Determinism and Quantum Theory Inside Black Holes

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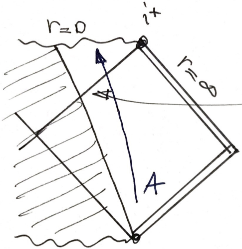
Schwarzschild black hole



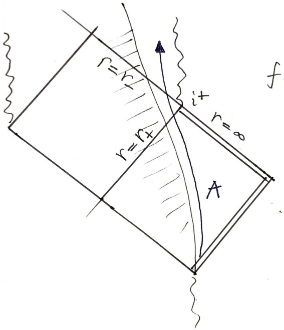
event horizon at $r = r_+ = 2M$

$$g = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

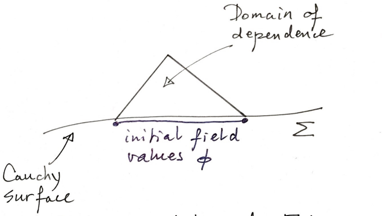
$$f(r) = 1 - \frac{2M}{r}$$

- Observer A ends her existence in singularity ($r=0$) after finite proper time.
- At $r=0$, classical spacetime picture not valid.
- Realistic spacetime contains a collapsing body and only part of horizon.
- Singularity is <u>spacelike</u>.

Reissner-Nordström black hole



$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$



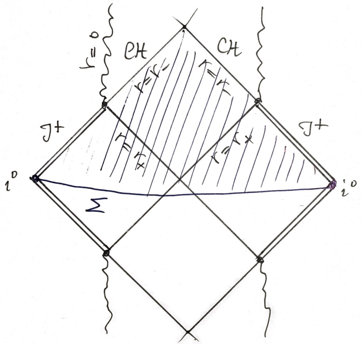
- Forward evolution of $\Box\phi = 0$ uniquely determined by field values inside "domain of dependence"

- A can enter BH but avoid singularity!
- There is an outer $(r=r_+)$ and an inner horizon $(r=r_-)$,
- Region beyond r_- is <u>not</u> <u>predictable</u>.
- Realistic spacetime contains only part of diagram.

$\Rightarrow$ Loss of predictability beyond r_-!

Loss of determinism

Loss of predictability beyond $r = r_-$ in RN spacetime



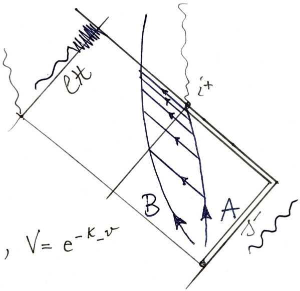
Domain of dependence of Σ' only reaches up to $r = r_-$ which is called "Cauchy horizon" (CH).

$\Rightarrow$ Initial field values on Σ fail to determine ϕ beyond CH !

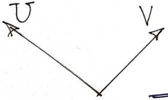
Cosmic censorship

Q: Is this a probem?

A: Yes, unless $\mathcal{CH}$ is unstable in the sense that remnant perturbations ϕ ($\leftrightarrow$ grav. pert., matter fields) from newly formed BH blow up "badly" at $\mathcal{CH}$



$v = t - r_*$, $u = t + r_*$, $V = e^{-\kappa_- v}$



- Text messages (or light signals) sent out periodically by A (according to her proper time) are received by B more and more frequently (according to his proper time) $\Rightarrow$ pile up

- If $\Box\phi = 0$ and ϕ oscillates moderately at $\mathcal{J}^-$, then it will oscillate very rapidly at $\mathcal{CH}$

$\Longrightarrow$ blue shift effect

$\Longrightarrow \partial_V \phi \sim V^{-1} \Rightarrow T_{VV} = (\partial_V \phi)^2 \sim V^{-2}$ at $\mathcal{CH}$

Cosmic censorship

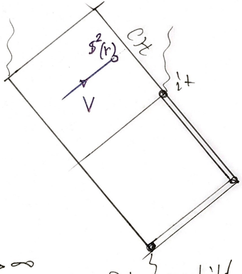
☆ We cannot trust classical picture at CH ⇒ determinism is restored. [Penrose]

☆ Raychaudhuri ; $\frac{d}{dV}\theta = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} - T_{VV}$

expansion shear $\sim V^{-2}$

⇒ castastrophic divergence of θ

☆ Mass inflation [Poisson & Israel]

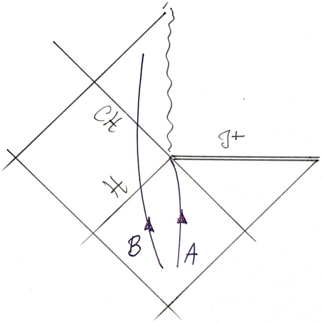


$m(r)$ = Hawking mass inside $S^2(r)$

$\sim \int^V T_{vv}\, \underbrace{T_{VV}}_{\sim V^{-2}} \longrightarrow \infty$

$\Longrightarrow \frac{m(r)}{Q(r)} \longrightarrow \infty \Longrightarrow$ Schwarzschild singularity

RN-deSitter



$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \Lambda r^2$$

$$\Lambda > 0$$

- Now, ϕ oscillates very rapidly at CH (blueshift) <u>but</u> amplitude goes down exponentially at H (redshift $\Leftrightarrow \Lambda$)
- Competing effect
- Not clear that $\partial_v \phi$ diverges at CH, and how.

Classical fields on RN-deSitter

For classical fields, the behavior of ϕ has recently been analyzed by

- [Mellor & Moss; Brady, Moss & Myers; Dias, Reall & Santos; Cardoso et al.; ...] (Physics)
- [Dafermos; Dafermos & Luk; Sbierski; Luk & Oh; Costa et al.; Franzen; Barreto & Zworski; Bony & Häffner; Dyatlov; Wunsch & Zworski, Nonnenmacher & Zworski; Vasy; Hintz & Vasy, ...] (Maths)

Conclusion [Hintz & Vasy] for $(\Box - \mu^2)\phi = 0$ on RNdS:

$\phi \in H^{\frac{1}{2}+\beta}_{loc}$ near $\mathcal{CH}$

$$\boxed{\beta = \frac{\alpha}{\kappa_-}}$$

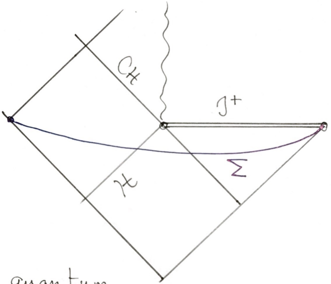
- $\mu^2 \geq 0$
- α = spectral gap of QNM frequencies
- κ_- = surface gravity of $\mathcal{CH}$ $(= f'(r_-))$

" $\Longleftrightarrow$ " $T_{VV} \sim \text{cst.}\, V^{-2+2\beta}$

Numerics [Cardoso et al.]: $\exists$ ranges of M, Q, Λ (near extremal range) s.t. $\beta > \frac{1}{2} \Rightarrow$ strong cosmic censorship violated!

Quantum fields on RN-deSitter

We therefore ask whether quantum effects can change this basic picture [Hollands, Wald & Zahn] (prev. work · [Ori et al.] RN
· [Dias et al.] BTZ 3d)



- We take a quantum state Ψ of the field that is regular near initial Cauchy surface
- Means that $\langle \phi(x_1) \dots \phi(x_n) \rangle_\Psi$ are of "Hadamard type" near Σ'
- Means that $\langle \phi(x_1)\phi(x_2) \rangle_\Psi \sim \frac{\Delta^{1/2}}{\sigma} + V \log \sigma + W_\Psi$

geometric state dep.

Main result

Main result [Hollands, Wald & Zahn]

$$\langle T_{VV} \rangle_\Psi = C V^{-2} + t_{VV}$$

- C is due entirely to quantum effects and only depends on BH parameters
- t_{VV} depends on Ψ but behaves as $t_{VV} \sim V^{2-2\beta}$ i.e. like classical stress tensor (more precise math. form. in paper)

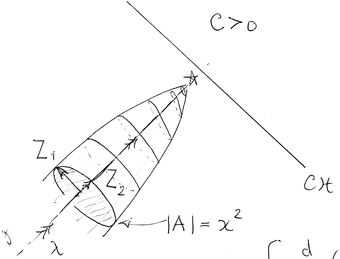
Methods

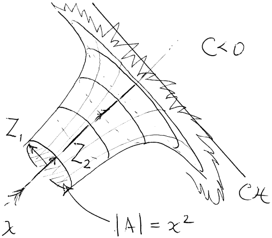
- Semi-analytic mode calculation $(\partial_{r_*}^2 - V_\ell) R = -\omega^2 R$
  $\longrightarrow$ Heun equation, MST-method $R \sim \sum a_n^\nu \, {}_2F_1(\ldots, n, \ldots)$
  [Schmidt; Suzuki, Takasugi & Umetsu, ...] (for C)
- Tools from "microlocal analysis" (for bound on t_{VV})

Comments

Strength of quantum singularity

- Impose semi-classical Einstein eq$^{\underline{n}}$ $G_{\mu\nu} = T^{cl}_{\mu\nu} + \langle T_{\mu\nu} \rangle_{\Psi}$
- Look at streching/crushing of bodies crossing $\mathcal{CH}$
- $\rightarrow$ "Strong" singularity in sense of [Clarke, Tipler]





null-geodesic $\gamma(\lambda)$

Z_1, Z_2 - Jacobi-fields

$$\begin{cases} \frac{d}{d\lambda}\left(x^2 \sigma^A{}_B\right) = -x^2 C^A{}_{\lambda B \lambda} \\ \frac{d^2}{d\lambda^2} x = -\left(\frac{1}{2}\sigma^A{}_B \sigma^B{}_A + R_{\lambda\lambda}\right) x \end{cases}$$

Proof

Outline of argument

① Write $$\langle T_{\mu\nu}\rangle_\Psi = \underbrace{\langle T_{\mu\nu}\rangle_\Psi - \langle T_{\mu\nu}\rangle_U}_{\substack{\text{treated with methods}\\ \text{of ``}\mu\text{-local analysis''}}} + \underbrace{\langle T_{\mu\nu}\rangle_U - \langle T_{\mu\nu}\rangle_F}_{\substack{\text{Schrödinger}\\ \text{scattering probem in 1D}}} + \underbrace{\langle T_{\mu\nu}\rangle_F}_{\substack{\text{smooth}\\ \text{at CH}}}$$

- U – Unruh type state F – "final" state

② $T_{VV} \sim \frac{1}{V^2} T_{vv}$ $(V = e^{-\kappa_- v})$

③ $$\langle T_{vv}\rangle_U - \langle T_{vv}\rangle_F \underset{\text{at CH}}{\sim} \sum_{\ell, m} \int d\omega \; \omega \underbrace{n_\ell(\omega)}_{\text{relative density of states}}$$

$=: C$

→ scattering problem for radial eq^n $(\partial_{r_*}^2 - V_\ell)R = -\omega^2 R$

④ $\langle T_{VV}\rangle_\Psi - \langle T_{VV}\rangle_U \sim \sum_j \pm |\partial_V \psi_j|^2$ where it is shown:

$\psi_j \in H^{\frac{1}{2}+\beta-\varepsilon}$ near CH, $\beta = \frac{\text{spec. gap of QNM}}{\text{surf. grav. of CH}}$, $\sum_j \|\psi_j\|^2_{H^{\frac{1}{2}-}} < \infty$

Final comments

Remarks:

- C generically not zero except fore fine-tuned M, Q, Λ
- $C = 0$ in case of BTZ (3d) consistent with [Dias et al.]. BTZ is not representative for strong cosmic censorship problem.

THANK YOU