Broadband Context of Group Delay: The Role of the Instantaneous Spectrum in the Behavior of Linear Dielectrics

Part I: Broadband Context of Group Delay

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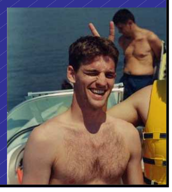
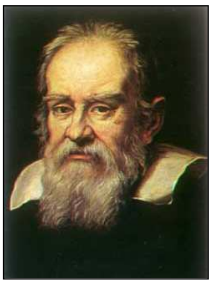
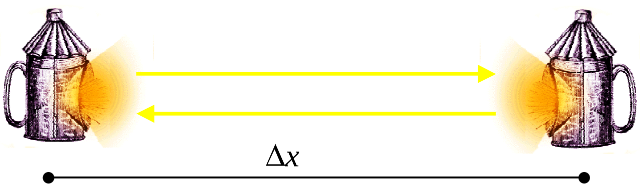


The speed of light is measured by noting the times when a light pulse reaches two separate points



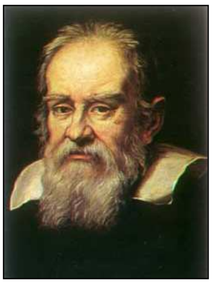
Galileo Galilei (1638)



$$velocity = 2\frac{\Delta x}{\Delta t} \qquad \Delta t = ??$$

"I was unable to make sure whether the facing light appeared instantaneously. But if not instantaneous, light is very swift."

The speed of light is measured by noting the times when a light pulse reaches two separate points



Galileo Galilei (1638)

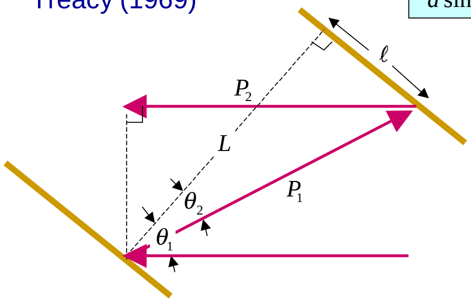
"But in what seas are we inadvertently engulfing ourselves, bit by bit? Among voids, infinities, indivisibles, and instantaneous movements, shall we ever be able to reach harbor even after a thousand discussions?"

Outline

- Group Delay in Angularly Dispersive Systems
- Broadband Context of Group Delay in Absorptive and/or Active Linear Media
- Bandwidth Dependent Transition from Superluminal to Subluminal Propagation

A representative ray propagates through a parallel grating pair setup

Treacy (1969)



$d\sin\theta_1 + d\sin\theta_2 = \lambda$ $\quad \lambda = \dfrac{2\pi c}{\omega}$

$$P_1 = \frac{L}{\cos\theta_2}$$

$$P_2 = \frac{L\cos(\theta_1 - \theta_2)}{\cos\theta_2}$$

$$\ell = L\tan\theta_2$$

Acquired Phase: $\phi = 2\pi\dfrac{P_1 + P_2}{\lambda} - 2\pi\dfrac{\ell}{d}$

$\tau \equiv \dfrac{d\phi}{d\omega} = \dfrac{P_1 + P_2}{c}$ Group Delay

Brorson and Haus (1988)

Group delay is usually understood in the context of an expansion on the k-vector

$$\mathbf{E}(\mathbf{r}_o, \omega) = \tfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathbf{E}(\mathbf{r}_o, t)e^{i\omega t}dt$$

$$\mathbf{E}(\mathbf{r}_o + \Delta\mathbf{r}, \omega) = \mathbf{E}(\mathbf{r}_o, \omega)e^{i\mathbf{k}(\omega)\cdot\Delta\mathbf{r}}$$

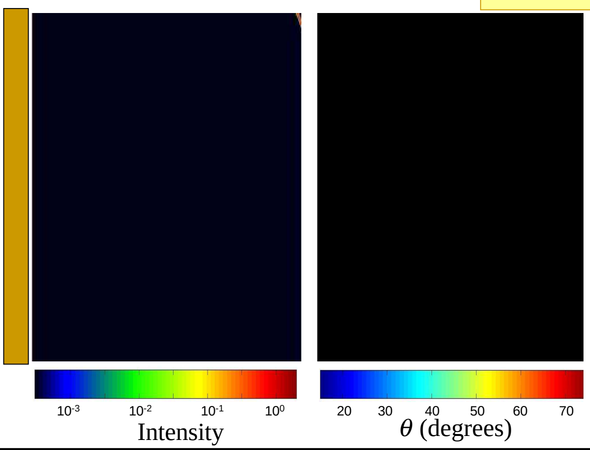
$$\mathbf{E}(\mathbf{r}_o + \Delta\mathbf{r}, t) = \tfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathbf{E}(\mathbf{r}_o + \Delta\mathbf{r}, \omega)e^{-i\omega t}dt$$

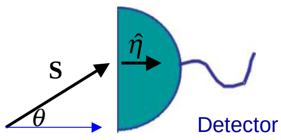
$$\phi = \mathbf{k}(\omega)\cdot\Delta\mathbf{r} \cong (\mathbf{k}\cdot\Delta\mathbf{r})\Big|_{\bar{\omega}} + \frac{d(\mathbf{k}\cdot\Delta\mathbf{r})}{d\omega}\Big|_{\bar{\omega}}(\omega - \bar{\omega}) + \ldots$$

We define the arrival time of the pulse to a point as a time expectation integral over the Poynting flux

$$\langle t\rangle_{\mathbf{r}} \equiv \int_{-\infty}^{\infty} t\rho(\mathbf{r},t)\,dt$$

$$\rho(\mathbf{r},t) \equiv \frac{\hat{\eta}\cdot\mathbf{S}(\mathbf{r},t)}{\hat{\eta}\cdot\int_{-\infty}^{\infty}\mathbf{S}(\mathbf{r},t)\,dt}$$



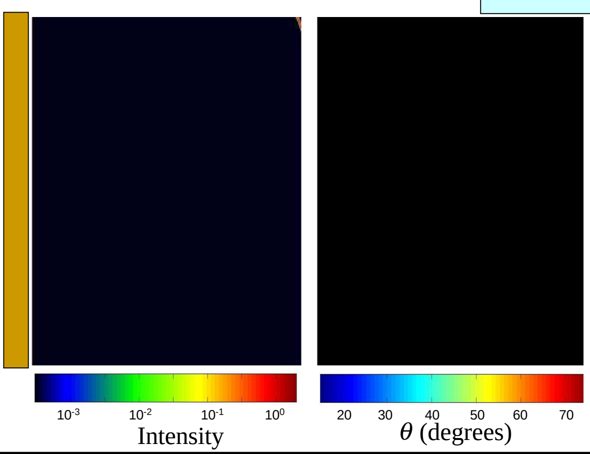


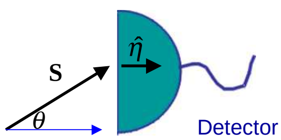
M. Ware, W. E. Dibble, S. A. Glasgow, J. Peatross, "Energy flow in angularly dispersive optical systems," J. Opt. Soc. Am. B (2001)

The difference between arrival times is a spectral superposition of the group delay function

$$\langle t\rangle_{\mathbf{r}+\Delta\mathbf{r}} - \langle t\rangle_{\mathbf{r}} = \int_{-\infty}^{\infty} \frac{d\mathbf{k}\cdot\Delta\mathbf{r}}{d\omega}\rho(\mathbf{r},\omega)\,d\omega$$

$$\rho(\mathbf{r},\omega) \equiv \frac{\hat{\eta}\cdot\mathbf{S}(\mathbf{r},\omega)}{\hat{\eta}\cdot\int_{-\infty}^{\infty}\mathbf{S}(\mathbf{r},\omega)\,d\omega}$$



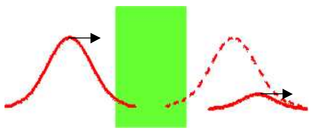


M. Ware, W. E. Dibble, S. A. Glasgow, J. Peatross, "Energy flow in angularly dispersive optical systems," J. Opt. Soc. Am. B (2001)

Simulations and experiments verify superluminal pulse propagation

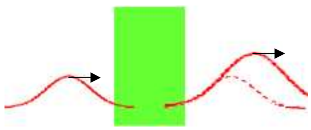
Absorbing resonance

Prediction: Garret and McCumber (1970).
Experiment: Chu and Wong (1982).



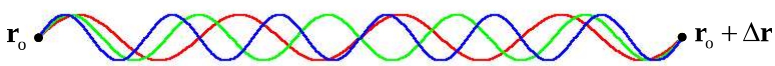
Amplifying resonance

Prediction: R. Chiao et al. (1993).
Experiment: Wang et al. (2000).



If the temporal profile is known at $\mathbf{r}_o$ the phase delay function gives the form of a pulse at $\mathbf{r}_o + \Delta\mathbf{r}$.

$$\mathbf{E}(\mathbf{r}_o, t) = waveform \qquad \mathbf{E}(\mathbf{r}_o + \Delta\mathbf{r}, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathbf{E}(\mathbf{r}_o + \Delta\mathbf{r}, \omega) e^{-i\omega t} d\omega$$



$$\mathbf{E}(\mathbf{r}_o, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathbf{E}(\mathbf{r}_o, t) e^{i\omega t} dt \longrightarrow \mathbf{E}(\mathbf{r}_o + \Delta\mathbf{r}, \omega) = \mathbf{E}(\mathbf{r}_o, \omega) e^{i\mathbf{k}(\omega)\cdot\Delta\mathbf{r}}$$

Phase Delay Function

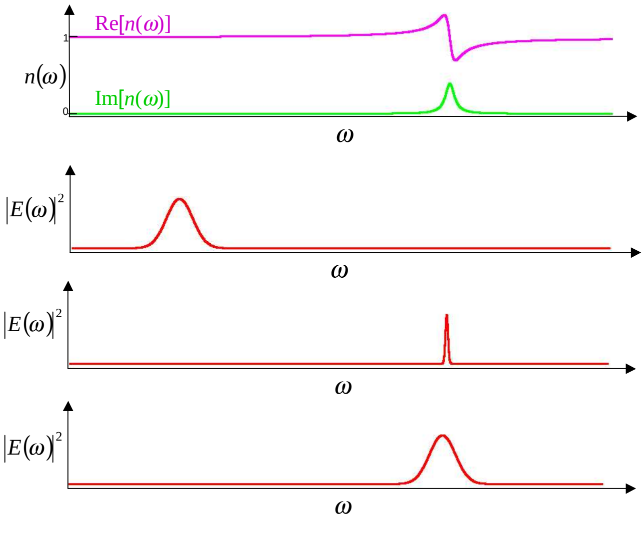
Group velocity can describe propagation of narrow band pulses by expanding the phase delay

$$\mathbf{E}(\mathbf{r}_o + \Delta\mathbf{r}, \omega) = \mathbf{E}(\mathbf{r}_o, \omega) e^{i\mathbf{k}(\omega)\cdot\Delta\mathbf{r}}$$

$$\mathbf{k}(\omega)\cdot\Delta\mathbf{r} \cong (\mathbf{k}\cdot\Delta\mathbf{r})\big|_{\bar{\omega}} + \left.\frac{d(\mathbf{k}\cdot\Delta\mathbf{r})}{d\omega}\right|_{\bar{\omega}} (\omega - \bar{\omega}) + \ldots$$

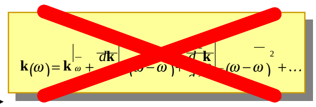
$$\mathbf{E}(\mathbf{r}_0 + \Delta\mathbf{r}, t) \cong \mathbf{E}(\mathbf{r}_0, t - \Delta t) e^{-\operatorname{Im}\mathbf{k}|_{\bar{\omega}}\cdot\Delta\mathbf{r}} \qquad \Delta t = \left.\frac{d\operatorname{Re}\mathbf{k}}{d\omega}\right|_{\bar{\omega}} \cdot \Delta\mathbf{r}$$

Traditional group delay fails when used to describe broadband pulses near resonances.



$$k(\omega) = \frac{n(\omega)\omega}{c}$$

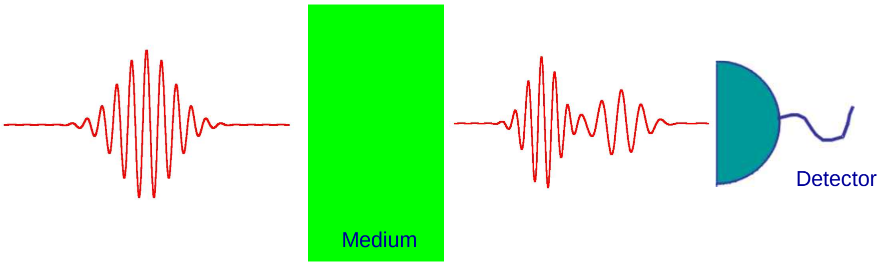
?



Might not converge!

Reshaping effects can also cause problems for the traditional concept of group velocity

Oughstun et al. (1997)



A precise definition of pulse arrival time is necessary before pulse transit time can be specified.

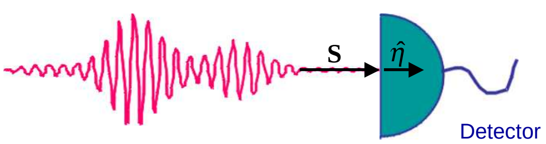
We define pulse arrival time to a point as the temporal expectation weighted by the energy transport flux

$$\langle t \rangle_{\mathbf{r}} \equiv \int_{-\infty}^{\infty} t\rho(\mathbf{r},t)\,dt$$

"Center of Mass"

$$\rho(\mathbf{r},t) \equiv \frac{\hat{\eta}\cdot\mathbf{S}(\mathbf{r},t)}{\hat{\eta}\cdot\int_{-\infty}^{\infty}\mathbf{S}(\mathbf{r},t)\,dt}$$

Normalized Temporal Distribution



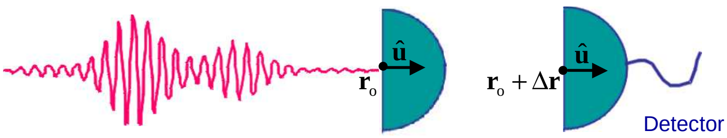
Definition proposed by R. L. Smith (1970)

Using this definition of arrival time, we can write an intuitive expression for the delay time between points.

$$\langle t \rangle_{\mathbf{r}} \equiv \int_{-\infty}^{\infty} t \rho(\mathbf{r}, t) dt$$

J. Peatross, S. Glasgow, M. Ware, "Average Energy Flow of Optical Pulses in Dispersive Media" Phys. Rev. Lett. (2000)

$$\Delta t \equiv \langle t \rangle_{\mathbf{r}} - \langle t \rangle_{\mathbf{r}_o} \quad \longrightarrow \quad \Delta t = \Delta t_G + \Delta t_R$$

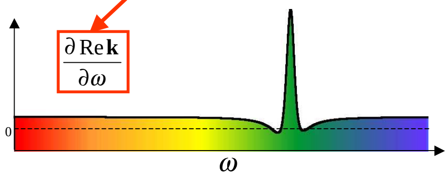


The net group delay term is a spectral superposition of the group delay function weighted by the final spectrum

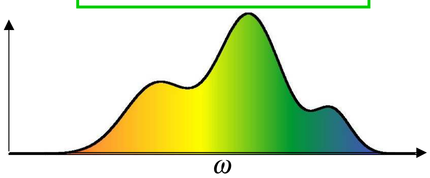
$$\Delta t \equiv \langle t \rangle_{\mathbf{r}} - \langle t \rangle_{\mathbf{r}_o} = \Delta t_G + \Delta t_R$$

$$\Delta t_G \equiv \int_{-\infty}^{\infty} \left[\frac{\partial \operatorname{Re} \mathbf{k}}{\partial \omega} \cdot \Delta \mathbf{r} \right] \rho(\mathbf{r}_o + \Delta \mathbf{r}, \omega) d\omega$$

$$\rho(\mathbf{r}_o + \Delta \mathbf{r}, \omega) \equiv \frac{\hat{\eta} \cdot \mathbf{S}(\mathbf{r}, \omega)}{\hat{\eta} \cdot \int_{-\infty}^{\infty} \mathbf{S}(\mathbf{r}, \omega) d\omega}$$



Group Delay Function



Normalized *Spectral* Distribution

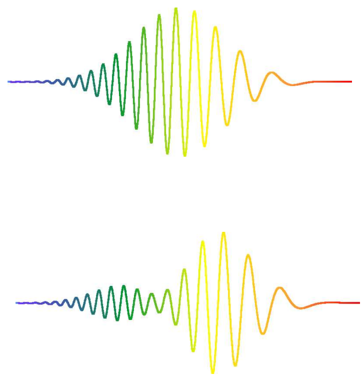
The reshaping term also plays a role in pulse delay time, especially in cases where the pulse is chirped.

$$\Delta t \equiv \langle t \rangle_{\mathbf{r}} - \langle t \rangle_{\mathbf{r}_o} = \Delta t_G + \Delta t_R$$

$$\Delta t_R \equiv T\left[\mathbf{E}(\mathbf{r}_o,\omega) e^{-\mathrm{Im}\,\mathbf{k}\cdot\mathbf{r}}\right] - T\left[\mathbf{E}(\mathbf{r}_o,\omega)\right]$$

where $\langle t \rangle_{\mathbf{r}} = T\left[\mathbf{E}(\mathbf{r},\omega)\right]$

Pulse reshaping through absorption



A model is employed to illustrate the new theorem

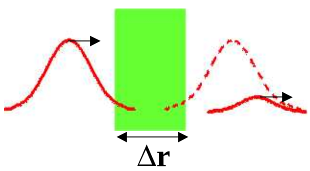
$$\mathbf{E}(\mathbf{r}_o,t) = \hat{\mathbf{x}} E_o \exp\left(-t^2/\tau^2\right)\cos(\varpi t) \qquad \tau = \begin{cases} 10/\gamma & \text{Narrowband} \\ 1/\gamma & \text{Broadband} \end{cases}$$

$$(n + i\kappa)^2 = 1 + \frac{f\omega_p^2}{\omega_o^2 - \omega^2 - i\gamma\omega}$$

$$\omega_o/\gamma = 100$$

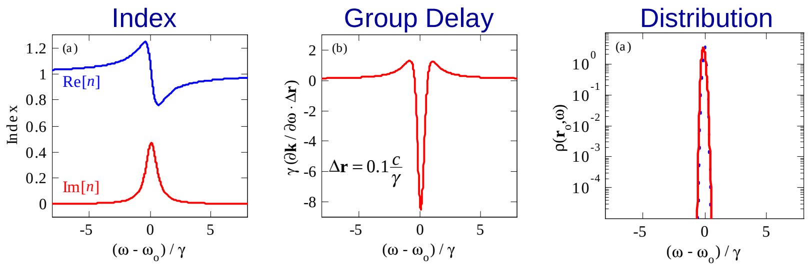
$$\omega_p/\gamma = 10$$

$$f = \begin{cases} 1 & \text{Absorbing} \\ -1 & \text{Amplifying} \end{cases}$$

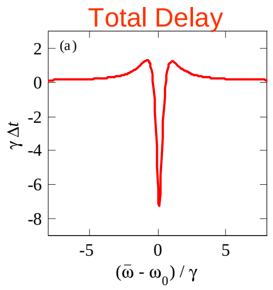


$$\Delta \mathbf{r} = \hat{\mathbf{z}} \frac{0.1c}{\gamma}$$

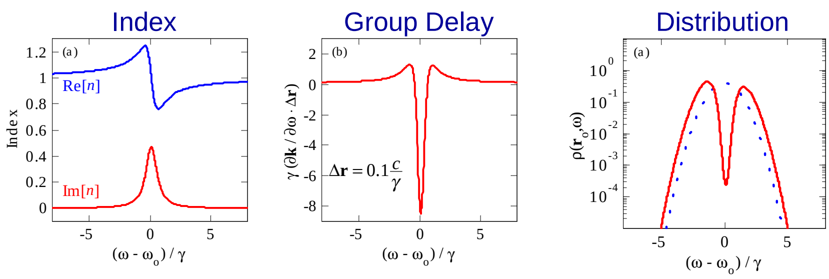
The total delay times for narrowband pulses is determined from the group delay function.



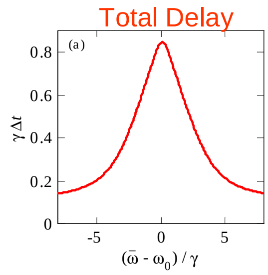
$$\Delta t = \int_{-\infty}^{\infty}\left[\frac{\partial \operatorname{Re}\mathbf{k}}{\partial\omega}\cdot\Delta\mathbf{r}\right]\rho(\mathbf{r}_o+\Delta\mathbf{r},\omega)\,d\omega$$



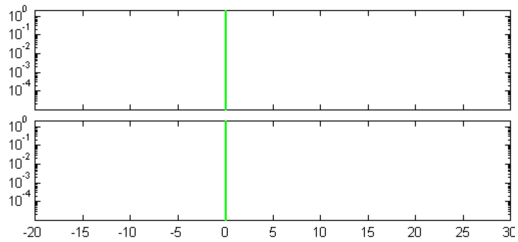
The total delay times for broadband pulses is determined from the group delay function.



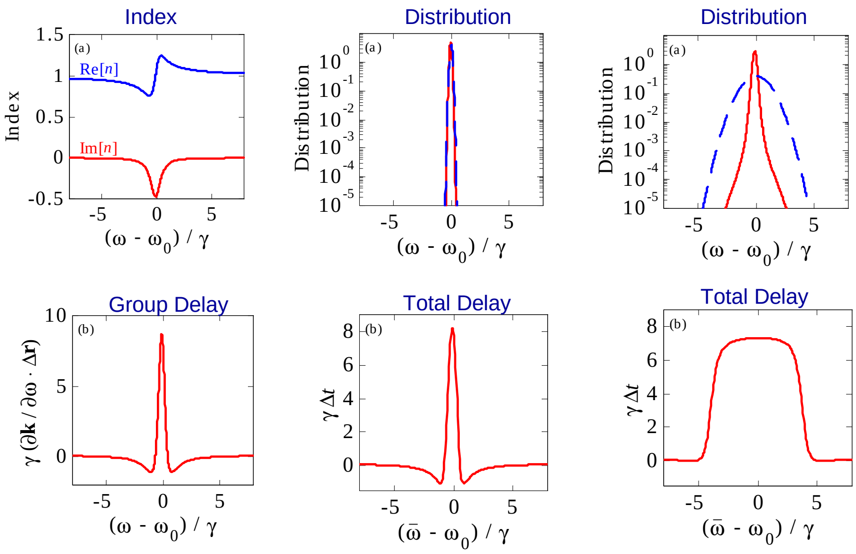
$$\Delta t = \int_{-\infty}^{\infty}\left[\frac{\partial \operatorname{Re}\mathbf{k}}{\partial\omega}\cdot\Delta\mathbf{r}\right]\rho(\mathbf{r}_o+\Delta\mathbf{r},\omega)\,d\omega$$



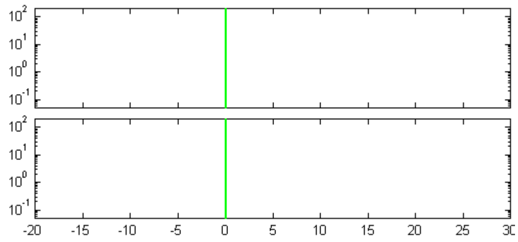
Broadband vs. Narrowband propagation



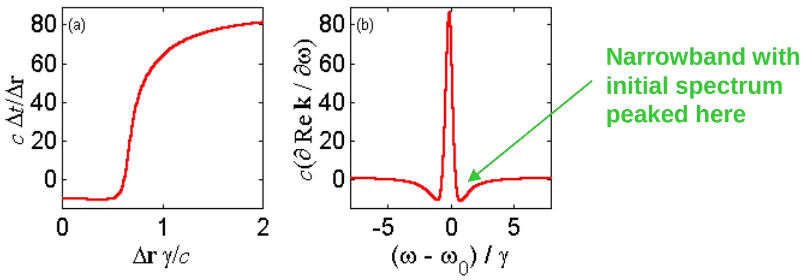
Pulses propagating on resonance through an *amplifying* medium



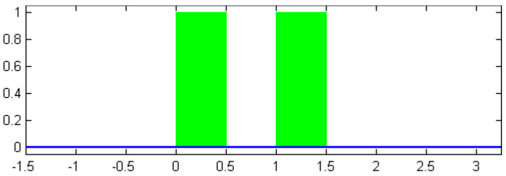
Broadband vs. Narrowband propagation

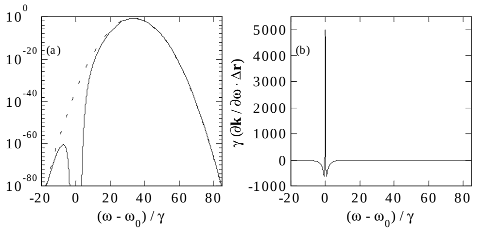


Superluminal transit does not persist as propagation distance is increased



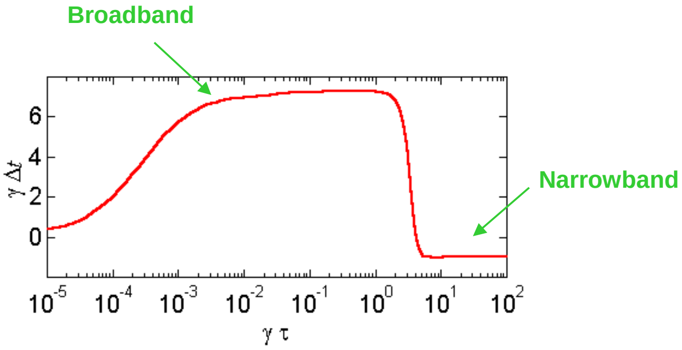
Superluminal propagation distance can be increased by preparing the pulse through absorption





R. Chiao et al. (1993)

Superluminal transit times are only observed for narrowband pulses



Summary of this approach

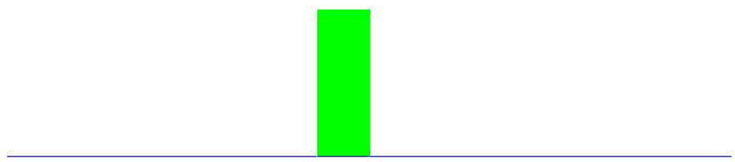
- The group delay function retains meaning even in a broadband context.
- Group delay is connected to the "center-of-mass" of the Poynting flux (with no approximation).
- The formalism naturally demonstrates that superluminal behavior does not occur for broadband pulses.
- No pulse can travel at superluminal speeds indefinitely.

I. Talukder, Y. Amagishi, and M. Tomita, "Superluminal to subluminal transition in the pulse propagation in a resonantly absorbing medium" **86**, 3546 (2001).

Part II: The Role of the Instantaneous Spectrum in the Behavior of Linear Dielectrics

(point-wise analysis)

Narrowband pulse traversing an amplifying medium(pulse frequency well off of resonance).



R. Chiao et al. (1993)

Comments

The inverted medium can temporarily loan part of its stored energy to the forward tail of the wave packet, in a pulse-reshaping process which moves the peak of the wave packet forward in time. One can think of this pulse-reshaping process as the virtual amplification of the forward tail of the wave packet, followed by the virtual absorption of the peak, resulting in an *advancement* of the wave packet.

R. Chiao, Progress in Optics (1997)

Comments

In the present experiment, … [the probe pulse contains] essentially no spectral components that are resonant with the Raman gain lines to be amplified. Therefore, the argument that the probe pulse is advanced by amplification of its front edge does not apply.

L. J. Wang, Nature (July 2000)

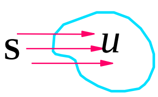
Poynting's theorem follows directly from Maxwell's Equations.

$$\nabla \cdot \mathbf{S} + \frac{\partial u}{\partial t} = 0$$

where $\mathbf{S}(t) \equiv \mathbf{E} \times \mathbf{B}/\mu_o$

$$u(t) \equiv \frac{\varepsilon_o \mathbf{E}^2}{2} + \frac{\mathbf{B}^2}{2\mu_o} + \varepsilon_o \int_{-\infty}^{t} \mathbf{E} \cdot \frac{\partial \mathbf{P}}{\partial t'} dt' + u(-\infty)$$

Energy stored in the medium before the arrival of the pulse should be included



$$\oint\!\!\oint_A \mathbf{S} \cdot d\mathbf{a} = -\frac{\partial}{\partial t} \iiint_V u \, dV$$

$$\mathbf{v}_E \equiv \mathbf{S}/u$$

Energy transport velocity is the effective speed of the energy density necessary to accomplish the flux S.

Causality requires the medium to interact with the instantaneous spectrum of the pulse.

$$u(t) = u_{field} + u_{exchange} + u(-\infty)$$

$$u_{field} \equiv \frac{B^2}{2\mu_o} + \frac{\varepsilon_o E^2}{2}$$

S. A. Glasgow, M. Ware, J. Peatross, "Poyntings theorem and luminal energy transport velocity in causal dielectrics" Phys. Rev. E (2001)

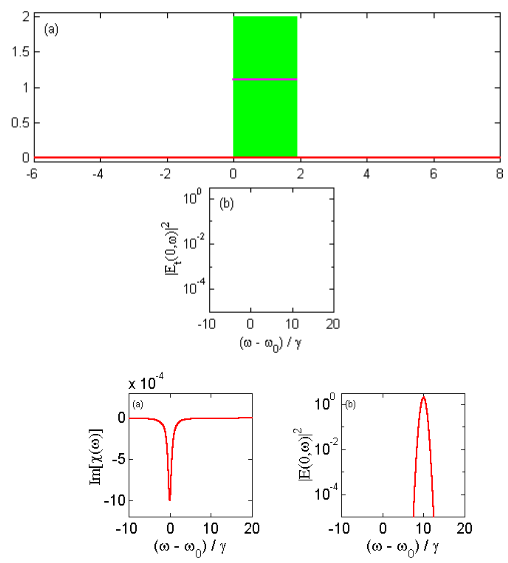
$$\mathbf{v}_E \equiv \frac{\mathbf{S}}{u} \le \frac{\mathbf{S}}{u_{field}} \le c$$

The energy transport velocity is strictly luminal as long as the material is not permitted to run an energy deficit.

Notice this

$$u_{exchange} \equiv \varepsilon_o \int_{-\infty}^{\infty} \left| E_t(\omega) \right|^2 \omega \, \mathrm{Im}[\chi(\omega)] d\omega \qquad E_t(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} dt' E(t') e^{i\omega t'}$$

The instantaneous spectrum demonstrates why the medium preferentially amplifies the leading edge.



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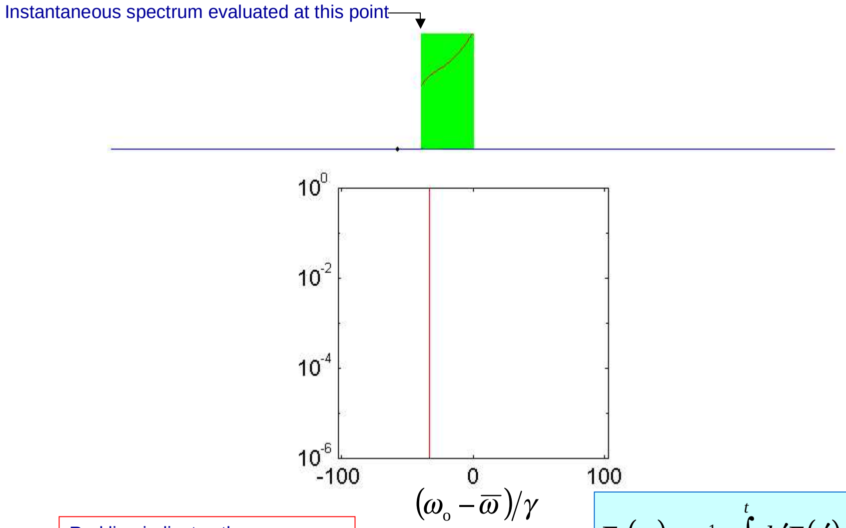

Red line indicates the resonance frequency: $(\omega_o-\bar{\omega})/\gamma = -33$

$$E_t(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} dt' E(t') e^{i\omega t'}$$

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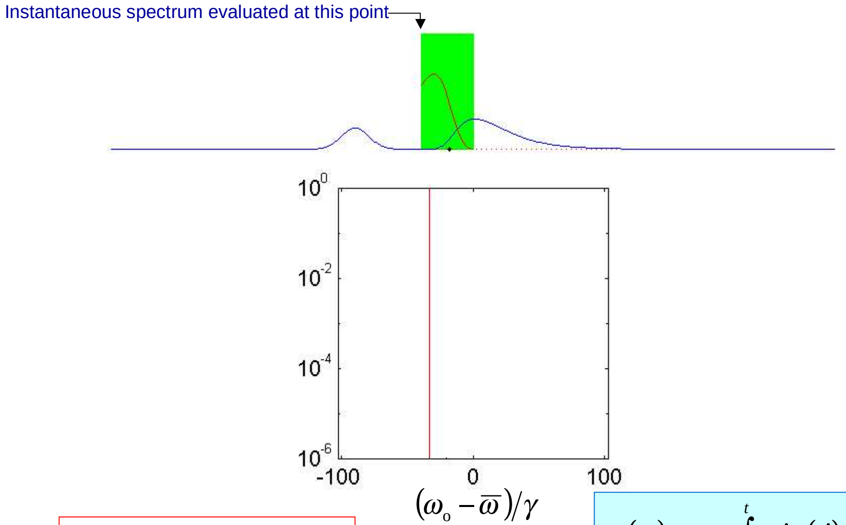
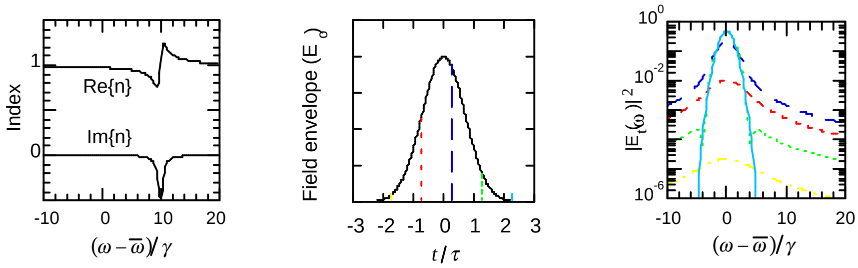
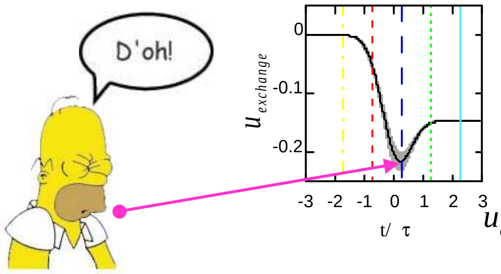


Red line indicates the resonance frequency: $(\omega_o-\bar{\omega})/\gamma = -33$

$$E_t(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} dt' E(t') e^{i\omega t'}$$

During the early portion of a pulse the medium perceives a wider spectrum and amplifies those spectral components.



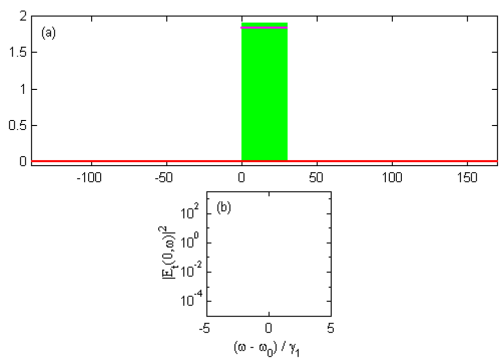


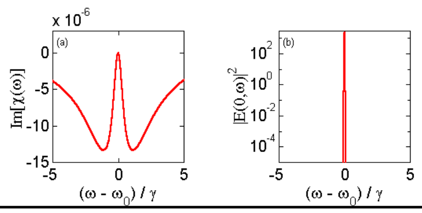
$$E_t(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} dt' E(t') e^{i\omega t'}$$

$$u_{exchange} \equiv \int_{-\infty}^{\infty} |E_t(\omega)|^2 \, \omega \, \mathrm{Im}[\varepsilon(\omega)] d\omega$$

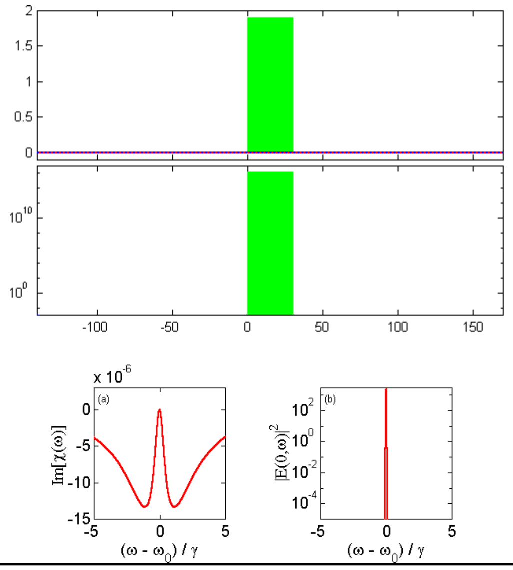
J. Peatross, M. Ware, S. A. Glasgow, "The role of the instantaneous spectrum on pulse propagation in causal linear dielectrics" , J. Opt. Soc. Am. A (2001)

The instantaneous spectrum demonstrates why the medium preferentially amplifies the leading edge.





The medium treats the leading edge of a pulse the same regardless of whether a termination occurs.



Summary

- The group delay function tracks the presence of field energy and can be superluminal.
- Energy transport velocity is never superluminal when all relevant energy is considered.
- The instantaneous spectrum governs how a causal medium interacts differently with the front of a pulse than with the back.