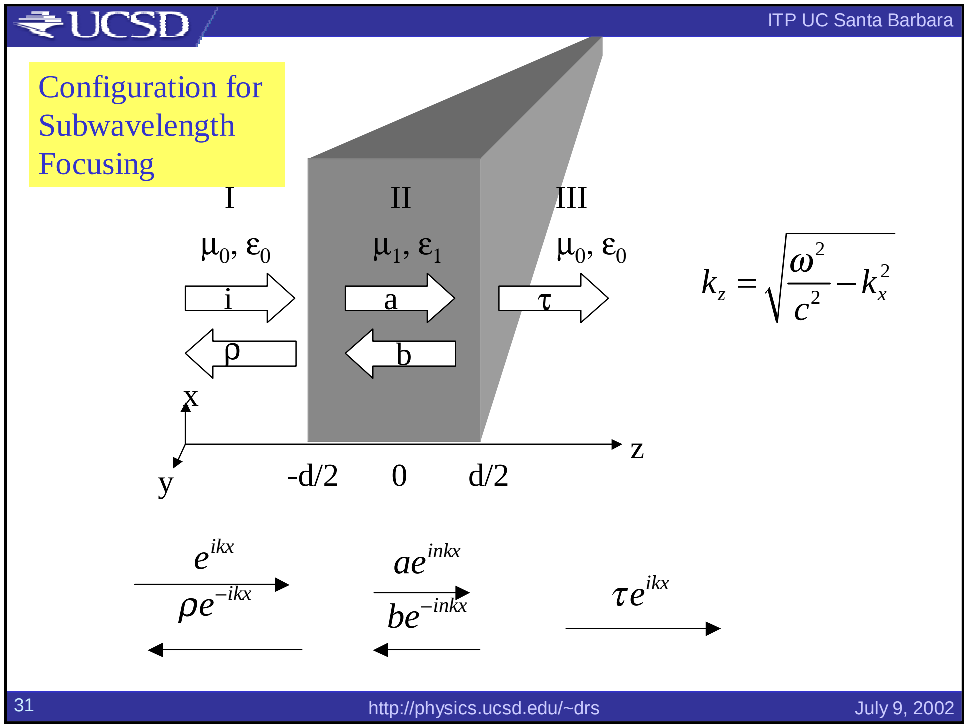
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Configuration for Subwavelength Focusing
I
II
III
μ_0, ε_0
μ_1, ε_1
μ_0, ε_0
i
a
τ
ρ
b
x
y
z
-d/2
0
d/2
$k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2}$
e^{ikx}
ρe^{-ikx}
ae^{inkx}
be^{-inkx}
τe^{ikx}
31
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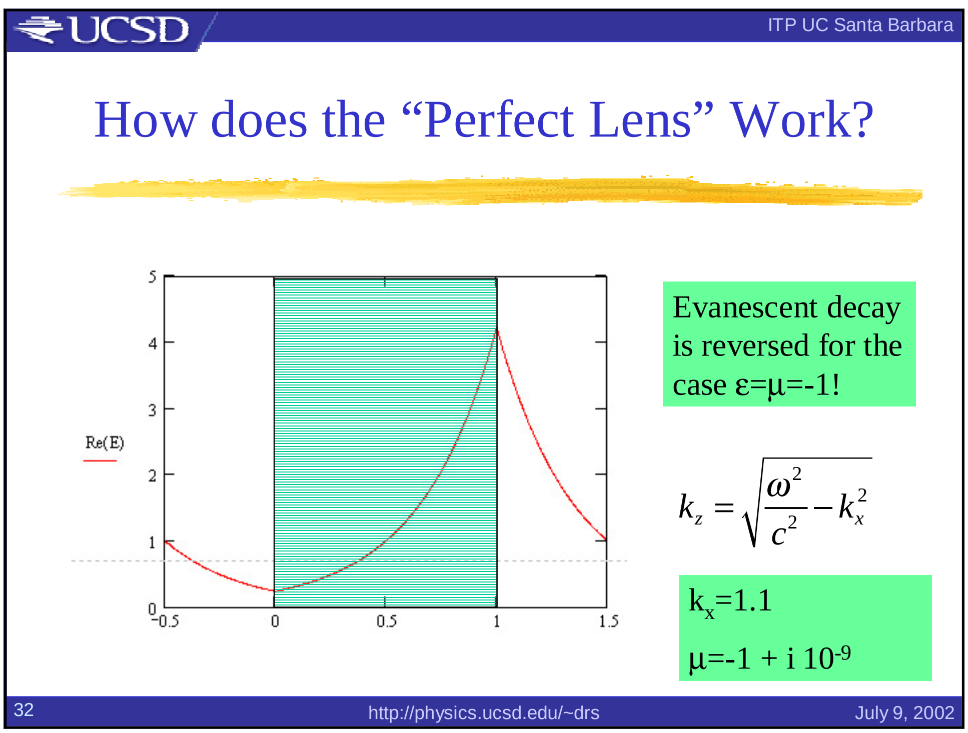
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How does the “Perfect Lens” Work?
Re(E)
Evanescent decay is reversed for the case ε=μ=-1!
$k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2}$
k_x=1.1
μ=-1 + i 10^{-9}
32
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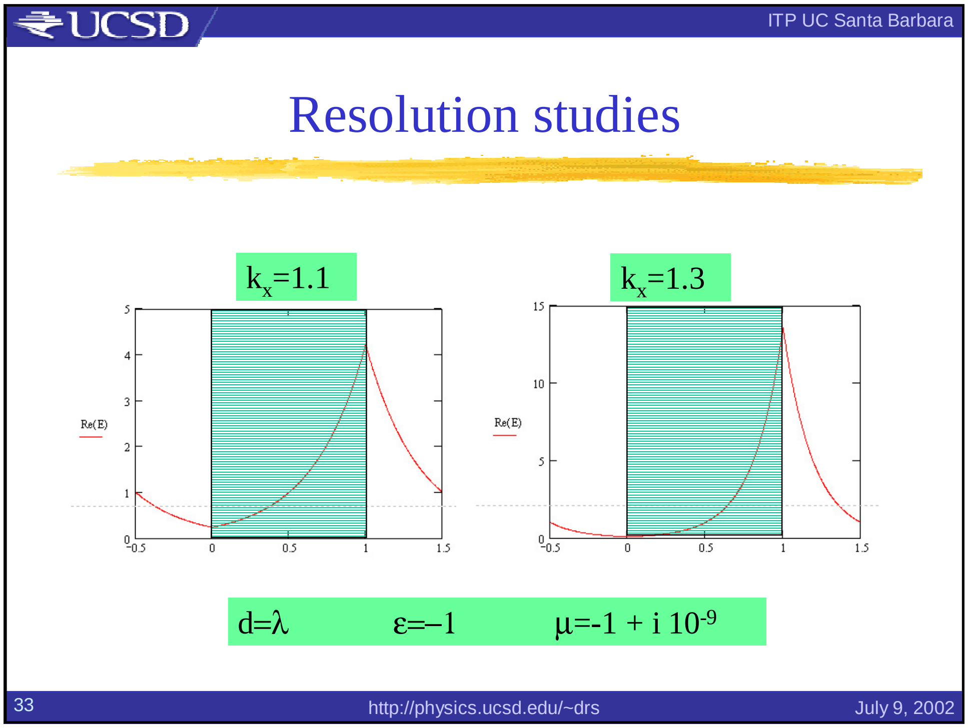
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Resolution studies
k_x=1.1
k_x=1.3
Re(E)
Re(E)
d=λ ε=−1 μ=-1 + i 10^{-9}
33
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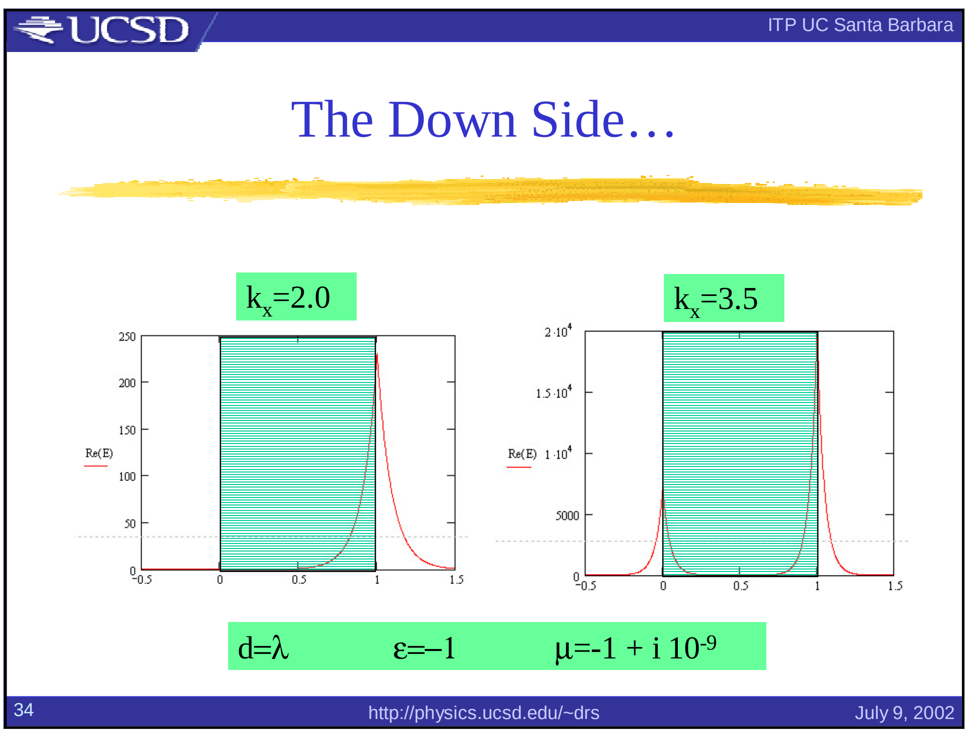
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The Down Side…
k_x=2.0
k_x=3.5
Re(E)
Re(E)
d=λ ε=−1 μ=-1 + i 10^{-9}
34
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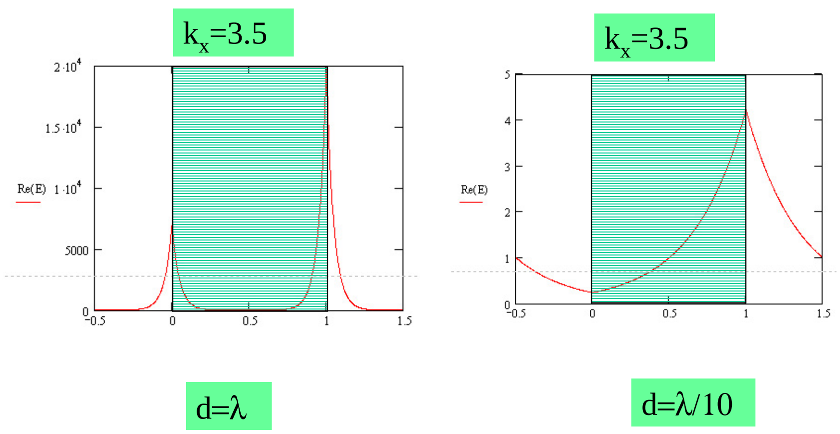
Thinner lenses give better resolution



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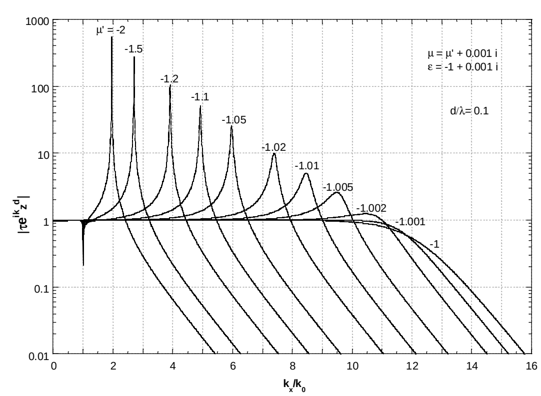
Resolution Limits on the Perfect Lens



Resolution is limited by losses, deviations from (-1,-1), and periodicity.

$$R \equiv \frac{\lambda}{\lambda_{\min}} = -\frac{\ln|\delta\mu|}{2\pi}\frac{\lambda}{d}$$

$$R \equiv \frac{\lambda}{\lambda_{\min}} = \frac{1}{2\pi}\ln\left(\frac{\lambda^2}{a^2\Delta^4}\right)\frac{\lambda}{d}$$

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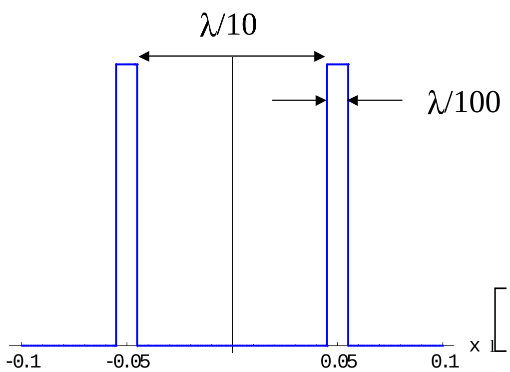
Numerical Simulations

Source object used in the parametric study…



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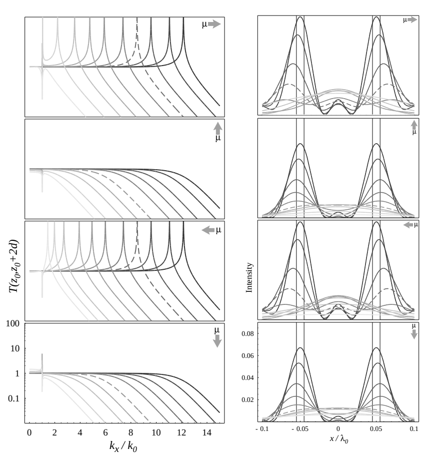
Parametric Studies of Imaging



μ	ϵ
$-0.999+0.00001i$	-1
$-0.998+0.00002i$	-1
$-0.995+0.00005i$	-1
$-0.99+0.0001i$	-1
$-0.98+0.0002i$	-1
$-0.95+0.0005i$	-1
$-0.9+0.001i$	-1
$-0.8+0.002i$	-1
$-0.6+0.004i$	-1
$-0.3+0.007i$	-1

μ	ϵ
$-1.001+0.00001i$	-1
$-1.002+0.00002i$	-1
$-1.005+0.00005i$	-1
$-1.01+0.0001i$	-1
$-1.02+0.0002i$	-1
$-1.05+0.0005i$	-1
$-1.1+0.001i$	-1
$-1.2+0.002i$	-1
$-1.5+0.005i$	-1
$-2+0.01i$	-1
$-3+0.02i$	-1

μ	ϵ
$-1+0.001i$	-1
$-1+0.002i$	-1
$-1+0.005i$	-1
$-1+0.01i$	-1
$-1+0.02i$	-1
$-1+0.05i$	-1
$-1+0.1i$	-1
$-1+0.2i$	-1
$-1+0.5i$	-1
$-1+i$	-1

μ	ϵ
$-1-0.001i$	-1
$-1-0.002i$	-1
$-1-0.005i$	-1
$-1-0.01i$	-1
$-1-0.02i$	-1
$-1-0.05i$	-1
$-1-0.1i$	-1
$-1-0.2i$	-1
$-1-0.5i$	-1

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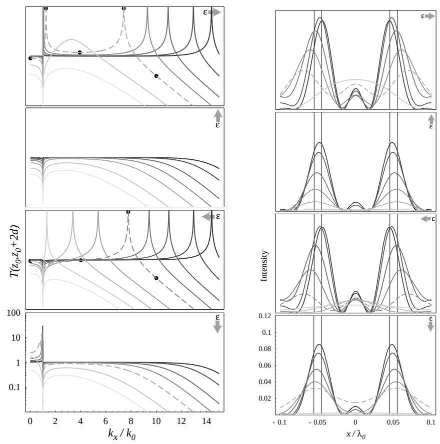
Parametric Studies of Imaging



μ	ϵ
-1	$-0.9+0.001i$
-1	$-0.8+0.002i$
-1	$-0.5+0.005i$
-1	$0.01i$
-1	$1+0.02i$
-1	$4+0.05i$
-1	$9+0.1i$

μ	ϵ
-1	$-1.1+0.001i$
-1	$-1.2+0.002i$
-1	$-1.5+0.005i$
-1	$-2+0.01i$
-1	$-3+0.02i$
-1	$-6+0.05i$
-1	$-11+0.1i$
-1	$-21+0.2i$
-1	$-51+0.5i$

μ	ϵ
-1	$-1+0.1i$
-1	$-1+0.2i$
-1	$-1+0.5i$
-1	$-1+i$
-1	$-1+2i$
-1	$-1+5i$
-1	$-1+10i$

μ	ϵ
-1	$-1-0.1i$
-1	$-1-0.2i$
-1	$-1-0.5i$
-1	$-1-i$
-1	$-1-2i$
-1	$-1-5i$
$, -1$	$-1-10i$

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Conclusion: Symmetric Perfect Lens is Difficult to Achieve!

- λ/d~10 is reasonable.
- For R~10, need parameter deviations <1/1000.
- Probably best demonstrated in other configurations.

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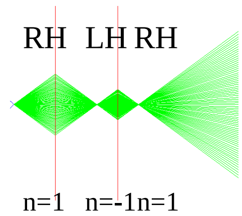
Time Dependent Solution: Motivation

- Information bandwidth.
- Transient information.
- Singularity paradox.



Nature of solution (i.e., exponential growth) suggests analytical treatment appropriate.

• Meshing issues in FDTD can lead to incorrect conclusions

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Source Object

$$E_s(x,t)=u(x)\theta(t)e^{i\Omega t}$$

$$E_s(k_x,s)=\frac{g(k_x)}{s+i\Omega}$$

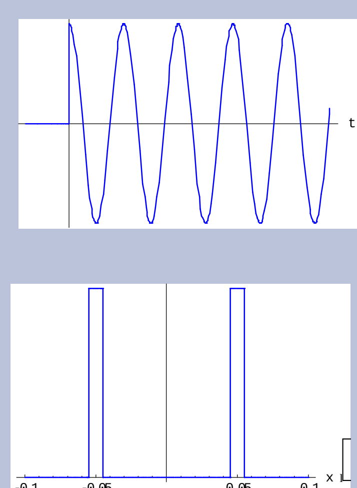

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Image Transfer Function

$$E_{image} = E_{\tau}(z = 2d)$$

$$E_{image}(k_x, s) = \frac{g(k_x)}{s + i\Omega} \frac{4\lambda e^{-pd}}{(\lambda+1)e^{qd} - (\lambda-1)e^{-qd}}$$

$$s = i\omega$$

$$\lambda = \frac{q}{\mu p}$$

Transfer function has branches, poles, essential singularities→convergence issues!

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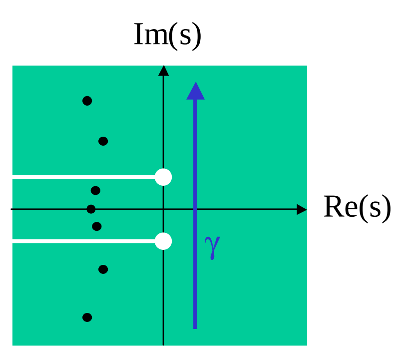
Inverse Laplace Transform frequency (s) →time (t)

$$E_{image}(k_x, t) = \frac{1}{2\pi i} \int_{\gamma} E_{image}(k_x, s) e^{st} ds$$



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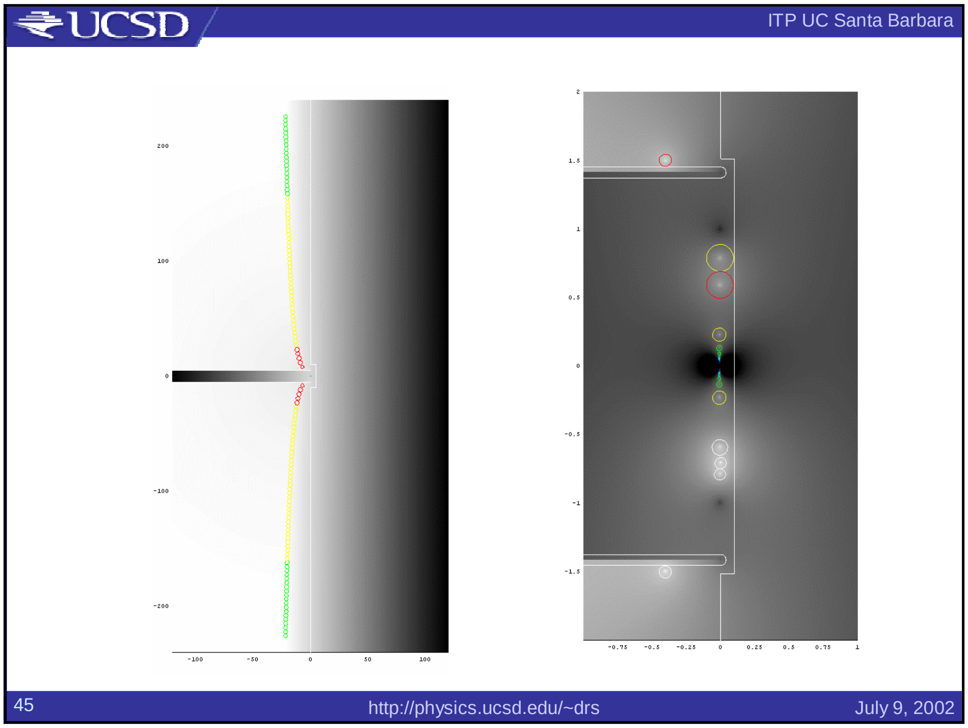
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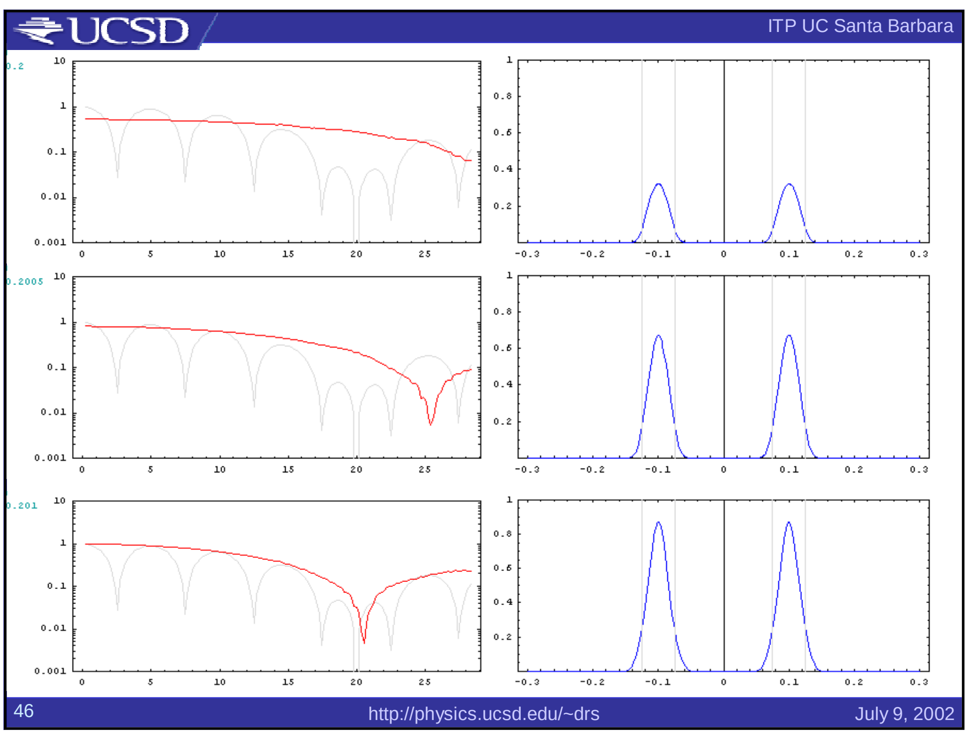
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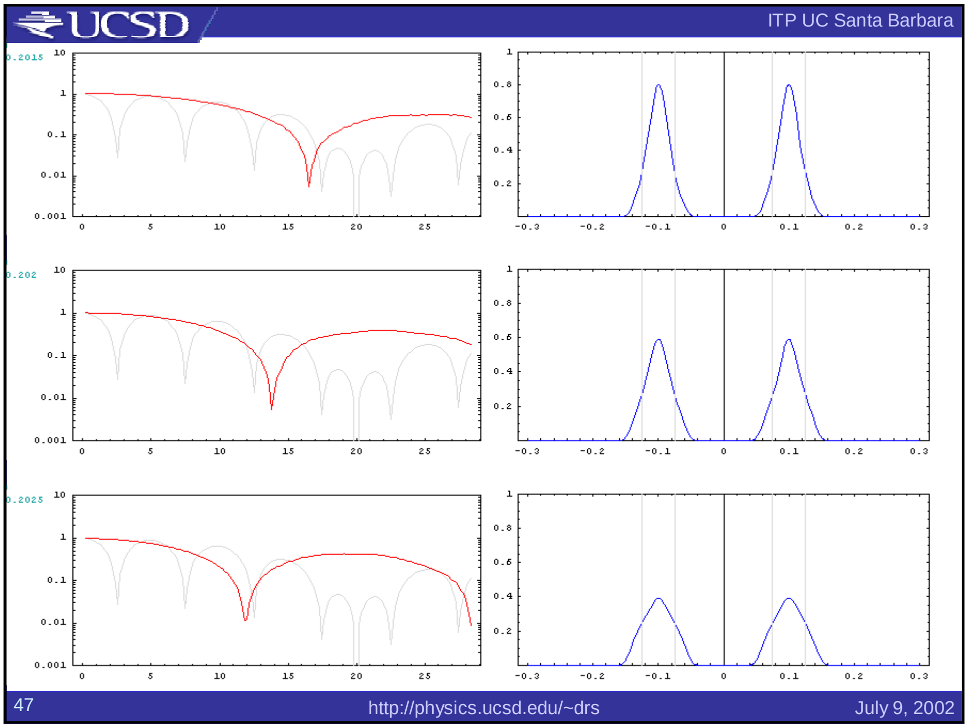
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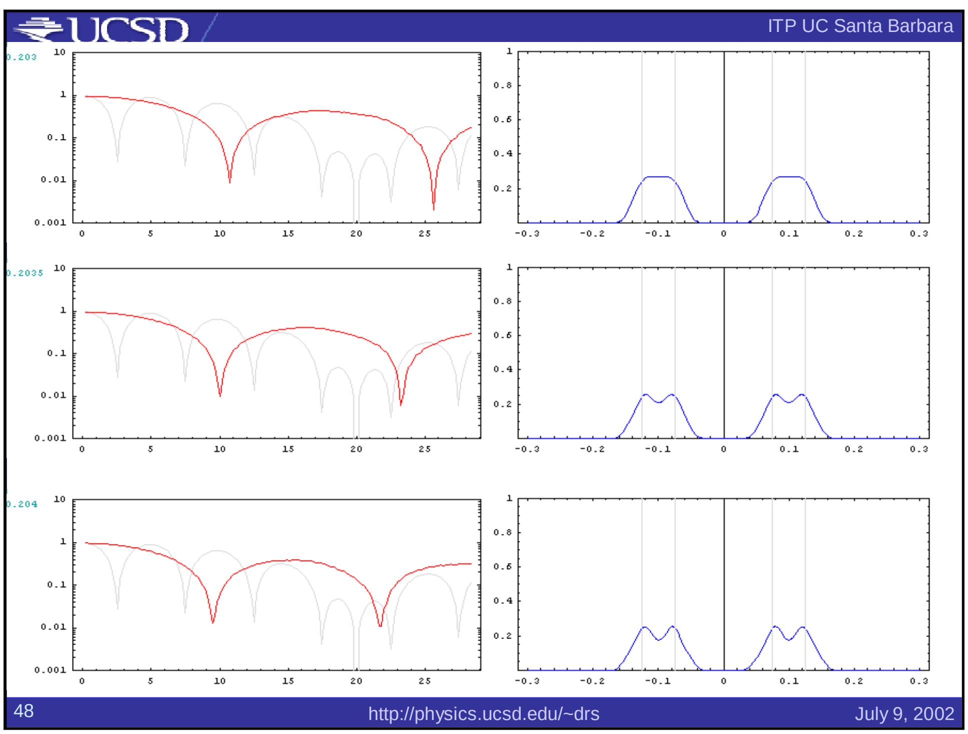
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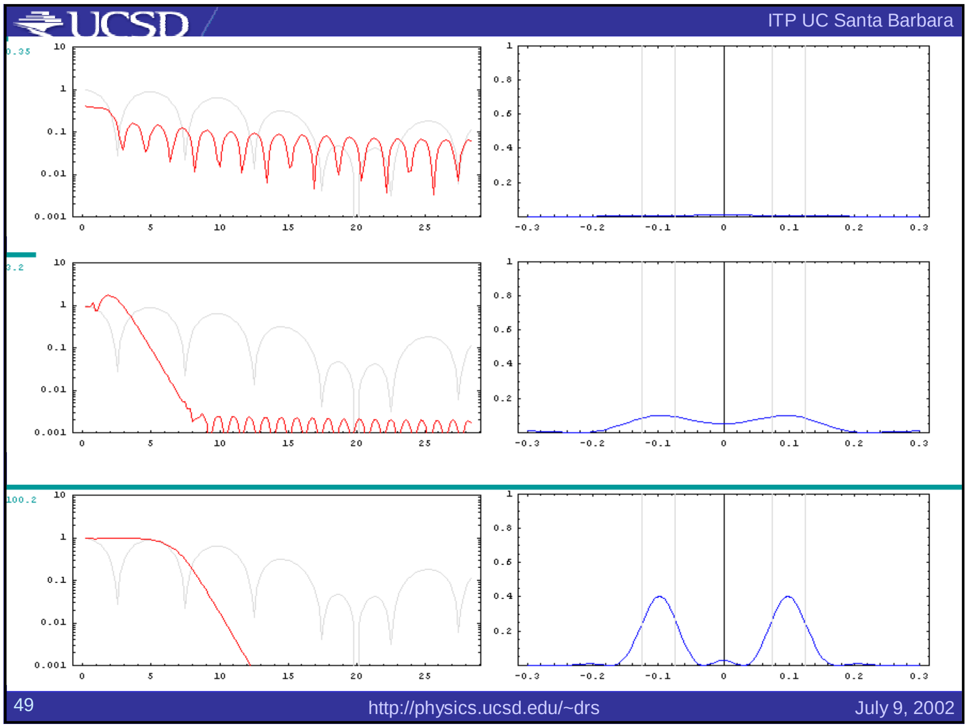

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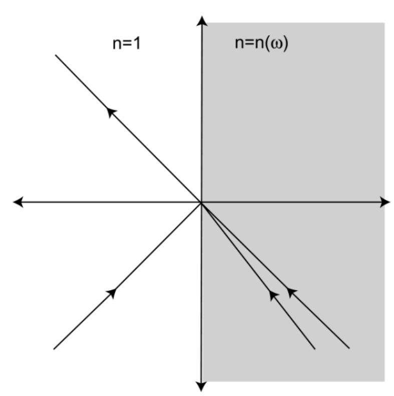
Refraction of Modulated Beams in LHM

$$E(\mathbf{r},t) = e^{i\mathbf{q}_c\cdot\mathbf{r}} e^{-i\Omega t} \cos(\Delta\mathbf{q}\cdot\mathbf{r} - \Delta\omega t)$$

$$\Delta\mathbf{q} = \left[k_x \hat{x} + \frac{1}{q_z}\left(k^2 n n_g - k_x^2\right)\hat{z} \right] \frac{\Delta\omega}{\Omega}$$

$$n_g \equiv \frac{\partial(n\omega)}{\partial\omega} \geq 1$$



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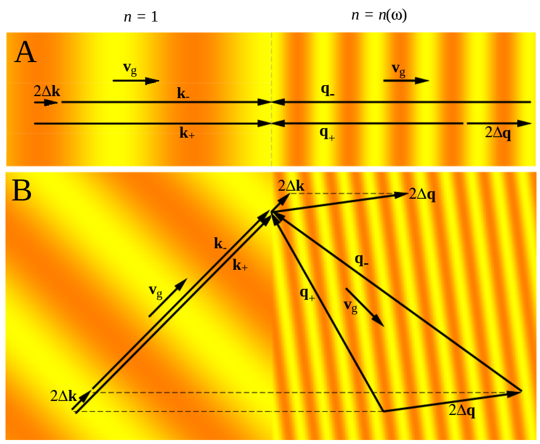
Waves at an angle produce interference patterns



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What is the Group Velocity?

The group velocity is an inherent property of the material. For an isotropic medium, it is antiparallel to the phase velocity.

$$\mathbf{v}_g \equiv \nabla_{\mathbf{q}} \omega(\mathbf{q})$$

$$\mathbf{v}_g \neq \frac{d\omega(\mathbf{q})}{d\mathbf{q}}$$

$$\mathbf{v}_g = \frac{\mathbf{q}}{q}\frac{d\omega(q)}{dq} = \frac{\mathbf{q}}{q}\,\mathrm{sign}(n)\frac{c}{n_g}$$

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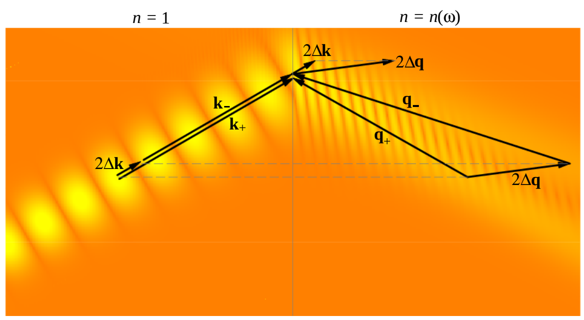
Modulated Gaussian Beams in LHM

Refraction of a Gaussian beam into a NIM. The angle of incidence is 30 degrees.



$$n(\omega_-) = -1.66 + 0.003i$$

$$n(\omega_+) = -1.00 + 0.002i$$

$$\Delta\omega/\omega = 0.07$$

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