eLOQC[a] II

Postselected Computation

- Postselected computation.
- Non-linear gate on one mode.
- Conditional sign flip.
- Creating entangled pairs.

[a] Efficient Linear Optics Quantum Computation [1]

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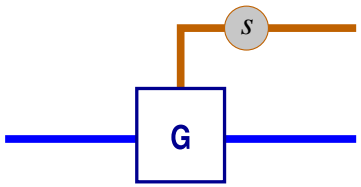
Postselected Computation

- Definition: G is a *postselected* gate with *failure probability* f if each time G is used, it succeeds with probability $1-f$ and fails otherwise, and whether it succeeded is known.



$s=1$: Success
$s=0$: Failure

- **Theorem 1:** f sufficiently small $\Rightarrow$ scalable quantum computation.

  Proof: By the accuracy threshold theorem. [2, 3, 4, 5]

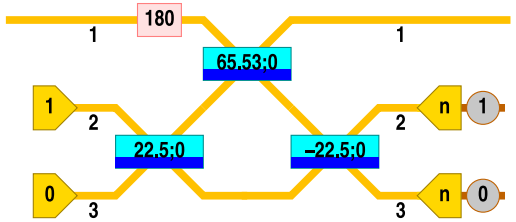
  - $f_{\text{threshold}} \gtrsim .01$. [6]

Non-linear Phaseshift I

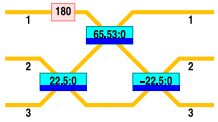
- Non-linear phaseshift on one mode:

$$U_\phi: \begin{array}{rcl} |0\rangle & \rightarrow & |0\rangle \\ |1\rangle & \rightarrow & |1\rangle \\ |2\rangle & \rightarrow & e^{i\phi}|2\rangle \end{array}$$

- Postselected network NS_{180° realizing U_{180°:



Non-linear Phaseshift II

- $\hat{u}$ for  :

$$\begin{pmatrix} 1-\sqrt[2]{2} & 1/\sqrt[4]{2} & \sqrt[2]{3/\sqrt[2]{2}-2} \\ 1/\sqrt[4]{2} & 1/2 & 1/2-1/\sqrt[2]{2} \\ \sqrt[2]{3/\sqrt[2]{2}-2} & 1/2-1/\sqrt[2]{2} & \sqrt[2]{2}-1/2 \end{pmatrix}$$

- Input $\mathbf{a}^{\dagger k}_1\mathbf{a}^\dagger_2$, measure modes $2,3$, accept only $1,0$:

$$\mathbf{a}^\dagger_2|\mathbf{0}\rangle_{123} \longrightarrow \left((1/\sqrt[4]{2})\mathbf{a}^\dagger_1 + (1/2)\mathbf{a}^\dagger_2 + (1/2-1/\sqrt[2]{2})\mathbf{a}^\dagger_3\right)|\mathbf{0}\rangle_{123}$$
$$\xrightarrow[=1,0]{\text{meas}_{2,3}} (1/2)|\mathbf{0}\rangle_1$$

  - Success probability $(1/2)^2 = 1/4$.

Non-linear Phaseshift III

- Analysis of $\hat{u} = \begin{pmatrix} 1-\sqrt[2]{2} & 1/\sqrt[4]{2} & \sqrt[2]{3/\sqrt[2]{2}-2} \\ 1/\sqrt[4]{2} & 1/2 & 1/2-1/\sqrt[2]{2} \\ \sqrt[2]{3/\sqrt[2]{2}-2} & 1/2-1/\sqrt[2]{2} & \sqrt[2]{2}-1/2 \end{pmatrix}$:

$$\mathbf{a}^\dagger{}_2|\mathbf{0}\rangle_{123} \longrightarrow (1/2)|\mathbf{0}\rangle_1$$

$$\mathbf{a}^\dagger{}_1\mathbf{a}^\dagger{}_2|\mathbf{0}\rangle_{123} \longrightarrow$$
$$\left((1-\sqrt[2]{2})\mathbf{a}^\dagger{}_1 + (1/\sqrt[4]{2})\mathbf{a}^\dagger{}_2 + (\sqrt[2]{3/\sqrt[2]{2}-2})\mathbf{a}^\dagger{}_3\right)$$
$$* \left((1/\sqrt[4]{2})\mathbf{a}^\dagger{}_1 + (1/2)\mathbf{a}^\dagger{}_2 + (1/2-1/\sqrt[2]{2})\mathbf{a}^\dagger{}_3\right)|\mathbf{0}\rangle_{123}$$
$$= \left(\left((1-\sqrt[2]{2})(1/2) + (1/\sqrt[4]{2})(1/\sqrt[4]{2})\right)\mathbf{a}^\dagger{}_1\mathbf{a}^\dagger{}_2 + \ldots\right)|\mathbf{0}\rangle_{123}$$
$$\xrightarrow[=1,0]{\text{meas}_{2,3}} (1/2)\mathbf{a}^\dagger{}_1|\mathbf{0}\rangle_1$$

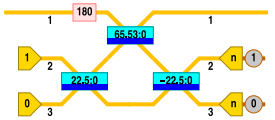
Non-linear Phaseshift III

- Analysis of $\hat{u} = \begin{pmatrix} 1-\sqrt[2]{2} & 1/\sqrt[4]{2} & \sqrt[2]{3/\sqrt[2]{2}-2} \\ 1/\sqrt[4]{2} & 1/2 & 1/2-1/\sqrt[2]{2} \\ \sqrt[2]{3/\sqrt[2]{2}-2} & 1/2-1/\sqrt[2]{2} & \sqrt[2]{2}-1/2 \end{pmatrix}$:

$$\mathbf{a}^\dagger{}_2|\mathbf{0}\rangle_{123} \longrightarrow (1/2)|\mathbf{0}\rangle_1 \;;\; \mathbf{a}^\dagger{}_1\mathbf{a}^\dagger{}_2|\mathbf{0}\rangle_{123} \longrightarrow (1/2)\mathbf{a}^\dagger{}_1|\mathbf{0}\rangle_1$$

$$\mathbf{a}^\dagger{}_1{}^2\mathbf{a}^\dagger{}_2|\mathbf{0}\rangle_{123} \longrightarrow$$
$$\left((1-\sqrt[2]{2})\mathbf{a}^\dagger{}_1 + (1/\sqrt[4]{2})\mathbf{a}^\dagger{}_2 + (\sqrt[2]{3/\sqrt[2]{2}-2})\mathbf{a}^\dagger{}_3\right)^2$$
$$* \left((1/\sqrt[4]{2})\mathbf{a}^\dagger{}_1 + (1/2)\mathbf{a}^\dagger{}_2 + (1/2-1/\sqrt[2]{2})\mathbf{a}^\dagger{}_3\right)|\mathbf{0}\rangle_{123}$$
$$= \Big(\left((1-\sqrt[2]{2})^2(1/2) + 2(1-\sqrt[2]{2})(1/\sqrt[4]{2})(1/\sqrt[4]{2})\right)\mathbf{a}^\dagger{}_1{}^2\mathbf{a}^\dagger{}_2$$
$$+ \ldots\Big)|\mathbf{0}\rangle_{123}$$
$$\xrightarrow[=1,0]{\text{meas}_{2,3}} -(1/2)\mathbf{a}^\dagger{}_2{}^2\mathbf{a}^\dagger{}_1|\mathbf{0}\rangle_1$$

Non-linear Phaseshift IV

- Analysis of NS_{180° = 

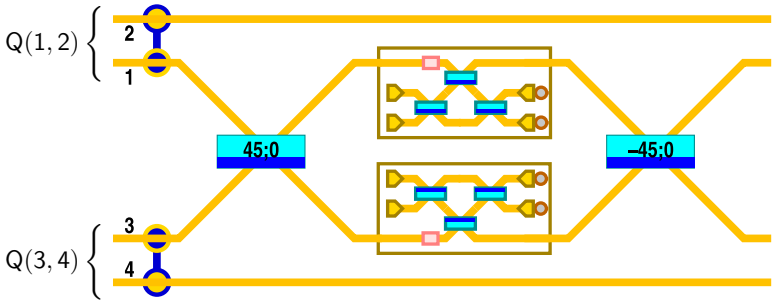
:

Conclusion:

- Realizes $\left\{\begin{array}{lcl} |0\rangle_1 & \rightarrow & |0\rangle_1 \\ |1\rangle_1 & \rightarrow & |1\rangle_1 \\ |2\rangle_1 & \rightarrow & -|2\rangle_1 \end{array}\right\}$ with probability $.25$.
- Mode 2 detector must distinguish 1 from $0, 2, 3$ photons.
- Resources for success in multiple attempts:
  - Contributing successfully: 3 beamsplitters, 1 photon, 2 detections.
  - Average total until success, with $f = 1/4$:
    Multiplier $= f + 2(1-f)f + 3(1-f)^2 f + 4(1-f)^3 f + \ldots$
    $= f(1/(1-(1-f))^2)$
    $= 1/f$ E.g. $4 * 2$ detections

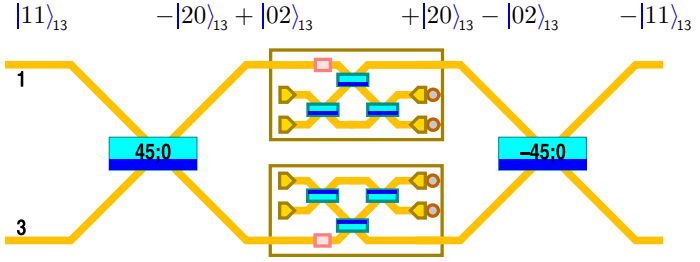
Conditional Sign Flip I

- Conditional sign flip for qubits:
  $\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \rightarrow \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle - \delta|11\rangle$
- Postselected $\text{CS}_{1/16}$ with $\text{prob}_{\text{succ}} = 1/16$:



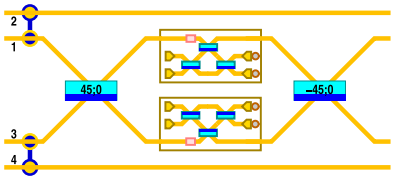
Conditional Sign Flip I

- Conditional sign flip for qubits:
  $\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \rightarrow \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle - \delta|11\rangle$
- Postselected two-mode nonlinear sign flip, $\text{prob}_{\text{succ}} = 1/16$:



Conditional Sign Flip II

- Resources for successful $\text{CS}_{1/16}$:
  - Contributing:



$$\begin{aligned} \text{b}(\text{CS}_{1/16}) &= 8 \text{ beamsplitters} \\ \text{p}(\text{CS}_{1/16}) &= 2 \text{ photons} \\ \text{d}(\text{CS}_{1/16}) &= 4 \text{ detections} \end{aligned}$$

  - Total, with parallel attempts of NS_{180°:

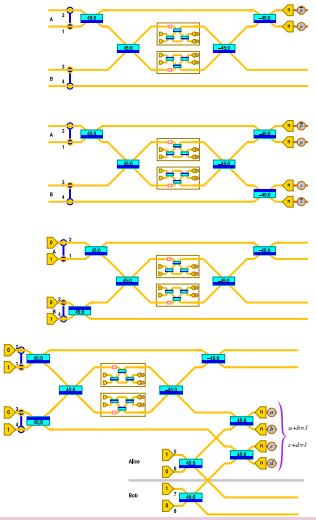
$$\begin{aligned} \overline{\text{b}}(\text{CS}_{1/16}) &= 1*16 + 6*16 + 1 = 113 \\ \overline{\text{p}}(\text{CS}_{1/16}) &= 2*16 = 32 \\ \overline{\text{d}}(\text{CS}_{1/16}) &= 4*16 = 64 \end{aligned}$$

  - Total, with sequential attempts of NS_{180°:

$$\begin{aligned} \overline{\text{b}}(\text{CS}_{1/16}) &= 1*16 + 3*16 + 3*4 + 1 = 77 \\ \overline{\text{p}}(\text{CS}_{1/16}) &= 1*16 + 1*4 = 20 \\ \overline{\text{d}}(\text{CS}_{1/16}) &= 2*16 + 2*4 = 40 \end{aligned}$$

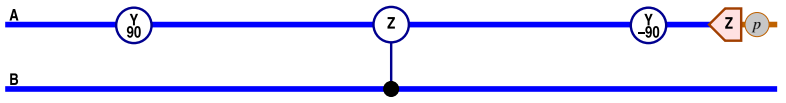
Applications

- Implement with success probability $1/16$:
  - Coherence preserving parity measurement.
  - Bell basis measurement.
  - Bell state preparation.
  - Entangling “without” interaction.

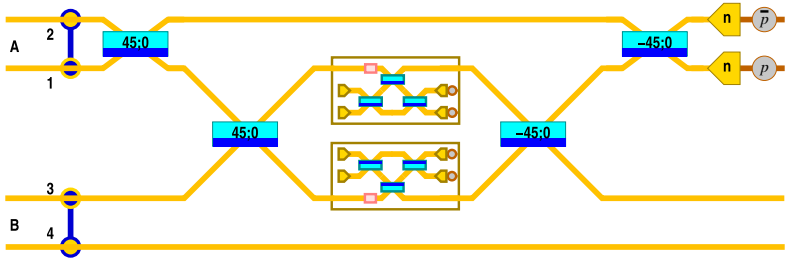


Coherence Preserving Parity

- Parity measurement for qubits:



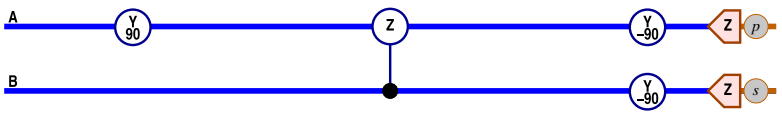
- For bosonic qubits, probability $1/16$:



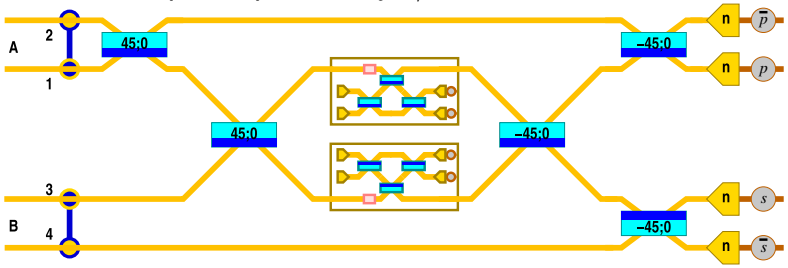
Back to Applications

Bell Measurement

- Bell basis measurement for qubits:



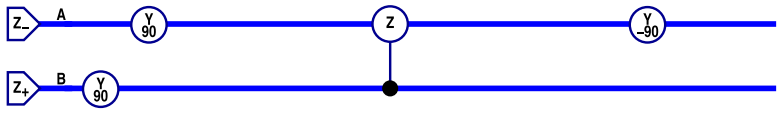
- For bosonic qubits, probability $1/16$:



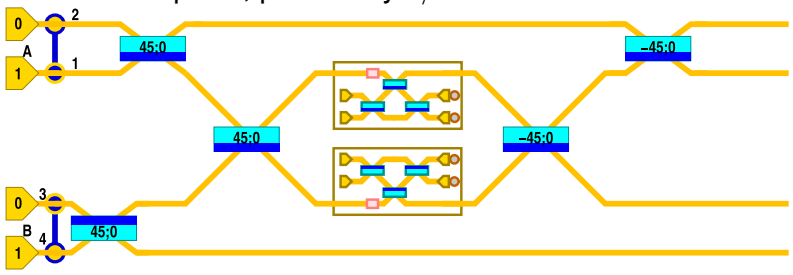
Back to Applications

Bell Preparation

- Bell state preparation for qubits:



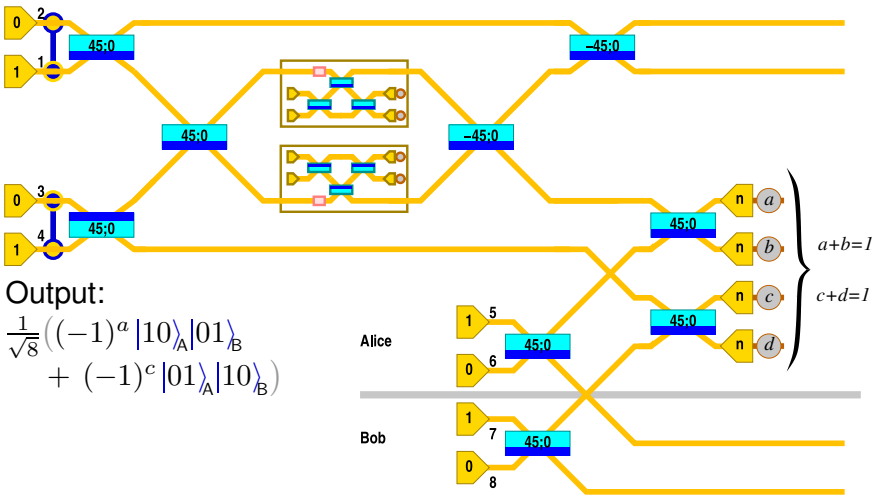
- For bosonic qubits, probability $1/16$:



Back to Applications

Entangling "without" Interaction

- Entangling photons, probability $1/4$.



Output:

$$\frac{1}{\sqrt{8}}\big((-1)^a\,|10\rangle_A|01\rangle_B + (-1)^c\,|01\rangle_A|10\rangle_B\big)$$

Efficient Conditional Sign Flip I

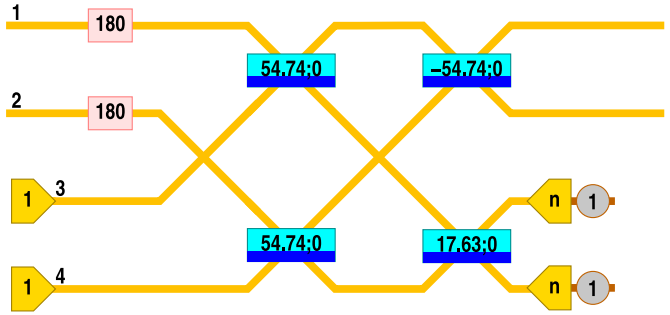
- $|ab\rangle_{12} \to (-1)^{a\cdot b}|ab\rangle_{12}$ with success probability $1/13.5$:

$$\hat{u} = \begin{pmatrix} -1/3 & -\sqrt{2}/3 & \sqrt{2}/3 & 2/3 \\ \sqrt{2}/3 & -1/3 & -2/3 & \sqrt{2}/3 \\ -\sqrt{3+\sqrt{6}}/3 & \sqrt{3-\sqrt{6}}/3 & -\sqrt{(3+\sqrt{6})/2}/3 & \sqrt{1/6-1/(3\sqrt{6})} \\ -\sqrt{3-\sqrt{6}}/3 & -\sqrt{3+\sqrt{6}}/3 & -\sqrt{1/6-1/(3\sqrt{6})} & -\sqrt{(3+\sqrt{6})/2}/3 \end{pmatrix}$$

Input: State on modes $3, 4$: $|11\rangle$.
Output: Measure modes $3, 4$, accept if $|11\rangle$.

Efficient Conditional Sign Flip I

- $|ab\rangle_{12} \rightarrow (-1)^{a \cdot b} |ab\rangle_{12}$ with success probability $1/13.5$:



Efficient Conditional Sign Flip II

- Decomposition into beamsplitter operations:

$$\hat{u} = \begin{pmatrix} .5774 & .8165 & 0 & 0 \\ -.8165 & .5774 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & .9530 & -.3029 \\ 0 & 0 & .3029 & .9530 \end{pmatrix}$$

$$\begin{pmatrix} -.5774 & 0 & .8165 & 0 \\ 0 & -.5774 & 0 & .8165 \\ -.7781 & .2473 & -.5502 & .1749 \\ -.2473 & -.7781 & -.1749 & -.5502 \end{pmatrix}$$

$$\begin{pmatrix} -.5774 & 0 & .8165 & 0 \\ 0 & -.5774 & 0 & .8165 \\ -.8165 & 0 & -.5774 & 0 \\ 0 & -.8165 & 0 & -.5774 \end{pmatrix}$$

Efficient Conditional Sign Flip II

- Decomposition into beamsplitter operations:

$$\hat{u} = \begin{pmatrix} -.3333 & -.4714 & .4714 & .6667 \\ .4714 & -.3333 & -.6667 & .4714 \\ -.7781 & .2473 & -.5502 & .1749 \\ -.2473 & -.7781 & -.1749 & -.5502 \end{pmatrix}$$

$$\hat{u} = \begin{pmatrix} .5774 & .8165 & 0 & 0 \\ -.8165 & .5774 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -.5774 & 0 & .8165 & 0 \\ 0 & -.5774 & 0 & .8165 \\ -.7781 & .2473 & -.5502 & .1749 \\ -.2473 & -.7781 & -.1749 & -.5502 \end{pmatrix}$$

Efficient Conditional Sign Flip II

- Decomposition into beamsplitter operations:

$$\hat{u} = \begin{pmatrix} .5774 & .8165 & 0 & 0 \\ -.8165 & .5774 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & .9530 & -.3029 \\ 0 & 0 & .3029 & .9530 \end{pmatrix}$$

$$\begin{pmatrix} -.5774 & 0 & .8165 & 0 \\ 0 & -.5774 & 0 & .8165 \\ -.8165 & 0 & -.5774 & 0 \\ 0 & -.8165 & 0 & -.5774 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & .5774 & 0 & -.8165 \\ 0 & 0 & 1 & 0 \\ 0 & .8165 & 0 & .5774 \end{pmatrix}$$

$$\begin{pmatrix} -.5774 & 0 & .8165 & 0 \\ 0 & -1 & 0 & 0 \\ -.8165 & 0 & -.5774 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

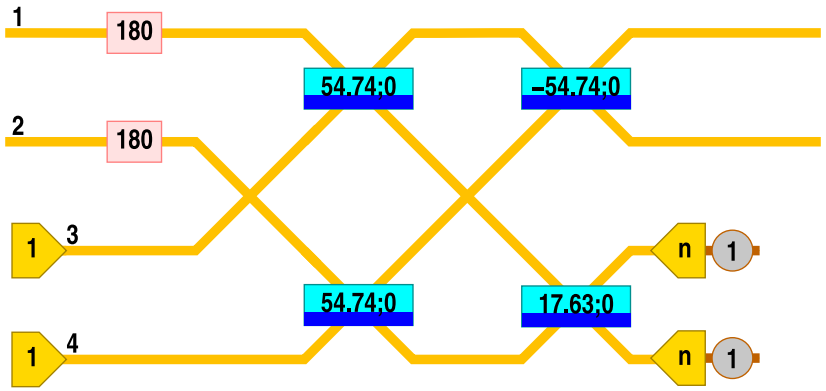
Efficient Conditional Sign Flip II

- Decomposition into beamsplitter operations:

$$\hat{u} = \begin{pmatrix} .5774 & .8165 & 0 & 0 \\ -.8165 & .5774 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & .9530 & -.3029 \\ 0 & 0 & .3029 & .9530 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & .5774 & 0 & -.8165 \\ 0 & 0 & 1 & 0 \\ 0 & .8165 & 0 & .5774 \end{pmatrix} \begin{pmatrix} .5774 & 0 & -.8165 & 0 \\ 0 & 1 & 0 & 0 \\ .8165 & 0 & .5774 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

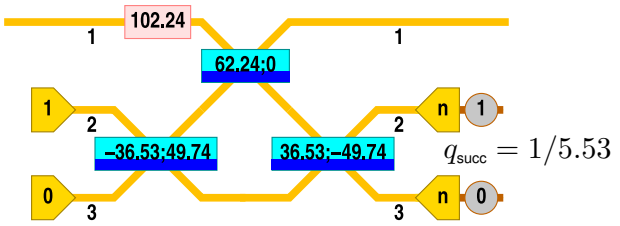
Efficient Conditional Sign Flip III

- Optical network realization:



Other Conditional Phases

- One qubit rotations + two conditional sign flips. Barenco&*al.* (1995) [7]
  ...low probability of success $(1/13.5)^2 = .0055$
- Conditional i-phase shift built on designed gate: $p_{\text{succ}} = 1/30.58$



$q_{\text{succ}} = 1/5.53$

- Best cond. i-phase shift found with two helpers: $p_{\text{succ}} = 1/19.37$

Problems

- **Problem 1:** What is the maximum probability of success for implementing
  $U_\phi : |0\rangle \to |0\rangle\ ,\ |1\rangle \to |1\rangle\ ,\ |2\rangle \to e^{i\phi}|2\rangle$
  with k helper photons and postselection (no feedback)?
- **Problem 2:** Problem 1 with U_ϕ replaced by
  $V_\phi : |00\rangle \to |00\rangle\ ,\ |10\rangle \to |10\rangle$
  $|01\rangle \to |10\rangle\ ,\ |11\rangle \to e^{i\phi}|11\rangle$
- Variations:
  - Conditional phase shift for bosonic qubits.
  - Consider other initial states like $|2\rangle$ or entangled photon pairs.

Knill 2001 [8]

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