Quantum simulation with superconducting qubits

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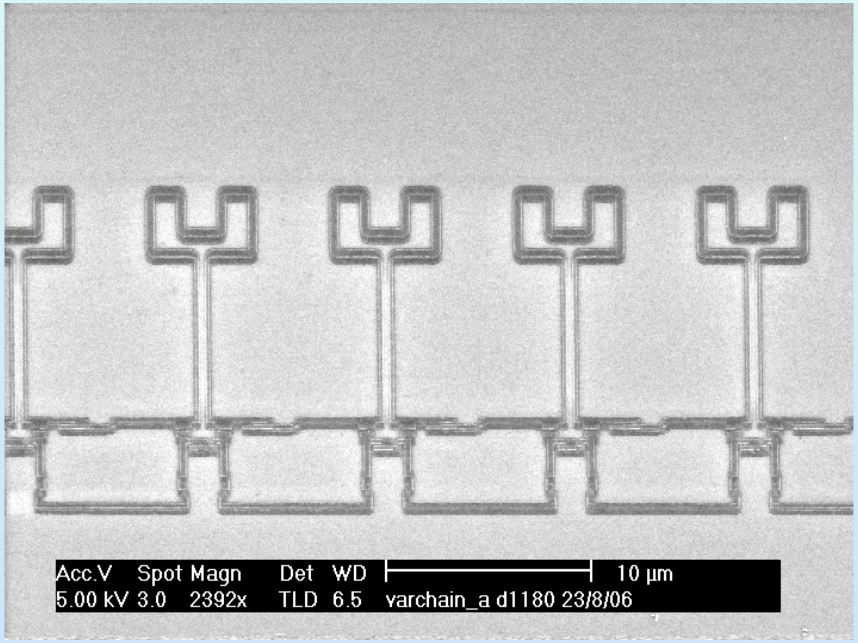


KITP Santa Barbara - *Workshop on Quantum Information Science*

November 4, 2009

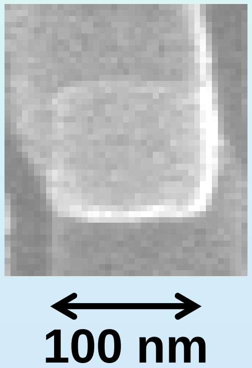
1. introduction superconducting qubits
2. flux qubits
3. quantum simulation with quantum vortices and flux qubits

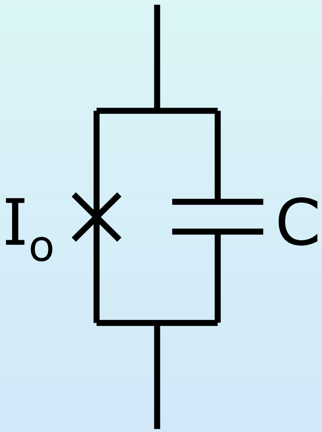
superconducting order parameter $\Psi = |\Psi| e^{i\varphi}$

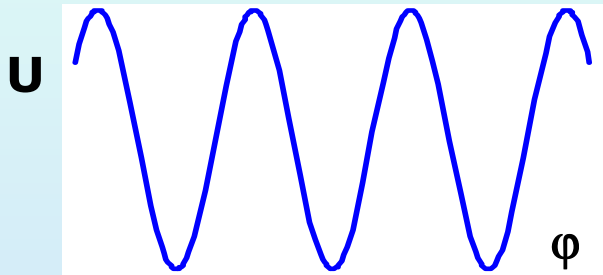
$$\Phi_o = \frac{h}{2e} \qquad V = \frac{\Phi_o}{2\pi}\frac{\partial \varphi}{\partial t}$$

loop $\sum_i \varphi_i = 2\pi \frac{\Phi}{\Phi_o} + 2\pi n$

Josephson junction







$$E_J = \frac{\Phi_o}{2\pi} I_o$$

$$E_C \equiv \frac{4e^2}{2C}$$

$$U = E_C \left(\frac{Q}{2e}\right)^2 + E_J(1 - \cos\varphi)$$

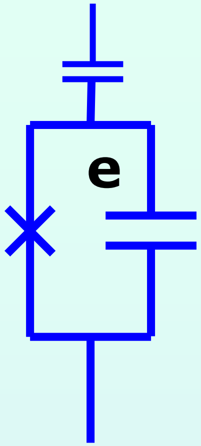
$$I = \frac{dQ}{dt} + I_o \sin\varphi$$

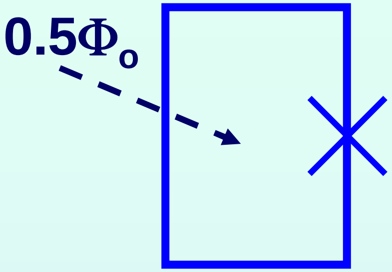
circuit with Josephson junctions

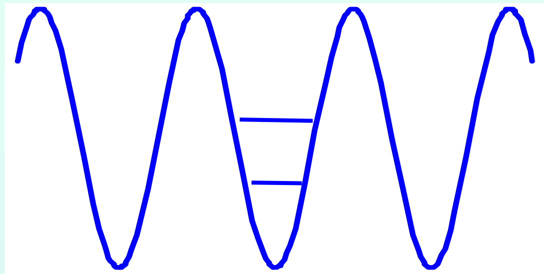
- $\varphi_1 .. \varphi_n$ phase differences+constraints
- $U(\varphi_i .. \varphi_n)$: potential energy
- $C_i(d\varphi_i/dt)^2$ terms: kinetic energy
- set of conjugate variables: Lagrangian, Hamiltonian
- quantization

set of islands with discrete numbers of Cooper pairs

- junctions provide off-diagonal coupling of charge states

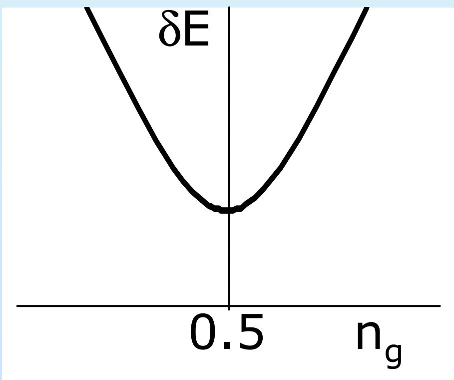


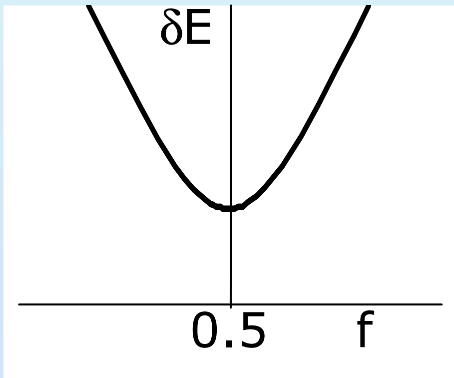




charge qubit | fluxoid qubit | oscillation qubit

charge qubit | *flux qubit* | *phase qubit transmon*





$$\delta E = \hbar\omega_{osc}$$

$\mathbf{n_g = CV_g/2e}$

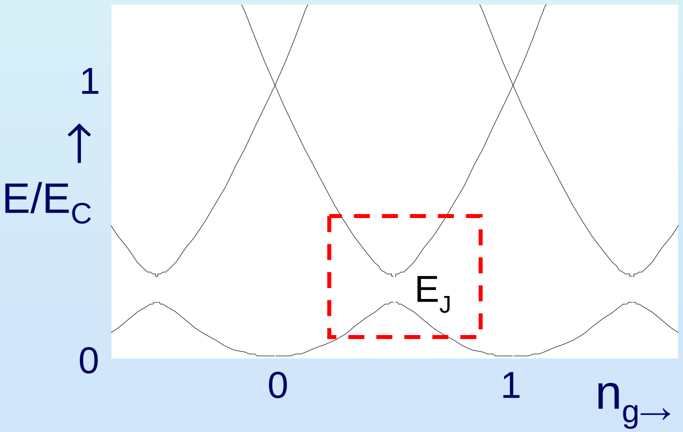
$\mathbf{f = \Phi/\Phi_0}$

quantum Cooper pair box: charge qubit

$$H = E_C(n - n_g)^2 - E_J \cos\varphi$$ (classical Hamiltonian)

$$\left[\hat{n}, \hat{\varphi}\right] = i$$

$$\hat{H} = E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\varphi}$$



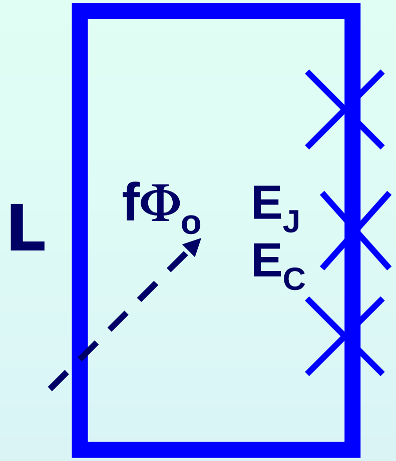
$E_C/E_J >> 1$

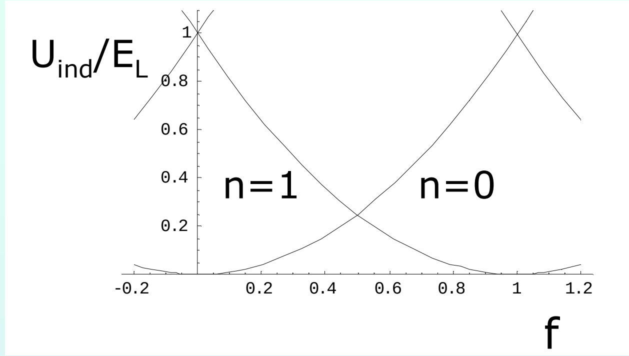
around $n_g = 0.5$

$$\hat{H} = -\{E_J \sigma_x + E_C(n_g - 0.5)\sigma_z\}/2$$

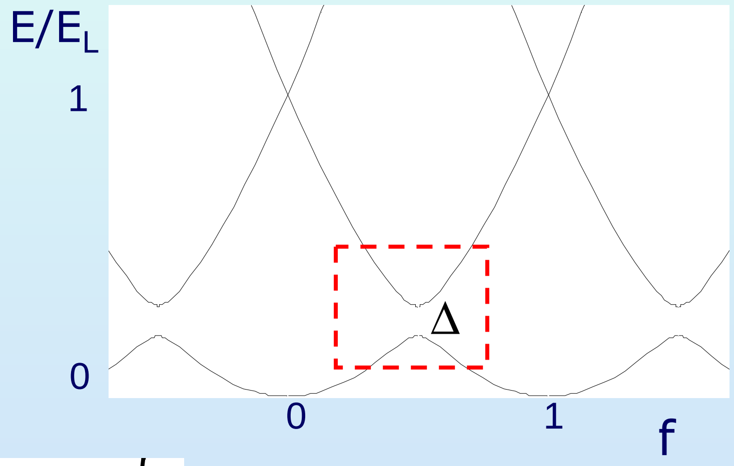
other levels extremely high

flux qubit $E_J >> E_C$



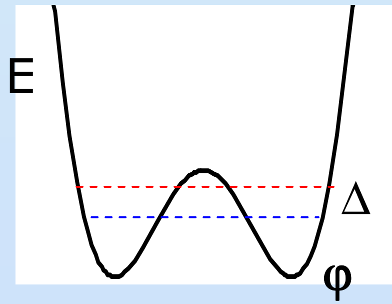


around f=0.5



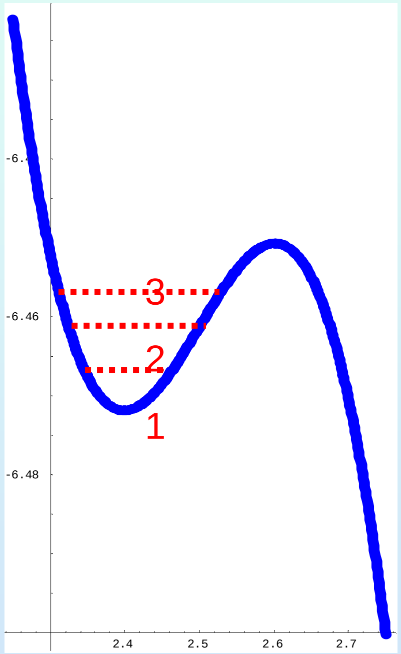
$$\hat{H} = -\{\Delta\sigma_x + E_L(f - 0.5)\sigma_z\}/2$$

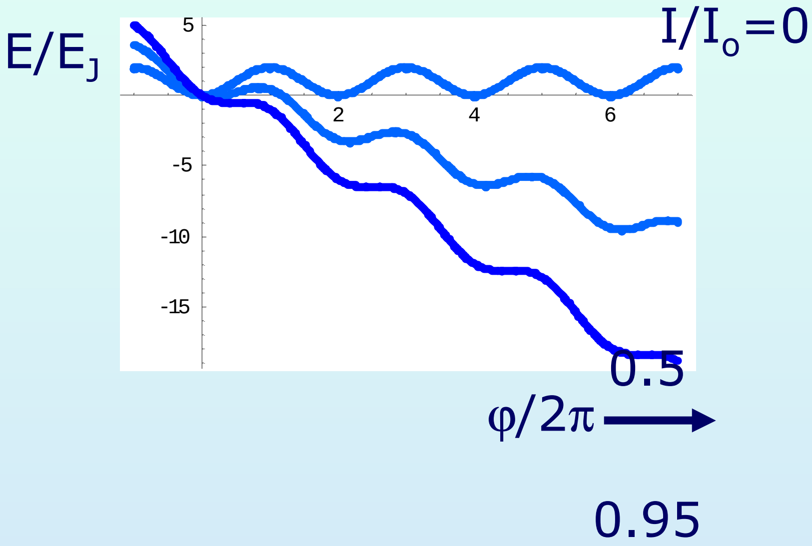
$$\Delta = a\sqrt{E_J E_C}\exp\left(-b\sqrt{\frac{E_J}{E_C}}\right)$$



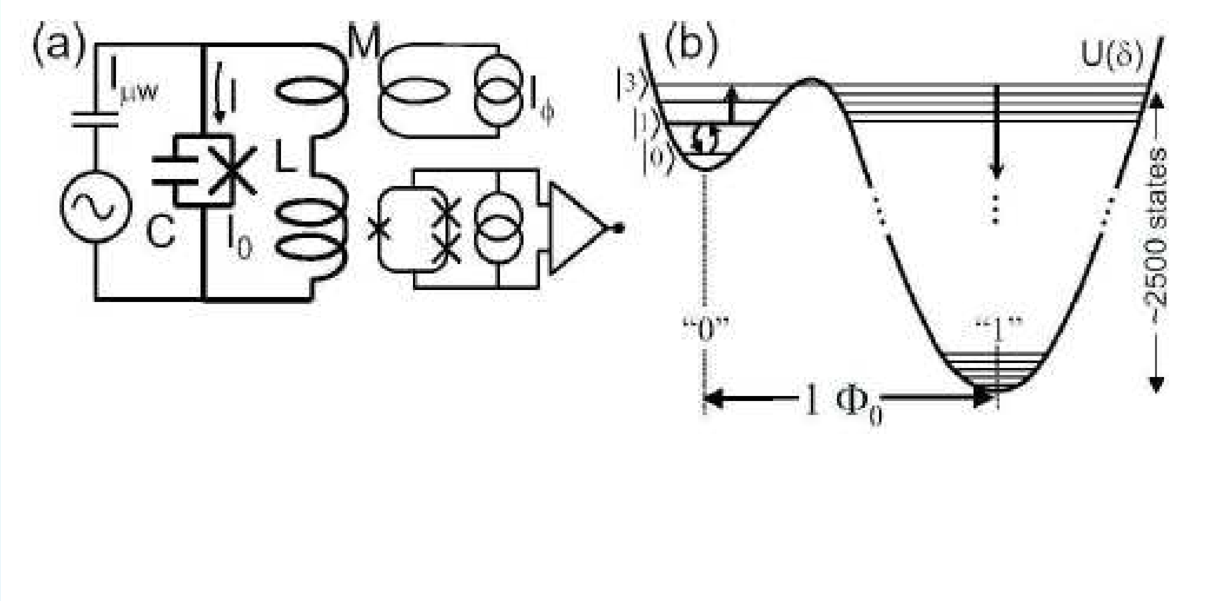
Phase Qubit

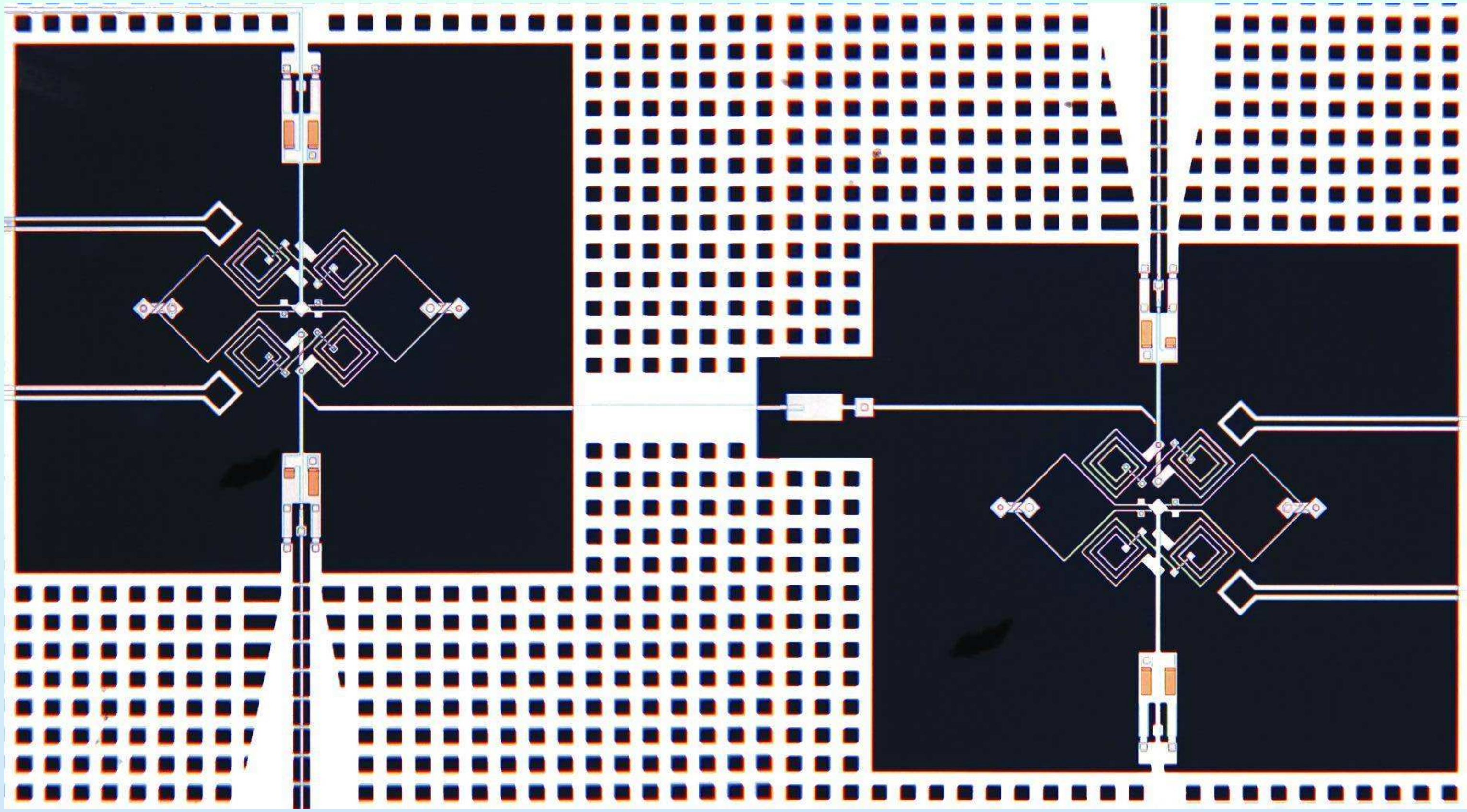
plasma oscillation of a current-biased single Josephson junction





Martinis group Santa Barbara

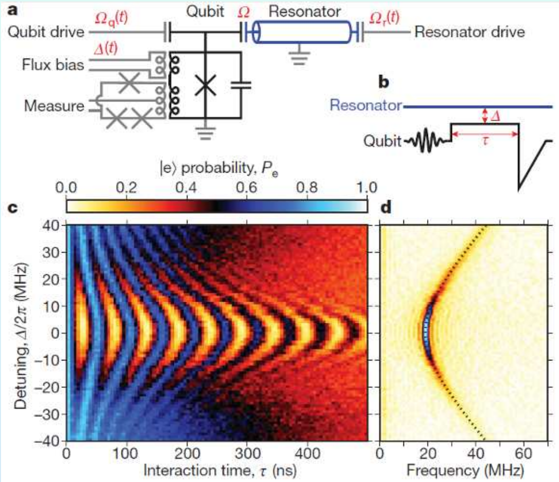


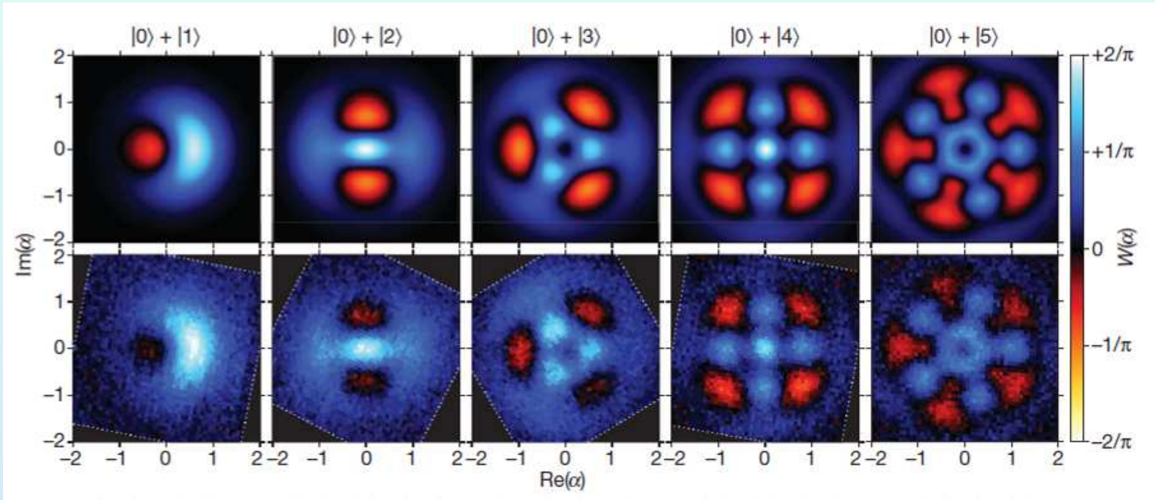


Synthesizing arbitrary quantum states in a superconducting resonator

Max Hofheinz[1], H. Wang[1], M. Ansmann[1], Radoslaw C. Bialczak[1], Erik Lucero[1], M. Neeley[1], A. D. O'Connell[1], D. Sank[1], J. Wenner[1], John M. Martinis[1] & A. N. Cleland[1]

Nature **459**, 546 (2009)





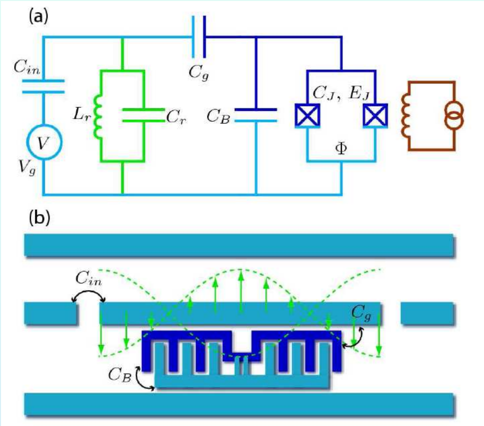
Violation of Bell's inequality in Josephson phase qubits

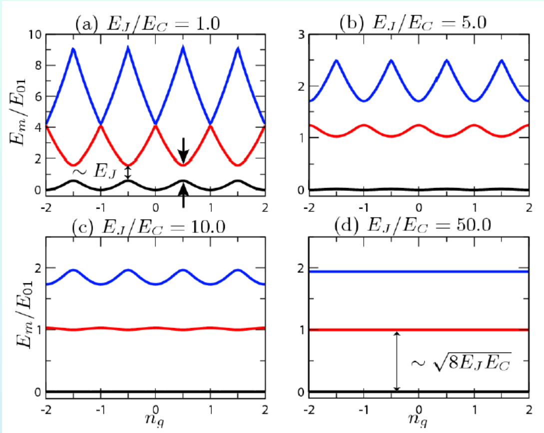
Markus Ansmann[1], H. Wang[1], Radoslaw C. Bialczak[1], Max Hofheinz[1], Erik Lucero[1], M. Neeley[1], A. D. O'Connell[1], D. Sank[1], M. Weides[1], J. Wenner[1], A. N. Cleland[1] & John M. Martinis[1]

Nature **461**, 504 (2009)

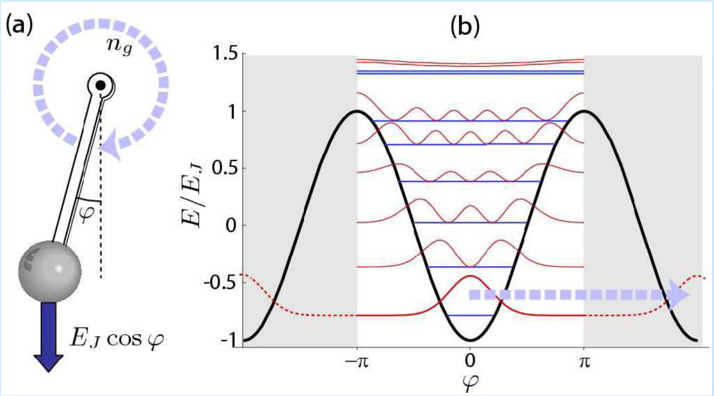
Yale group: Schoelkopf, Girvin

transmon: Cooper pair box with strongly reduced E_C
coupled to high Q harmonic oscillator





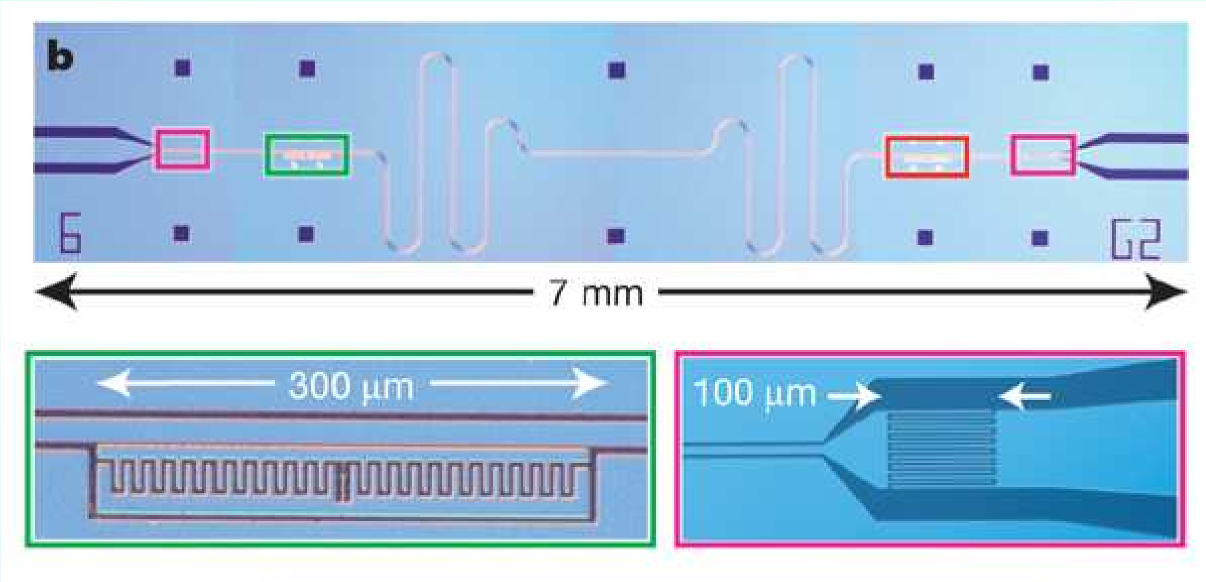
external capacitive shunt
E_{Jmax}/h= 18 GHz
E_{Ceff}/h= 1.4 Ghz
ν_{pl}~ 6 GHz

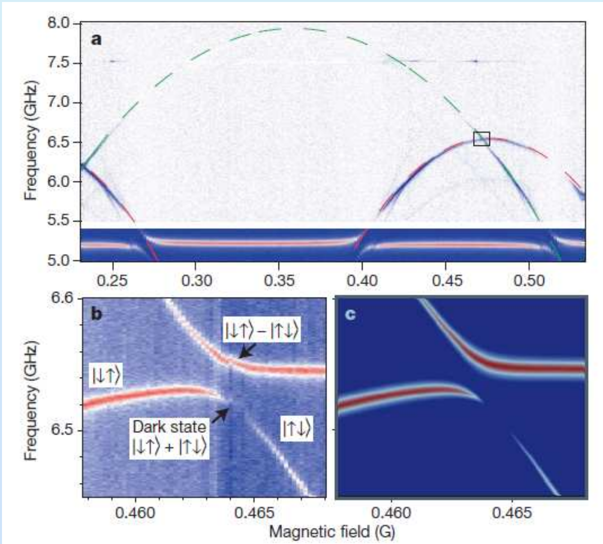


increasing E_J leads to
- gradually decreasing anharmonicity
- exponentially decreasing dependence of δE on n_g

Koch et al. PRA **76**, 042319 (2007), Schreier et al., PRB **77**, 180502 (2008)

circuit quantum electrodynamics





Demonstration of two-qubit algorithms with a superconducting quantum processor

L. DiCarlo[1], J. M. Chow[1], J. M. Gambetta[2], Lev S. Bishop[1], B. R. Johnson[1], D. I. Schuster[1], J. Majer[3], A. Blais[4], L. Frunzio[1], S. M. Girvin[1] & R. J. Schoelkopf[1]

Nature **460**, 240 (2009)

sources of decoherence

driving circuits: gate, bias flux, microwave lines, measurement

engineering design, calculation possible

microscopic defects

1/f noise due to many fluctuators

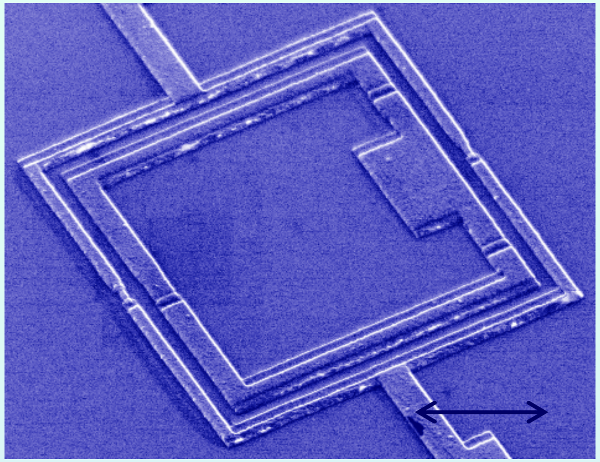
parasitic two-level systems with same energy splitting

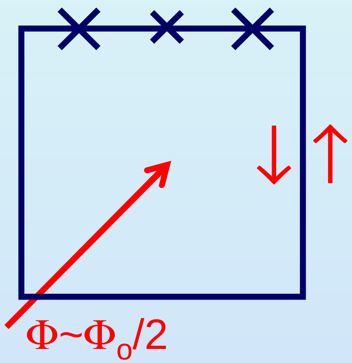
smaller qubits, fewer defects

dephasing times > 1 μs reached in most qubit types

at optimal conditions

level separation 3-15 GHz, operation time 5-50 ns

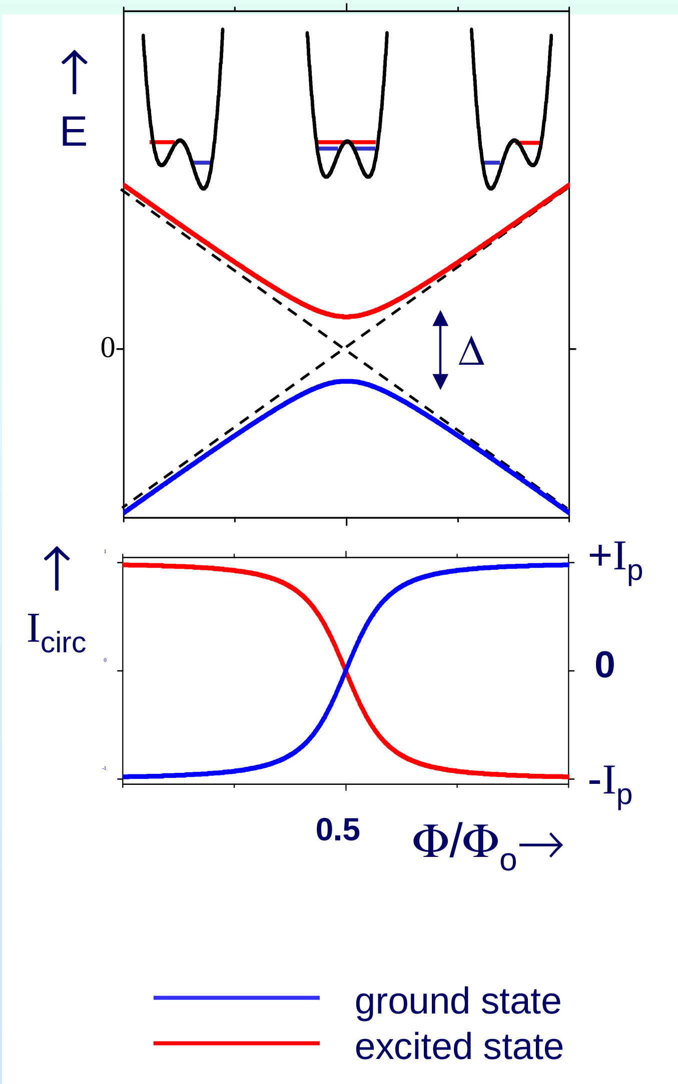




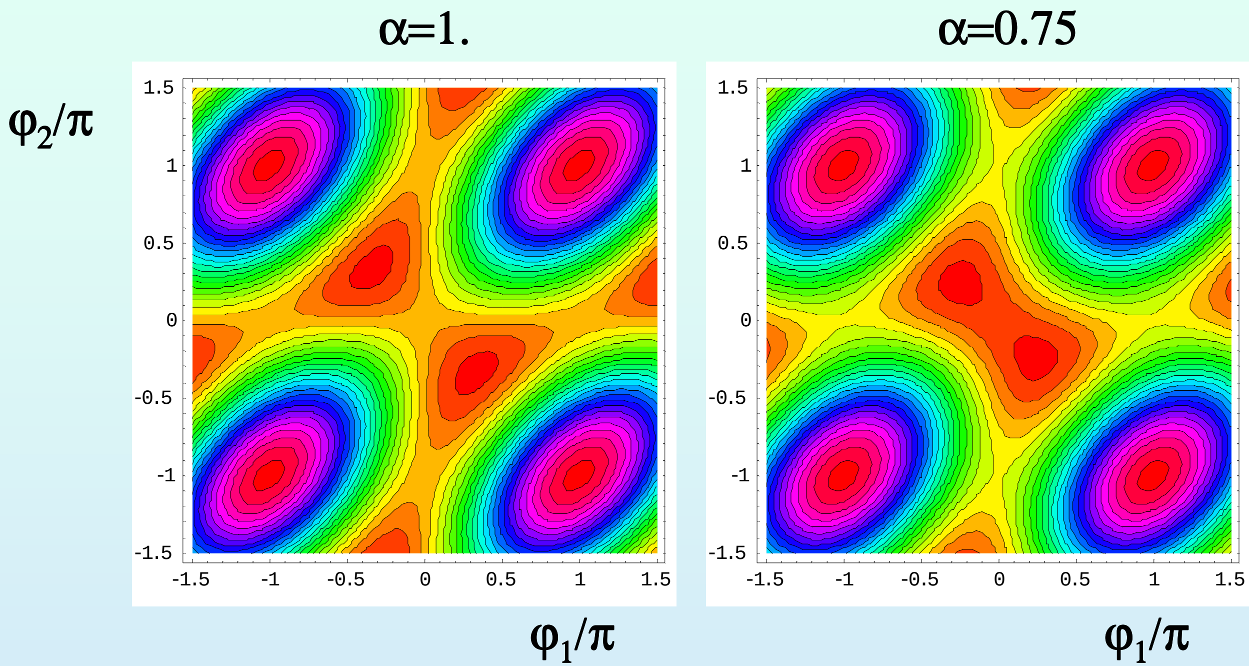
flux bias $\Phi_o/2$
currents ± I_p

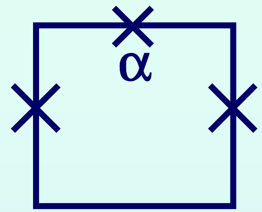
$$H = ½(\varepsilon\sigma_z + \Delta\sigma_x)$$

$$\varepsilon = (\Phi/\Phi_o - 0.5)2\Phi_o I_p$$



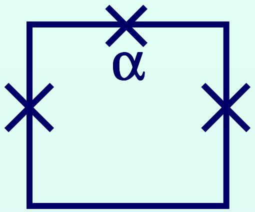
Mooij et al. Science **285**, 1036 (1999), Orlando et al. PRB (1999)
Van der Wal et al. Science **290** 1140 (2000)

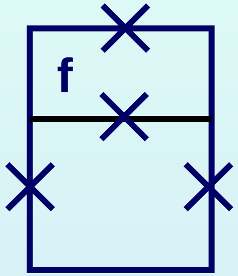


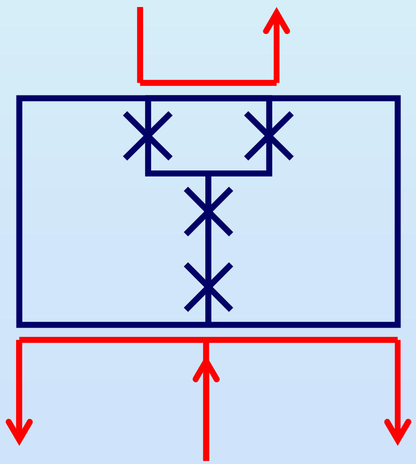


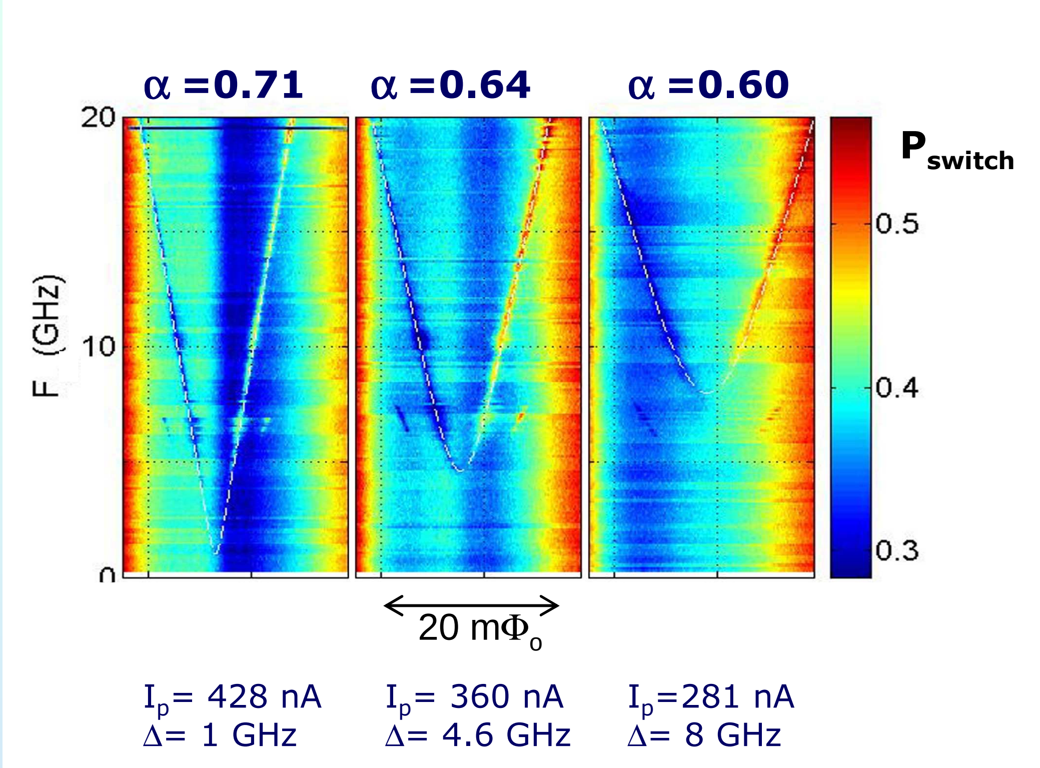
$$\Delta = a\sqrt{E_J E_C}\exp\left(-b\sqrt{\frac{\alpha E_J}{E_C}}\right)$$

tunable Δ





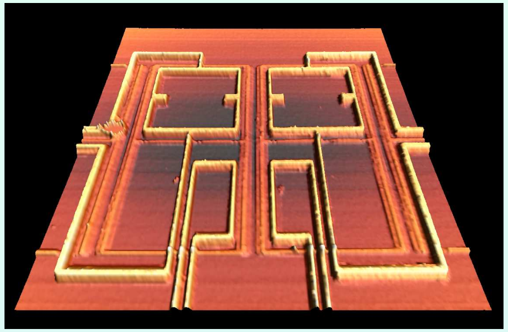




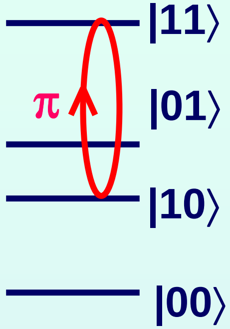
F.G. Paauw et al. PRL **102**, 090501 (2009)

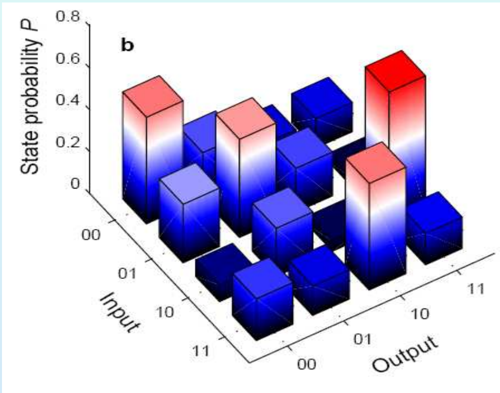
$$H = \tfrac{1}{2}(\varepsilon\sigma_z + \Delta\sigma_x)$$

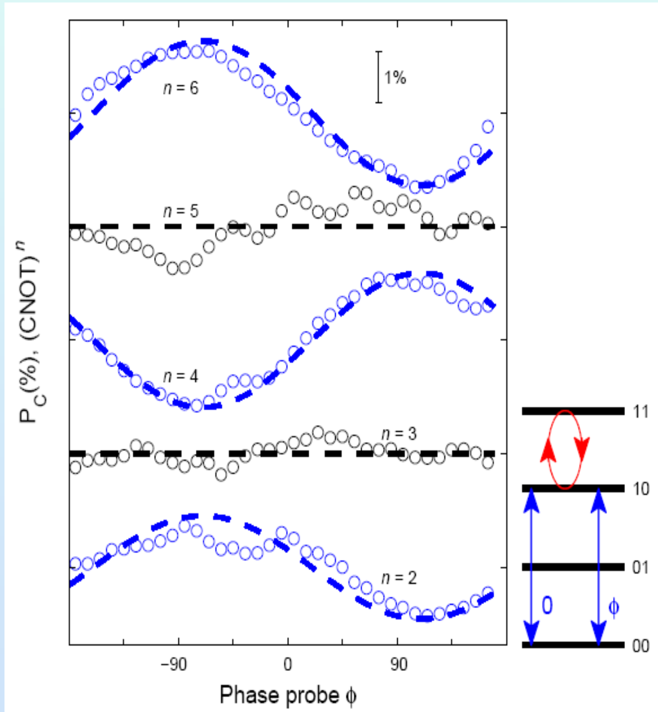
$$\varepsilon = (\Phi/\Phi_o - 0.5)2\Phi_o I_p$$



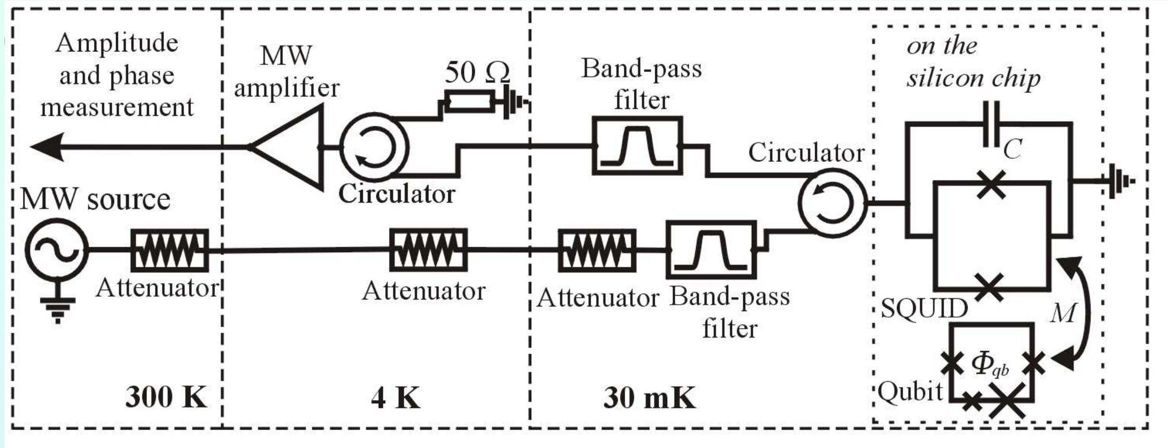
controlled-NOT gate



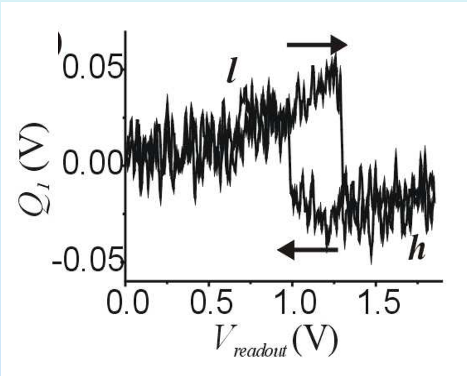


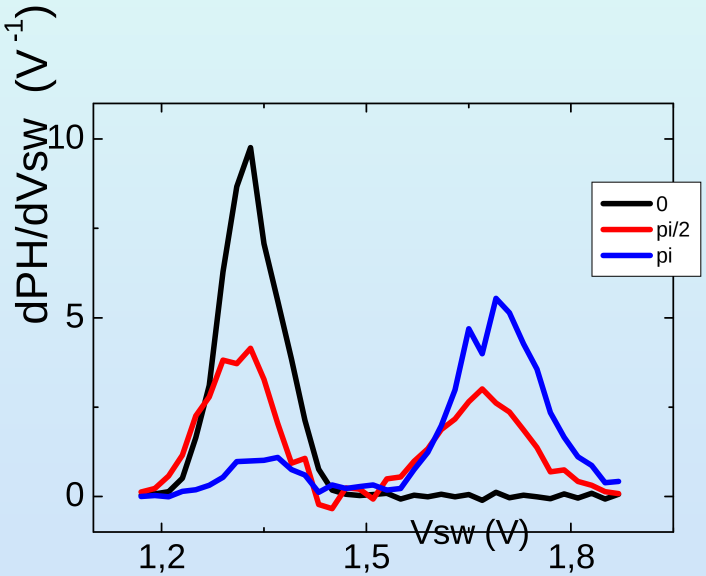


Plantenberg et al. Nature **447**,836 (2007)

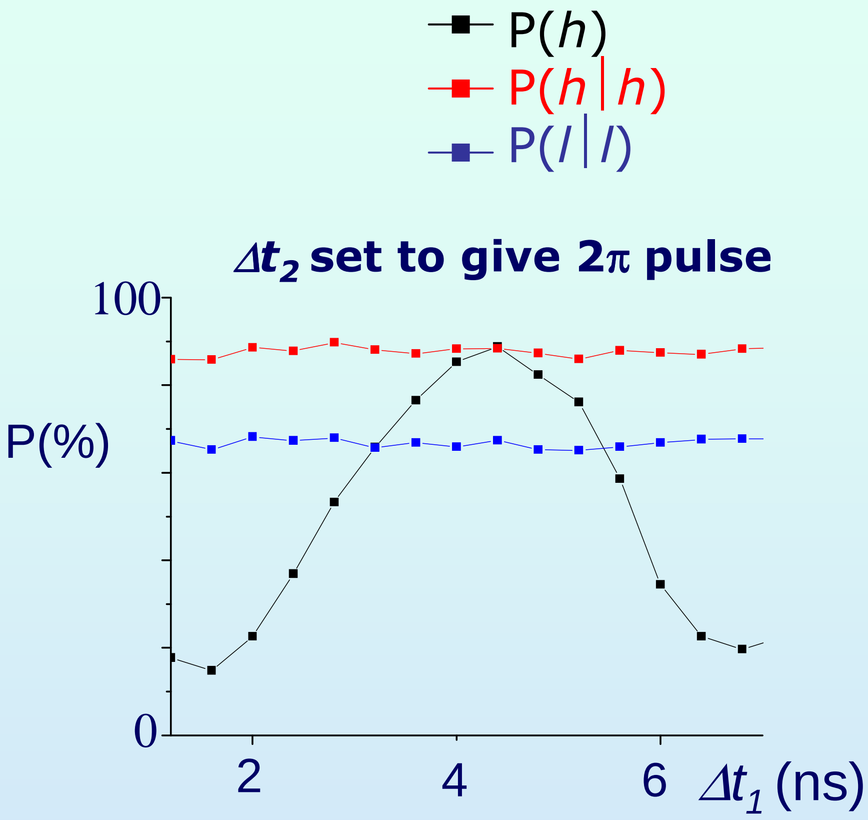


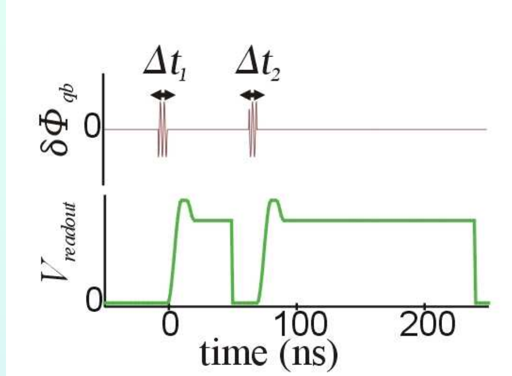
$$L_k = \frac{\Phi_o}{2\pi I_o}$$





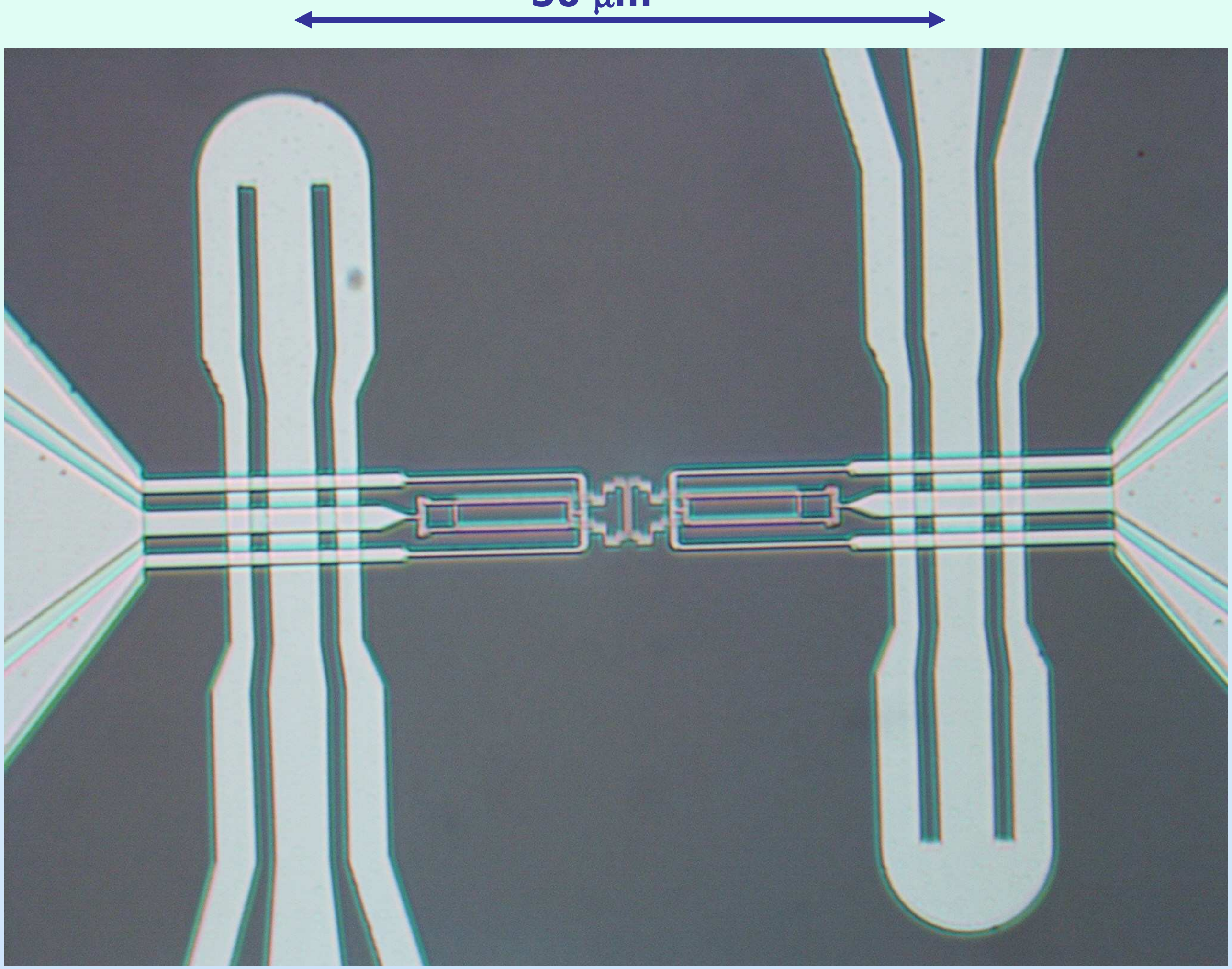
A. Lupascu et al. Nature Physics **3**, 119 (2007)



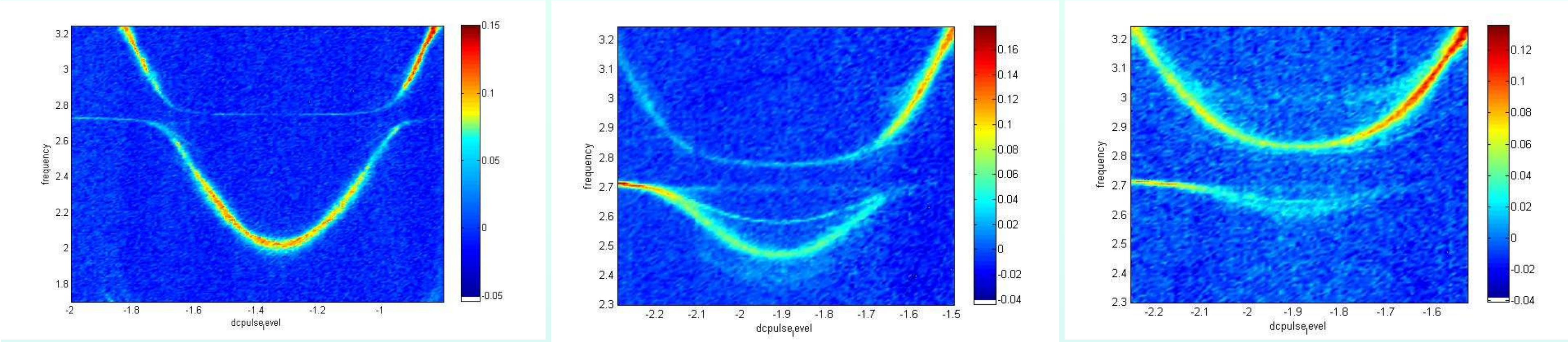


oscillator state *h* ⇔qubit ground state
oscillator state *l* ⇔ qubit excited state

Pieter de Groot - 2-qubit sample with bifurcative readout

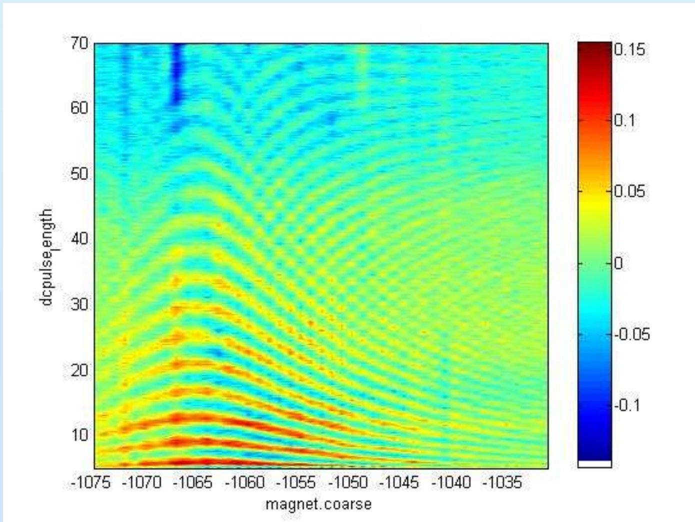


cross-talk between readout systems < 0.1 %



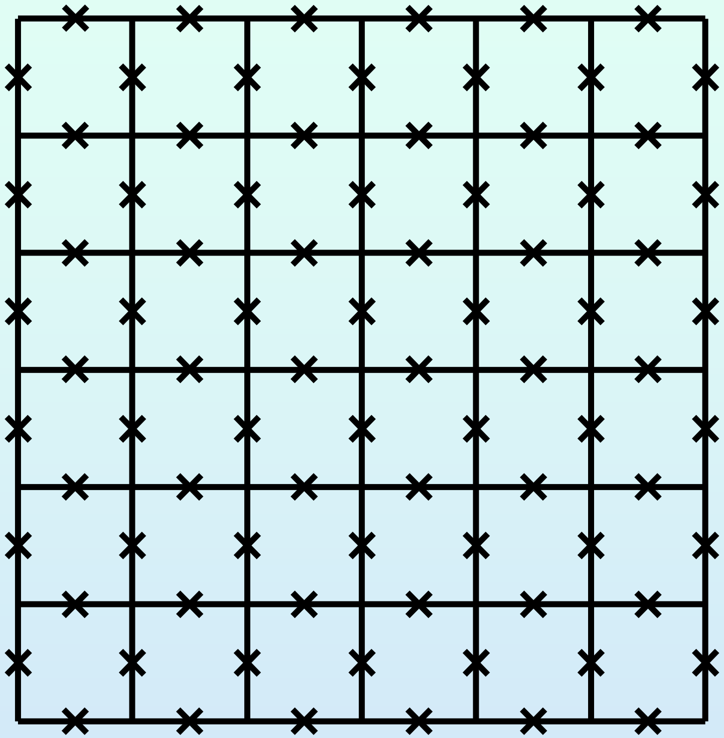
tunable-α qubit coupled to low Q oscillator

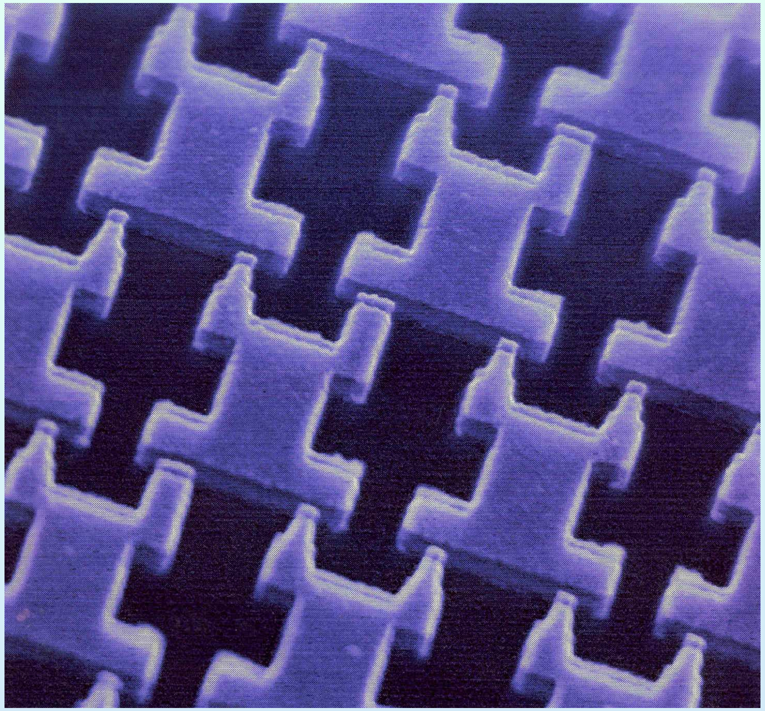
Arkady Federov



vacuum Rabi oscillation around symmetry point

quantum vortices
in 2D or quasi-1D array of Josephson junctions

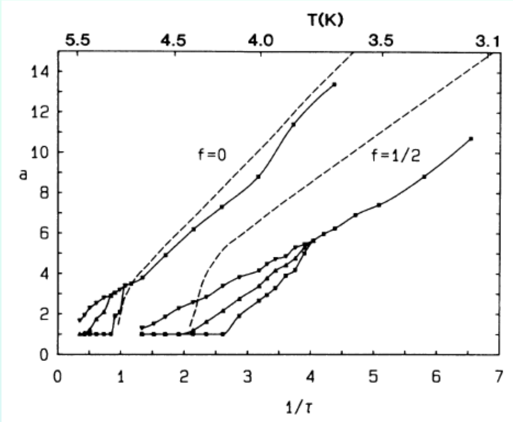




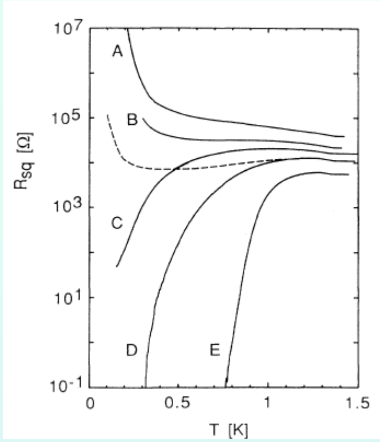
vortex in 2D Josephson junction array:

particle moving in 2D potential E_J

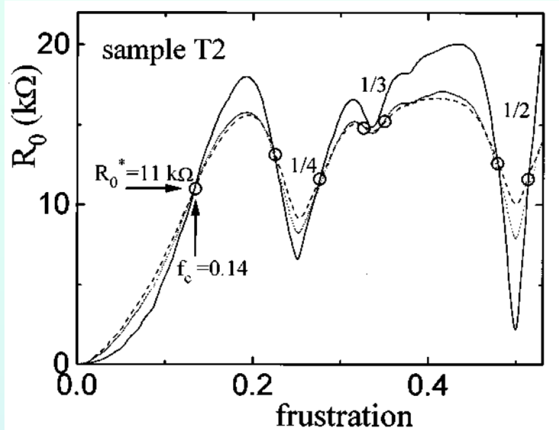
mass determined by junction capacitance $1/E_C$



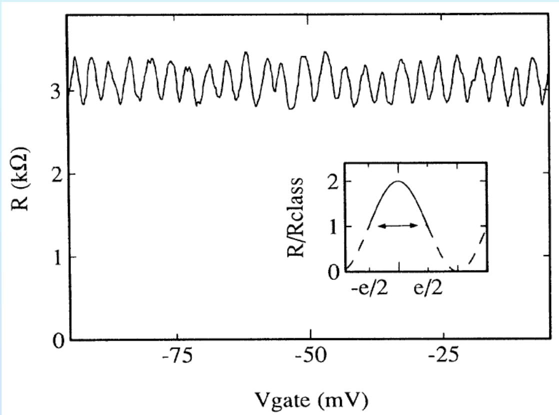
Berezinskii, Kosterlitz,Thouless transition
PRB 35, 7291 (1987)



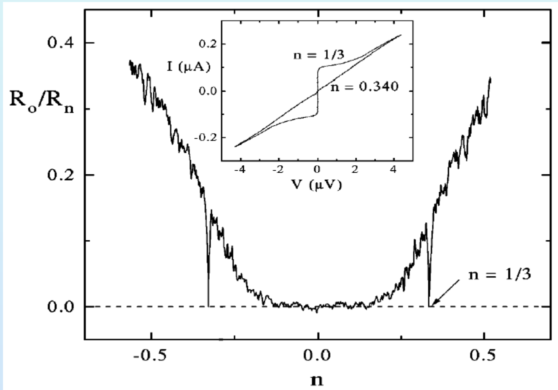
superconductor-insulator transition E_J/E_C
PRL 63, 326 (1989)



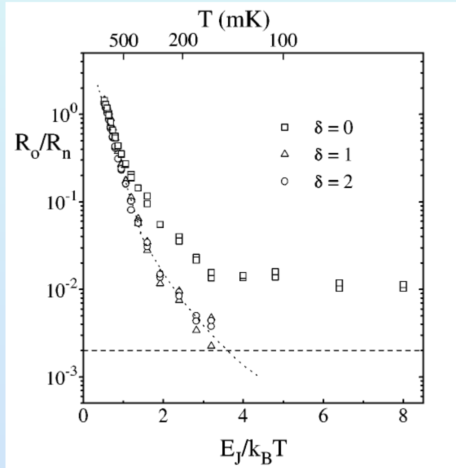
Bose Einstein condensation
PRL 69, 2971 (1992)



quantum interference around charge
PRL 71, 2311 (1993)



Mott insulator
PRL 76, 4947 (1996)



Anderson localization
PRL 77, 4257 (1996)

arXiv:cond-mat/0108266 (2001)

Quantum spin chains and Majorana states in arrays of coupled qubits

L. S. Levitov[1], T. P. Orlando[2], J. B. Majer[3], J. E. Mooij[3]
(1) *Physics Department and Center for Materials Science & Engineering, Massachusetts Institute of Technology, 77 Massachusetts Ave, Cambridge, MA 02139*
(2) *Electrical Engineering Department, Massachusetts Institute of Technology, 77 Massachusetts Ave, Cambridge, MA 02139*
(3) *Applied Physics and DIMES, Delft Technical University, P.O.Box 5046, 2600 GA Delft, the Netherlands*

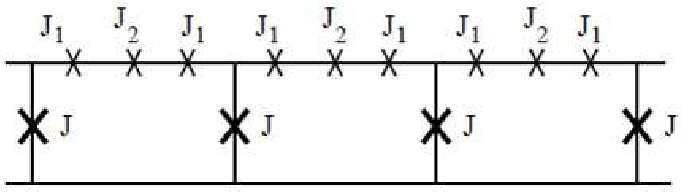
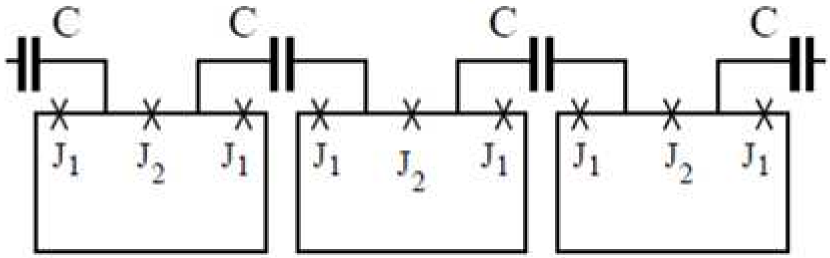
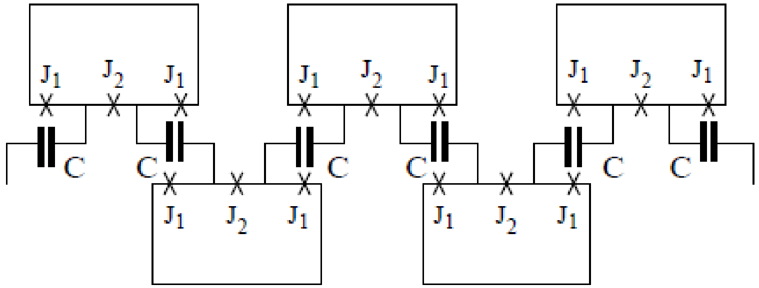


FIG. 2. 1D array of qubits coupled by shared Josephson junctions: a realization of the $\sigma_1^z \sigma_2^z$ interaction.



$$\mathcal{H} = \sum_{i=-\infty}^{\infty} t(\sigma_i^+ \sigma_{i+1}^- + \sigma_i^- \sigma_{i+1}^+) - (\Delta \sigma_i^x + h \sigma_i^z)$$

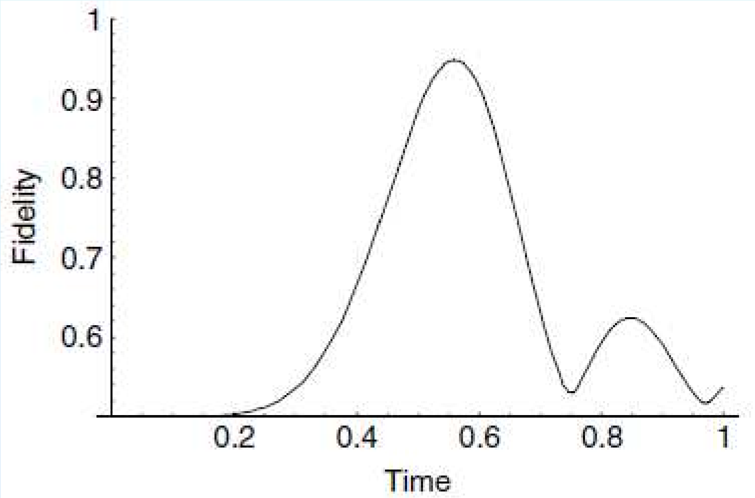


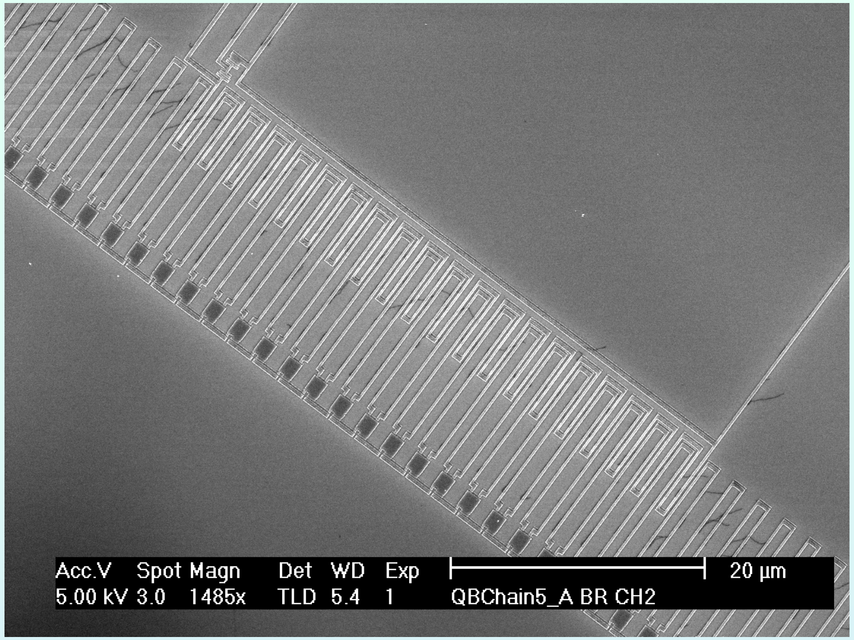
$$\mathcal{H} = \sum_{i=-\infty}^{\infty} t(\sigma_i^+ \sigma_{i+1}^+ + \sigma_i^- \sigma_{i+1}^-) - (\Delta \sigma_i^x + h \sigma_i^z)$$

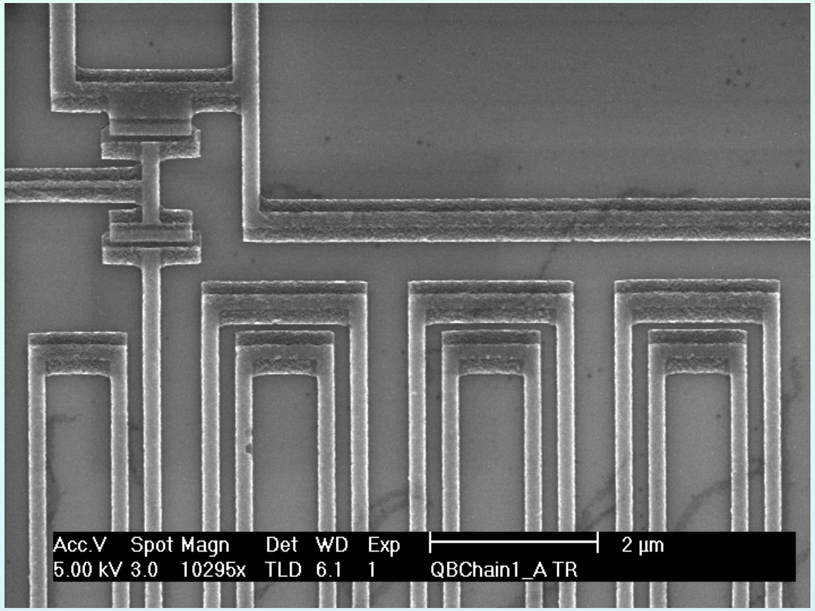
Quantum state transfer in arrays of flux qubits

A Lyakhov and C Bruder **New J of Phys 7, 181 (2005)**

chain of flux qubits: excitation at one end, readout at the other end

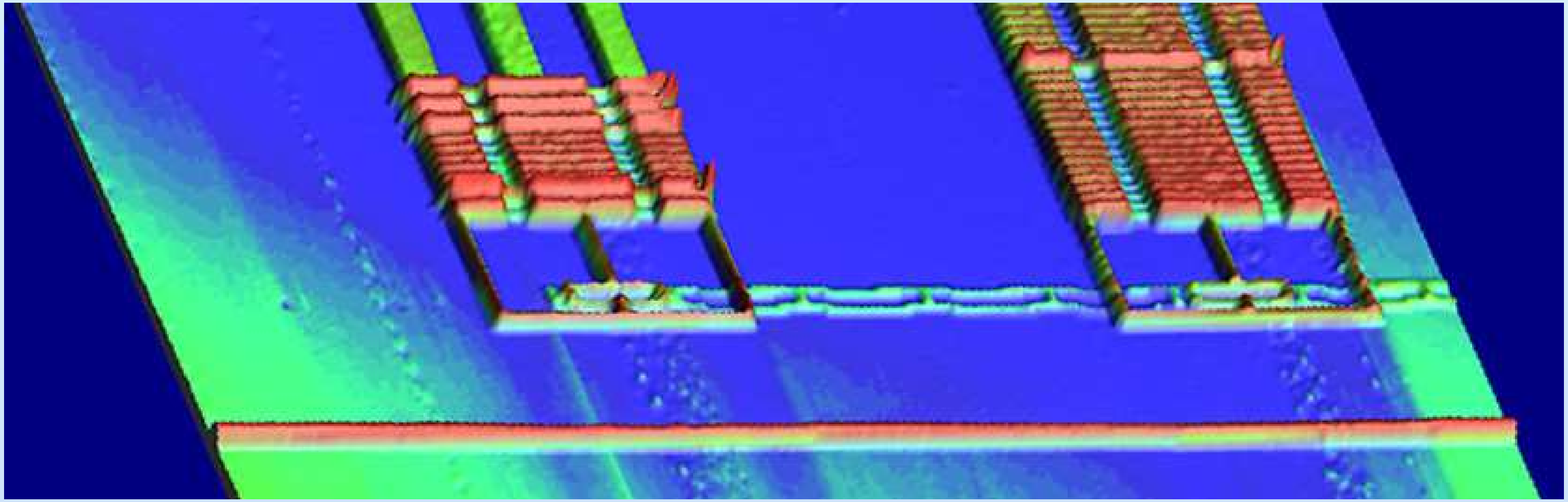






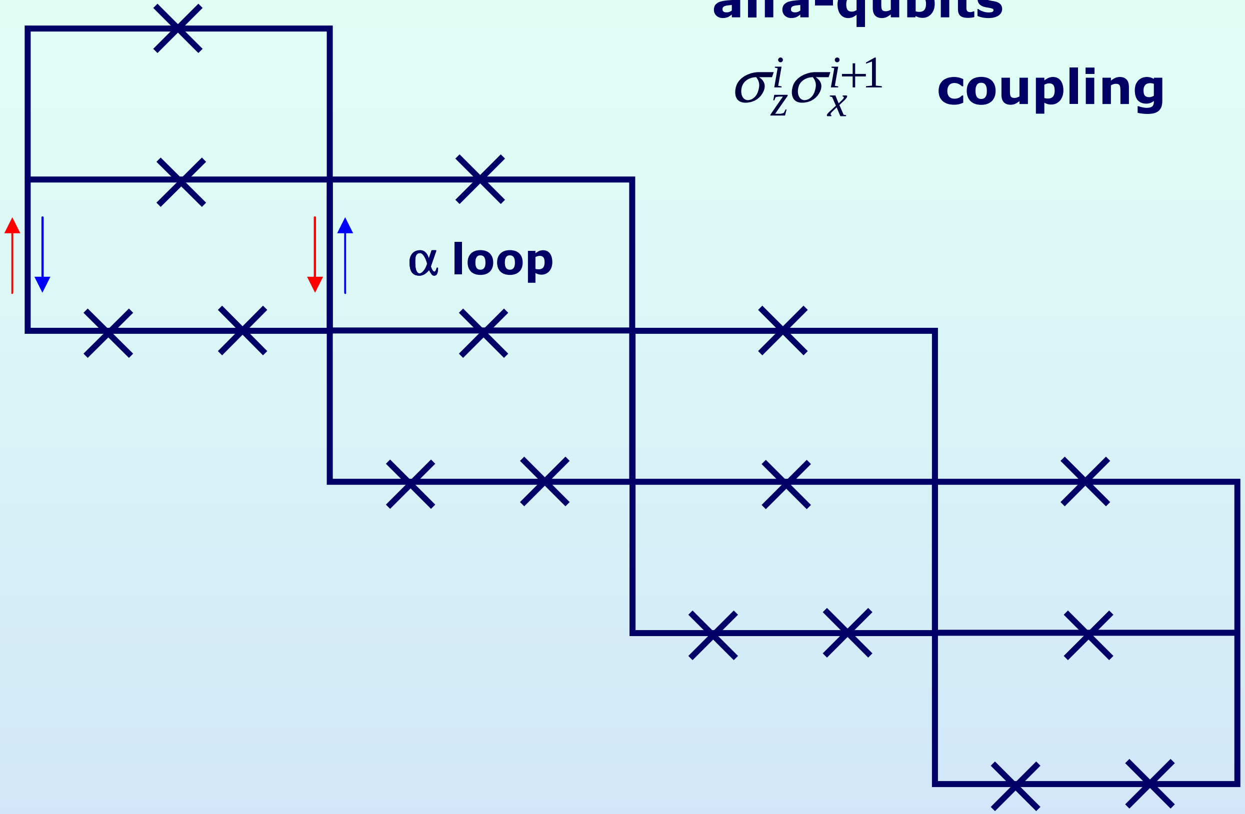
1D coupled chains of flux qubits

Floor Paauw, Delft

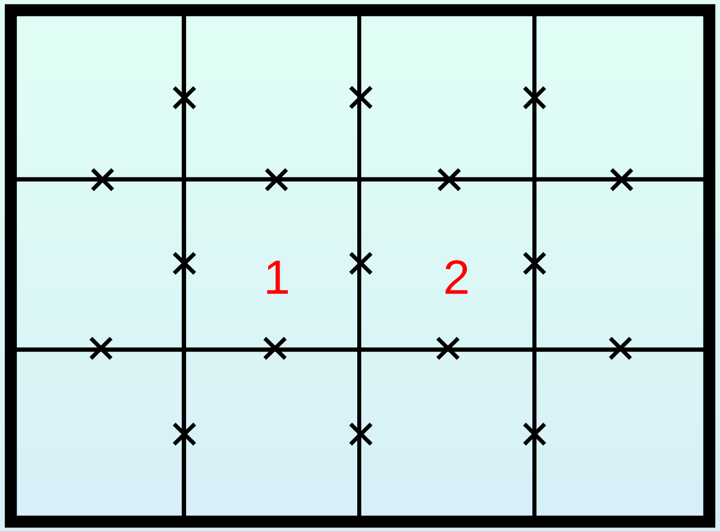


alfa-qubits

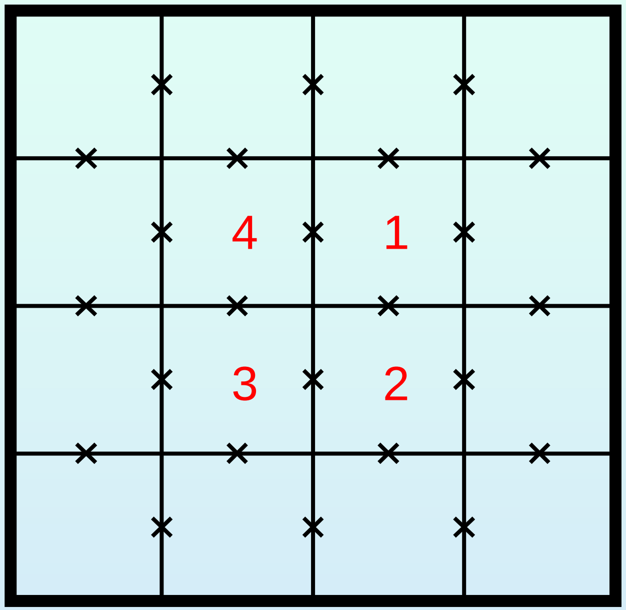
$\sigma_z^i \sigma_x^{i+1}$ **coupling**



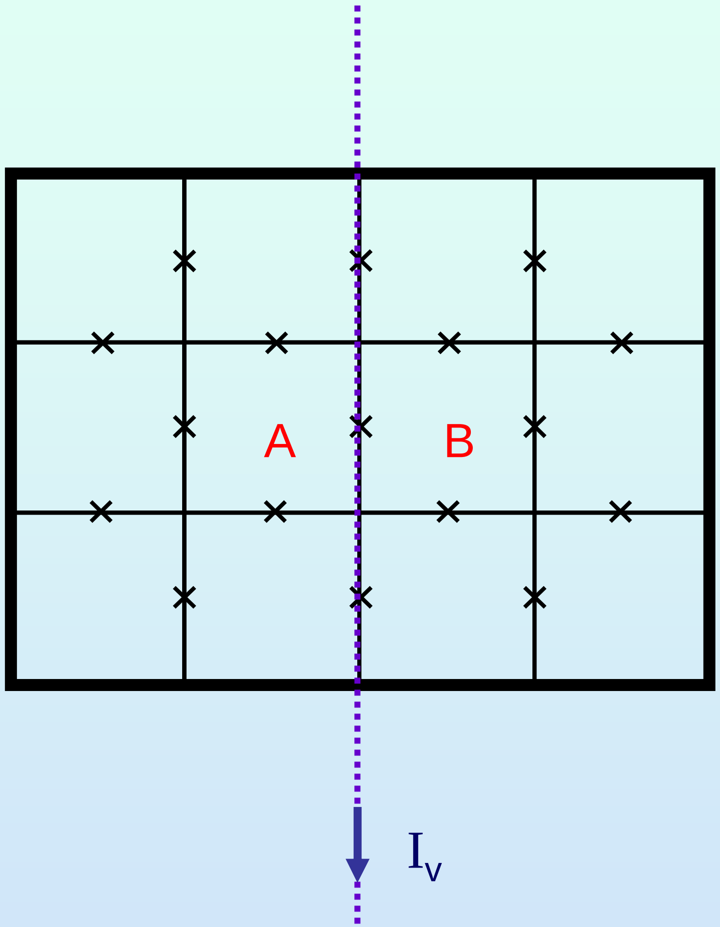
trapped vortex in junction array



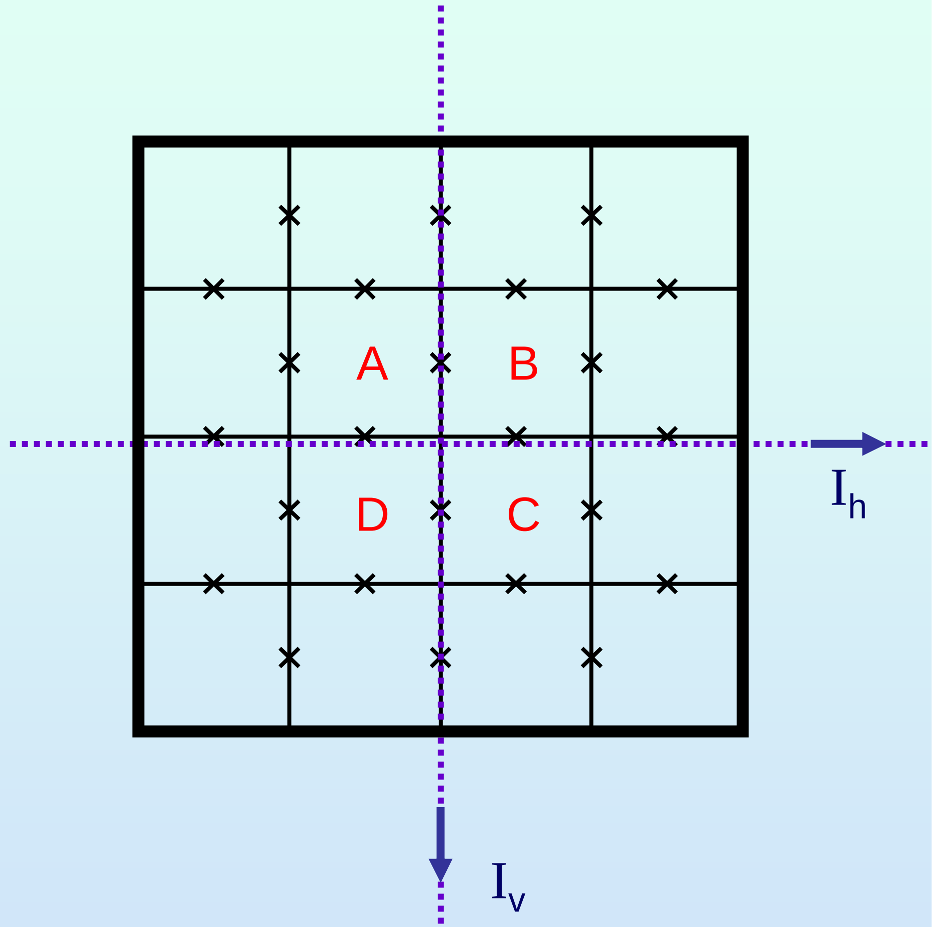
2 degenerate states



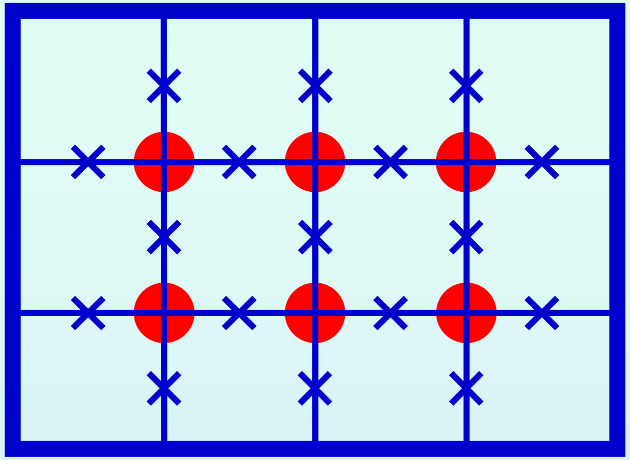
4 degenerate states

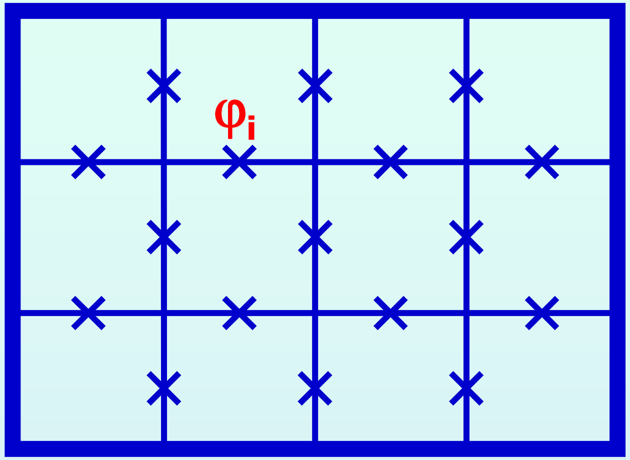


2 degenerate states



4 degenerate states





6 charge variables

$E_C >> E_J$

17 junctions

(12-1) loop equations

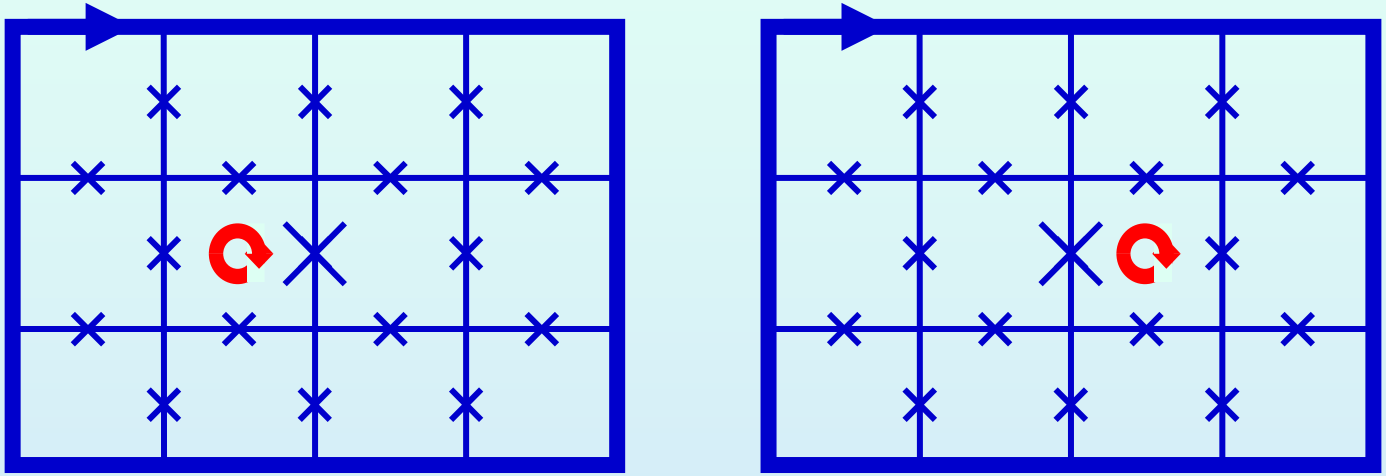
6 phase variables

$E_J >> E_C$

if vertical symmetry is assumed:
3 variables

trapped fluxoid: two positions

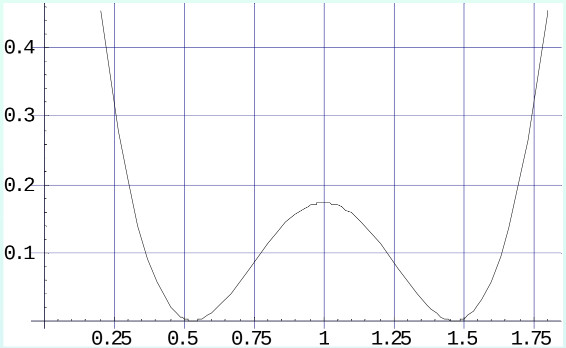
central junction q times larger



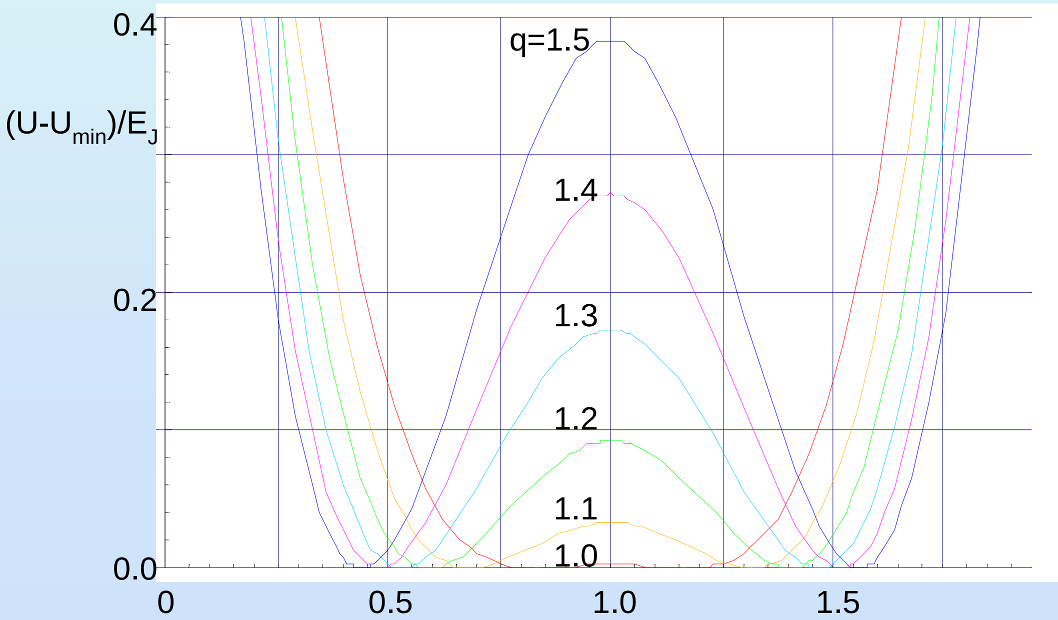
potential energy in phase landscape, symmetric solution
phase difference across central junction imposed, rest optimized

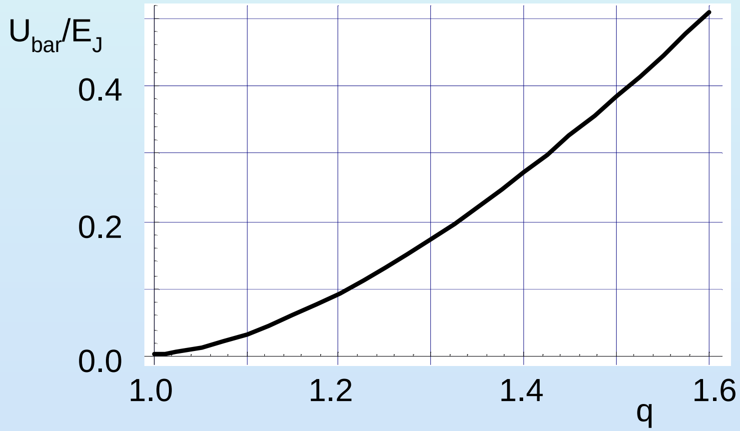
potential energy/E_J

q=1.3

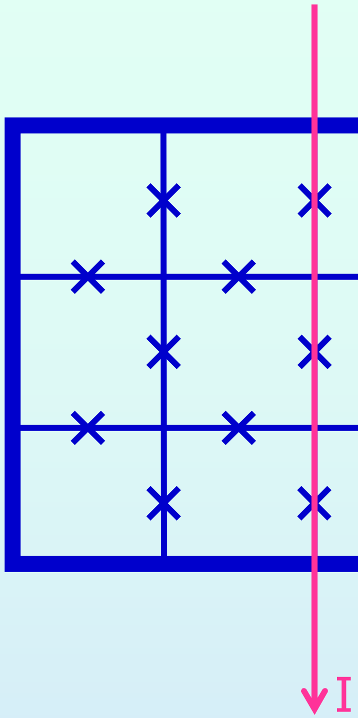


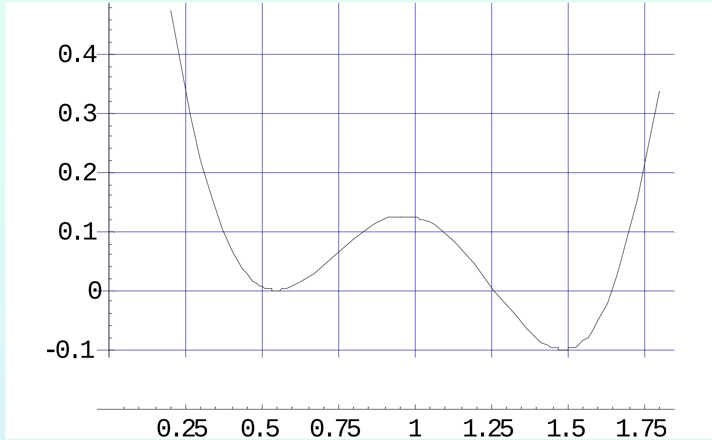
$\varphi_{central}/\pi$





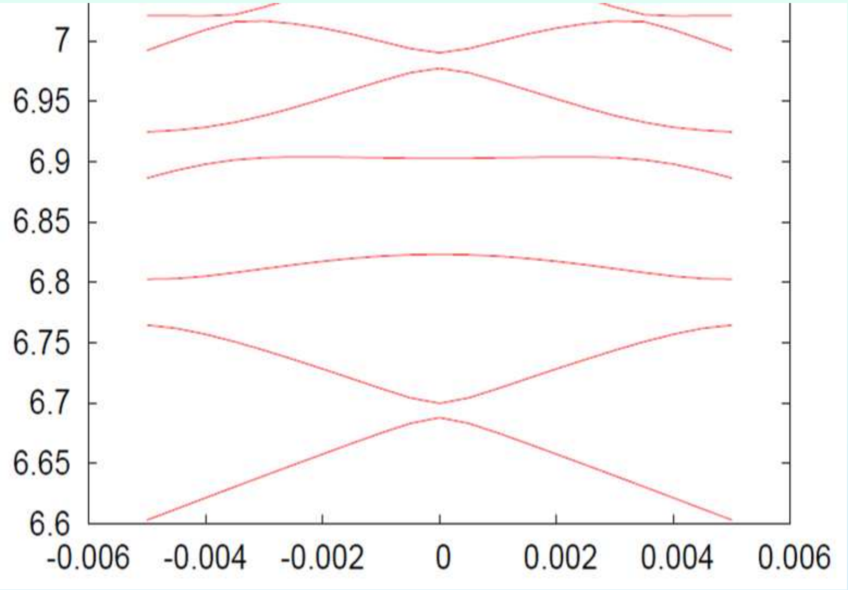
flux tilt

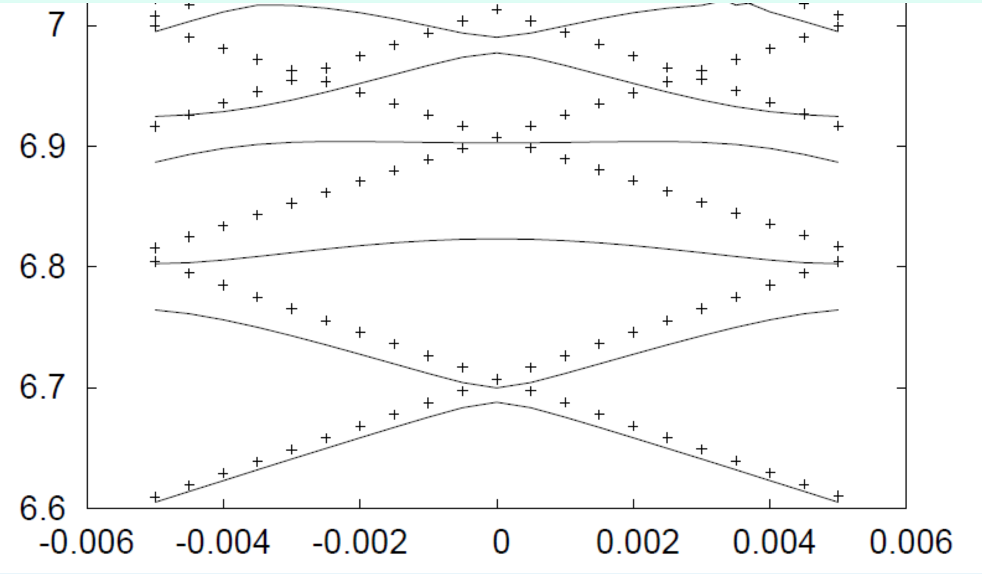




quantum calculation (Jos Thijssen et al.)

classical : crosses



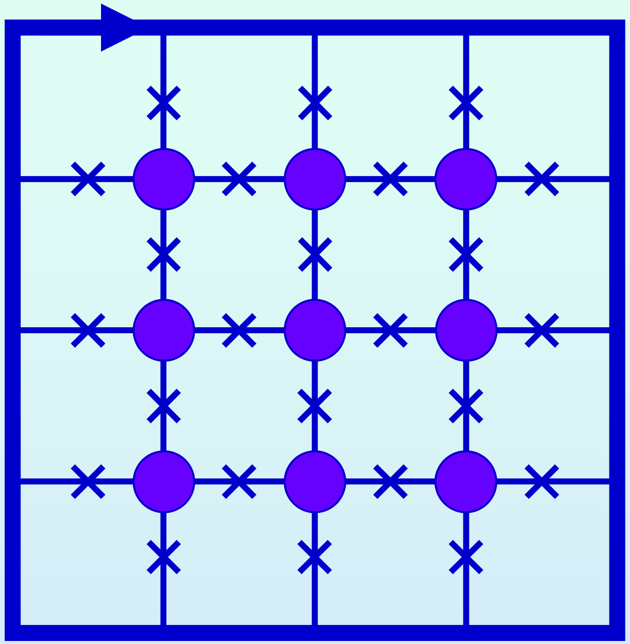


q=1.3, m=5, symmetric case (3 degrees of freedom)

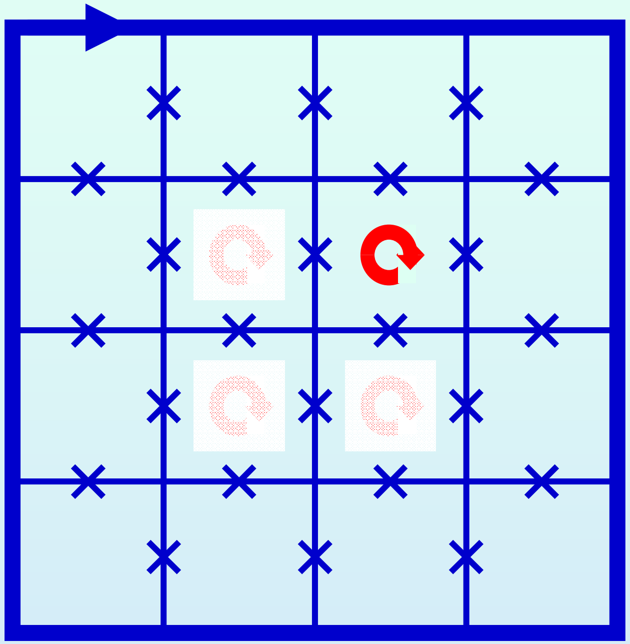
↑

energy **current tilt →**

square array with one trapped fluxoid

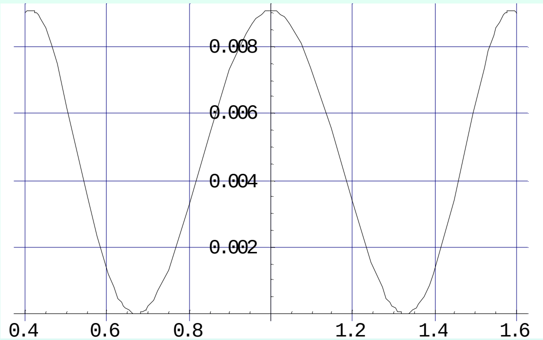


9 degrees of freedom

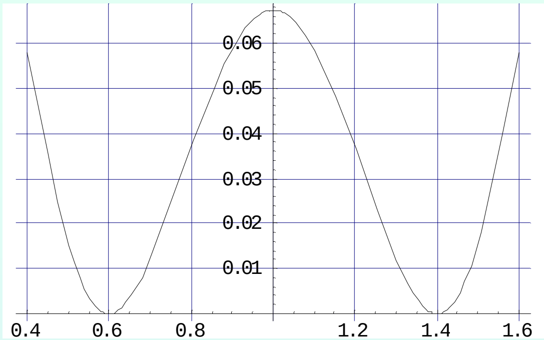


4 equivalent fluxoid positions

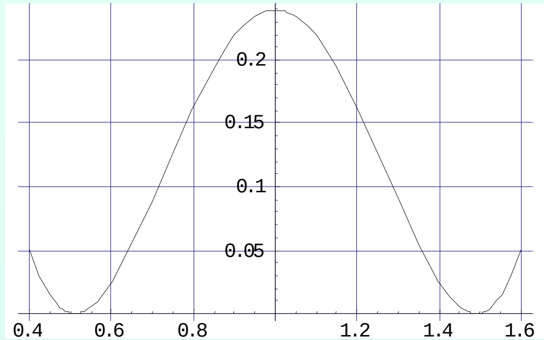
q=0. 6



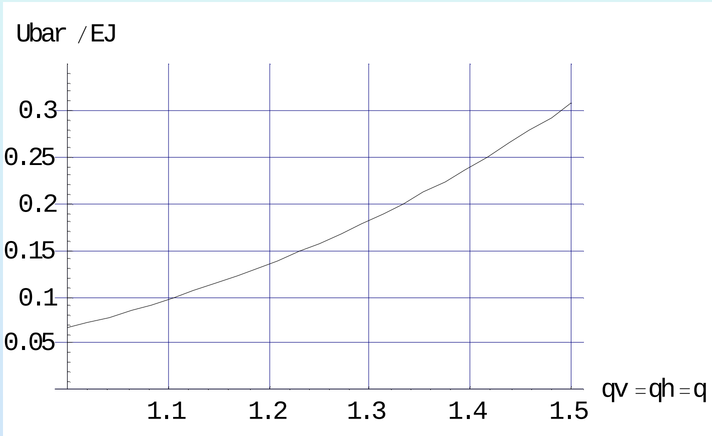
q=1.0



q=1.4



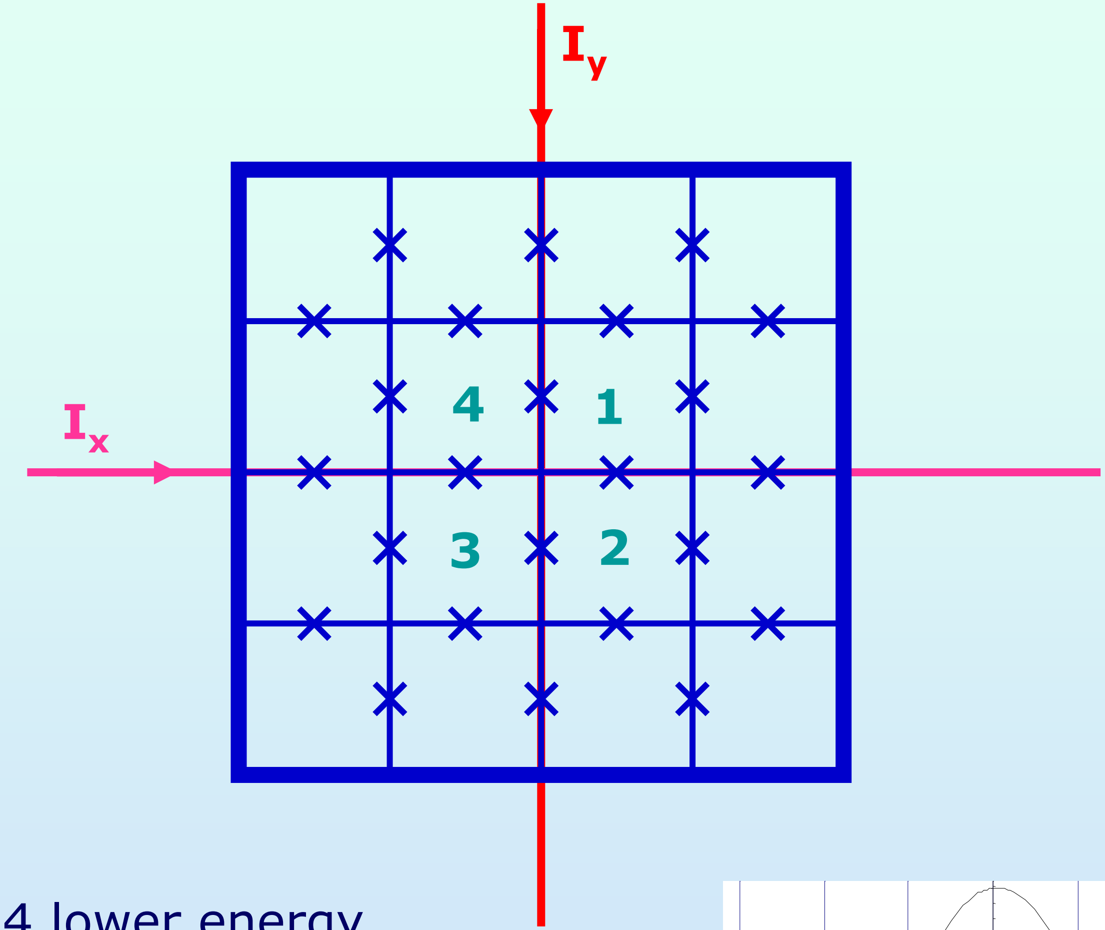
barrier as a function of central junction strength



barriers

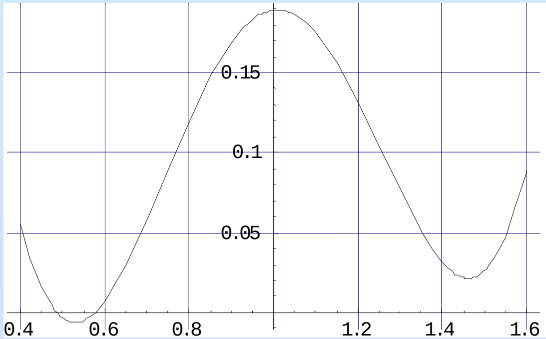
center junctions q times larger

than other junctions

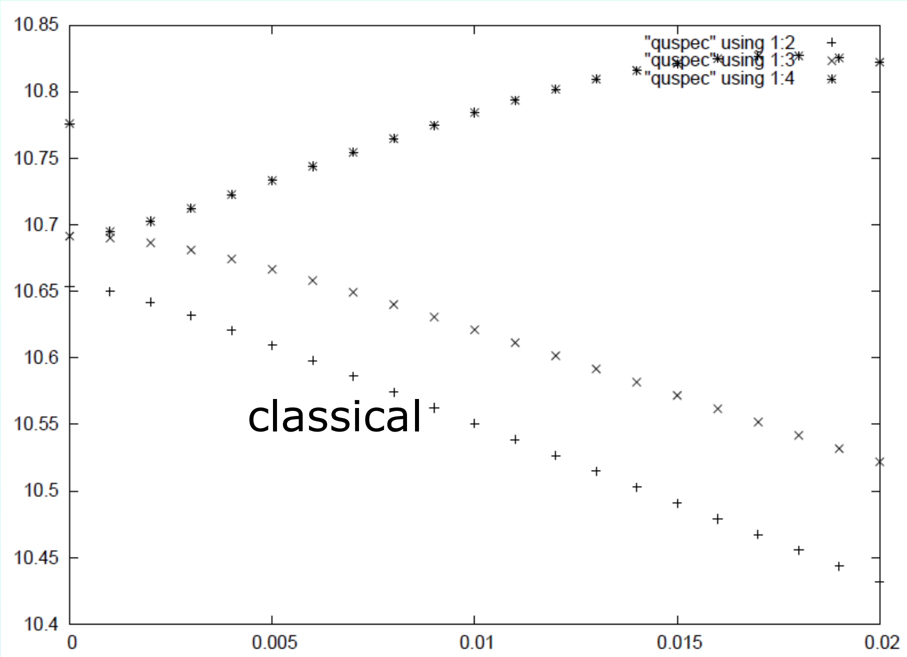


$I_x > 0$: 1 and 4 lower energy

$I_y > 0$: 1 and 2 lower energy



energy

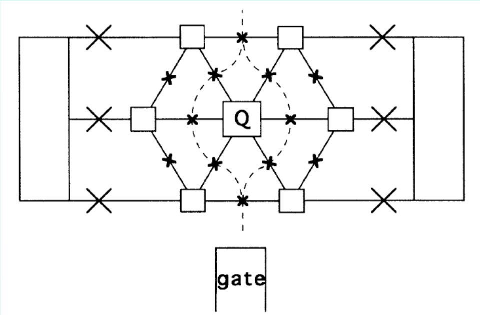


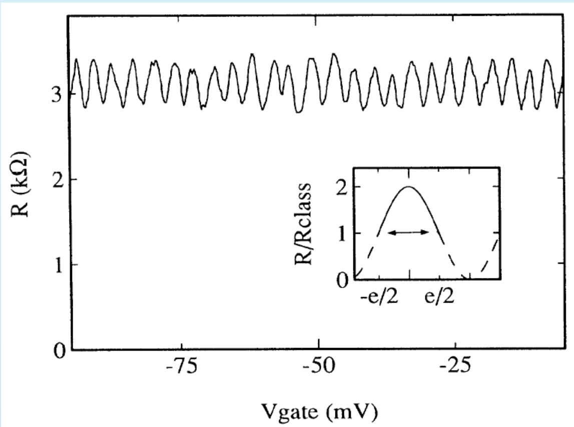
tilt induced by current

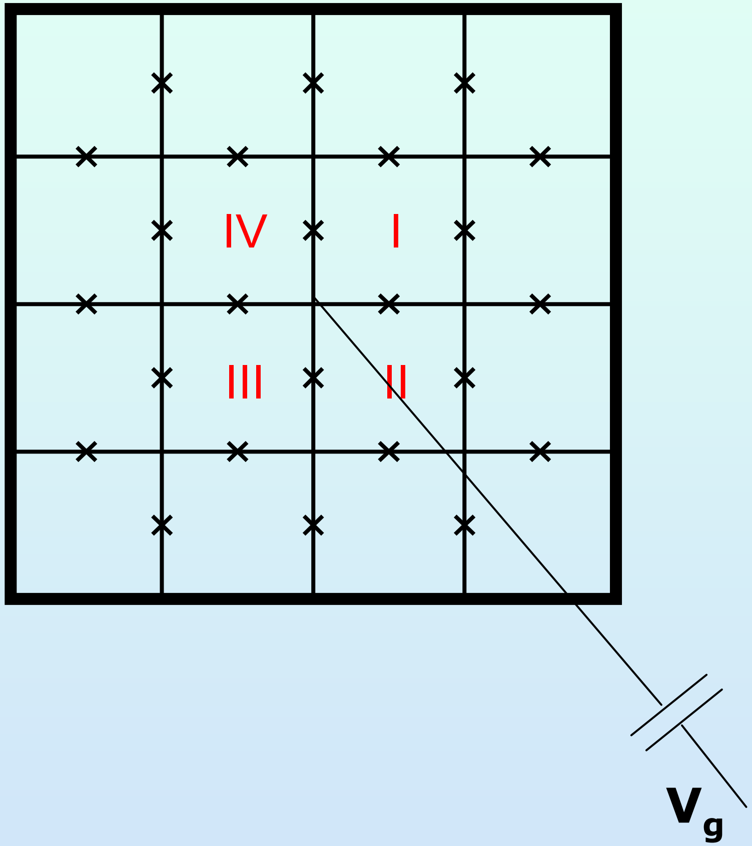
quantum calculation Jos Thijssen, 9 degrees of freedom

Quantum interference of vortices around charge, Aharonov-Casher

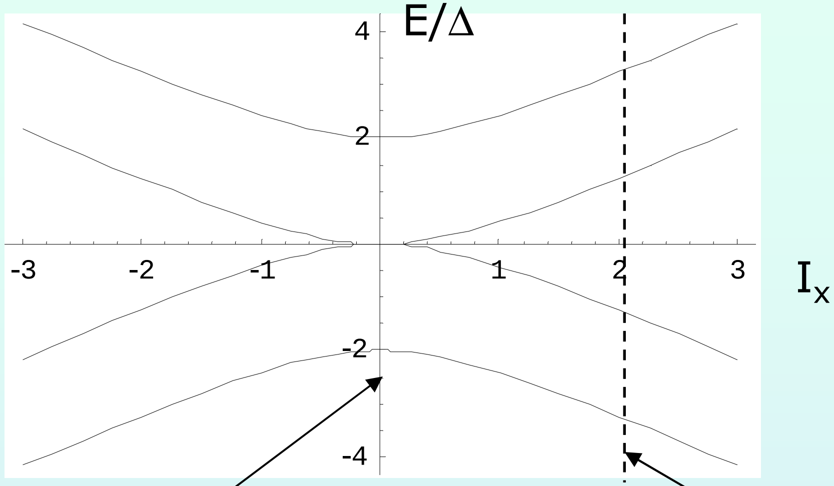
W.J. Elion, J.J. Wachters, L.L. Sohn, J.E. Mooij, PRL **71**, 2311 (1993)

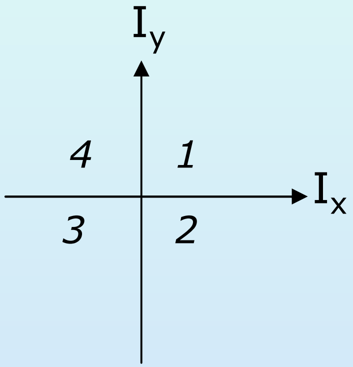






$$\begin{pmatrix} e_1 & \Delta & 0 & \Delta \\ \Delta & e_2 & \Delta & 0 \\ 0 & \Delta & e_3 & \Delta \\ \Delta & 0 & \Delta & e_4 \end{pmatrix}$$



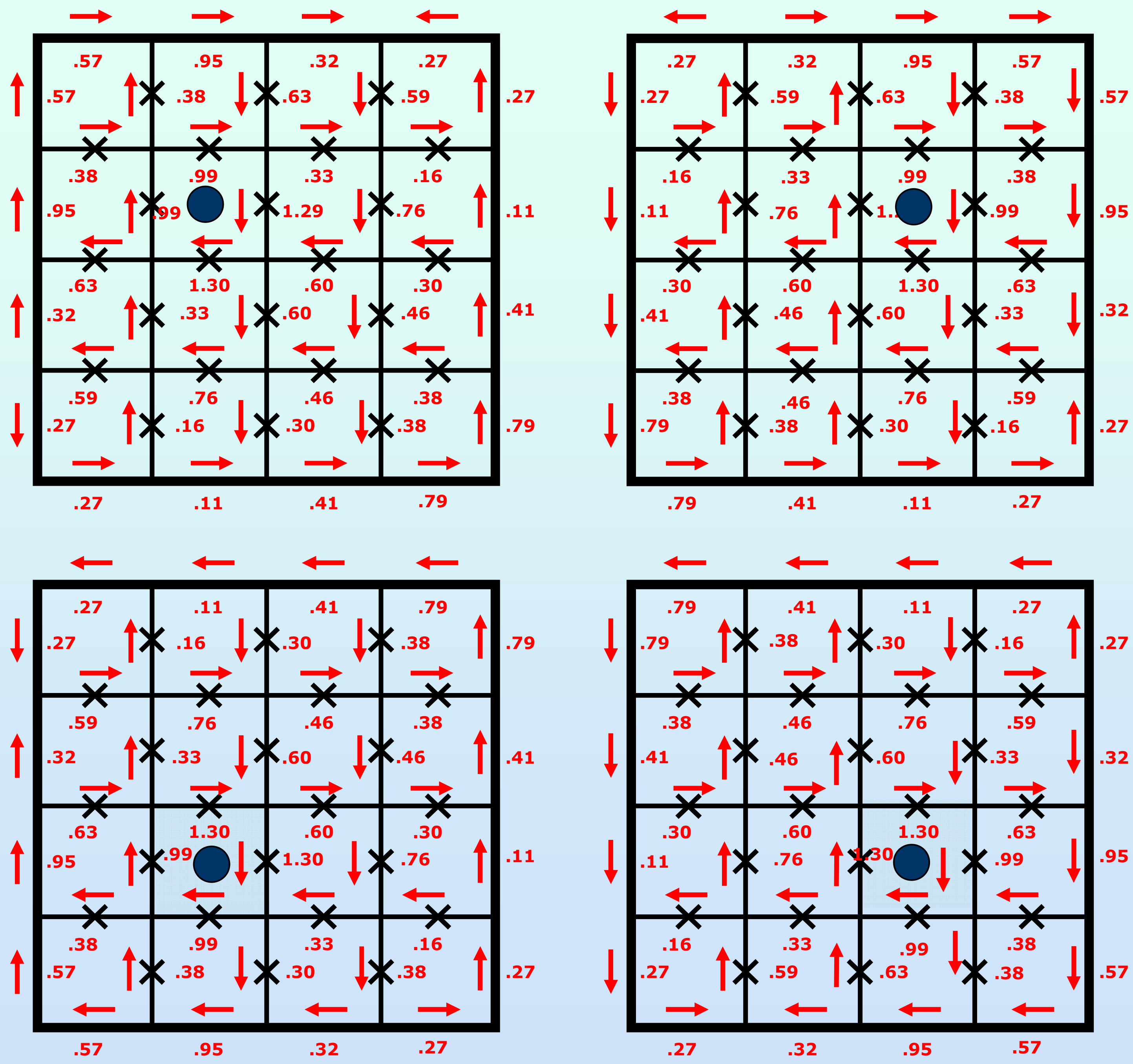


E/Δ	eigenvector
2	(-1,1,-1,1)
0	(-1,0,1,0)
0	(0,-1,0,1)
-2	(1,1,1,1)

$I_x=0,\ I_y=0$

E/Δ	eigenvector
3.2	(-1,4.2,-4.2,1)
1.2	(1,-4.2,-4.2,1)
-1.2	(-4.2,-1,1,4.2)
-3.2	(4.2,1,1,4.2)

$I_x=2,\ I_y=0$



.57 .95 .32 .27
.57 .38 .63 .59 .27
.38 .99 .33 .16
.95 .99 1.29 .76 .11
.63 1.30 .60 .30
.32 .33 .60 .46 .41
.59 .76 .46 .38
.27 .16 .30 .38 .79
.27 .11 .41 .79
.27 .32 .95 .57
.27 .59 .63 .38 .57
.16 .33 .99 .38
.11 .76 .99 .95
.30 .60 1.30 .63
.41 .46 .60 .33 .32
.38 .46 .76 .59
.79 .38 .30 .16 .27
.79 .41 .11 .27
.27 .11 .41 .79
.27 .16 .30 .38 .79
.59 .76 .46 .38
.32 .33 .60 .46 .41
.63 1.30 .60 .30
.95 .99 1.30 .76 .11
.38 .99 .33 .16
.57 .38 .30 .38 .27
.57 .95 .32 .27
.79 .41 .11 .27
.79 .38 .30 .16 .27
.38 .46 .76 .59
.41 .46 .60 .33 .32
.30 .60 1.30 .63
.11 .76 1.30 .99 .95
.16 .33 .99 .38
.27 .59 .63 .38 .57
.27 .32 .95 .57

classical

nearest neighbor interaction

a

4	1
3	2

b

4	1
3	2

$$
\begin{array}{c} \\ b=1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{cccc} a=1 & 2 & 3 & 4 \\ \end{array}
$$

$$
\begin{pmatrix} -1.0 & -4.9 & +3.3 & +5.6 \\ -4.9 & -1.0 & +5.6 & +3.3 \\ +2.1 & +8.9 & -1.0 & -4.9 \\ +8.9 & +2.1 & -4.9 & -1.0 \end{pmatrix}
$$

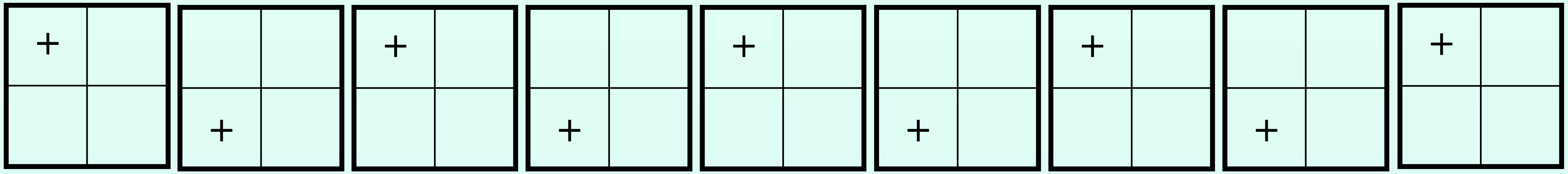
	+

+8.9	−1.0
+2.1	−4.9

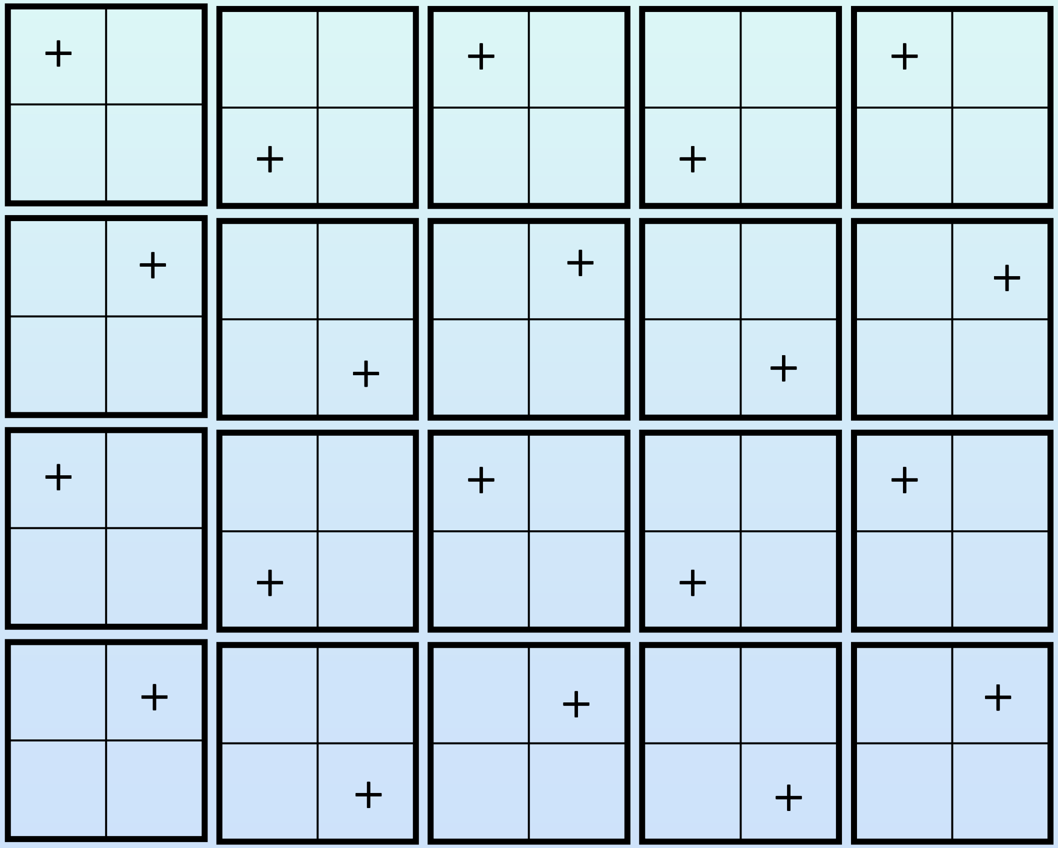
+	

−1.0	+5.6
−4.9	+3.2

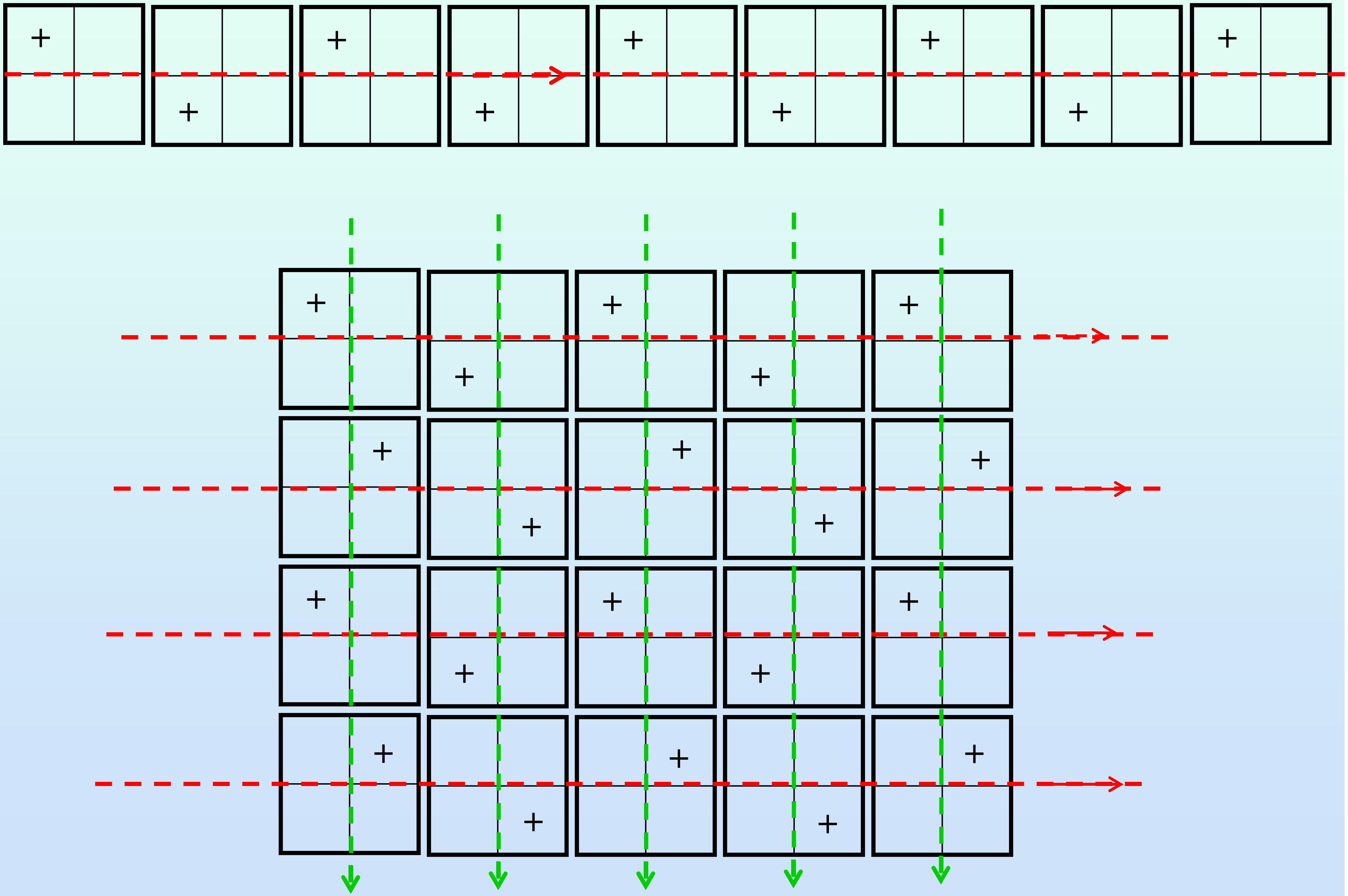
1D chain

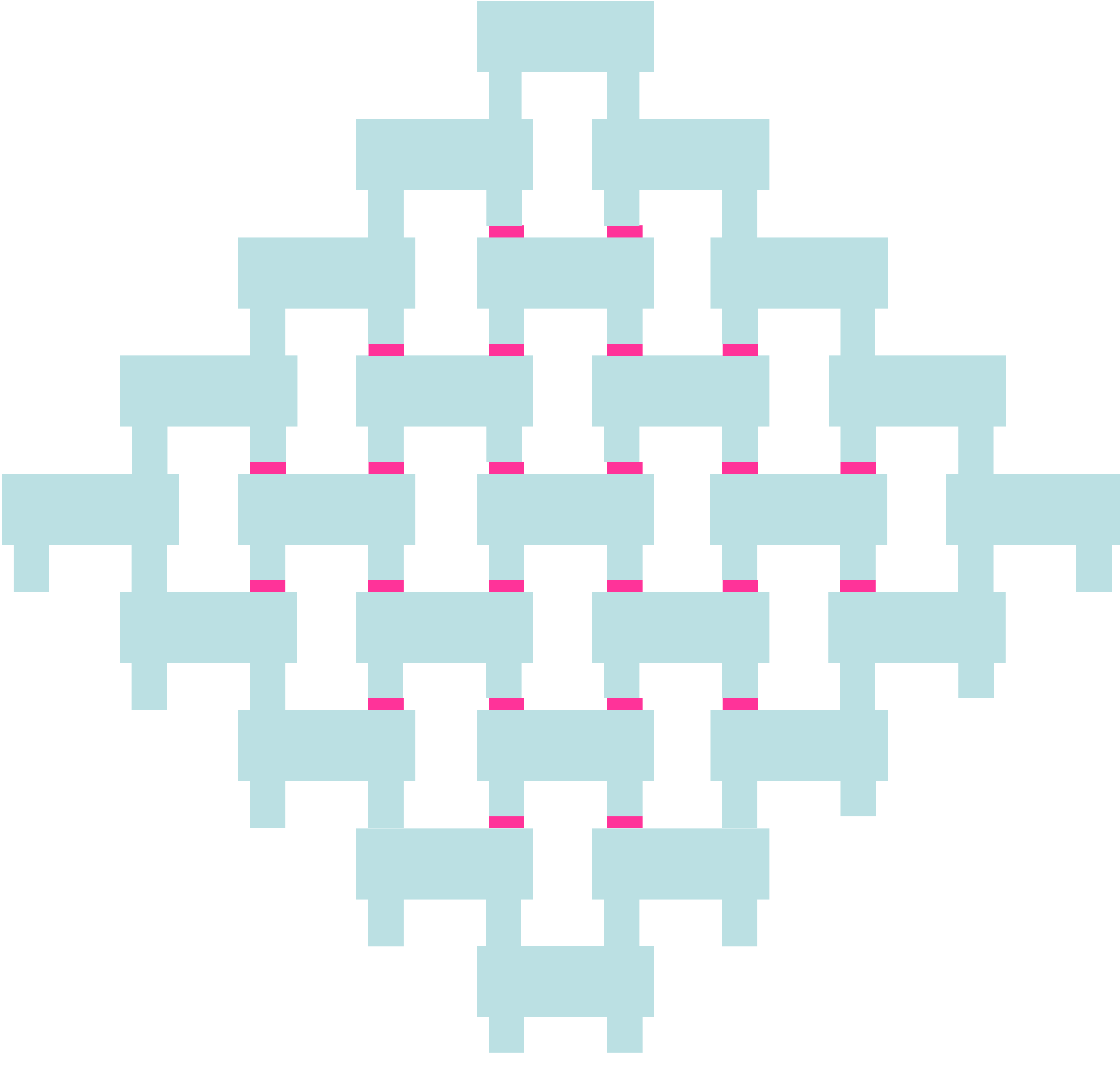


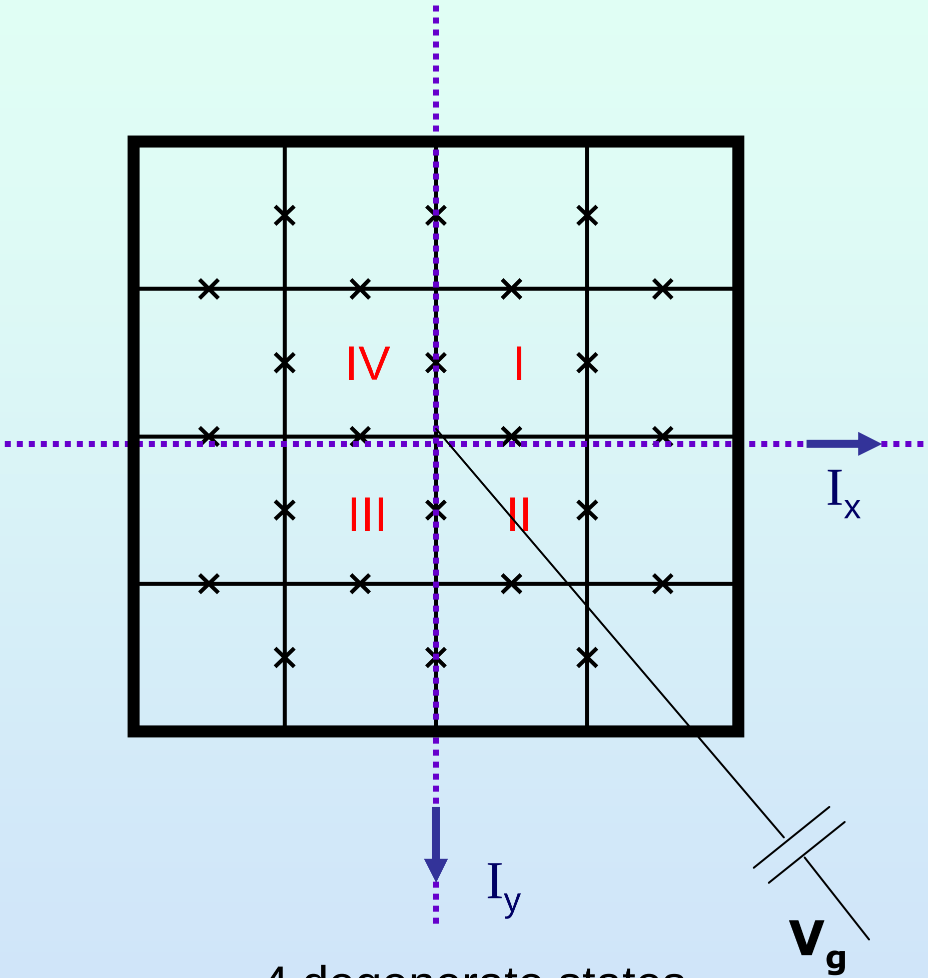
2D array



1D chain







4 degenerate states

Berry phase

H(p1,p2)

make closed loop through parameter space

wave function picks up geometric phase and dynamical phase

retrace loop backwards to eliminate dynamical phase, but do something smart to double geometric phase

e-change on island charge?