Closed Timelike Curves: What they're good or bad for

Graham Smith, Univ. Birmingham

Charles Bennett, IBM

Debbie Leung, Univ. Waterloo

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35 minutes + discussion.

Todd assured me there'd be plenty to talk about!

Time Travel Jokes

“Didn’t you get my draft? I sent it to you tomorrow.”

“It seems like our discussions always go around and around and around.”

…

Time Travel Jokes

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“It seems like our discussions always go around and around and around.”

…

The best thing about time travel jokes is they never get old.

Outline

Closed Timelike Curves

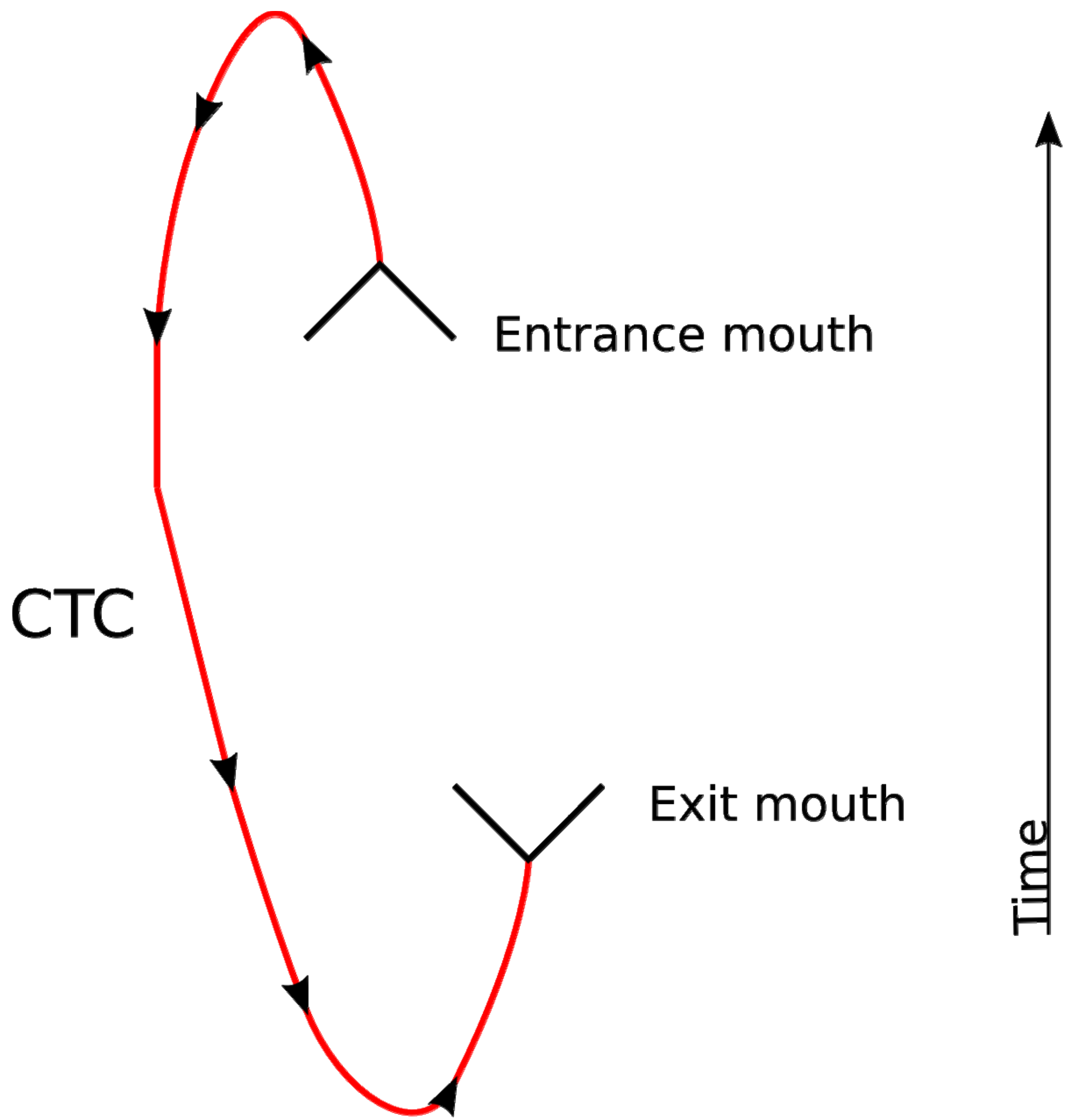
Grandfather Paradox

Deutsch model

State distinguishing

Computational power

Closed Timelike Curves



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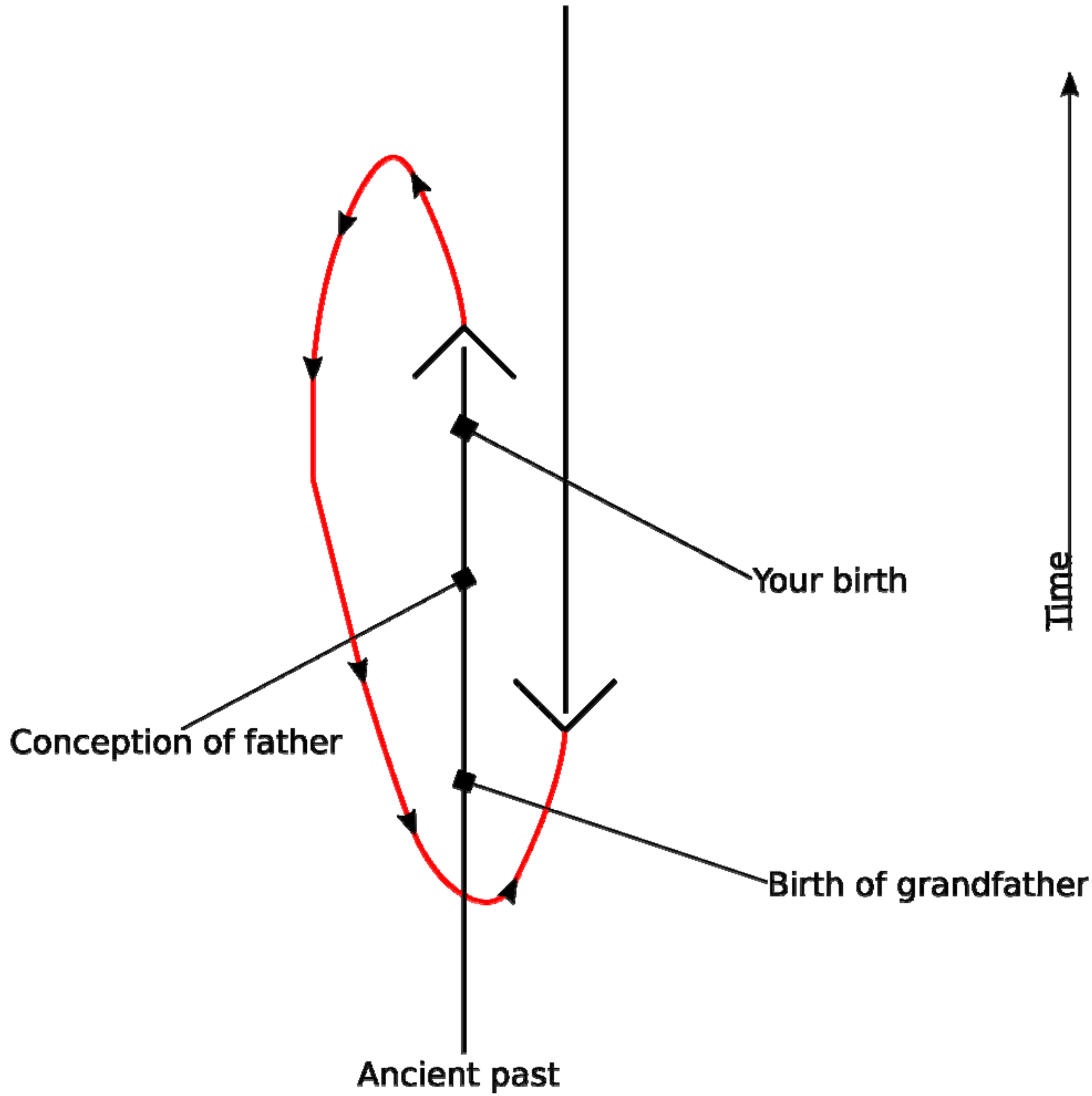
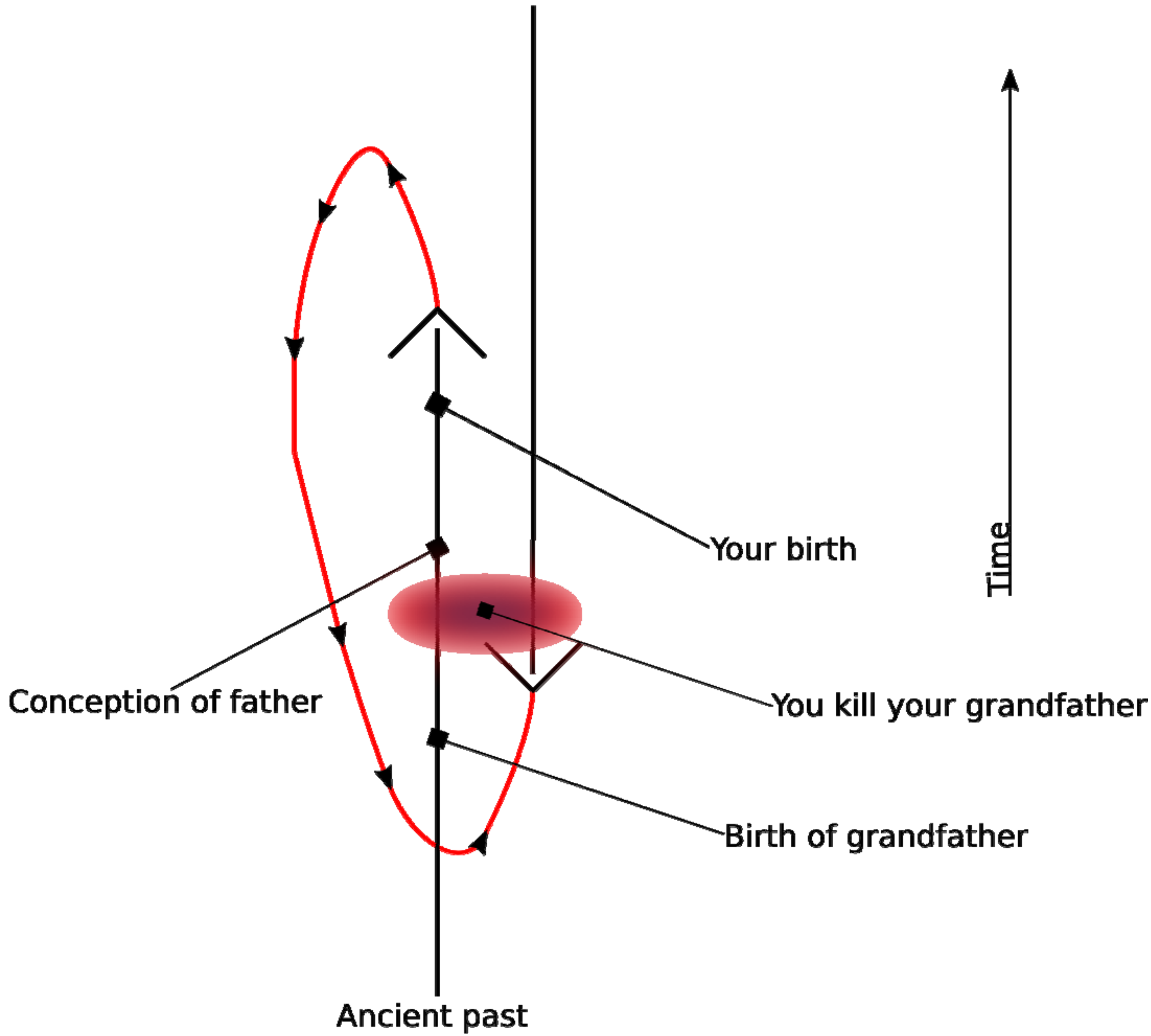
First law of time travel

Whenever anyone gets a time machine, the first thing they do is go back in time and kill their grandfather.


Your birth
Time
Conception of father
Birth of grandfather
Ancient past



Your birth
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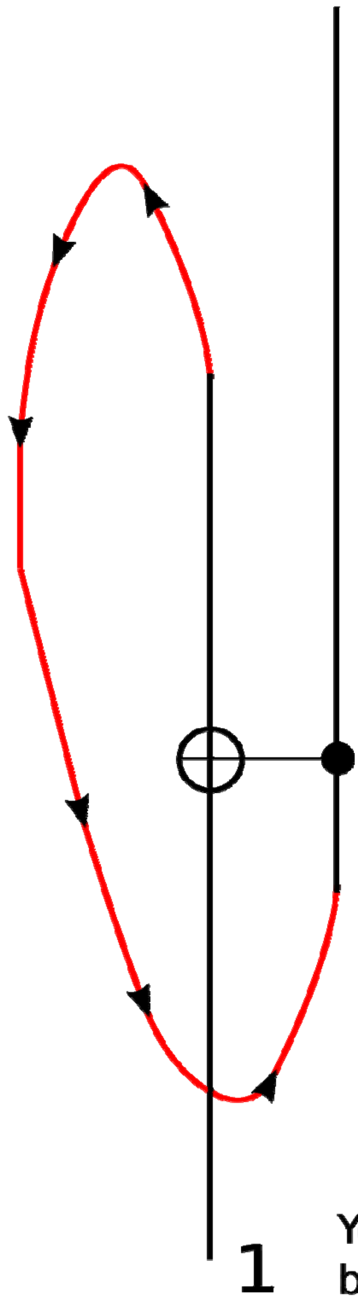


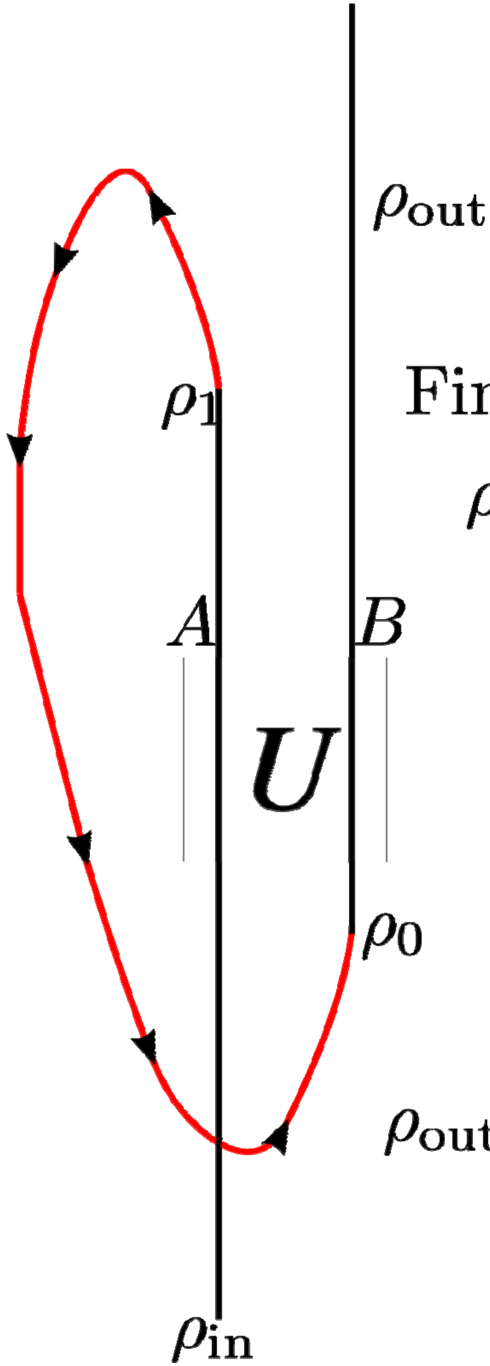
1: I'm alive
0: I don't exist

Controlled-Not

00: 00
01: 01
10: 11
11: 10

Your grandfather was born before all this nonsense with the CTC happens

Deutsch's Model



$$\rho_1 = \rho_0$$

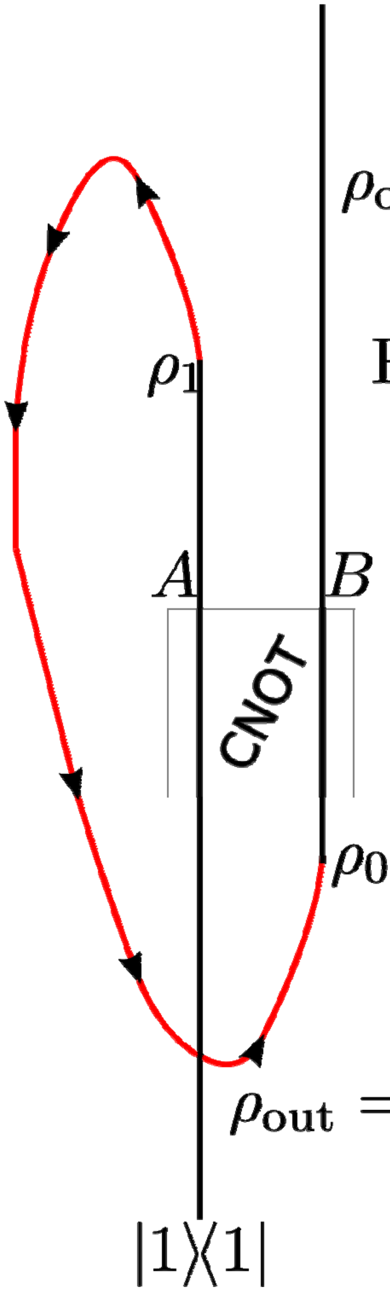
Gives nonlinear evolution

Find the fixed point of:

$$\rho_1 = \mathrm{Tr}_B U \rho_{\mathrm{in}} \otimes \rho_0 U^\dagger$$

$$\rho_{\mathrm{out}} = \mathrm{Tr}_A U \rho_{\mathrm{in}} \otimes \rho_0 U^\dagger$$

Fixing the Paradox



Alive with
probability half

Find the fixed point of:

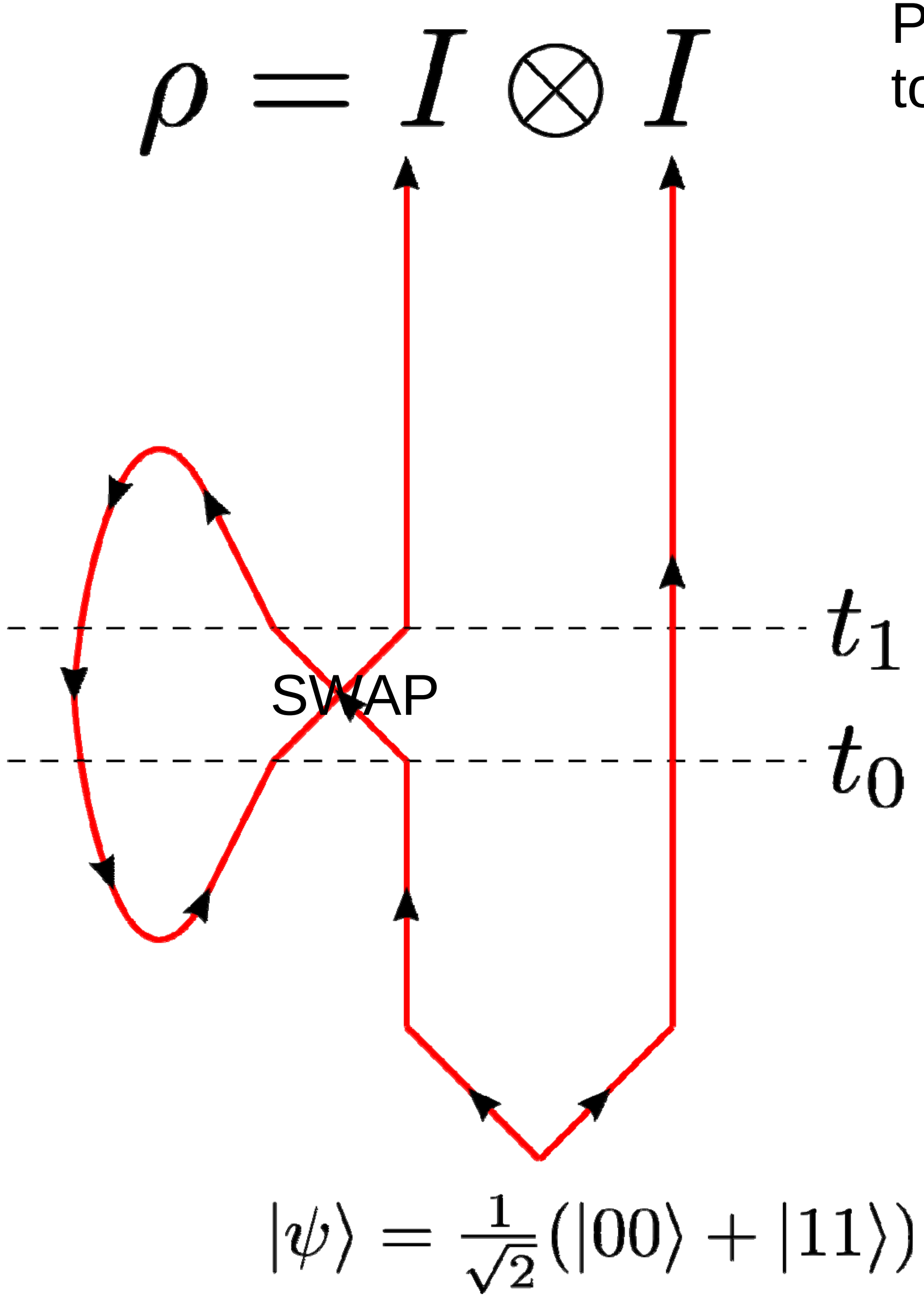
$$\rho_1 = \mathrm{Tr}_B \mathrm{CNOT} \rho_0 \otimes |1\rangle\langle 1| \mathrm{CNOT}$$

$$I = \mathrm{Tr}_B \mathrm{CNOT}(I \otimes |1\rangle\langle 1|)\mathrm{CNOT}$$

$$\rho_{\text{out}} = \mathrm{Tr}_A \mathrm{CNOT}(I \otimes |1\rangle\langle 1|)\mathrm{CNOT} = I$$

Deutsch Model Swap

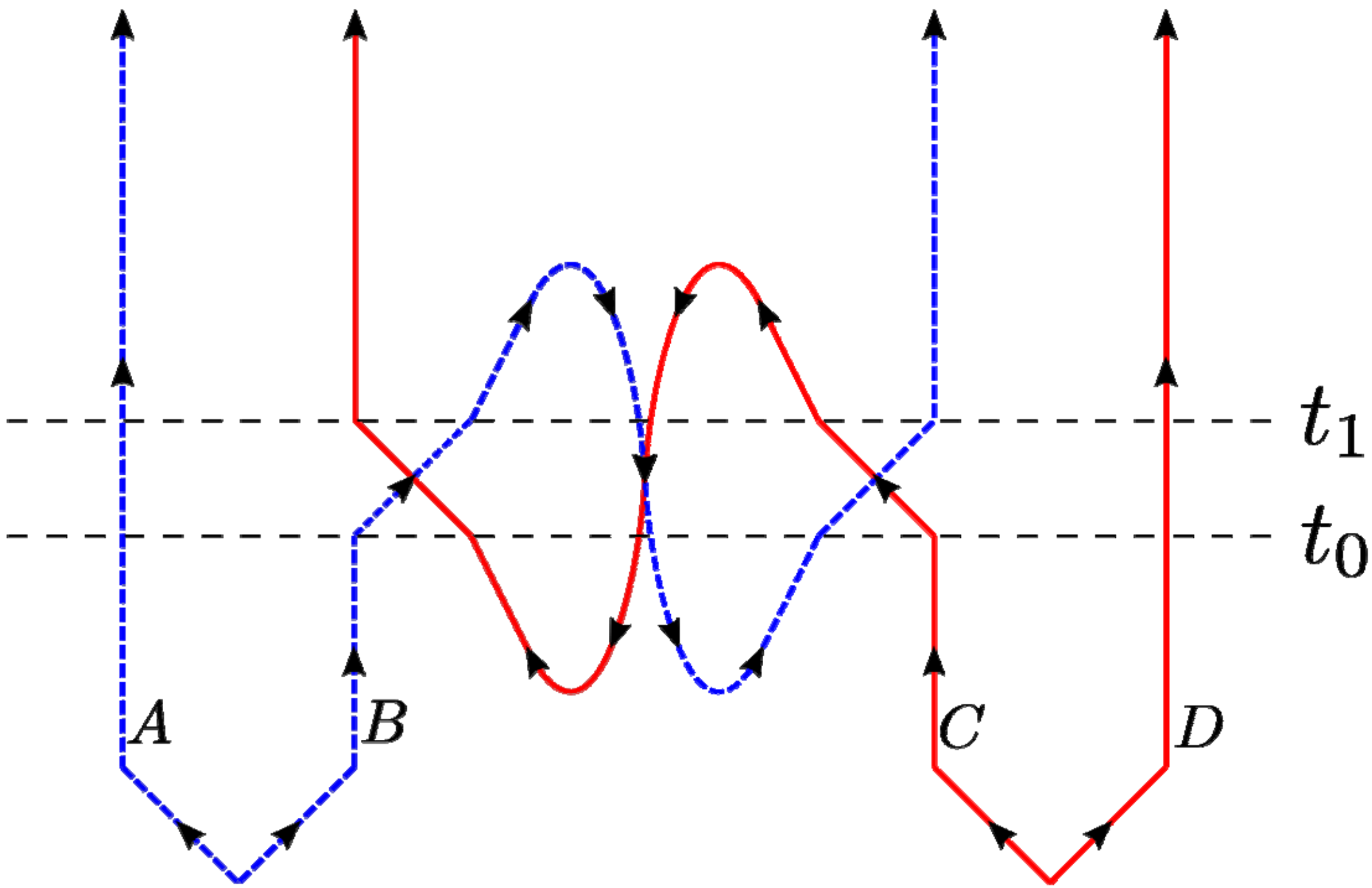
Pure state evolves to mixed state



Two Universes

Pure state evolution restored!

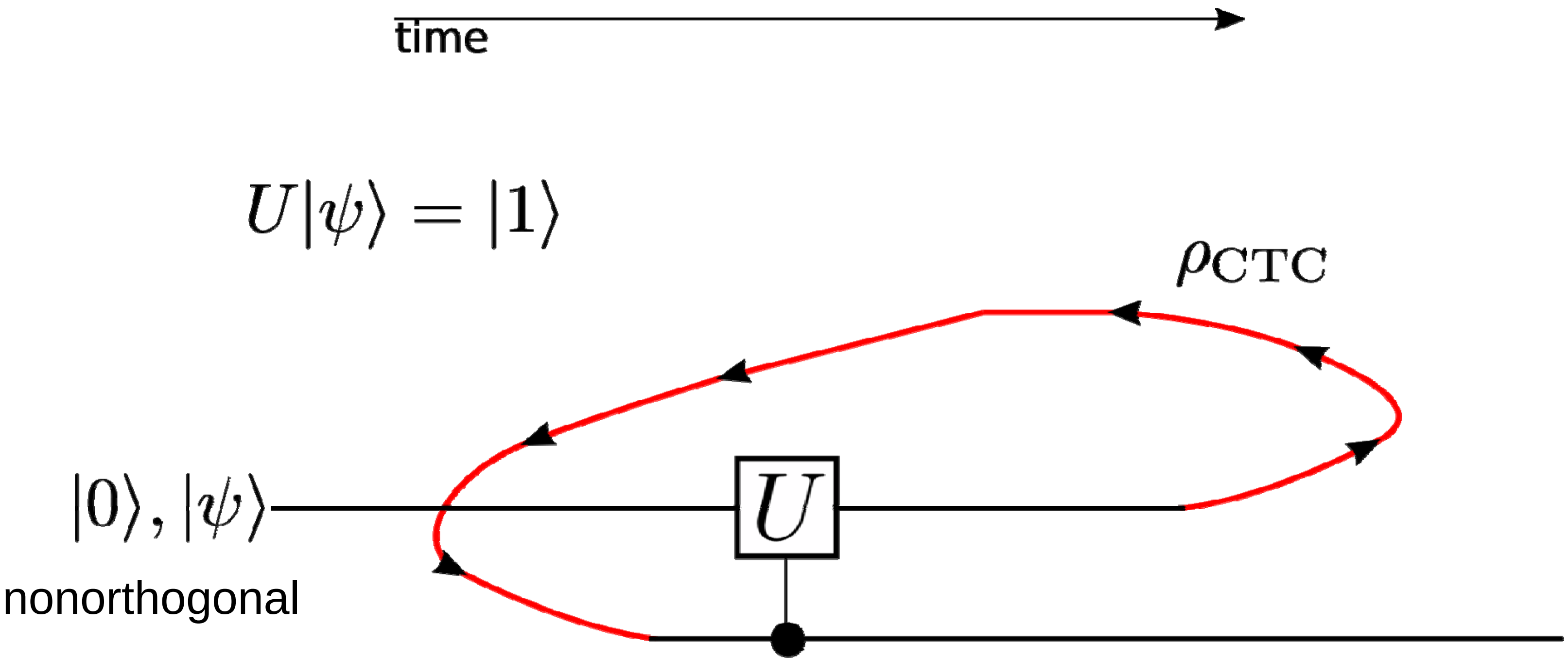
$$|\psi_f\rangle = \frac{1}{\sqrt{2}}(|00\rangle_{AC} + |11\rangle_{AC}) \otimes \frac{1}{\sqrt{2}}(|00\rangle_{BD} + |11\rangle_{BD})$$



$$|\psi_i\rangle = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) \otimes \frac{1}{\sqrt{2}}(|00\rangle_{CD} + |11\rangle_{CD})$$

State Distinguishing

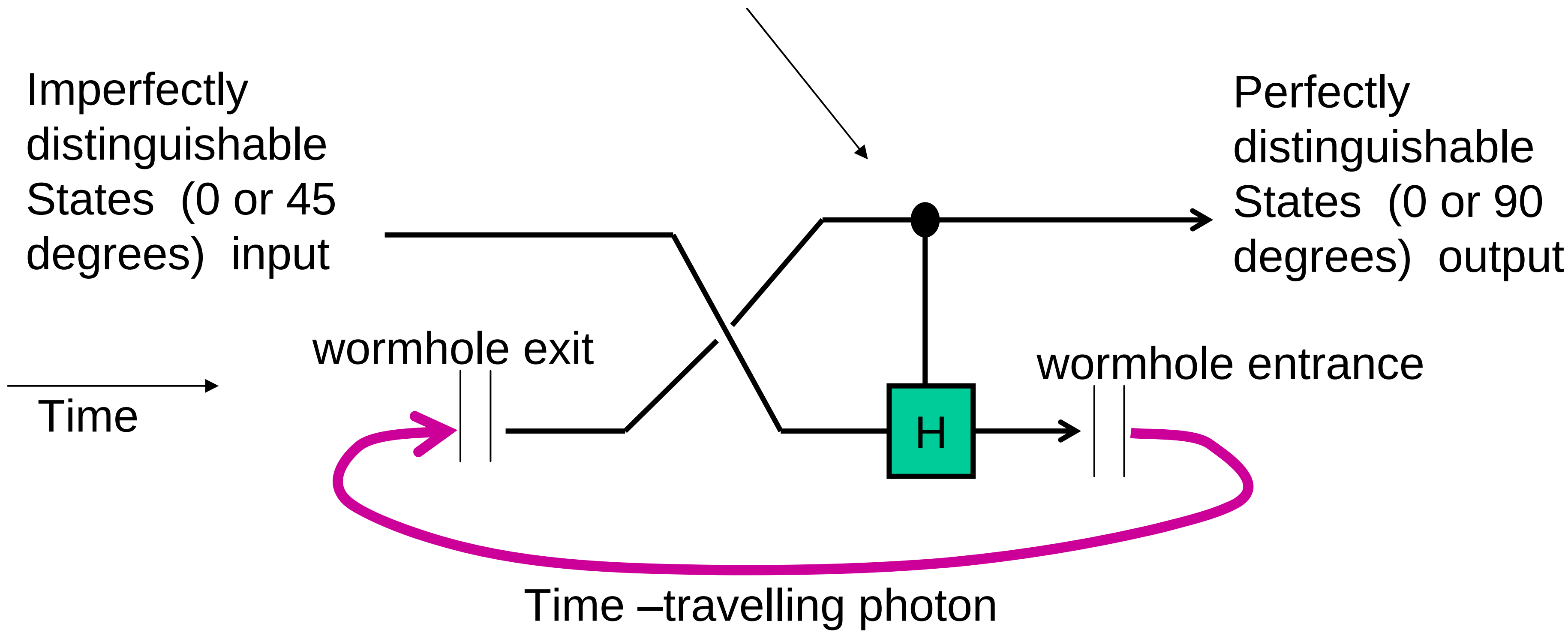
$$U|\psi\rangle = |1\rangle$$



$$\rho_{\mathrm{CTC}}(0) = |0\rangle$$
$$\rho_{\mathrm{CTC}}(\psi) = |1\rangle$$

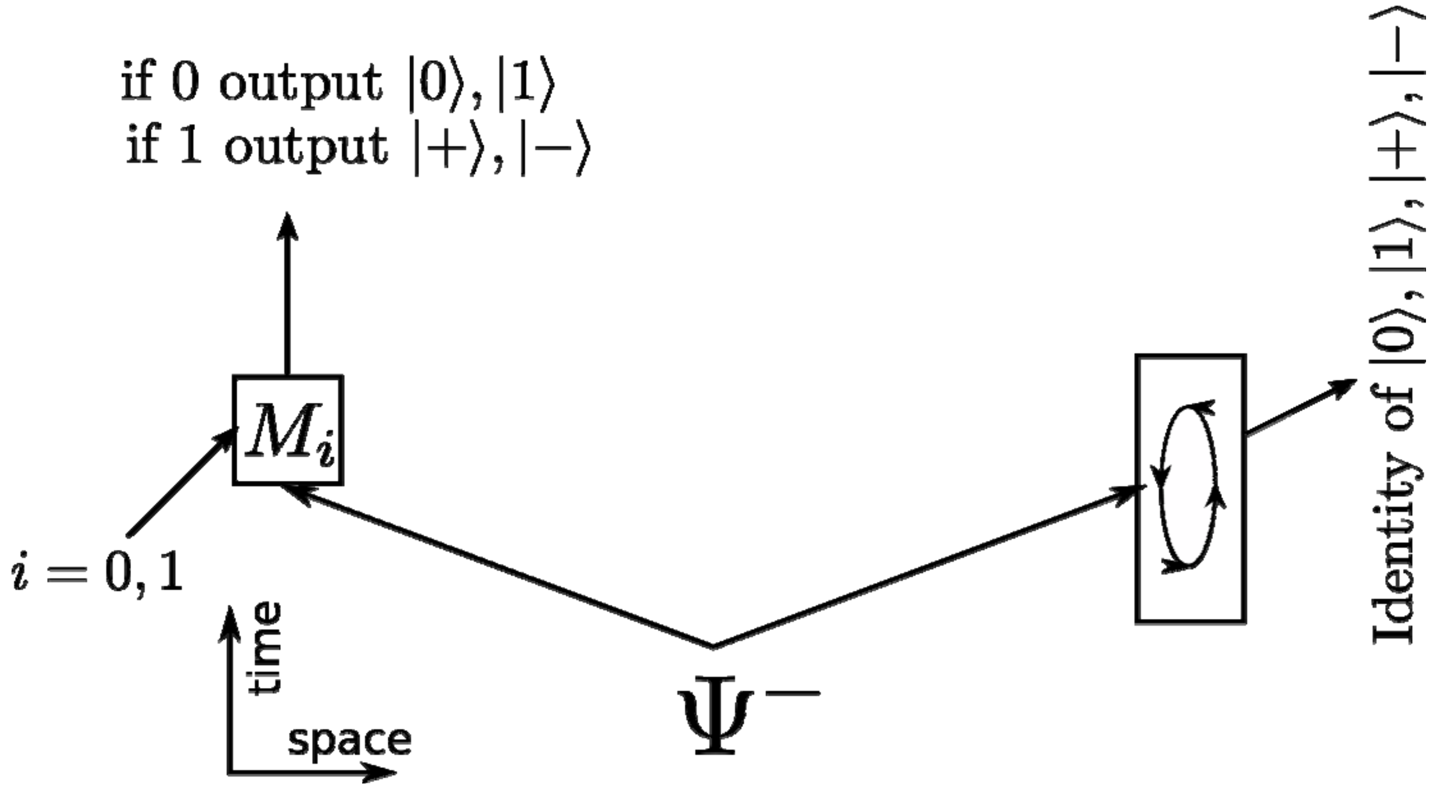
Brun, Harrington, Wilde

Using a CTC to reliably distinguish nonorthogonal states: If upper photon is vertical (90 degrees), rotate the lower photon by +45 degrees. If upper photon is horizontal (0 degrees), leave the lower one alone.



Exotic interaction with its own earlier self, made possible by the CTC, stretches two initially non orthogonal states of the photon apart and makes them orthogonal. This capacity to perform this stretching would break supposedly secure cryptosystems and greatly speed up some hard computations.

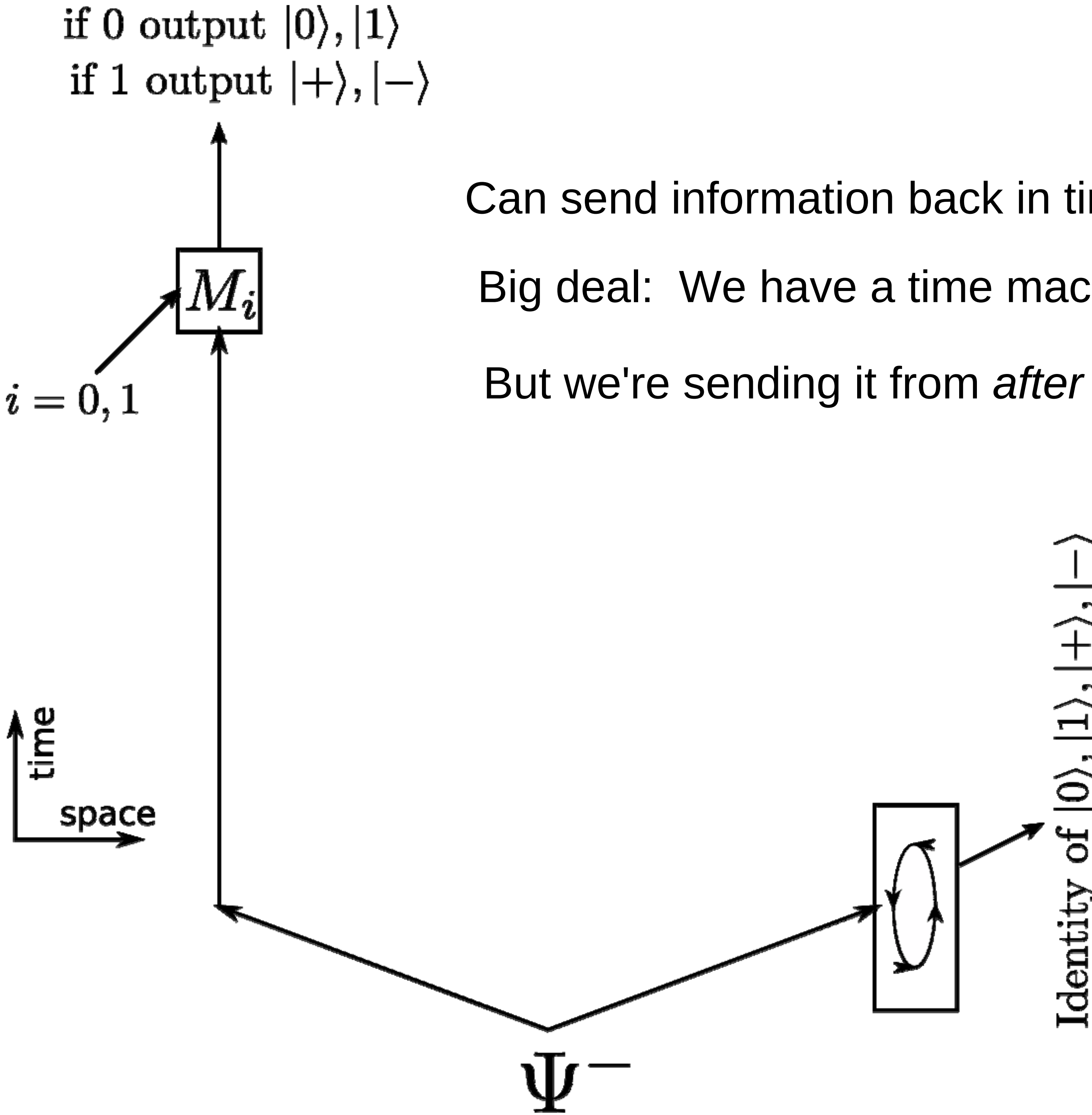
Communicate Faster Than Light

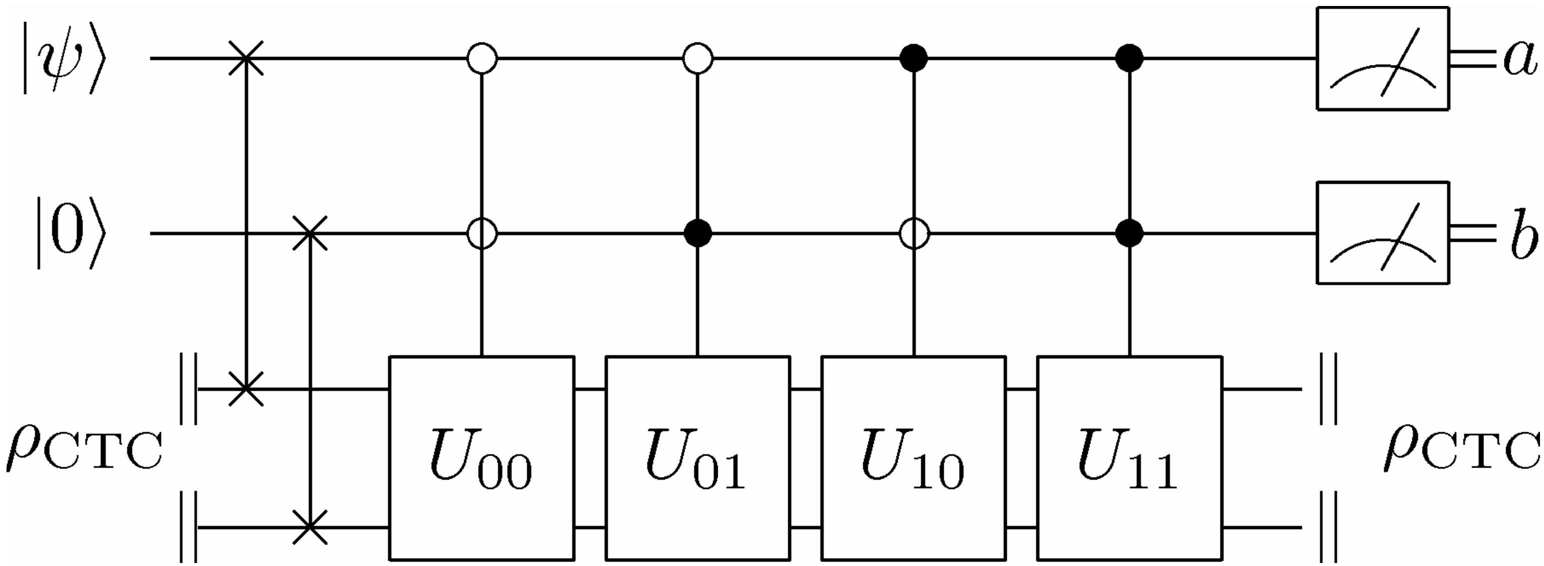


Can send information back in time

Big deal: We have a time machine

But we're sending it from *after* the CTC ends!





$$|00\rangle \rightarrow |00\rangle \quad |+0\rangle \rightarrow |10\rangle$$
$$|10\rangle \rightarrow |01\rangle \quad |-0\rangle \rightarrow |11\rangle$$

$$U_{00} = \mathrm{SWAP}$$
$$U_{01} = X \otimes X$$
$$U_{10} = (X \otimes I) \cdot (H \otimes I)$$
$$U_{11} = (X \otimes X) \cdot (\mathrm{SWAP})$$

Breaks BB84 Cryptography

A generalization can distinguish *arbitrarily* many states.

--Allows encoding infinite classical information in one qubit!

Can distinguish the same state from itself!

Recall: The circuit can distinguish

$$|0\rangle, |1\rangle, |+\rangle, |-\rangle$$

Therefore it can also distinguish

$$\frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$

from $\frac{1}{2}(|+\rangle\langle +| + |-\rangle\langle -|)$

but

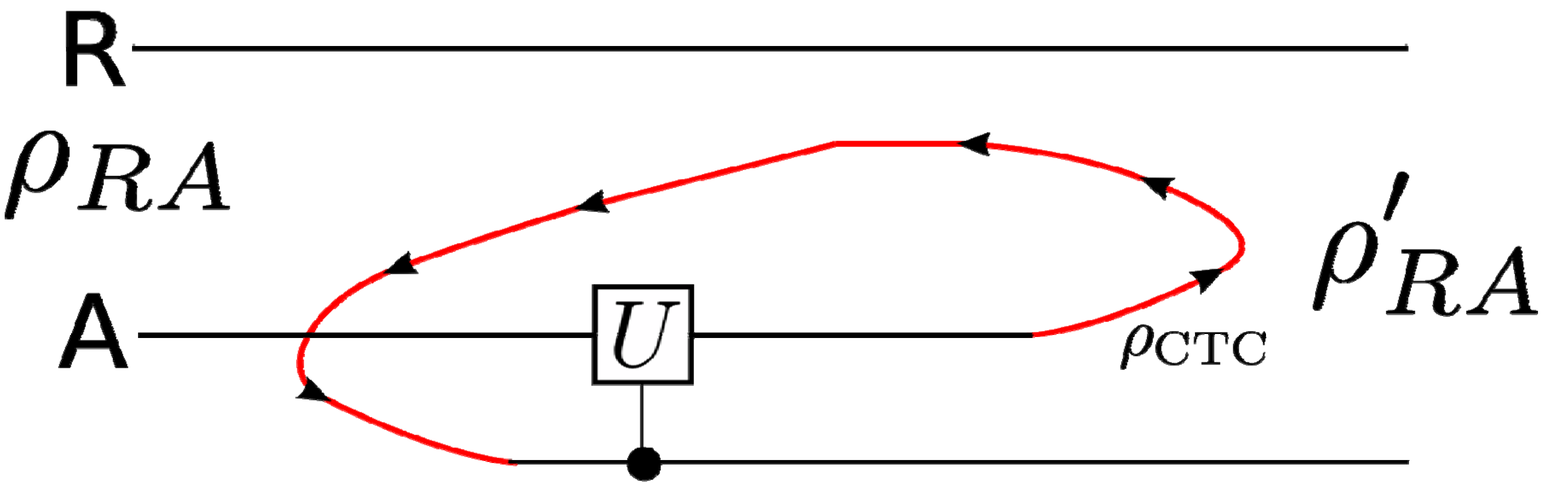
$$\frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) = \frac{1}{2}|+\rangle\langle +| + |-\rangle\langle -|) = I/2$$

Definition of state discrimination

$$\rho_{RA} = \sum_{x=0}^{1} p_x |x\rangle\langle x|_R \otimes |\phi_x\rangle\langle\phi_x|_A$$

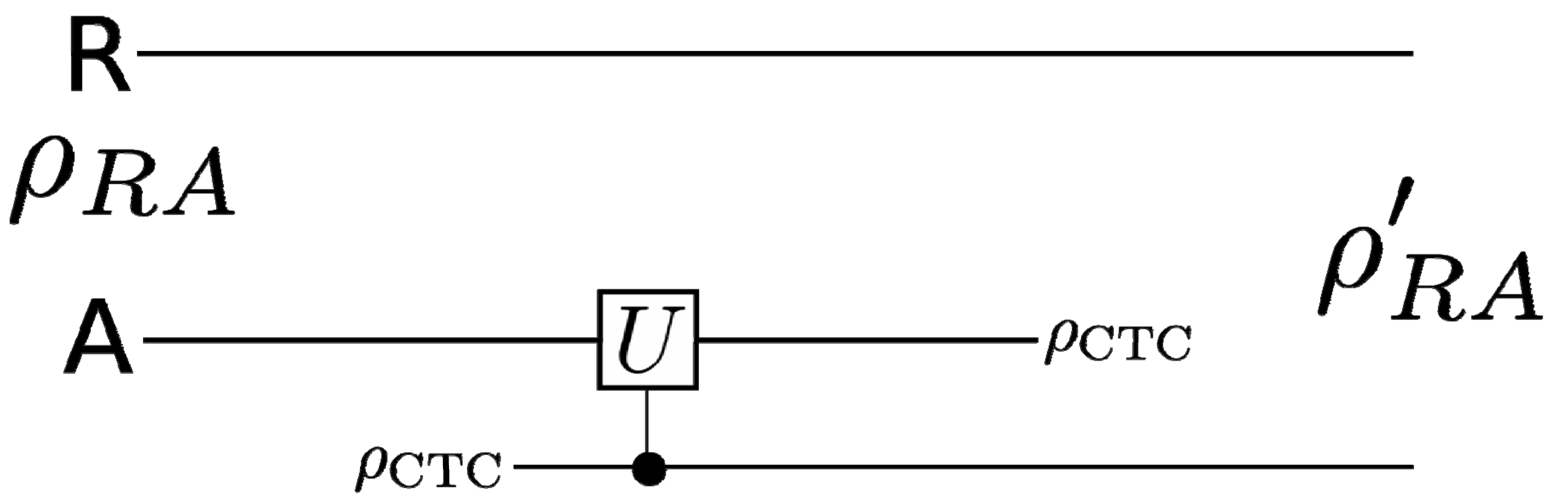
$$\rho'_{RA} = \sum_{x=0}^{1} p_x |x\rangle\langle x|_R \otimes |x\rangle\langle x|_A$$

Circuit with Referee



Alice simply calculates ρ_{CTC}

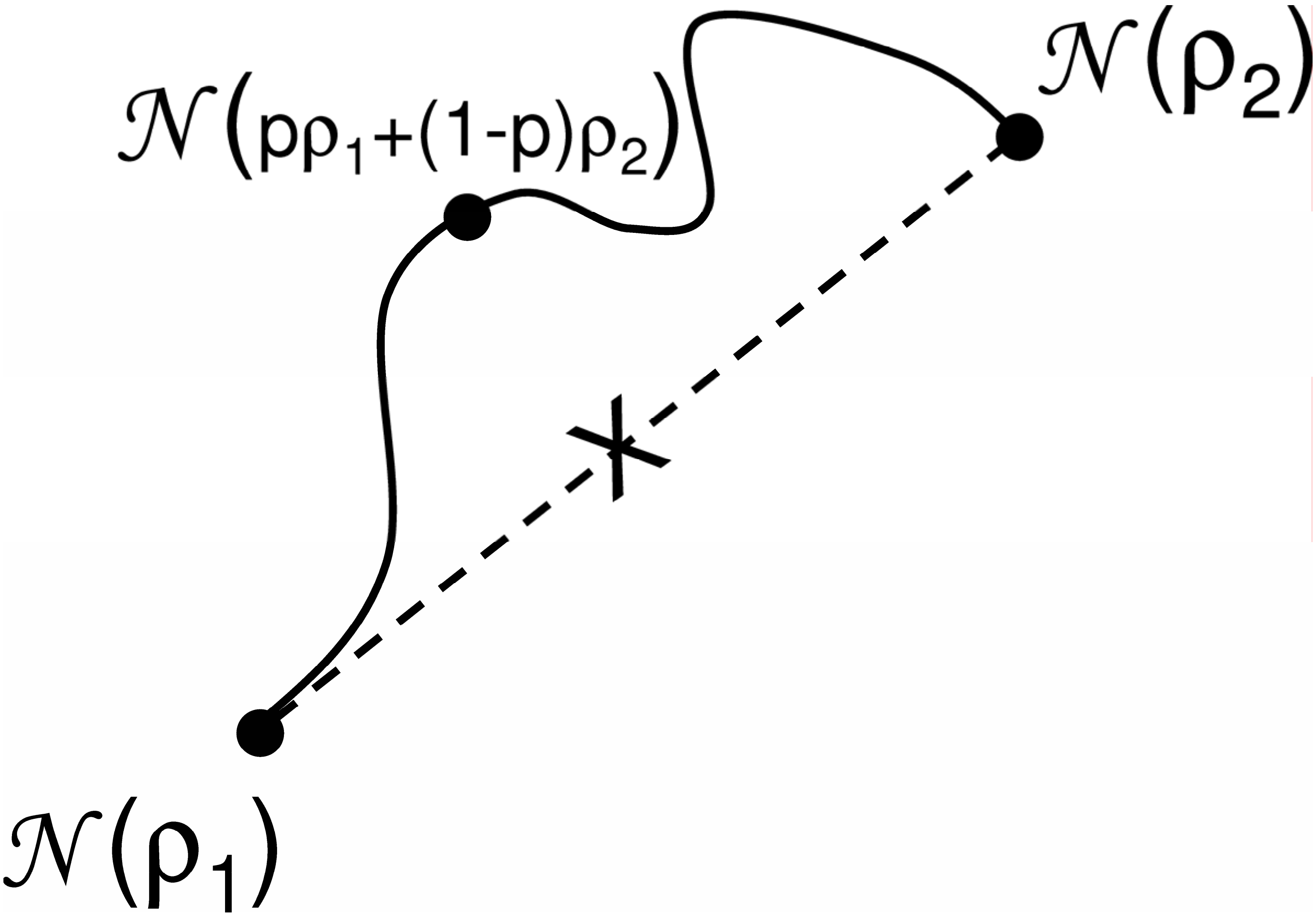
No CTCs need apply



Alice simply calculates ρ_{CTC}
and feeds it into the circuit herself

Cleary, the CTC wasn't needed or useful

The Linearity Trap



Recall: We want

$$\rho_{RA} = \sum_{x=0}^{1} p_x |x\rangle\langle x|_R \otimes |\phi_x\rangle\langle\phi_x|_A$$

$$\downarrow$$

$$\rho'_{RA} = \sum_{x=0}^{1} p_x |x\rangle\langle x|_R \otimes |x\rangle\langle x|_A$$

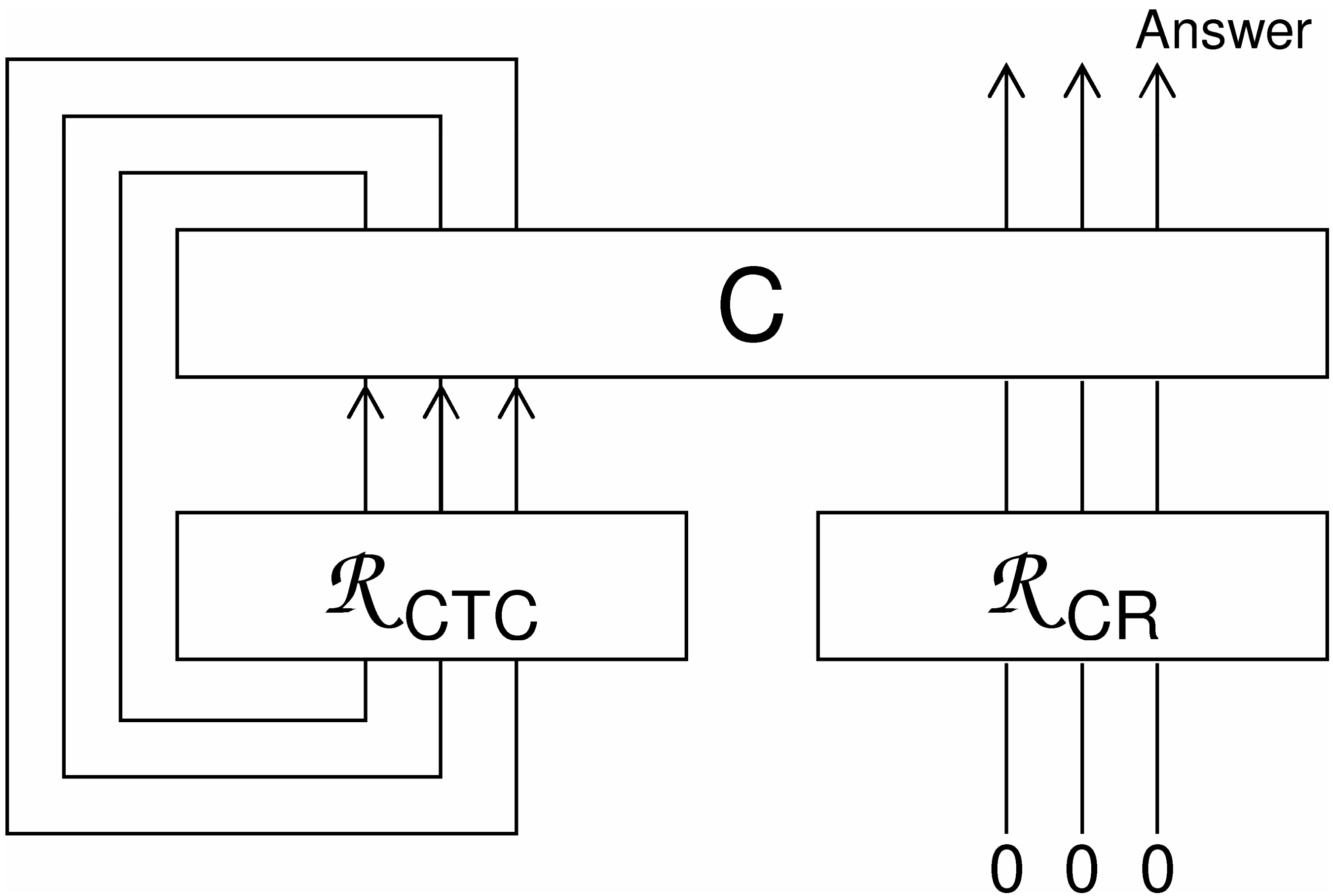
Even though the circuit does map

$$|x\rangle_R|\phi_x\rangle_A \to |x\rangle_R|x\rangle_A$$

On the labeled mixture we find:

$$\rho_{RA} \to \left(\sum_{x=0}^{1} |x\rangle\langle x|_R\right) \otimes \rho'_A$$

A CTC-assisted Computer



Bacon's algorithm for NP-complete problems

$$Prepare \quad \psi = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i\rangle$$

$$If\ satisfied\ by\ i\ \ output\ |1\rangle$$

$$else\ \ output\ |0\rangle$$

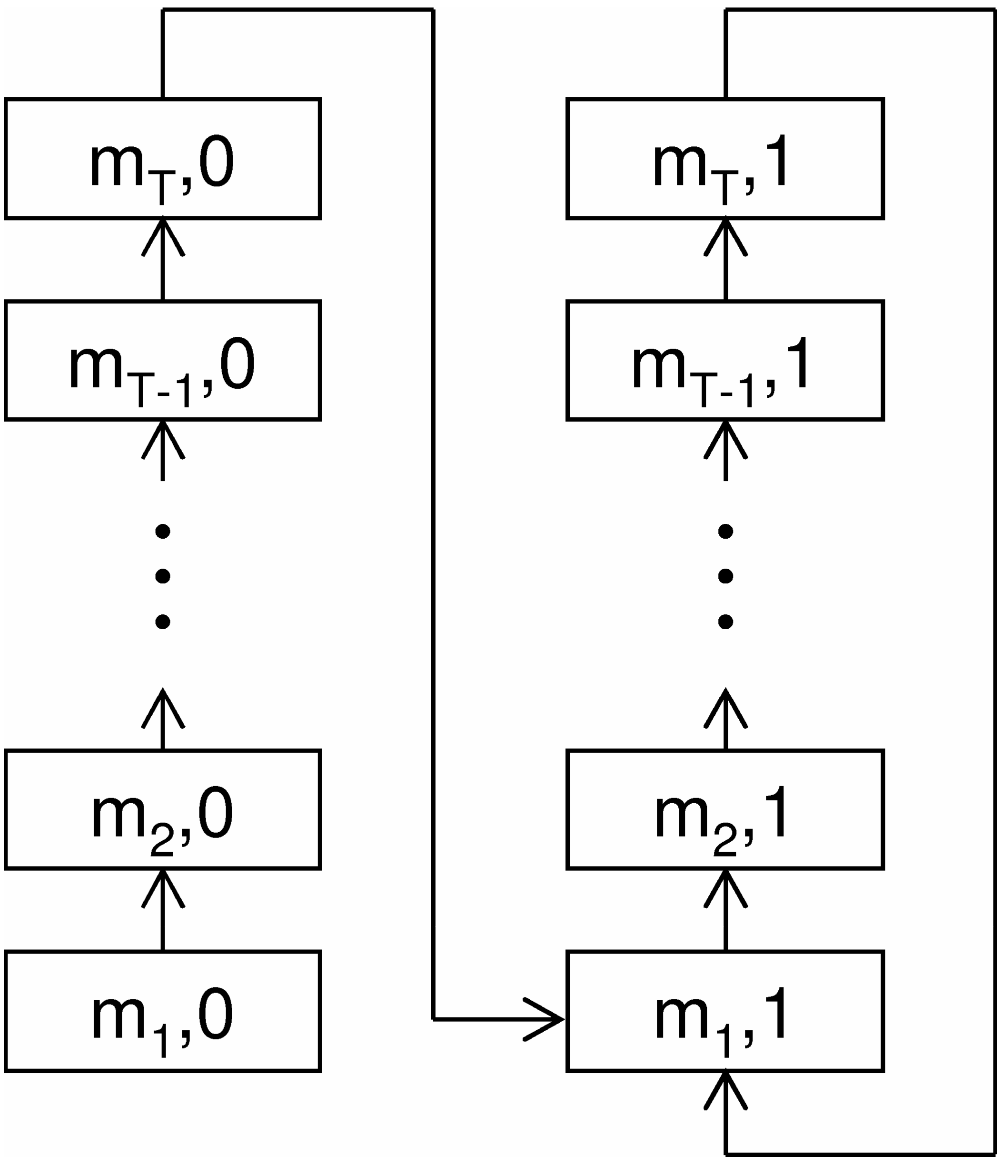
$$this\ \ leads\ \ to$$

$$\psi \rightarrow |0\rangle, \alpha|0\rangle + \beta|1\rangle \quad \alpha \simeq 1$$

then distinguish the nonorthonal states using a CTC

Abrams and Lloyd argued you can do something similar with any nonlinear extension to quantum mechanics

Aaronson-Watrous-Fortnow



BQP_{CTC}=PSPACE

$BQPP_{CTC}$

?=BQP\qpoly

The Principle of Universal Inclusion

The evolution of a nonlinearly evolving system may depend on parts of the universe with which it does not interact.

In regular quantum mechanics this holds:

$$I \otimes \mathcal{N}\left(\sum p_i |i\rangle\langle i| \otimes \phi_i\right) = \sum p_i |i\rangle\langle i| \otimes \mathcal{N}(\phi_i)$$

But CTCs give:

$$\left(\sum p_i |i\rangle\langle i|\right) \otimes \rho_{\{p_i,\phi_i\}}$$

Objections

It works for every input state/I made a firm decision

Fine but boring---more like a one-entry look-up table than a computer; 23x23 Go?, (Computation is subtler than state discrimination)

Same density matrix is not the same as the same mixture.

We are restricting to theories that reduce to QM away from the nonlinear region

Clintonian arguments—what the meaning of “is” is.

i.e., What state is the input in and how is it chosen?

Questions

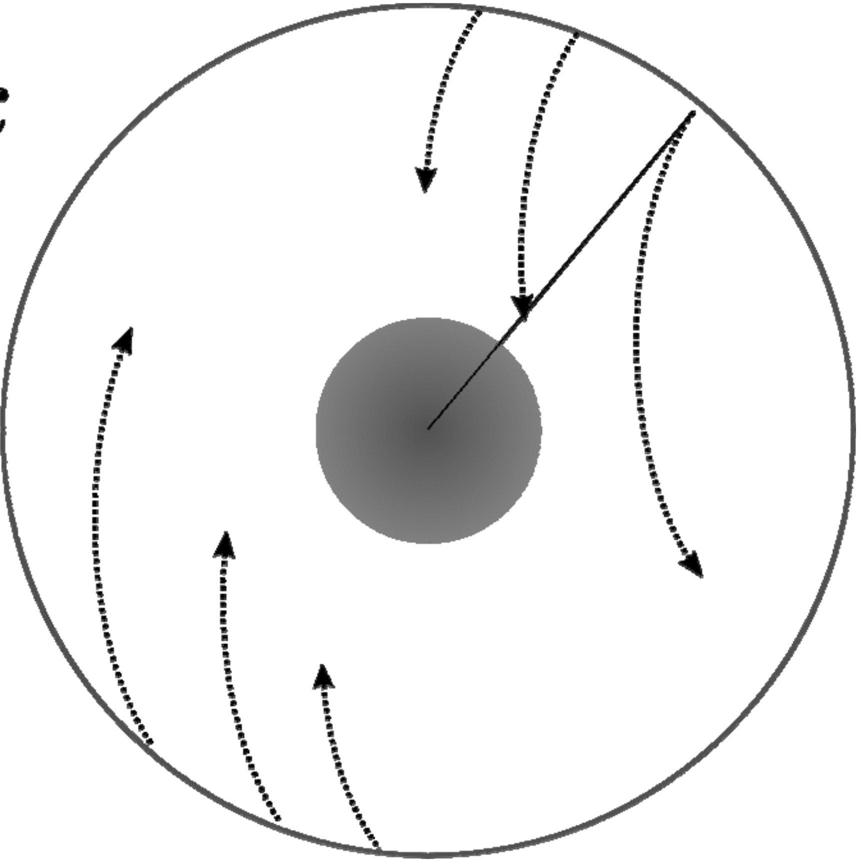
- What’s the “right” way to do quantum mechanics with closed timelike curves?
- Is there some sensible model where flipping a coin and throwing it into a CTC has output correlated with the input?
- Do we have to go to second quantization?

  (Can a CTC give linear evolution here?)
- How do we know we’ve chosen an input ensemble? Can we test this? Shouldn’t we include in the analysis the process by which we chose the input ensemble?
- What’s the right way to define computation in a nonlinear theory?

Weinberg-Polchinski

Nonlinear extension which maps smoothly to regular QM

$$\theta = \alpha x$$



Can distinguish nonorthogonal states

“EPR-Phone”
(FTL)

“Everett Phone”??