Entangled States and Quantum Algorithms in Circuit QED

Applied Physics + Physics
Yale University

PI's:
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Michel Devoret
Steven Girvin

IARPA

Expt.
Leo DiCarlo
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U. Sherbrooke ← Alexandre Blais
Munich ← Florian Marquardt
Eli Luberoff
Göteborg ↔ Lars Tornberg
Terri Yu

Recent Reviews

‘Wiring up quantum systems’
R. J. Schoelkopf, S. M. Girvin
Nature **451**, 664 (2008)

‘Superconducting quantum bits’
John Clarke, Frank K. Wilhelm
Nature **453**, 1031 (2008)

Quantum Information Processing **8** (2009)
ed. by A. Korotkov

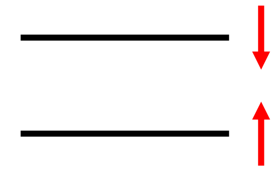
Overview

- Noise and how to ignore it
- Circuit QED: using cavity bus to couple qubits
- Two qubit gates and generation of Bell's states
- "Metrology of entanglement" – using joint cQED msmt.
- Demonstration of Grover and Deutsch-Josza algorithms

  DiCarlo et al., *Nature* **460**, 240 (2009)

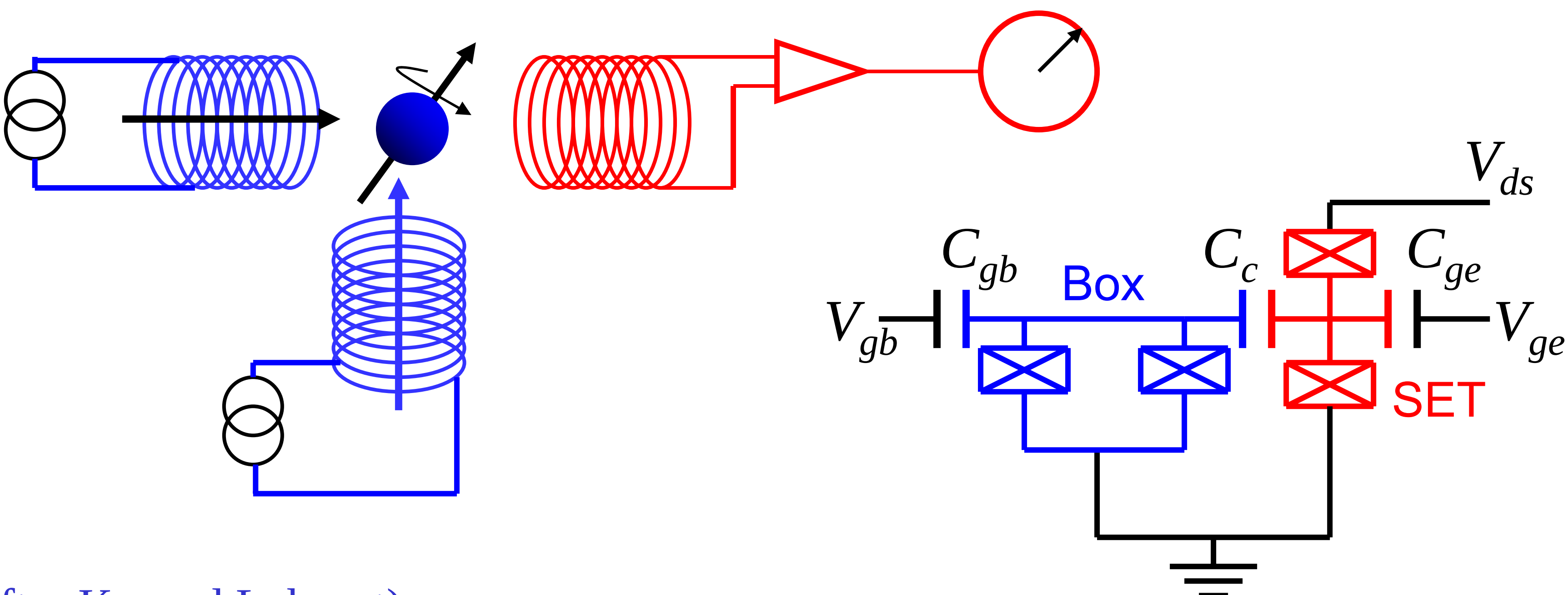
Quantum Computation and NMR of a Single ‘Spin’

Electrical circuit with two quantized energy levels is like a spin -1/2.



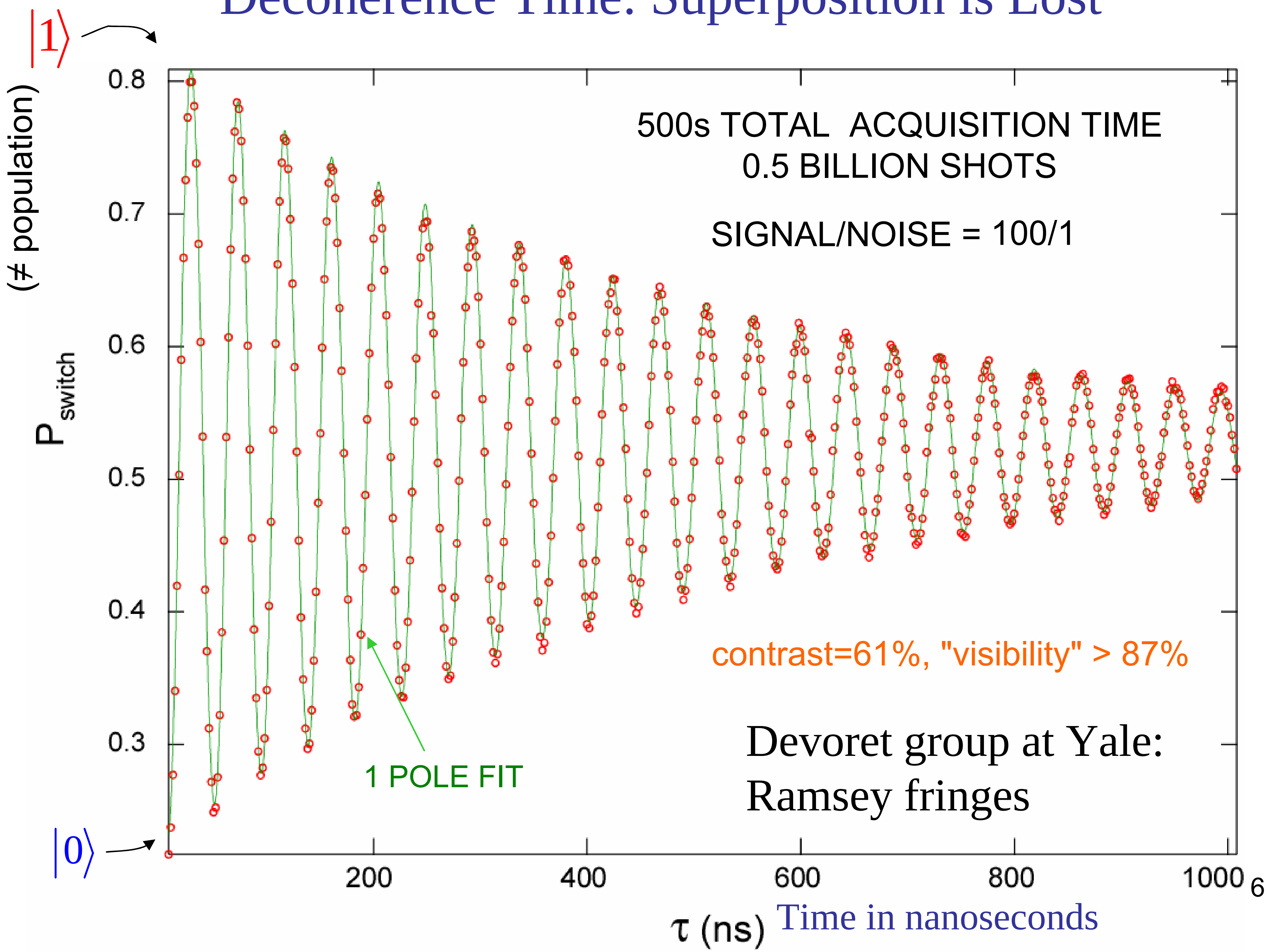
Single Spin ½

Quantum Measurement



(After Konrad Lehnert)

Decoherence Time: Superposition is Lost

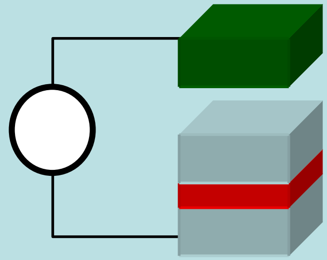


Different types of SC qubits

► Nonlinearity from Josephson junctions

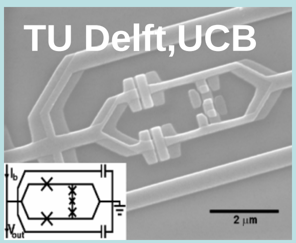
$E_J = E_C$

charge qubit (CPB)

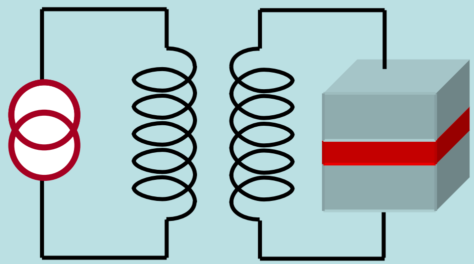


Nakamura et al., NEC Labs
Vion et al., Saclay
Devoret et al., Schoelkopf et al., Yale,
Delsing et al., Chalmers

$E_J = 40\text{-}100E_C$



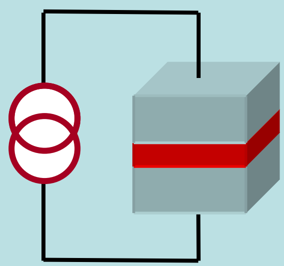
flux qubit



Lukens et al., SUNY
Mooij et al., Delft
Orlando et al., MIT
Clarke, UC Berkeley

$E_J = 10{,}000E_C$

phase qubit



Martinis et al., UCSB
Simmonds et al., NIST
Wellstood et al., U Maryland
Koch et al., IBM

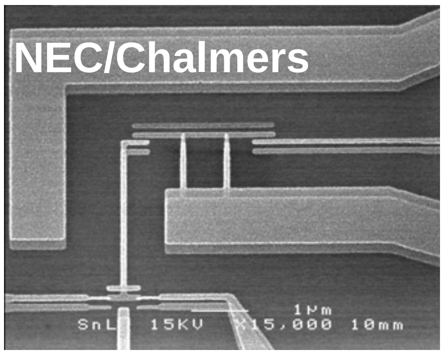
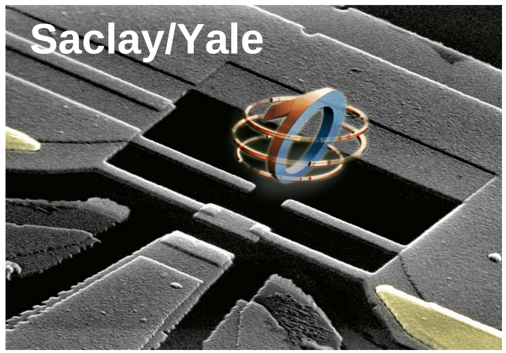
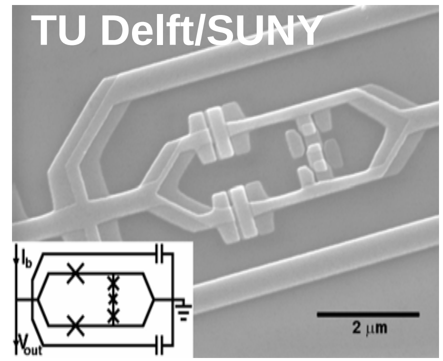
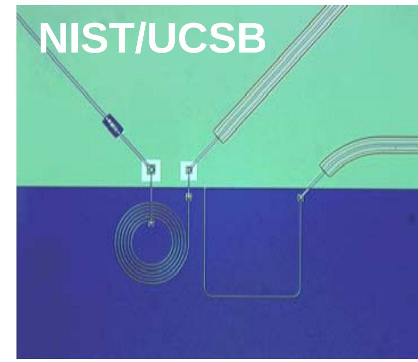
…and more…

Reviews:

Yu. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001)
M. H. Devoret, A. Wallraff and J. M. Martinis, *cond-mat/0411172* (2004)
J. Q. You and F. Nori, Phys. Today, Nov. 2005, 42

State of the Art in Superconducting Qubits

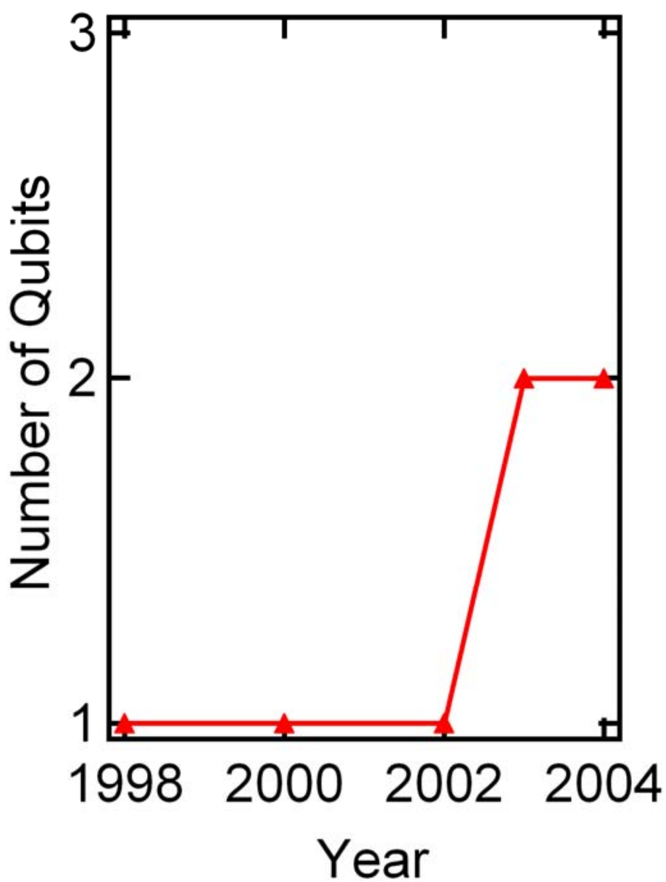
• Nonlinearity from Josephson junctions (Al/AlO_x/Al)

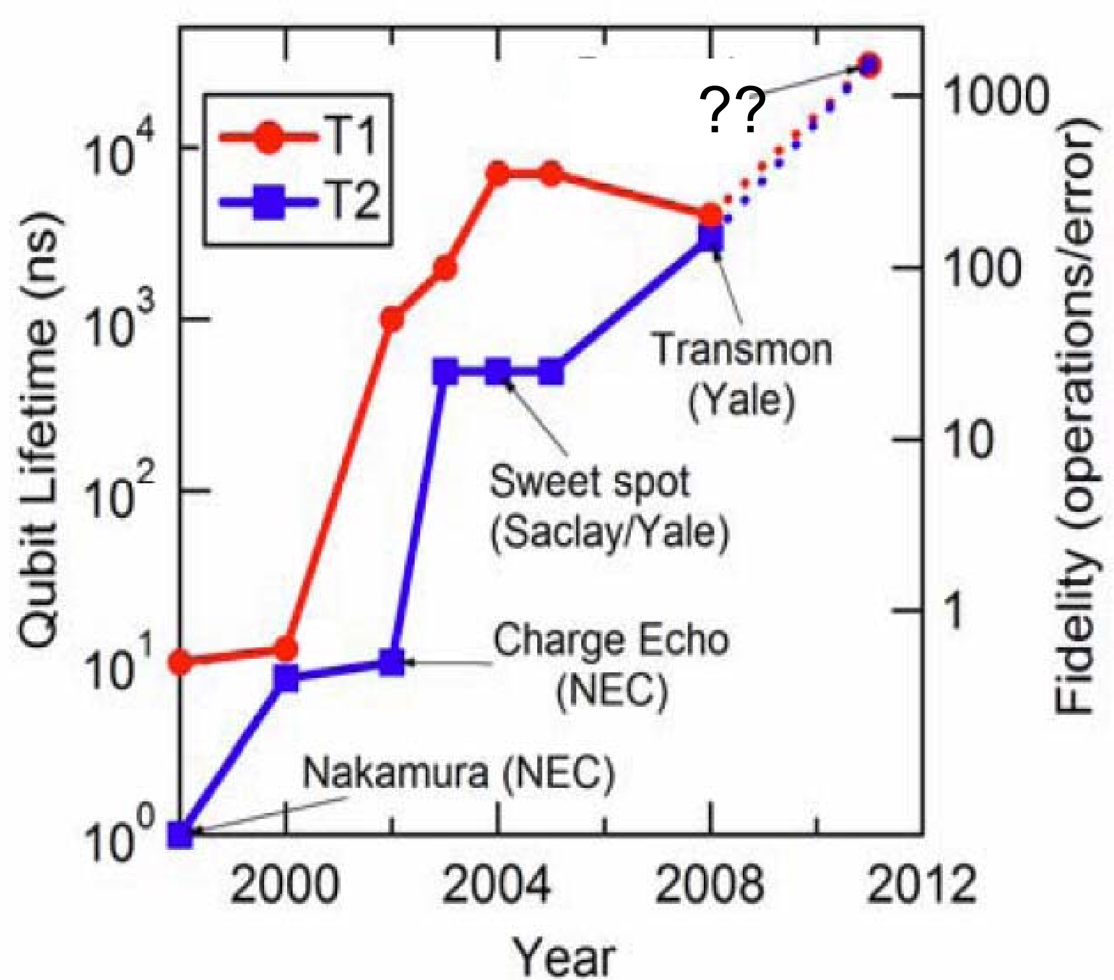
Charge	Charge/Phase	Flux	Phase
			

Junction size ⟶ $E_J = E_C$ ⟵ # of Cooper pairs

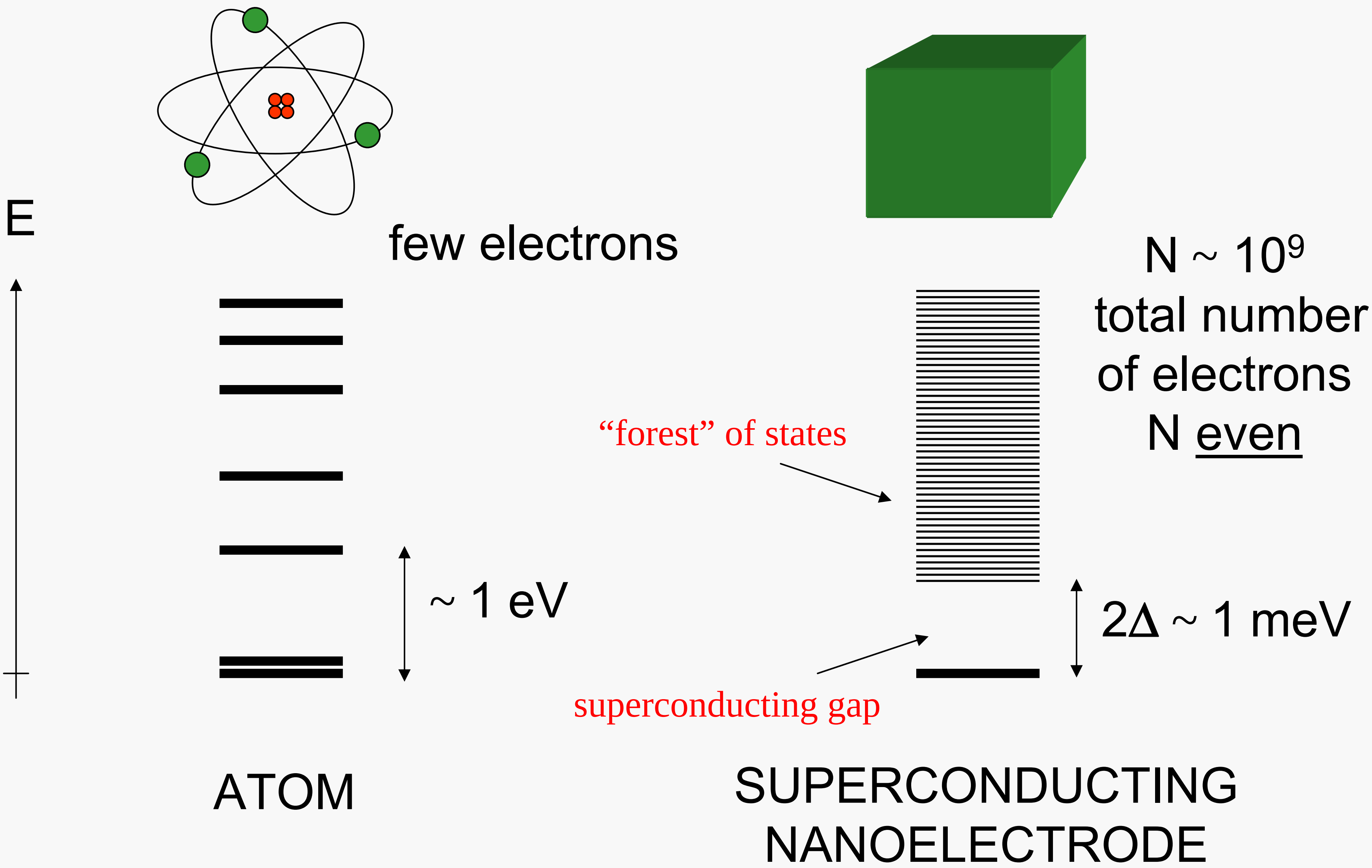
- 1st qubit demonstrated in 1998 (NEC Labs, Japan)
- "Long" coherence shown 2002 (Saclay/Yale)
- Several experiments with two degrees of freedom
- C-NOT gate (2003 NEC, 2006 Delft and UCSB)
- CHSH Bell inequality violation (2009, UCSB, Yale [w/meas. Loophole])
- 2 qubit Grover search and Deutsch-Josza algorithms (2009, Yale)

Progress in Superconducting QC…





WHY SUPERCONDUCTIVITY?

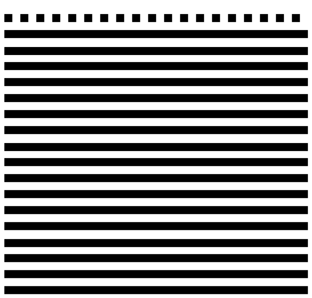


Collective Quantization easiest (?) to understand for charge qubits

An isolated superconductor has definite charge.

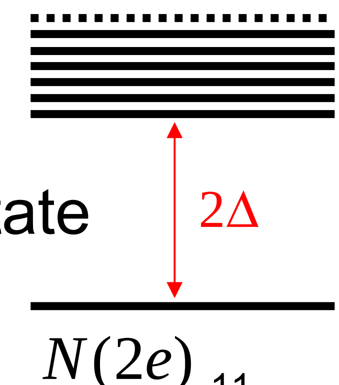
For an even number of electrons there are <u>no</u> low energy degrees of freedom!

<u>Unique</u> non-degenerate quantum ground state.



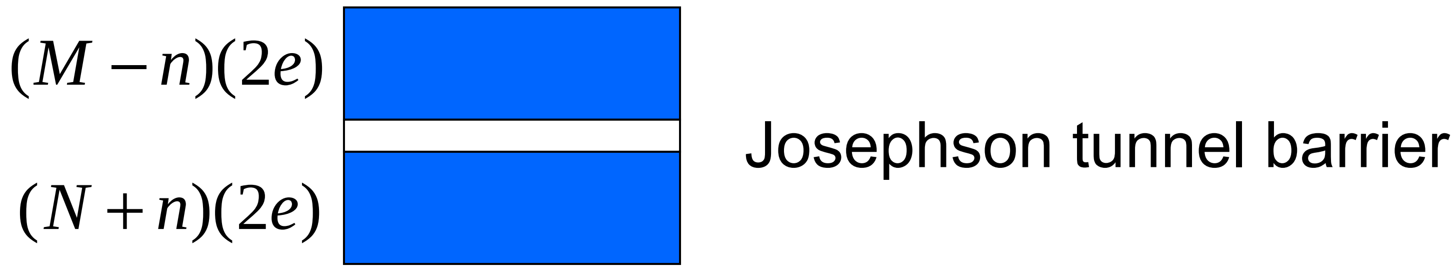
Normal State

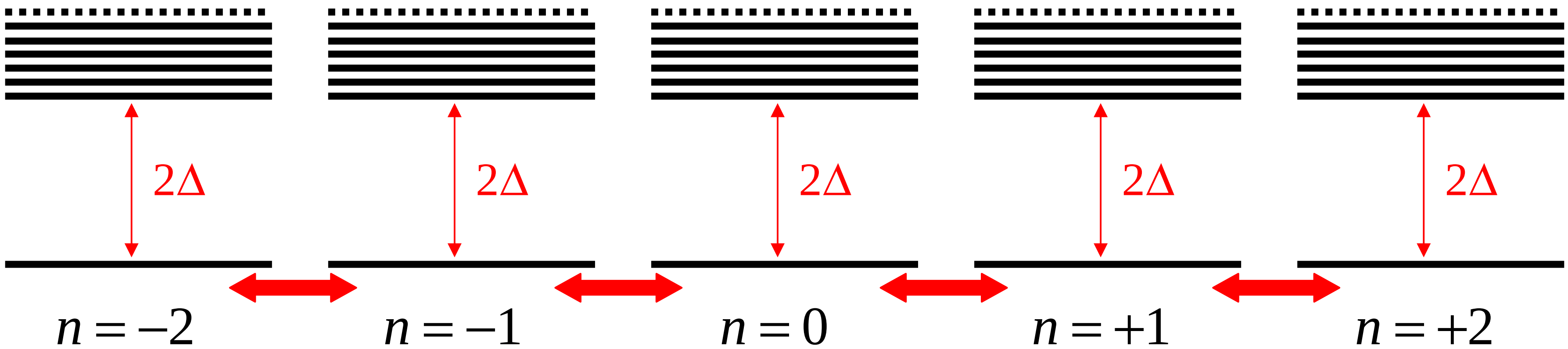
Superconducting State



To have any degrees of freedom, we need a junction.

charge qubits





Tunnel coupling: $-E_J \sum_n |n\rangle\langle n+1| + |n+1\rangle\langle n|$

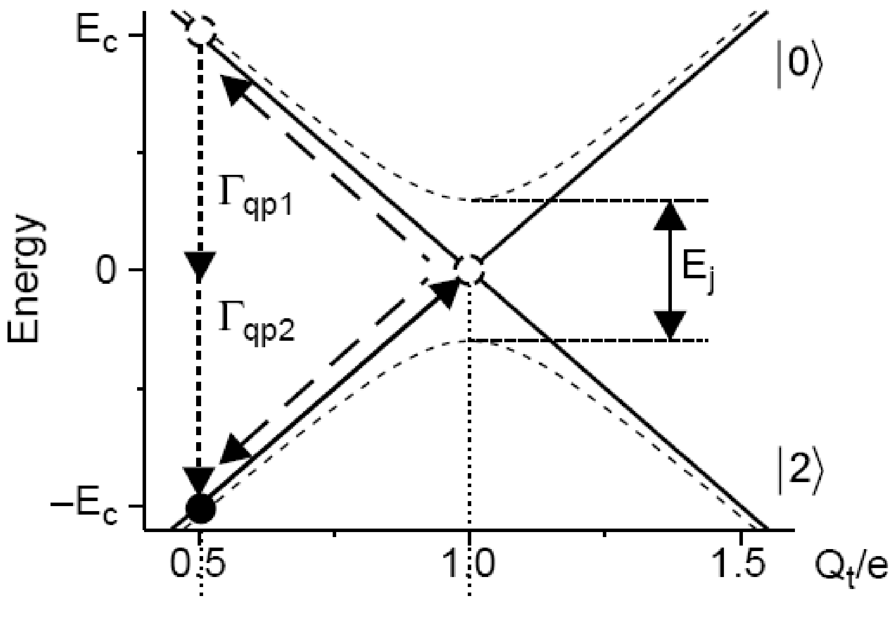
Charging energy: $+4E_c \sum_n n^2 |n\rangle\langle n|$

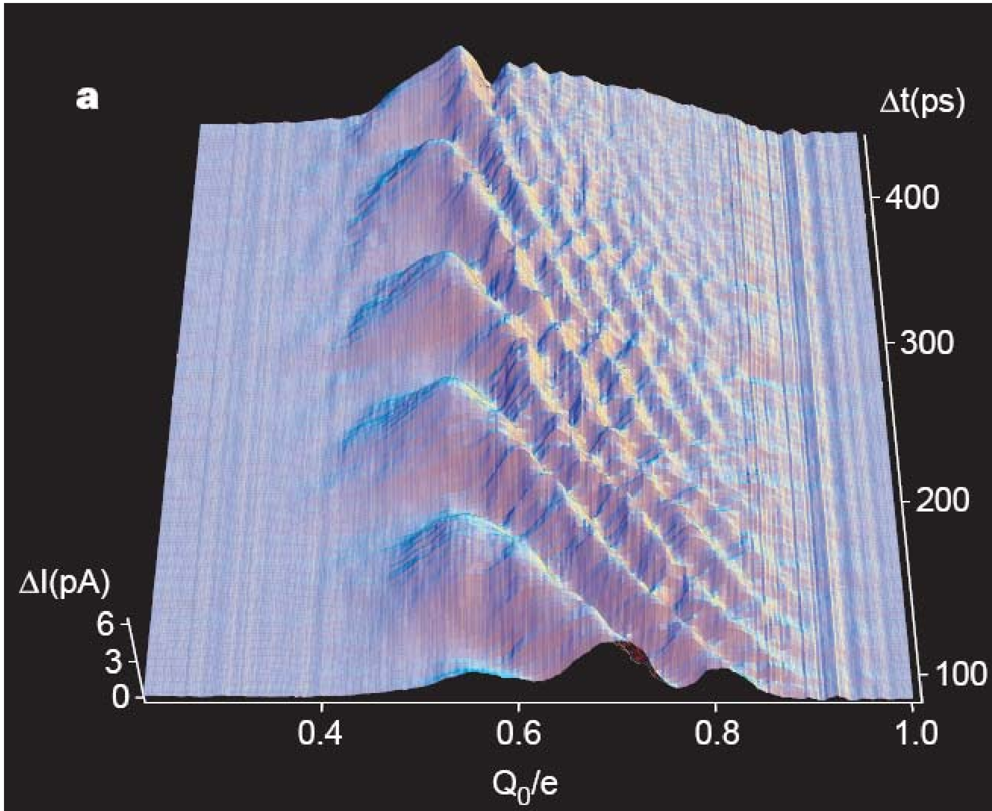
Coherent control of macroscopic quantum states in a single-Cooper-pair box

Y. Nakamura*, Yu. A. Pashkin† & J. S. Tsai*

* NEC Fundamental Research Laboratories, Tsukuba, Ibaraki 305-8051, Japan
† CREST, Japan Science and Technology Corporation (JST), Kawaguchi, Saitama 332-0012, Japan

First Rabi oscillations: 1999





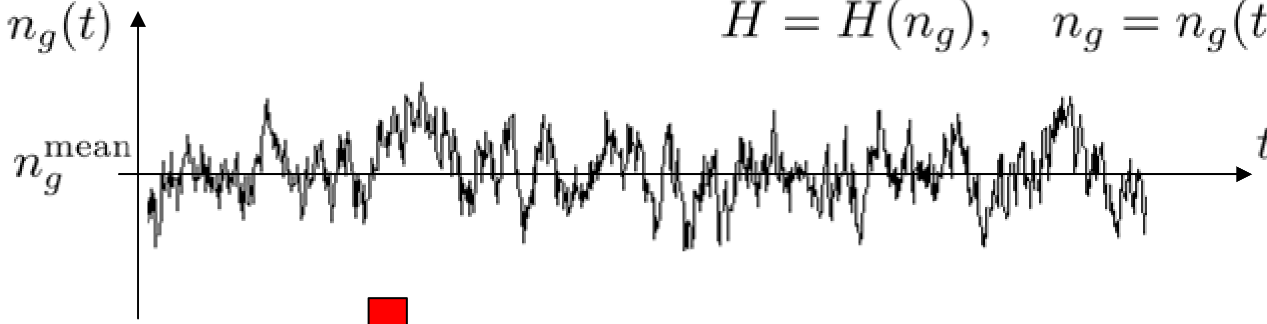
First SC charge qubit works!
But…
coherence time extremely short.

Generic asymmetry means qubit has different static dipole moments in ground and excited states.

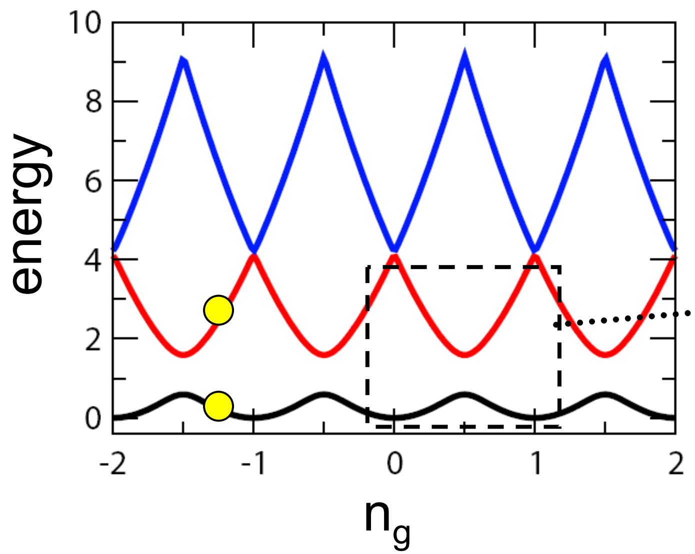
Environment can measure the qubit state via stray electric fields. What to do?

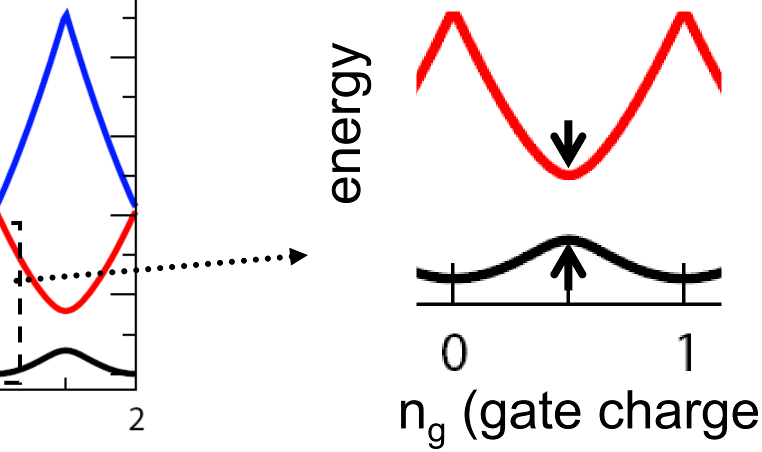
Outsmarting noise: CPB sweet spot

$$\hat{H} = \hat{H}(n_g), \quad n_g = n_g(t)$$ ◀ charge fluctuations



$$\omega_a = \omega_a(t)$$





sweet spot

only sensitive to 2nd order fluctuations in gate charge!

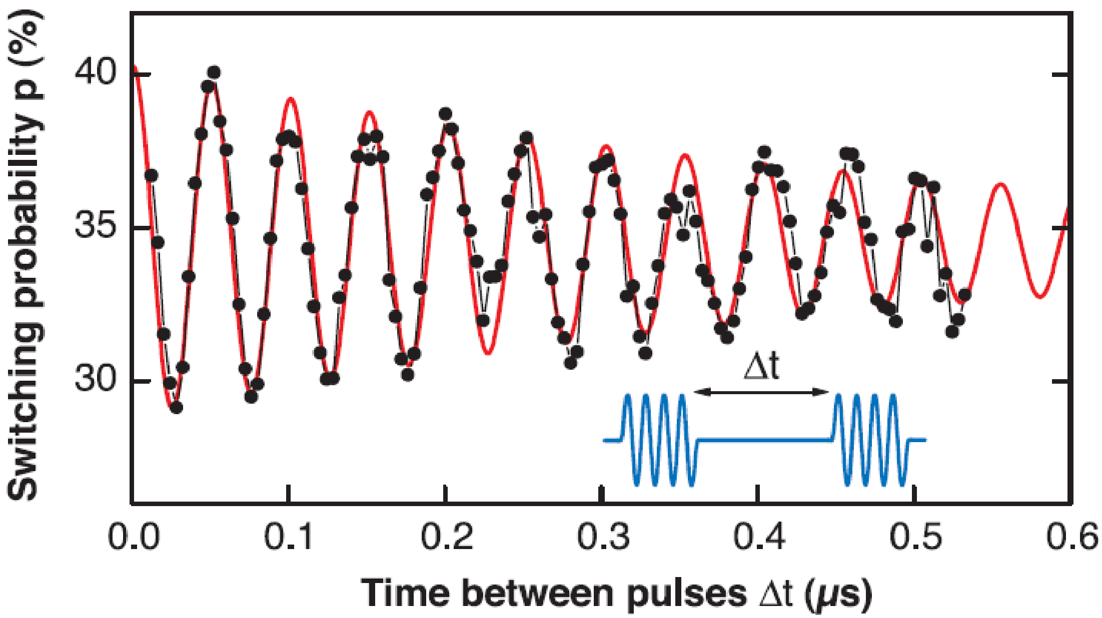
Vion et al., Science **296**, 886 (2002)

First Ramsey Fringe Experiment Proves True Coherence of Superpositions

Science **296**, 886 (2002)

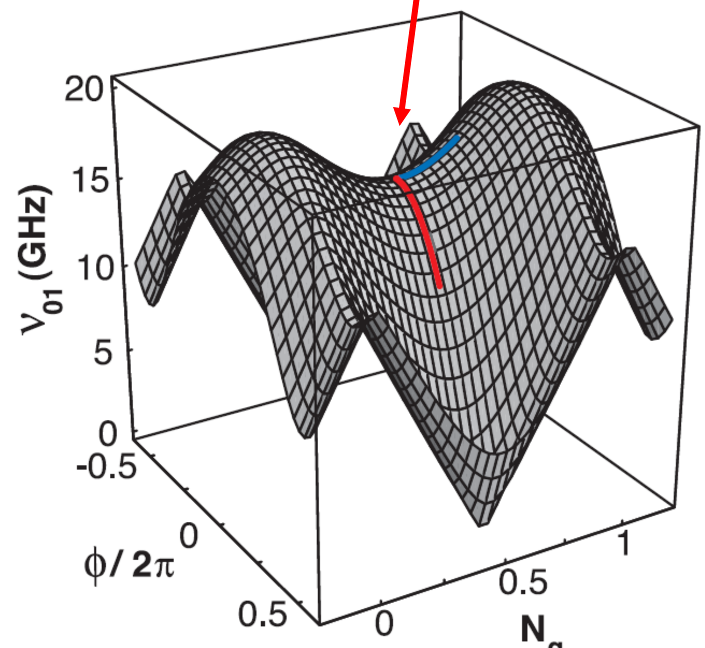
Manipulating the Quantum State of an Electrical Circuit

D. Vion,* A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina,† D. Esteve, M. H. Devoret‡



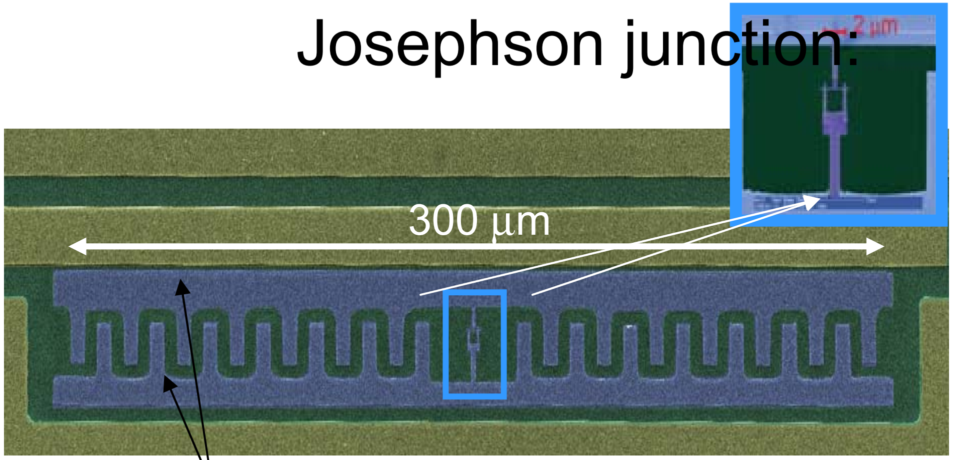
T_2* rises to 300-500 ns

Double sweet spot!



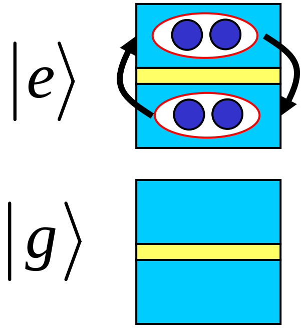
‘Transmon’ Cooper Pair Box: Charge Qubit that Beats Charge Noise

Josephson junction:



Added metal = capacitor & antenna

$E_J >> E_C$



plasma oscillation of 2 or 3 Cooper pairs: exponentially small static dipole

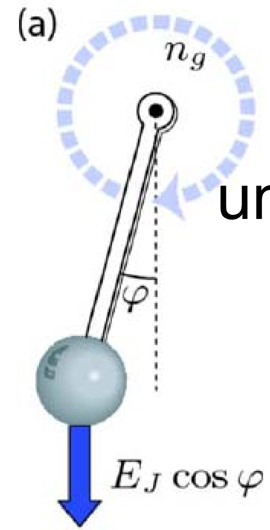
Transmon qubit insensitive to 1/f electric fields

* Theory: J. Koch et al., PRA (2007); Expt: J. Schreier et al., P

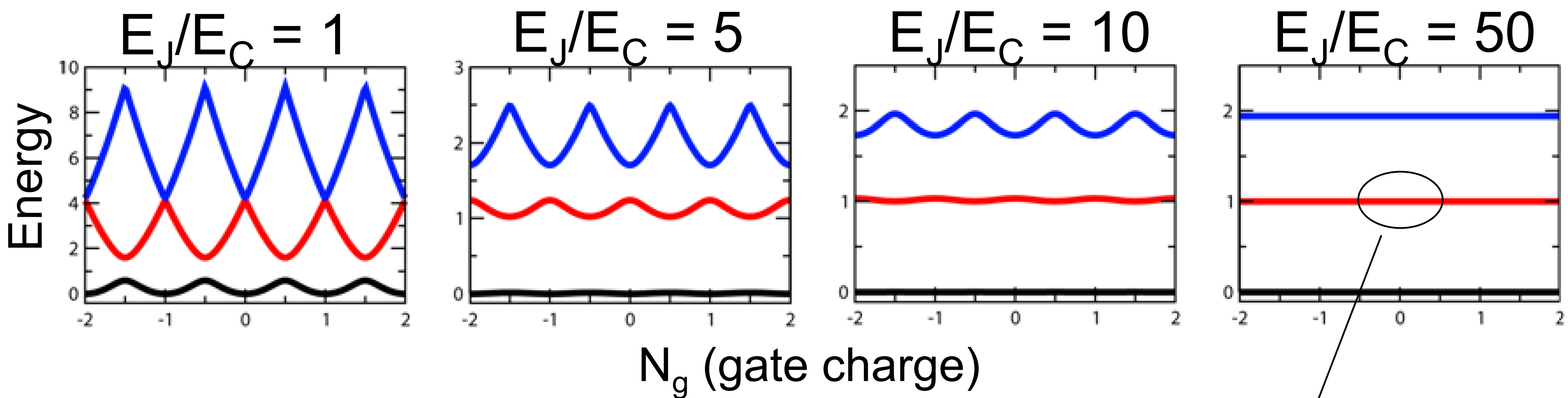
Flux qubit + capacitor: F. You et al., PRB (2006)

Transmon Qubit: Sweet Spot Everywhere!

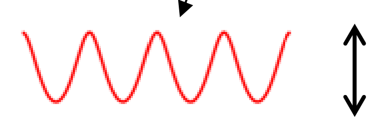
$$\hat{H} = 4E_C\left(\hat{n} - n_g\right)^2 - E_J \cos\hat{\varphi}$$



Nota Bene!
unlike phase qubit
$\Psi(\varphi) = \Psi(\varphi + 2\pi)$



$$\epsilon_m \to (-1)^m E_C \frac{2^{4m+5}}{m!}\sqrt{\frac{2}{\pi}}\left(\frac{E_J}{2E_C}\right)^{\frac{m}{2}+\frac{3}{4}} e^{-\sqrt{8E_J/E_C}}$$

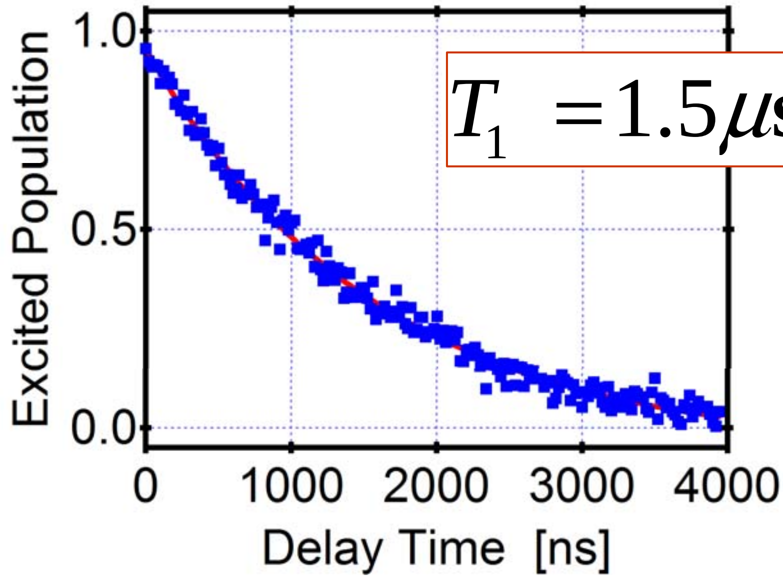


$\epsilon_m \equiv$ charge dispersion

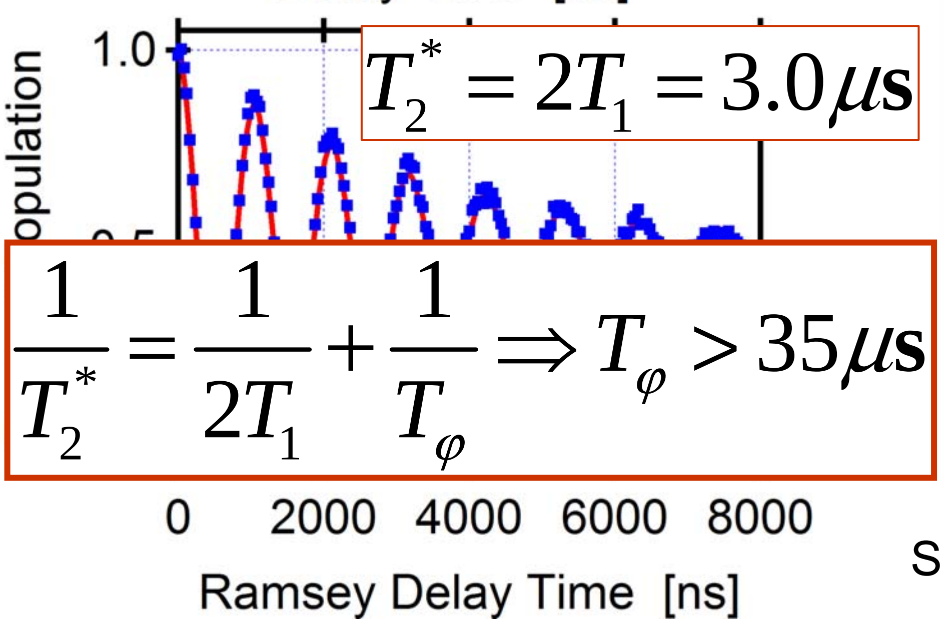
Exponentially small charge dispersion!

Adequate anharmonicity: $\omega_{12} - \omega_{01} \approx -E_c$

Coherence in Transmon Qubit



$$T_1 = 1.5\mu s$$

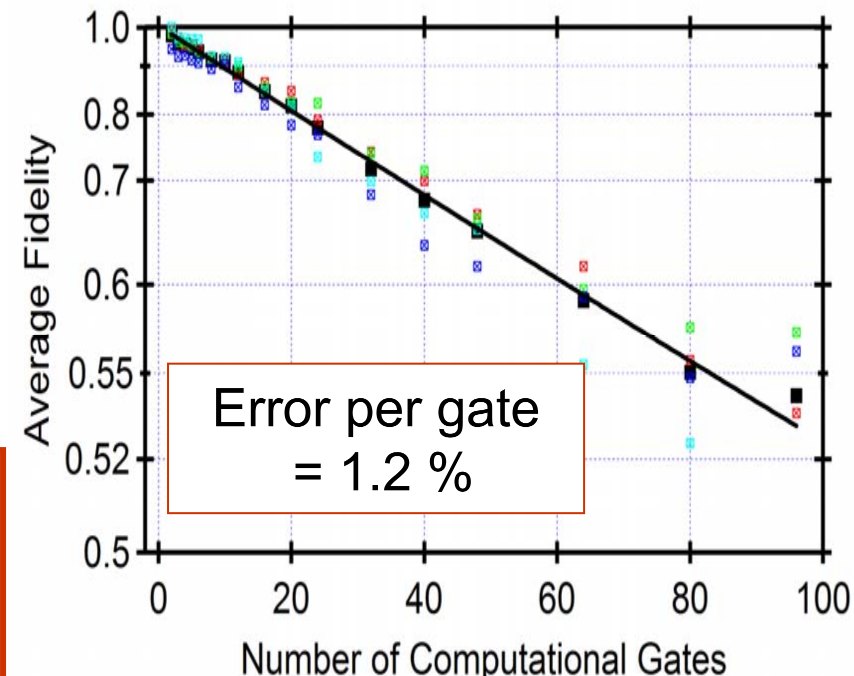


$$T_2^* = 2T_1 = 3.0\mu s$$

$$\frac{1}{T_2^*} = \frac{1}{2T_1} + \frac{1}{T_\varphi} \Rightarrow T_\varphi > 35\mu s$$

Random benchmarking of 1-qubit ops

Chow et al. *PRL* 2009:
Technique from Knill et al. for ions

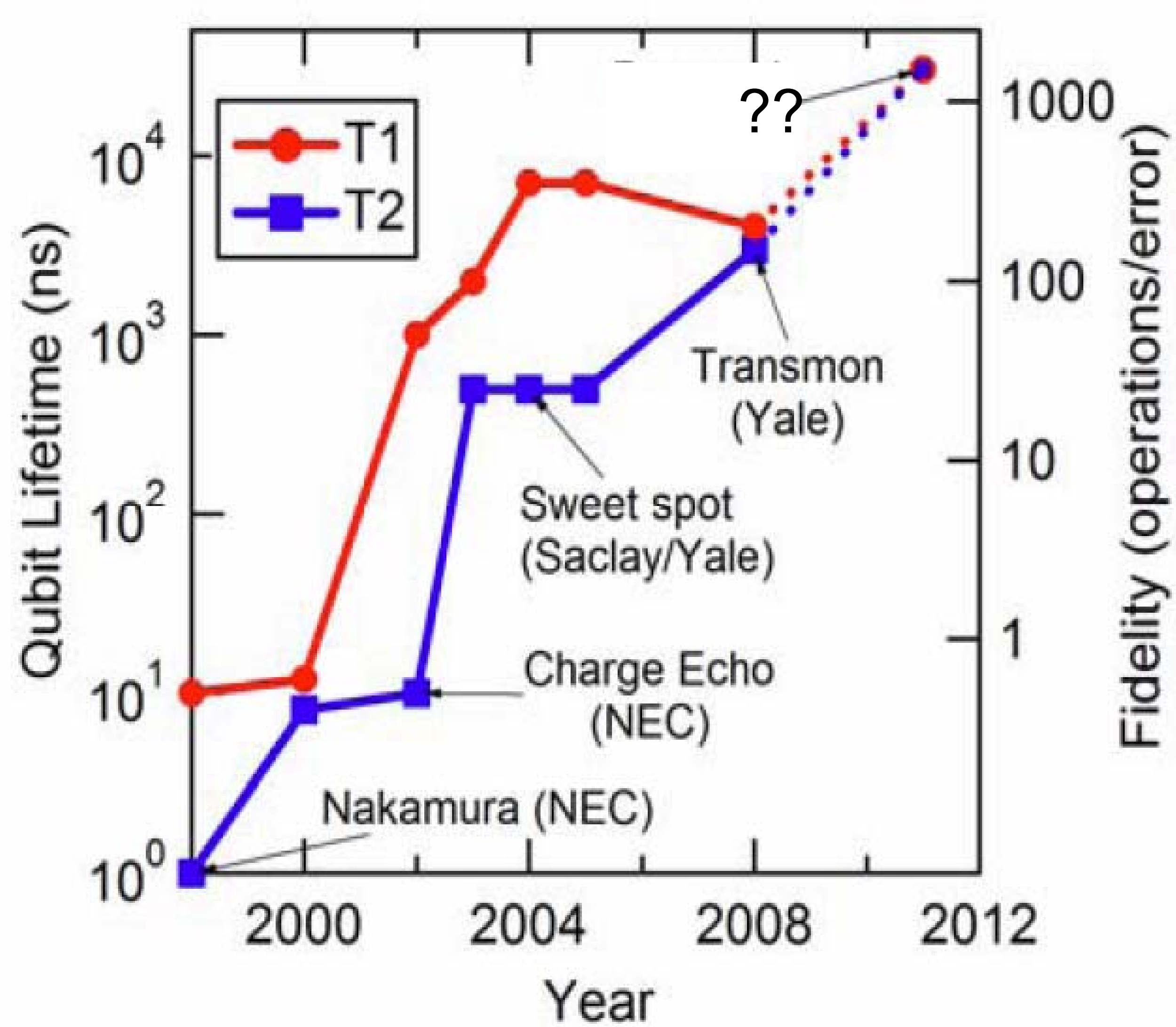


Error per gate
= 1.2 %

Similar error rates in phase qubits (UCSB):
Lucero et al. *PRL* 100, 247001 (2007)

‘Moore’s Law’ for Charge Qubit Coherence Times

(No Echo)

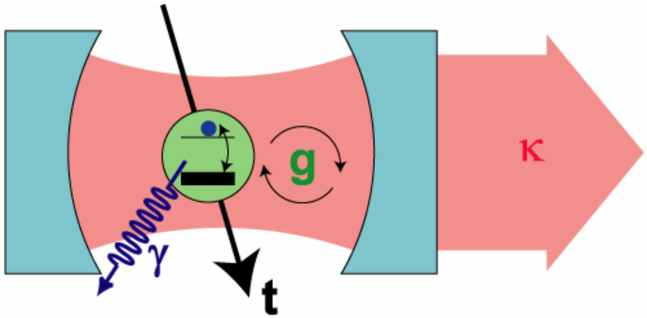


T2 now limited largely by T1

$T_\varphi \geq 30\mu s$

Qubits Coupled with a Quantum Bus

use microwave photons guided on wires!

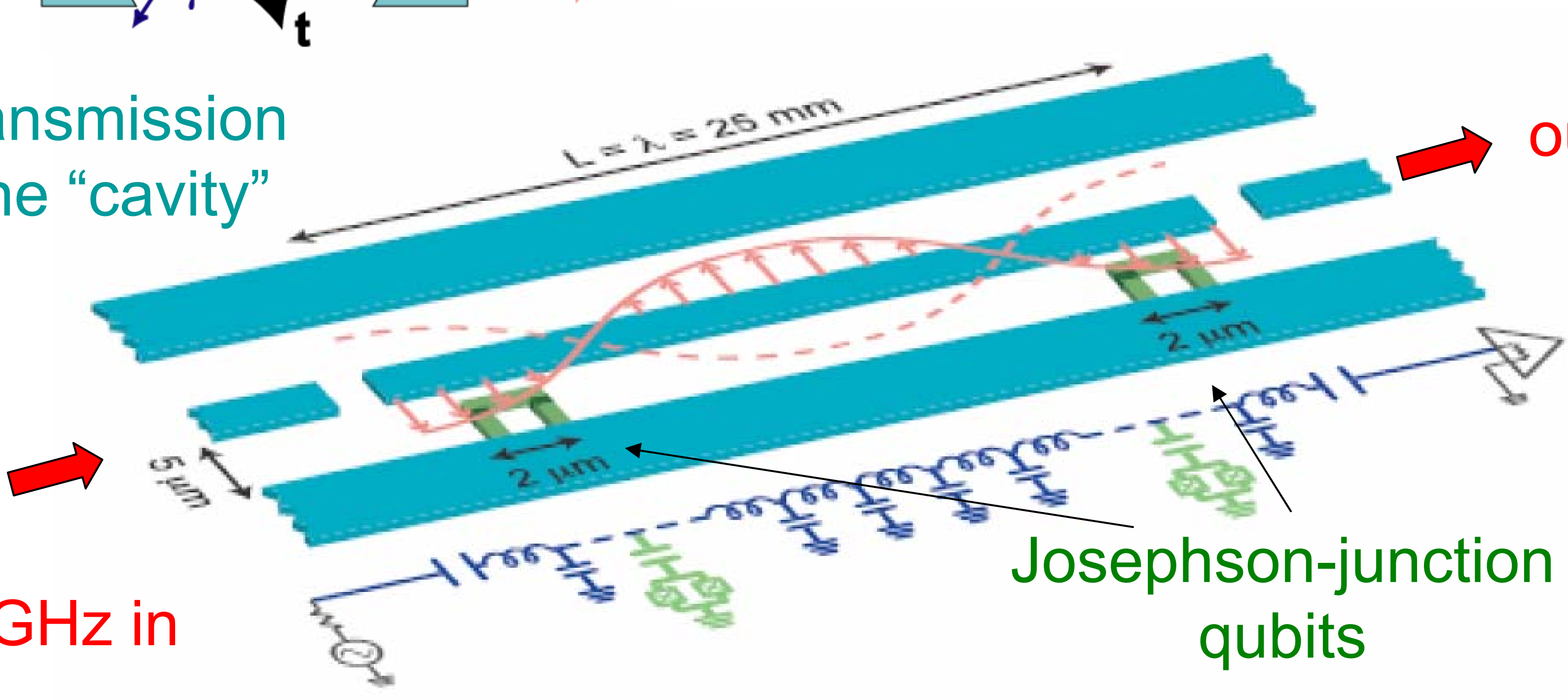


“Circuit QED”

Blais *et al.*, *Phys. Rev. A* (2004)

transmission line “cavity”

out

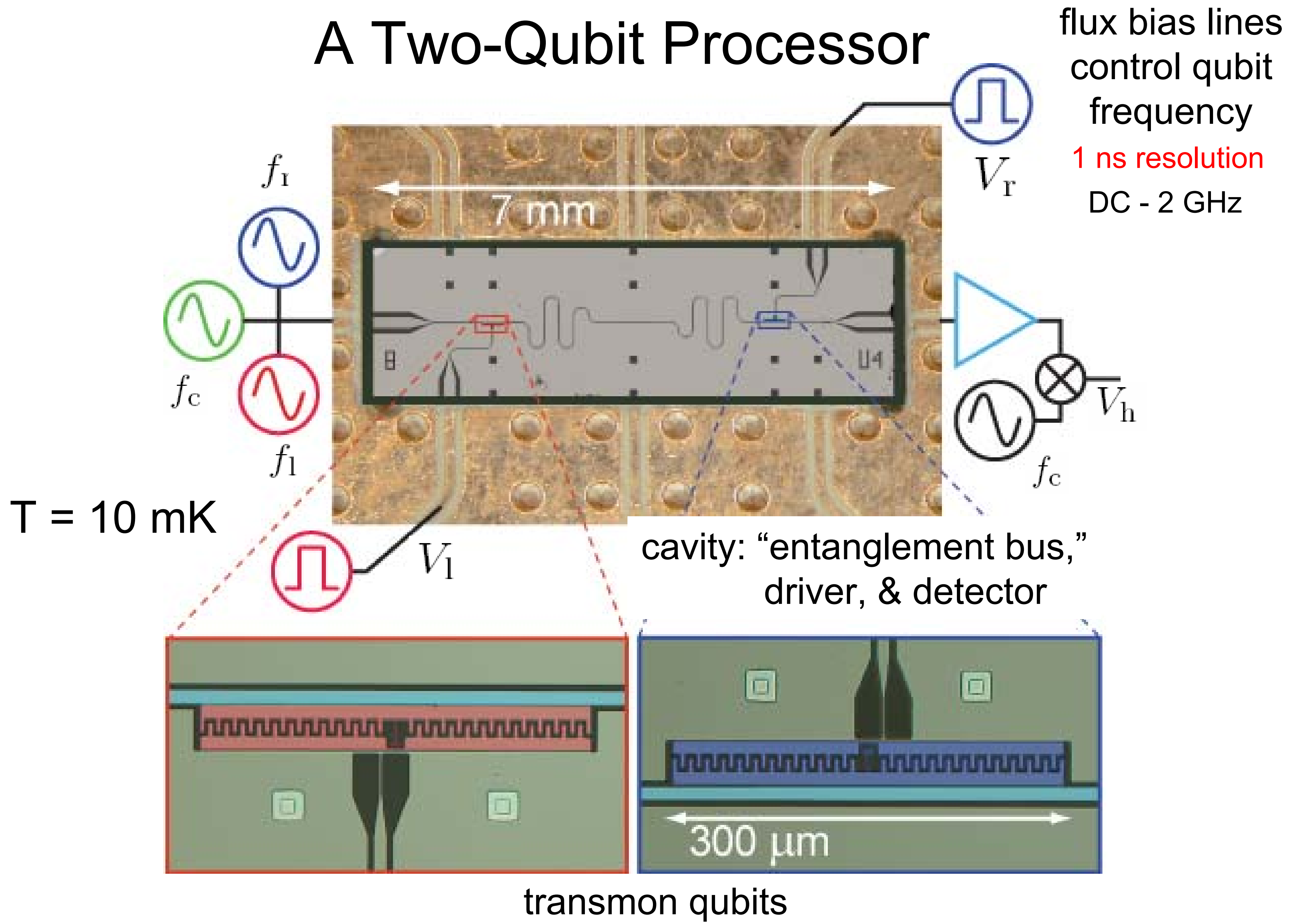


7 GHz in

Josephson-junction qubits

<u>Expts:</u> Majer et al., *Nature* 2007 (Charge qubits / Yale)
Sillanpaa et al., *Nature* 2007 (Phase qubits / NIST)

A Two-Qubit Processor



How do we entangle two qubits?

$R_Y(-\pi/2)$ rotation on each qubit yields superposition:

$$|\Psi\rangle = \frac{1}{2}\left(|0\rangle + |1\rangle\right) \otimes \left(|0\rangle + |1\rangle\right)$$

$$= \frac{1}{2}\left(|00\rangle + |10\rangle + |01\rangle + |11\rangle\right)$$

'Conditional Phase Gate' entangler:

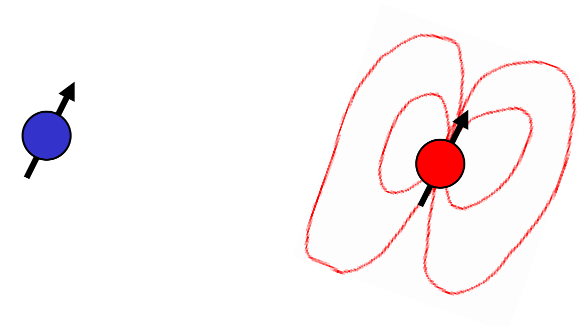
$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} |\Psi\rangle = \frac{1}{2}\left(|00\rangle + |10\rangle + |01\rangle - |11\rangle\right)$$

No longer a product state!

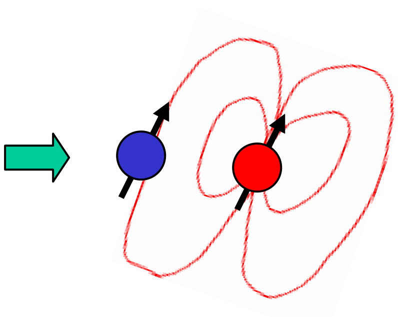
How do we realize the conditional phase gate?

$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} |\Psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right)$$

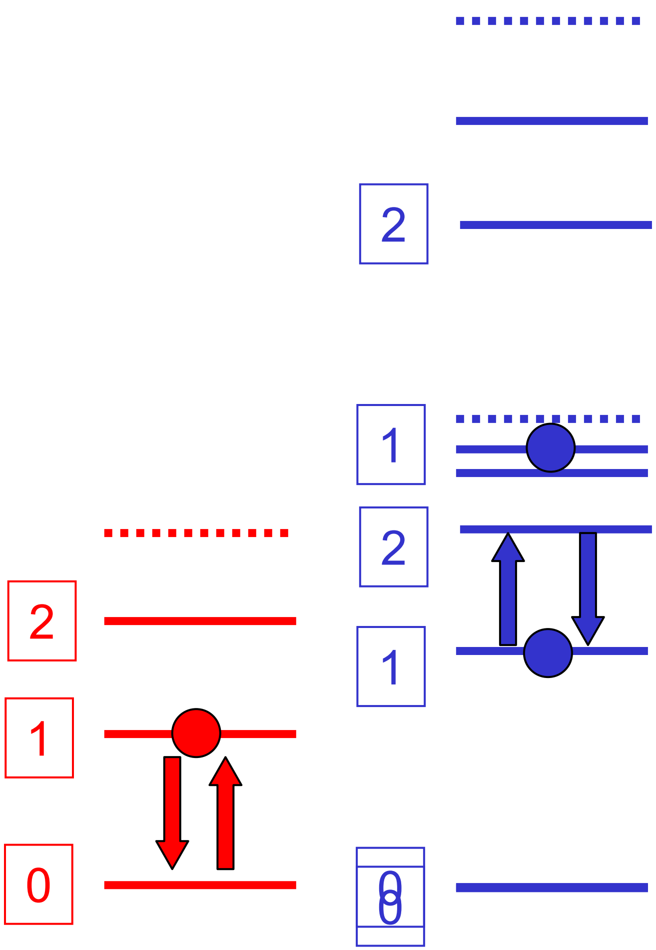
Use control lines to push qubits near a resonance:



A controlled z-z interaction also à la NMR



Key is to use 3rd level of transmon (outside the logical subspace)



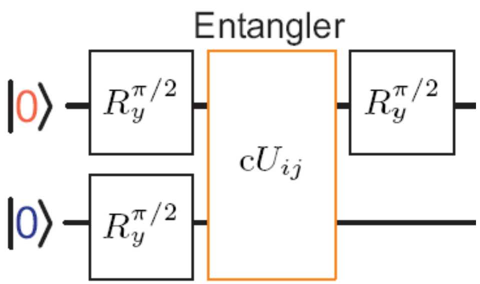
Coupling turned off.

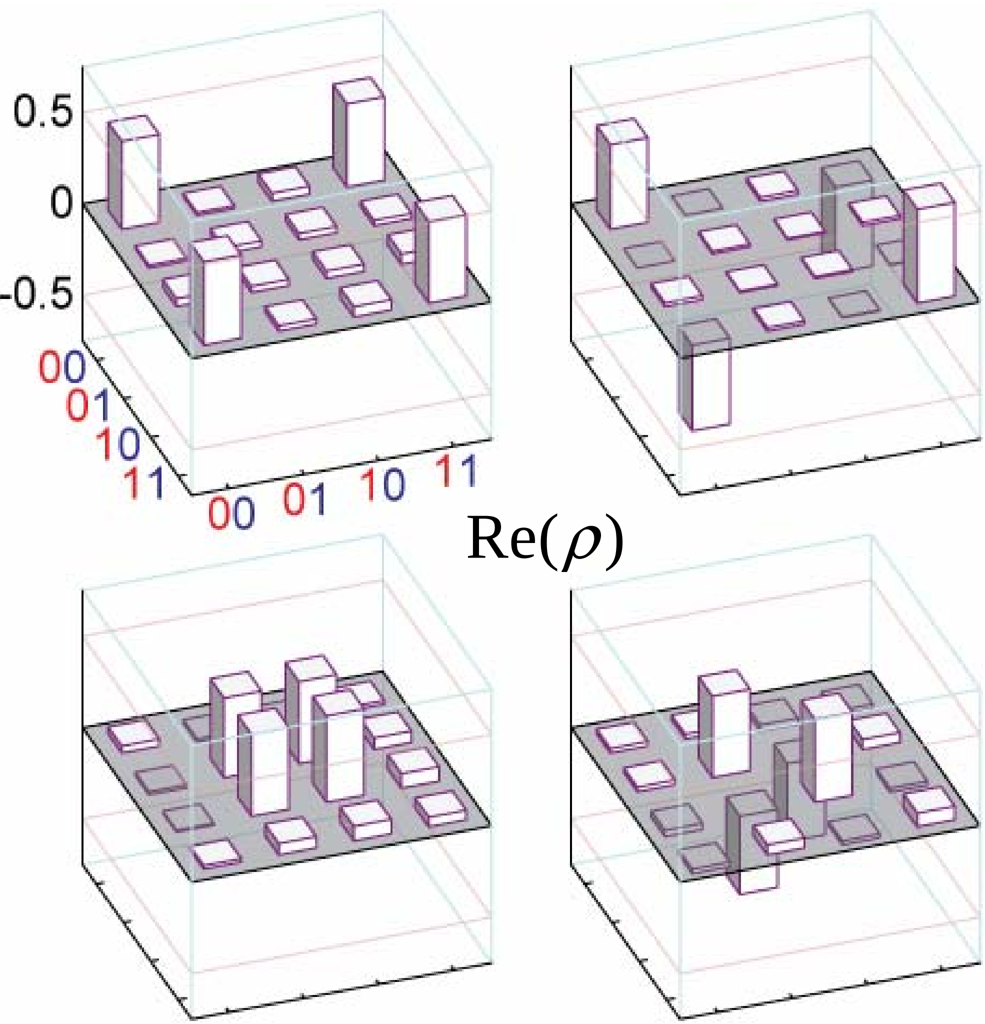
Coupling turned on:
Near resonance with 3rd level

$$\omega_{01} \approx \omega_{12}$$

Energy is shifted if and only if both qubits are in excited state.

Entanglement on demand using controlled phase gate





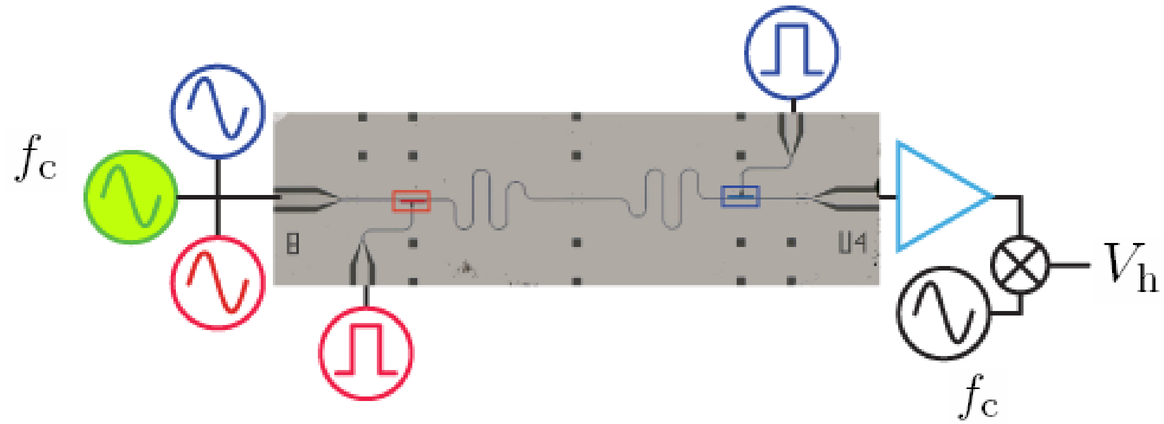
Bell state	Fidelity	Concurrence
$\lvert 00\rangle + \lvert 11\rangle$	91%	88%
$\lvert 00\rangle - \lvert 11\rangle$	94%	94%
$\lvert 01\rangle + \lvert 10\rangle$	90%	86%
$\lvert 01\rangle - \lvert 10\rangle$	87%	81%

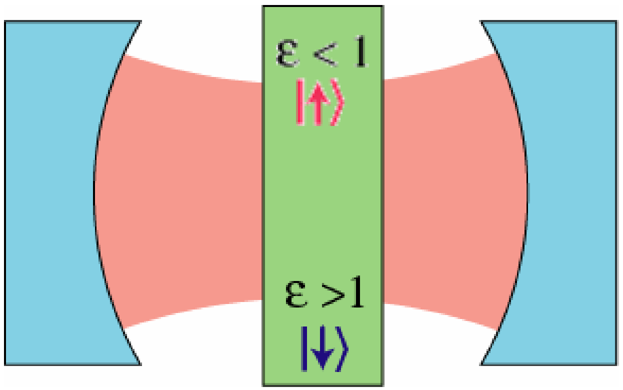
UCSB: Steffen *et al., Science* (2006)
ETH: Leek *et al., PRL* (2009)
Yale: DiCarlo *et al., Nature* (2009)

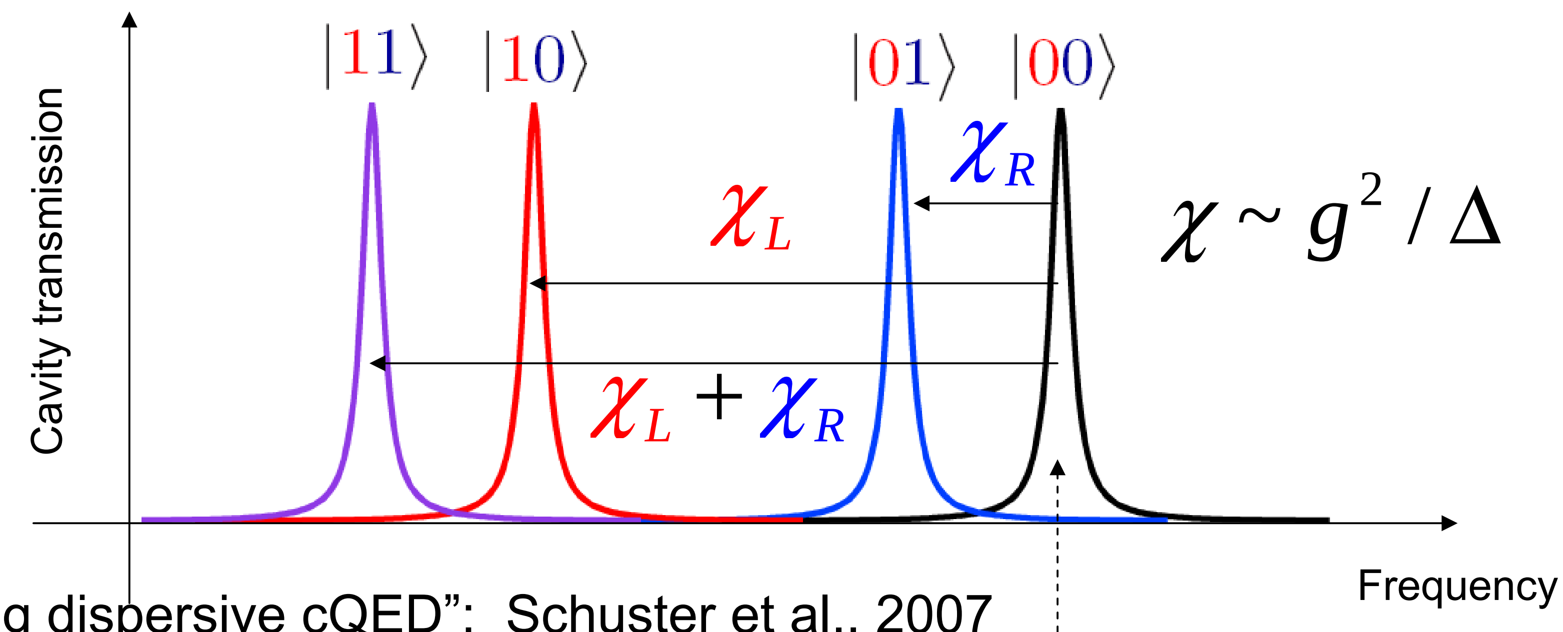
How do we read out the qubit state and measure the entanglement?

Two Qubit Joint Readout via Cavity

Reverse of Haroche et al.: photons measure qubits



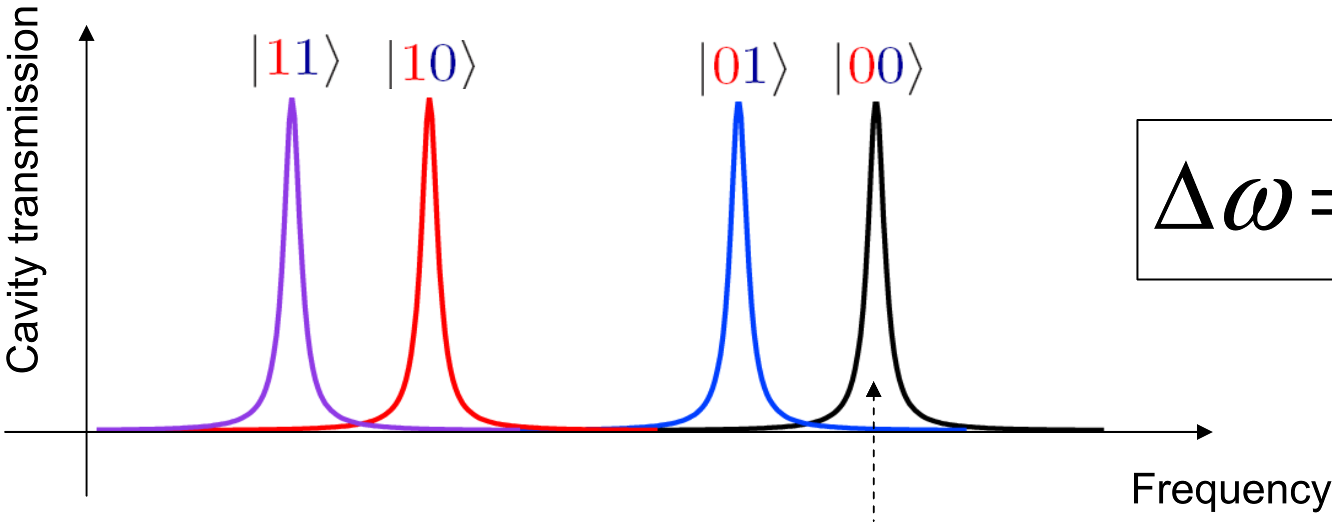




"Strong dispersive cQED": Schuster et al., 2007

Basic two-qubit readout demo'd in Majer et al., 2007

Cavity Pull is linear in spin polarizations



$$\Delta\omega = a\sigma_L^z + b\sigma_R^z$$

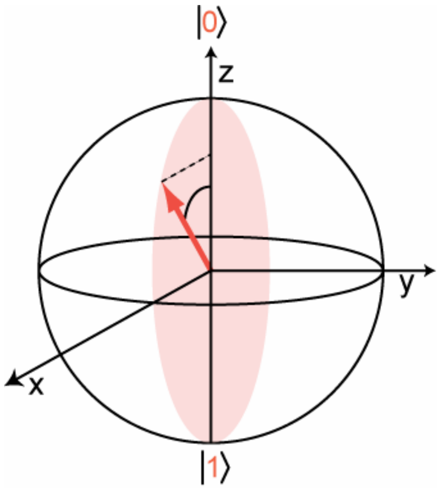
Complex transmitted amplitude is non-linear in cavity pull:

$$t = \frac{\kappa/2}{\omega_{\text{drive}} - \omega_{\text{cavity}} - \Delta\omega + i\kappa/2}$$

Most general non-linear function of two Ising spin variables:

$$t = \beta_0 + \beta_1\,\sigma_L^z + \beta_2\,\sigma_R^z + \beta_{12}\,\sigma_L^z \otimes \sigma_R^z$$

State Tomography



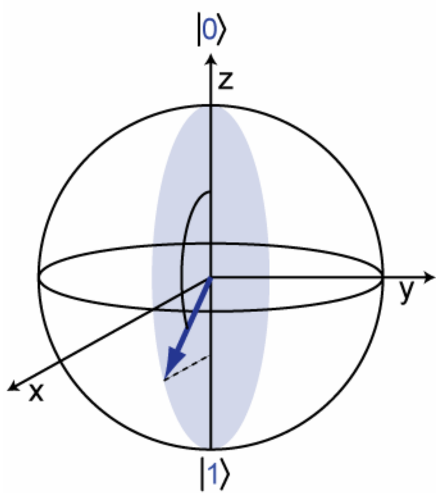
$$V_{\mathrm{H}} \sim \langle M \rangle = \beta_1 \langle \sigma_z^{\mathrm{L}} \rangle + \beta_2 \langle \sigma_z^{\mathrm{R}} \rangle + \beta_{12} \langle \sigma_z^{\mathrm{L}} \otimes \sigma_z^{\mathrm{R}} \rangle$$

Combine joint readout with one-qubit “analysis” rotations

Do nothing:

$$\beta_1 \langle \sigma_z^{\mathrm{L}} \rangle + \beta_2 \langle \sigma_z^{\mathrm{R}} \rangle + \beta_{12} \langle \sigma_z^{\mathrm{L}} \otimes \sigma_z^{\mathrm{R}} \rangle$$

+



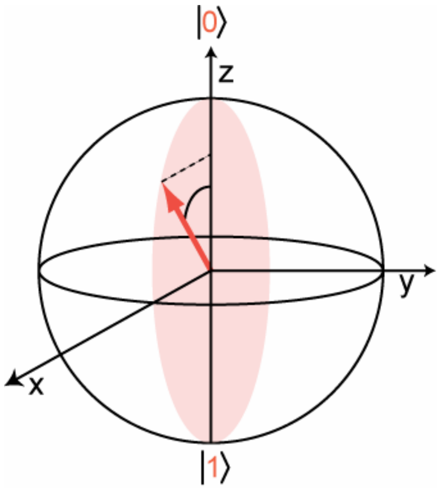
π-pulse on right qubit

$$\beta_1 \langle \sigma_z^{\mathrm{L}} \rangle - \beta_2 \langle \sigma_z^{\mathrm{R}} \rangle - \beta_{12} \langle \sigma_z^{\mathrm{L}} \otimes \sigma_z^{\mathrm{R}} \rangle$$

$$2\beta_1 \langle \sigma_z^{\mathrm{L}} \rangle$$

See similar from Zurich group: Fillip et al., PRL **102**, 200402 (2009).

State Tomography



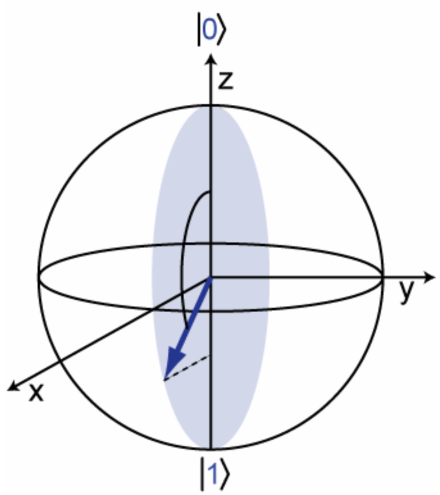
$$V_H \sim \langle M \rangle = \beta_1 \langle \sigma_z^L \rangle + \beta_2 \langle \sigma_z^R \rangle + \beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle$$

Combine joint readout with one-qubit "analysis" rotations

Do nothing: $\beta_1 \langle \sigma_z^L \rangle + \beta_2 \langle \sigma_z^R \rangle + \beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle$

+

π-pulse on ***both*** qubits $-\beta_1 \langle \sigma_z^L \rangle - \beta_2 \langle \sigma_z^R \rangle + \beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle$



$$2\beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle$$

Possible to acquire correlation info.,
even with single, ensemble averaged msmt.!

(single-shot fidelity ~ 10% here)

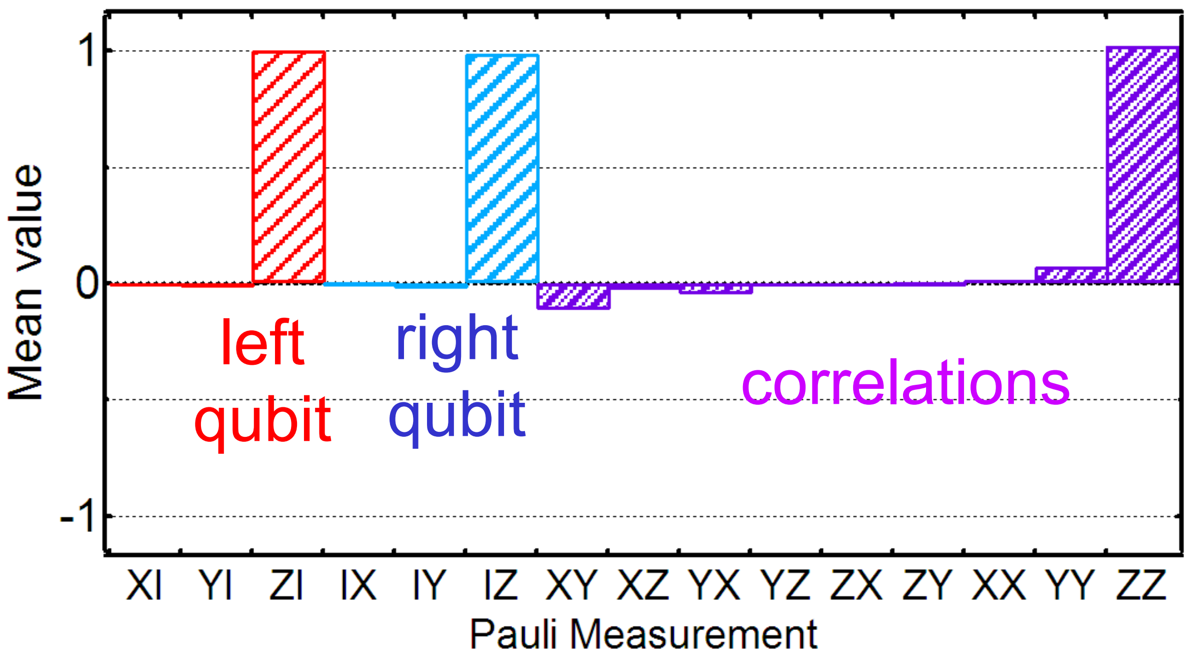
Measuring the Full Two-Qubit State

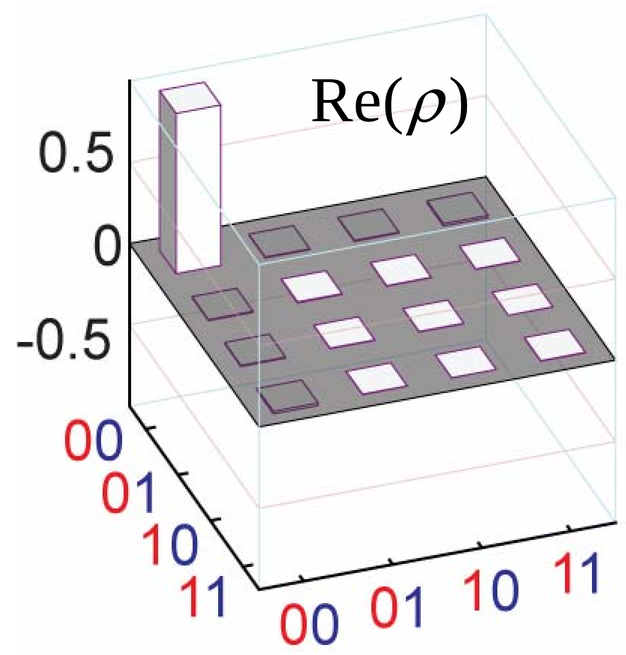
Total of 16 msmts.: $I, Y_{\pi}^{L}, X_{\pi/2}^{L}, Y_{\pi/2}^{L}$ and combinations

$I, Y_{\pi}^{R}, X_{\pi/2}^{R}, Y_{\pi/2}^{R}$

max. likelihood (nonlinear!)

(almost) raw data



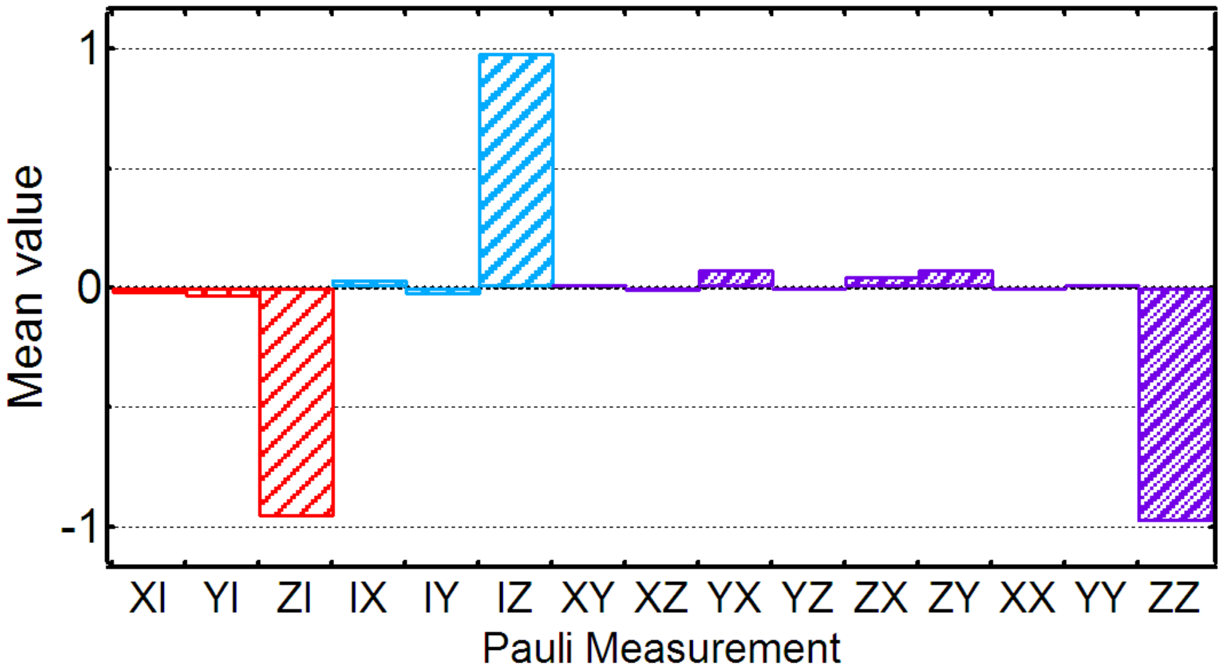


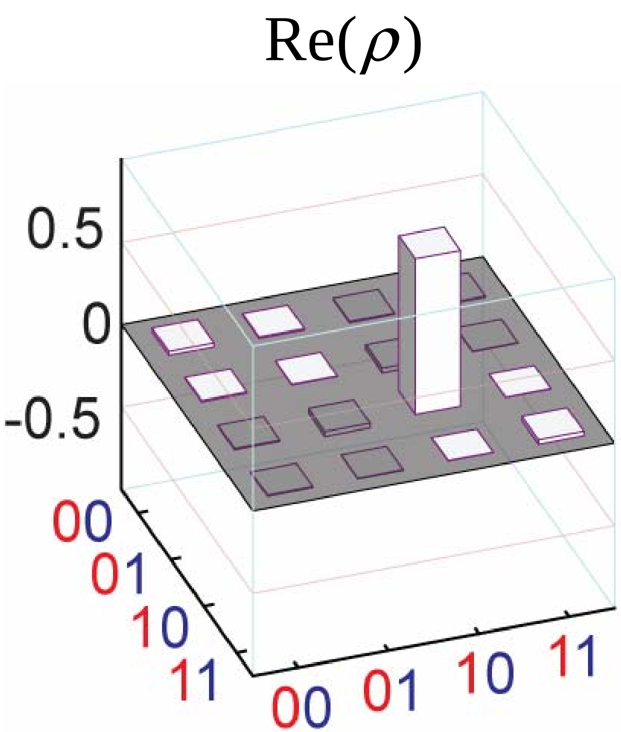
Density matrix

Ground state: $|\psi\rangle = |00\rangle$

Measuring the Two-Qubit State

Apply π-pulse to invert state of left qubit



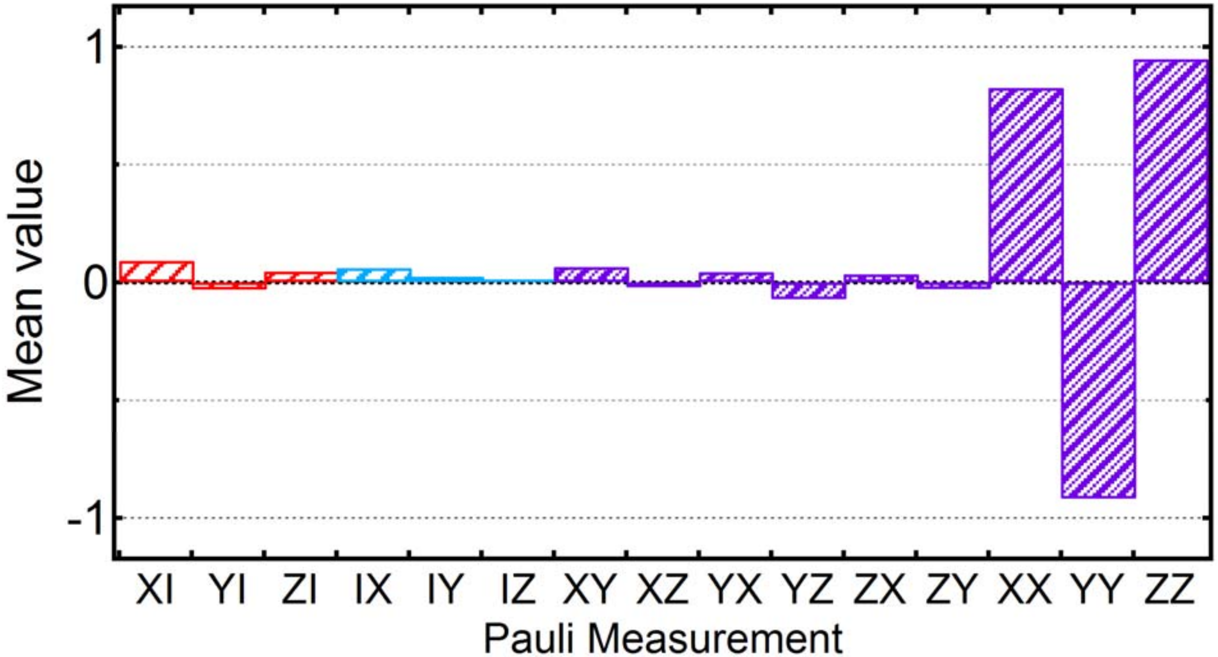


One qubit excited: $|\psi\rangle = |10\rangle$

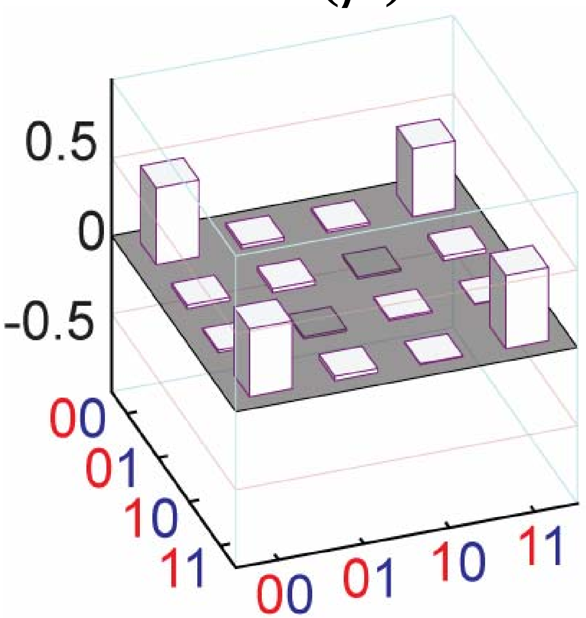
Measuring the Two-Qubit State

Now apply a two-qubit gate to *entangle* the qubits

Entangled state: $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$



Re(ρ)



C = 0.94 +-??

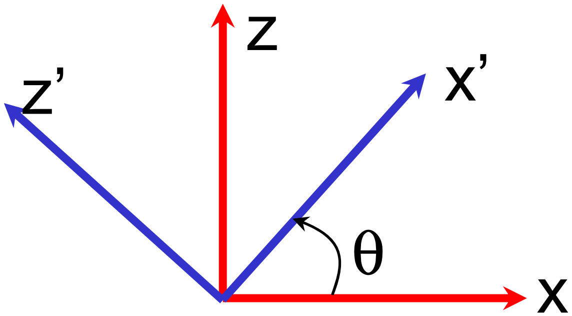
What's the entanglement metric?

"Concurrence":

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

λ are e-values of $\sqrt{\rho\tilde{\rho}}$

Witnessing Entanglement



CHSH operator = entanglement witness

$$\langle CHSH \rangle = \langle XX' \rangle - \langle XZ' \rangle + \langle ZX' \rangle + \langle ZZ' \rangle$$

Clauser, Horne, Shimony & Holt (1969)

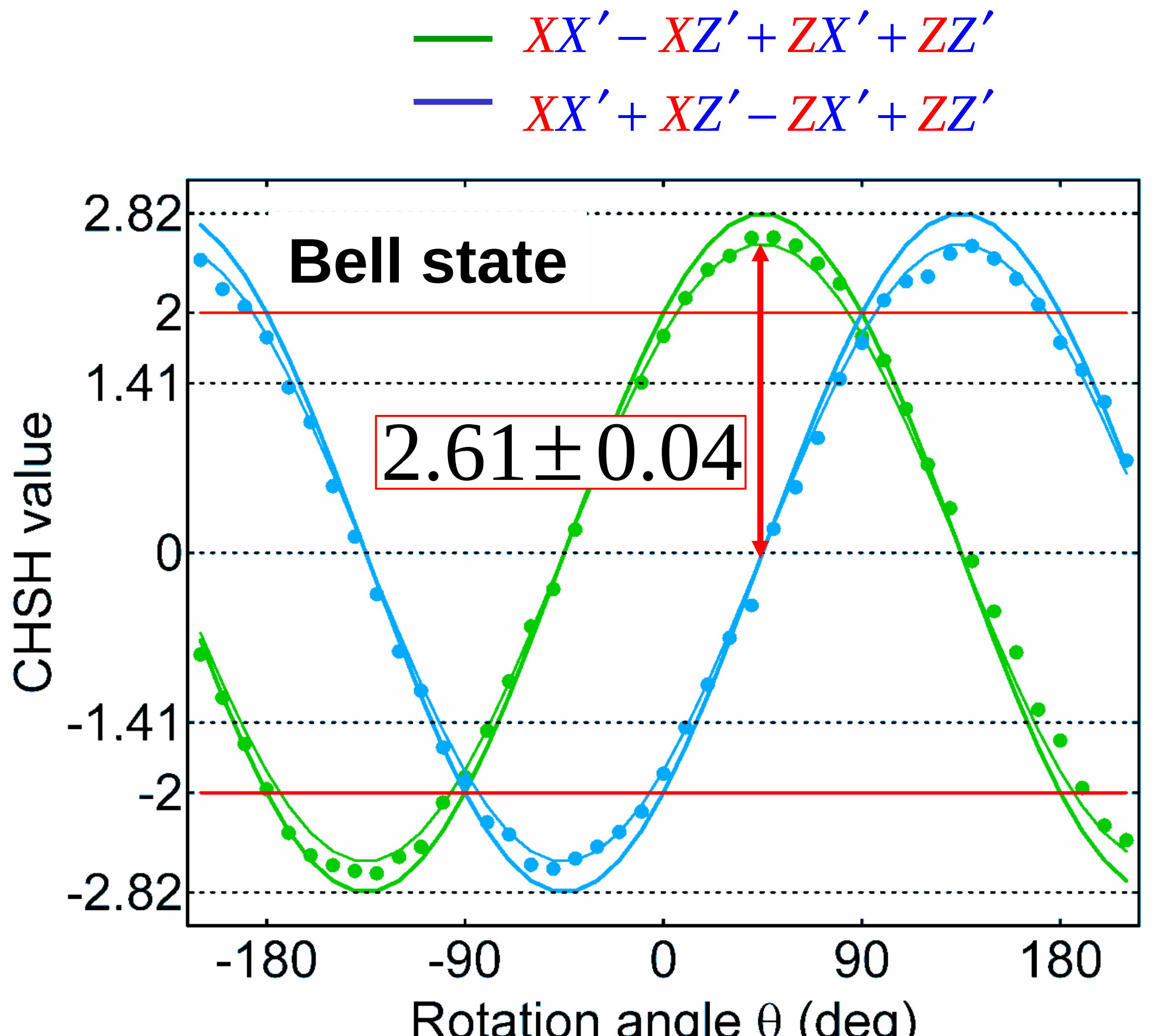
Separable bound:

$$|CHSH| \leq 2$$

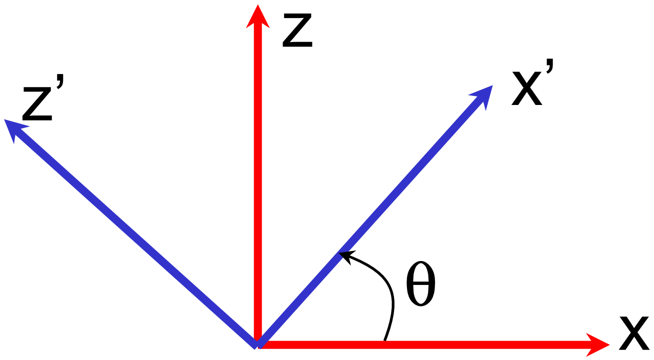
not test of hidden variables… (loopholes abound)

but state is clearly highly entangled!

Chow et al., arXiv:0908.1955



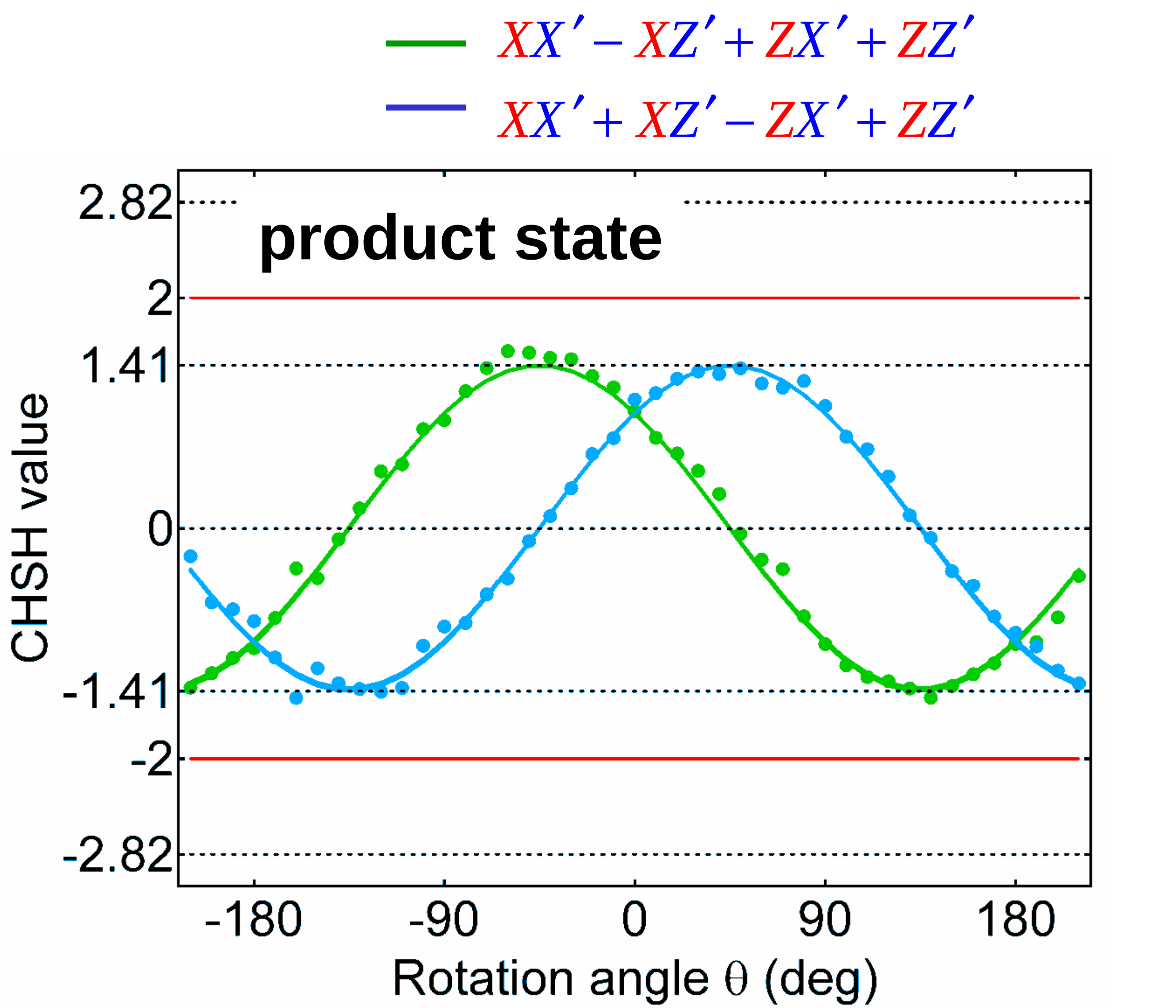
Control: Analyzing Product States



CHSH operator = entanglement witness

$$\langle CHSH \rangle = \langle XX' \rangle - \langle XZ' \rangle + \langle ZX' \rangle + \langle ZZ' \rangle$$

Clauser, Horne, Shimony & Holt (1969)



control experiment shows no entanglement

Using entanglement on demand to run first quantum algorithm on a solid state quantum processor

Skip to Summary

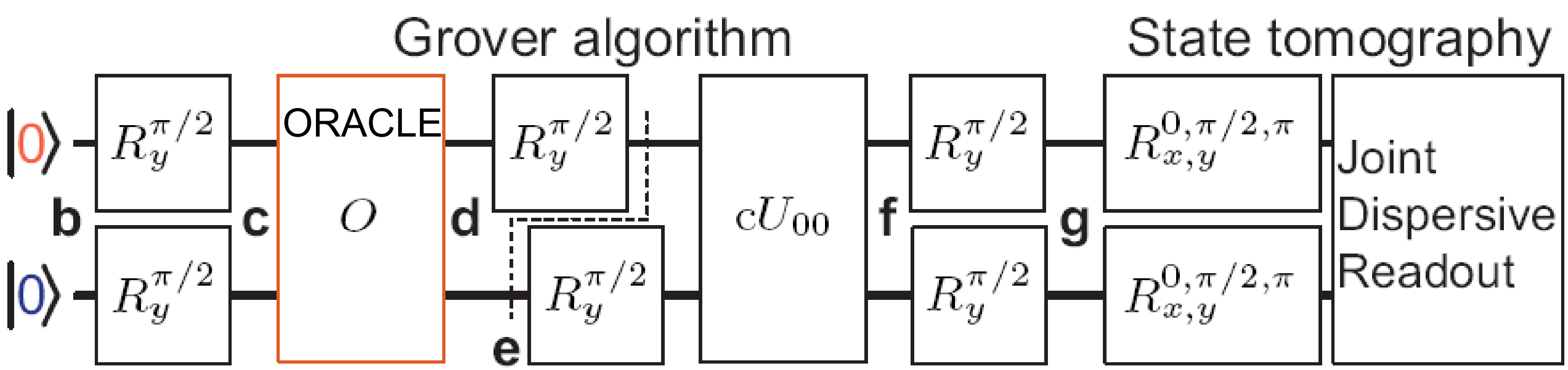
Grover's Algorithm

"unknown" unitary operation: →

$$O|\psi\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} |\psi\rangle$$

Challenge: Find the location of the -1 !!!

Previously implemented in NMR: Chuang et al., 1998
Optics: Kwiat et al., 2000
Ion traps: Brickman et al., 2003

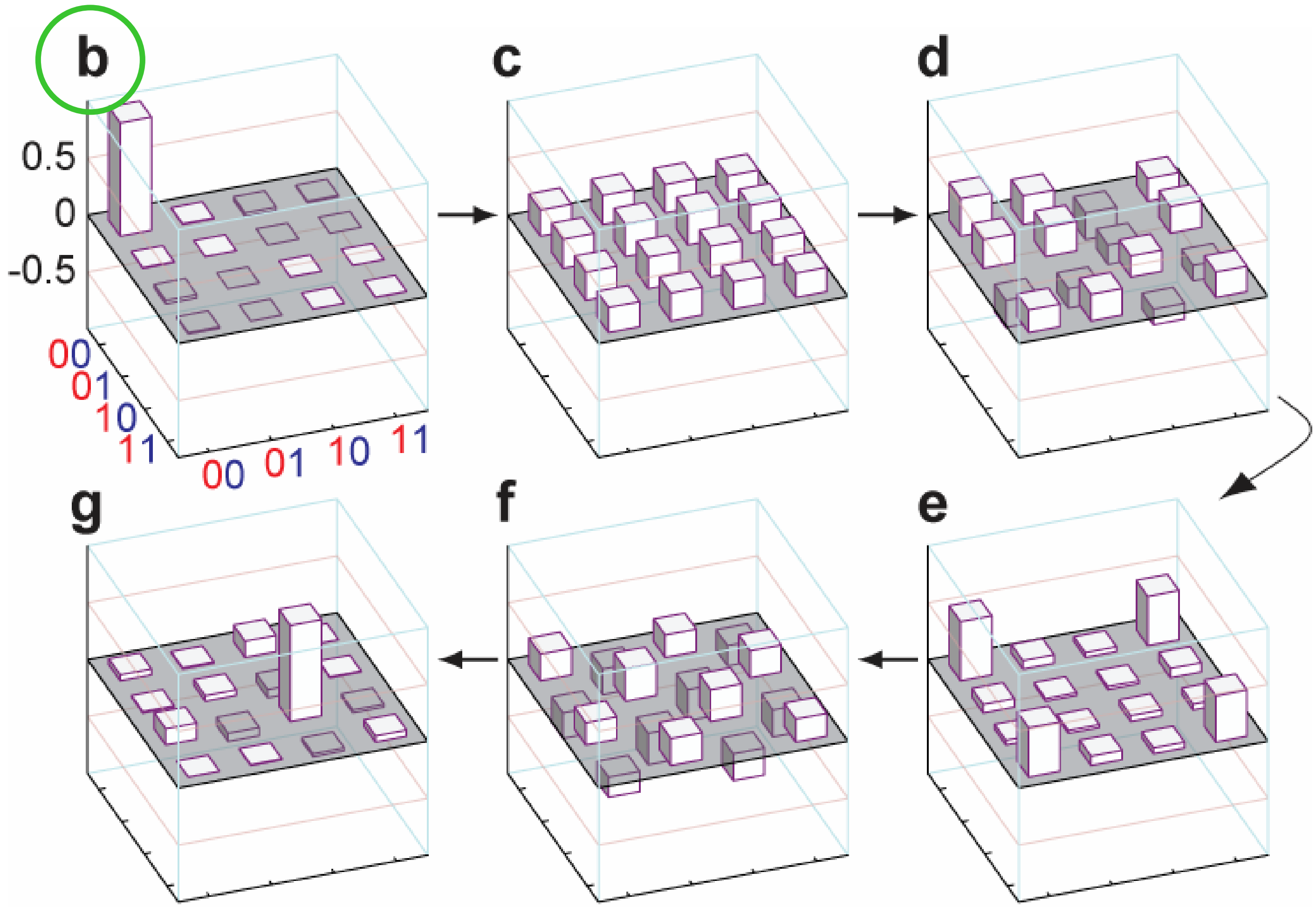


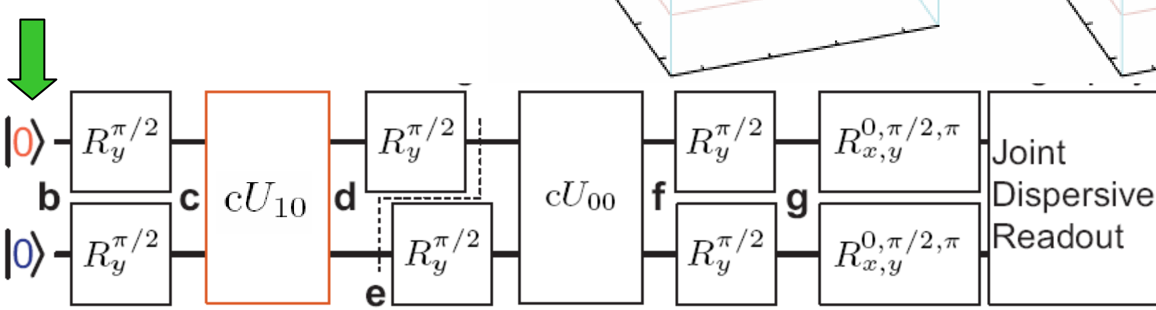
10 pulses w/ nanosecond resolution, total 104 ns duration

Grover Step-by-Step

$$\left|\psi_{\text{ideal}}\right\rangle = \left|00\right\rangle$$

Begin in ground state:

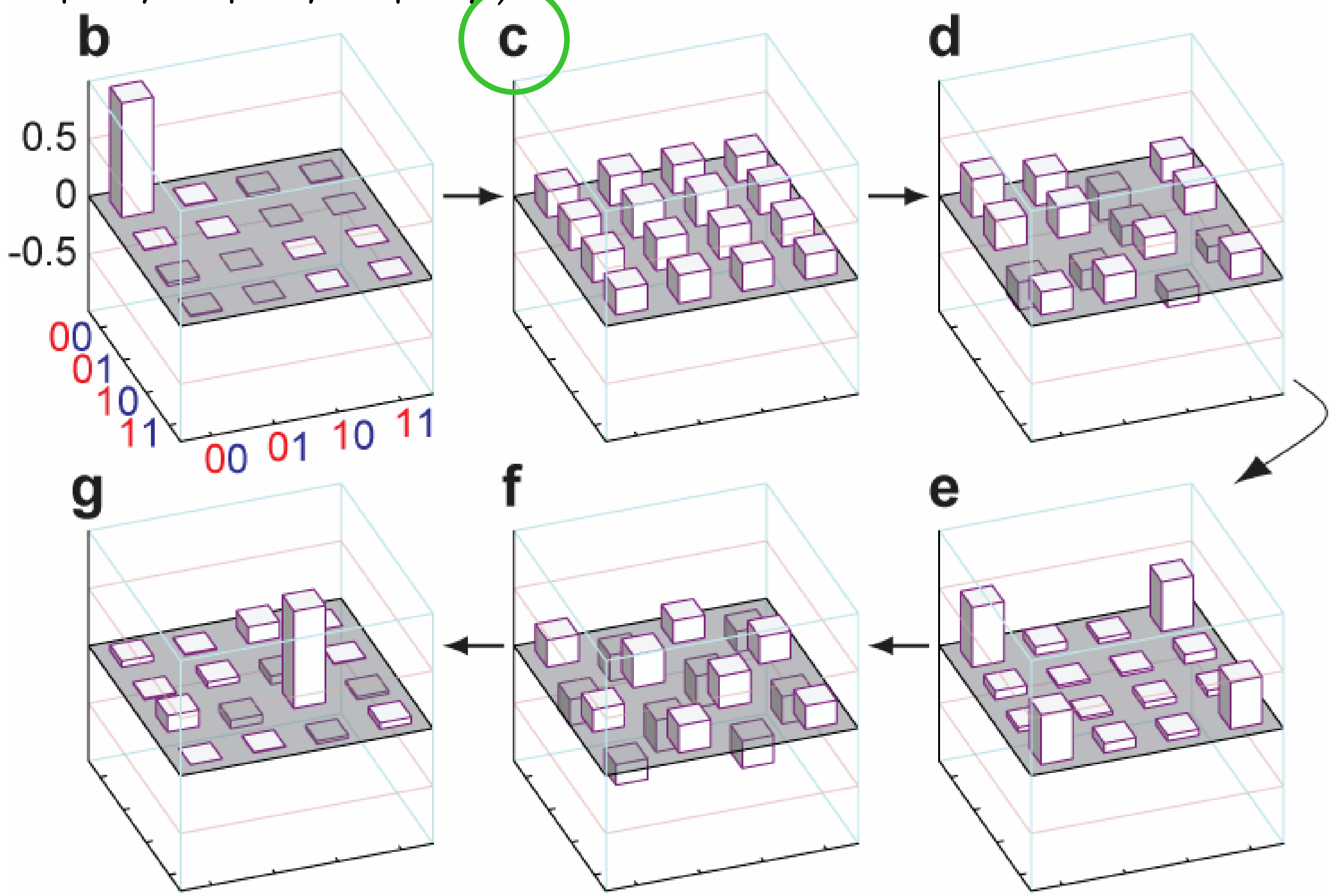


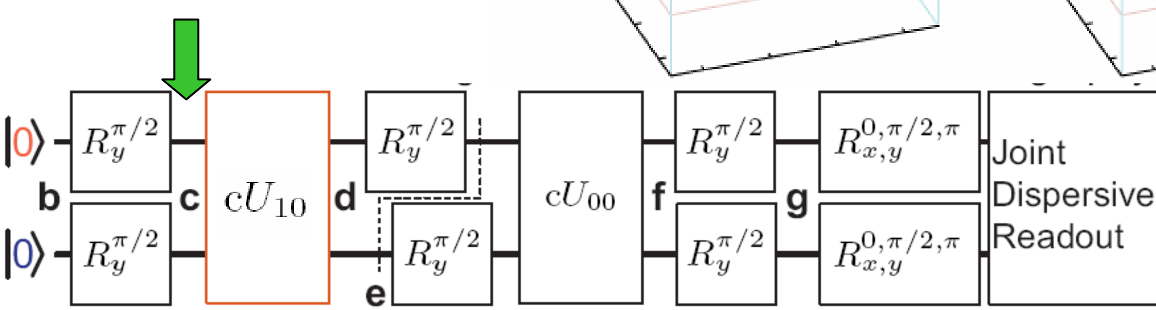


Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

Create a maximal superposition: look everywhere at once!



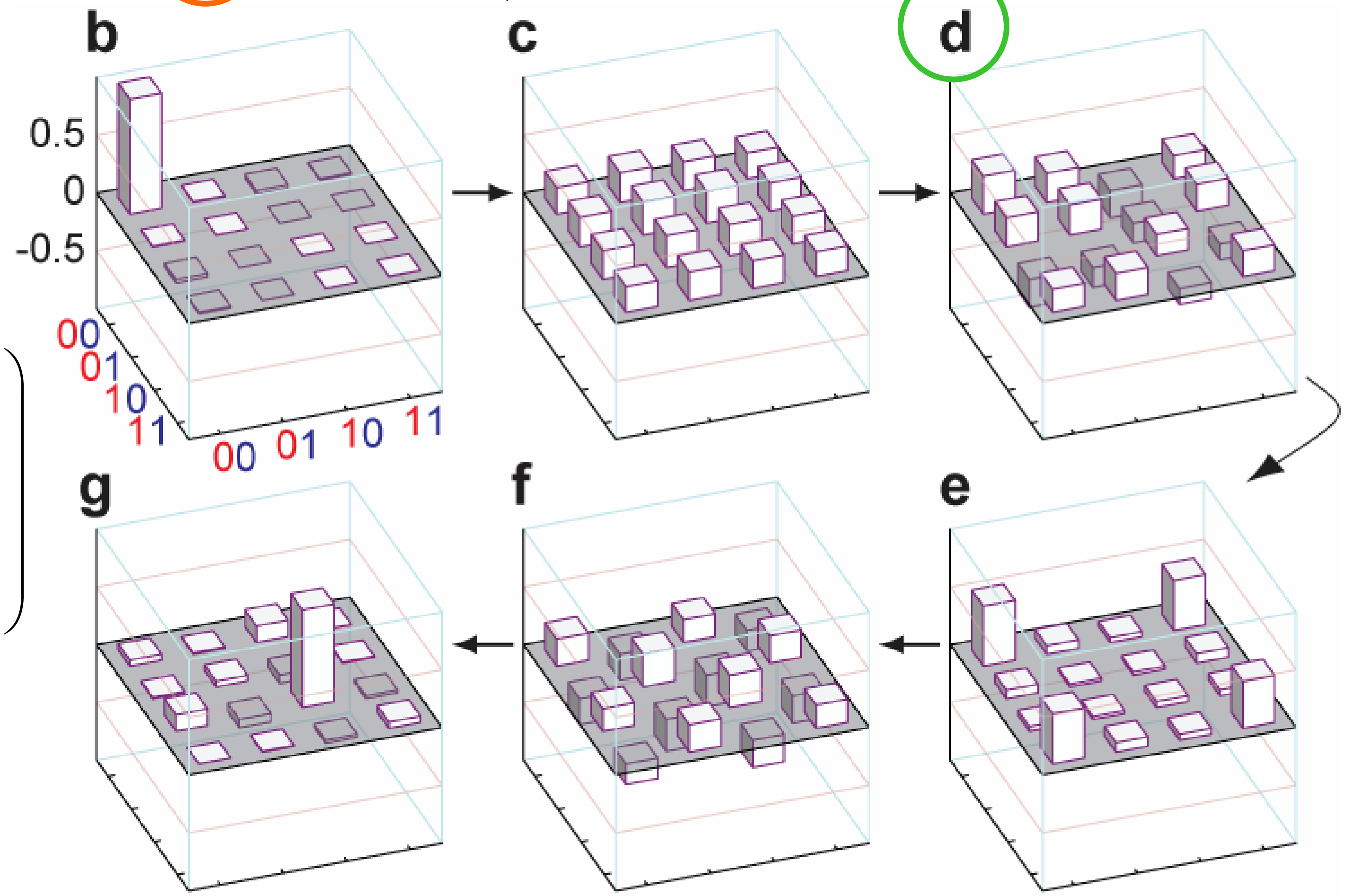


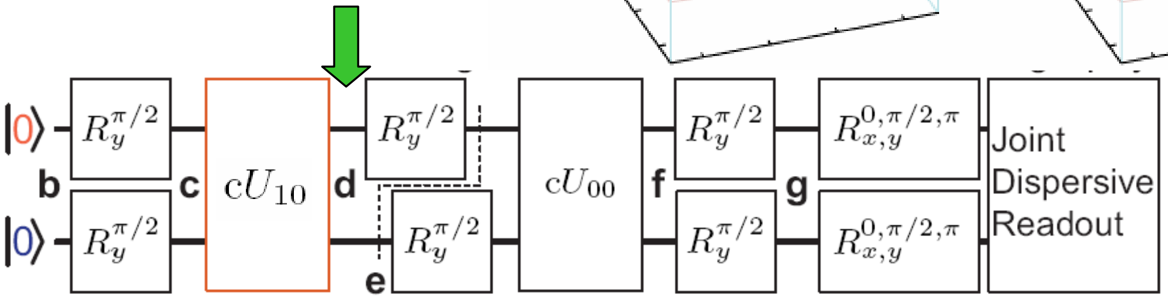
Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right)$$

Apply the "unknown" function, and mark the solution

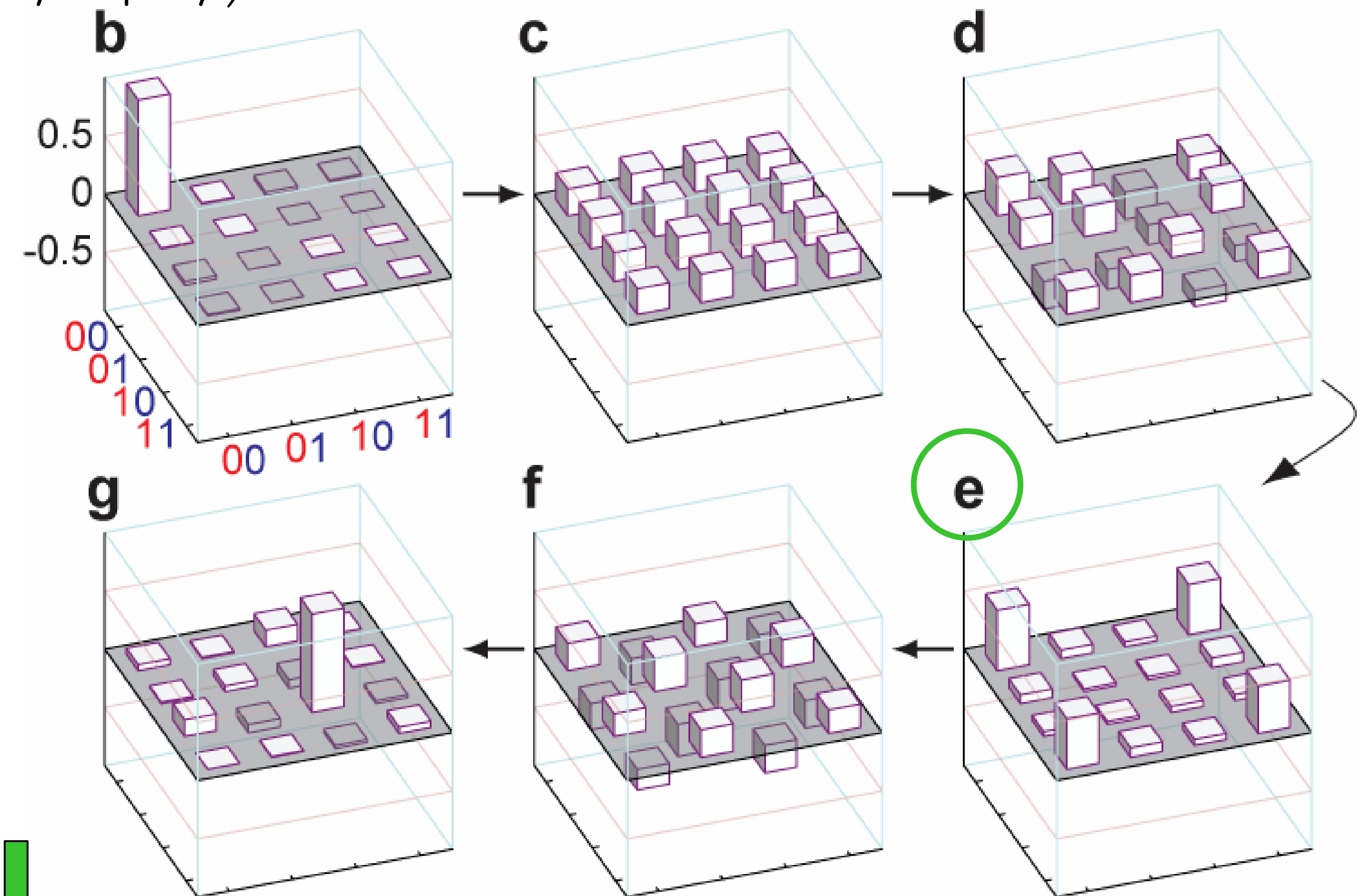
$$cU_{10} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$





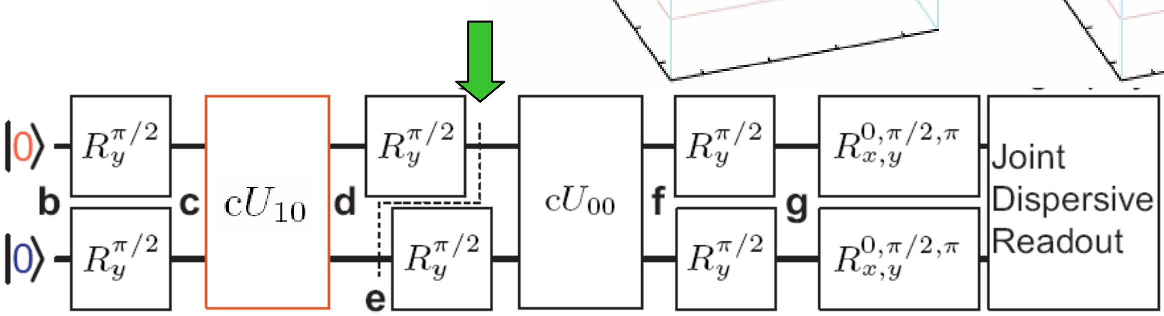
Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$



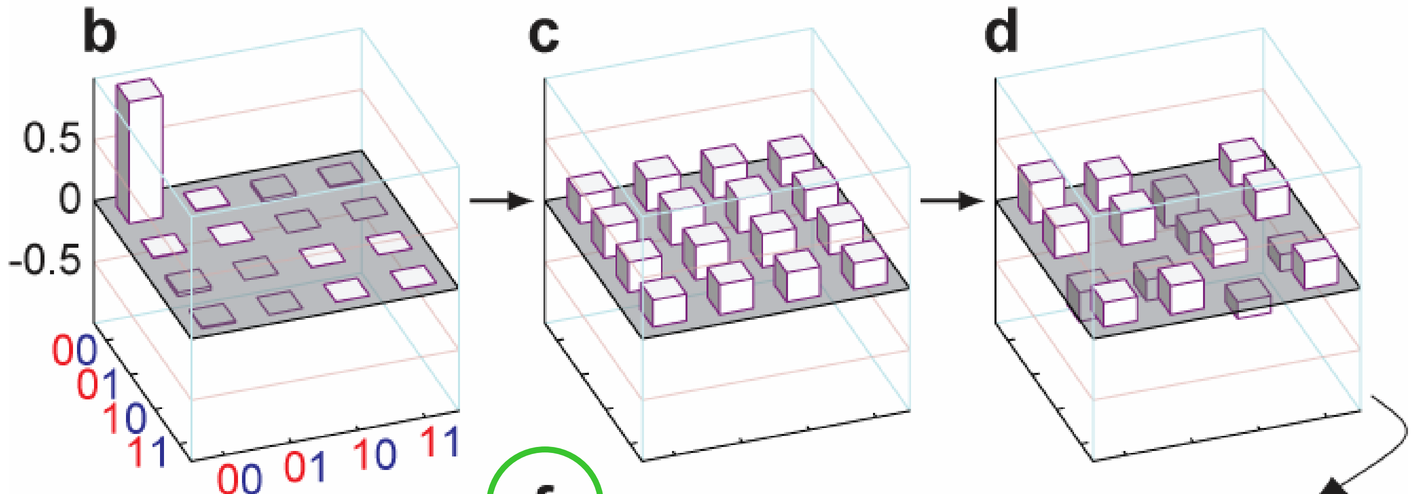
Some more 1-qubit rotations…

Now we arrive in one of the four Bell states

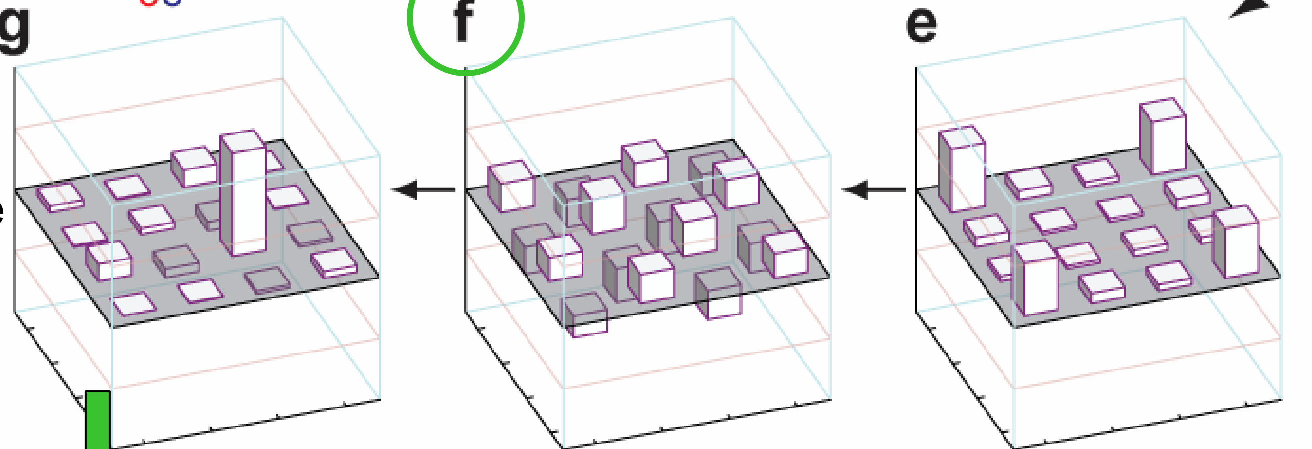


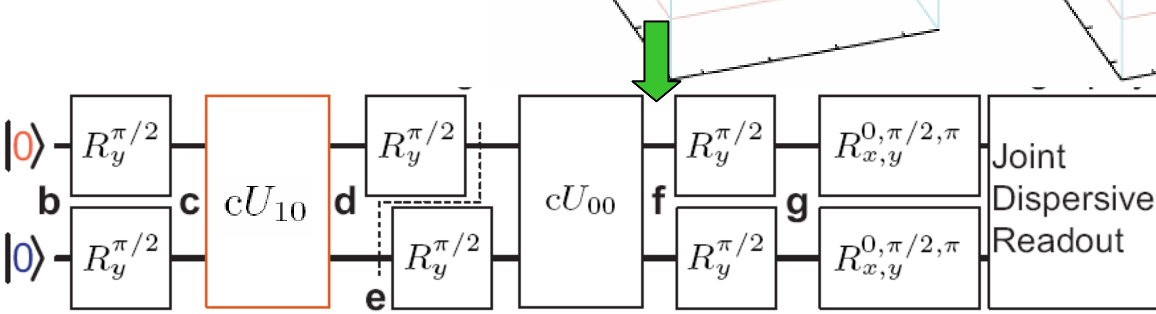
Grover Step-by-Step

$$|\psi_{\text{ideal}}\rangle = \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right)$$



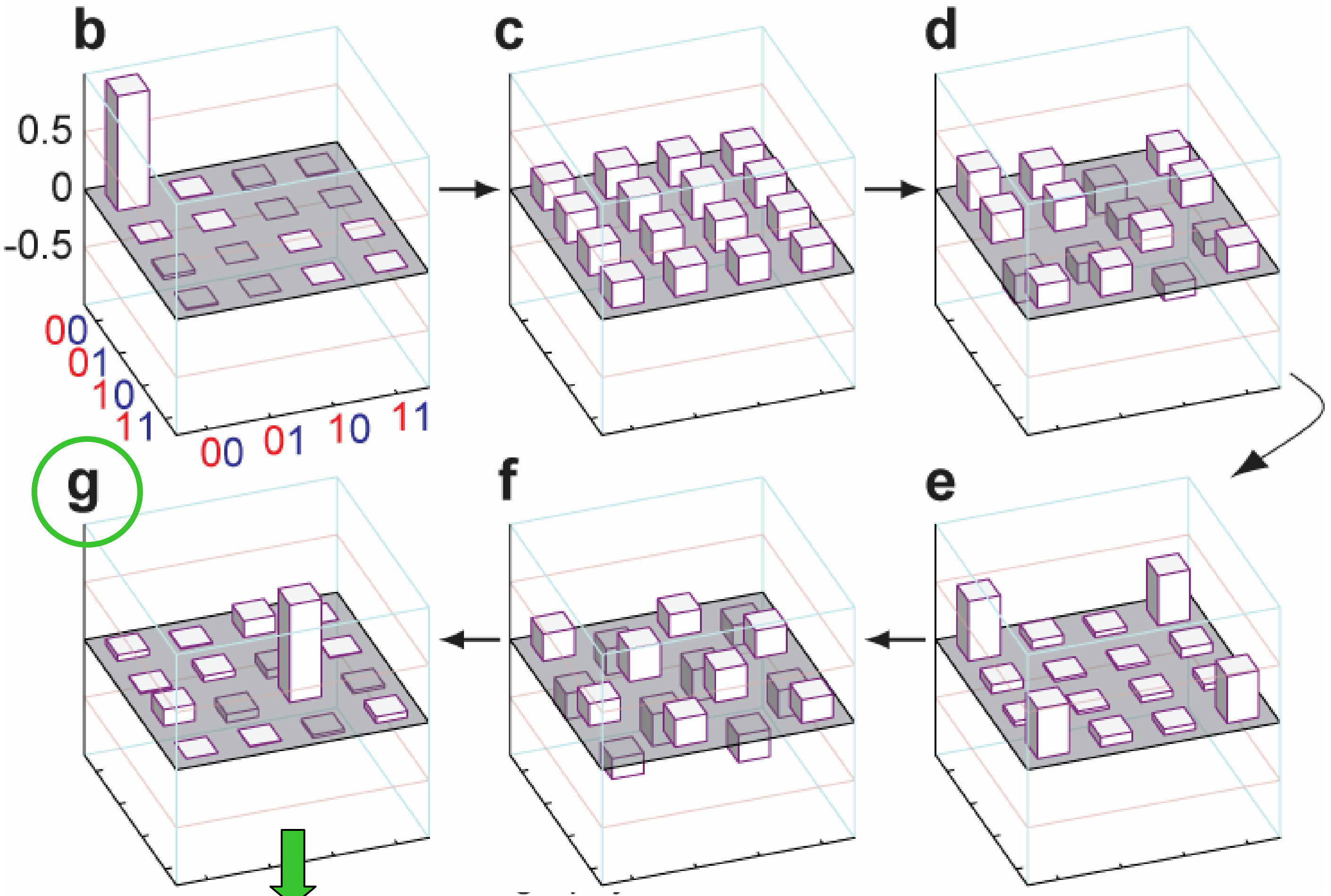
Another (but known) 2-qubit operation now undoes the entanglement and makes an interference pattern that holds the answer!





Grover Step-by-Step

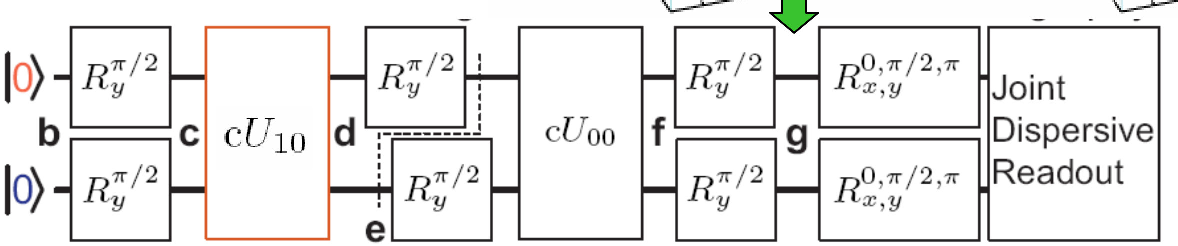
$$|\psi_{\text{ideal}}\rangle = |10\rangle$$



Final 1-qubit rotations reveal the answer:

The binary representation of “2”!

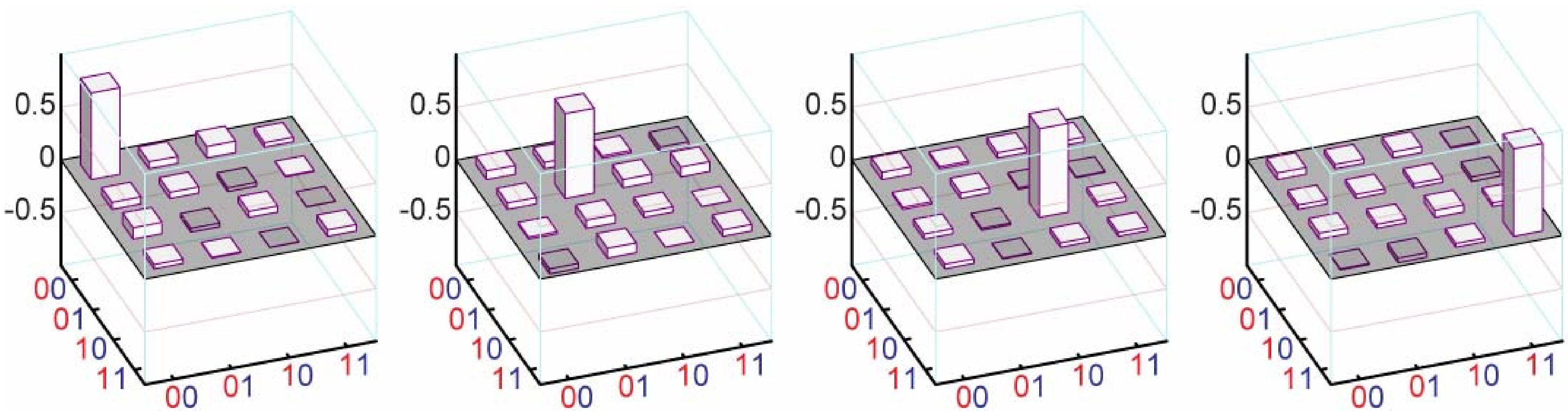
The correct answer is found
>**80%** of the time!



Grover with Other Oracles

Oracle

$\hat{O} = cU_{00}$ cU_{01} cU_{10} cU_{11}



$\overline{F} = 81\%$ 80% 82% 81%

Fidelity $F = \langle \psi_{\text{ideal}} | \rho | \psi_{\text{ideal}} \rangle$ to ideal output

(average over 10 repetitions)

The cost of entanglement

- 1 Cryogenic HEMT amp
- 2 Room Temp Amps
- 1 Two-channel digitizer
- 1 Two-channel AWG
- 1 Four-channel AWG
- 2 Scalar signal generators
- 2 Vector signal generators
- 1 Low-frequency generator
- 1 Rubidium frequency standard
- 2 Yokogawa DC sources
- 1 DC power supply
- 1 Amp biasing servo
- 1 Computer
- 10^3 Coffee pods

"Where a calculator on the ENIAC is equipped with 18,000 vacuum tubes and weighs 30 tons, computers in the future may have only 1,000 vacuum tubes and perhaps only weigh one and a half tons."

Popular Mechanics, 1949

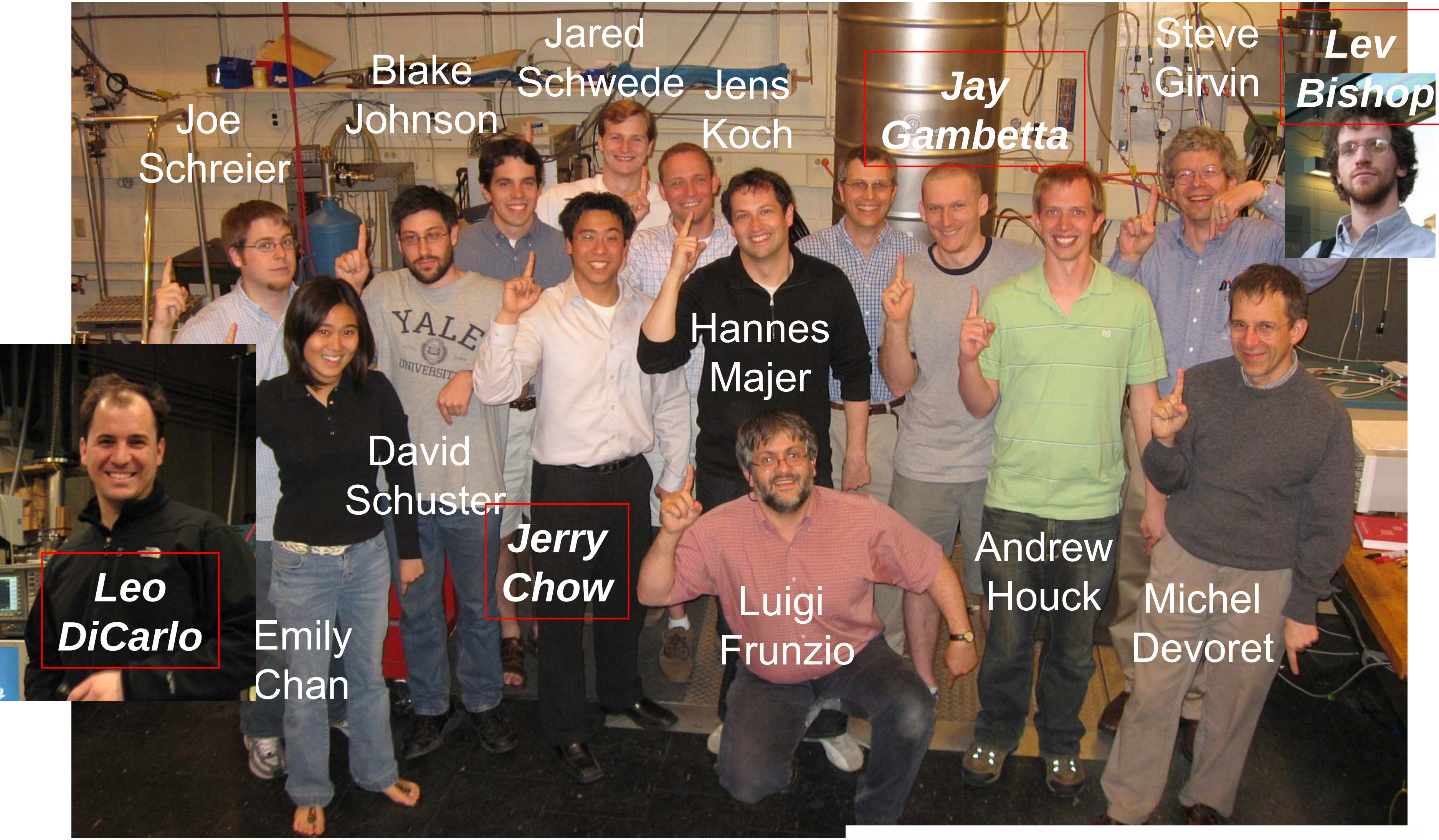
Many thanks to: Jay Gambetta, Jens Koch, Ren-Shou Huang, Alexandre Blais, Krishnendu Sengupta, Aashish Clerk, Andreas Wallraff, David Schuster, Andrew Houck, Florian Marquardt, Jens Koch, Terri Yu, Lev Bishop, **Robert Schoelkopf, Michel Devoret**

We still have a long way to go.

Theorist

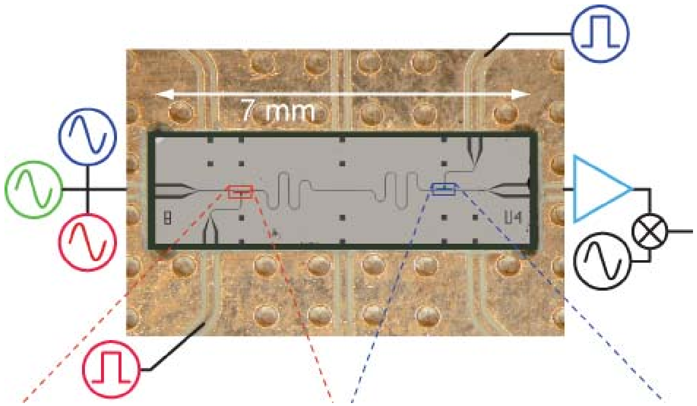
Experimentalist

Circuit QED Team Members

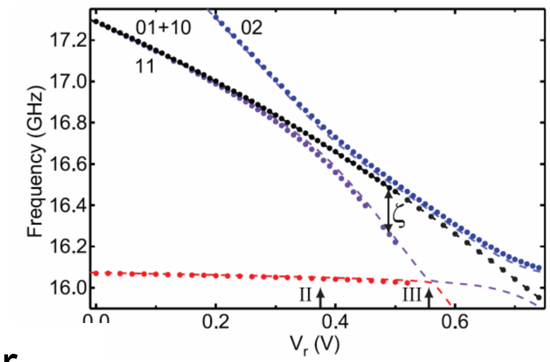


Funding:

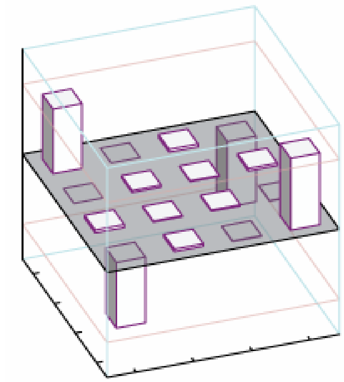
Summary



Rudimentary two-qubit processor



Adiabatic C-phase gate

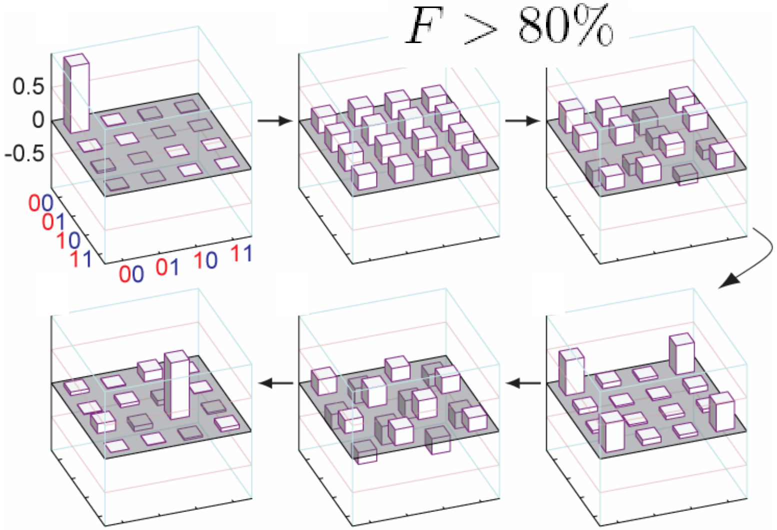


Entanglement on demand

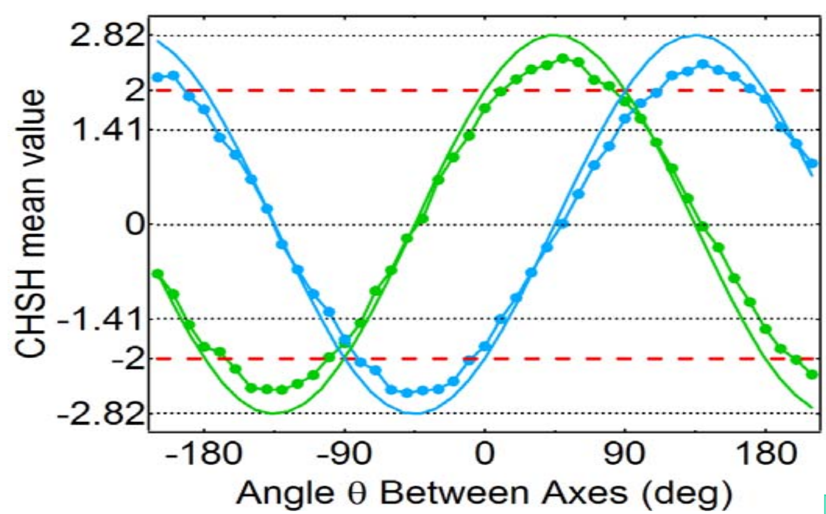
$$F = 87\text{–}94\%$$
$$C = 81\text{–}94\%$$

- Grover algorithm with Fidelity

$F > 80\%$



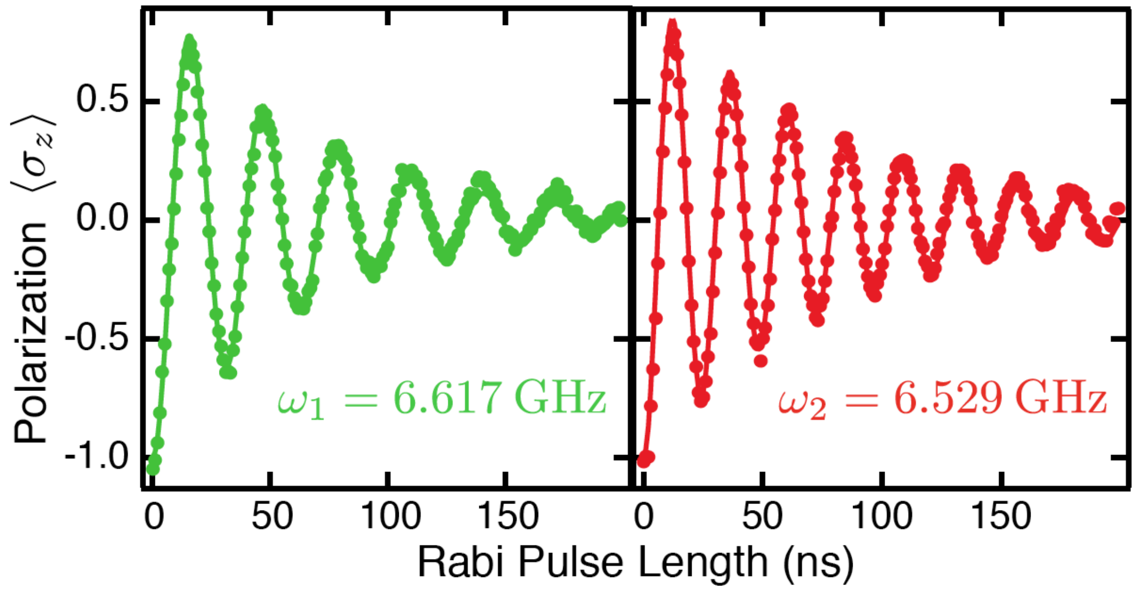
CHSH as entanglement witness



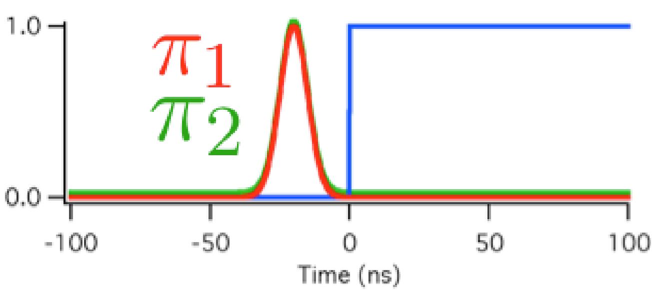
DiCarlo et al., cond-mat 0903.2030, *Nature* in press

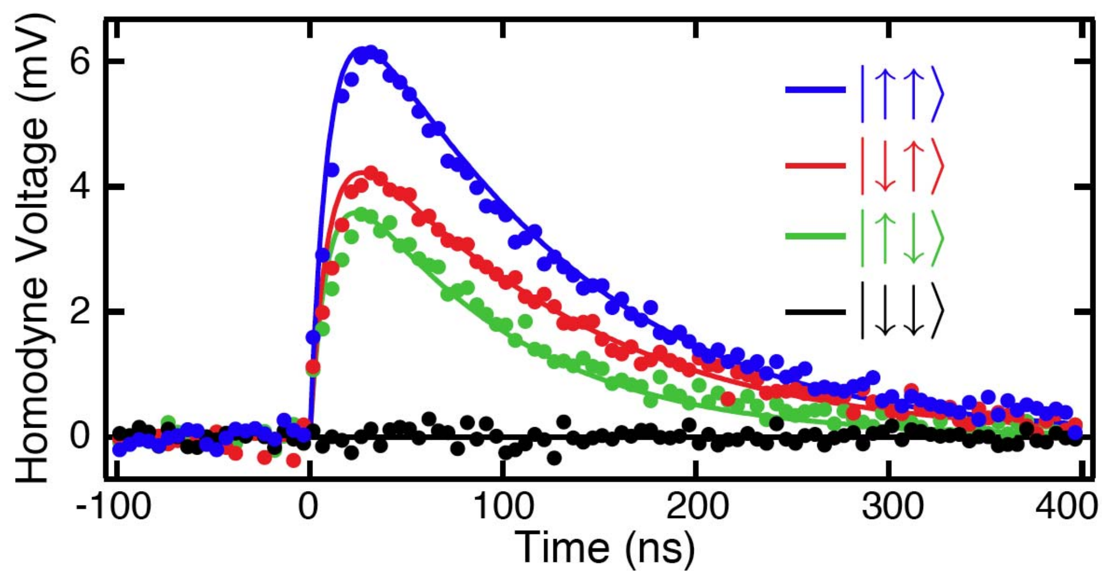
Additional Slides Follow

Multiplexed Qubit Control and Read-Out

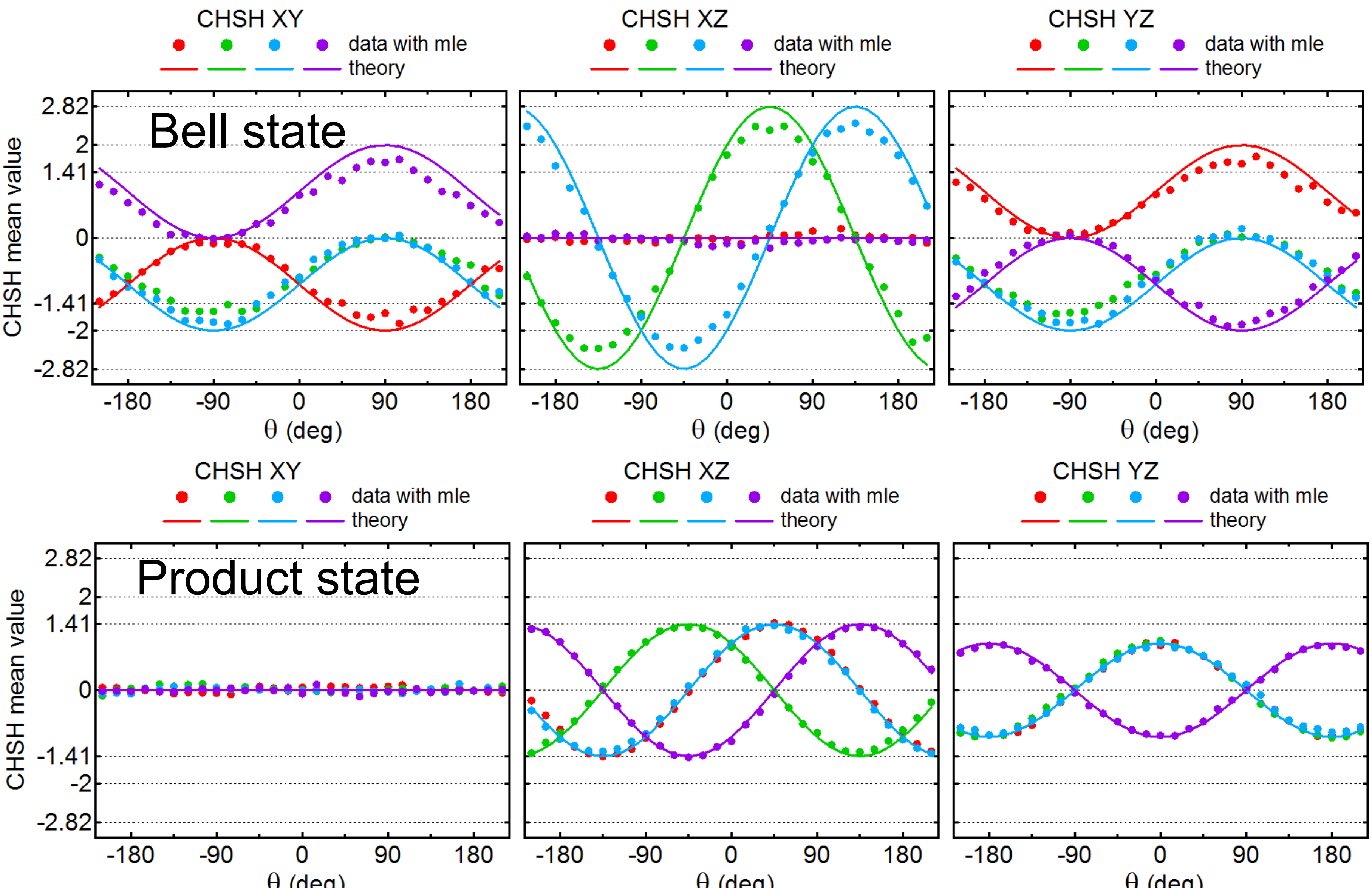


Single Oscillation



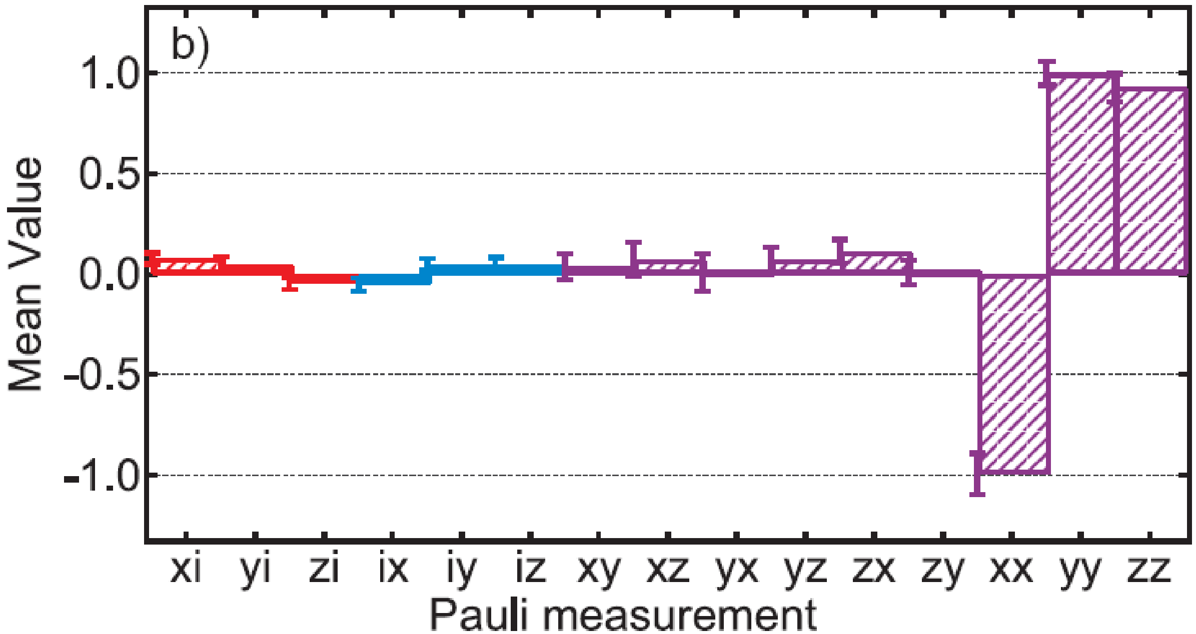


Witnessing Entanglement



Measuring the Two-Qubit State

Now apply a two-qubit gate to entangle the qubits



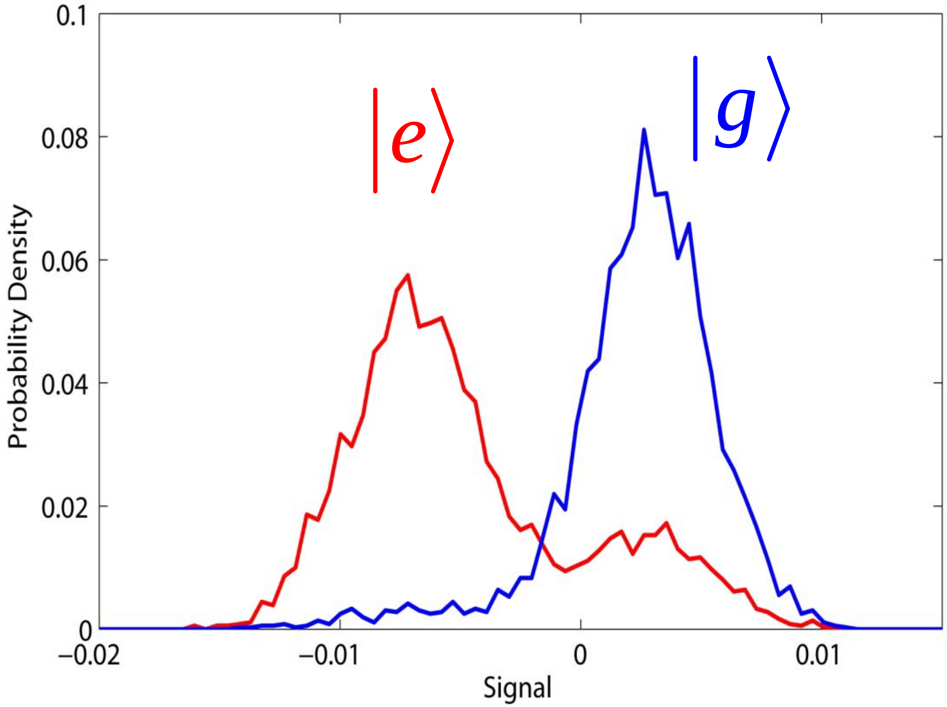
Concurrence ***directly***:
(for pure states)

$$C = \sqrt{\frac{Q-1}{2}}$$

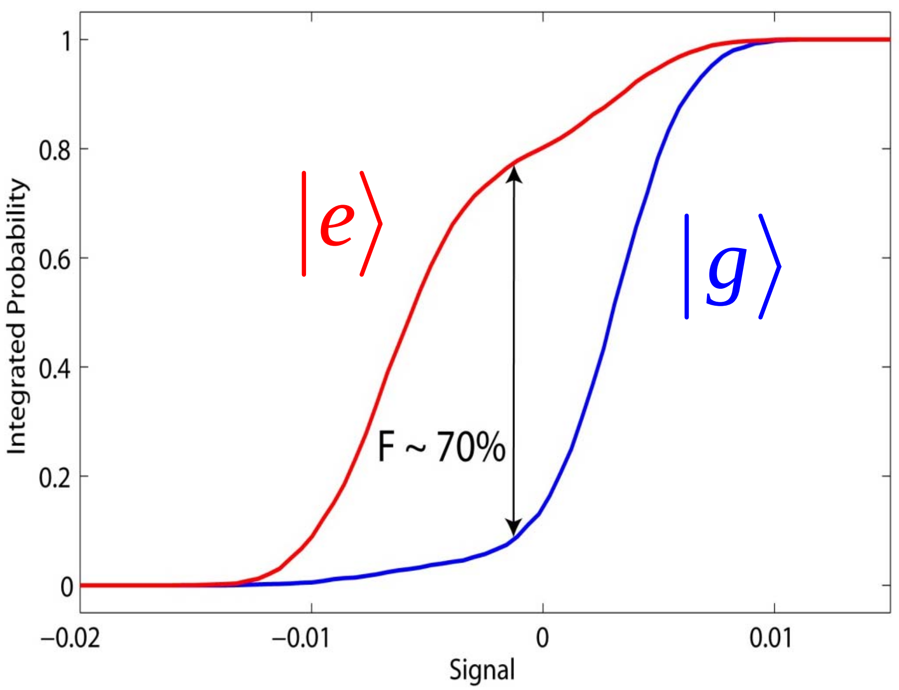
$$Q = \langle XX \rangle^2 + \langle XY \rangle^2 + \langle XZ \rangle^2 + \langle YX \rangle^2 + \langle YY \rangle^2 + \ldots + \langle ZZ \rangle^2$$

Single shot readout fidelity

Histograms of single shot msmts.



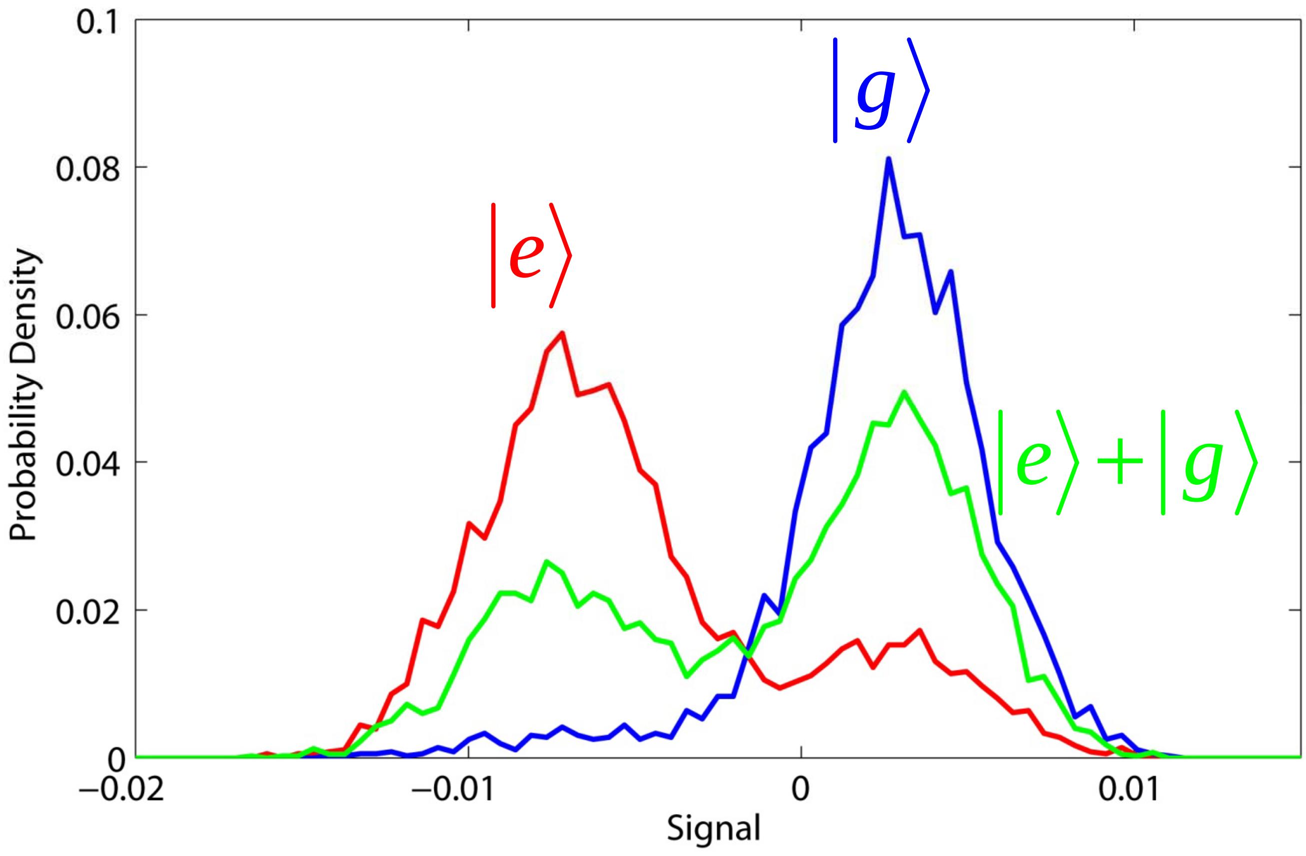
Integrated probabilities



Measurement with ~ 5 photons in cavity;
SNR ~ 4 in one qubit lifetime (T_1)

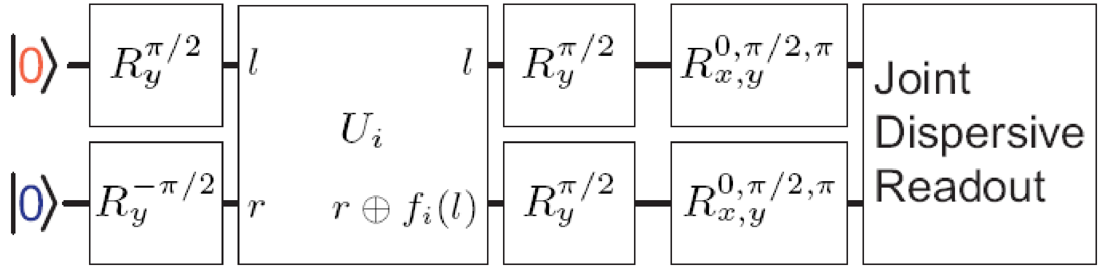
T1 ~ 300 ns, low Q cavity on sapphire

Projective measurement

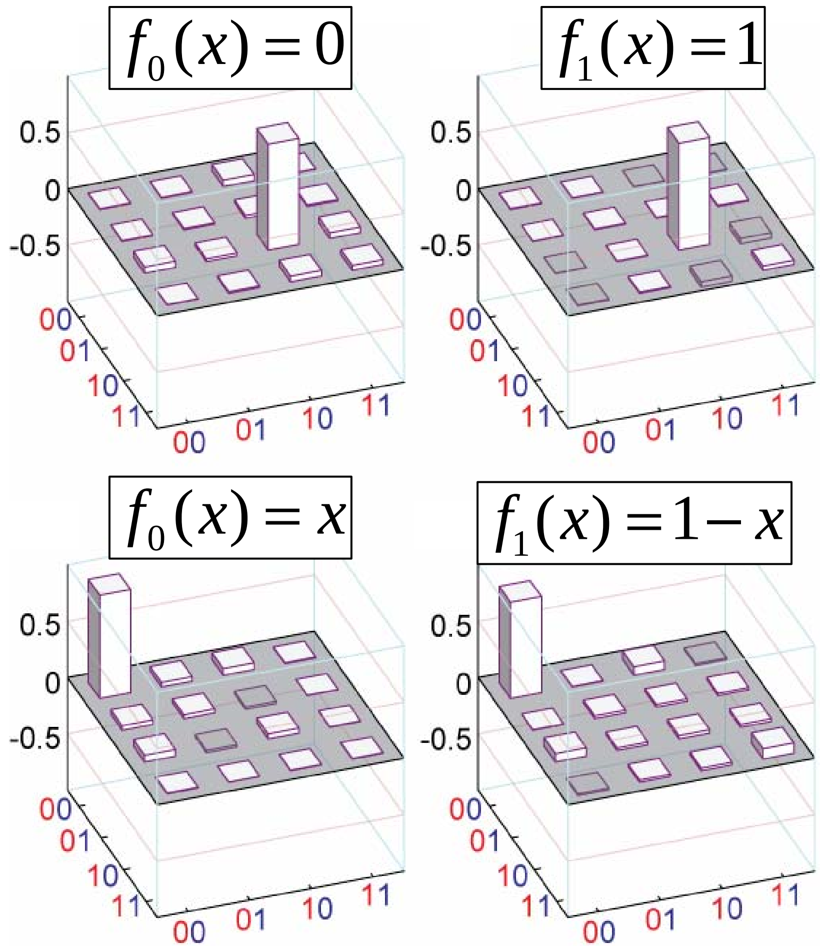


- Measurement after pi/2 pulse bimodal, halfway between

Deutsch-Jozsa Algorithm



Constant functions →



→ $|10\rangle$

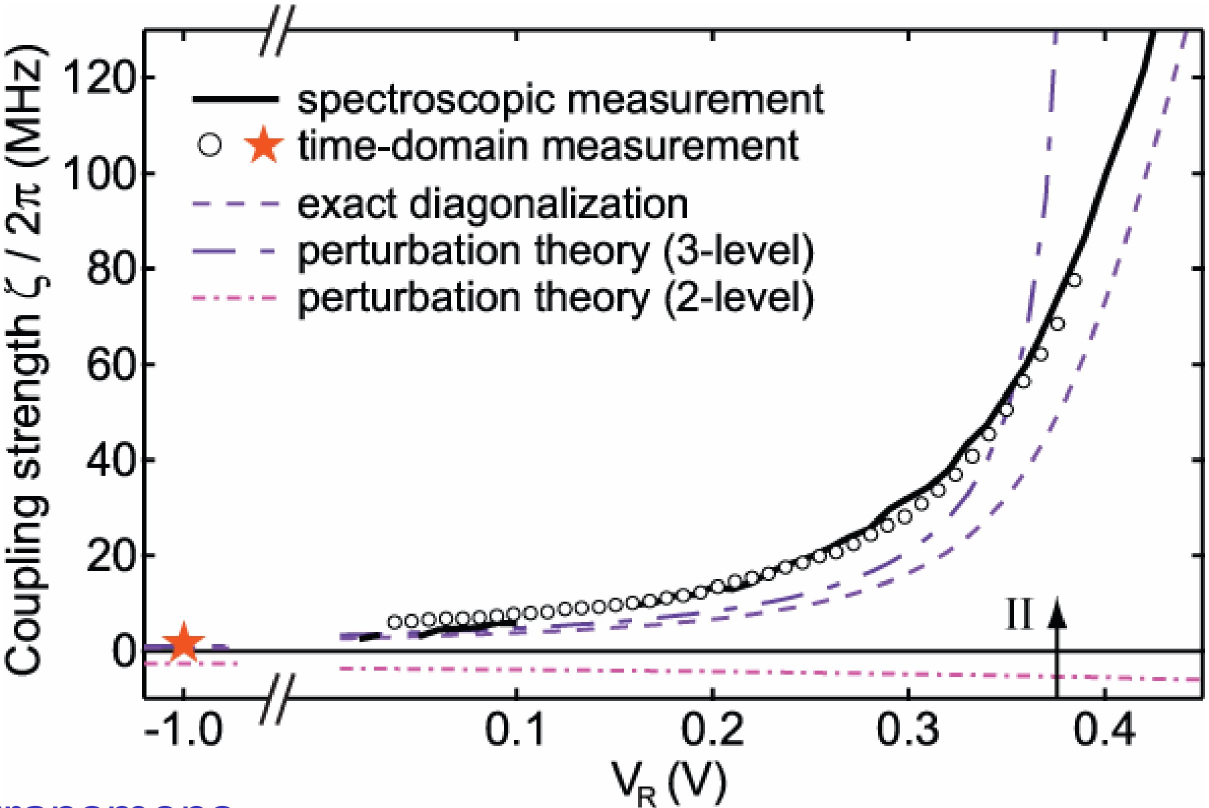
Answer is encoded in the state of left qubit

Balanced functions →

→ $|00\rangle$

The correct answer is found **>84%** of the time.

On/Off Ratio for Two-Qubit Coupling



← > 100 MHz

← 1 MHz

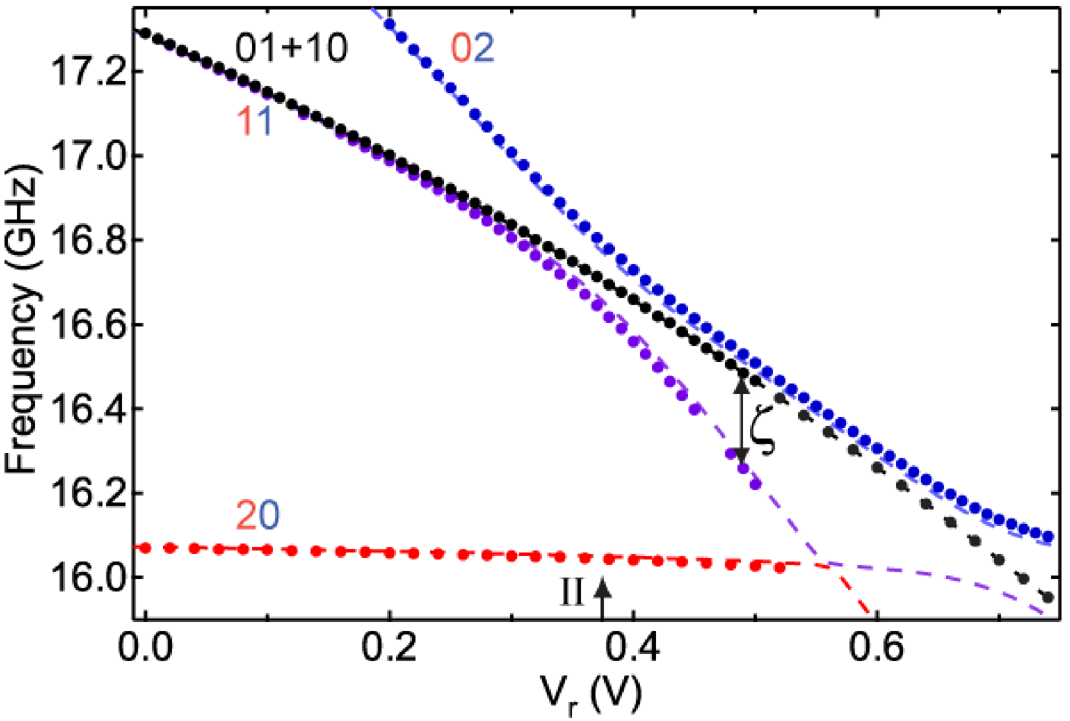
3-level transmons

$$\zeta = -2g_{\mathrm{L}}^{2}g_{\mathrm{R}}^{2}\left(\frac{1}{\left(\omega_{01}^{\mathrm{L}}-\omega_{\mathrm{C}}\right)\left(\omega_{01}^{\mathrm{R}}-\omega_{\mathrm{C}}\right)^{2}}+\frac{1}{\left(\omega_{01}^{\mathrm{R}}-\omega_{\mathrm{C}}\right)\left(\omega_{01}^{\mathrm{L}}-\omega_{\mathrm{C}}\right)^{2}}+\frac{1}{\left(\omega_{01}^{\mathrm{R}}-\omega_{12}^{\mathrm{L}}\right)\left(\omega_{01}^{\mathrm{L}}-\omega_{\mathrm{C}}\right)^{2}}+\frac{1}{\left(\omega_{01}^{\mathrm{L}}-\omega_{12}^{\mathrm{R}}\right)\left(\omega_{01}^{\mathrm{R}}-\omega_{\mathrm{C}}\right)^{2}}\right)$$

Diverges at Point II

4^{th}-order in qubit-cavity coupling!

Adiabatic Conditional Phase Gate



- Avoided crossing (160 MHz)

$$|11\rangle \leftrightarrow |02\rangle$$

- A frequency shift

$$\zeta/2\pi = f_{01} + f_{10} - f_{11}$$

$$1.2\,\mathrm{MHz} \leqslant \zeta/2\pi \lesssim 150\,\mathrm{MHz}$$

On/off ratio ≈ 100:1

Use large on-off ratio of ζ to implement 2-qubit phase gates.

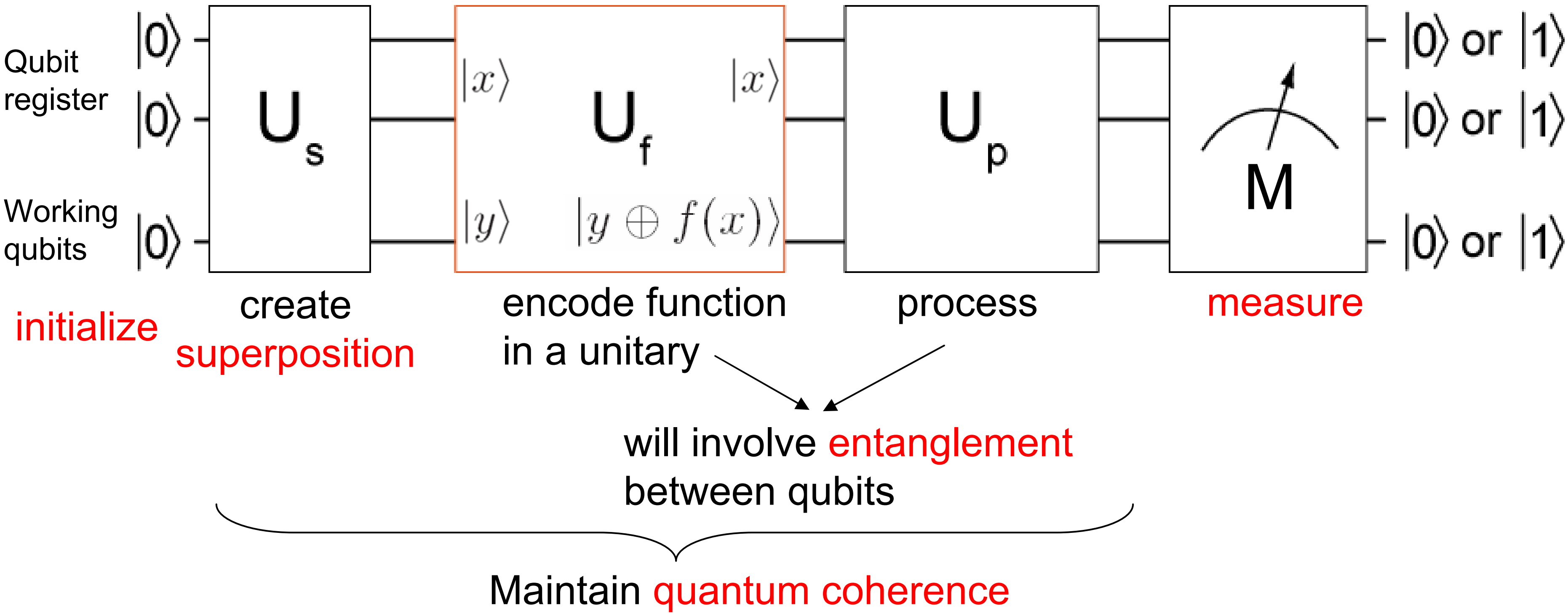
$$\int \zeta(t)\,\mathrm{d}t = (2n+1)\pi$$

Strauch et al. *PRL* (2003): proposed use of excited states in phase qubits

Adjust timing so that amplitude for both qubits to be excited acquires a minus sign:

$$\begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \left|\Psi\right\rangle = \frac{1}{2}\left(\left|00\right\rangle + \left|10\right\rangle + \left|01\right\rangle - \left|11\right\rangle\right)$$

General Features of a Quantum Algorithm



1) Start in superposition: all values at once!
2) Build complex transformation out of one-qubit and two-qubit "gates"
3) Somehow* make the answer we want result in a definite state at end!

*use interference: the magic of the properly designed algorithm