Cross Kerr Non-linearity and other tricks with the superconducting ring resonator

David DiVincenzo, IBM
KITP, 12/2009

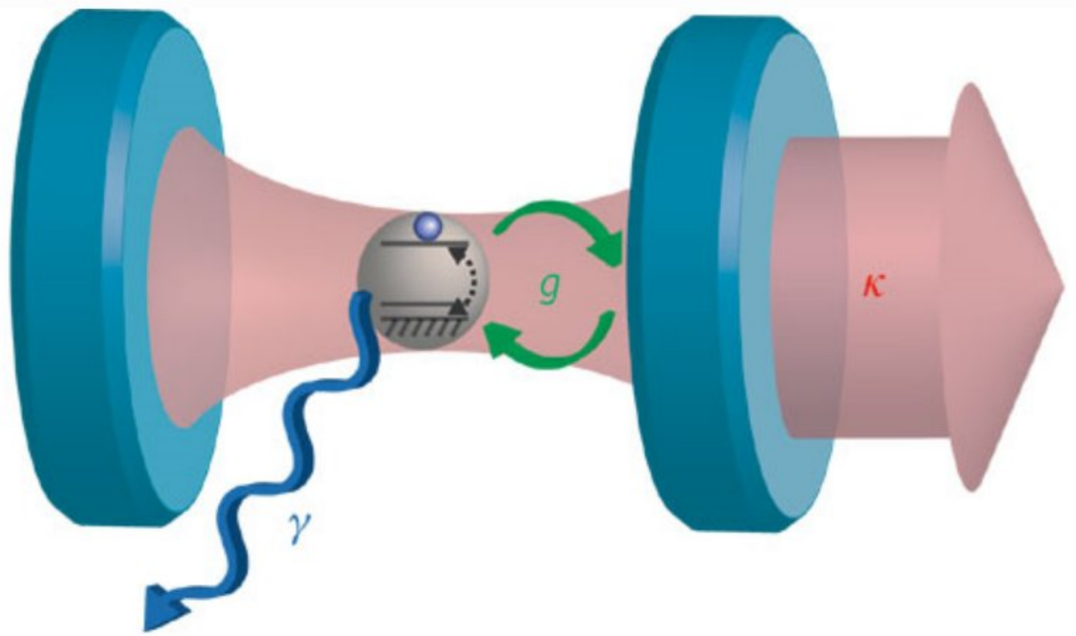
With Shwetank Kumar, IBM

Also: Luca Chirolli
Guido Burkard

Outline

-- Circuit Quantum Electrodynamics:
  the surprising quantumness of simple
  electric circuits – a new quantum optics
-- Regimes of application:
  resonant, dispersive, **parametric**
-- Our parametric circuit: ring resonator
-- nondemolition quantum measurement
-- another application: beam splitter
-- Prospect: qubitless circuits
  for quantum computing

Cavity QED: A central theme of quantum optics & atomic physics

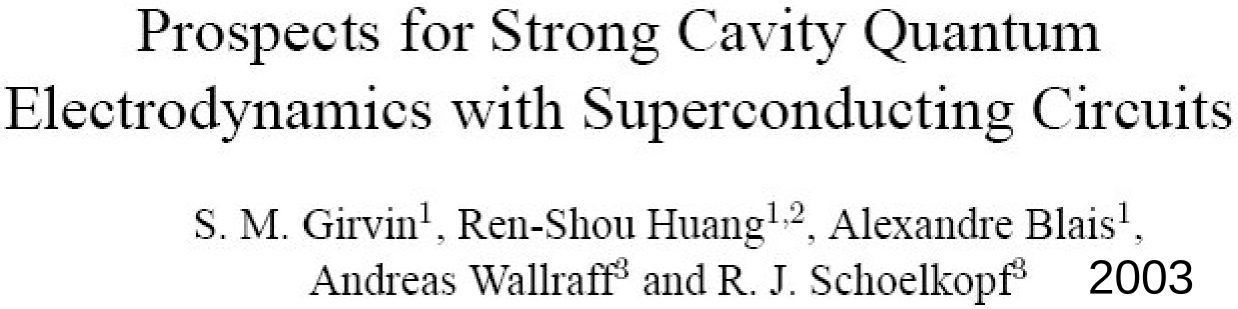


Kimble, Walther, Haroche
--heroic experiments to
achieve

Strong coupling: g >> γ, κ

-- entangled states achieved between
atoms and single photons

Cavity QED → Circuit QED



Prospects for Strong Cavity Quantum
Electrodynamics with Superconducting Circuits

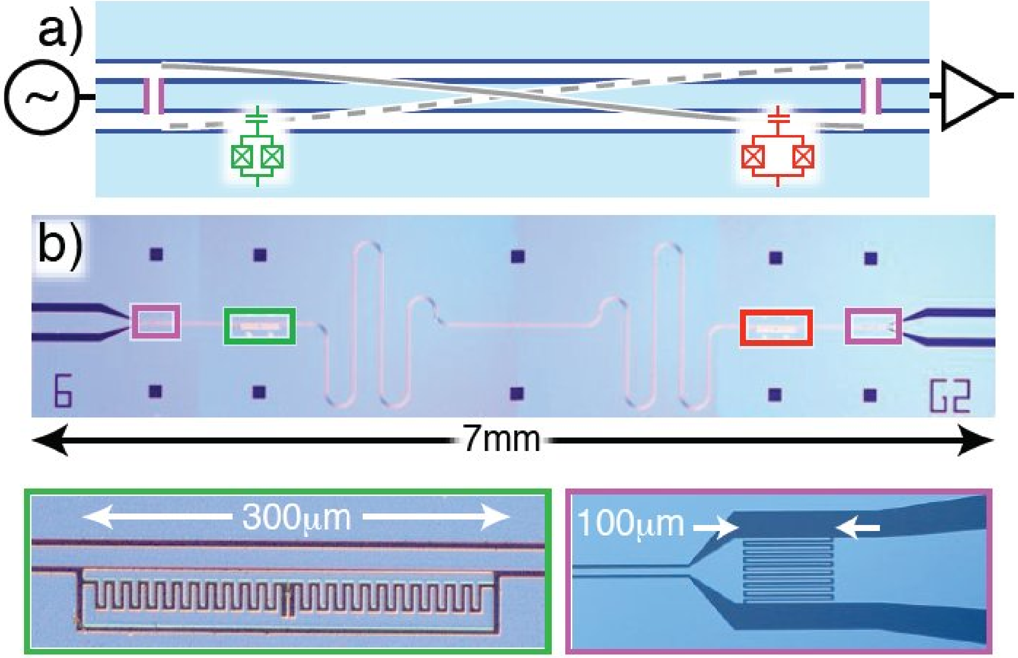
S. M. Girvin[1], Ren-Shou Huang[1,2], Alexandre Blais[1],
Andreas Wallraff[3] and R. J. Schoelkopf[3] 2003

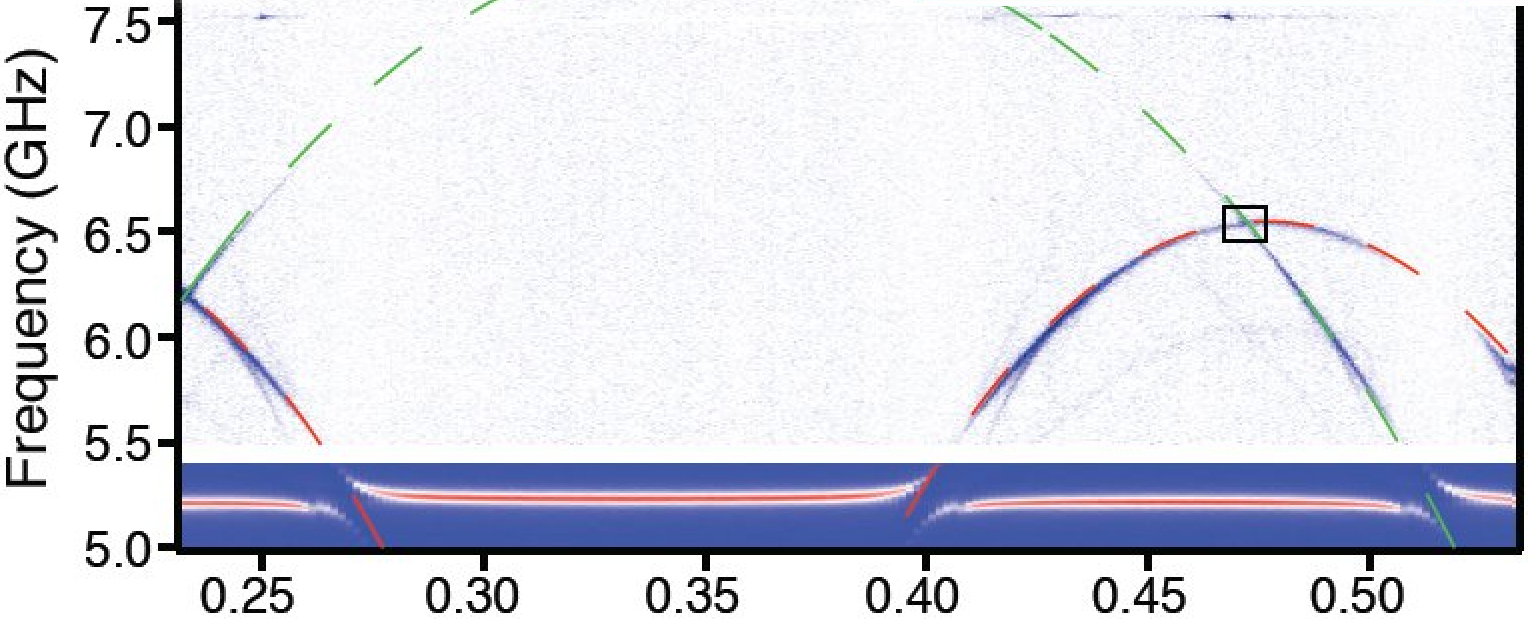
Coupling Superconducting Qubits via a Cavity Bus

Nature **449**, 443 (2007)

J. Majer,[1,2] J. M. Chow,[1,2] J. M. Gambetta,[1] Jens Koch,[1] B. R. Johnson,[1] J. A. Schreier,[1] L. Frunzio,[1]
D. I. Schuster,[1] A. A. Houck,[1] A. Wallraff,[1,3] A. Blais,[1,4] M. H. Devoret,[1] S. M. Girvin,[1] and R. J. Schoelkopf[1]

Cavity mode/qubit anticrossing
-- strong coupling immediately
achieved!

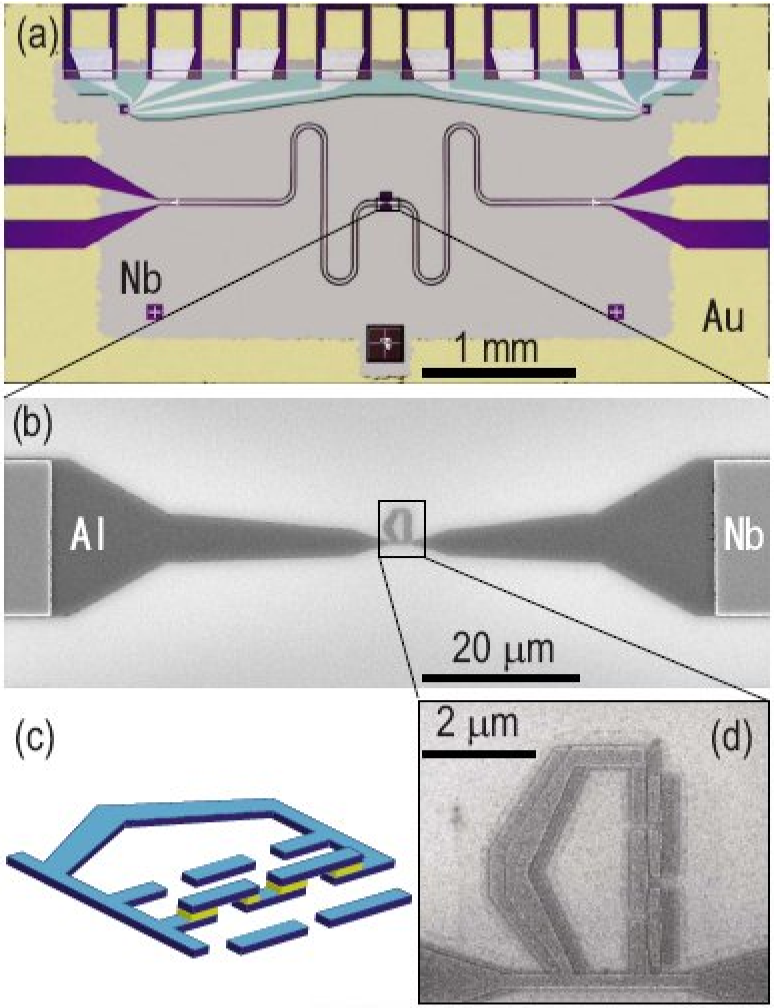


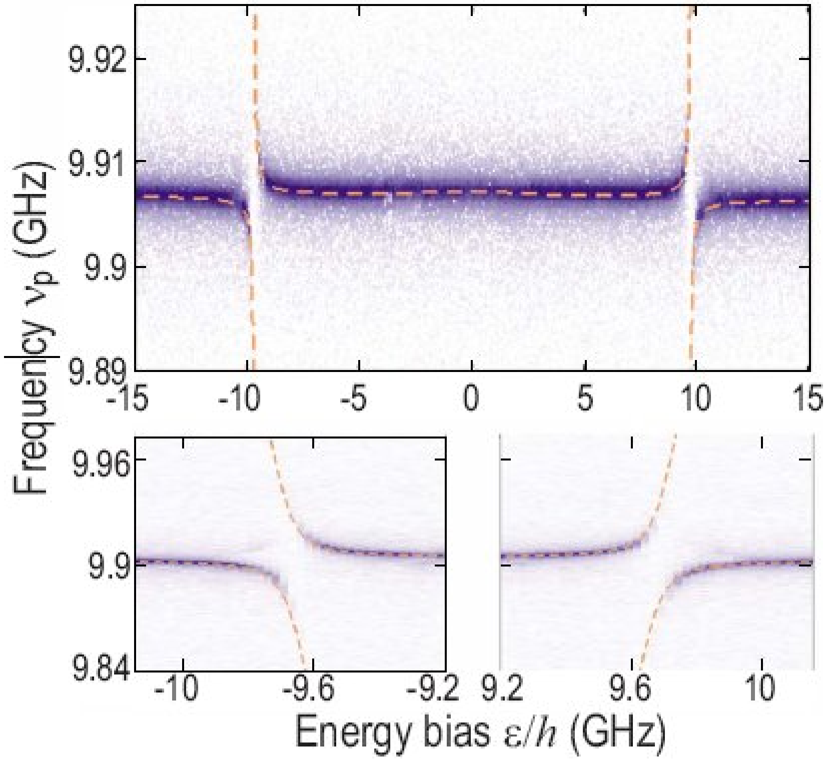


PHYSICAL REVIEW B **78**, 180502(R) (2008)

Vacuum Rabi splitting due to strong coupling of a flux qubit and a coplanar-waveguide resonator

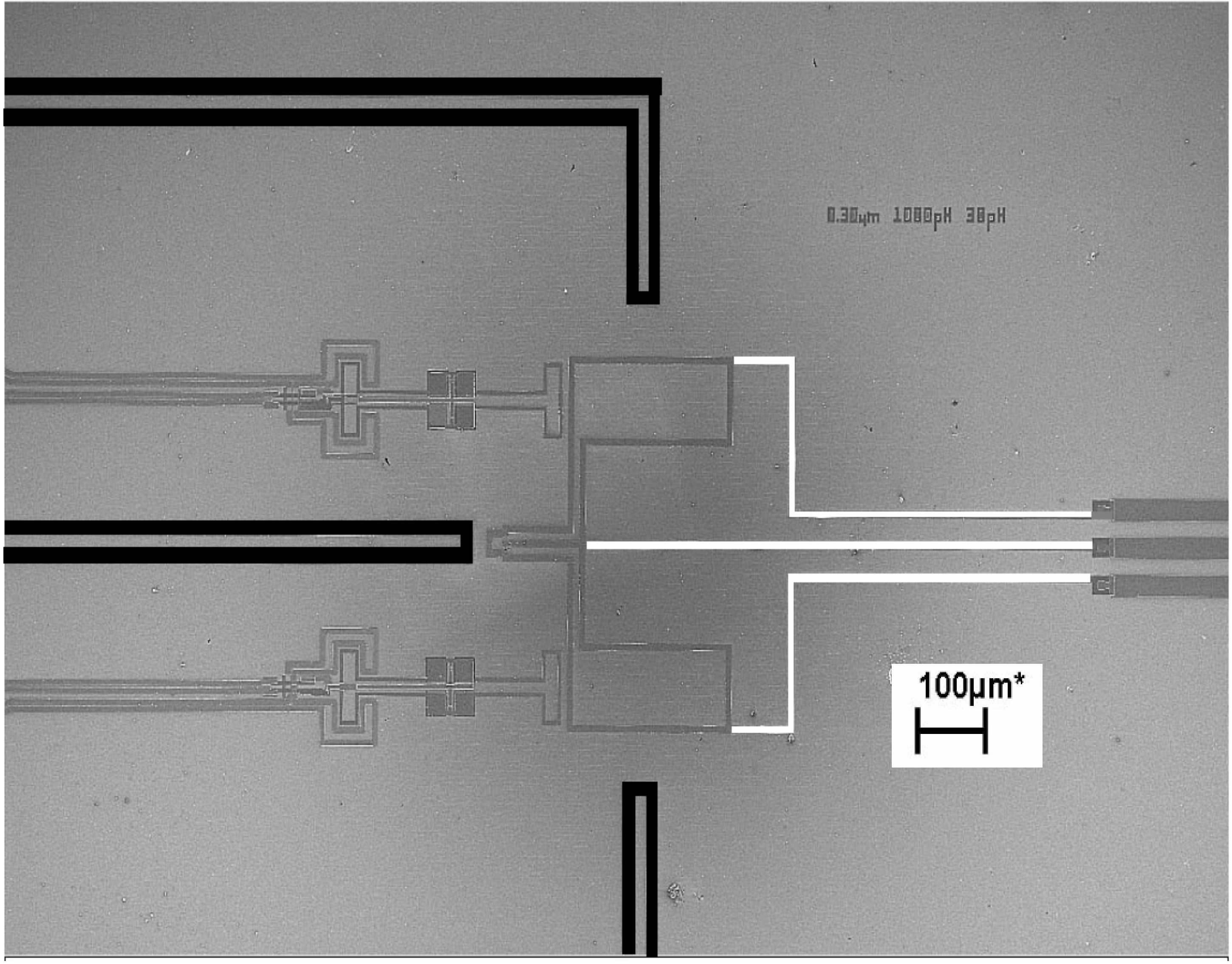
Abdufarrukh A. Abdumalikov, Jr.,[1,†,*] Oleg Astafiev,[1,2] Yasunobu Nakamura,[1,2,3]
Yuri A. Pashkin,[1,2,†] and JawShen Tsai[1,2,3]





Strong coupling seen in
“Delft” qubit

Efficient one- and two-qubit pulse gates for an oscillator stabilized Josephson flux qubit



I_c=1.3 µA

L_1=32pH

L_3=680pH

M_{1cf}=0.8pH

M_{3flux}=0.5pH

ω_T=2π 3.1 GHz

Z_0= 110 Ω

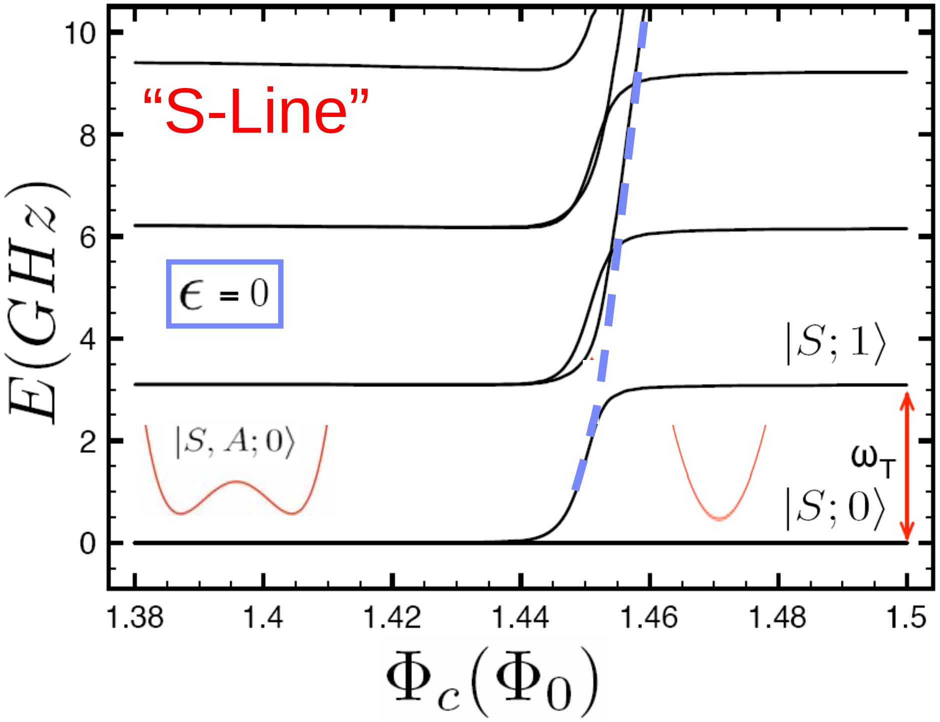
L_T=5.6 nH

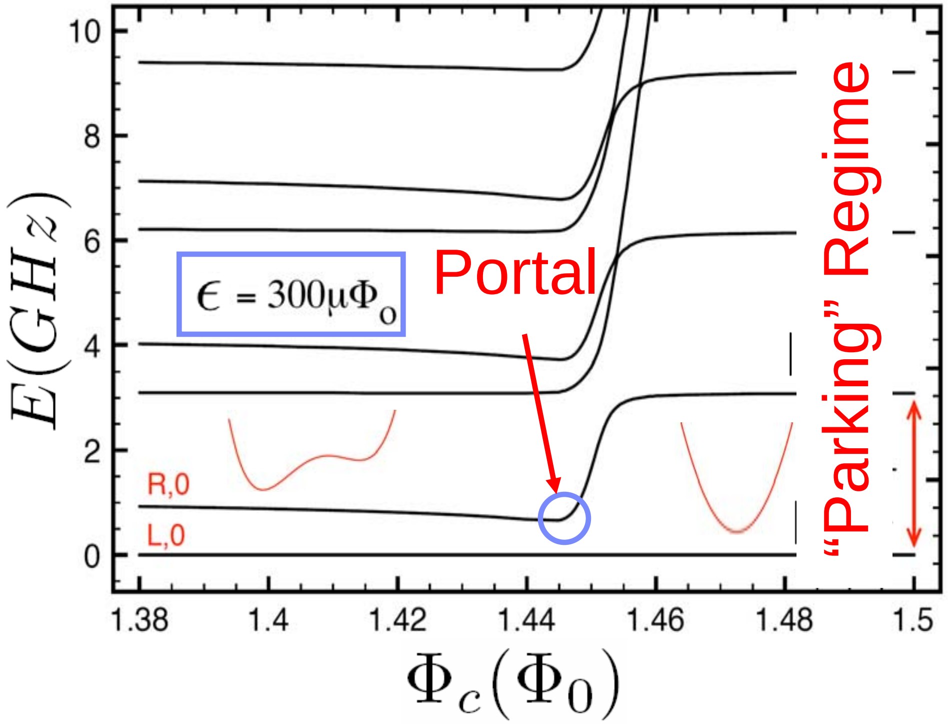
M_{qT}=200 pH

.R. H. Koch, J. R. Rozen, G. A. Keefe, F. M. Milliken, C. C. Tsuei, J. R. Kirtley, and D. P. DiVincenzo, “Low-bandwidth control scheme for an oscillator stabilized Josephson qubit,” cond-mat/0411380, Phys. Rev. B 72, 092512 (2005).

- Level Dynamics

$$H = -\frac{1}{2}\Delta(\Phi_c)\hat{\sigma}_x + \frac{1}{2}\epsilon b(\Phi_c)\hat{\sigma}_z + \hbar\omega_T \hat{a}^\dagger \hat{a} \xcancel{+ g(\Phi_c)(\hat{a} + \hat{a}^\dagger)\hat{\sigma}_z}$$

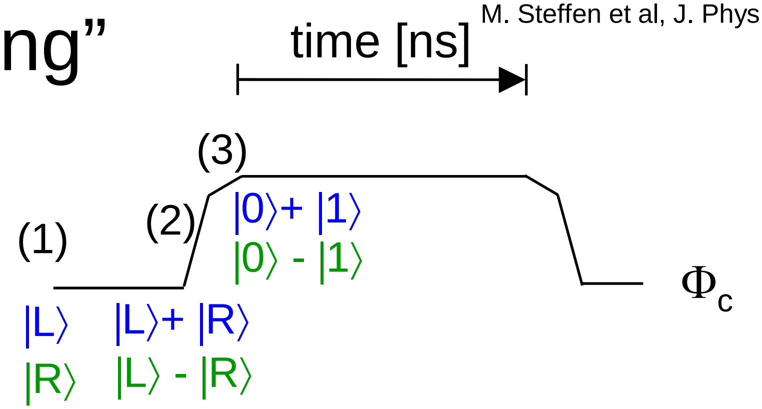


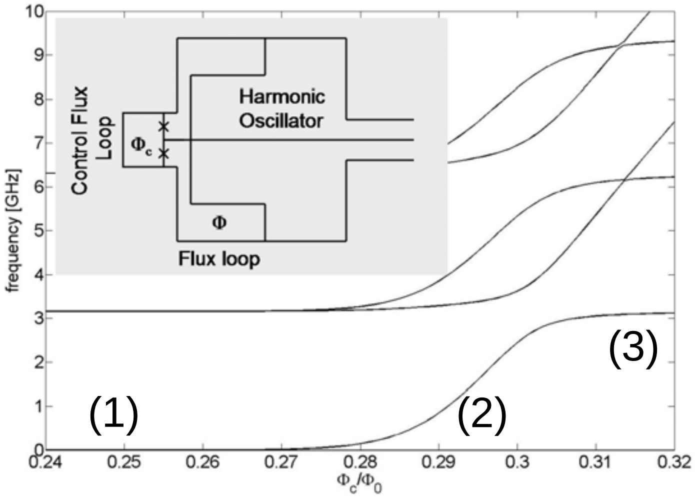


Follow-up Experiment, March 2007 (unpublished)

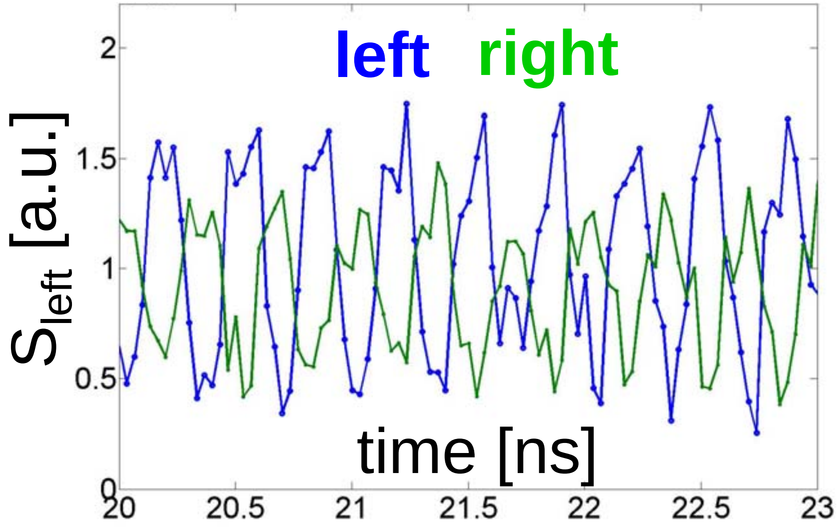
“parking”

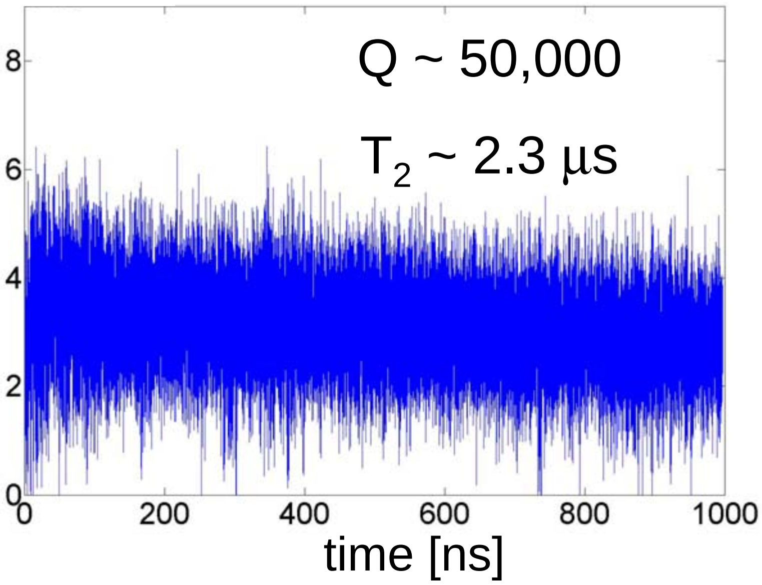
M. Steffen et al, J. Phys. CM (2010)





Should observe “Larmor precessions” which measure quality of harmonic oscillator





R. H. Koch, J. R. Rozen, G. A. Keefe, F. M. Milliken, C. C. Tsuei, J. R. Kirtley, and D. P. DiVincenzo, “Experimental observation of an oscillator stabilized Josephson qubit,” Phys. Rev. Lett. **96**, 127001 (2006).

Two styles of circuit QED

Resonant/dispersive

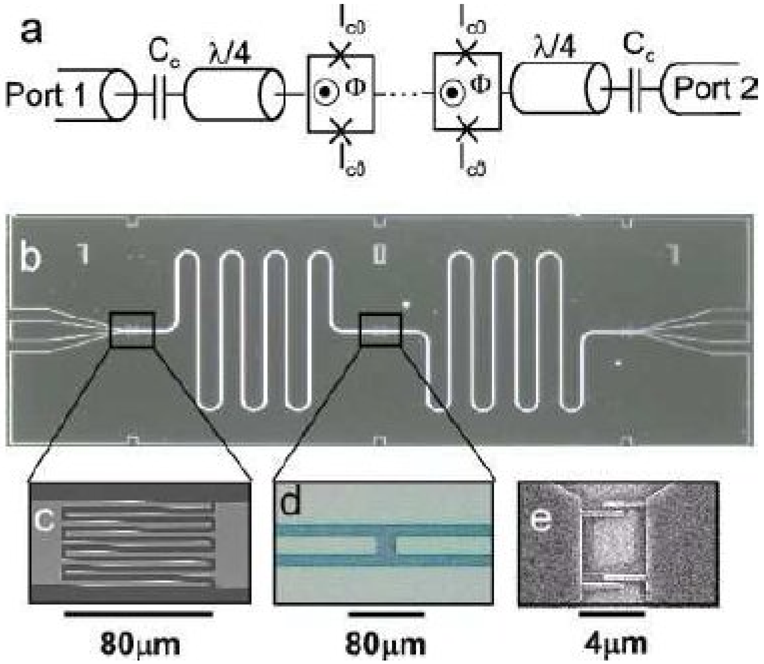
-- qubit levels coupled to resonator modes
-- “low impedance”
-- shares decoherence problems of qubits
-- many strong-coupling QED effects observed

Parametric

-- no qubits
--”moderate/high impedance”
-- coherence determined by resonator physics (largely)
-- various electrodynamic effects observed
-- what about quantum?

Tunable resonators for quantum circuits 2008

A. Palacios-Laloy · F. Nguyen · F. Mallet ·
P. Bertet · D. Vion · D. Esteve



--SQUID inductance tuned from <<50Ω to
$\lesssim$ 50Ω by flux biassing
--SQUID is not a qubit! No quantum states
of its own. Capacitance of SQUID irrelevant

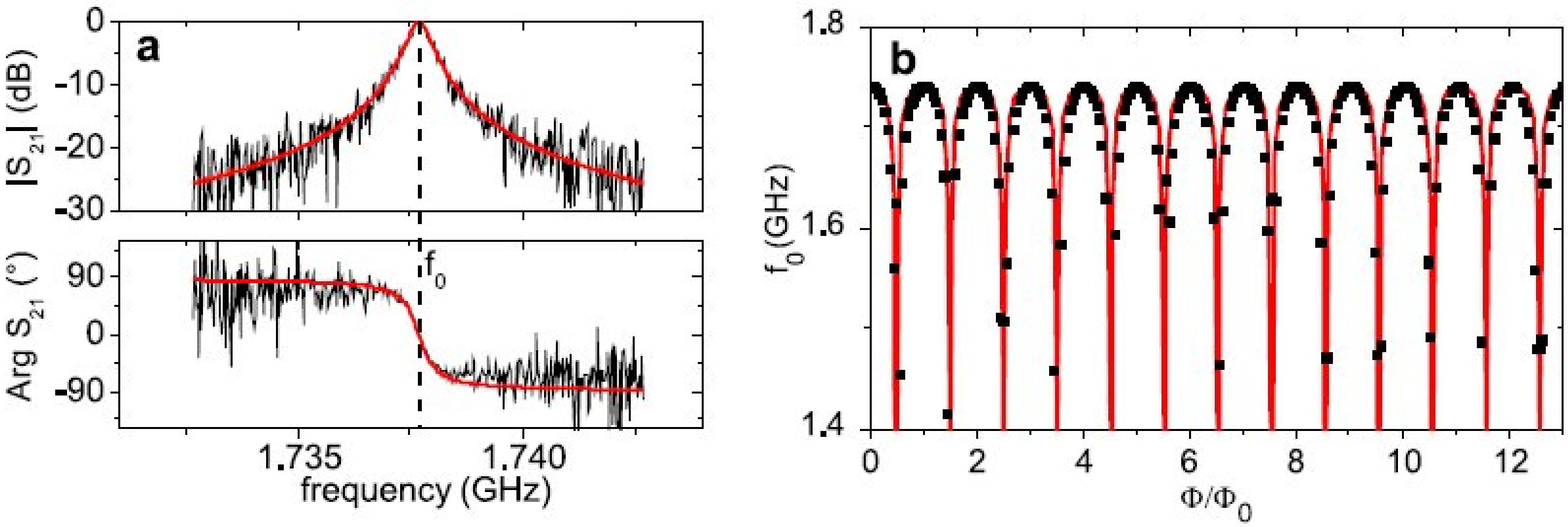
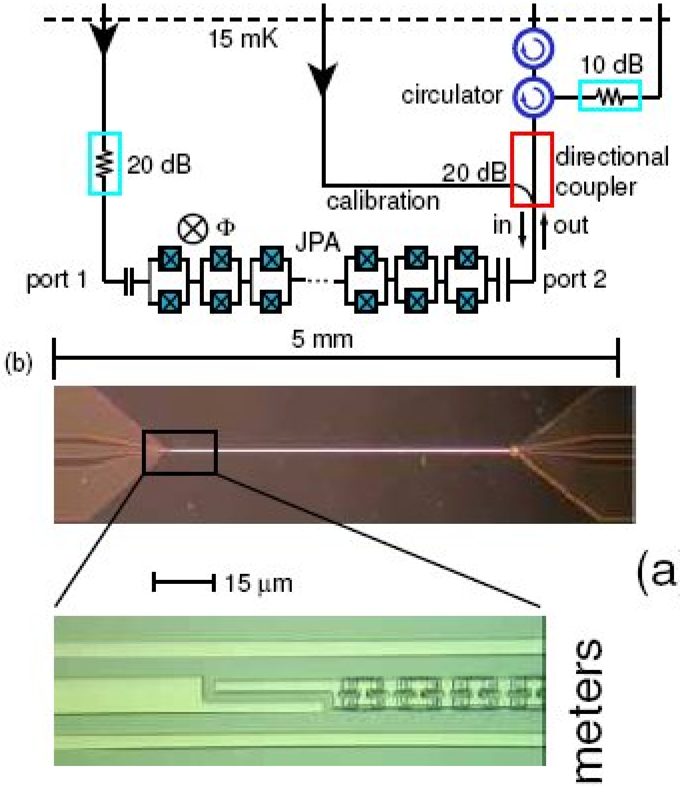


Fig. 4 (color online) **a:** Measured (thin line) amplitude (top) and phase (bottom) transmission of sample A for $\Phi = 0$ and fit (bold line) yielding a quality factor $Q = 3300$. **b:** Measured resonance frequency of sample A (squares) as a function of applied magnetic flux and corresponding fit (full line) according to Eq. 7.

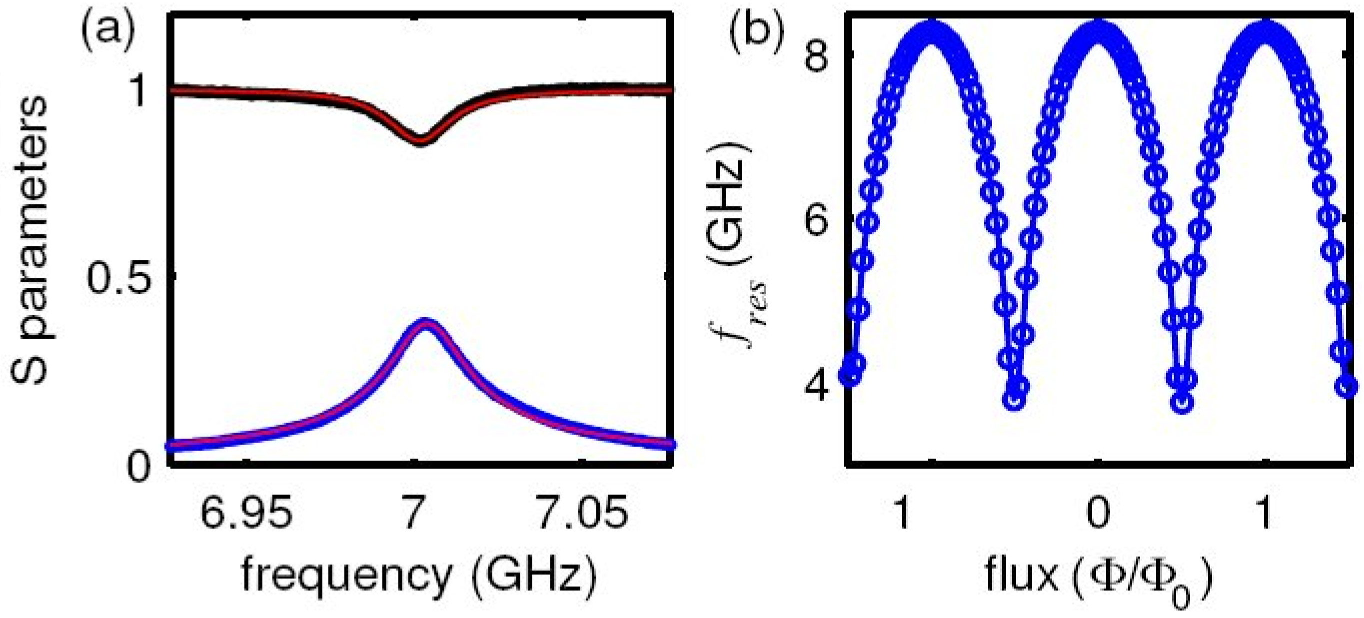
Amplification and squeezing of quantum noise with a tunable Josephson metamaterial

2008

M. A. Castellanos-Beltran,[1,*] K. D. Irwin,[2] G. C. Hilton,[2] L. R. Vale,[2] and K. W. Lehnert[1]



-- parametric effect and nonlinearity of impedance to produce low-noise parametric amplifier

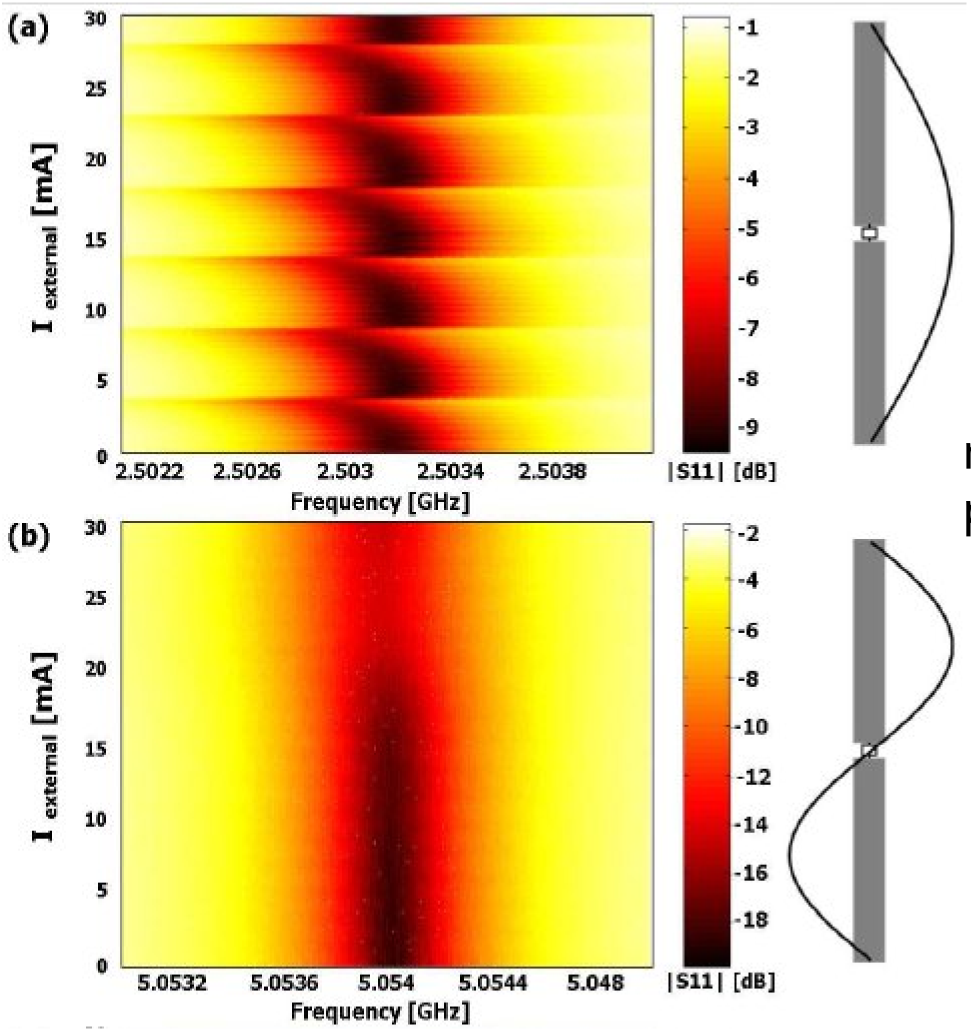


2009

Intermode Dephasing in a Superconducting Stripline Resonator

Oren Suchoi,[1] Baleegh Abdo,[1] Eran Segev,[1] Oleg Shtempluck,[1] M. P. Blencowe,[2] and Eyal Buks[1]

Flux bias



modal current profiles

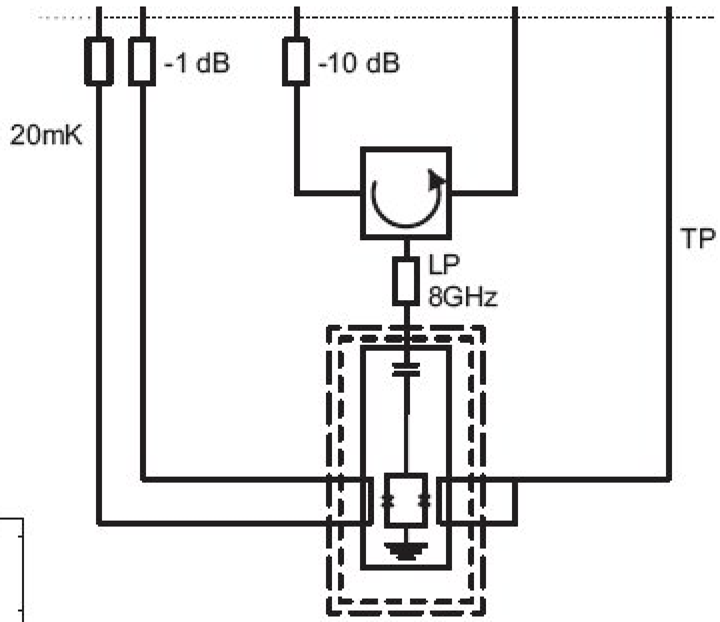
Parametric effect
is mode-dependent

Fast tuning of superconducting microwave cavities

2008

M. Sandberg, C. M. Wilson, F. Persson, G. Johansson, V. Shumeiko, T. Bauch, and P. Delsing
Department of Microtechnology and Nanoscience, Chalmers University of Technology.

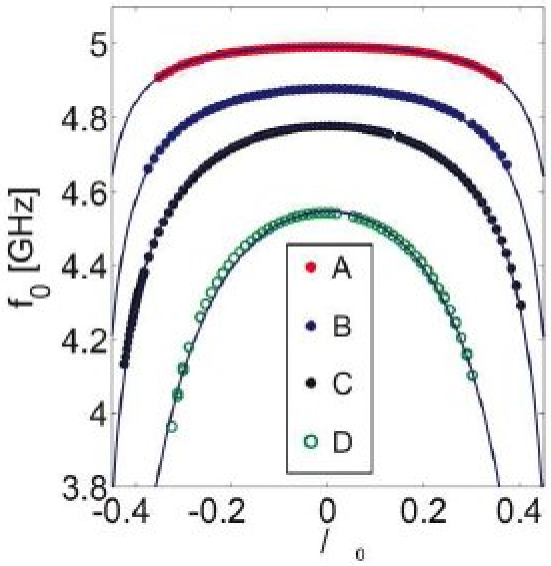
T. Duty
University of Queensland, School of Physical Sciences, Brisbane, QLD 4072 Australia.

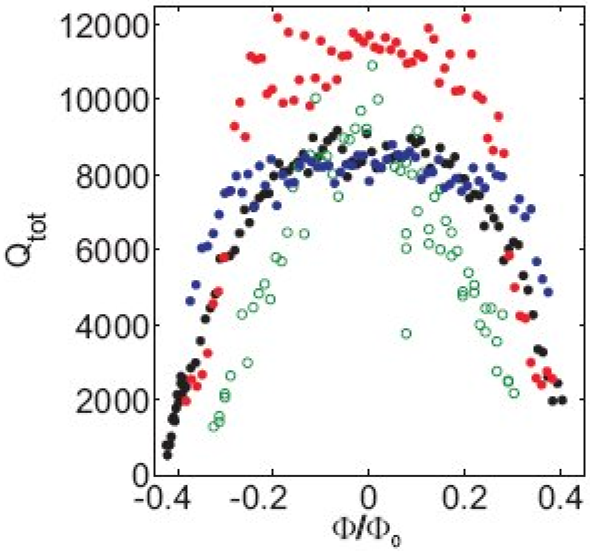


Q factor corresponds to decoherence time
of 2μsec.

20μsec may be feasible

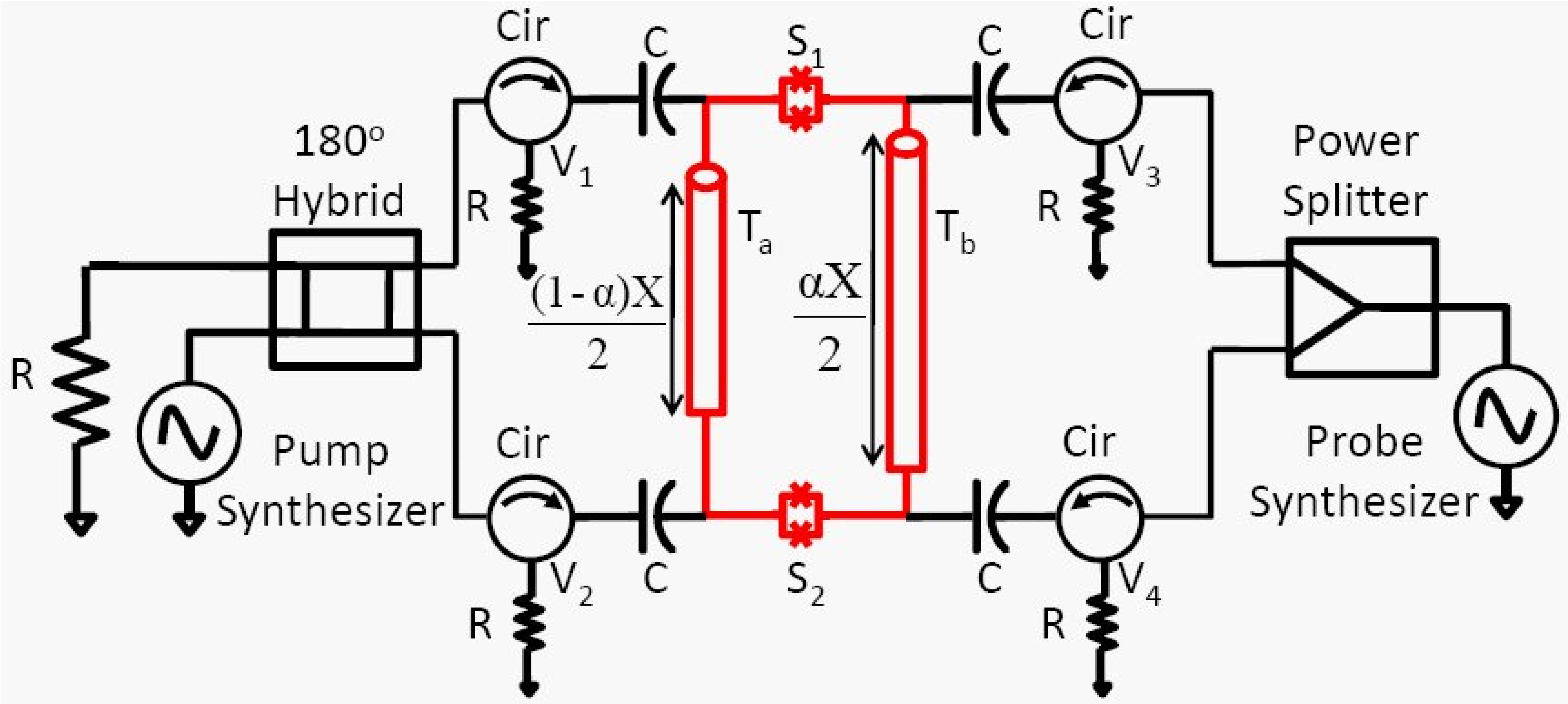
But, what is quantum?





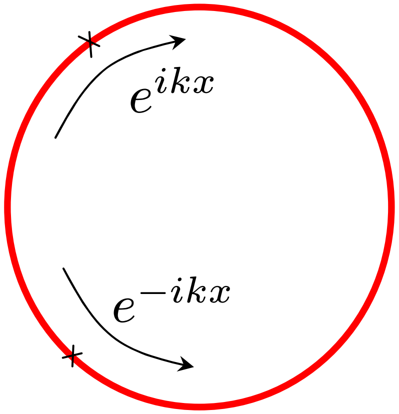
Exploiting Kerr Cross Non-linearity in Circuit Quantum Electrodynamics for Non-demolition Measurements

Shwetank Kumar and David P. DiVincenzo* arXiv:0906:2979

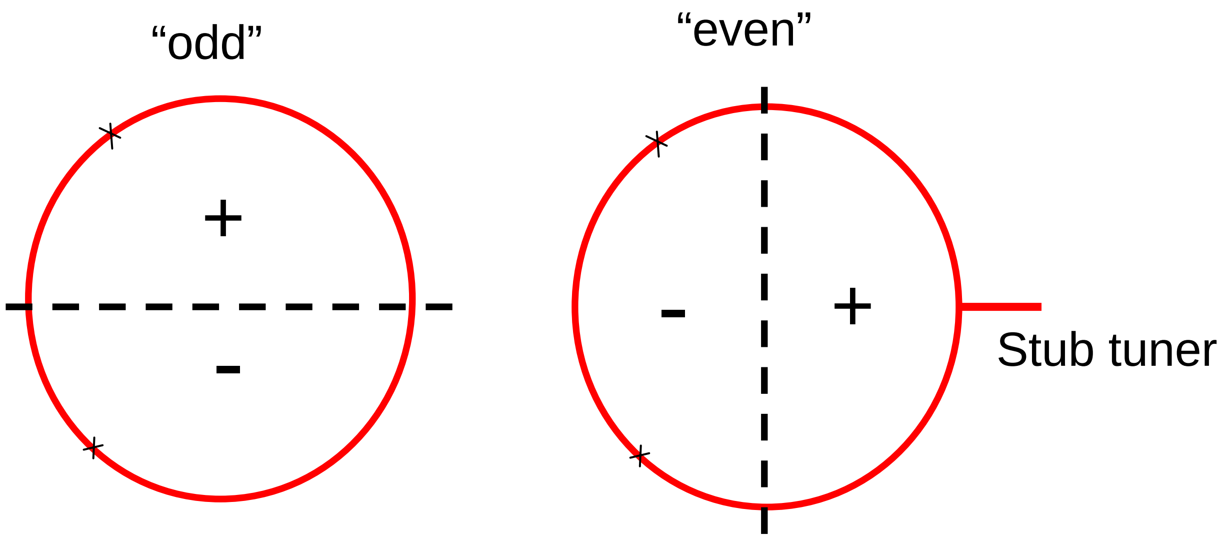


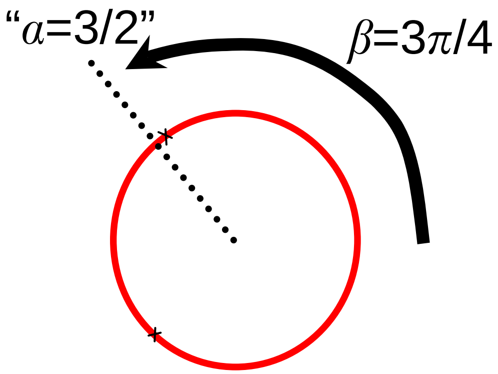
In red: ring resonator – different topology from all before

Nearly degenerate modes of ring resonator:



Degeneracy lifted by SQUID impedance: $f_{o,e} = f_0 \,\S\, f_J \cos(\pi\alpha)$





Stub tuner can further manipulate mode frequencies

Two-mode Hamiltonian, with lowest-order effect of nonlinear inductance of SQUID

$$\beta = \pi\alpha/2$$

$$H_{total} = E_{Ce}n_e^2 + E_{Le}\phi_e^2 + E_{Co}n_o^2 + E_{Lo}\phi_o^2 + \frac{16\pi^2 L_{sq1}}{\Phi_0^4}\left[E_{Le}^4\phi_e^4\cos^4\beta + E_{Lo}^4\phi_o^4\sin^4\beta + 6E_{Le}^2E_{Lo}^2\phi_e^2\phi_o^2\sin^2\beta\cos^2\beta\right]$$

Quantized, in rotating-wave approximation:

$$H_{total}^{RW} = \hbar\omega_e a_e^\dagger a_e + \hbar\omega_o a_o^\dagger a_o + \frac{4\pi^2 L_{sq1}}{\Phi_0^4}\Big\{12\sqrt{E_{Le}^3E_{Lo}^3E_{Ce}E_{Co}}\left[2a_e^\dagger a_e a_o^\dagger a_o - a_o^\dagger a_o - a_e^\dagger a_e\right]\sin^2\beta\cos^2\beta + E_{Le}^3E_{Ce}\left[(a_e^\dagger a_e)^2 - 6a_e^\dagger a_e\right]\cos^4\beta + E_{Lo}^3E_{Co}\left[(a_o^\dagger a_o)^2 - 6a_o^\dagger a_o\right]\sin^4\beta\Big\}$$

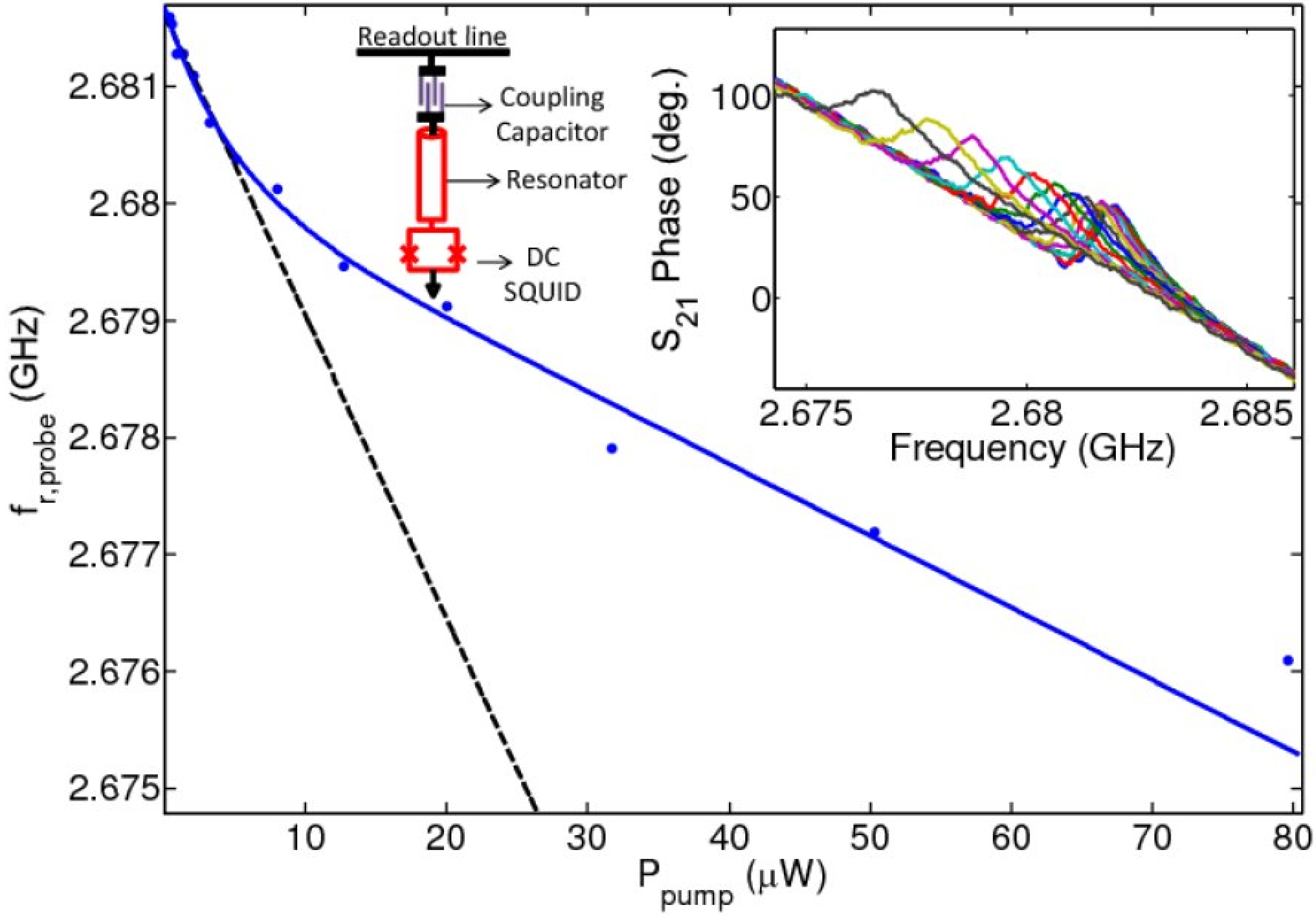
Many terms vanish because of the symmetry of the structure, e.g., $\phi_e\phi_o^3$

Mode frequencies not commensurate → no multiphoton processes

Most important term: cross Kerr nonlinearity

Simple preliminary experiment:
Kerr shift of probe frequency easily seen

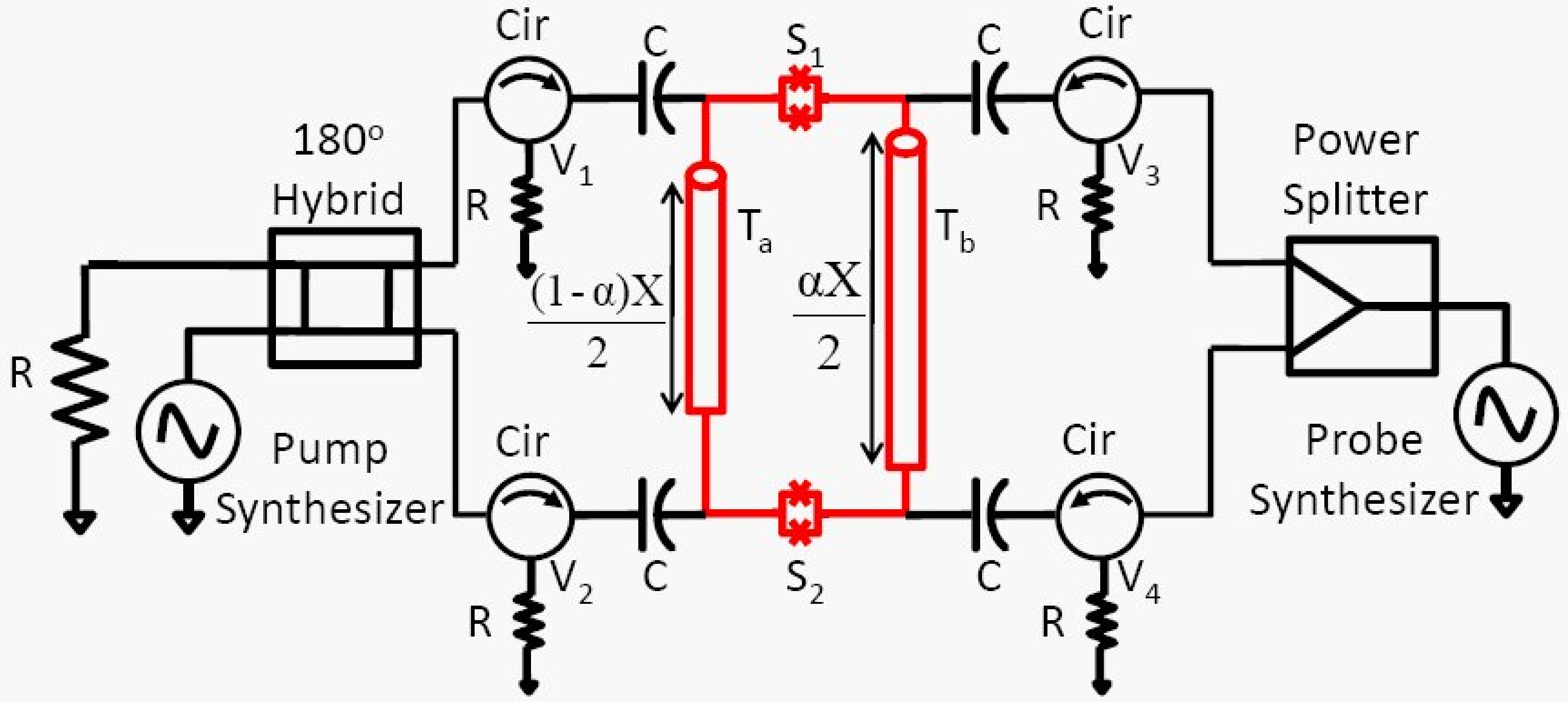
Ni resonator, Al SQUID
350 mK



Exploiting Kerr Cross Non-linearity in Circuit Quantum Electrodynamics for Non-demolition Measurements

Shwetank Kumar and David P. DiVincenzo*

arXiv:0906:2979

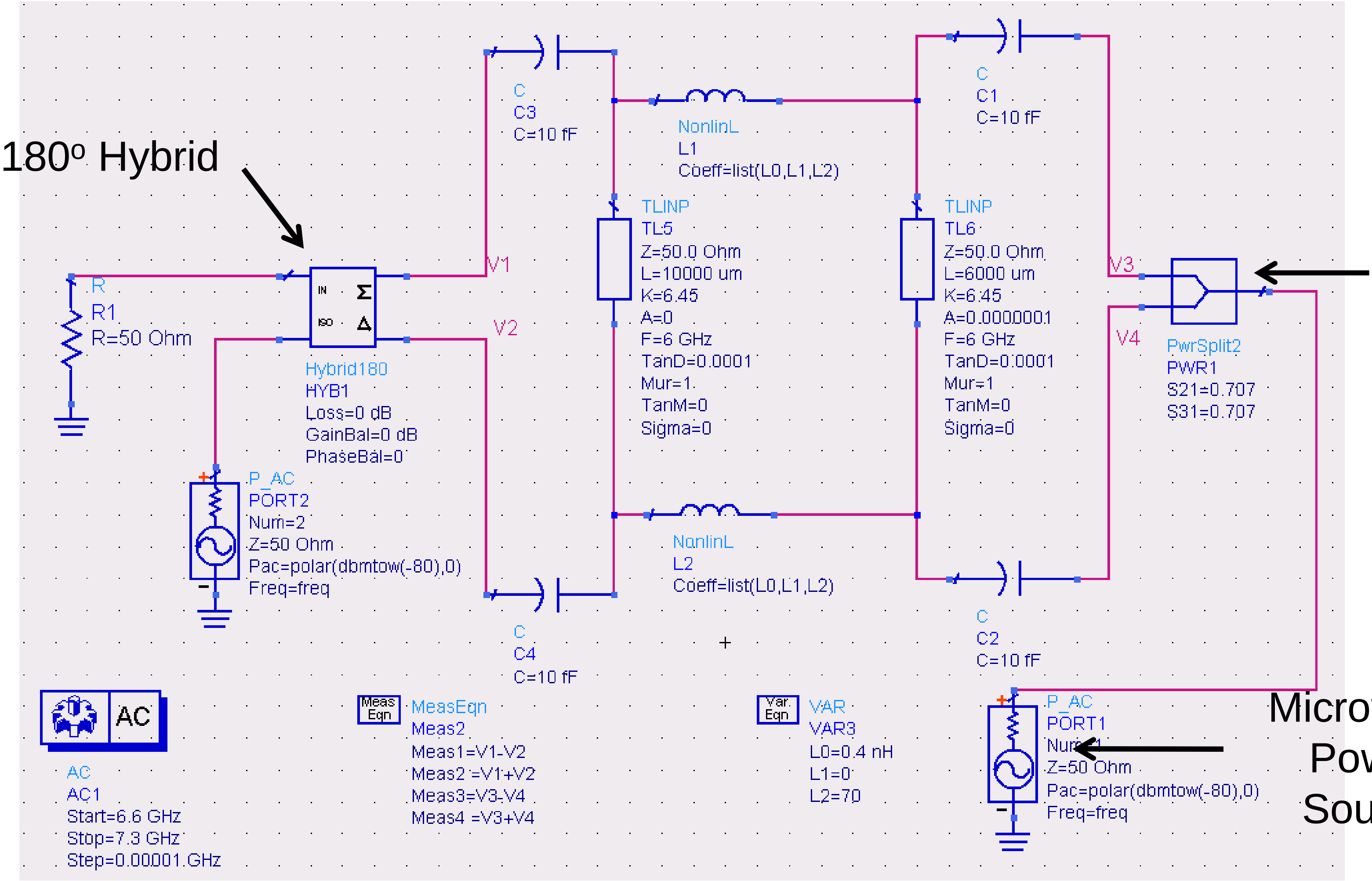


Pump excites only odd mode
Probe excites only even mode

Lossy TRline Model

Agilent ADS simulations – very realistic

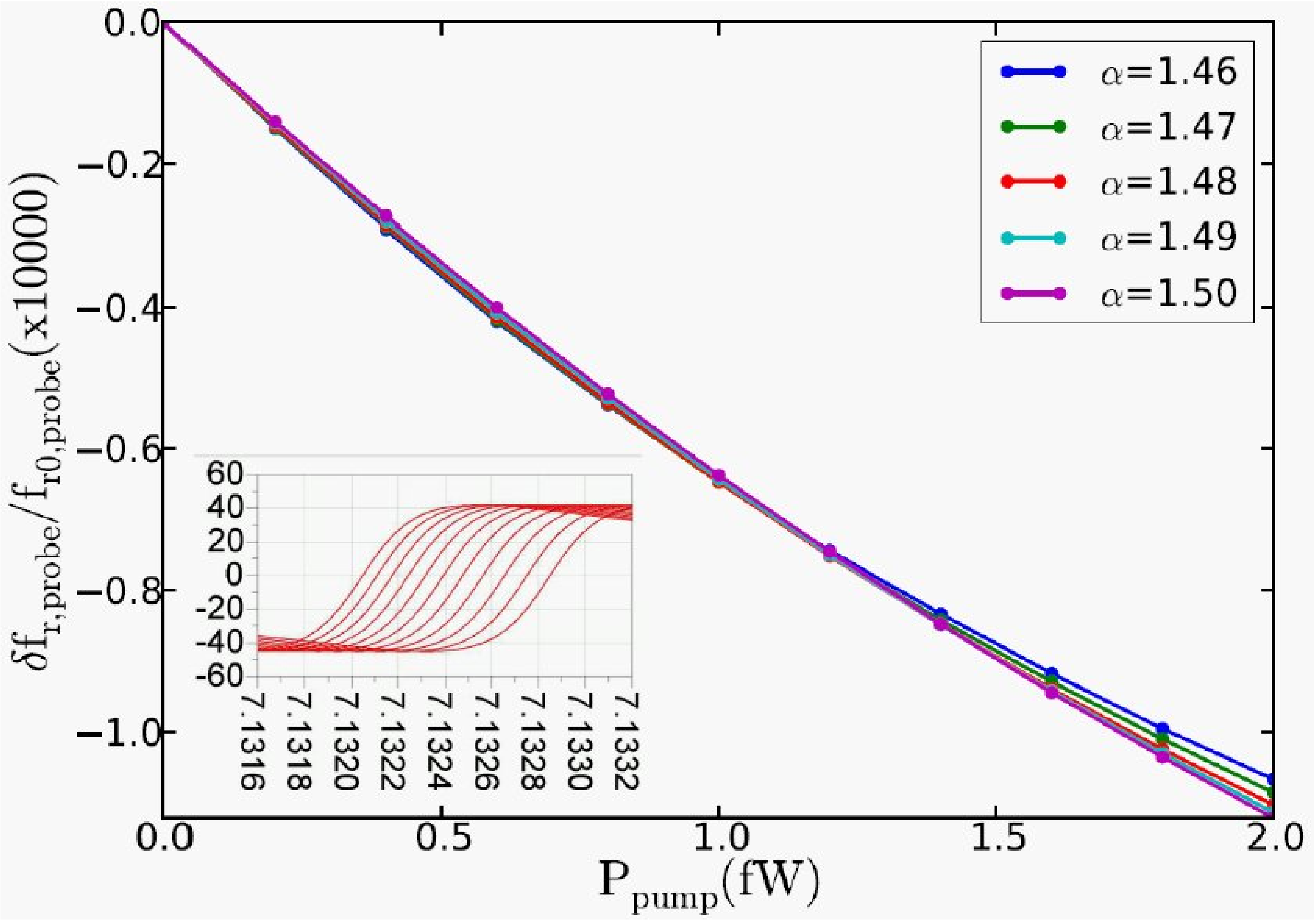
Present value of coupling caps ~ 10 fF for a Q of approx.3,000



180º Hybrid

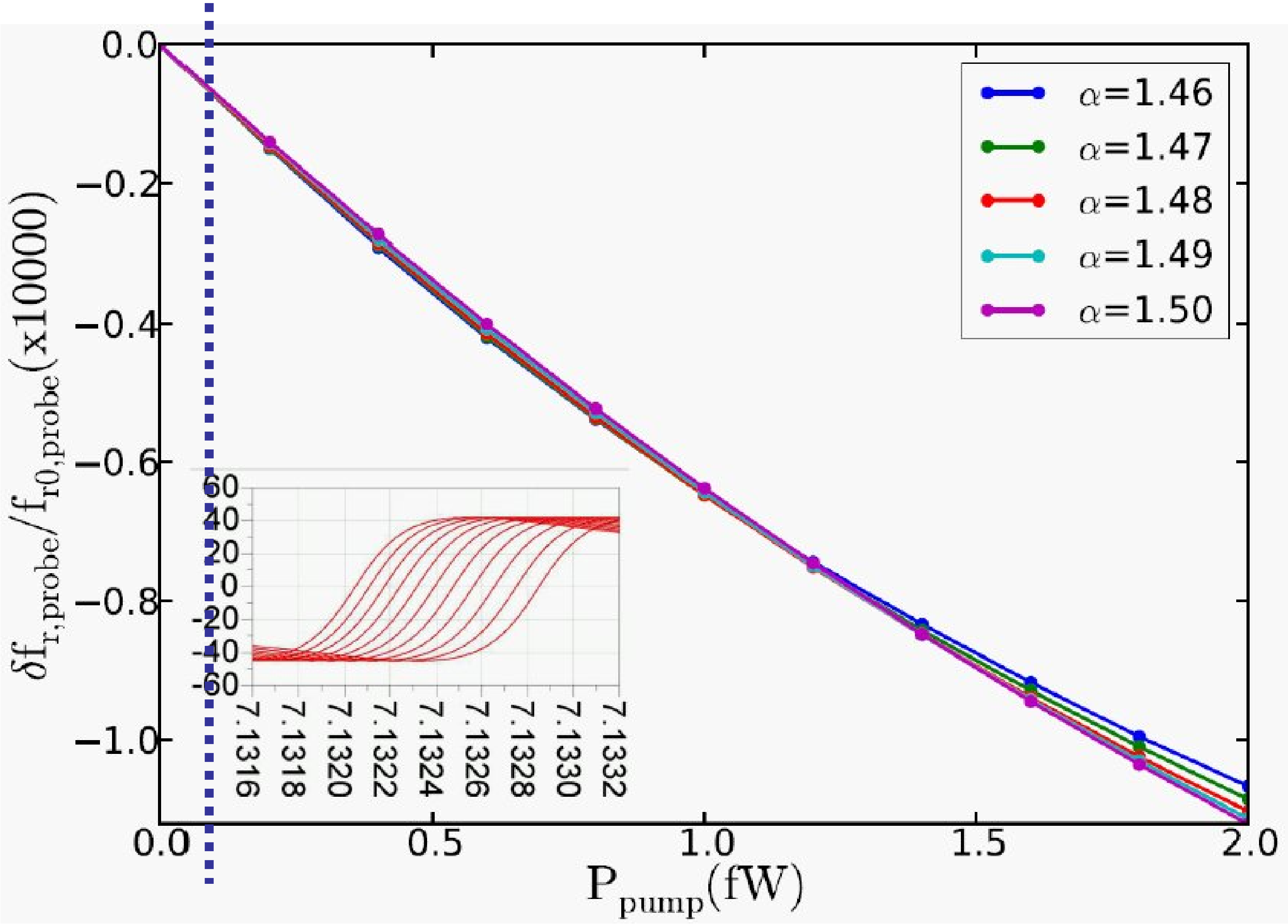
Power Splitter

Microwave Power Source



Inset: nearly linear shift of probe line with pump power – clear signature of cross Kerr effect

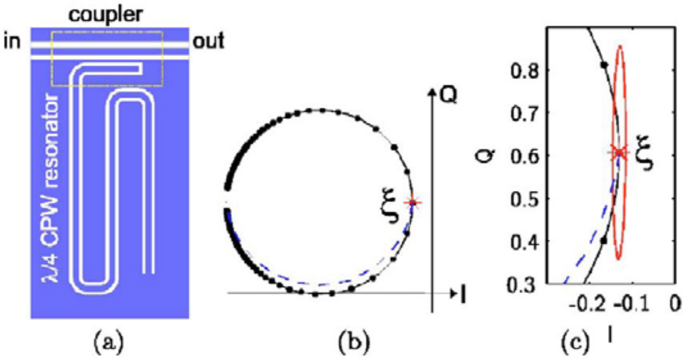
Is the measurement in the quantum regime?



Phase signal for one photon ≃ 7°

But, how big is the noise?

Evaluation of existing art (no theory)

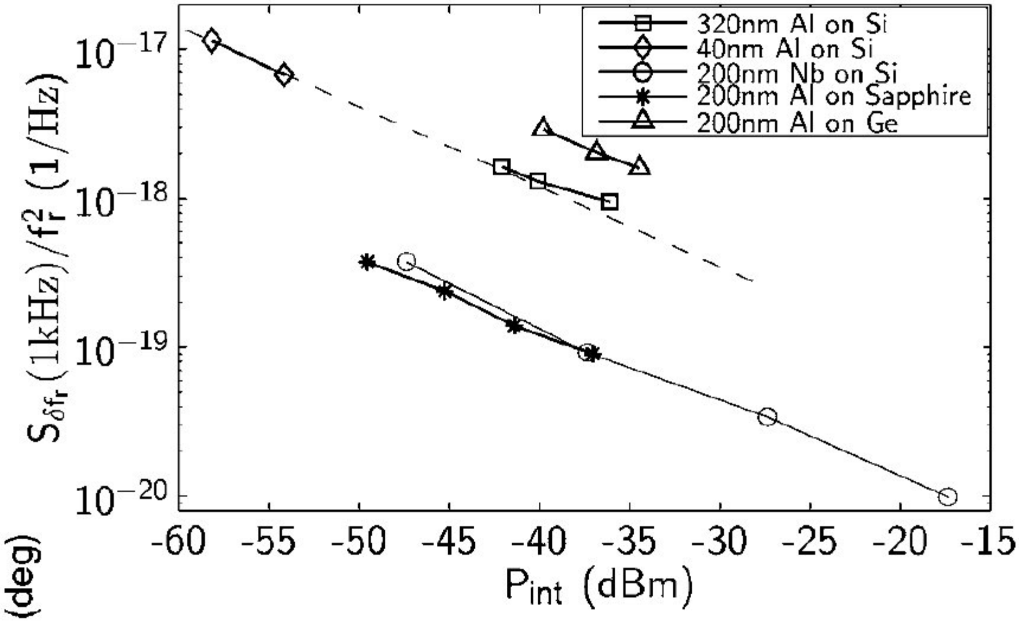


APPLIED PHYSICS LETTERS **90**, 102507 (2007)

Noise properties of superconducting coplanar waveguide microwave resonators

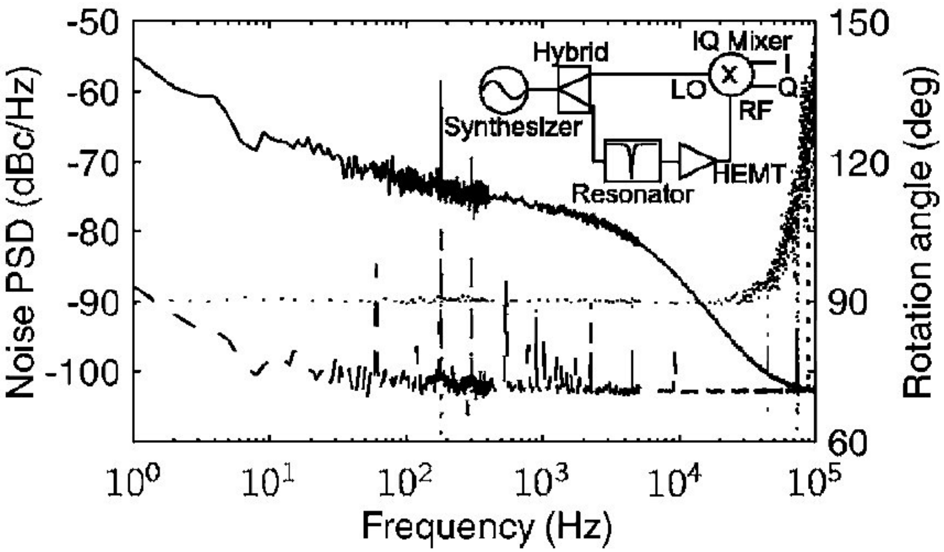
Jiansong Gao[a)] and Jonas Zmuidzinas
Physics Department, California Institute of Technology, Pasadena, California 91125

Benjamin A. Mazin, Henry G. LeDuc, and Peter K. Day
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109



Mostly phase noise, “colored”

Noise increases at lower excitation power
(saturated two-level systems?)



Assessment of signal to noise ratio (SNR)

$$S_{\pm f}(\nu_0,\ P_0)/f_r^2 = 10^{-18}/\mathrm{Hz}$$

Observed by Gao et al. Measurement bandwidth of ν_0=1kHz, measurement power

$$P_0 = -60\ \mathrm{dBm} = 1\ \mathrm{nW}$$

$$S_\theta(\nu_0,\ P_0) = 4Q_r^2 S_{\pm f}(\nu_0)/f_r^2 = 3.6 \times 10^{-10}\ \mathrm{rad}^2/\mathrm{Hz}.$$

Convert to resonator phase noies

$$S_\theta(\nu,\ P_0) = \left(\frac{\nu}{\nu_0}\right)^{-1/2} S_\theta(\nu_0,\ P_0) = 1.1 \times 10^{-12}\ \ \mathrm{rad}^2/\mathrm{Hz}$$

Convert to measurement bandwidth of ν=1 MHz, colored-noise correction

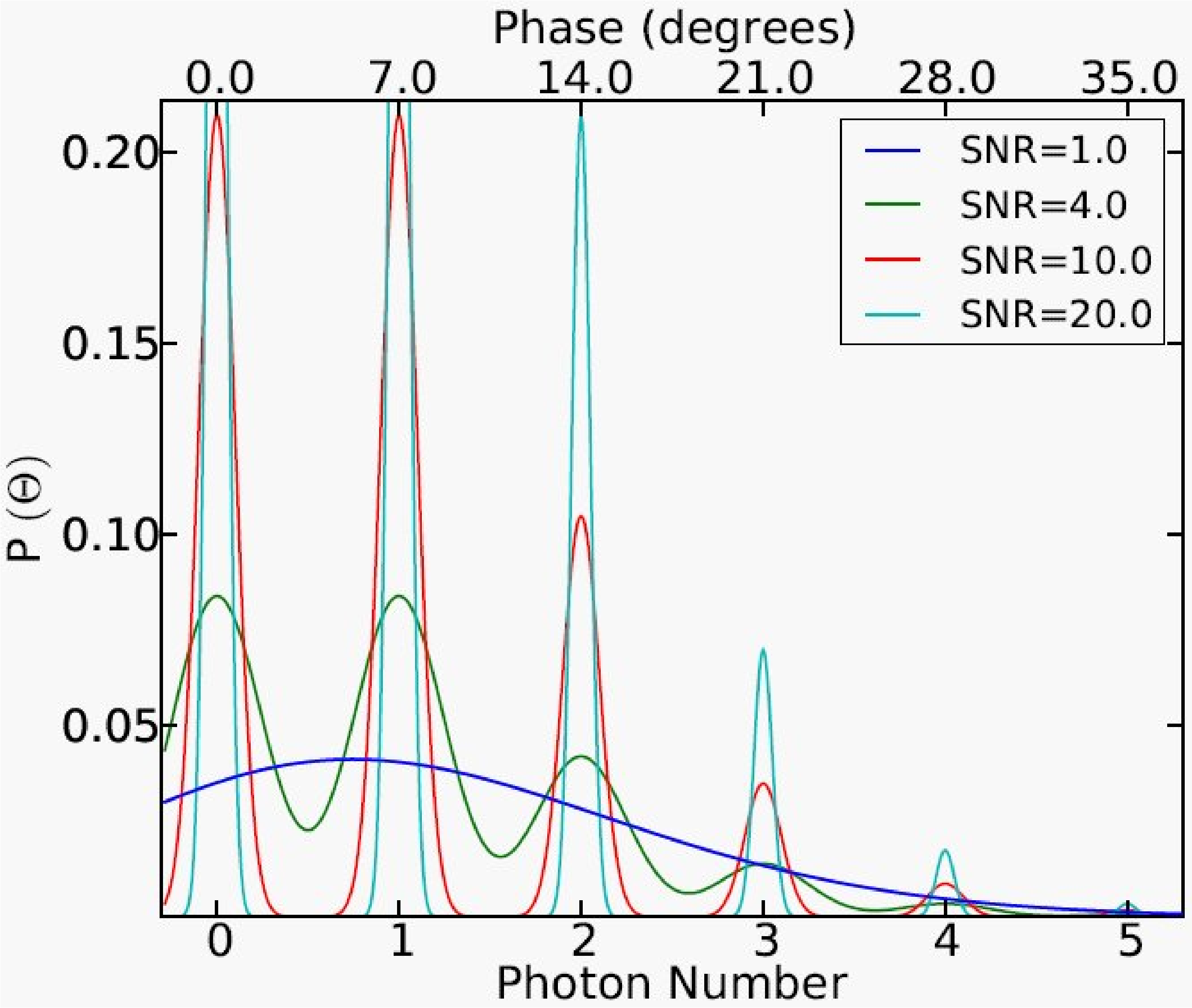
$$S_\theta(\nu,\ P) = \left(\frac{P_{photon}}{P_0}\right)^{-1/2} S_\theta(\nu,\ P_0) = 2.7 \times 10^{-11}\mathrm{rad}^2/\mathrm{Hz}$$

Extrapolate to single-photon power

$\sigma_\theta = \sqrt{S_\theta(\nu,\ P)/\tau} = 0.3^{\pm}$ Phase measurement error. Measurement time τ=1μsec

$$\boxed{\mathrm{SNR} = \theta_{sig}/\sigma_\theta = 7^{\pm}/0.3^{\pm} = 20.}$$

Expected measurement stats for pump in coherent state



Average photon number=1

w/ Chirolli & Burkard

Another application of ring resonator:
  in-place beamsplitter (or, maximal mode entangler)

SQUID modulation trick

Circuit model

Symplectic evolution formalism

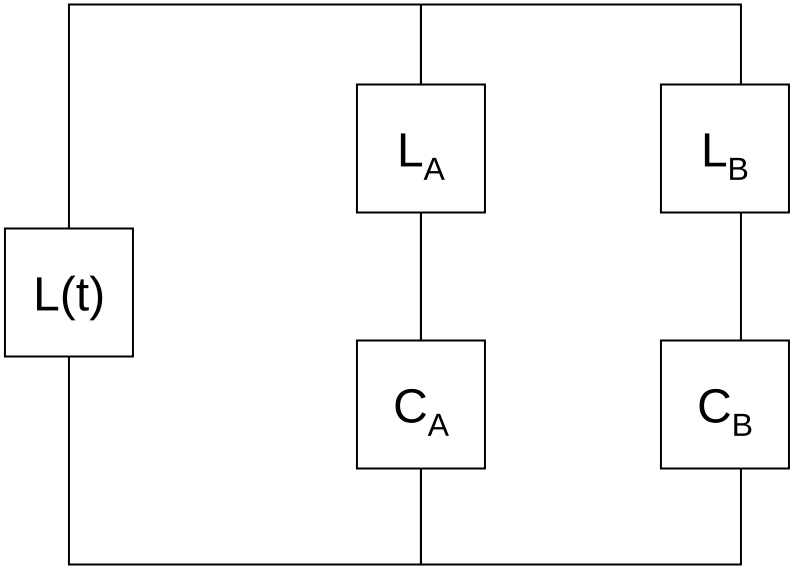
Avoidance of squeezing

Great fidelity expected!

Outline

-- Circuit Quantum Electrodynamics:
 the surprising quantumness of simple
 electric circuits – better than quantum optics
-- Regimes of application:
 resonant, dispersive, **parametric**
-- Parametric: many new effects, no qubits,
 not so dramatically quantum
-- Our result: A quantum thing that the
 parametric circuit can do: nondemolition
 quantum measurement
-- Prospect: qubitless circuits
 for quantum computing

Description: A.J. Rutgers and Hendrik Casimir in an automobile they bought for $50.00 to drive from Ann Arbor, Michigan to New York City where they abandoned it. (photo: S. Goudsmit.)


L(t)
L_A
L_B
C_A
C_B