tions connecting two generic inertial frames f, f', *a priori* with $-\infty < |\boldsymbol{u}| < +\infty$

$$\mathrm{d}x'_\mu = G^\nu{}_\mu \, \mathrm{d}x_\nu \tag{14}$$

must (cf. sect. **2**)

i) transform inertial motion into inertial motion,

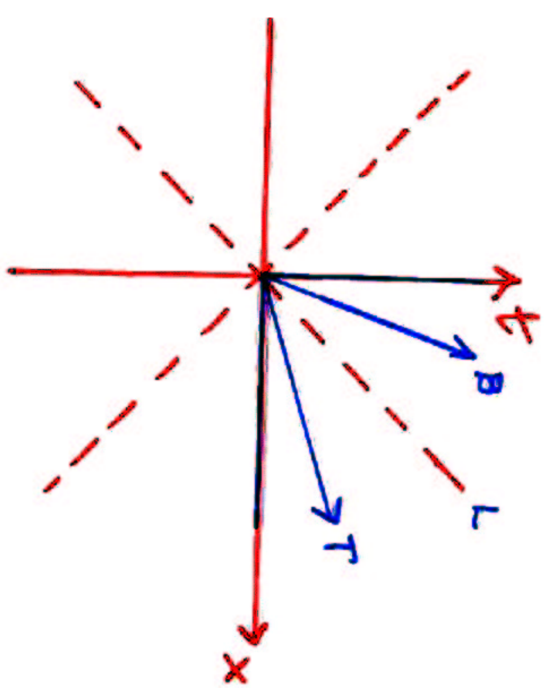
ii) form a group **G**,

iii) preserve space isotropy,

iv) leave the quadratic form invariant, *except for its sign* (RINDLER, 1966, p. 16; LANDAU and LIFSHITZ, 1966*a*, *b*):

$$\mathrm{d}x'_\mu \, \mathrm{d}x'^\mu = \pm \, \mathrm{d}x_\mu \, \mathrm{d}x^\mu . \tag{15}$$

Notice that eq. (15) imposes—among the others—the light speed to be invariant (JAMMER, 1979). Equation (15) holds for any quantity $\mathrm{d}x_\mu$ (position, momentum, velocity, acceleration, current, etc.) that is a G-*four-vector*, *i.e.*



3

(27b) $\quad dt'^2 - dx'^2 = -(dt^2 - dx^2) \quad (u^2 > 1)$

5·3. *Energy-momentum space.* – Since tachyons are just usual particles w.r.t. their own rest frames f, where the f's are Superluminal w.r.t. us, they will possess *real* rest masses m_0 (RECAMI and MIGNANI, 1972; LEITER, 1971a; PARKER, 1969). From eq. (27b) applied to the energy-momentum vector p^μ one derives for free tachyons the relation

(28) $\quad E^2 - p_x^2 = -m_0^2 < 0 \quad (m_0 \text{ real})$.

provided that p^μ is so defined to be a G-vector (see the following); so that one has (cf. fig. 5)

(29a), (29b), (29c) $\quad p_\mu p^\mu = \begin{cases} +m_0^2 > 0 & \text{for bradyons (timelike case)}, \\ 0 & \text{for luxons (lightlike case)}, \\ -m_0^2 < 0 & \text{for tachyons (spacelike case)}. \end{cases}$

$E'^2 - p_x'^2 = \pm(E^2 - p_x^2) = \pm m_0^2$

SLT: → $dx'_\mu dx'^\mu = -dx_\mu dx^\mu$; $p'_\mu p'^\mu = -p_\mu p^\mu$; $x'_\mu p'^\mu = -x_\mu p^\mu$

3'

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fore, finding out the (Superluminal) Lorentz transformations—if they exist—connecting the observations made by S with the observations by s.

5·2. *Sub- and Super-luminal Lorentz transformations: preliminaries.* – We neglect space-time translations, *i.e.* consider only *restricted* Lorentz transformations. All frames are supposed to have the same event as their origin. Let us also recall that in the chronotopical space Bs are characterized by timelike, ℓs by lightlike, and Ts by spacelike world-lines.

The ordinary, subluminal Lorentz transformations (LT) from s_1 to s_2, or from S_1 to S_2, are known to preserve the four-vector type. After subsect. 5·1, on the contrary, it is clear that the «Superluminal Lorentz transformations» (SLT) from s to S, or from S to s, must transform timelike into spacelike quantities, and *vice versa*. With assumption (25) it follows that in eq. (15) the *plus* sign has to hold for LTs and the minus sign for SLTs:

(15) $$ds'^2 = \pm ds^2 \qquad (u^2 \lessgtr 1);$$

therefore, in «extended relativity» (ER), with assumption (25), the quadratic form

$$ds^2 \equiv dx_\mu dx^\mu$$

is a *scalar* under LTs, but is a *pseudoscalar* under SLTs. In the present case, we shall write that LTs are such that

(27a) $$dt'^2 - dx'^2 = +(dt^2 - dx^2) \qquad (u^2 < 1),$$

while for SLTs it must be

(27b) $$dt'^2 - dx'^2 = -(dt^2 - dx^2) \qquad (u^2 > 1).$$

5·3. *Energy-momentum space.* – Since tachyons are just usual particles w.r.t. their own rest frames f, where the f's are Superluminal w.r.t. us, they will possess *real* rest masses m_0 (RECAMI and MIGNANI, 1972; LEITER, 1971a; PARKER, 1969). From eq. (27b) applied to the energy-momentum vector p^μ, one derives for free tachyons the relation

(28) $$E^2 - p_x^2 = -m_0^2 < 0 \qquad (m_0 \text{ real}),$$

provided that p^μ is so defined to be a G-vector (see the following); so that one has (cf. fig. 5)

(29a), (29b), (29c) $$p_\mu p^\mu = \begin{cases} +m_0^2 > 0 & \text{for bradyons (timelike case)}, \\ 0 & \text{for luxons (lightlike case)}, \\ -m_0^2 < 0 & \text{for tachyons (spacelike case)}. \end{cases}$$

4

$$\begin{cases} dt' = \textcircled{+}\pm \dfrac{dt - u\,dx}{\sqrt{1-u^2}} \\ dx' = \pm \dfrac{dx - u\,dt}{\sqrt{1-u^2}} \end{cases} \qquad \begin{cases} E' = \textcircled{+1}\pm \dfrac{E - u\,p_x}{\sqrt{1-u^2}} \\ p'_x = \pm \dfrac{p_x - u\,E}{\sqrt{1-u^2}} \end{cases}$$

$$ds'^2 = \textcircled{+}\, ds^2 \quad [u^2 < c^2]$$

$$\mathcal{L}_+^{\uparrow} \cup \mathcal{L}_+^{\downarrow} \quad [u^2 < c^2]$$

4

the contrary, any antichronous (= nonorthochronous) LT
sign—among the others—to the time components of *all the*
ciated with P. Any $L^{\downarrow}$ will transform P into a particle P' en
lar with negative energy *and* motion backwards in time (fig

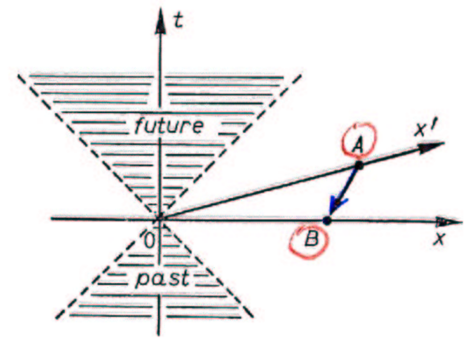

Fig. 1.

In other words, SR together with the natural assumpti
that a particle going backwards in time (GÖDEL, 1973) (f
in the four-momentum space (fig. 2) to a particle carrying
and, *vice versa*, that changing the energy sign in one spa
changing the sign of time in the dual space. It is then easy
two paradoxical occurrences (« negative energy » and « mot
time ») give rise to a phenomenon that any observer will d
orthodox way, when they are—as they actually are—simu
1978*c*, 1979*a* and references therein).

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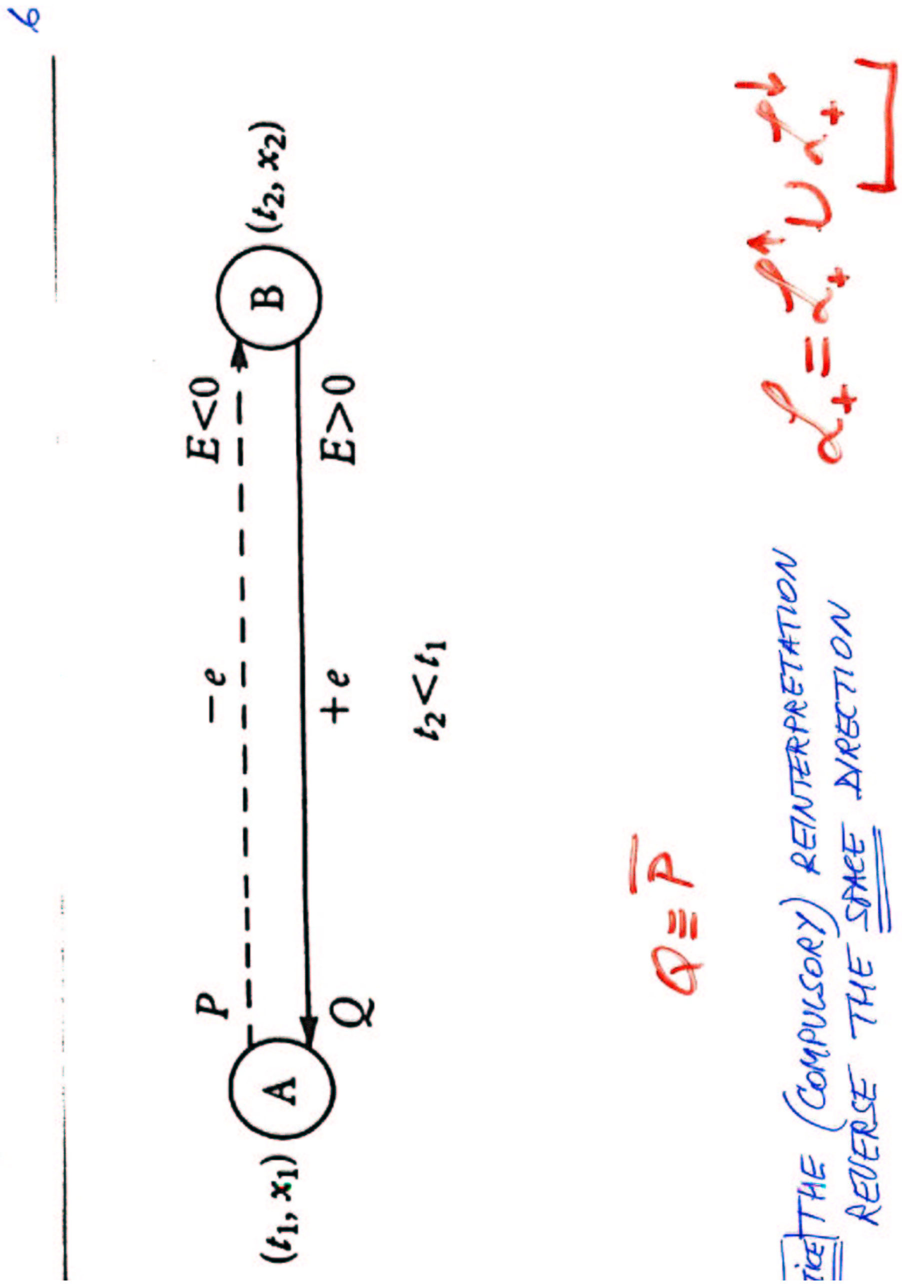

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RIVISTA DEL NUOVO CIMENTO VOL. 9, N. 6 1986

Classical Tachyons and Possible Applications (*).

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(ricevuto il 26 Aprile 1985)

« Quone vides citius debere et longius ire
Multiplexque loci spatium transcurrere eodem
Tempore quo Solis pervolgant lumina coelum? » (**).
LUCRETIUS (50 B.C., ca.)

« .... should be thoughts,
Which ten times faster glide than the Sun's beams
Driving back shadows over low'ring hills. »
SHAKESPEARE (1597)

(*) Work partially supported by C.N.R. and M.P.I.

(**) « Don't you see that they must go *faster* and farther/And travel a larger interval of space in the same amount of/Time *than the Sun's light* as it spreads across the sky? »

for details on such an interesting theory—which corresponds to assuming $F^{\mu\nu}$ to be a G-tensor—see CORBEN (1975, 1976, 1978*a*).

ii) On the contrary, one can try to generalize the subluminal transformations (202) for the Superluminal case, and only *a posteriori* deduce if $F^{\mu\nu}$ is a G-tensor or not, and finally derive how Maxwell equations get generalized. In eqs. (202) each couple of components E_y, H_z and E_z, H_y transforms just as the couple of co-ordinates x, t (cf. fig. 7*a*)); and the components E_x, H_x both transform just as the co-ordinate y or z.

Substituting the plane (E_y, H_z), or the plane (E_z, H_y), for the plane (x, t), it is then natural (cf. fig. 7*b*)) to extend the subluminal transformations by allowing the axes E'_y, H'_z (or E'_z, H'_y) to « rotate » beyond 45°, until when E'_y coincides with H_z and H'_z with E_y for $U \to \infty$: see fig. 46. This corresponds to extend the two-dimensional Lorentz transformations so as in subsect. **5**·6, eq. (39″).

Then, we may extend the transformations for E_x (and H_x) by analogy with the last two equations in (154 *bis*) or in (160); that is to say: $E_x = iE'_x$, $H_x = iH'_x$, where for simplicity we confined ourselves to $-\pi/2 < \theta < +\pi/2$. In such an approach, the quantities $T^{\mu\nu}$, $F^{\mu\nu}$, $A_{\mu,\nu}$ are *not* G-tensors, since under SLTs they transform as tensor *except* for an *extra* i (see, *e.g.*, review I and RECAMI and MIGNANI, 1976, 1977). Notice that, due to the invariance of $T^{\mu\nu}$ under the « duality » transformations, we may identify $iE'_x \equiv -H_x$, $iH'_x \equiv E_x$, in Heaviside-Lorentz units (*i.e.* in rationalized Gaussian units).

In review I it has been shown that the assumption of the previous Superluminal transformations for the components of $\boldsymbol{E}$ and $\boldsymbol{H}$ leads to generalize eqs. (200) in the following (G-covariant) form:

$$\text{(204)}\qquad \begin{cases} \partial_\nu T^{\mu\nu} = j^\mu(s) - ij^\mu(S) \\ \tilde{T}^{\mu\nu} = T^{\mu\nu} \end{cases} \qquad (v^2 \gtrless 1),$$

which constitute the « extended Maxwell equations »—valid in the presence of both sub- and Super-luminal « electric » currents—according to MIGNANI and RECAMI (1975*b*, *c*, 1974*d*) and RECAMI and MIGNANI (1976, 1974*a*, *b*).

If we confine ourselves to subluminal observers, eqs. (204) can easily be writen as (RECAMI and MIGNANI, 1974*a*)

$$\text{(205)}\qquad \begin{cases} \operatorname{div}\boldsymbol{D} = +\varrho(s) \\ \operatorname{div}\boldsymbol{B} = -\varrho(S) \\ \operatorname{rot}\boldsymbol{E} = -\partial\boldsymbol{B}/\partial t + \boldsymbol{j}(S) \\ \operatorname{rot}\boldsymbol{H} = +\partial\boldsymbol{D}/\partial t + \boldsymbol{j}(s) \end{cases} \qquad (v^2 \gtrless 1,\ s \leftrightarrow v^2 < 1,\ S \leftrightarrow v^2 > 1).$$

Therefore, according to the present theory, if both sub- and Super-luminal

« electric » charges exist, Maxwell equations get fully symmetrized, even if (ordinary) magnetic monopoles do not exist.

Actually, the generalization of eq. (202) depicted in fig. 46, as well as the extended Maxwell equations (204)-(205) seem to comply with the very spirit of SR and to « complete » it.

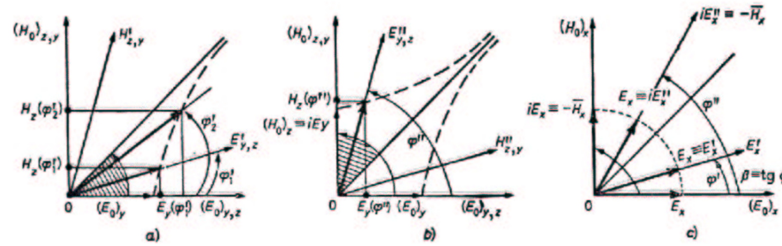

Fig. 46.

15·2. *Tachyons and magnetic monopoles.* – The « subluminal » equations (102*b*) seem to suggest that a multiplication by i carries electric into magnetic current, and *vice versa*. Comparison of eqs. (201*b*) with the generalized equations (204) suggests that:

i) the covariance of eqs. (201*b*) under the duality transformations, *e.g.* under eqs. (198), besides under LTs, corresponds to the covariance of eqs. (204) under the operation $\mathscr{S} \equiv \mathscr{S}_4$ (subsect. **14**·2), *i.e.* under SLTs. In other words, the covariance of eq. (201*b*) under the transition charges ⇄ monopoles corresponds to the covariance of eqs. (204) under the transition bradyons ⇄ tachyons;

ii) when transforming eqs. (201*b*) under SLTs (in particular, under the Superluminal transformations previously defined for the electric and magnetic field components) electric and magnetic currents go one into the other. Equations (205) show, more precisely, that a Superluminal « electric » *positive* charge will contribute to the field equations in a way similar to the one expected to come from a magnetic *south* pole; and analogously for the currents. This does *not* mean, of course, that a Superluminal charge is expected to behave just as an ordinary monopole, due to the difference in the speeds (one sub-, the other Super-luminal!). Since eqs. (205) are symmetric even if ordinary monopoles would not exist, ER seems to suggest—at least in its most economical version—that only a unique type of charge exists (let us call it the *electromagnetic charge*), which, if you like, may be called « electric » when subluminal, and « magnetic » when Superluminal (MIGNANI and RECAMI, 1975*b*; RECAMI and MIGNANI, 1976, 1977). The universality of electromagnetic interactions seems, therefore, recovered even at the classical level (*i.e.* in SR).

SLT's:

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It follows in particular that, if we consider a particle P_T which is *a tachyon* with respect to the Superluminal frames, to us it will behave as an ordinary particle (*bradyon*). Let us initially assume such a particle P to be spherical (in particular pointlike) when at rest:

$$0 \leqslant x^2 + y^2 + z^2 \leqslant r . \qquad \text{(at rest)} . \tag{2}$$

In the frame in which P moves with subluminal speed $v = \beta c$ along x ($P = P_B$), the equation of its *world-tube* becomes (with the metric $(+ - - -)$, and in natural units)

$$0 \leqslant \frac{(x - vt)^2}{1 - v^2} + y^2 + z^2 \leqslant r^2 \qquad (v^2 < 1) , \tag{3}$$

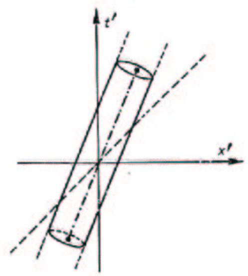
which in Lorentz-invariant form reads (cf. fig. 2)

$$0 \leqslant \frac{[(x_\mu - c_\mu)u^\mu]^2}{u_\mu u^\mu} - (x_\mu - c_\mu)(x^\mu - c^\mu) \leqslant r^2 \qquad (v^2 < 1) , \tag{4}$$

SLT

$x'_\mu x'^\mu = - x_\mu x^\mu$

$p'_\mu p'^\mu = - p_\mu p^\mu$



$x'_\mu p'^\mu = - x_\mu p^\mu$

Fig. 2. – The «world-tube» of an ordinary (bradyonic) particle $P \equiv P_B$, assumed to be spherical—or ellipsoidal—in its rest frame. For simplicity, particle P is assumed to move along the x-axis and the world-line of the centre C of P to pass through the space-time origin, so that $C \equiv O$ for $t = 0$. Notice, however, that eqs. (4), (6), (7), (9) of the text have been written down for the most general case.

where $x_\mu \equiv (t, x, y, z)$, the co-ordinates c_μ refer to the *centre* C of P_B, and [1,4] the four-velocity u_μ is defined as $u_\mu \equiv dx_\mu / d\tau_0$ (see appendix A). Equation (4) reduces to eq. (3) in the special case in which the world-line of C passes through the space-time origin, and, moreover,

$$x_\mu \equiv (t, x, y, z) , \qquad c_\mu \equiv (t, vt, 0, 0) . \tag{4'}$$

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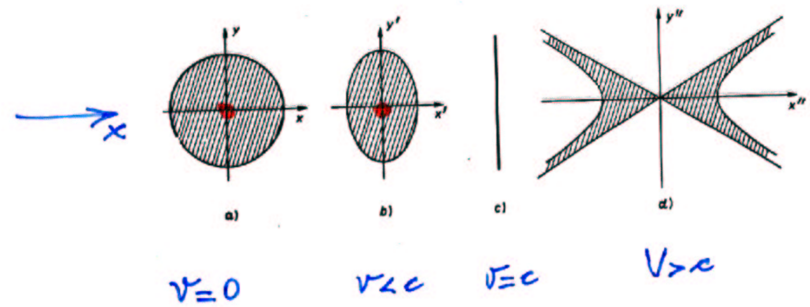
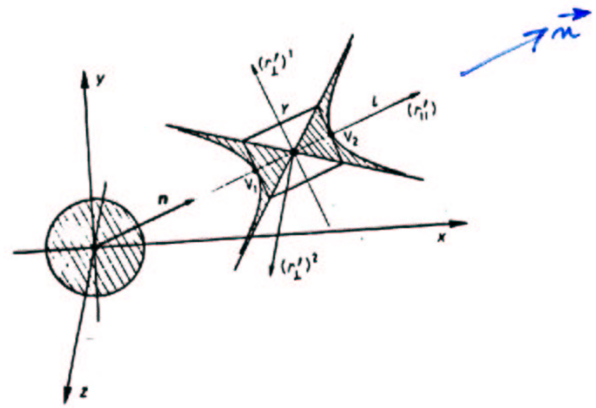




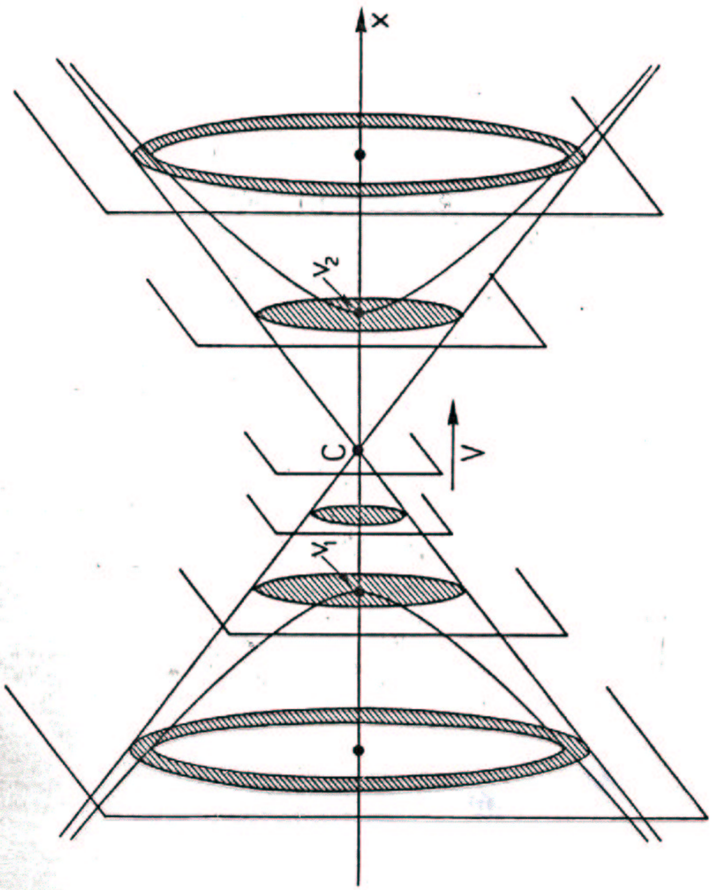
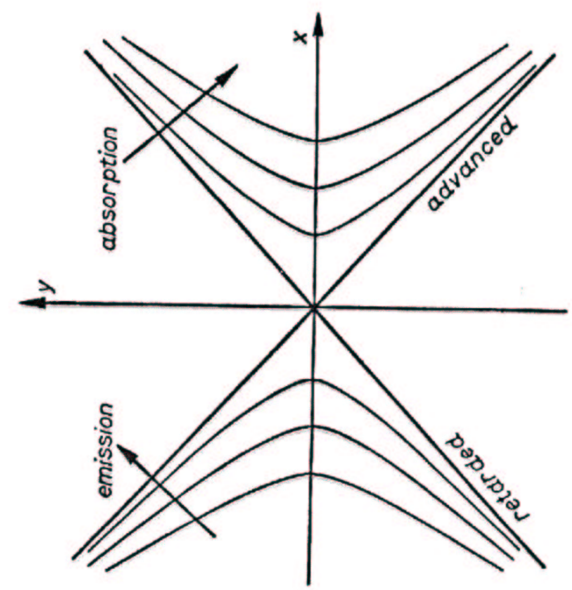


Fig.6

... hyperbolas asymptotic to s_0. Such is the result of the « distortion » to the high relative speed V; cf. fig. 27 (see also GLADKIKH, 1978*a*, *b*; TERLETS



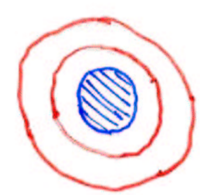


Fig. 27.

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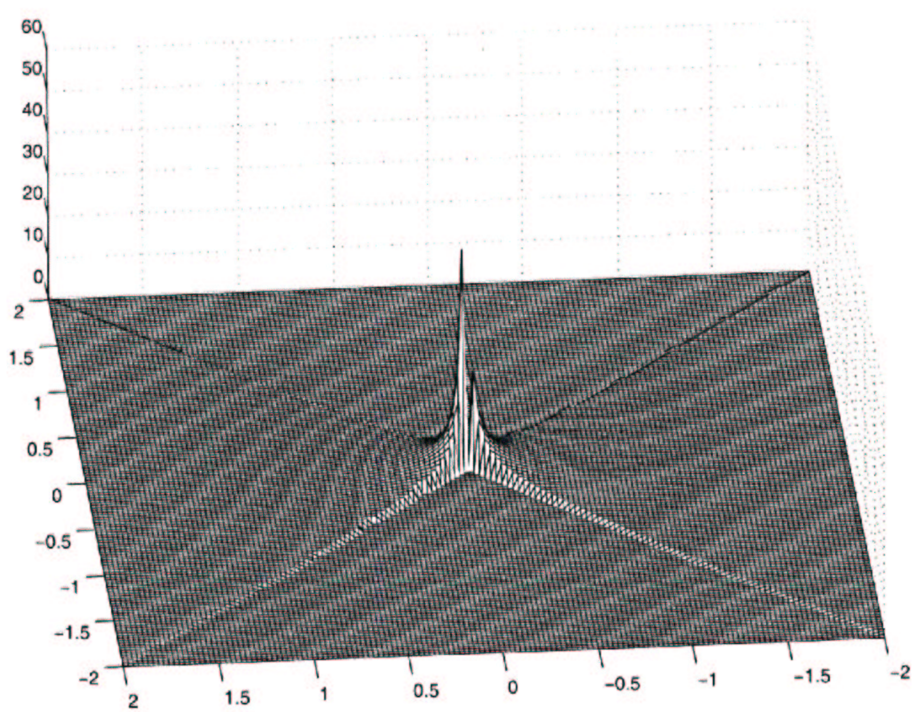

Cargasup. eps

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IL NUOVO CIMENTO Vol. 71 A, N. 4 21 Ottobre 1982

On the Shape of Tachyons (*).

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(ricevuto il 14 Giugno 1982)

Summary. — We study some aspects of the experimental behaviour of tachyons, in particular by finding out their « apparent » shape. A Superluminal particle, which in its own rest frame is spherical or ellipsoidal (and with an infinite lifetime), would « appear » to a laboratory frame as occupying the whole region of space bound by a double cone and a two-sheeted hyperboloid. Such a structure (the tachyon « shape ») rigidly travels with the speed of the tachyon. However, if the Superluminal particle has a finite lifetime *in its rest frame*, then in the laboratory frame it gets a *finite* space extension. As a by-product, we are able to interpret physically the imaginary units entering—as is well known—the transverse co-ordinates in the Superluminal Lorentz transformations. The various particular or limiting cases of the tachyon shape are thoroughly considered. Finally, some brief considerations concerning possible experiments to look for tachyons are added.

1. – Introduction.

Tachyons (or spacelike states) are already known to exist as *internal states*. Can they also exist as asymptotically free states? Here we shall address ourselves to this latter possibility.

(*) Work partially supported by M.P.I. and C.N.R.

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the propagation axis [5,31,32]. The X-shaped waves are different from the Bessel beams because they contain multiple frequencies, but possess the extremely important characteristic of being *non-dispersive* in isotropic-homogeneous media or free space [4,5,23]. Let us recall that Bessel beams are "localized" at a single frequency, but become dispersive for multiple frequencies because the phase velocity of each frequency component of theirs is different [33].

Even more important, for us, is the fact that the X-shaped waves are Superluminal [5,6,9,23], i.e., propagate rigidly with Superluminal speed, ~~while Bessel beams are subluminal~~.

In cylindrical coordinates, localized beams propagating along the z axis can be written in the following form:

$$\Phi(r, \phi, z - c_1 t)\,, \tag{1}$$

where r, ϕ, z and t represent the radial distance, polar angle, axial distance, and time, respectively; Φ represents the Hertz potential (or, in other cases, the acoustic pressure, or the velocity potential); $z - c_1 t$ is a propagation term, and c_1 is the velocity of the beam. Because the variables, z and t, appear only in the propagation term in Eq. (1), localized beams are only a function of r and ϕ if $z - c_1 t = \text{const.}$; that is to say, travelling with the beam at the speed c_1, one sees a constant beam pattern. This is different from conventional focused beams [34] and from the localized waves studied by Brittingham [19] and other investigators [20,21,35].

In Section 3, we shall extend the theory of the X-shaped beams to electromagnetic waves, i.e., we shall consider X-shaped wave solutions to the free Maxwell equations.

Let us start by recalling the well-known fact that, even if Maxwell's are vector equations, in various cases, such as optics and microwaves, they can be simplified, i.e., only the scalar amplitude of one transverse component of either the electric field $\boldsymbol{E}$ or the magnetic field $\boldsymbol{H}$ strength is considered, and any other components of interest are treated independently in a similar fashion (treating light and microwaves as a scalar phenomenon). This is approximately true – for ordinary experimental setups – under the following conditions [36]: (i) the diffracting aperture must be large compared with a wavelength, and (ii) the diffracted fields must not be observed too close to the aperture. In this case, localized beams developed in acoustics [3,16–21] can be directly applied to electromagnetism because they share the same wave equation (this was verified even experimentally in optics by Durnin for Bessel beams [14]).

Another way to solve the Maxwell equations is to use the (magnetic) Hertz *vector* potential $\boldsymbol{\Phi} = \Phi \hat{\boldsymbol{n}}$, where $\hat{\boldsymbol{n}}$ is a unit vector (this implies that the electromagnetic wave given by the Maxwell equation $\nabla \cdot \boldsymbol{E} = 0$ is a TE ("transverse electric field") polarization wave that is perpendicular to $\hat{\boldsymbol{n}}$; for TM ("transverse megnetic field") polarization, the procedure is similar). This approach is rigorous, as opposed to the scalar method above. But one easily gets (cf., e.g., Ref. [24]) expressions for $\boldsymbol{E}$ and $\boldsymbol{H}$ in terms of Φ, where the Hertz vector potential is still a solution of the scalar wave equation. Of course, not all the solutions of the scalar wave equation are localized. However, if Φ

14''

by use of the Hertz potential. Let us recall that, in terms of Φ, when $\hat{\boldsymbol{n}}$ is chosen in the z-direction, the quantities $\boldsymbol{E}$ and $\boldsymbol{H}$ read [24]

$$\boldsymbol{E} = -\mu_0 \frac{1}{r}\frac{\partial^2 \Phi}{\partial t\, \partial \phi}\hat{r} + \mu_0 \frac{\partial^2 \Phi}{\partial \phi\, \partial r}\hat{\phi}\,, \tag{3'}$$

$$\boldsymbol{H} = \frac{\partial^2 \Phi}{\partial r\, \partial z}\hat{r} + \frac{1}{r}\frac{\partial^2 \Phi}{\partial \phi\, \partial z}\hat{\phi} + \left(\frac{\partial^2 \Phi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2}\right)\hat{z}\,, \tag{3''}$$

respectively.

3.1. X-shaped wave solutions

Let us consider Eq. (3). When $T(k) = B(k)\exp[-a_0 k]$; $A(\theta) = \mathrm{i}^n \exp[\mathrm{i}n\theta]$; $\alpha_0(k,\zeta) = -\mathrm{i}k\sin\zeta$; $b(k,\zeta) = \mathrm{i}k\cos\zeta$ and $f(s) = \exp[s]$, we obtain an nth-order scalar localized "X-wave" [24] that has an X-like shape in a plane (r,z) passing through the propagation axis z:

$$\Phi_{X_n}(r,\phi,z-c_1 t) = \mathrm{e}^{\mathrm{i}n\phi}\int_0^\infty B(k) J_n(kr\sin\zeta)\mathrm{e}^{-k[a_0 - \mathrm{i}\cos\zeta(z-c_1 t)]}\,\mathrm{d}k \quad (n = 0, 1, 2, \ldots)\,, \tag{4}$$

where $B(k)$ is any well-behaved function of k (representing as before the transfer function of an electromagnetic antenna); the quantity $J_n(\cdot)$ is the nth-order *Bessel function* of the first kind; $c_1 = c/\cos\zeta$; $k = \omega/c$; ω is the angular frequency; while $a_0 > 0$ and $0 < \zeta < \pi/2$ are constant.

It can be immediately noticed that c_1 is larger than the light speed in the medium; i.e., in vacuum, is *larger* than c. Let us recall that one can get the group velocity v_g by the stationary phase method [38] (provided the considered wave-packet presents a clear bump), i.e., by equating to zero the partial derivative with respect to k of the unitary phase factor entering Eq. (4): $\partial[k\cos\zeta - c_1 k\cos\zeta t]/\partial k = 0$, which yields $z - c_1 t = 0$ and therefore $v_g = c_1$. Notice that for each component it is $v_g = d\mathscr{E}/dk$, and v_g depends only on the relation $\mathscr{E} = \mathscr{E}(k)$ (the quantity $\mathscr{E}$ being the energy).

If $B(k) = a_0$, from Eq. (4) we get the nth-order broadband [4,24] X-wave

$$\Phi_{XBB_n}(r,\phi,z-c_1 t) = \frac{a_0 (r\sin\zeta)^n \mathrm{e}^{\mathrm{i}n\phi}}{\sqrt{M}(\tau + \sqrt{M})^n} \quad (n = 0, 1, 2, \ldots)\,, \tag{4'}$$

where the subscript "BB" means "broadband"; $M = (r\sin\zeta)^2 + r^2$, and $\tau = a_0 - \mathrm{i}\cos\zeta(z - c_1 t)$.

If $B(k)$ is a band-limited function [4,24], we obtain a nth-order band-limited X-wave which is a convolution of functions $\mathscr{F}^{-1}[B(\omega/c)]/a_0$ and $\Phi_{XBB_n}(r,\phi,z-c_1 t)$ with respect to time t

$$\Phi_{XBL_n}(r,\phi,z-c_1 t) = \frac{1}{a_0}\mathscr{F}^{-1}\left[B\left(\frac{\omega}{c}\right)\right] * \Phi_{XBB_n} \quad (n = 0, 1, 2, \ldots)\,, \tag{4''}$$

14'

is a localized solution, then also the solution of the Maxwell equations is localized (since the derivatives with respect to the variables do not change [16] the propagation term $z - c_1 t$). Because of this, numerous limited diffraction (relativistic) *electromagnetic waves* can be obtained from the scalar localized (non-relativistic) beams studied in acoustics.

Actually, families of generalized solutions of the equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Phi = 0\,, \tag{2}$$

were discovered recently in the acoustic case [4]. One of the families of solutions is given by [24]

$$\Phi_\zeta(s) = \int_0^\infty T(k)\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} A(\theta)\, f(s)\,\mathrm{d}\theta\right]\mathrm{d}k\,, \tag{3}$$

where

$$s \equiv \alpha_0(k,\zeta) r\cos(\phi-\theta) + b(k,\zeta)[z \pm c_1(k,\zeta)t]\,, \tag{3a}$$

$$c_1(k,\zeta) \equiv c\sqrt{1+[\alpha_0(k,\zeta)/b(k,\zeta)]^2}\,. \tag{3b}$$

The quantity $T(k)$ is any complex function (well behaved) of k and can include the temporal frequency transfer function of a practical *electromagnetic* antenna (or acoustic transducer); $A(\theta)$ is any complex function (well behaved) of θ and represents a weighting function of the integration with respect to θ; quantity $f(s)$ is any complex function (well behaved) of s; quantities $\alpha_0(k,\zeta)$ and $b(k,\zeta)$ are any complex function of k and ζ; while c is the *speed of light* (or of sound) entering Eq. (2), and k, ζ are variables that are independent of the spatial position $\boldsymbol{r} = (r\cos\phi,\, r\sin\phi,\, z)$ and of time t. At last, ζ is the Axicon angle [4,37], that we confine in the range $0 < \zeta < \pi/2$.

If $c_1(k,\zeta)$ in Eq. (3b) is real, then "$\pm$" in Eq. (3a) represent forward and backward propagating waves, respectively (in the following analysis, we consider only the forward propagating waves and all results will be the same for the backward propagating waves). Furthermore, $\Phi_\zeta(s)$ will represent a family of localized waves if $c_1(k,\zeta)$ is independent of k (containing, that is, the same propagation terms $z - c_1(\zeta)t$ for all frequency components k). It must be noticed that $\Phi_\zeta(s)$ in Eq. (3) is very general. It contains some of the localized solutions known previously, such as the plane wave and Durnin's localized beams, in addition to a quantity of new beams.

3. Electromagnetic X-shaped waves

We shall now find that the localized X-shaped waves discovered in the scalar case [4] exist also in electromagnetism, i.e., hold also as solutions to Maxwell equations,

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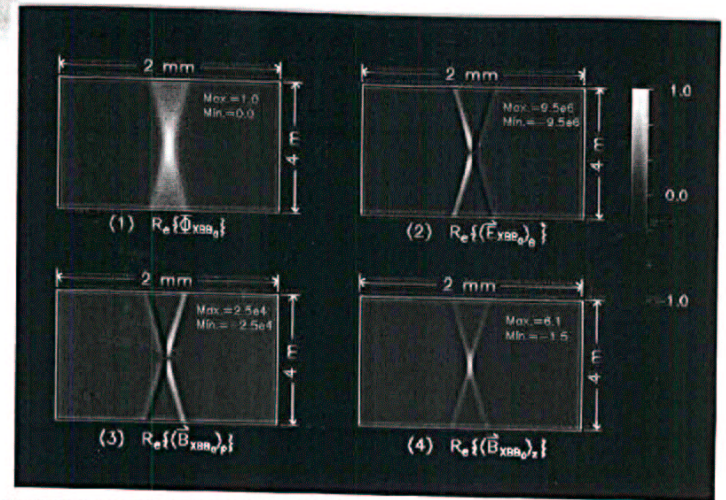

Fig. 1. Real part of the Hertz potential and field components of the zeroth-order ($n = 0$) localized electromagnetic X-shaped wave at time $t = z/c_1$. Panel (1) is the Hertz potential $\mathrm{Re}\{\Phi_{XBB_0}\}$; Panel (2) is the ϕ component of the electric field strength, $\mathrm{Re}\{(E_{XBB_0})_\phi\}$; and Panels (3) and (4) are the r and z components of the magnetic field strength, $\mathrm{Re}\{(H_{XBB_0})_r\}$ and $\mathrm{Re}\{(H_{XBB_0})_z\}$, respectively. The dimension of each panel is 4 m (r direction) × 2 mm (z direction). The free parameters ζ and a_0 are 0.005° and 0.05 mm, respectively. The values shown on the right-top corner of each panel represent the maxima and the minima of the images before normalization for display [MKSA units] (see also Table 1).

also shown in Table 2, where the axial component of the Poynting flux is at least four orders larger than its lateral components. Lateral line plots of Figs. 1 and 2 along X branches are shown in Fig. 3.

3.4. *Finite-aperture approximation of X-shaped waves and their depth of field*

The localized electromagnetic X-shaped waves obtained above by us are exact solutions to the free-space Maxwell wave equations. In these equations, there are no boundary conditions and thus the apertures required to produce the waves are infinite; therefore, they cannot be realized in practice. However, these waves can be approximated very well, over a certain "depth of field", by truncating them in both space and time.

One important question is how far the truncated X-shaped waves can travel without appreciable distortion, i.e., how long is their field depth. The answer is that they travel practically undistorted along a *large* depth of field, and then they suddenly decay. This has been shown, even experimentally, in the acoustic case [4–6,16,17], and mathematically in the more general case of the slingshot pulses (X-shaped-type waves, found

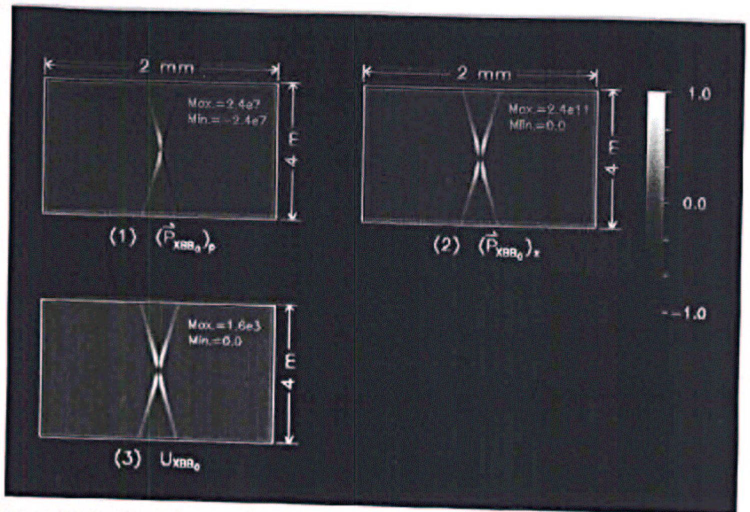

Fig. 2. Poynting flux and energy density of the zeroth-order localized electromagnetic X-shaped wave at time $t = z/c_1$. Panels (1) and (2) are the r and z components of the Poynting flux, $(\boldsymbol{P}_{XBB_0})_r$ and $(\boldsymbol{P}_{XBB_0})_z$, respectively; and Panel (3) is the energy density U_{XBB_0}. The dimension of each panel and the parameters of the X-waves are the same as those in Fig. 1. The values shown on the right-top corner of each panel represent the maxima and the minima of the images before normalizing for display [MKSA units] (see also Table 2).

independently in Ref. [23] for the case of homogeneous, general scalar wave-equations) as well as for other pulses [15,26]. Let us address the same question in the present (electromagnetic) case.

Since $|\boldsymbol{E}_{XBB_n}(r,\phi,z-c_1t)| \leqslant |\boldsymbol{E}_{XBB_n}(r,\phi)|$ and $|\boldsymbol{H}_{XBB_n}(r,\phi,z-c_1t)| \leqslant |\boldsymbol{H}_{XBB_n}(r,\phi)|$ for $|z-c_1t| > d_z/2$ within a finite transverse aperture, where d_z is a constant quantity, the X-shaped waves *may* be truncated e.g. within the axially moving window $[c_1t - d_z/2, c_1t + d_z/2]$. The truncated waves do not meet the problems of the theoretical (infinitely extended) ones. If the diameter of the aperture is D, the depth of field (which, following Durnin, is defined as the axial distance at which the field amplitude falls to half of that at the surface of the source) of the X-wave Hertz potential (Eqs. (4′) and (4″)) is given by [4,16,24,42]

$$Z_{\max} = \frac{D}{2}\,\frac{1}{\sqrt{(c_1/c)^2-1}} = \frac{D}{2}\cot\zeta\,. \tag{8}$$

Because the derivatives in Eqs. (3′) and (3″) do not change the cone angle ζ, the field depth of the electromagnetic X-waves is the same as that of the Hertz potential produced by the same aperture. In addition, Eq. (8) is also valid for band-limited electromagnetic X-waves [16]. As an example, if the diameter of the aperture is 20 m

ELSEVIER Physica A 252 (1998) 586–610 PHYSICA A

On localized "X-shaped" Superluminal solutions to Maxwell equations[1]

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Abstract

In this paper we extend for the case of Maxwell equations the "X-shaped" solutions previously found in the case of scalar (e.g., acoustic) wave equations. Such solutions are localized in theory: i.e., diffraction-free and particle-like (wavelets), in that they maintain their shape as they propagate. In the electromagnetic case they are particularly interesting, since they are expected to be Superluminal. We address also the problem of their practical, approximate production by finite (dynamic) radiators. Finally, we discuss the appearance of the X-shaped solutions from the purely geometric point of view of the Special Relativity theory. © 1998 Elsevier Science B.V. All rights reserved.

PACS: 03.50.De; 41.20.Jb; 03.30.+p; 03.40.Kf; 14.80.-j
Keywords: X-shaped waves; Localized solutions to Maxwell equations; Superluminal waves; Bessel beams; Limited-dispersion beams; Electromagnetic wavelets; Special Relativity; Extended Relativity

1. Introduction

Starting with the 1915 pioneering work by H. Bateman [1], it became slowly known that all the relativistic homogeneous wave equations – in a general sense: scalar, electromagnetic and spinorial – admit also solutions with group velocities slower than the ordinary wave velocity in the considered medium. More recently, solutions had been constructed for those homogeneous wave equations with group velocities even higher than the ordinary wave velocity in the medium [2].

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Erasmo Recami 1999

Experimental Verification of Nondiffracting X Waves

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"X-shaped" Waves!

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***Abstract*—The authors have embedded descriptions for localized nondiffracting waves such as Durnin's wave in a generalized family of exact solutions to the isotropic/homogeneous wave equation. The first experimental production of acoustic forms of a subset of these solutions that the authors term, "X waves" are reported. Our generalized expression includes a term for the frequency response of the system and parameters for varying depth of field versus beam width of the resulting family of beams. Excellent agreement between theoretical predictions and experiment was obtained. An X wave of finite aperture driven with realizable (causal, finite energy) pulses travels with a large depth of field (nondiffracting length).**

I. Introduction

The propagation of acoustic waves in isotropic/homogeneous media and electromagnetic waves in free space is governed by the isotropic/homogeneous (or free space) scalar wave equation. The first localized solution of the free-space scalar wave equation was discovered by J. N. Brittingham in 1983 and was called a focus wave mode [1]. In 1985, R. W. Ziolkowski discovered a new localized solution of the free-space scalar wave equation and found a way to construct new solutions from this localized solution by Laplace transform [2]. In 1989, a localized solution using the so-called modified power spectrum was constructed and the predicted localized wave was experimentally realized through acoustical superposition [3]. These localized solutions were further studied by several investigators [4]–[10].

The first nondiffracting beam that was an exact solution of the free-space scalar wave equation was discovered by J. Durnin in 1987 [11]. A finite aperture approximation of this beam was expressed in continuous wave form and was realized by optical experiments (treating the scalar amplitude of one transverse component of either the electrical or magnetic field as the solution of the scalar wave equation) [12]. Durnin's beams were further studied in optics in a number of papers [13]–[19]. Hsu *et al.* [20] realized a J_0 Bessel beam with a narrow-band PZT ceramic ultrasonic transducer of nonuniform poling. We made the first J_0 Bessel annular array transducer [21] using a PZT ceramic/polymer composite and applied it to medical acoustic imaging and tissue characterization [22]–[25]. Campbell *et al.* had a similar idea to use an annular array to realize a J_0 Bessel beam and compared the J_0 Bessel beam to the Axicon [26].

We have recently discovered families of generalized nondiffracting solutions of the free-space scalar wave equation [27]. One subset of these solutions represents waves that have X-like shapes in a plane along the axis of the waves and we term them "X waves" [27]. The nondiffracting X waves propagate without changing their waveforms in both space and time provided they are produced by an infinite aperture. Even with finite aperture, nearly exact X waves can be realized with either broadband or band-limited radiators over deep depth of field (nondiffracting distance). In comparison with Durnin's beam [11], X waves contain multiple frequencies and are localized in both space and time. X waves are nondiffracting in nature and have a constant peak amplitude as they propagate. This is different from Brittingham's [1] and Ziolkowski's [2], [3] localized waves that recover their amplitude periodically or aperiodically.

In this paper, we report the experimental production of an axially symmetric acoustic X wave and compare it with computer simulations.

II. Theoretical Preliminaries

The isotropic/homogeneous scalar wave equation in cylindrical coordinates is given by

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\Phi=0 \quad (1)$$

where $r=\sqrt{x^2+y^2}$ represents radial coordinate, ϕ is the azimuthal angle, z is the axial axis, which is perpendicular to the plane defined by r and ϕ, t is the time, c is the speed of sound, and Φ represents acoustic pressure that is a function of r, ϕ, z, and t. One of the families of generalized solutions of (1) discovered recently by us [27] is of the form

$$\Phi_\zeta(s)=\int_0^\infty T(k)\left[\frac{1}{2\pi}\int_{-\pi}^{\pi}A(\theta)f(s)d\theta\right]dk \quad (2)$$

where

$$s=\alpha_0(k,\zeta)r\cos(\phi-\theta)+b(k,\zeta)[z\pm c_1(k,\zeta)t] \quad (3)$$

and where

$$c_1(k,\zeta)=c\sqrt{1+[\alpha_0(k,\zeta)/b(k,\zeta)]^2}. \quad (4)$$

where $T(k)$ can be any complex function (well behaved) of k ($k=\omega/c$ is a wave number and ω is angular frequency) and could include the frequency response of an acoustic transducer. $A(\theta)$ is any complex function (well behaved) of θ and represents a weighting function of the integration with respect to θ, which is the angle around the aperture of the

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and $\mathbf{n}_/=[\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta]$, $\mathbf{n}_\backslash=[\sin\theta\cos(\phi+\pi),\sin\theta\sin(\phi+\pi),\cos\theta]$ is a pair of unit vectors forming a cone as ϕ runs from 0 to π, which has the top angle 2θ. These unit vectors, as they stand in the scalar-product argument of the field function A, determine directions of pulsed plane wave constituents of the X wave. Hence, according to Eq. (3) the field is built up from interfering pairs of identical bursts of plane waves. Every plane wave pair makes up an X-shaped propagation-invariant interference pattern moving along the axis z with speed $c'=c/\cos\theta$ which is both the phase and the group velocity of the wave field in the direction of the axis z. This speed is superluminal in a similar way as one gets a faster-than-light movement of a bright stripe on a screen when a plane wave light pulse is falling at the angle θ onto the screen plane [23]. The shorter the source pulse, the better the separation and resolution of the branches of the X-shaped profile. In the superposition of all pairs the only point of completely constructive interference is a point of z axis, which becomes the pulse center. In other words, as a result of the integration one gets a field profile reminiscent of the one obtainable by revolving the letter X around its horizontal axis [24]. The highly localized energy "bullet" arises in the center, while the intensity falls off as ρ^{-1} along the branches and much faster in all other directions (see Fig. 1 and Refs. [19,20]). The optical carrier manifests itself as one or more (depending on the pulse length) halo toroids which are nothing but residues of the concentric cylinders of intensity characteristic of the Bessel beam. That is why we use the term "Bessel-X pulse" (or wave) to draw a distinction from carrierless or unipolar pulses.

According to Fig. 1 a straightforward method for recording the field shape would use a CCD camera with a gate in front of it, which should possess a temporal resolution and a variable firing delay both in subfemtosecond range. As such a gate is not realizable, any workable idea of experiment has to resort to a field cross-correlation technique. Fortunately, we can take advantage of the superluminal speed of the Bessel-X pulse, which allows the latter to catch up with a reference plane wave pulse generated in the same optical scheme as depicted in Fig. 2.

A simple geometric consideration shows that the catch-up point, which we further take as the origin of the z axis, is near the rear focal point of the L3 lens (more exactly at the distance $f/\cos\theta$ from the L3 lens, where $f_{L3}=56.5$ cm and $\theta=0.006$ rad in our experiment). If one performs a time-integrated recording of the cross-correlation interference patterns in the radial plane at successive points along the z axis, one gets the spatiotemporal profile of the Bessel-X wave field. Indeed, a 2D photodetector placed in an x-y plane at some distance z records time-integrated intensity distribution given by $\langle U^*U\rangle=\langle U_P^*U_P+U_X^*U_X+2\,\mathrm{Re}\,U_X^*U_P\rangle$, where the brackets denote the time (or ensemble) averaging, U_P is the plane wave field, the first terms are the intensities of

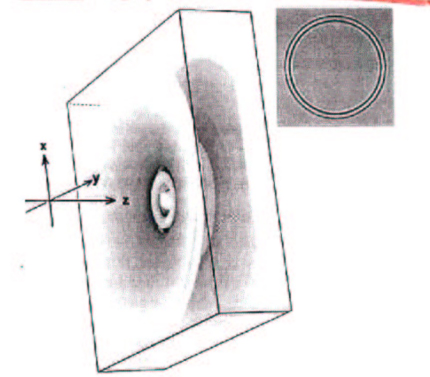

FIG. 1. Intensity profile of a computer-simulated Bessel-X pulse flying in space, shown as surfaces on which the field intensity is equal to a fraction $0.13(\simeq 1/e^2)$ of its maximum value in the central point. The field intensity outside the central bright spot has been multiplied by the radial distance in order to reveal the weak off-axis sidelobes. Inset: Amplitude distribution in the plane shown as intersecting the pulse. The plots have been computed for a 3-fsec near-Gaussian-spectrum source pulse [19,20] with carrier wavelength $\lambda_0=0.6$ μm and the angle $\theta=14°$. For these parameters the dimensions of the plot xyz box are $20\times 20\times 6$ μm.

the cross-correlated fields, and the last term corresponds to the interference pattern. By making use of Eq. (3), we obtain

$$\langle U_X^*U_P\rangle=\int_0^\pi d\phi[\Gamma_{AA}(\mathbf{r}_\theta\mathbf{n}_/)+\Gamma_{AA}(\mathbf{r}_\theta\mathbf{n}_\backslash)]. \quad (4)$$

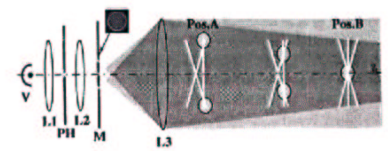

FIG. 2. Optical scheme of the experiment. Mutual instantaneous placement of the Bessel-X pulse and the plane wave pulse is shown for three recording positions (two of which labeled in accordance with Fig. 3). The ovals indicate toroidlike correlation volumes where copropagating Bessel-X and plane wave pulses interfere at different propagation distances along the z axis. L's, lenses; M, mask with Durnin's annular slit and an additional central pinhole for creating the plane wave; PH, cooled pinhole 10 μm in diameter to assure the transversal coherence of the light from the source V. In case source V generates a non-transform-limited pulse or a white cw noise, the bright shapes depict propagation of the correlation functions instead of the pulses.

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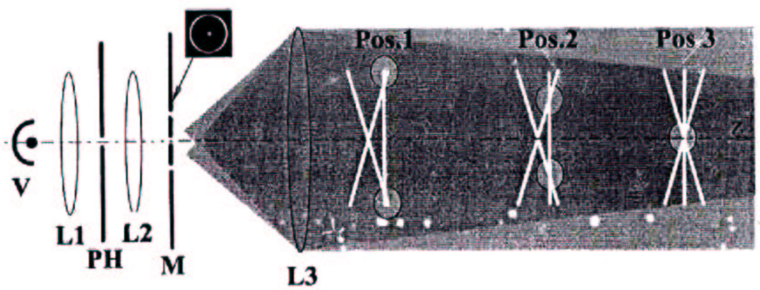
Fig.2, P.Saari, Phys.Rev.Lett.



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NEW LOCALIZED SUPERLUMINAL SOLUTIONS TO THE WAVE EQUATIONS WITH FINITE TOTAL ENERGIES AND ARBITRARY FREQUENCIES †

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Abstract

By a generalized bidirectional decomposition method, we obtain new Superluminal localized solutions to the wave equation (for the electromagnetic case, in particular) which are suitable for arbitrary frequency bands; various of them being endowed with *finite* total energy. We construct, among the others, an infinite family of generalizations of the so-called *X-shaped* waves. Results of this kind may find application in the other fields in which an essential role is played by a wave-equation (like acoustics, seismology, geophysics, etc.)

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Keywords: Wave equations; Wave propagation; Localized beams; Superluminal waves; Bidirectional decomposition; Bessel beams; X-shaped waves; Microwaves; Optics; Specialrelativity; Acoustics; Seismology; Mechanical waves; Elastic waves; Gravitational waves

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1. – Introduction

Since many years it has been known that localized (non-dispersive) solutions exist to the (homogeneous) wave equation[1], endowed with subluminal or Superluminal[2] velocities.

Particular attention has been paid to the localized Superluminal solutions, which seem to propagate not only in vacuum but also in media with boundaries[3], like normal-sized metallic waveguides[4] and possibly optical fibers.

It is well known that such Superluminal Localized Solutions (SLS) have been *experimentally* produced in acoustics[5], in optics[6] and recently in microwave physics[7].

However, all the analytical SLSs considered till now and known to us, with one exception[8], are superposition of Bessel beams with a frequency spectrum starting with $\nu = 0$ and suitable for low frequency regions. In this paper we shall set forth a new class of SLSs with a spectrum starting at any arbitrary frequency, and therefore well suited for the construction also of high frequency (microwave, optical,...) pulses.

2. – "V-cone" variables: A generalized bidirectional expansion

Let us start from the axially symmetric solution (Bessel beam) to the wave equation in cylindrical co-ordinates:

$$\psi(\rho, z, t) \; = \; J_0(k\rho)\, e^{+ik_z z}\, e^{-i\omega t} \qquad (1)$$

with the conditions

$$k^2 = \frac{\omega^2}{c^2} - k_z^2 \, ; \qquad k^2 \geq 0 \, , \qquad (2)$$

where J_0 is the zeroth-order ordinary Bessel function, and k_z (of course) the relevant wavenumber. The second condition (2) excludes the non-physical solutions.

It is essential to stress right now that the dispersion relation (2), with positive (but

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not constant, a priori) k^2, while enforcing the consideration of the truly propagating waves only (with exclusion of the evanescent ones), does allow for both subluminal and Superluminal solutions!; the latter being the ones of interest here for us. Conditions (2) correspond in the (ω, k_z) plane to confining ourselves to the sector shown in Fig.1; that is, to the region delimited by the straight lines $\omega = \pm c k_z$.

A general, axially symmetric superposition of Bessel beams (with Φ as spectral weight-function) will therefore be:

$$\Psi(\rho, z, t) \; = \; \int_0^\infty \mathrm{d}k \int_0^\infty \mathrm{d}\omega \int_{-\omega/c}^{+\omega/c} \mathrm{d}k_z \; \psi(\rho, z, t) \; \delta\Big(k - \sqrt{\frac{\omega^2}{c^2} - k_z^2}\Big) \; \Phi(k, k_z, \omega) \, . \qquad (3)$$

Notice that it is $k \geq 0$; $\omega \geq 0$ and $-\omega/c \leq k_z \leq +\omega/c$. The question of the negative k_z values entering expansion (3) will soon be considered below.

The base functions $\psi(\rho, z, t)$ can be however rewritten as

$$\psi(\rho, \zeta, \eta) \; = \; J_0(k\rho) \; \exp i[\alpha\zeta - \beta\eta] \, ,$$

where (α, β), which will substitute in the following for the parameters (ω, k_z), are

$$\alpha \; \equiv \; \frac{1}{2V}(\omega + V k_z) \, ; \qquad \beta \; \equiv \; \frac{1}{2V}(\omega - V k_z) \, , \qquad (4)$$

in terms of the new "V-cone" variables:

$$\begin{cases} \zeta \; \equiv \; z - Vt \\ \eta \; \equiv \; z + Vt \end{cases} \qquad (5)$$

The present procedure is a generalization of the so-called "bidirectional decomposition" technique[9], which was previously devised for $V = c$ only.

The "V-cone" of Fig.2a corresponds in the (ω, k_z) plane to the sector limited by the straight-lines $\omega \pm V k_z = 0$, that is, by the lines $\alpha = 0$ and $\beta = 0$ (Fig.2b); while conditions (2) become [let us put $c = 1$ *whenever convenient*, throughout this paper]:

$$k^2 \; = \; V^2(\alpha + \beta)^2 - (\alpha - \beta)^2 \; \equiv \; (\alpha^2 + \beta^2)(V^2 - 1) + 2(V^2 + 1)\alpha\beta \, ; \quad k^2 \geq 0 \qquad (2')$$

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Inside the allowed region shown in Fig.1, we can choose for simplicity the sector delimited by the straight-lines $\omega = \pm V k_z$ shown in Fig.2b, *provided that* $V \geq 1$.

Let us observe that integrating over the ranges $\alpha, \beta \geq 0$ corresponds in eq.(3) to integrating over k_z between $-\omega/V$ and $+\omega/V$. But we shall choose in eq.(3) spectral weights $\Phi(k, k_z, \omega)$, and therefore spectral weights $\Phi(k, \alpha, \beta)$ in eq.(3') below, such as to either eliminate or make negligible the contribution from the negative values of k_z, that is, from the backwards moving waves: thus curing from the start the problem met by the "bidirectional decomposition" technique in connection with the so-called non-causal components. Therefore, our SLSs will all be physical solutions.

Let us recall also that each Bessel beam is associated with an ("axicone") angle θ, linked to its speed by the relations[10]:

$$\tan\theta = \sqrt{V^2 - 1}; \quad \sin\theta = \sqrt{\frac{V^2 - 1}{V}}; \quad \cos\theta = \frac{c}{V}, \tag{6}$$

where $V \to c$ when $\theta \to 0$, while $V \to \infty$ when $\theta \to \pi/2$.

Therefore, instead of eq.(3) we shall consider the (more easily integrable) Bessel beam superposition in the new variables [with $V \geq 1$]

$$\Psi(\rho, \zeta, \eta) = \int_0^\infty dk \int_0^\infty d\alpha \int_0^\infty d\beta \; J_0(k\rho)\, e^{i\alpha\zeta}\, e^{-i\beta\eta} \times$$
$$\times\, \delta\left(k - \sqrt{(\alpha^2 + \beta^2)(V^2 - 1) + 2(V^2 + 1)\alpha\beta}\right)\; \Phi(k, \alpha, \beta) \tag{3'}$$

where the integrations over α, β between 0 and ∞ just correspond to the dashed region of Fig.2b.

Let us now go on to constructing new Superluminal Localized Solutions for arbitrary frequencies, various of them possessing finite total energy.

3. – Some new Superluminal Localized Solutions for arbitrary frequencies and/or with finite total energy

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Superluminal solutions of the d'Alembert equation

a simple example:

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Consider the complex function

$$\Phi(x,y,z,t) = \frac{a}{\sqrt{[b - ic(x - ut)]^2 + (u^2 - c^2)(y^2 + z^2)}} \tag{1}$$

where a and b are nonzero constants, c is the usual "velocity of light" parameter, and

$$u > c \tag{2}$$

is any given superluminal velocity. Obviously the function (1) represents a signal propagating along the x axis with velocity u.

With a direct calculation I will confirm [1] that $\Phi(x,y,z,t)$ as given in (1) is a solution of the well known d'Alembert equation

$$\nabla^2\Phi(x,y,z,t) - \frac{1}{c^2}\frac{\partial^2\Phi(x,y,z,t)}{\partial t^2} = 0 \tag{3}$$

Proof: Setting

$$R = \sqrt{[b - ic(x - ut)]^2 + (u^2 - c^2)(y^2 + z^2)} \tag{4}$$

one has $\Phi = a/R$ and one can easily calculate the first derivatives

$$\begin{cases} \dfrac{1}{a}\dfrac{\partial\Phi}{\partial x} = ic\,\dfrac{b - ic(x - ut)}{R^3} \\ \dfrac{1}{a}\dfrac{\partial\Phi}{\partial y} = -(u^2 - c^2)\,\dfrac{y}{R^3} \\ \dfrac{1}{a}\dfrac{\partial\Phi}{\partial z} = -(u^2 - c^2)\,\dfrac{z}{R^3} \\ \dfrac{1}{a}\dfrac{\partial\Phi}{\partial t} = -icu\,\dfrac{b - ic(x - ut)}{R^3} \end{cases} \tag{5}$$

20 IV

whence the second derivatives follow:

$$\begin{cases} \frac{1}{a}\frac{\partial^2 \Phi}{\partial x^2} = \frac{c^2}{R^3} - \frac{3c^4}{R^5}\left[b - ic(x-ut)\right]^2 \\ \frac{1}{a}\frac{\partial^2 \Phi}{\partial y^2} = -\frac{u^2-c^2}{R^3} + 3\left(u^2-c^2\right)^2 \frac{y^2}{R^5} \\ \frac{1}{a}\frac{\partial^2 \Phi}{\partial z^2} = -\frac{u^2-c^2}{R^3} + 3\left(u^2-c^2\right)^2 \frac{z^2}{R^5} \\ \frac{1}{a}\frac{\partial^2 \Phi}{\partial t^2} = \frac{c^2u^2}{R^3} - \frac{3c^2u^2}{R^5}\left[b - ic(x-ut)\right]^2 \end{cases} \qquad (6)$$

From these equations we get:

$$\frac{1}{a}\left[\frac{\partial^2 \Phi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2}\right] = -\frac{u^2-c^2}{R^3} + 3\left(u^2-c^2\right)\frac{\left[b - ic(x-ut)\right]^2}{R^5} \qquad (7)$$

and

$$\frac{1}{a}\left[\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2}\right] = -2\,\frac{u^2-c^2}{R^3} + 3\left(u^2-c^2\right)^2 \frac{y^2+z^2}{R^5} \qquad (8)$$

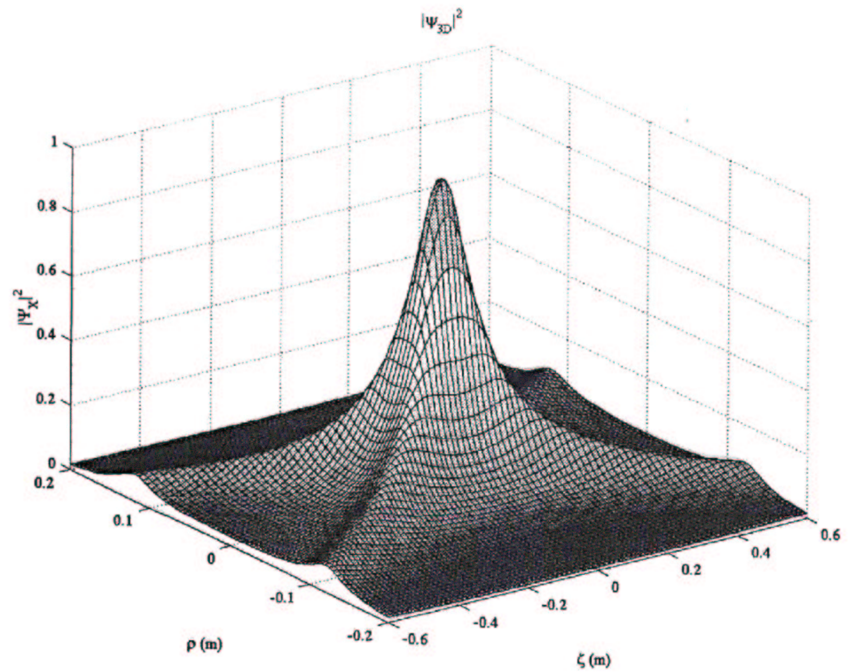
whence, remembering (4):

$$\frac{1}{a}\left[\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2}\right] = 0 \qquad (9)$$

The d'Alembert equation is satisfied!

[1] J. E. Maiorino and W. A. Rodrigues Jr., What is Superluminal motion? RP 59/99 IMECC-UNICAMP

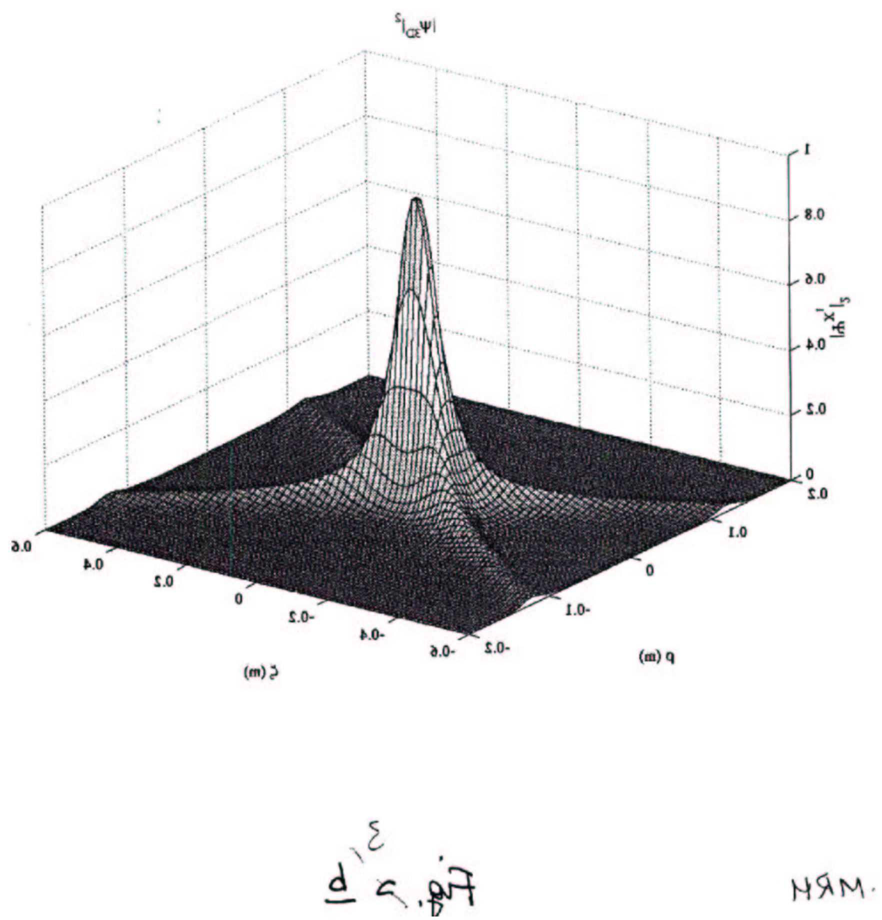
21 a



MRH

Fig. 3a

21 b



21c

3.1 – *The classical "X-shaped solution" and its generalizations.*

Let us start by choosing the spectrum [with $a > 0$]:

$$\Phi(\alpha, \beta) = \delta(\beta - \beta') \, e^{-a\alpha} \quad ??????????????????? \text{ ok} \qquad (7)$$

$a > 0$ and $\beta' \geq 0$ being constants (related to the transverse and longitudinal localization of the pulse).

In the simple case when $\beta' = 0$, one completely dispenses with the "non-causal" (backwards-moving) components of the bidirectional Fourier-type expansion (3'). For the sake of clarity, let us go back to examining Fig.2b: The $\delta(\beta)$ factor in spectrum (7) does actually imply the integrations over α and β in eq.(3') to run along the α-line only, i.e., along the $\beta = 0$ line (where $\omega = +Vk_z$). Let us, then, choose $\beta' = 0$, and observe that for $\beta = 0$ all the solutions $\Psi(\rho, \zeta, \eta)$ are actually functions only of ρ and $\zeta = z - Vt$. [Let us also notice that in empty space such solutions $\Psi(\rho, \zeta = z - Vt)$ can be transversally localized only if $V \neq c$, because if $V = c$ the function Ψ has to obey the Laplace equation on the transverse planes. Let us recall that in this paper we always assume $V > 0$].
In the present case, eq.(3') can be easily integrated over β and k by having recourse to identity (6.611.1) of ref.[11], yielding

$$\Psi_X(\rho, \zeta) = \int_0^\infty d\alpha \, J_0(\alpha\sqrt{V^2 - 1}) \, e^{-\alpha(a - i\zeta)} = \qquad (8)$$

$$= [(a - i\zeta)^2 + \rho^2(V^2 - 1)]^{-1/2} \,,$$

which is exactly the classical X-shaped solution proposed by Lu & Greenleaf[12] in acoustics, and later on by others[12] in electromagnetism, once relations (6) are taken into account.

Many other SLSs can be easily constructed. For instance, by inserting into the weight function (7) the extra factor α^m, while it is still $\beta' = 0$. Then an infinite family of new SLSs is obtained (for $m \geq 0$), by using this time identity (6.621.4) of the same ref.[11]:

21d

$$\Psi_{X,m}(\rho,\zeta) = -(-i)^{m-1}\frac{d^m}{d\zeta^m}\left[(a-i\zeta)^2+\rho^2(V^2-1)\right]^{-1/2} \qquad (9)$$

which generalize the classical X-*shaped solution*, corresponding to $m=0$: namely, $\Psi_X \equiv \Psi_{X,0}$. Notice that all the derivatives of the latter with respect to ζ lead to new SLSs. In the particular case $m=1$, one gets the SLS

$$\Psi_{X,1}(\rho,\zeta) = \frac{(a-i\zeta)}{\left[(a-i\zeta)^2+\rho^2(V^2-1)\right]^{3/2}} \qquad (10)$$

which is the first derivative of the X-shaped wave[14].

The present solutions are well suited for low frequencies only, since their frequency spectrum (exponentially decreasing) starts from zero. One can see this for instance by writing eq.(7) in the (ω, k_z) space: by eqs.(4) one obtains

$$\Phi(\omega,k_z) = \delta\left(\frac{\omega - Vk_z}{2V} - \beta'\right)\ \exp[-a\frac{\omega+Vk_z}{2V}]$$

and can observe that $\beta' = 0$ in the delta implies $\omega = Vk_z$. So that the spectrum becomes $\Phi = \exp[-a\omega/V]$, which starts from zero and has a width given by $\Delta\omega = V/a$.

Nevertheless, it can be worthwhile noticing that, when the factor α^m is present, the frequency spectrum of the solutions can be bumped in correspondence with any value ω_M of the angular frequency, provided that m is large, or a/V is small: in fact, ω_M results to be $\omega_M = 2mV/a$.

Moreover, let us mention here that also in the spectra of the following pulses (considered in subsections 3.2 and 3.3 below) one can insert the α^m factor; in fact, in correspondence with the spectrum

$$\Phi(\alpha,\beta) = \alpha^m\ \Phi_0(\beta)\ e^{-a\alpha}\ , \qquad (7')$$

one obtains *as further solutions* the m-th order derivatives of the basic ($m=0$) solution below considered. This is due to the circumstance that our integrations over α (as in eq.(3')) are always Fourier-type transformations. We shall not write them down explicitly, however, for the sake of conciseness.

Different SLSs can be obtained also by *modifying* (still with $\beta' = 0$) the spectrum (7). Some interesting solutions are reported in Appendix A.

6

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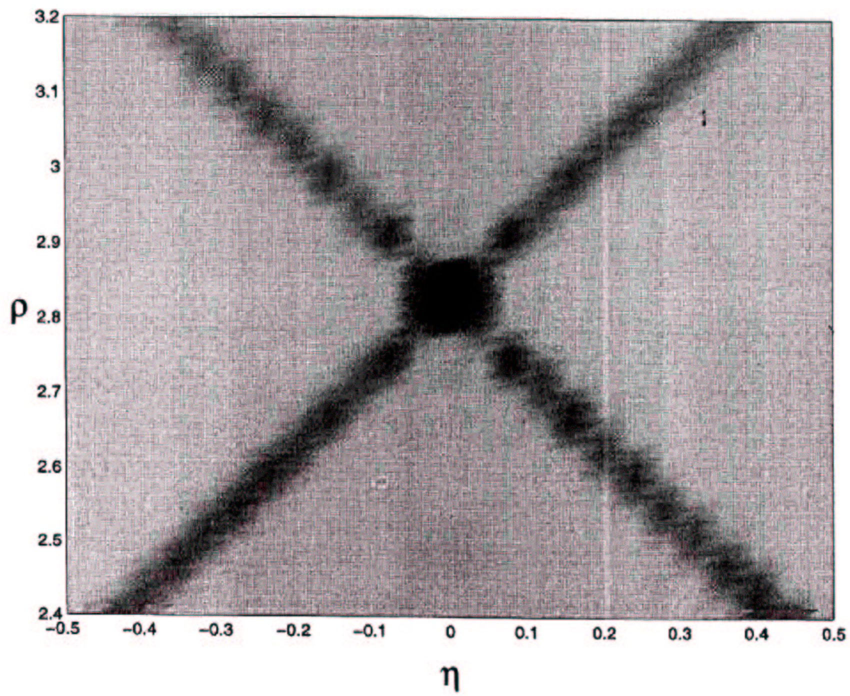
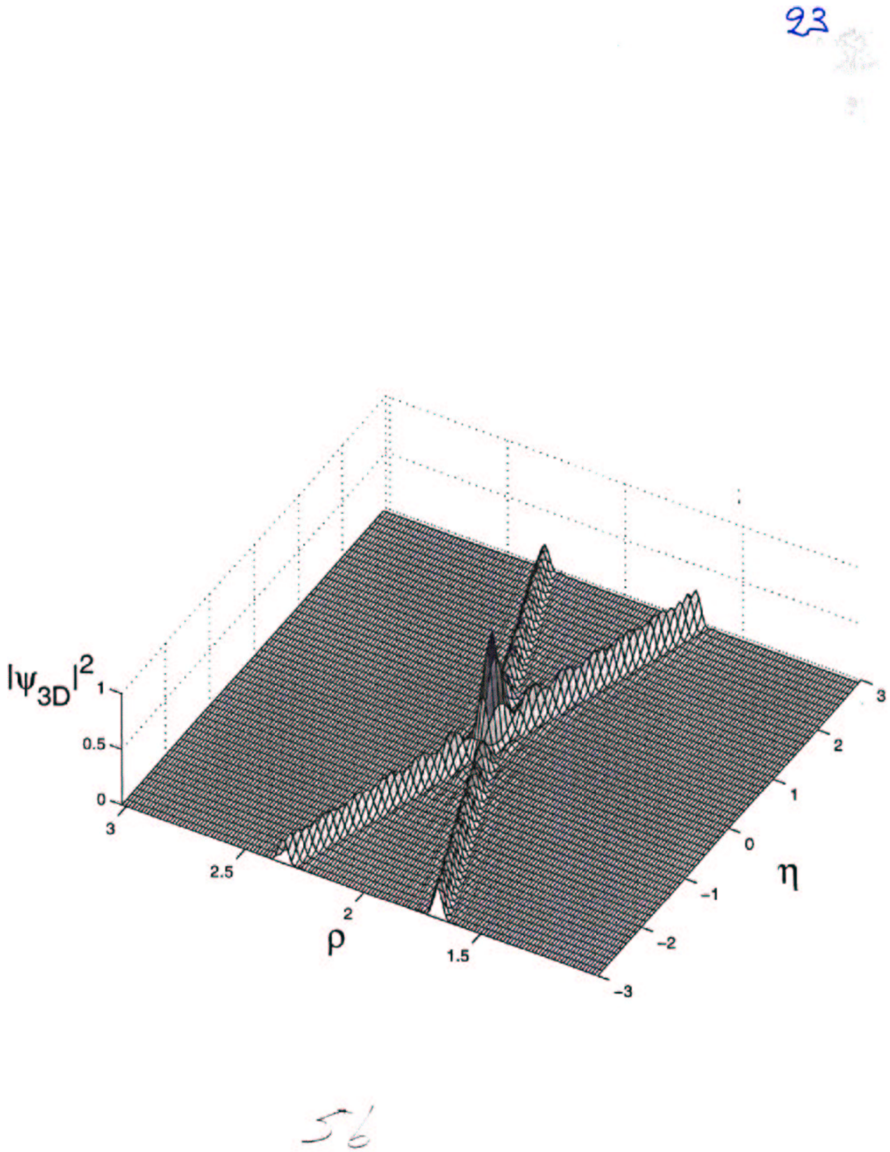
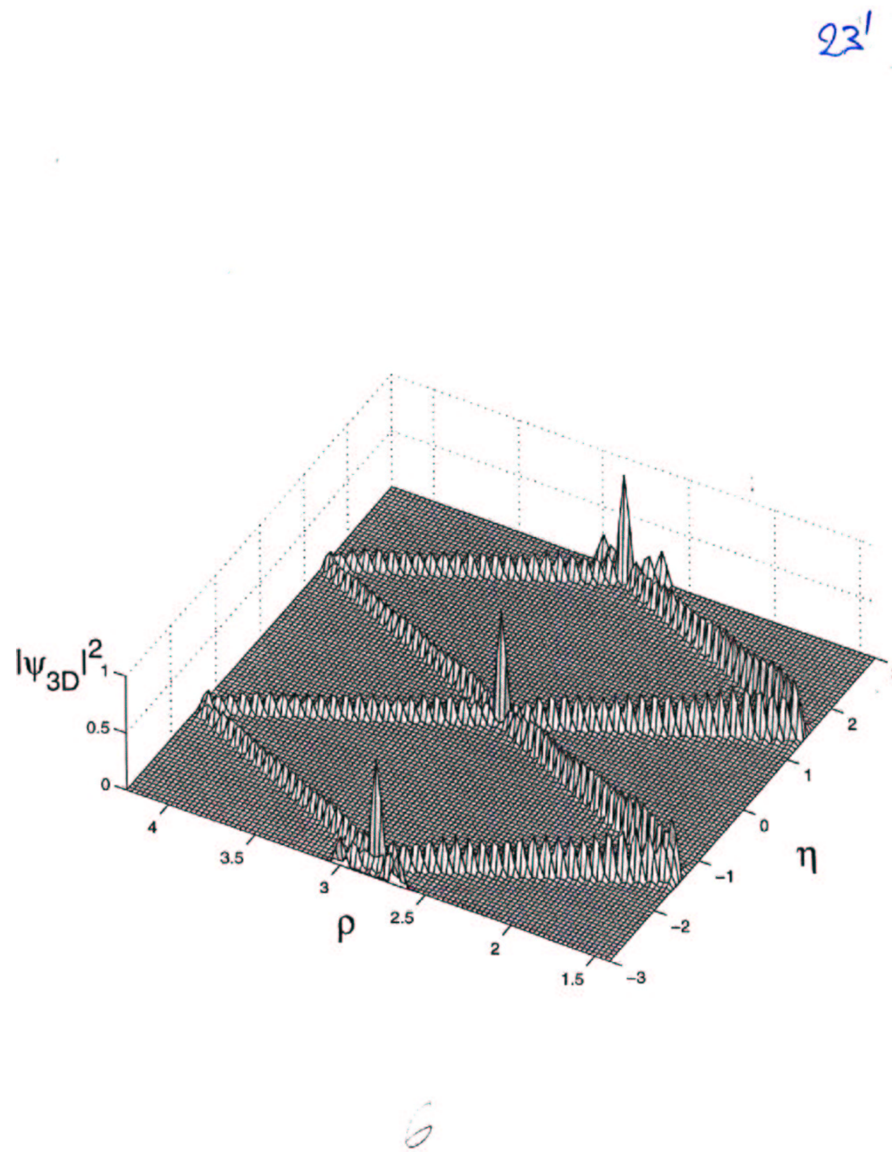



23
$|\psi_{3D}|^2$
1
0.5
0
3
2.5
2
1.5
ρ
3
2
1
0
−1
−2
−3
η
56


23'
$|\psi_{3D}|^2$
1
0.5
0
4
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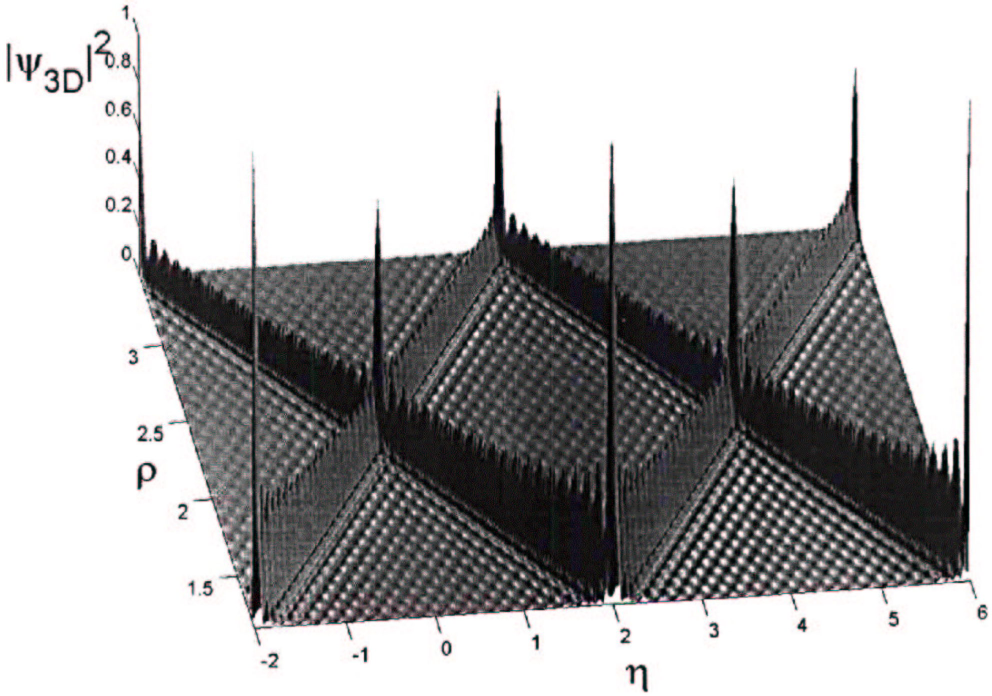
2
1
0
−1
−2
−3
η
6

23''



24a

finite energy:

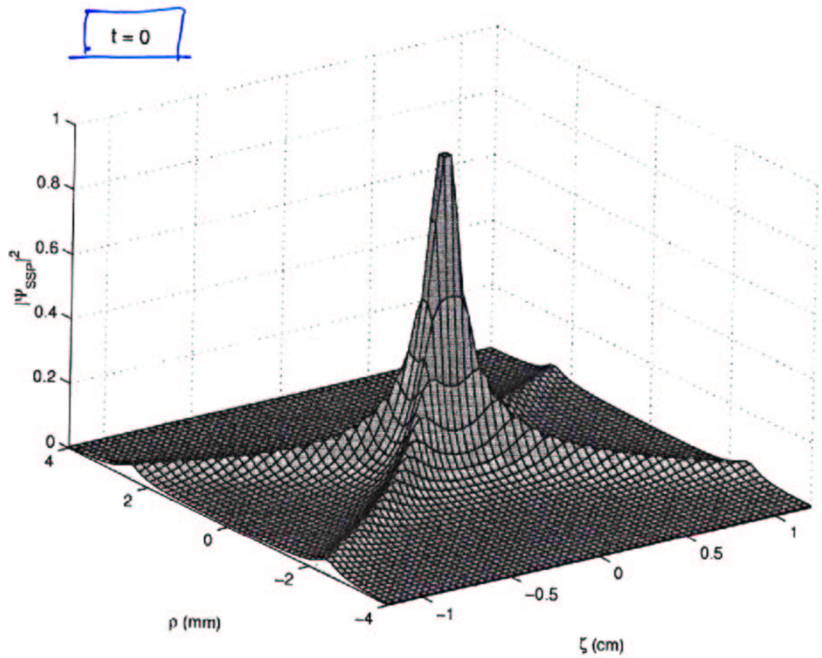


Fig. a

24b

t = 10 s

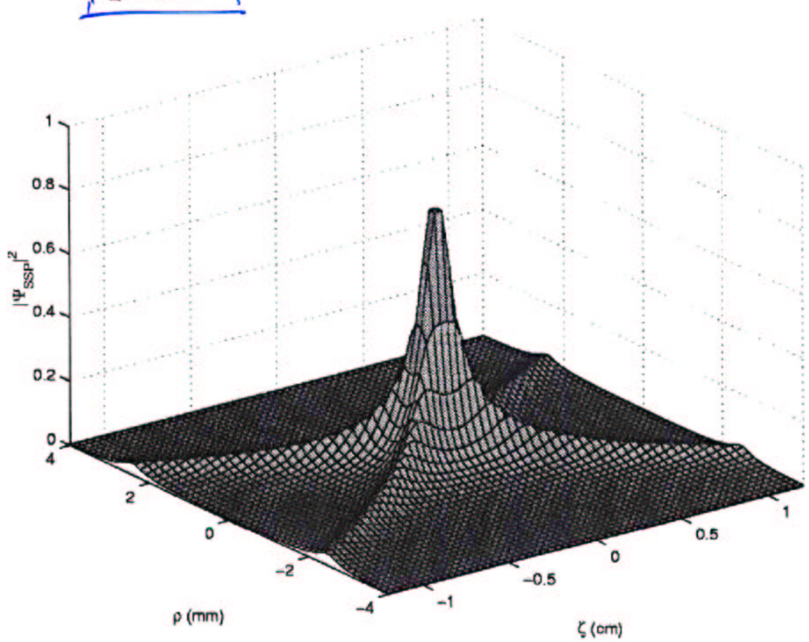


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Fig. b

25

CLASSICAL TACHYONS

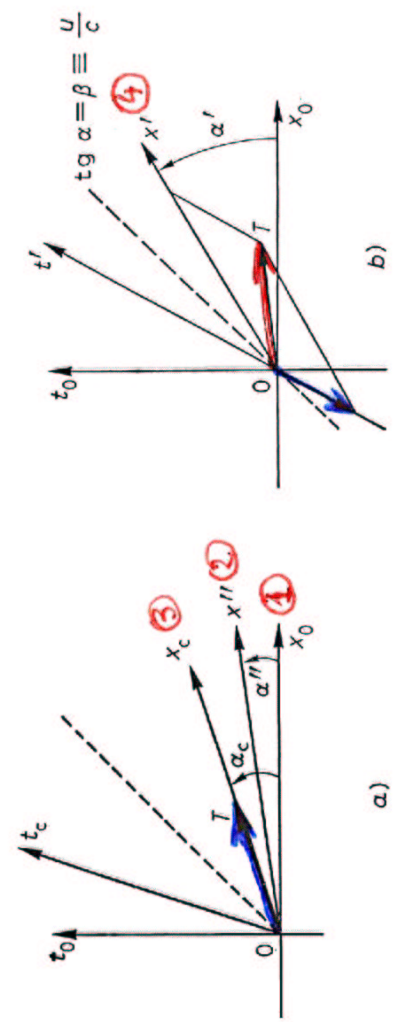

Fig. 11.

is simply the one whose space axis x is superposed to the world-line OT; speed u_c w.r.t. s_0, along the positive x-axis, is evidently

$$u_c = c^2/V_0\,, \qquad u_c V_0 = c^2 \tag{52}$$

(« critical frame dual to the tachyon speed V_0. Finally, from fig. 10 and 11b) we concl

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26 the same transformation L which inverts the energy sign will also reverse the motion direction in time (review I; RECAMI, 1973, 1975, 1979*a*; CALDIROLA and RECAMI, 1978; see also GARUCCIO *et al.*, 1980). In fact, from fig. 10 we can see that for going from a positive-energy state T_i to a negative-energy state T'_f it is necessary to by-pass the « transcendent » state T_∞ (with $V = \infty$). From fig. 11*a*) we see moreover that, given in the initial frame s_0 a tachyon T travelling, *e.g.*, along the positive x-axis with speed V_0, the « critical observer » (*i.e.* the ordinary subluminal observer $s_c \equiv (t_c, x_c)$, seeing T with *infinite* speed)

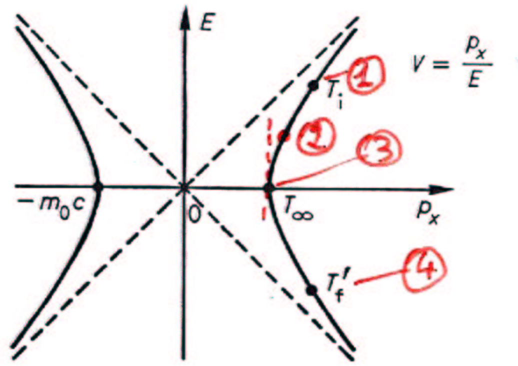
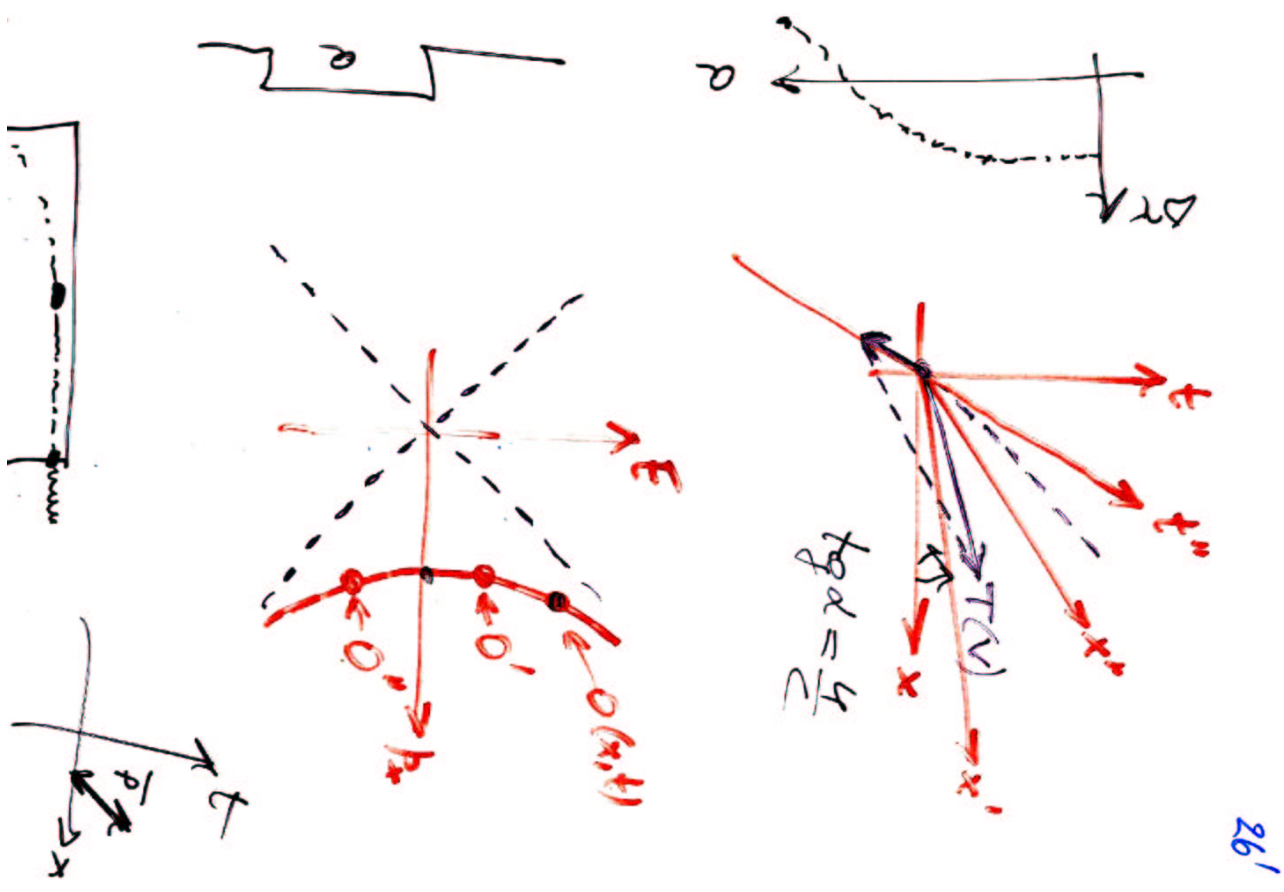


Fig. 10.

→ A Superluminal object, after overcoming the infinite speed appears as an anti-object traveling in the opposite space direction = with negative speed



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LETTERE AL NUOVO CIMENTO VOL. 44, N. 8 16 Dicembre 1985

The Tolman-Regge *Antitelephone* Paradox: Its Solution by Tachyon Mechanics (*).

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(ricevuto il 14 Settembre 1985)

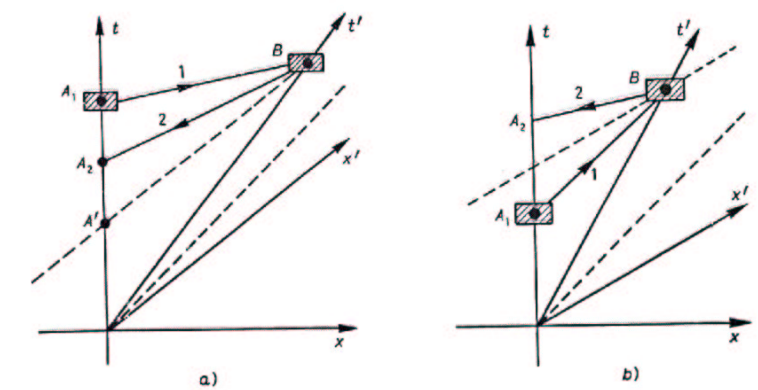
PACS. 03.30. – Special relativity.

Summary. – The possibility of solving (at least « in microphysics ») all the ordinary causal paradoxes devised for tachyons is not yet widely recognized; on the contrary, the effectiveness of the Stuckelberg-Feynman « switching principle » is often misunderstood. We want, therefore, to show in detail and rigorously how to solve the oldest causal paradox, originally proposed by Tolman, which is the kernel of so many further tachyon paradoxes. The key to the solution is a careful application of *tachyon kinematics*, which can be unambiguously derived from special relativity. A systematic, thorough analysis of all tachyon paradoxes is going to appear elsewhere.

Introduction. – It has been claimed since long (1) that all the ordinary causal paradoxes proposed for tachyons can be solved (at least « in microphysics » (2)) on the basis of the « *switching procedure* » (SWP ») by STÜCKELBERG (3) and FEYNMAN (3), also known as the « reinterpretation principle »: a principle which has been given the status of a

(*) Work supported in part by CIME/IILA and INFN.
(**) Temporary address.
(***) Permanent address.
(1) O. M. B. BILANIUK, V. K. DESHPANDE and E. C. G. SUDARSHAN: *Am. J. Phys.*, **30**, 718 (1962); R. G. ROOT and J. S. TREFIL: *Lett. Nuovo Cimento*, **3**, 412 (1970); J. A. PARMENTOLA and D. D. H. YEE: *Phys. Rev. D*, **4**, 1912 (1971); E. RECAMI and R. MIGNANI: *Lett. Nuovo Cimento*, **7**, 388 (1973); E. RECAMI: in *Enciclopedia EST-Mondadori, Annuario 73*, edited by E. MACORINI (Mondadori, Milano, 1973), p. 85; E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209, 398 (1974); E. RECAMI: *Lett. Nuovo Cimento*, **18**, 501 (1977); *Found. Phys.*, **8**, 329 (1978); G. D. MACCARRONE and E. RECAMI: *Found. Phys.*, **10**, 949 (1980); A. GARUCCIO, G. D. MACCARRONE, E. RECAMI and J. P. VIGIER: *Lett. Nuovo Cimento*, **27**, 60 (1980); P. CALDIROLA and E. RECAMI: in *Italian Studies in the Philosophy of Science*, edited by M. DALLA CHIARA (Reidel, Boston, Mass., 1980), p. 249.
(2) E. RECAMI: *Classical tachyons and possible applications: a review*, Report INFN/AE-84/8 (Frascati, August 1984), to appear in *Riv. Nuovo Cimento*.
(3) E. C. G. STÜCKELBERG: *Helv. Phys. Acta*, **14**, 321, 588 (1941); R. P. FEYNMAN: *Phys. Rev.*, **76**, 749, 769 (1949). See also O. KLEIN: *Z. Phys.*, **53**, 157 (1929).

27'



. 22.

ne w.r.t. B, that is to say its world-line BA_2 must have a slope smaller than slope BA' of the x'-axis (where $BA' \| x'$); this means that A_2 must stay ove A'. If the speed of tachyon 2 is such that A_2 falls between A' and A_1, seems that 2 reaches back to A (event A_2) *before* the emission of 1 (event A_1). is appears to realize an *antitelephone*.

9'1.2. The solution. First of all, since tachyon 2 moves backwards time w.r.t. A, the event A_2 will appear to A as the emission of an antitachyon $\bar{2}$. e observer « t » will see his apparatus A (able to exchange tachyons) emit ccessively towards B the antitachyon $\bar{2}$ and the tachyon 1.

At this point, some supporters of the paradox (overlooking tachyon kinetics, as well as relations (66)) would say that, well, the description forwarded observer « t » can be orthodox, but then the device B is no longer working cording to the premises, because B is no longer emitting a tachyon 2 on receipt tachyon 1. Such a statement would be wrong, however, since the fact that » sees an « intrinsic emission » at A_2 *does not mean* that « t' » will see an trinsic absorption » at B. On the contrary, we are just in the case of bsect. **6'10**: intrinsic emission by A, at A_2, with $\boldsymbol{u}\cdot\boldsymbol{V}_{\bar{2}} > c^2$, where $\boldsymbol{u}$ and $\boldsymbol{V}_{\bar{2}}$ the velocities of B and $\bar{2}$ w.r.t. A, respectively; so that *both* A *and* B fer an intrinsic *emission* (of tachyon 2 or of antitachyon $\bar{2}$) in their own t frames.

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Tachyon Kinematics and Causality: A Systematic Thorough Analysis of the Tachyon Causal Paradoxes[1]

Erasmo Recami[2,3]

Received November 18, 1985

The chronological order of the events along a spacelike path is not invariant under Lorentz transformations, as is well known. This led to an early conviction that tachyons would give rise to causal anomalies. A relativistic version of the Stückelberg–Feynman "switching procedure" (SWP) has been invoked as the ...

subluminal electric charges are present, then $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ and eq. (210) does not depend on the selected surface Σ (it depends only on its boundary $P - P'$). If also subluminal magnetic monopoles are present, then $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} - \varepsilon_{\mu\nu\rho\sigma} B^{\rho,\sigma}$, where B^μ is a *second* four-potential, and the following *condition* must be explicitly imposed:

$$\exp\left[-\frac{ie}{2}\oint_{\Sigma-\Sigma'} F_{\mu\nu}\,\mathrm{d}\sigma^{\mu\nu}\right] = 1\,, \qquad (211)$$

wherefrom Dirac relation $eg = nhc/2$ follows.

However, if « magnetic monopoles » cannot be put at rest, as in the case of « *tachyon monopoles* », then eq. (210) is again automatically satisfied, without any recourse to the Dirac condition.

15·4. *Further remarks.*

i) It may be interesting to quote that the possible connection between tachyons and « monopoles » in the sense outlined above (RECAMI and MIGNANI, 1974*a*) was first heuristically guessed by ARZELIÈS (1958)—who predicted that $E \rightleftarrows H$ for $U > c$—and later on by PARKER (1969), in its important and pioneering two-dimensional theory (see also WEINGARTEN, 1973).

ii) As to the first considerations about the motion of a charged tachyon in an external field, see BACRY (1972) and BACRY *et al.* (1974). Notice, incidentally, that even a zero-energy charged tachyon may radiate (RHEE, 1969) subtracting energy to the field.

iii) The interactions of tachyon soliton charges have been studied, *e.g.*, by VAN DER MERWE (1978), by means of Bäcklund transformations.

iv) If we consider the quanta inside the Cauchy-Fresnel « evanescent waves », since the momentum component normal to the reflecting plane is imaginary, the one parallel to that plane is larger than the energy. Such partial « tachyon properties » of those quanta have been studied particularly by COSTA DE BEAUREGARD (1973; see also COSTA DE BEAUREGARD *et al.*, 1971), whose research group even performed an experimental investigation (HUARD and IMBERT, 1978). Further experimental work is presently being performed, for example, by ALZETTA at Pisa.

v) AGUDIN and PLATZECK (1982*b*) observed that the motion of an *accelerated* charge can be expanded into uniform-motion components, via Fourier analysis. They claimed that, when the acceleration is not zero, both « tachyonic » and « bradyonic » components *must* be present.

15·5. « *Experimental* » *considerations*. – The very first experiments looking for tachyons, by ALVÄGER *et al.* (1963, 1965, 1966), have been already mentioned n.b.

E. Recami

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Physica B 175 (1991) 257–262
North-Holland

Analogies between electron and photon tunneling

A proposed experiment to measure photon tunneling times

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Department of Physics, University of California, Berkeley, CA 94720, USA

The phenomenon of tunneling is a well-known fundamental consequence of quantum mechanics. *All* particles can in principle tunnel. In particular, both electrons and photons can tunnel through classically forbidden regions of space known as "barriers". However, there have been numerous controversies over how long it takes a particle to cross a barrier. Exploiting an analogy between electrons and photons, we suggest an experiment to infer the characteristics of an electron's barrier-traversal time by measuring the time it takes a photon to traverse a similar barrier. Electron tunneling experiments are in general much more difficult to perform than analogous optical ones. With an optical technique one can construct optical barriers on the scale of microns, in contrast to the ångstrom-scale barriers required for electron tunneling. Our experiment may help settle the controversies over tunneling times.

By means of a newly developed quantum optical technique, we should be able to measure the tunneling times of individual photons with sub-picosecond resolution. In our experiments we are using a two-photon light source, in which a pair of tightly correlated photons is generated by the process of spontaneous parametric down-conversion. Hong, Ou and Mandel have already achieved a sub-picosecond comparison between the arrival times of two such photons at a beam splitter placed at the intersection of their paths. In our geometry, one member of the photon pair tunnels through a barrier, while the other does not. Then *coincidence* detection of this photon pair constitutes the detection of an individual tunneling event. The particle aspect of photon tunneling can thus be clearly observed. We propose to use the Hong–Ou–Mandel technique to measure the tunneling time. We have chosen for our tunneling barrier two glass prisms placed in close proximity, utilizing the phenomenon of frustrated total internal reflection.

The question of how much time it takes a particle to tunnel across a barrier has been a controversial one [1]. There have been several plausible answers proposed to this question. Here we shall only briefly list some of the main contenders: (1) the phase time, i.e. the time that it takes the peak of a wave packet to appear on the other side of the barrier, as given by the stationary phase method; (2) the dwell time, i.e. the time spent by the particle in the barrier region averaged over all scattering channels; (3) the local Larmor time, i.e. the traversal time as measured by spin precession of the tunneling particle in a uniform magnetic field; and (4) the Büttiker–Landauer time, i.e. the time calculated for an oscillating barrier, which has the form of a quasiclassical WKB integral. Not only do these times contradict each other in general, but in some limits they also contradict certain basic principles, such as Einstein causality, or the uncertainty principle. Clearly, these contradictions are symptoms of some fundamental problems in our understanding of the tunneling process. Towards the resolution of this problem, we suggest a photon tunneling experiment.

We begin by establishing an analogy between electron and photon tunneling. Although at present electron tunneling is more relevant to actual devices, photon tunneling is more convenient for experimental study to resolve the above contradictions. This is true for two main reasons. First, the size of tunneling barriers is typically much larger for photon tunneling than for electron tunneling. This stems from the fact that the wavelength of light is usually much larger than that of electrons. Second, there has been a recent development of a quantum optical technique by which one can perform high resolution

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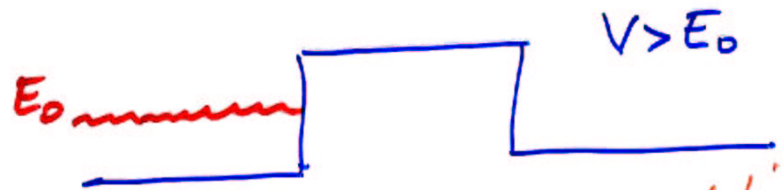
29a

imaginary wave-number →
→ imaginary momentum



Classical case (QUANTUM, non-relativistic)

$$\frac{p_i^2}{2m} = E_0 - V < 0 \Rightarrow p_i \text{ imagin.}$$

Relativistic case:

$$E_i = \pm\sqrt{\vec{p}^{\,2} \pm m_0^2}$$; for photons: small

p_i imaginary $\Longrightarrow$ E_i imaginary too

(IN THE ULTRA-RELATIVISTIC CASE)

29b

$$\frac{e^{imr}}{r} \longrightarrow \frac{e^{-m'r}}{r}$$

$$e^{ikz} \longrightarrow e^{-k'z}$$

se $k = ik'$

30

– Capitolo 2 –

Prima di prendere in rassegna gli esperimenti, ed i risultati da questi ottenuti, occupiamoci dell'equivalenza tra modi elettromagnetici evanescenti e tunneling di particelle, in particolare nel caso delle guide d'onda. Considerando una particella di massa m ed energia cinetica $E = \hbar^2 k^2 / 2m$. Nel caso (unidimensionale) di attraversamento di un potenziale uniforme V_0, l'equazione di *Schrödinger* per tale particella sarà:

Schrödinger:
$$\frac{\partial^2 \psi}{\partial x} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0. \tag{2.0.0}$$

Posto allora:

$$\kappa^2 = \frac{2m}{\hbar^2}(E - V_0), \tag{2.0.1}$$

la (2.0.0) risulta formalmente identica all'equazione di Helmholtz per la componente scalare relativa al campo elettrico, o a quello magnetico, di un campo (e.m.) che si propaghi in un mezzo dispersivo:

Helmholtz:
$$\frac{\partial^2 \psi}{\partial x} + \kappa^2 \psi = 0, \tag{2.0.2}$$

dove in questo caso:

$$\kappa = \frac{2\pi}{\lambda_m} = \frac{2\pi}{\lambda} n,$$

λ_m è la lunghezza d'onda all'interno del mezzo, λ è la lunghezza d'onda nel vuoto, e n è l'indice di rifrazione del mezzo in cui il campo si propaga.

Il confronto tra le due equazioni suggerisce la sostituzione:

$$\sqrt{\frac{2m}{\hbar^2}(E - V_0)} \rightarrow \frac{2\pi}{\lambda} n = \frac{2\pi\nu}{v} = \frac{E}{\hbar v}$$

Nel caso di una guida d'onda rettangolare di dimensioni $a \times b$ ($a < b$), e con pareti perfettamente conduttrici, sappiamo che:

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EUROPHYSICS LETTERS

Europhys. Lett., **57** (6), pp. 879–884 (2002) 15 March 2002

Superluminal tunneling through two successive barriers(*)

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PACS. 03.65.-w – Quantum mechanics.
PACS. 03.30.+p – Special relativity.

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... by a segment of normal region. For suitable frequency bands —*i.e.*, far from resonances— it was found that the total crossing time does not depend on the length of the intermediate (normal) region: namely, that the beam speed along it is infinite. The related experimental results [8] have been already confirmed by numerical simulations, based on Maxwell equations only [9]. But they are so amazing that we want to check whether they agree also with what is predicted by quantum mechanics in the analogous case of two successive potential barriers.

In this note we are actually going to show that, for non-resonant tunneling trough two successive, rectangular (opaque) potential barriers (fig. 1), the (total) phase time does depend neither on the barrier widths *nor on the distance between the barriers*. In other words, far from resonances the tunneling phase time, which does depend on the entering energy, can be shown to be *independent* of the distance between the two barriers.

Phase time evaluation. – Let us consider the (quantum-mechanical) stationary solution for the one-dimensional (1D) tunneling of a non-relativistic particle, with mass m and kinetic energy $E = \hbar^2 k^2/2m = mv^2/2$, through two equal rectangular barriers with height V_0 ($V_0 > E$) and width a, the quantity $L - a \geq 0$ being the distance between them. The Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\,\psi(x) = E\,\psi(x), \tag{1}$$

where $V(x)$ is zero outside the barriers, while $V(x) = V_0$ inside the potential barriers. In the various regions I ($x \leq 0$), II ($0 \leq x \leq a$), III ($a \leq x \leq L$), IV ($L \leq x \leq L + a$) and V ($x \geq L + a$), the stationary solutions to eq. (1) are the following:

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$$\begin{cases} \psi_{\rm I} = e^{+ikx} + A_{1\rm R}\, e^{-ikx}\,, & (2a)\\ \psi_{\rm II} = \alpha_1\, e^{-\chi x} + \beta_1\, e^{+\chi x}\,, & (2b)\\ \psi_{\rm III} = A_{1\rm T}\,\left[e^{ikx} + A_{2\rm R}\, e^{-ikx}\right], & (2c)\\ \psi_{\rm IV} = A_{1\rm T}\,\left[\alpha_2\, e^{-\chi(x-L)} + \beta_2\, e^{+\chi(x-L)}\right], & (2d)\\ \psi_{\rm V} = A_{1\rm T}\, A_{2\rm T}\, e^{ikx}\,, & (2e)\end{cases}$$

where $\chi \equiv \sqrt{2m(V_0 - E)}/\hbar$, and quantities $A_{1\rm R}$, $A_{2\rm R}$, $A_{1\rm T}$, $A_{2\rm T}$, α_1, α_2, β_1 and β_2 are the reflection amplitudes, the transmission amplitudes, and the coefficients of the "evanescent" (decreasing) and "anti-evanescent" (increasing) waves for barriers 1 and 2, respectively. Such quantities can be easily obtained from the matching (continuity) conditions:

$$\begin{cases} \psi_{\rm I}(0) = \psi_{\rm II}(0)\,, & (3a)\\ \left.\dfrac{\partial\psi_{\rm I}}{\partial x}\right|_{x=0} = \left.\dfrac{\partial\psi_{\rm II}}{\partial x}\right|_{x=0}, & (3b)\end{cases}$$

$$\begin{cases} \psi_{\rm II}(a) = \psi_{\rm III}(a)\,, & (4a)\\ \left.\dfrac{\partial\psi_{\rm II}}{\partial x}\right|_{x=a} = \left.\dfrac{\partial\psi_{\rm III}}{\partial x}\right|_{x=a}, & (4b)\end{cases}$$

$$\begin{cases} \psi_{\rm III}(L) = \psi_{\rm IV}(L)\,, & (5a)\\ \left.\dfrac{\partial\psi_{\rm III}}{\partial x}\right|_{x=L} = \left.\dfrac{\partial\psi_{\rm IV}}{\partial x}\right|_{x=L}, & (5b)\end{cases}$$

$$\begin{cases} \psi_{\rm IV}(L+a) = \psi_{\rm V}(L+a)\,, & (6a)\\ \left.\dfrac{\partial\psi_{\rm IV}}{\partial x}\right|_{x=L+a} = \left.\dfrac{\partial\psi_{\rm V}}{\partial x}\right|_{x=L+a}. & (6b)\end{cases}$$

Equations (3)-(6) are eight equations for our eight unknowns ($A_{1\rm R}$, $A_{2\rm R}$, $A_{1\rm T}$, $A_{2\rm T}$, α_1, α_2, β_1 and β_2). First, let us obtain the four unknowns $A_{2\rm R}$, $A_{2\rm T}$, α_2, β_2 from eqs. (5) and (6) in the case of *opaque* barriers, *i.e.*, when a is large enough (and χ not too small) so that one can assume that $\chi a \to \infty$:

$$\begin{cases} \alpha_2 \longrightarrow e^{ikL}\,\dfrac{2ik}{ik-\chi}\,, & (7a)\\ \beta_2 \longrightarrow e^{ikL-2\chi a}\,\dfrac{-2ik(ik+\chi)}{(ik-\chi)^2}\,, & (7b)\\ A_{2\rm R} \longrightarrow e^{2ikL}\,\dfrac{ik+\chi}{ik-\chi}\,, & (7c)\\ A_{2\rm T} \longrightarrow e^{-\chi a} e^{-ika}\,\dfrac{-4ik\chi}{(ik-\chi)^2}\,. & (7d)\end{cases}$$

Then, we may obtain the other four unknowns $A_{1\rm R}$, $A_{1\rm T}$, α_1, β_1 from eqs. (3) and (4). Again in the case of large enough barriers (and $\chi a \to \infty$), one gets:

$$\begin{cases} \alpha_1 \longrightarrow \dfrac{2ik}{ik-\chi}\,, & (8a)\\ \beta_1 \longrightarrow e^{-2\chi a}(k - i\chi)\,\dfrac{\sin k(L-a)}{\chi}\,A\,, & (8b)\\ A_{1\rm R} \longrightarrow \dfrac{ik+\chi}{ik-\chi}\,, & (8c)\\ A_{1\rm T} \longrightarrow e^{-\chi a} e^{-ikL}\,A\,, & (8d)\end{cases}$$

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Fig. 1

Fig. 1 – The non-resonant tunneling process, through *two* successive (opaque) potential barriers, considered in this paper. We show that, far from resonances, the (total) phase time for tunneling through the two barriers does depend neither on the barrier widths *nor on the distance between the barriers.*

x, as a consequence. In other words, it becomes an *evanescent* wave (going back to normal propagation, even if with reduced amplitude, when the narrowing ends and the guide returns to its initial transverse size). Thus, a tunneling experiment can be simulated by having recourse to evanescent waves (for which the concept of group velocity can be properly extended [51]).

And the fact that evanescent

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where

$$A \equiv \frac{2\chi k}{2\chi k \ \cos k(L-a) + (\chi^2 - k^2) \ \sin k(L-a)} \tag{9}$$

results, incidentally, to be real.

At this point, by applying the well-known definition of phase time (see, for instance, refs. [1–3]), we can derive that the tunneling time

$$\tau^{\rm ph}{}_{\rm tun} \equiv \hbar \frac{\partial \arg\left[A_{1\rm T} A_{2\rm T} e^{ik(L+a)}\right]}{\partial E} = \hbar \frac{\partial}{\partial E} \arg\left[\frac{-4ik\chi}{(ik-\chi)^2}\right] = \hbar \frac{\partial}{\partial E} \arctan\left[\frac{k^2-\chi^2}{k\,\chi}\right] = \frac{1}{\hbar\chi}\,\frac{2m}{k}, \tag{10}$$

while depending on the energy of the tunneling particle, *does not depend* on $L + a$ (being it actually independent both of a and of L).

This result does *not only* confirm the so-called "Hartman effect" [2,3] for the two opaque barriers —*i.e.*, the independence of the tunneling time from the opaque barrier widths— but it does *also* extend such an effect by implying the total tunneling time to be independent even of L (see fig. 1). This might be regarded as a further evidence of the fact that quantum systems seem to behave as non-local; but is has a more general meaning, being it associated with the properties of any waves (and, in fact, something very similar happens also ...

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quots! file

PHYSICAL REVIEW A, VOLUME 65, 052124

Superoscillations and tunneling times

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It is proposed that superoscillations play an important role in the interferences that give rise to superluminal effects. To exemplify that, we consider a toy model that a wave packet to travel in zero time and negligible distortion, a distance arbitrarily larger than the width of the wave packet. The peak is shown to result from a superoscillatory superposition at the tail. Similar reasoning applies to the dwell time.

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I. INTRODUCTION

Superluminal effects have been predicted in conjunction with various quantum systems propagating in a forbidden zone. In these regimes, the (semiclassical) kinetic energy is negative, making the semiclassical tunneling time ill defined, and various operational definitions of the velocity of a wave packet have been proposed, in many examples, giving differing values. In recent years, a number of experiments with superluminal photons have been performed, reviving interest in the problem, as well as controversy. The theoretical ...

subjected to a nondisturbing, "weak" measurement, the outcome of the measurement, known as the "weak value," can attain values that lie outside the spectrum of eigenvalues of the measured observable [13,20]. Weak values may hence be naturally related to the superluminal phenomenon, as indeed, Steinberg has already argued that the dwell time is a weak value of a projector to the tunneling domain. The appearance of unusual weak values has been associated with a unique interference structure [14], for which Berry [15] coined the term "*superoscillations*." As an instructive example of a superoscillatory function ...

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PHYSICAL REVIEW E, VOLUME 65, 046610

Measurement of superluminal optical tunneling times in double-barrier photonic band gaps

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Tunneling of optical pulses at 1.5 μm wavelength through double-barrier periodic fiber Bragg gratings is experimentally investigated in this paper. Tunneling time measurements as a function of the barrier distance show that, far from resonances of the structure, the transit time is paradoxically short—implying superluminal propagation—and almost independent of the barrier distance. This result is in agreement with theoretical predictions based on phase-time analysis and provides, in the optical context, an experimental evidence of the analogous phenomenon in quantum mechanics of nonresonant superluminal tunneling of particles across two successive potential barriers.

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I. INTRODUCTION

the analogy between electron and photon tunneling [16,17], resonant-tunneling phenomena have been also studied and

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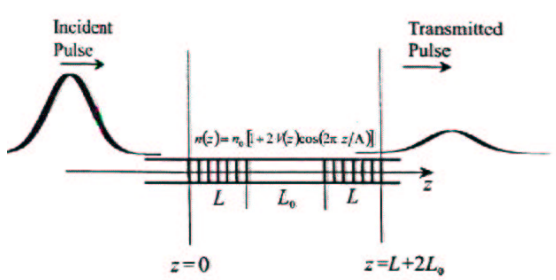


FIG. 1. Schematic of tunneling through a rectangular DB photonic structure.

generalized Hartman effect). The paper is organized as follows. In Sec. II, the basic model of tunneling in a DB rectangular FBG is reviewed and the quantum-mechanical analogy of electron tunneling is outlined. In Sec. III the experimental measurements of tunneling times are presented. Finally, in Sec. IV the main conclusions are outlined.

II. OPTICAL TUNNELING IN A DB FBG: BASIC EQUATIONS AND QUANTUM-MECHANICAL ANALOGY

We consider tunneling of optical pulses through a DB

TABLE I. Analogies between tunneling of optical waves and electrons in a symmetric rectangular DB potential.

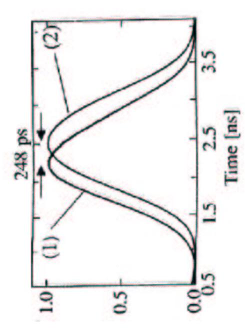
Photons	Electrons
Equations	
$du/dz=i\delta u+ik_BV(z)v$ $dv/dz=-i\delta v-ik_BV(z)u$	$\frac{d^2\psi}{dz^2}+\frac{2m}{\hbar^2}[E-V(z)]\psi=0$
DB Transmission[a] (off-resonance)	
$T=\|t\|^2=1/\cosh^2(2k_BV_0L_0)$	$T=\|t\|^2=1/\cosh^2(2\chi L_0)$
Phase time[a] (off-resonance)	
$\tau=\mathrm{Im}\left\{\frac{\partial \ln(t)}{\partial\omega}\right\}=\tau_1+\tau_2$	$\tau=\hbar\,\mathrm{Im}\left\{\frac{\partial \ln(t)}{\partial E}\right\}=\tau_1+\tau_2$
$\tau_1=[n_0/(c_0k_BV_0)]\tanh(2k_BV_0L_0)$ $\tau_2=(n_0L/c_0)/\cosh(2k_BV_0L_0)$	$\tau_1=[2/(\chi v_g)]\tanh(2\chi L_0)$ $\tau_2=(L/v_g)/\cosh(2\chi L_0)$

[a]For electrons, calculations are made assuming a mean energy of incident wave packet equal to half of the barrier height, i.e., $E=V_0/2$, and assuming off-resonance tunneling, i.e., χL is an integer multiple of $\pi/2$, where $\chi=\sqrt{mV_0}/\hbar$ is the wave number of oscillatory wave function between the two barriers. $v_g=\hbar\chi/m$ is the group velocity of free wave packet.

dv

38b

notice!



traces recorded on the sampling oscilloscope cor-
ıe transmitted pulse for off-resonance tunneling
erence pulse propagating outside the stop band of
ve 2) for the 42-mm separation DB FBG.

ıe pulse train was sent to the DB FBG through
tical circulator that enables both transmitted
gnals to be simultaneously detected. The sig-
through the DB FBG was sent to a low-noise
fiber amplifier (OptoCom Mod. OI LNPA;
3) with a low saturation power (≃30 μW at
maintains the average power level of the out-
nal at a constant level (≃18 mW). In this
r levels transmitted through the DB FBG, for
ion tuned either at Fabry-Perot resonances or

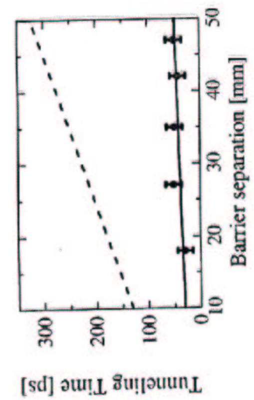


FIG. 5. Off-resonance tunneling time versus barrier separation L for a rectangular symmetric DB FBG structure. The solid line is the theoretical prediction based on group delay calculations (Table I); dots are the experimental points as obtained by time delay measurements; the dashed curve is the transit time from input ($z=0$) to output ($z=L+2L_0$) planes for a pulse tuned far away from the stopband of the FBGs.

≃248 ps; repeated measurements showed that the measured pulse peak advancement is accurate within ≃±15 ps, the main uncertainty in the measure being determined by the achievement of the optimal tuning condition. We checked that propagation through EDFA2 does not introduce any appreciable pulse distortion nor any measurable time delay dependence on the amplification level. Time delay measure-

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RECENT DEVELOPMENTS IN THE TIME ANALYSIS OF TUNNELLING PROCESSES

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NORTH-HOLLAND

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PHYSICS REPORTS

RECENT DEVELOPMENTS IN THE TIME ANALYSIS OF TUNNELLING PROCESSES*

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Contents:

Abstract:

In this paper we critically review and analyse the main theoretical definitions and calculations of the sub-barrier tunnelling and reflection *times*. Moreover, we propose a new, physically sensible definition of such durations, on the basis of a recent general formalism (already tested for other types of quantum collisions). Finally, we discuss some surprising results regarding the temporal evolution of the tunnelling processes.

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initial position x_i to the final position x_f) and to a particle reflected (from the initial position x_i to the same position), respectively; cf., e.g. ref. [1]. For a rectangular barrier with height V_0, the phase times (10) and (11), when linearly extrapolated [1] to the barrier region ($x_i = 0$; $x_f = a$) become

$$\tau_T^{Ph}(0, a; E) = \tau_R^{Ph}(0, a; E) = (m/\hbar k\kappa D)[2\kappa a k^2(\kappa^2 - k^2) + k_0^4 \operatorname{Sinh}(2\kappa a)] , \tag{12a}$$

which, for $\kappa a \gg 1$, would simply yield $2/v\kappa$. In eqs. (12a) one has $D \equiv 4\kappa^2 k^2 + k_0^4 \operatorname{Sinh}^2(\kappa a)$, and $k_0 \equiv 2mV_0/\hbar$. In other words [7], for sufficiently wide – i.e., opaque – (or high) barriers, eqs. (12a) do not depend on the barrier width a, and the effective tunnelling velocity a/τ^{Ph} may become arbitrarily large (Hartmann and Fletcher effect [9, 10]).

One of the main objections against the extrapolations (12a) is that they do not describe the actual asymptotic behaviour of the phase times. The reason is that they disregard the fact that both the (magnitude of the) initial packet mean position, $|x_i|$, and quantity $x_f - a$ (where x_f is the transmitted packet mean position) must be large with respect to the packet spatial extension (of the order of $\hbar v/\Delta E$), in order to avoid "interference" effects between physically quite different processes (i.e., between incident and reflected waves). Therefore, it is not completely correct to attribute to the extrapolated phase times the physical meaning of "times spent in the barrier region (= inside the barrier)". Moreover, one cannot separate in τ_T^{Ph} and τ_R^{Ph} the self-interference delays from the time spent inside the barrier.

Before going on, let us clarify the behaviour of the phase times at the very *top* of the barrier, and check whether there is any continuity there between the values of the sub-barrier tunnelling time and those for the above-barrier case. Let us compare eqs. (12a) with the following expression for the above-barrier transmission time:

$$\tau_T^{Ph}(0, a; E > V_0) = \frac{2m}{\hbar kq} \frac{-(k^2 - q^2)^2 \tan(qa) + 4qak^2(k^2 + q^2)/\cos^2(qa)}{4k^2q^2 + [(k^2 + q^2)\tan(qa)]^2} , \tag{13a}$$

which was obtained*¹, by the stationary-phase method, for the case of a rectangular barrier. In such a case, $\psi_{II} = \gamma e^{iqx} + \delta e^{-iqx}$ with $q \equiv [2m(E - V_0)]^{1/2}/\hbar$, and the coefficients γ and δ can be evaluated analytically. By comparing eqs. (12a) and (13a) one gets

$$\lim_{\kappa \to 0} \tau_T^{Ph}(0, a; E > V_0) = \frac{mka^3}{6\hbar(1 + k^2a^2/4)} \xrightarrow[a\to\infty]{} \frac{2ma}{3\hbar k} . \tag{12b}$$

$$\lim_{\kappa \to \infty} \tau_T^{Ph}(0, a; E < V_0) = \frac{mka^3}{6\hbar(1 + k^2a^2/4)} \xrightarrow[a\to\infty]{} \frac{2ma}{3\hbar k} . \tag{13b}$$

In other words, we find that the two limits (12b), (13b) coincide, and depend linearly on a for "opaque" barriers (provided that the condition $\kappa a \to 0$ holds). Notice that such a result does not contradict the Hartmann–Fletcher effect, since the latter takes place only when $\kappa a \to \infty$, while it is absent for finite values of κa.

*¹ These calculations have been explicitly performed by V.S. Sergeyev

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Nevertheless those authors[6] noticed that definitions (10.1) hold only when the incident and transmitted wave-packets are completely separated either in space or in time. Indeed when x_i and x_f are not far enough from the barrier walls it is possible to have interference effects between the incident part and the reflected one. Moreover, the current density $J(x,t)$ can, in general, change its sign during the packet time-evolution (for instance when the peak of the incident wave reaches the barrier). In such a way the integral $\int_{-\infty}^{\infty} \mathrm{d}t\, t J(x,t)$, that represents the algebraic sum of positive and negative quantities (fluxes), and the probability densities $w(x,t)$, can be no more positive-defined. In such a case, each probability density owns an immediate physical meaning only during the time intervals in which the related current does not change direction. Then it is necessary to break the previous integral into several integrals, each of them being considered on a time interval in which the sign of $J(x,t)$ is only positive or only negative. In such a way we will obtain probability densities everywhere positive-defined:

$$w_+(x,t) = \frac{J_+(x,t)\mathrm{d}t}{\int_{-\infty}^{\infty} \mathrm{d}t J_+(x,t)}\,, \qquad w_-(x,t) = \frac{J_-(x,t)\mathrm{d}t}{\int_{-\infty}^{\infty} \mathrm{d}t J_-(x,t)}$$

where J_+ and J_- represent, respectively, the positive and negative values of $J(x,t)$. Taking in account these considerations, the two authors [6] proposed as average transmission and reflection times the following expressions:

$$\overline{\tau_{\mathrm{T}}} = \overline{t(x_f)}_+ - \overline{t(x_i)}_+ = \frac{\int_{-\infty}^{\infty} \mathrm{d}t\, t J_+(x_f,t)}{\int_{-\infty}^{\infty} \mathrm{d}t\, J_+(x_f,t)} - \frac{\int_{-\infty}^{\infty} \mathrm{d}t\, t J_+(x_i,t)}{\int_{-\infty}^{\infty} \mathrm{d}t\, J_+(x_i,t)} \qquad (10.4a)$$

and

$$\overline{\tau_{\mathrm{R}}} = \overline{t(x_i)}_- - \overline{t(x_i)}_+ = \frac{\int_{-\infty}^{\infty} \mathrm{d}t\, t J_-(x_i,t)}{\int_{-\infty}^{\infty} \mathrm{d}t\, J_-(x_i,t)} - \frac{\int_{-\infty}^{\infty} \mathrm{d}t\, t J_+(x_i,t)}{\int_{-\infty}^{\infty} \mathrm{d}t\, J_+(x_i,t)} \qquad (10.4b)$$

Before going further on, starting from the continuity equation

$$\frac{\partial\rho(x,t)}{\partial t} + \frac{\partial J(x,t)}{\partial x} = 0,$$

and by means of the probabilistic standard interpretation of $\rho(x,t)$, we want to prove that the above quantities $w_\pm(x,t)$ correspond just to the probability that our particle, moving forwards or coming backwards, cross in the time interval $(t, t+\mathrm{d}t)$ the point x. In each time interval in which $J = J_+$ or $J = J_-$, we can write the continuity equation applying to $J_\pm$ (the continuity equation however still holds):

$$\frac{\partial\rho_>(x,t)}{\partial t} = -\frac{\partial J_+(x,t)}{\partial x}\,, \qquad (10.5a)$$

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N°10 TUNNELLING TIMES AND THE "HARTMAN EFFECT" 1359

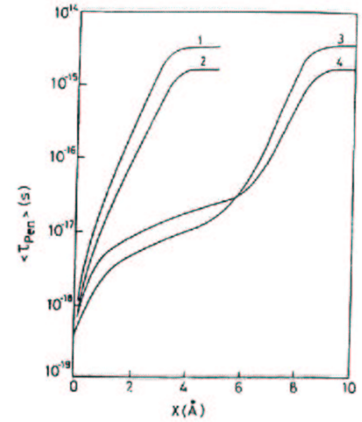


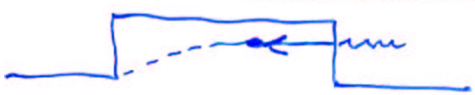
Fig. 4. — Behaviour of $<\tau_{\mathrm{Pen}}(0,x)>$ (in seconds) as a function of x (in Angstroms) for $\overline{E} = 5$ eV and different values of a and Δk. (curve 1): $a = 5$ Å and $\Delta k = 0.02$ Å^{-1}; (curve 2): $a = 5$ Å and $\Delta k = 0.04$ Å^{-1}; (curve 3): $a = 10$ Å and $\Delta k = 0.02$ Å^{-1}; (curve 4): $a = 10$ Å and $\Delta k = 0.04$ Å^{-1}.

$a = 5$ Å, with $\Delta k = 0.02$ Å^{-1} and 0.04 Å^{-1} (plots 1 and 2, respectively); and to $a = 10$ Å, with $\Delta k = 0.02$ Å^{-1} and 0.04 Å^{-1} (plots 3 and 4, respectively), the mean kinetic energy $\overline{E}$ being 5 eV, i.e., one half of V_0.

In Figure 5 the plots are shown of $<\tau_{\mathrm{Ref}}(x,x)>$. The curves 1, 2 and 3 correspond to $\overline{E} = 2.5$ eV, 5 eV and 7.5 eV, respectively, for $\Delta k = 0.02$ Å^{-1} and $a = 5$ Å; the curves 4, 5 and 6 correspond to $\overline{E} = 2.5$ eV, 5 eV and 7.5 eV, respectively, for $\Delta k = 0.04$ Å^{-1} and $a = 5$ Å; while the curves 7, 8 and 9 correspond to $\Delta k = 0.02$ Å^{-1} and 0.04 Å^{-1}, respectively, for $\overline{E} = 5$ eV and $a = 10$ Å.

Also from the new Figures 3–5 one can see that: 1) at variance with reference [8], no plot considered by us for the mean penetration duration $<\tau_{\mathrm{Pen}}(0,x)>$ of our wave packets presents any interval with negative values, nor with a decreasing $<\tau_{\mathrm{Pen}}(0,x)>$ for increasing x; and, moreover, that 2) the mean tunnelling duration $<\tau_{\mathrm{Tun}}(0,a)>$ does not depend on the barrier width a ("Hartman effect"); and finally that 3) quantity $<\tau_{\mathrm{Tun}}(0,a)>$ decreases when the energy increases. Furthermore, it is noticeable that also from Figures 3–5 we observe: 4) a rapid increase for the value of the electron penetration time in the initial part of the barrier region (near $x = 0$); and 5) a tendency of $<\tau_{\mathrm{Pen}}(0,x)>$ to a saturation value in the final part of the barrier, near $x = a$.

Feature 2, firstly observed for quasi-monochromatic particles, [2] does evidently agree with the predictions made in reference [1] for arbitrary wave packets. Feature 3 is also in agreement with previous evaluations performed for quasi-monochromatic particles and presented, for instance, in references [1, 2, 15]. Features 4 and 5 can be apparently explained by interference



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More about Tunnelling Times, the Dwell Time and the "Hartman Effect" (*)

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Abstract. — In a recent review paper [*Phys. Reports* **214** (1992) 339] we proposed, within conventional quantum mechanics, new definitions for the sub-barrier tunnelling and reflection times. Aims of the present paper are: i) presenting and analysing the results of various numerical calculations (based on our equations) on the penetration and return times $<\tau_{\rm Pen}>$, $<\tau_{\rm Ret}>$, during tunnelling *inside* a rectangular potential barrier, for various penetration depths x_f; ii) putting forth and discussing suitable definitions, besides of the mean values, also of the *variances* (or dispersions) $D\tau_T$ and $D\tau_R$ for the time durations of transmission and reflection processes; iii) mentioning, moreover, that our definition $<\tau_T>$ for the average transmission time results to constitute an *improvement* of the ordinary dwell-time $\bar{\tau}^{Dw}$ formula: iv) commenting, at last, on the basis of our *new* numerical results, upon some recent criticism by C.R. Leavens. We stress that our numerical evaluations *confirm* that our approach implied, and implies, the existence of the *Hartman effect*: an effect that in these days (due to the theoretical connections between tunnelling and evanescent-wave propagation) is receiving —at Cologne, Berkeley, Florence and Vienna— indirect, but quite interesting, experimental verifications. Eventully, we briefly analyze some other definitions of tunnelling times.

1. Introduction

In our review article [1] we put forth an analysis of the main theoretical definitions of the sub-barrier tunnelling and reflection times, and proposed new definitions for such durations

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UNIFIED TIME ANALYSIS OF PHOTON AND (NONRELATIVISTIC) PARTICLE TUNNELLING, AND THE SUPERLUMINAL GROUP-VELOCITY PROBLEM[1]

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Abstract

A unified approach to the time analysis of tunnelling of nonrelativistic particles is presented, in which Time is regarded as a quantum-mechanical observable, canonically conjugated to Energy. The validity of the Hartman effect (independence of the Tunnelling Time of the opaque barrier width, with Superluminal group velocities as a consequence) is verified for *all* the known expressions of the mean tunnelling time.

Moreover, the analogy between particle and photon tunnelling is suitably exploited. On the basis of such an analogy, an explanation of some recent microwave and optics experimental results on tunnelling times is proposed. Attention is devoted to some aspects of the causality problem for particle and photon tunnelling.

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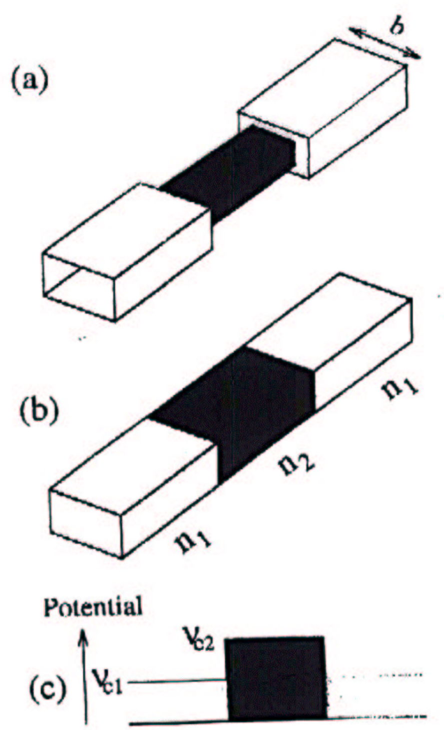

Fig. 1 [illegible] & Nimtz

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PHYSICAL REVIEW E VOLUME 62, NUMBER 6 DECEMBER 2000

Propagation speed of evanescent modes

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(Received 23 July 1999; revised manuscript received 6 March 2000)

The group velocity of evanescent waves (in undersized waveguides, for instance) was theoretically predicted, and has been experimentally verified, to be superluminal ($v_g > c$). By contrast, it is known that the precursor speed in vacuum cannot be larger than c. In this paper, by computer simulations based on Maxwell equations only, we show the existence of both phenomena. In other words, we verify the actual possibility of superluminal group velocities, without violating the so-called (naive) Einstein causality.

PACS number(s): 03.50.De, 84.40.Az, 41.20.Jb, 73.40.Gk

I. INTRODUCTION

A series of recent experiments, performed at Cologne [1], Berkeley [2], Florence [3], and in other places [4], revealed that evanescent waves seem to travel with a superluminal group velocity ($v_g > c$). This originated a lot of discussion, since it is known, on the other hand, that the speed of the precursors cannot be larger than c. For instance, the existence of Sommerfeld's and Brillouin's precursors (the so-called first and second precursors) has been recently stressed in Refs. [5], while studying the transients in metallic waveguides.

In this paper we would like to address simultaneously both such problems, relevant for the understanding of the propagation of a signal; namely, the question of the (superluminal) value of v_g in the evanescent case, and the question of the arrival time of the transients (which implies a nonviolation of the so-called Einstein causality).

From a historical point of view, let us recall that for a long time the topic of the electromagnetic wave propagation velocity was regarded as already settled down by the works of Sommerfeld [6] and Brillouin [7]. Some authors, however, studying the propagation of light pulses in anomalous dispersion (absorbing) media both theoretically [8] and experimentally [9], found their envelope speed to be the group velocity v_g, even when v_g exceeds c, equals $\pm\infty$, or becomes negative. In the meantime, evanescent waves were predicted [10] to be faster-than-light just on the basis of special relativistic considerations.

But evanescent waves in suitable ("undersized") waveguides, in particular, can be regarded also as tunnelling photons [11], due to the known formal analogies [12] between the Schrödinger equation in presence of a potential barrier and the Helmholtz equation for a wave-guided beam. And it was known since long that tunneling particles (wave packets) can move with superluminal group velocities inside opaque barriers [13]; therefore, even from the quantum theoretical point of view, it was expected [13,11,10] that evanescent waves could be superluminal.

In Sec. II of this paper we shall first show how the first electric perturbation, reaching any point P, always travels with the speed c of light in vacuum, *independently of the medium*. Some comments will be added about the instant of appearance, and the behavior in time, of the Sommerfeld's and Brillouin's precursors. The results of a computer simulation will be presented for free propagation in a dispersive medium, with the precursors arriving before the (properly said) signal.

In Sec. III, however, we shall deal by further computer simulations (always based on Maxwell equations only) with evanescent guided waves, showing their group velocity to be superluminal.

Finally, in Secs. IV and V we shall deal with the transients associated with superluminal evanescent waves: a study that, to our knowledge, was not carried on in the past.

II. PRECURSORS AND CAUSALITY

Every perturbation passes through a transient state before reaching the stationary regime. This happens also when transmitting any kind of wave. In the case of electromagnetic waves, such a transient state is associated with the propagation of precursors, arriving before the principal signal. This fact seems to be enough to satisfy the requirements of the naive "Einstein causality."

In particular, when investigating the *free* propagation of an electromagnetic wave, in a dispersive medium with resonances in correspondence with some discrete angular frequencies ω_j, we can easily observe the arrival of the first and second precursors, followed by the arrival of the properly said signal. Let us consider for instance the motion in the z direction of a harmonic beam, such that at $z=0$ one has

$$f(0,t) = \frac{1}{2\pi}\int_{\gamma - i\infty}^{\gamma + i\infty} \frac{e^{i\omega t}}{s + i\omega}\, ds = e^{i\omega t} \quad \text{for } t \geqslant 0 \qquad (1)$$

and $f(0,t)=0$ for $t<0$; where s is the complex integration variable, and $\gamma>0$ in order that the function be transformable. Let us then consider a dispersive medium whose dielectric constant ε (electric permittivity) as a function of ω is

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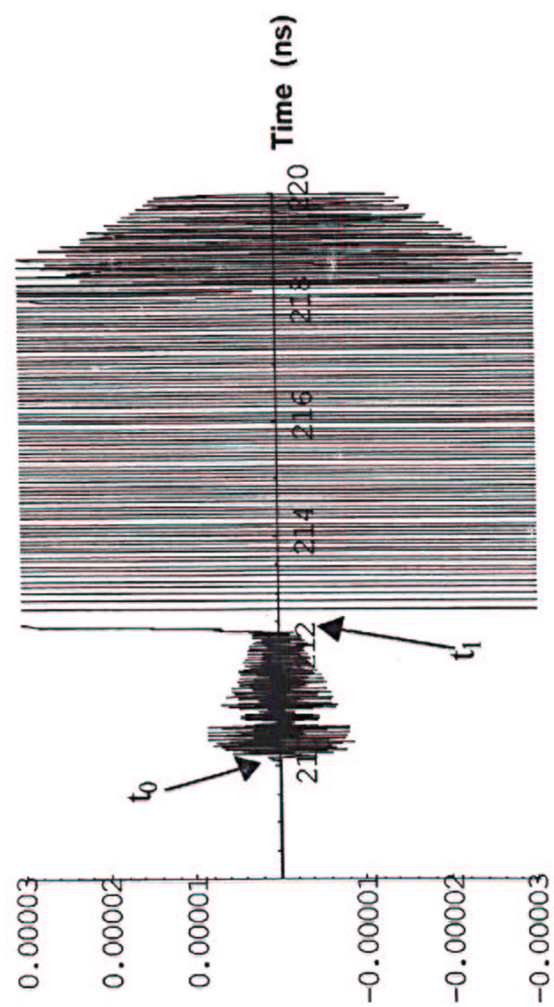

— Fig. 3 —

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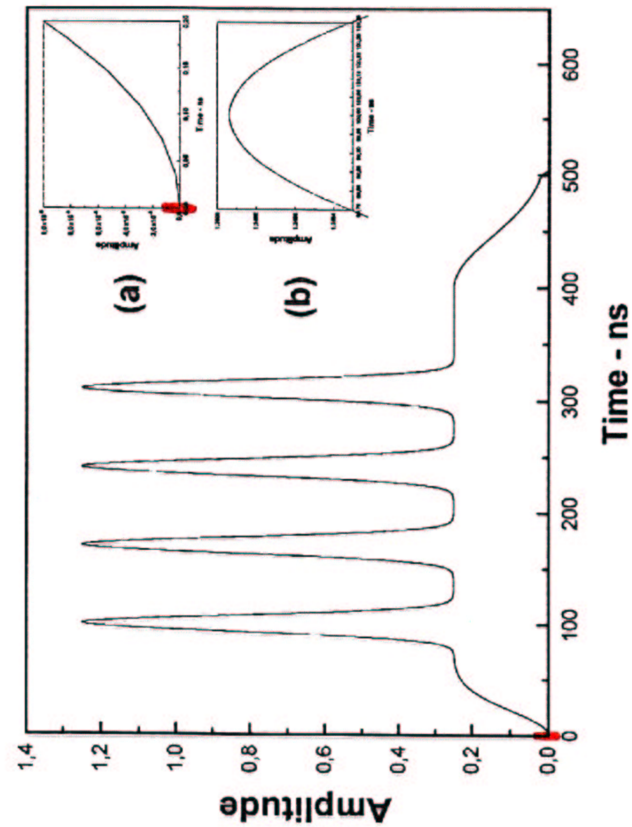

— Fig. 11 —

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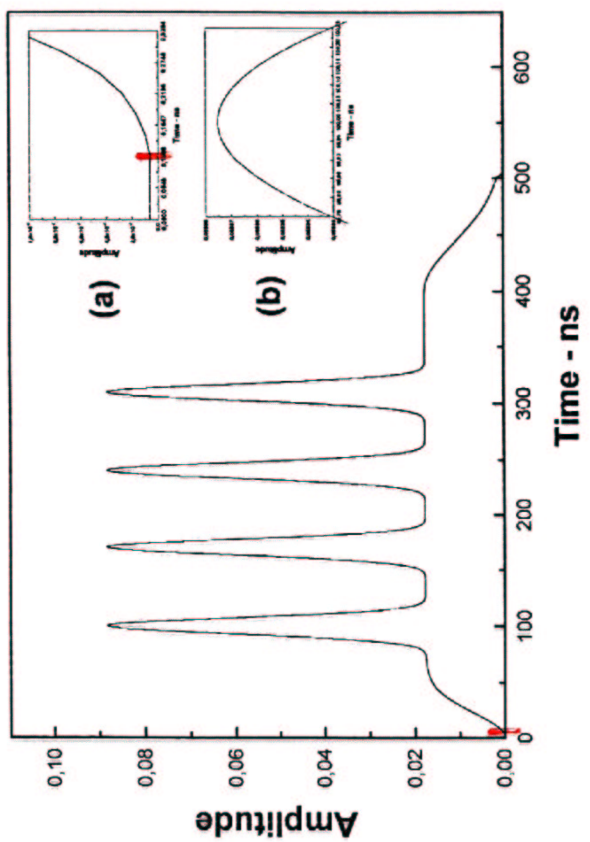

– Fig. 13 –

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PHYSICAL REVIEW E, VOLUME 64, 066603

Superluminal localized solutions to Maxwell equations propagating along a normal-sized waveguide

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We show that localized (nonevanescent) solutions to Maxwell equations exist, which propagate without distortion along normal waveguides with superluminal speed.

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I. INTRODUCTION: LOCALIZED SOLUTIONS TO THE WAVE EQUATIONS

Already in 1915 Bateman [1] showed that Maxwell equations admit (besides of the ordinary solutions, endowed in vacuum with speed c) of wavelet-type solutions, endowed in vacuum with group velocities $0 \leqslant v \leqslant c$. But Bateman's work went practically unnoticed. Only few authors, as Barut and co-workers [2], followed such a research line; incidentally, Barut and coworkers constructed even a wavelet-type solution traveling with superluminal group velocity [3] $v > c$.

In recent times, however, many authors have discussed the fact that all (homogeneous) wave equations admit solutions with $0 < v < \infty$: see, e.g., Donnelly and co-workers [4]. Most of these authors confined themselves to investigate (subluminal or superluminal) *localized* nondispersive solutions in vacuum: namely, those solutions that were called "undistorted progressive waves" by Courant and Hilbert. Among localized solutions, the most interesting appeared to be the so-called "X-shaped" waves, which —predicted to exist even by special relativity in its extended version [5]— had been mathematically constructed by Lu and Greenleaf [6] for acoustic waves, and by Ziolkowski *et al.* [7], and later Recami [8], for electromagnetism.

Let us recall that such "X-shaped" localized solutions are superluminal (i.e., travel with $v > c$ in the vacuum) in the electromagnetic case; and are "supersonic" (i.e., travel with a speed larger than the sound speed in the medium) in the acoustic case. The first authors to produce X-shaped waves *experimentally* were Lu and Greenleaf [9] for acoustics, Saari and co-workers [10] for optics, and Mugnai and co-workers and for microwaves.

II. ABOUT EVANESCENT WAVES

Notwithstanding all that work, still it is not yet well understood what solutions (let us now confine ourselves to Maxwell equations and to electromagnetic waves) have to enter into the play in some experiments. Actually, most of the experimental results did not refer to the above-mentioned localized, subluminal or superluminal, solutions, which in vacuum are expected to propagate rigidly (or almost rigidly, when suitably truncated). They referred, on the contrary, to measurements of the group velocity of *evanescent waves* (cf., e.g., Refs. [11,12]). In fact, tunneling wave packets (tunneling photons too) and/or evanescent waves had been predicted to be superluminal by *both* quantum mechanics [13] and special relativity [5].

For instance, experiments [12] with evanescent waves traveling down an undersized *waveguide* revealed that evanescent modes are endowed with superluminal group velocities [14].

A *problem* arises in connection with experiments with two "barriers" (i.e., segments of *undersized* waveguide) 1 and 2 separated by a *normal-sized* waveguide 3. In fact, it was found that for suitable frequency bands the wave coming out from barrier 1 goes on having a practically infinite speed, and crosses the intermediate (normal) waveguide 3 in zero time [15]. Even if this can be theoretically understood by looking at the relevant transfer function (see the computer simulations, based on Maxwell equations only, in Refs. [14,16,17]), it is natural to ask ourselves whether solutions to the Maxwell equations can actually exist, that travel with superluminal speed in a normal waveguide (where one ordinarily meets propagating, subluminal modes only).

Namely, the dispersion relation in undersized guides is $\omega^2/c^2 - k^2 = -\Omega^2$, so that the standard formula $v \simeq d\omega/dk$ yields a $v > c$ group velocity [17,18]. However, in normal guides the dispersion relation becomes $\omega^2/c^2 - k^2 = +\Omega^2$, so that the same formula yields values $v < c$ only.

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[19] J.-y.Lu, H.-h.Zou and J.F.Greenleaf: *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control* 42 (1995) 850-853; and J.-y.Lu: (unpublished).

[20] S.Ramo, J.R.Whinnery,T.Van Duzer: *Fields and Waves in Communication Electronics*, Chapt. 8 (John Wiley; New York, 1984).

[21] Cf., e.g., ref.[8] and refs. therein.

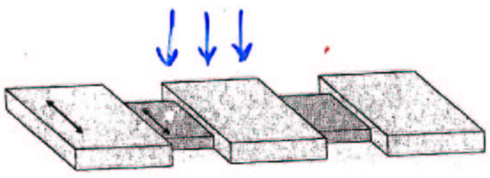

Fig.1 – The experiment with two "barriers" (i.e., segments of *undersized* waveguide) 1 and 2. It had been found that, for suitable frequency bands, the wave coming out from barrier 1 can go on with practically infinite speed, crossing the intermediate (normal-sized) waveguide 3 in zero time. We show in this paper that localized solutions to the Maxwell equations do actually exist, which travel along a normal waveguide with Superluminal speed.

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1. – Introduction: Localized solutions to the wave equation

Already in 1915 Bateman[1] showed that Maxwell equations admit (besides of the ordinary solutions, endowed in vacuum with speed c) of wavelet-type solutions, endowed in vacuum with group-velocities $0 \le v \le c$. But Bateman's work went practically unnoticed. Only few authors, as Barut et al.[2], followed such a research line; incidentally, Barut et al. constructed even a wavelet-type solution travelling with Superluminal group-velocity[3] $v > c$.

In more recent times, however, many authors discussed the fact that all (homogeneous) wave equations admit solutions with $0 < v < \infty$: see, e.g., refs.[4]. Most of those authors confined themselves to investigate (sub- or Super-luminal) *localized* non-dispersive solutions in vacuum: namely, those solutions that were called "undistorted progressive waves" by Courant & Hilbert. Among localized solutions, the most interesting appeared to be the so-called "X-shaped" waves, which —predicted to exist even by Special Relativity in its extended version[5]— had been mathematically constructed by Lu & Greenleaf[6] for acoustic waves, and by Ziolkowski et al.[6], and later Recami[6], for electromagnetism. Let us recall that such "X-shaped" localized solutions are Superluminal (i.e., travel with $v > c$ in the vacuum) in the electromagnetic case; and are "super-sonic" (i.e., travel with a speed larger than the sound-speed in the medium) in the acoustic case. The first authors to produce X-shaped waves *experimentally* were Lu & Greenleaf[7] for acoustics, Saari et al.[7] for optics, and Mugnai et al. for microwaves[7].

In a recent paper of ours, appeared in this journal[8], we showed that solutions to the Maxwell equations exist, that displace themselves with Superluminal speed even along a normal waveguide: where one ordinarily expects to meet propagating, subluminal modes only. Actually, a segment of "undersized" waveguide constitutes an evanescence region[9], and evanescent waves are known to travel Superluminally[5,9-11]; however, it was rather unexpected that (localized) waves could propagate Superluminally down a normal-sized waveguide. In fact, the dispersion relation in undersized guides is $\omega^2/c^2 - \beta^2 = -K^2$, so that the standard formula $v \simeq d\omega/d\beta$ yields a $v > c$ group-velocity[12]; by contrast, in normal guides the dispersion relation becomes $\omega^2/c^2 - \beta^2 = +K^2$, so that it seems to yield values $v < c$ only. Instead, in our paper[8] we have shown that localized solutions

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to Maxwell equations do exist, propagating with $v > c$ even in normal waveguides; but their group-velocity v cannot be given[#1] by the approximate formula $v \simeq d\omega/d\beta$. [Let us recall that the group-velocity is well defined only when the pulse has a clear bump in space; but it can be calculated by the approximate, elementary relation $v \simeq d\omega/d\beta$ only when ω as a function of β is also clearly bumped)].

2. – The infinite-energy solutions

Namely, in ref.[8] we construced localized solutions to the Maxwell equations (which propagate undistorted, with Superluminal speed along a *cylindrical* waveguide located along the z-direction) for the TM (transverse magnetic) case and for a dispersion-free medium. The case with dispersion has been treated elsewhere[13], as well as the case of a co-axial cable[14]. Here, let us call attention to two points, which received just a mention in ref.[8], with regard to eq.(9) and Fig.2 therein: (i) those solutions consist in *trains* of pulses (similar to the one depicted in Fig.2 of ref.[1]); (ii) each of such pulses is X-*shaped*: See our Fig.1 below. Let us notice, incidentally, that we are referring ourselves to the electromagnetic case, but the same would hold for all situations in which a fundamental role is played by the wave equation (as in acoustics, geophysics, gravitational wave physics, etc.).

For intance, in the case of axial symmetry, let us consider a metallic waveguide with radius[8]

$$r \equiv R\,.$$

Let us also put $\boldsymbol{\rho} \equiv (x, y)$, and $\rho = |\boldsymbol{\rho}|$ and the bondary condition $\Psi(\rho = r, z; t) = 0$. In the previous paper[8] we constructed the following solution,

$$\Psi(\rho, z; t) \;=\; \sum_{n=1}^{N} \left(\frac{2}{a^2 \sin^2\theta J_1^2(\lambda_n)} \right) J_0(K_n\,\rho)\; \cos\left[\frac{\omega_n}{V}\,(z - V\,t)\right]\,, \qquad (1)$$

where Ψ represents the longitudinal component of the electric field, E_z, while N is an integer, the quantities λ_n are the roots of the Bessel function, $K_n = \lambda_n/R$, $\omega_n = K_n\,c/\sin\theta$ and $V = c/\cos\theta$. These solutions are therefore Fourier-Bessel–type sums over different

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The initial conditions (4) imply that $\sum A_n J_0(\lambda_n \rho/a) = \delta(\rho)/\rho$, and $\sum B_n J_0(\lambda_n \rho/a) = 0$, so that all B_n must vanish, while $A_n = 2[a^2 J_1^2(\lambda_n)]^{-1}$; and eventually one gets:

$$\Psi_{2D}(\rho; t) \;=\; \textstyle\sum_{n=1}^{\infty} \left(\frac{2}{a^2 J_1^2(\lambda_n)} \right)\; J_0(\frac{\lambda_n}{a}\rho)\;\cos\omega_n t\,, \qquad (8)$$

where $\omega_n = \lambda_n c/a$.

Let us explicitly notice that we can pass from such a formal solution to more physical ones, just by considering a finite number N of terms. In fact, each partial expansion will satisfy (besides the boundary condition) the second initial condition $\partial_t\psi = 0$ for $t = 0$, while the first initial condition gets the form $\phi(\rho) = f(\rho)$, where $f(\rho)$ will be a (well) localized function, but no longer a delta-type function. Actually, the "localization" of $\phi(\rho)$ increases with increasing N. We shall come back to this point below.

4. – Localized waves propagating Superluminally down (normal-sized) waveguides.

We have now to apply transformations (3) to solution (8), in order to pass to threedimensional waves propagating along a cylindrical (metallic) waveguide with radius $r = a/\sin\theta$. We obtain that Maxwell equations admit in such a case the solutions

$$\Psi_{3D}(\rho, z; t) \;=\; \textstyle\sum_{n=1}^{\infty} \left(\frac{2}{a^2 J_1^2(\lambda_n)} \right)\; J_0(\frac{\lambda_n}{a}\rho \sin\theta)\; \cos\left[\frac{\lambda_n \cos\theta}{a}\,(z - \frac{c}{\cos\theta}t)\right] \qquad (9)$$

where $\omega_n = \lambda_n c/a$, which are sums over different propagating modes.

Such solutions propagate, down the waveguide, rigidly with Superluminal velocity[#2] $v = c/\cos\theta$. Therefore, (non-evanescent) solutions to Maxwell equations exist, that are waves propagating undistorted along *normal* waveguides with Superluminal speed (even if in normal-sized waveguides the dispersion relation for each mode, i.e. for each term of the Fourier-Bessel expansion, is the ordinary "subluminal" one, $\omega^2/c^2 - k_z^2 = +\Omega^2$).

[#2] Let us stress that each eq.(9) represents a *multimodal* (but *localized*) propagation, as if the geometric dispersion compensated for the multimodal dispersion.

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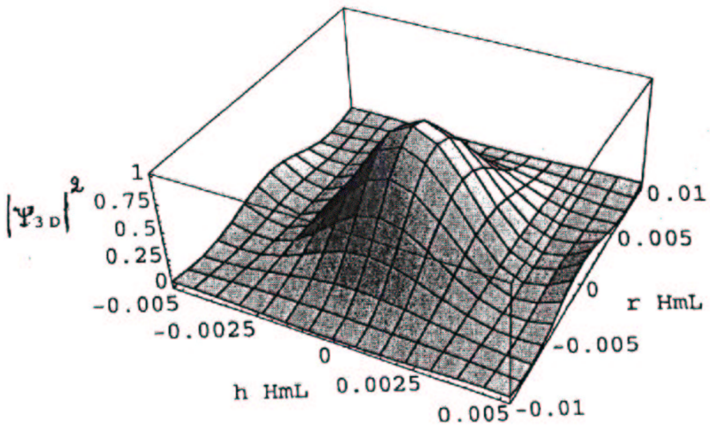
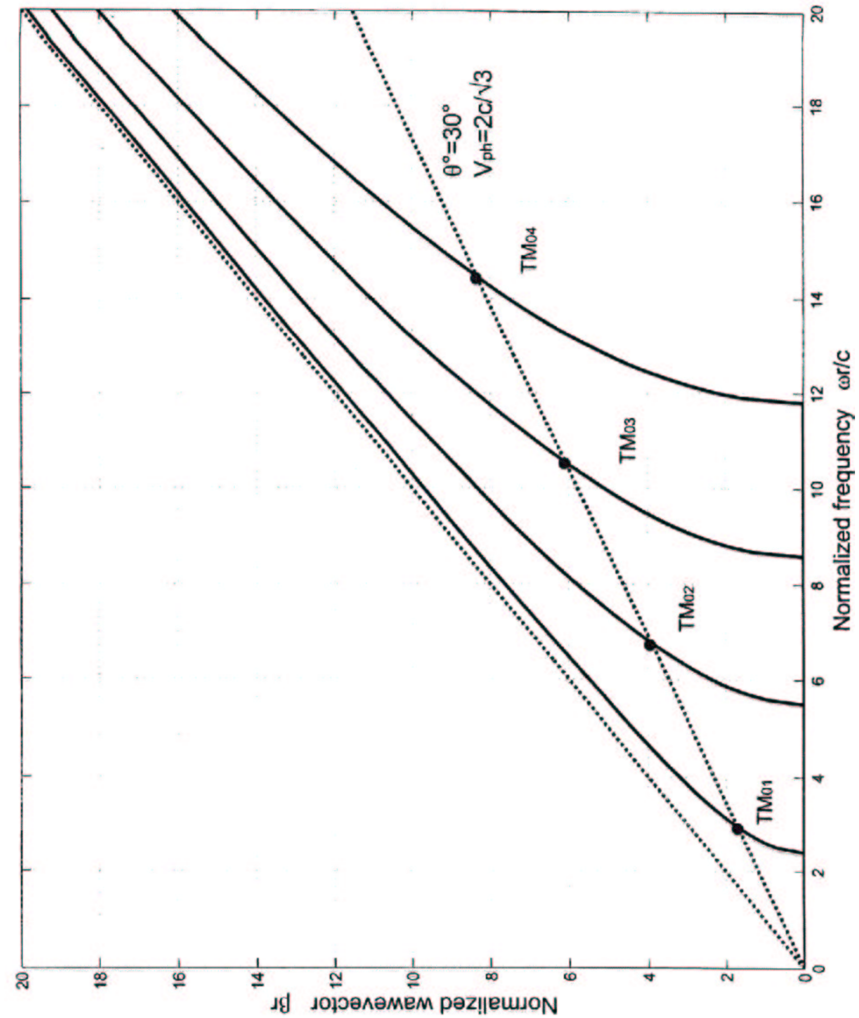


– Fig. 3 –

ZAMBONI-RACHED et al.



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Superluminal Localized Solutions to Maxwell Equations propagating along a waveguide: The finite-energy case(†)

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Abstract – In a previous paper we have shown localized (non-evanescent) solutions to Maxwell equations to exist, propagating without distortion with Superluminal speed along normal-sized waveguides, and consisting in trains of "X-shaped" beams. Those solutions possessed therefore infinite energy. In this note we show how to obtain, by contrast, *finite*-energy solutions, with the same localization and Superluminality properties.

PACS nos.: 03.50.De ; 41.20.Jb ; 03.30.+p ; 84.40.Az ; 42.82.Et .
Keywords: Wave-guides; Localized solutions to Maxwell equations; Superluminal waves; Bessel beams; Limited-dispersion beams; Finite-energy waves; Electromagnetic wavelets; X-shaped waves; Evanescent waves; Electromagnetism; Microwaves; Optics; Special relativity; Localized acoustic waves; Seismic waves; Mechanical waves; Elastic waves; Guided gravitational waves.

(†) Electronic Lanl Arquives # physics/******. Work partially supported by CAPES (Brazil), and by MIUR/MURST and INFN (Italy). E-mail address for contacts: recami@mi.infn.it

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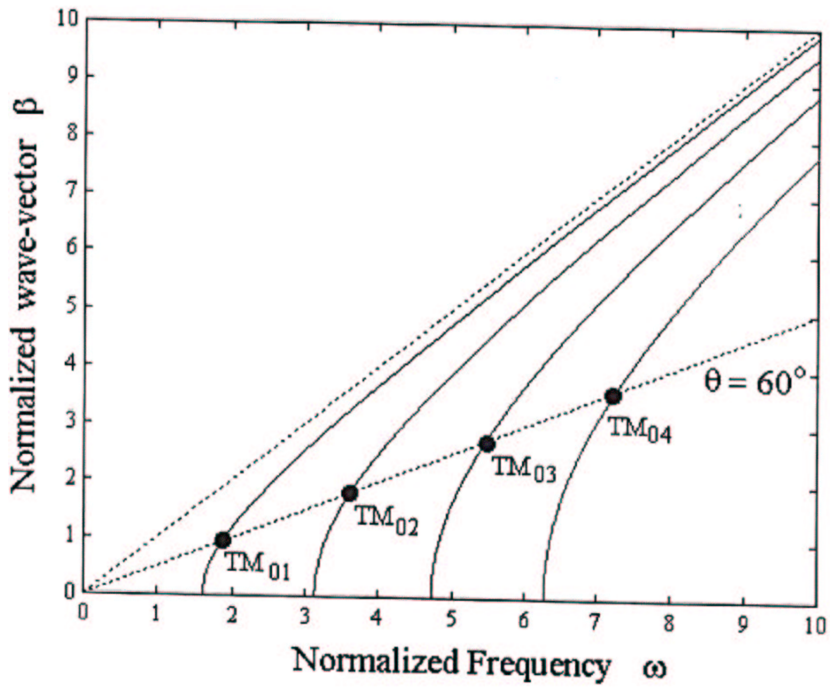

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MKRH 48

Superluminal X-shaped beams propagating without distortion along a co-axial guide [†]

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and

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Abstract – In a previous paper we showed that localized Superluminal solutions to the Maxwell equations exist, which propagate down (non-evanescence) regions of a metallic cylindrical waveguide. In this paper we construct analogous non-dispersive waves propagating along co-axial cables. Such new solutions, in general, consist in trains of (undistorted) Superluminal "X-shaped" pulses. Particular attention is paid to the construction of finite total energy solutions. Any results of this kind may find application in the other fields in which an essential role is played by a wave-equation (like acoustics, geophysics, etc.).

PACS nos.: 03.50.De ; 41.20;Jb ; 83.50.Vr ; 62.30.+d ; 43.60.+d ; 91.30.Fn ; 04.30.Nk ; 42.25.Bs ; 46.40.Cd ; 52.35.Lv .

Keywords: Wave equations; Wave propagation; Localized beams; Superluminal waves; Co-axial cables; Bidirectional decomposition; Bessel beams; X-shaped waves; Maxwell equations; Microwaves; Optics; Special relativity; Co-axial metallic waveguides; Acoustics; Seismology; Mechanical waves; Elastic waves; Gravitational waves.

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Localized Superluminal solutions to the wave equation in (vacuum or) dispersive media, for arbitrary frequencies and with adjustable bandwidth [†]

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Abstract – In this paper we set forth new exact analytical Superluminal localized solutions to the wave equation, for arbitrary frequencies and adjustable bandwidth. The formulation presented here is rather simple, its results being obtainable starting from the ordinary, so-called "X-shaped waves". Our solutions may find application in different fields, like radio waves, microwaves, optics, and so on. Actually, by the present formalism we obtain the first *analytical* localized Superluminal approximated solutions which represent beams propagating *in dispersive media*.

PACS nos.: 03.50.De ; 41.20;Jb ; 83.50.Vr ; 62.30.+d ; 43.60.+d ; 91.30.Fn ; 04.30.Nk ; 42.25.Bs ; 46.40.Cd ; 52.35.Lv .

Keywords: Wave equation; Wave propagation; Localized beams; Superluminal waves; Bessel beams; X-shaped waves; Optics; Acoustics; Mechanical waves; Dispersion compensation.

(†) Work supported by FAPESP (Brazil). E-mail address for contacts: mzamboni@ifi.unicamp.br

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SPECIAL RELATIVITY AND SUPERLUMINAL MOTIONS: A DISCUSSION OF SOME RECENT EXPERIMENTS*

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Some experiments, performed at Berkeley, Cologne, Florence, Vienna, Orsay and Rennes led to the claim that something seems to travel with a group velocity larger than the speed c of light in vacuum. Various other experimental results seem to point in the same direction: For instance, localized wavelet-type solutions of Maxwell equations have been found, both theoretically and experimentally, that travel with Superluminal speed. Even muonic and electronic neutrinos — it has been proposed — might be "tachyons," since their square mass appears to be negative. With regard to the first-mentioned experiments, it was very recently claimed by Guenter Nimtz that those results with evanescent waves or "tunneling photons" — implying Superluminal signal and impulse transmission — violate Einstein causality. In this note, on the contrary, we want to stress that all such results do *not* place relativistic causality in jeopardy, even if they refer to actual tachyonic motions: In fact, special relativity can cope even with Superluminal objects and waves. For instance, it is possible (at least in microphysics) to solve also the known causal paradoxes, devised for "faster than light" motion, even if this is not widely recognized. Here we show, in detail and rigorously, how to solve the oldest causal paradox, originally proposed by Tolman, which is the kernel of many further tachyon paradoxes. The key to the solution is a careful application of tachyon mechanics, as it unambiguously follows from special relativity.

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etc. are more general than the ordinary ones (see Recami and Mignani,[43] hereafter called Review I; and Refs. 26 and 28). Incidentally, such a switching procedure has been shown[44] to be equivalent to applying the chirality operation γ_5.

Matter and Antimatter from SR — A close inspection shows that the application of any antichronous transformation $L^{\downarrow}$, together with the switching procedure, transforms P into an object

$$Q \equiv \overline{P} \tag{A.1}$$

which is indeed the *antiparticle* of P. We are saying that *the concept of antimatter is a purely relativistic one*, and that, on the basis of the double sign in $[c = 1]$

$$E = \pm\sqrt{p^2 + m_0^2}\,, \tag{A.2}$$

the existence of antiparticles could have been predicted already in 1905, exactly with the properties they actually exhibited when later discovered, *provided that* recourse to the "switching procedure" had been made. We therefore maintain that the points of the lower hyperboloid sheet in Fig. 4 — since they correspond not only to negative energy but also to motion backwards in time — represent the kinematical states of the *antiparticle* $\overline{P}$ *of* the particle P represented by the upper hyperboloid sheet.

Let us stress that the switching procedure not only can, but *must* be enforced, since any observer can do nothing but explore space-time along the positive time direction. That procedure is an improved translation into a purely relativistic language of the Stückelberg–Feynman[30] "Switching principle." Together with our Assumption above, it can take the form of a "Third Postulate": ⟪Negative-energy objects traveling forward in time do *not* exist; any negative-energy object P traveling backwards in time can and must be described as its antiobject $\overline{P}$ going the opposite way *in space* (but endowed with positive energy and motion forward in time)⟫. Cf. e.g. Refs. 45 and 28 and references therein.

Concluding remarks of Appendix A

Let us go back to Fig. 3. In SR, when based only on the two ordinary postulates, nothing prevents *a priori* the event A from influencing the event B. Just to forbid such a possibility we introduced our Assumption together with the switching procedure. As a consequence, not only we eliminate any particle-motion backwards in time, but we also "predict" and naturally explain within SR the existence of antimatter.

In the case of tachyons the switching procedure was first applied by Sudarshan and coworkers;[27] see e.g. Ref. 28 and references therein.

At last, it is necessary — however — to observe the following: Whenever it is met an object (or wavelet), $\mathcal{O}$, traveling at Superluminal speed, negative contributions should be expected to the "tunneling" times:[46,5] and this ought not to be regarded as unphysical or at variance with the expectations of SR. In fact we have just

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seen above that, whenever an "object" $\mathcal{O}$ *overcomes* the infinite speed[26,28] with respect to a certain observer, it will afterwards appear to the same observer as its "*anti*-object" $\overline{\mathcal{O}}$ traveling in the opposite *space* direction. For instance, when passing from the lab to a frame $\mathcal{F}$ moving in the *same* direction as the waves (or particles) entering the undersized waveguide (or the barrier region), the objects $\mathcal{O}$ penetrating through that waveguide or barrier (with almost infinite speeds[5,46]) will appear in the frame $\mathcal{F}$ as antiobjects $\overline{\mathcal{O}}$ crossing the waveguide or barrier *in the opposite space direction*. In the new frame $\mathcal{F}$, therefore, such antiobjects $\overline{\mathcal{O}}$ would yield a *negative* contribution to the traversal time: which could even result, in total, to be *negative*.

Appendix B. Group Velocity of the Evanescent Waves

Let us here define the group velocity of an electromagnetic signal in the evanescence region, as well as in a normal (not undersized) waveguide. We shall follow Ref. 47. Let us start with a normal waveguide. The wave equation will be

$$\phi(t, x, y, z) = \phi_0(t, x)e^{i\omega t - \gamma z}\,, \tag{B.1}$$

where γ is the propagation constant inside the waveguide

$$\gamma^2 \equiv k_z^2 \equiv -\beta^2\,, \tag{B.2}$$

so that

$$\beta = ik_z = \pm i\sqrt{k_c^2 - k^2} = \pm\sqrt{k^2 - k_c^2} \tag{B.3}$$

where $k_c^2 = -k_x^2 - k_y^2$. The cutoff frequency (i.e. the lowest frequency that can propagate, in the normal way, along a waveguide having width a) is given by the condition $\beta^2 = 0$; which yields:

$$\sqrt{k^2 - k_c^2} = 0 \;\Rightarrow\; k^2 - k_c^2 = 0 \;\Rightarrow\; k^2 = k_c^2\,. \tag{B.4}$$

Let us now recall that

$$k_x = -\frac{im\pi}{a} \quad \text{and} \quad k_y = -\frac{in\pi}{b}$$

and that

$$-k_c^2 = k_x^2 + k_y^2 \;\Rightarrow\; k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2. \tag{B.5}$$

If we adopt the propagation mode TE_{10}, relation (B.5) reduces to

$$k_c^2 = \left(\frac{\pi}{a}\right)^2$$

which (since $\omega^2 = k^2c^2$) yields the critical frequency:

$$\frac{\omega_c^2}{c^2} = \left(\frac{\pi}{a}\right)^2 \;\Rightarrow\; \omega_c = \frac{\pi}{a}c \tag{B.6}$$

representing the minimum frequency that a (carrier)-wave can possess in order to be able to carry a signal, for the mentioned mode, along the waveguide with width a. The wavepacket (group)-velocity is obtained through its dispersion relation, which in the considered case is

$$\beta^2 = k^2 - k_c^2$$

and can be rewritten as

$$\beta^2 = \frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}.$$

By definition, the group velocity is given by $\frac{d\omega}{d\beta}$. Quantity $\beta(\omega)$ is known. One gets:

$$\frac{d\beta}{d\omega} = \frac{\omega}{c^2}\sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}}$$

and, by use of relation (B.6),

$$\frac{d\beta}{d\omega} = \frac{\omega}{c\sqrt{\omega^2 - \omega_c^2}}.$$

Finally, by inverting, we obtain

$$\frac{d\omega}{d\beta} = v_{gr} = c\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \quad \text{with} \quad \omega > \omega_c$$

which shows that the packet propagates in the waveguide with a *subluminal* velocity.

Velocity of an evanescent wave — If at a certain point the signal meets a "barrier," i.e. a waveguide segment with a smaller width a', such that the lowest propagating frequency is larger than the carrier's, the dispersion relation does then change, assuming (as requested by Extended Relativity)[26] the form:

$$\beta^2 = \frac{\omega^2}{c^2} + \frac{\pi^2}{a'^2}.$$

Following the same procedure as before, we arrive at

$$v_{gr} = c\sqrt{1 + \left(\frac{\omega_c}{\omega}\right)^2} \quad \text{with} \quad \omega < \omega_c$$

wherefrom it is evident that the signal is now endowed with a *Superluminal* velocity.

Just for completeness' sake, let us add that simple simulations, based on Maxwell equations (by using *Mathematica*™), have been performed also at Milan university[47] *confirming these evaluations*. For instance, a numerical experiment was performed for microwave frequencies between 5 and 10 GHz in a rectangular waveguide. Let us examine the case with a single barrier (region II). In the normal barrier segments (regions I and III, respectively), the propagating wave is described by the equation

$$\phi = \phi_0 \cos\left(\frac{\pi}{a}x\right) e^{i(\omega t - kz)},$$

while, in the undersized (*evanescent*) region, that equation becomes

$$\phi = \phi_0 \cos\left(\frac{\pi}{a'}x\right) e^{i\omega t} e^{-k'z}.$$

When the system is in a stationary state, the complete set of equations is:

$$\begin{aligned}
\psi_{\rm I} &= A\cos\left(\frac{\pi}{a}x\right)e^{ik_1z} + R\cos\left(\frac{\pi}{a}x\right)e^{-ik_1z};\\
\psi_{\rm II} &= B\cos\left(\frac{\pi}{a'}x\right)e^{k_2z} + C\cos\left(\frac{\pi}{a'}x\right)e^{k_2z};\\
\psi_{\rm III} &= T\cos\left(\frac{\pi}{a}x\right)e^{ik_1z} + E\cos\left(\frac{\pi}{a}x\right)e^{-ik_1z}.
\end{aligned} \tag{B.7}$$

If the entering waves come from $z = -\infty$ (and none come from $z = +\infty$), one has $A = 1$ and $E = 0$. The continuity conditions, which allow determining the coefficients R, T, B, C, are (for the regions I and II)

$$\psi_{\rm I}(0) = \psi_{\rm II}(0); \qquad \left.\frac{d\psi_{\rm I}}{dz}\right|_0 = \left.\frac{d\psi_{\rm II}}{dz}\right|_0, \tag{B.8}$$

and (for the regions II and III), quantity L being the evanescence region length,

$$\psi_{\rm II}(L) = \psi_{\rm III}(L); \qquad \left.\frac{d\psi_{\rm II}}{dz}\right|_L = \left.\frac{d\psi_{\rm III}}{dz}\right|_L. \tag{B.9}$$

For a carrier-wave frequency of 7 GHz, the group velocity in the normal and in the evanescent regions resulted to be $0.7c$ and $1.7c$, respectively.

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Superluminal Motions? A Bird's-Eye View of the Experimental Situation[1]

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In this article, after a theoretical introduction and a sketch of some related long-standing predictions, a bird's-eye view is presented—with the help of nine figures—of the various experimental sectors of physics in which Superluminal motions seem to appear (thus contributing support to those past predictions). In particular, a panorama is presented of the experiments with evanescent waves and/or tunnelling photons, and with the "localized Superluminal solutions" to the Maxwell equations (like the so-called X-shaped beams). The present review is brief, but is followed by a large enough bibliography to allow the interested reader deepening the preferred topic.

1. INTRODUCTION

The question of Superluminal ($V^2 > c^2$) objects or waves,[3] has a long story, starting perhaps in 50 B.C. with Lucretius' *De Rerum Natura* (cf., e.g., book 4, line 201: [≪ Quone vides *citius* debere et longius ire/Multiplexque loci spatium transcurrere eodem/Tempore *quo Solis* pervolgant *lumina* coelum? ≫]). Still in pre-relativistic times, one meets various related works, from those by J. J. Thomson to the papers by the great A. Sommerfeld. With Special Relativity, however, since 1905 the conviction spread over that the speed c of light in vacuum was the *upper* limit of any possible speed. For instance, R. C. Tolman in 1917 believed to have shown by his "paradox" that the existence of particles endowed with speeds larger than

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[3] It is an old use of ours to write Superluminal with a capital S.

e.g., refs.11,12]. In fact, tunnelling wavepackets (tunnelling photons too) and/or evanescent waves had been predicted to be Superluminal by *both* Quantum Mechanics[13] and Special Relativity[5].

For instance, experiments[12] with evanescent waves travelling down an undersized *waveguide* revealed that evanescent modes are endowed with Superluminal group-velocities[14].

A *problem* arises in connection with the experiment in Fig.1, with two "barriers" 1 and 2 (i.e., segments of *undersized* waveguide). In fact, it was found that for suitable frequency bands the wave coming out from barrier 1 goes on having a practically infinite speed, and crosses the intermediate (normal) waveguide 3 in zero time[15]. Even if this can be theoretically understood by looking at the relevant transfer function (see the computer simulations, based on Maxwell equations only, in refs.[14,16,17]), it is natural to ask ourselves *whether solutions to the Maxwell equations can actually exist, that travel with Superluminal speed in a normal waveguide* (where one ordinarily meets propagating, subluminal modes!).

Namely, the dispersion relation in undersized guides is $\omega^2 - k^2 = -\Omega^2$, so that the standard formula $v \simeq \mathrm{d}\omega/\mathrm{d}k$ yields a $v > c$ group-velocity[17,18]. However, in normal guides the dispersion relation becomes $\omega^2 - k^2 = +\Omega^2$, so that the same formula yields values $v < c$ only.

In this paper we are going to show that actually localized solutions to Maxwell equations propagating with $v > c$ do exist even in normal waveguides; but their group-velocity v *cannot* be given[#1] by the approximate formula $v \simeq \mathrm{d}\omega/\mathrm{d}k$. One of the main motivations of the present note is just contributing to the clarification of this question, even if our localized Superluminal solutions are *not* strictly related with the particular case in ref.[15] and Fig.1.

3. – About some localized solutions to Maxwell equations.

Let us start by considering localized solutions to Maxwell equations in vacuum. A

[#1] Let us recall that the group-velocity is well defined only when the pulse has a clear bump in space; but it can be calculated by the approximate, elementary relation $v \simeq \mathrm{d}\omega/\mathrm{d}k$ *only* when some extra conditions are satisfied (namely, when ω as a function of k is also clearly bumped).