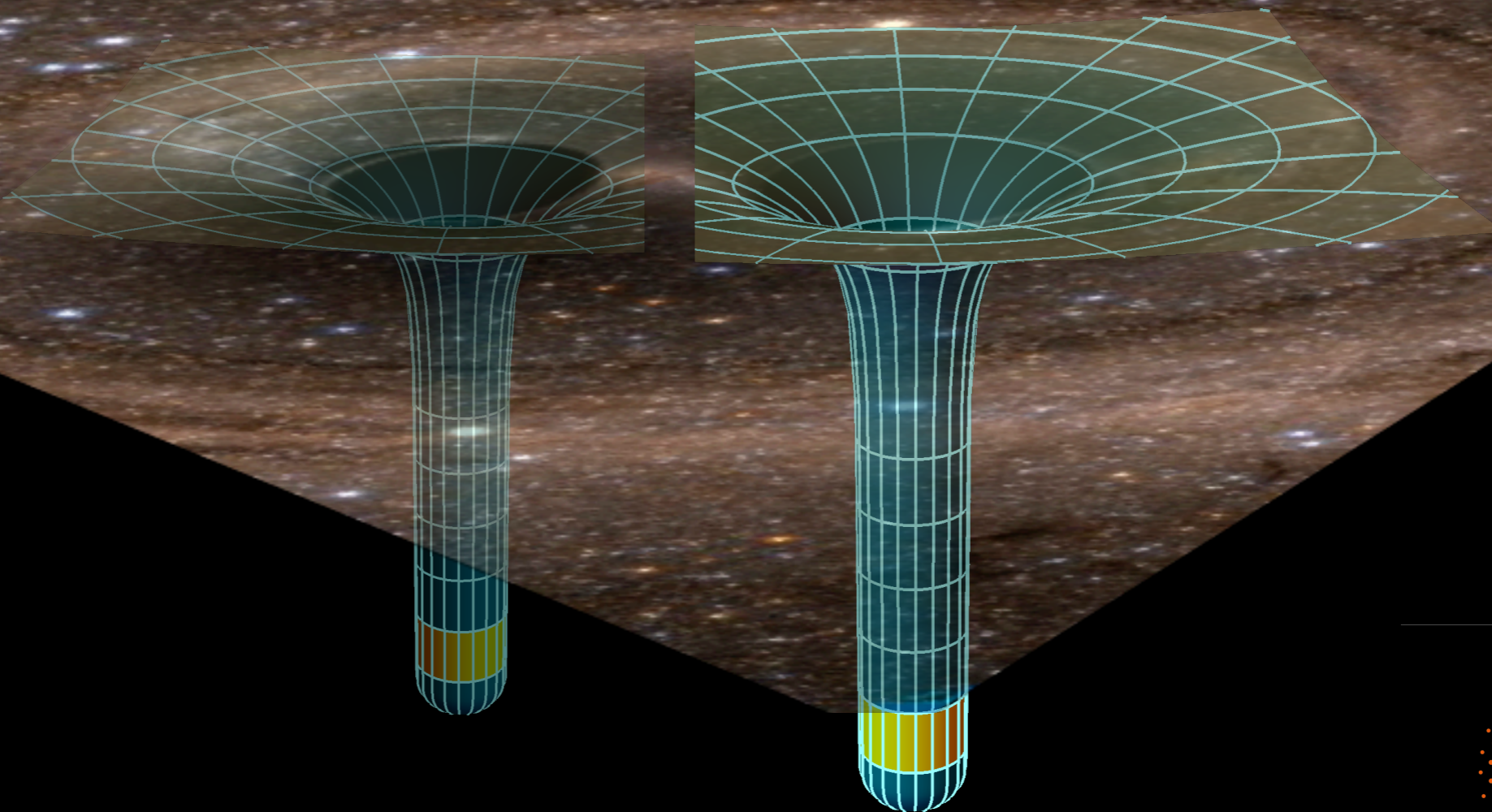


The Quantum from Geometry:

The States of the of Microstate Geometries



European Research Council
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“Geometry from the Quantum”

Nick Warner, KITP Santa Barbara, *January 13-17, 2020.*

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**Original photo credit:
LIGO/Caltech**

Outline

The present state

- ★ Why microstate geometries are an essential part of black-hole physics

Illustrate these ideas:

- ★ The structure of microstate geometries; *superstrata*
- ★ The holographic dictionary for *superstrata*
- ★ Probing superstrata

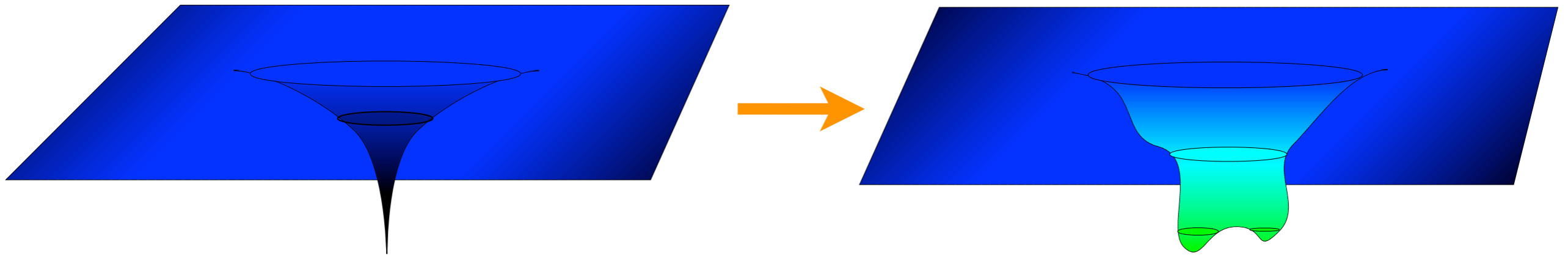
The (hopefully near) future state

- ★ Infall and energy exchange; viscosity
- ★ The gravity dual of twisted-sector states
- ★ Non-extremal microstate geometries *and gauged supergravity*

Microstate Geometries

Smooth, horizonless “solitonic” solutions to the bosonic sector of supergravity (the low-energy limit of string theory) *with the same asymptotic structure as a given black hole or black ring*

Singularity resolved; Horizon removed



Looks exactly like a classical black hole until arbitrarily close to horizon scale

No horizons \Leftrightarrow No information problem

Fundamental mechanism: *Replace singular sources by topology and topological fluxes from geometric transitions*

Now have huge “zoos” of examples

... almost all are BPS

Why you *must* care about *Microstate Geometries I*

They embody the *only gravitational mechanism that can support structure at the horizon scale*

Gibbons and Warner, arXiv:1305.0957

Whatever quantum system you use to model a black hole:

- ★ It must look and behave like the astrophysical black hole described by GR close to the horizon scale ... *it better have some gravity*
- ★ It must have microstructure accessible from near the horizon scale

This strongly suggest that such a quantum system

- has a *semi-classical limit* that supports horizon-scale microstructure against gravitational collapse \Rightarrow *This limit* must be a *microstate geometry*
- there should be a limit in which some of the quantum states are described as stringy fluctuations in a *microstate geometry*
- ... and coherent combinations of such quantum states should be describable as supergravity fluctuations about a *microstate geometry*

Microstate geometries provide a gravitational laboratory for studying horizon-scale microstructure

Why you *must* care about *Microstate Geometries II*

Holographic RG flows, phase transitions and geometric transitions

The problem: Start from a **UV CFT**, perturb the Lagrangian, determine the correct **IR physics**

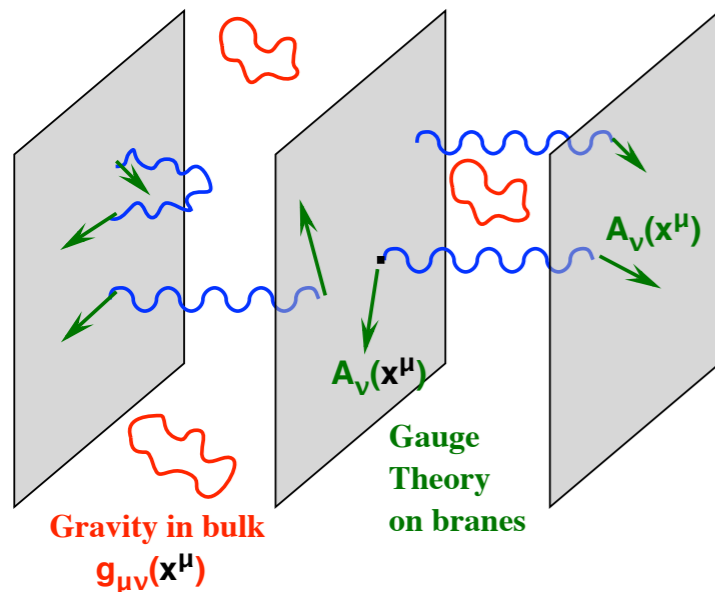
The challenges: What is the *new IR phase*; *Multiple phases*? What are the *order parameters* and their *vevs* and *how do they evolve* into the IR? Are there *new scales generated in the IR*? Is there a *mass gap*, and how do you *compute it*?

Last 20 years: Large range of examples and some very important and general lessons for the gravity side of this duality.

- ★ **IR geometry:** Branes often undergo phase transition to create non-trivial cohomological cycles
 - *Magnetic fluxes* on those cycles dual to *order parameters*
 - *Scale of the cycle* is the emergent *scale of new phase*
 - *Scale of cycle* + *redshift* between UV and IR geometry → **Mass gap**
- ★ You *should impose too much symmetry*:
you may be forbidding the transition to the new phase
- ★ You must *activate all appropriate IR degrees of freedom*:
or you may be forbidding the order parameters of the new phase
- ★ **Singular IR Geometry:** usually the *missing essential physics*/*wrong IR phase*

The holographic dual of a flow to a confining gauge theory

$N = 2$ or $N = 4$ Yang Mills

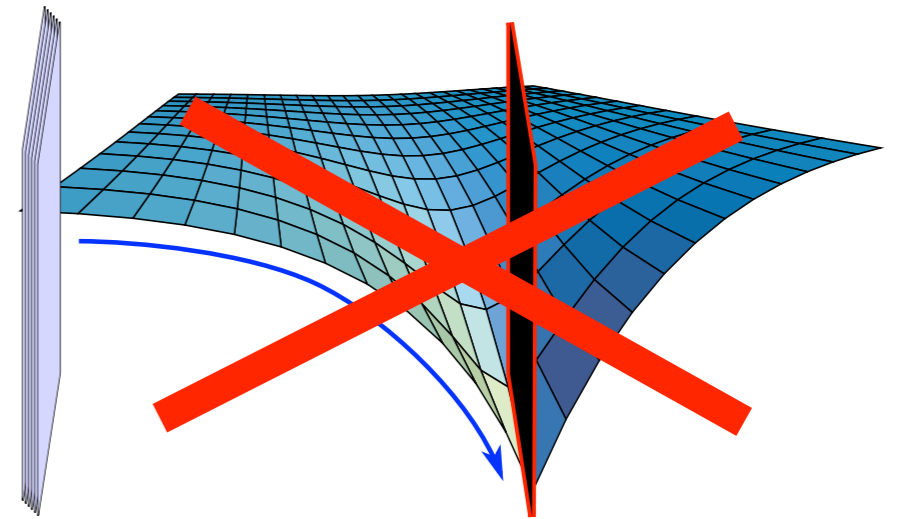


Turn on masses and flow to $N=1$ SQCD

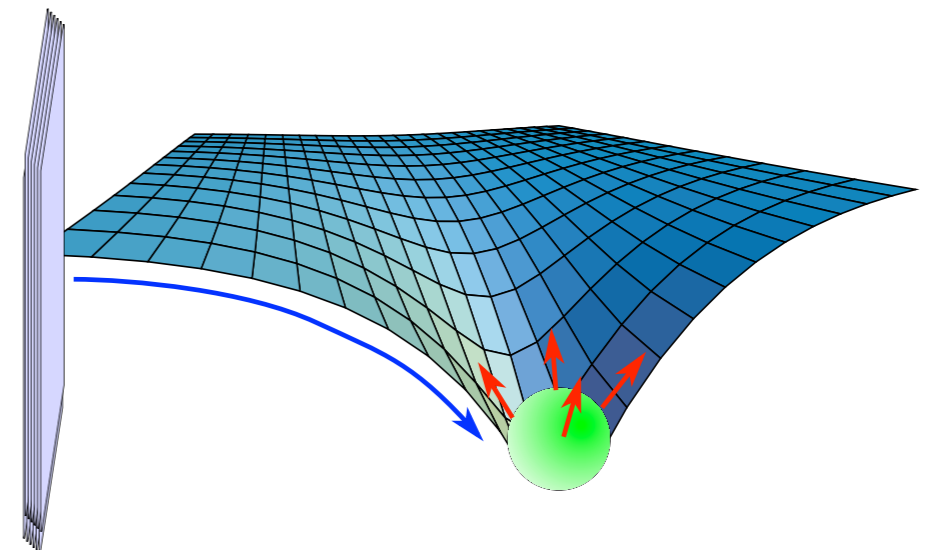
Interesting failures: Singular flows.

- **Gauged supergravity (GPPZ)**
- **Klebanov-Tseytlin**

Limited/Wrong IR Physics



Polchinski-Strassler flow
Klebanov-Strassler flow



The first attempts failed because they were missing critical degrees of freedom essential to the underlying physics ... and too much symmetry

Correct holographic description

- *Magnetic fluxes* dual to *gaugino condensates*
- *Scale of the cycle* dual to *SQCD scale*
- *Scale of cycle + redshift* \rightarrow *Mass gap*

Microstate geometries:

This collection of ideas applied to black holes ...

Black-hole physics

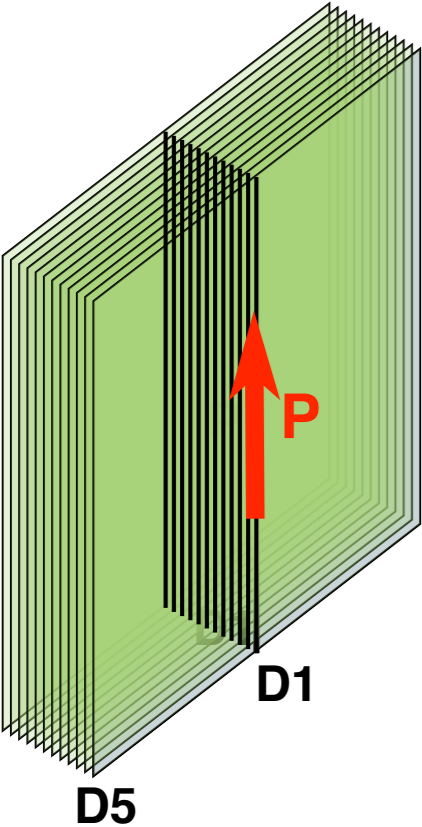
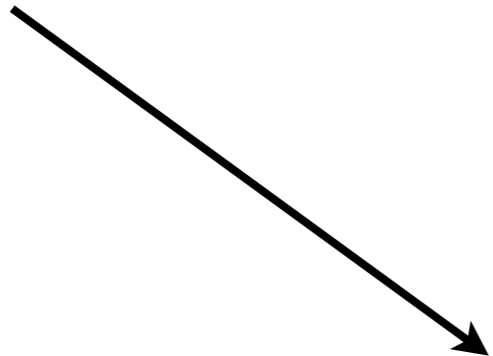
Spherically symmetric solution to gravity

Physics obscured by imposing too much symmetry

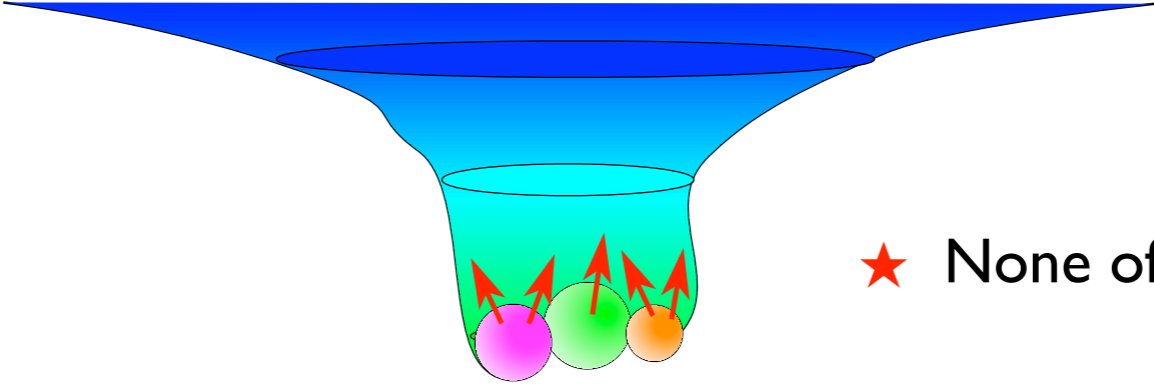
- ★ No scale, no mass gap
- ★ An information problem
- ★ Degrees of freedom completely inaccessible

Only the thermodynamic average of IR Physics

Finite string coupling, g_s



Microstate geometry



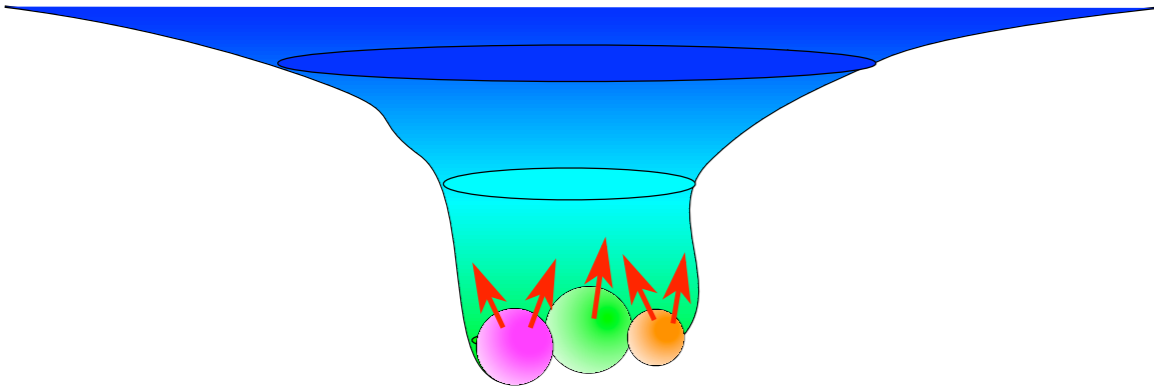
- ★ None of the above

The Microstate Geometry Program so far ...

Two basic mechanisms underlying microstate geometries

Pure topology:

Charges from cohomological fluxes



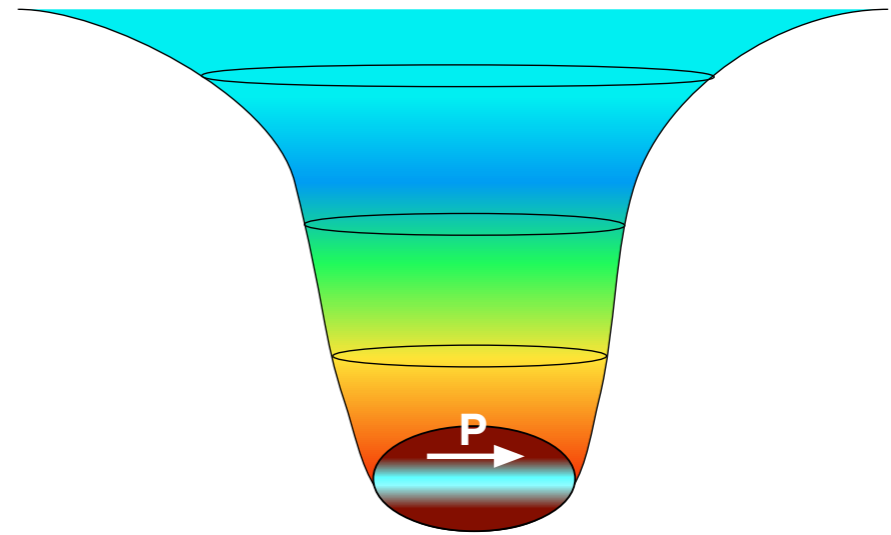
$$d * F^{(p)} \sim \sum_k G^{(k)} \wedge G^{(D-p-k)}$$

Charges from Chern-Simons interaction

Holographic Dictionary = Open problem!

Superstrata:

Momentum waves on topological “bubbles”



Third charge adjusted via momentum waves on bubbled geometry

Well-developed holographic dictionary

⇒ Central focus for last few years

+ Hybrids = Momentum waves on multi-bubble geometries = Open problem!

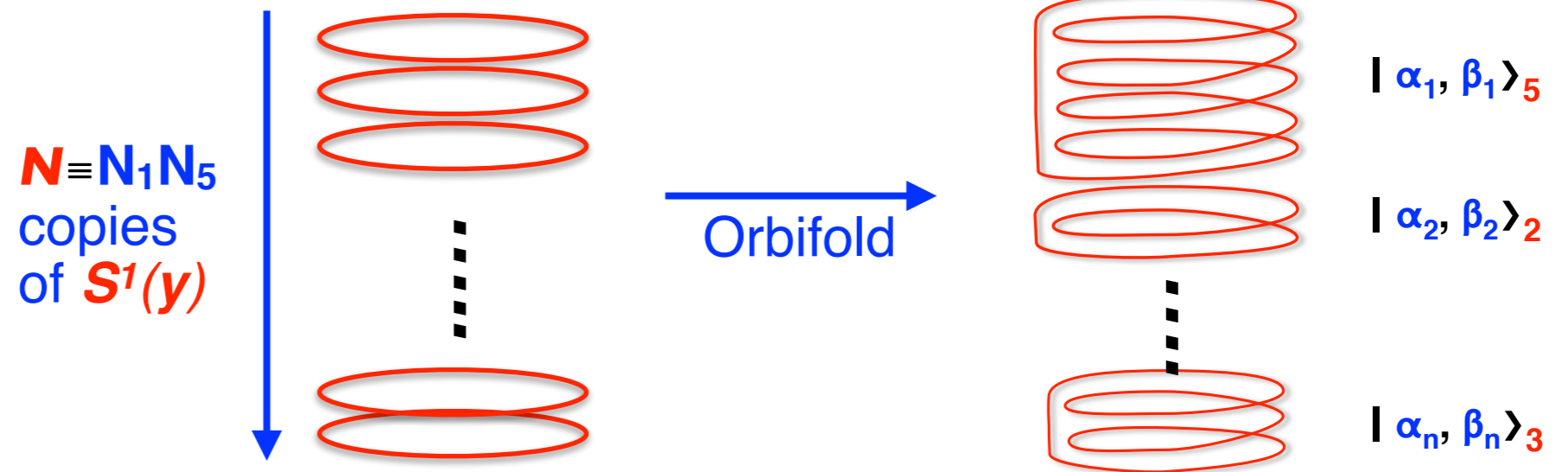
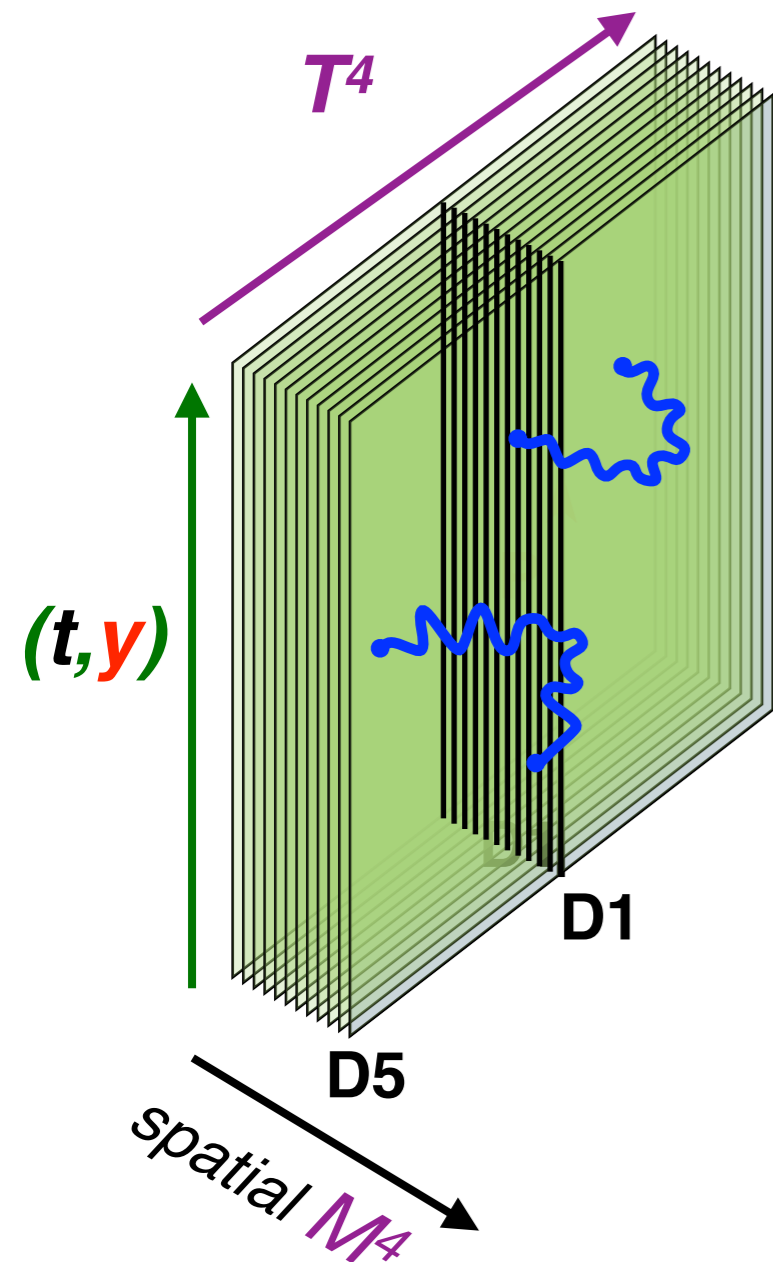
Superstrata

The Field Theory: The D1-D5 system wrapped on $T^4 \times S^1(y)$

Open D1-D5 superstrings moving in T^4 with $N \equiv N_1 N_5$ Chan-Paton labels: $(T^4)^N / S_N$

\Rightarrow Supersymmetric CFT with $c = 6 N_1 N_5$

Ramond-Ramond Ground States



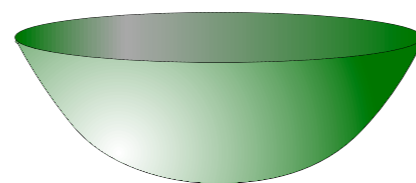
Ground-state spins on strand of length k: $|\alpha_i, \beta_i\rangle_k$

These are $\frac{1}{4}$ BPS States with (4,4) supersymmetry

Gravity dual: $AdS_3 \times S^3$

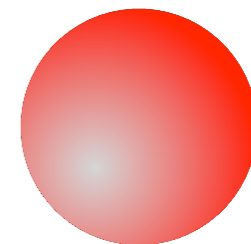
Maximally spinning ground state:

$$\left(|+\frac{1}{2}, +\frac{1}{2}\rangle_1\right)^N$$



(t, y, r)

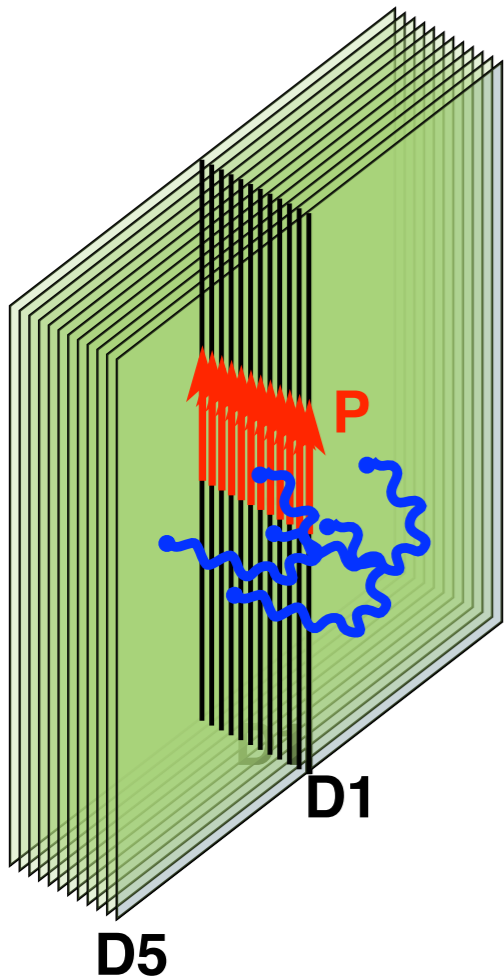
\times



(θ, ψ, ϕ)

Mode excitations on sphere

Momentum Excitations:



Add purely **left-moving** momentum:

$$Q_P \sim N_P = L_{0,\text{left}} \neq 0$$

Right moving sector: **Ramond ground state**

\Rightarrow **1/8 BPS states**

At vanishing string coupling, $g_s \rightarrow 0$, Cardy formula:

$$\begin{aligned}
 S &\equiv \log(\Omega(Q_P)) = 2\pi \sqrt{\frac{c}{6} L_0} \\
 &= 2\pi \sqrt{N_1 N_5 N_P} = 2\pi \sqrt{Q_1 Q_5 Q_P}
 \end{aligned}$$

Finite string coupling, g_s

★ **Spherical symmetry:** Matter/microstate structure disappears inside black-hole horizon

$$S = \frac{1}{4} A = 2\pi \sqrt{Q_1 Q_5 Q_P}$$

Perfect match! Declare victory

Strominger, Vafa 1996

Horizon Area, A

Thermodynamic average of IR Physics

D5

Too much symmetry

Superstrata and the holography of the D1-D5 CFT

We now know the holographic dual of a very specific coherent subsets of the BPS states counted by Strominger and Vafa: **“Supergraviton gas”**

Left-moving states: (Right moving sector: *Ramond ground state*; $\frac{1}{8}$ BPS)

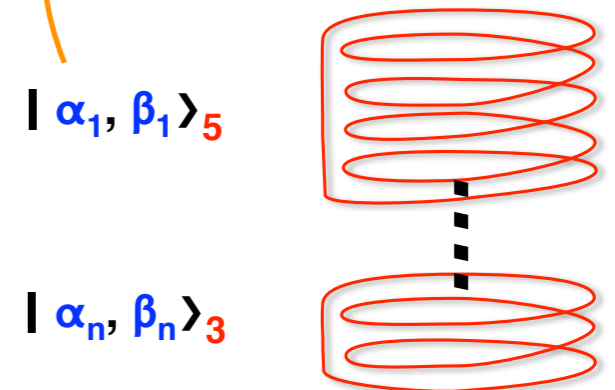
$$\left(\left| +\frac{1}{2}, +\frac{1}{2} \right\rangle_1 \right)^{N_0} \otimes \left[\bigotimes_{k_i, m_i, n_i} \left(\frac{1}{m_i! n_i!} \underbrace{(J_{-1}^+)^{m_i} (L_{-1} - J_{-1}^3)^{n_i}}_{\text{excitations}} |00\rangle_{k_i} \right)^{N_{k_i, m_i, n_i}} \right]$$

$i = 1, \dots, N \equiv N_1 N_5$

Momentum and R-charge excitations

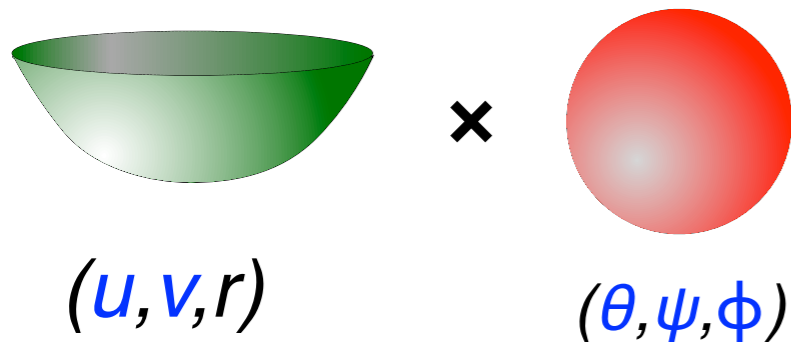
Geometric modes:

$$\chi_{k_j, m_j, n_j} \equiv R^{-1} (m_j + n_j) v + \frac{1}{2} (k_j - 2m_j) \psi - \frac{1}{2} k_j \phi$$

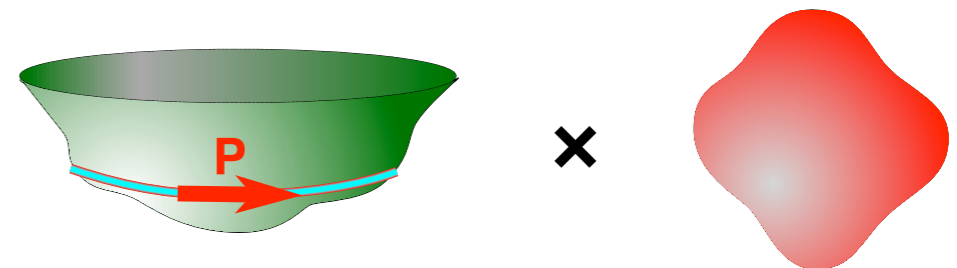


Coherent states $N_{k_i, m_i, n_i} \gg 1$

$AdS_3 \times S^3$



Harmonic deformation of $AdS_3 \times S^3$



Superstrata: Holomorphic waves of microstructure

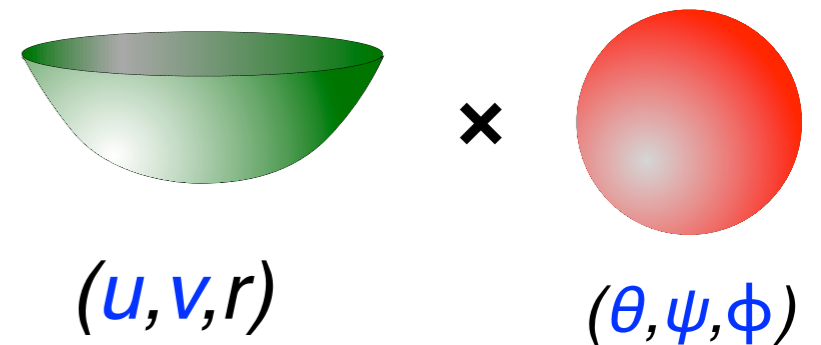
Coherent states of the supergraviton gas:

Geometric modes:

$$\chi_{k_j, m_j, n_j} \equiv R^{-1} (m_j + n_j) v + \frac{1}{2} (k_j - 2m_j) \psi - \frac{1}{2} k_j \phi$$

with arbitrary Fourier coefficients

⇒ Supergravity solutions based on arbitrary functions of **three variables**



Generic **BPS** Superstrata ⇒ arbitrary **holomorphic** functions of three variables

$$\xi \equiv \frac{r}{\sqrt{r^2 + a^2}} e^{i \frac{\sqrt{2}}{Ry} v}, \quad \chi \equiv \frac{a}{\sqrt{r^2 + a^2}} \sin \theta e^{i \frac{1}{2} (\psi + \phi)}, \quad \eta \equiv \frac{a}{\sqrt{r^2 + a^2}} \cos \theta e^{i \left(\frac{\sqrt{2}v}{Ry} - \frac{1}{2} (\psi - \phi) \right)}$$

Hugely simplifies the construction ...

P. Heidmann, N.P. Warner: 1903.07631

P. Heidmann, D. Mayerson, R. Walker, N.P. Warner: 1910.10714

Superstrata → largest classes of solutions ever built = States of the supergraviton gas

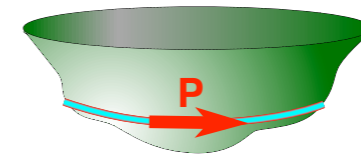
Counting superstrata:

$$S_{superstrata} \sim \sqrt{Q_1 Q_5} (Q_P)^{1/4} \ll \sqrt{Q_1 Q_5 Q_P} \sim S_{black\ hole}$$

M. Shigemori, 1907.03878

What are superstrata missing? **The twisted sector states of the CFT ...**

The structure of the (2+1)-geometry



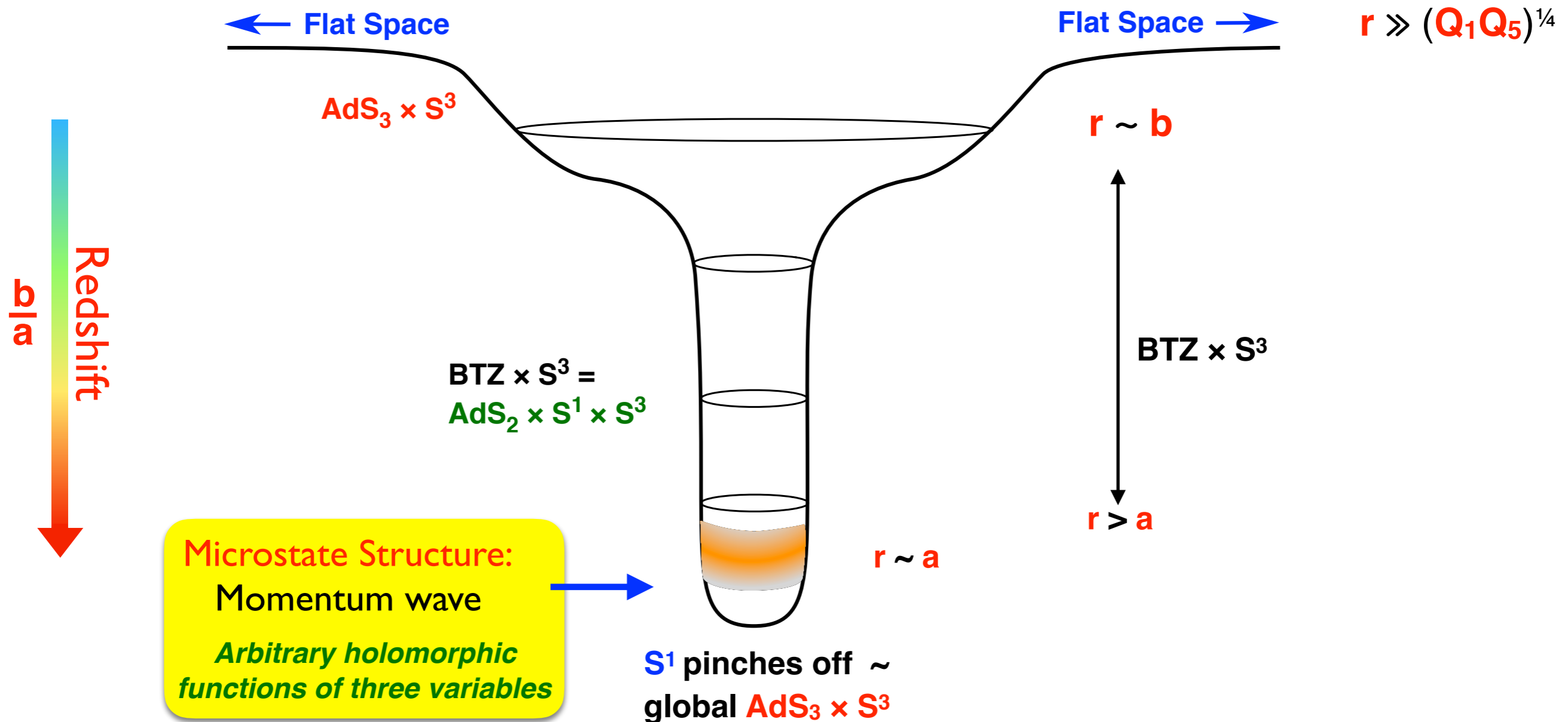
Asymptotically flat $R^{4,1} \times S^1 \rightarrow$ BTZ ($\times S^3$) \rightarrow Capped in IR by global AdS_3
 Looks just like a five-dimensional black hole

Important scales related to charges

b \leftrightarrow momentum charge, Q_P

a \leftrightarrow angular momenta, j_L, j_R

Low angular momentum: **b** \gg **a**



A Simple Class of D1-D5 States

$$\left(|+\frac{1}{2}, +\frac{1}{2}\rangle_1\right)^{N_0} \otimes \left(\frac{1}{n!} (L_{-1} - J_{-1}^3)^n |00\rangle_1\right)^{N_n} \quad \text{with} \quad N_1 N_5 = N_0 + N_n$$

(0,4) supersymmetry; $\frac{1}{8}$ BPS states

Supergravity Parameters: Fourier Coefficients **a, b**; Momentum mode, **n**

$$\text{Compactification/Planck Scale:} \quad \mathcal{K} \equiv \frac{\text{Vol}(T^4) R_y^2}{\ell_{10}^8}$$

Quantized Parameters:

$$j_L = \tilde{j}_R = \frac{1}{2} \mathcal{K} a^2 \quad N_P = \frac{1}{2} \mathcal{K} n b^2$$

Probing Microstate Geometries: Tides

Consider a family of *time-like* geodesics with velocity vectors, V^μ , and proper time, τ

A *deviation vector*, S^μ , is any *space-like* unit vector (at $\tau = 0$) and *everywhere* perpendicular to the geodesics:

$$S^\mu S_{\mu\tau=0} = 1, \quad V^\mu S_\mu = 0$$

The *relative acceleration* between geodesics in a family is then given by:

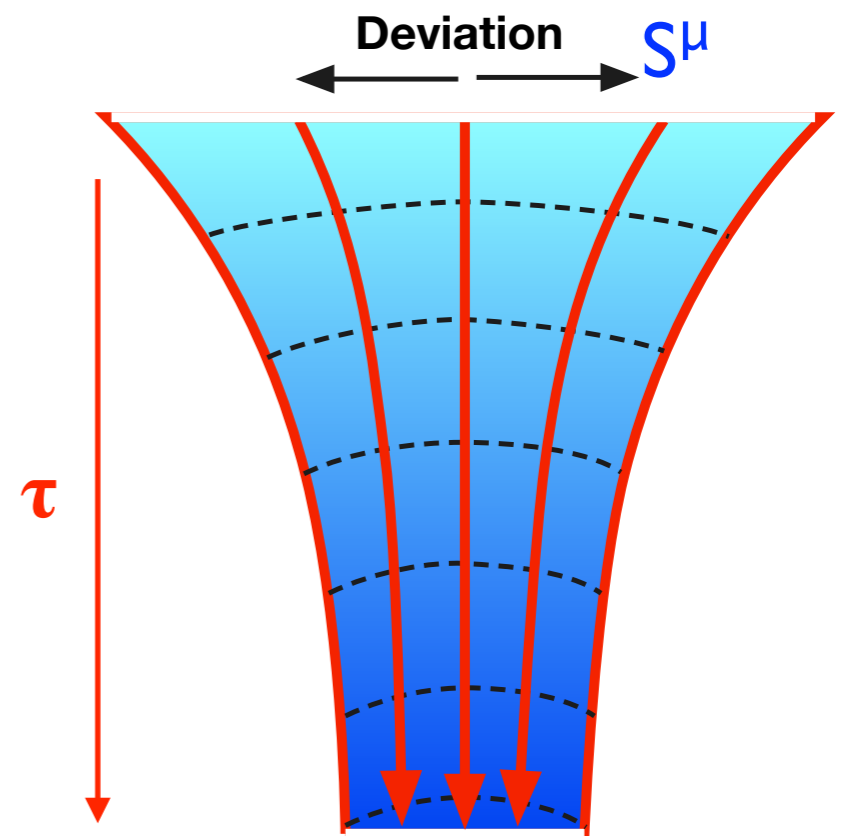
$$a^\mu \equiv \frac{d^2 S^\mu}{d\tau^2} = \mathcal{A}^\mu{}_\nu S^\nu$$

where $\mathcal{A}^\mu{}_\nu \equiv -R^\mu{}_{\rho\nu\sigma} V^\rho V^\sigma$ is the *tidal tensor*.

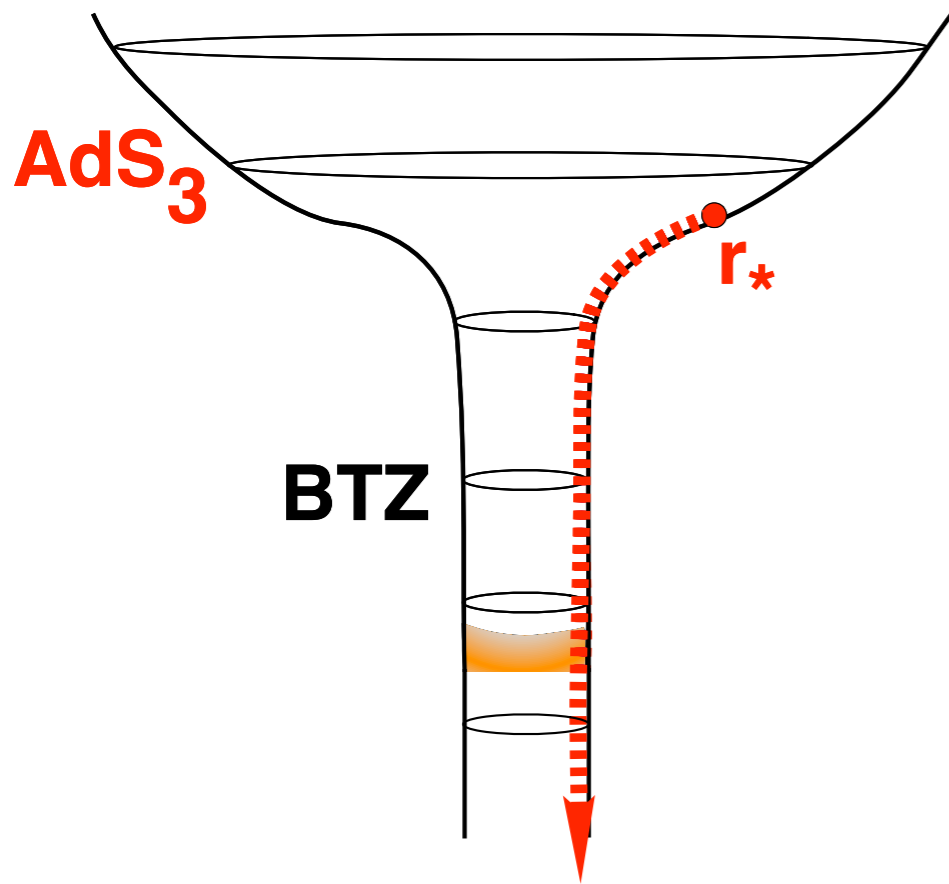
The tidal tensor satisfies $\mathcal{A}^\mu{}_\nu V^\nu = 0$ and so only has space-like components

The norm of the tidal tensor is defined by: $|\mathcal{A}| \equiv \sqrt{\mathcal{A}^\mu{}_\nu \mathcal{A}^\nu{}_\mu}$

and sets the scale of the tidal force per unit length, per unit mass



Tidal Forces: BTZ



BTZ Metric: $a = 0$

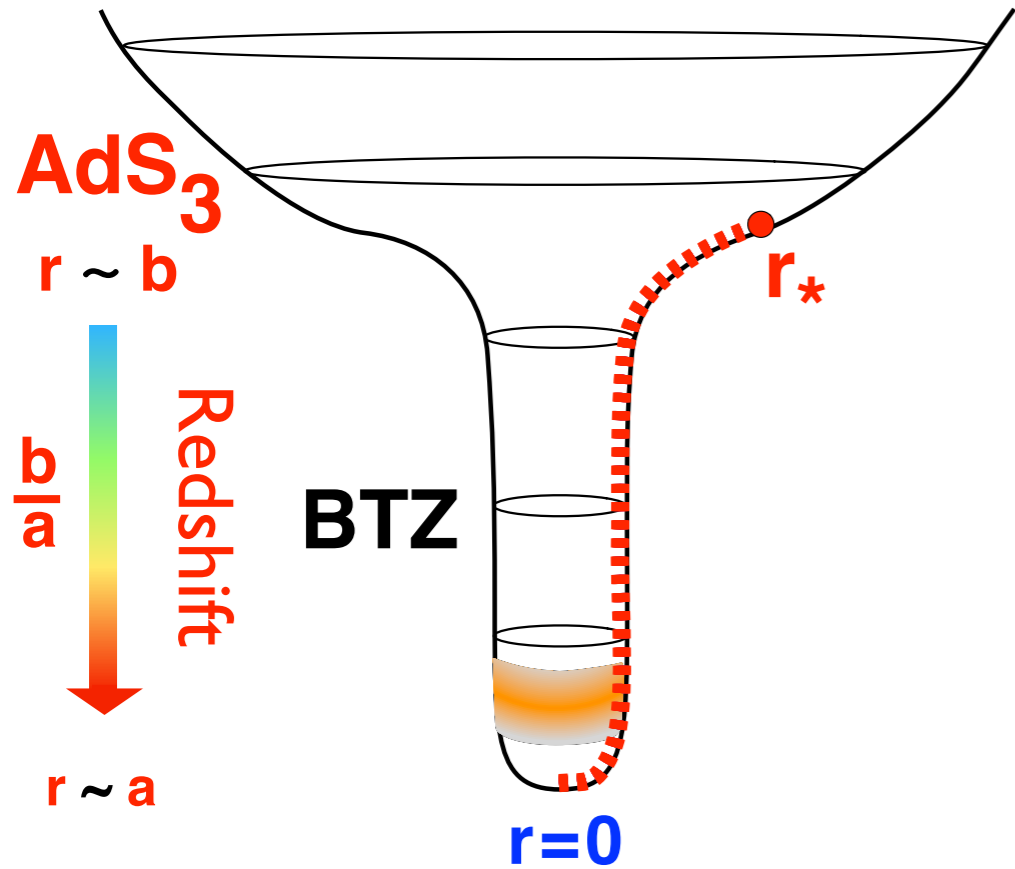
- No “drama at the horizon”
- Locally same curvature as AdS_3

Tidal tensor magnitude along radial infall:

$$|\mathcal{A}| = \frac{2}{R_y b} = \frac{\sqrt{2}}{\sqrt{Q_1 Q_5}} = \frac{\sqrt{2}}{\sqrt{N_1 N_5}} \frac{\sqrt{\text{Vol}(T^4)}}{\ell_{10}^4}$$

Vanishes for large $N \equiv N_1 N_5$

Tidal Forces in Superstrata



Tidal tensor has BTZ term: $|\mathcal{A}| = \frac{2}{R_y b}$

Tidal tensor also has higher multipole moments:

$$|\mathcal{A}|_{\text{throat}} \sim \frac{a^2 b^2 E^2}{r^6}$$

$E = \text{energy of geodesic particle}$

Note that this vanishes for BTZ ($a=0$)

$$r_* \approx b \sqrt{n} \Rightarrow |E| \approx \sqrt{\frac{bn}{R_y}} \Rightarrow |\mathcal{A}|_{\text{throat}} \sim \frac{a^2 b^3 n}{R_y r^6}$$

At $r \sim \sqrt{ab}$ this gives $|\mathcal{A}|_{\text{throat}} \sim \frac{n}{a R_y} \sim \frac{n}{\sqrt{2} j_L} \frac{\sqrt{\text{Vol}(T^4)}}{\ell_{10}^4}$

At $r \sim \sqrt{ab}$ in the deepest possible throats: $|\mathcal{A}|_{\text{throat}} \sim \frac{\sqrt{\text{Vol}(T^4)}}{\ell_{10}^4}$

\Rightarrow Tidal forces hit the Planck/Compactification scale at $r \sim \sqrt{ab}$

Penrose limit of ultra-relativistic *radially infalling* string:

Light-cone gauge: $x^- = \alpha' p^+ \tau$

Transverse oscillations

$$(\partial_\tau^2 - \partial_\sigma^2) z^i = (\alpha' p^+)^2 \mathcal{A}_{ij}(\alpha' p^+ \tau) z^j$$

\mathbf{A}_{ij} is a “time-dependent” mass term.

AdS or BTZ: $\mathbf{A}_{ij} \equiv 0$

Superstratum at large r , \mathbf{A}_{ij} has eigenvalues $\sim +\frac{a^2 b^2}{r^6}$ and $-\frac{a^2 b^2}{r^6}$

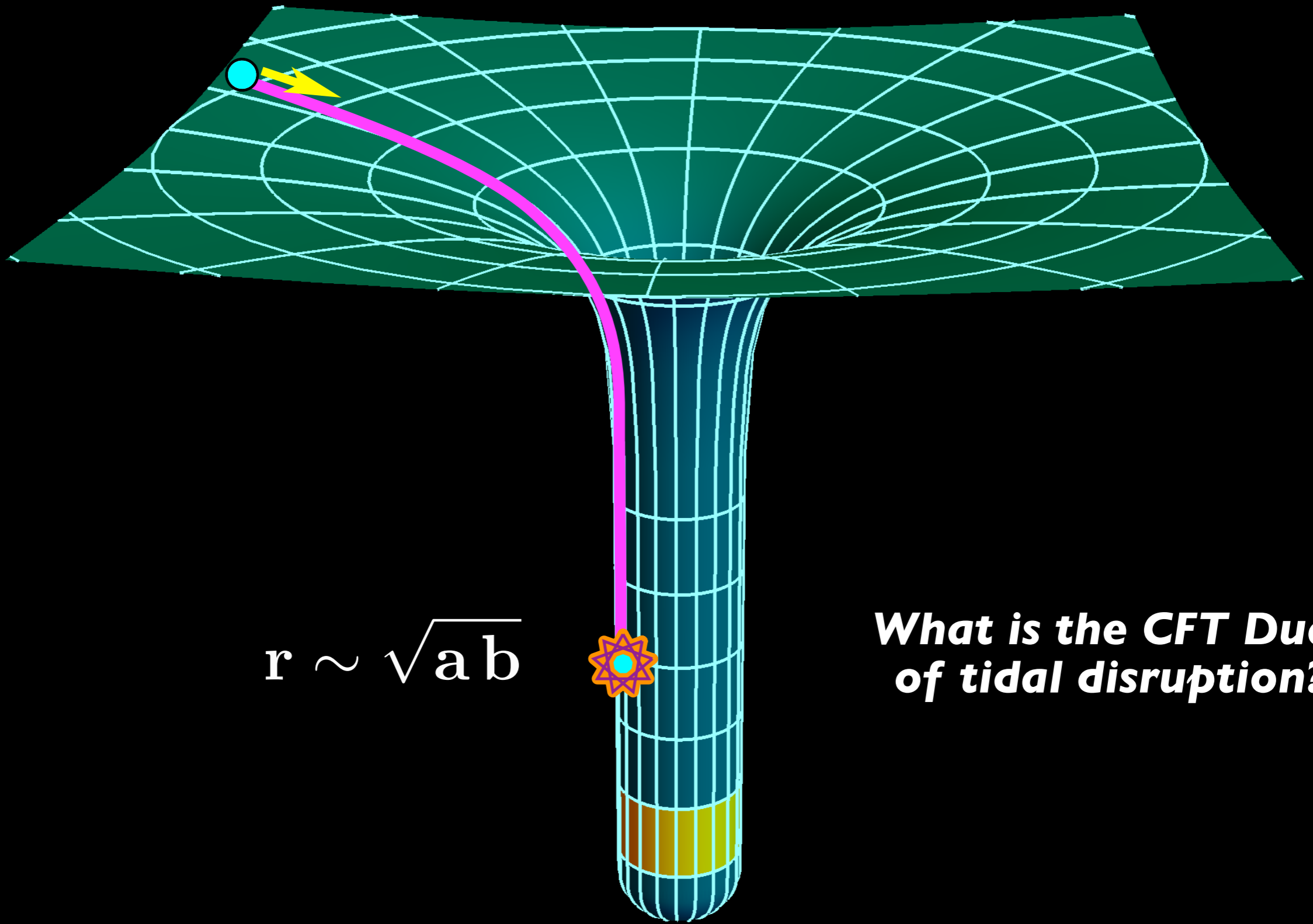
\Rightarrow Exponentially growing mode ..

Recall tidal tensor: $|\mathcal{A}|_{\text{throat}} \sim \frac{a^2 b^2 E^2}{r^6}$ with $E \sim \alpha' p^+$

Ultra-relativistic speeds enhance multipole moments of microstate geometries

\Rightarrow Tidal forces scramble infalling matter* into stringy excitations

*falling from a large distance



$$r \sim \sqrt{a b}$$

**What is the CFT Dual
of tidal disruption?**

Next steps and open problems ...

Tidal forces and viscosity

(*Why you should care about Microstate Geometries III*)

Supergravity can be a powerful tool in understanding the *large-scale effective hydrodynamics* of strongly coupled quantum systems

- e.g.
- ◆ *Viscosity and jet-quenching in quark-gluon plasmas*
 - ◆ *Trailing strings and viscous drag from microstate geometries*

I. Bena, A. Tyukov: 1911.12821

- ★ Tides \Rightarrow Energy transfer to probes
- ★ Back-reaction of tidal effects on probes
 - \rightarrow Energy transfer between probes and background
- ★ Use D1-brane probes
 - \rightarrow background/probe on same footing
- ★ Estimate energy exchange with background as probe scrambles
 - \rightarrow effective viscosity of interactions with microstates
- ★ ***What is the CFT analogue of this tidal disruption?***

Twisted sector states: The bulk of black hole entropy

- Very limited supergravity examples from orbifolds
- “W-branes:” wrapping branes around cycles of microstate geometries

Martinec and Niehoff, 1509.00044

Entropy of such states grows as $\sim Q^{3/2}$

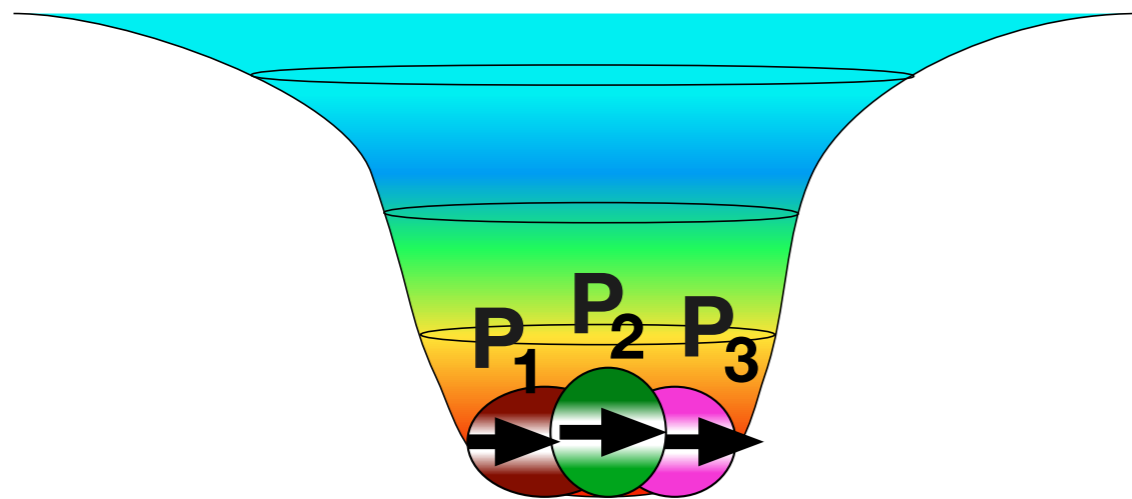
Massless limit \rightarrow Semi-classical description of black-hole microstructure

Generic coherent twisted sector excitations should be visible in supergravity.

- States of open strings stretched between branes \rightarrow **Magnons**

Related to (?) open problem on the gravity side:

The holography of multi-centered microstate geometries: multi-superstrata



UV: D1-D5 CFT with $c = 6 N_1 N_5$

Holographic RG flow

IR: State/Phase of D1-D5 CFT
Complex “product of CFTs” each
with some momentum excitation

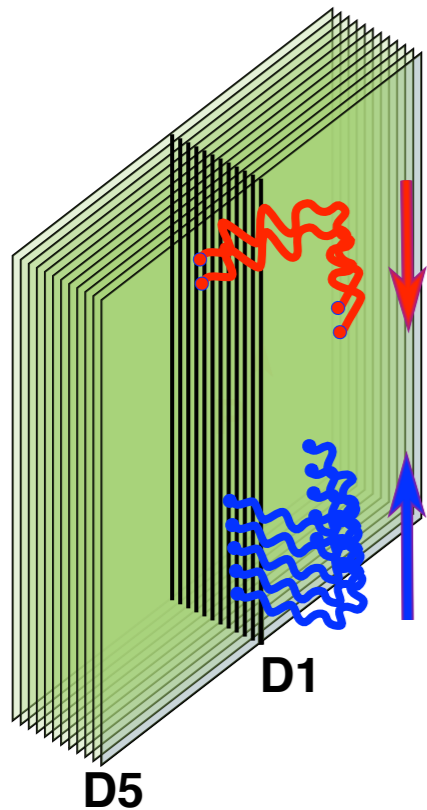
Magnon condensates?

Bena, Shigemori, NPW

***Non-extremal microstate geometries:
Colliding superstrata***

Following the results of Strominger and Vafa in 1996, there was an industry analyzing such *near BPS* states in the CFT and at vanishing string coupling, $g_s \rightarrow 0$

Near BPS \leftrightarrow *Small number, N_{Right} , of right movers*



- ★ Non BPS: $M \sim N_{\text{Left}} + N_{\text{Right}}$, $Q_P \sim N_{\text{Left}} - N_{\text{Right}}$
- ★ State counting worked well, including both left-moving and right-moving states
- ★ Small Hawking Temperature
- ★ Hawking radiation is correctly captured by open string scattering into closed strings
- ★ Grey-body factors work nicely ...

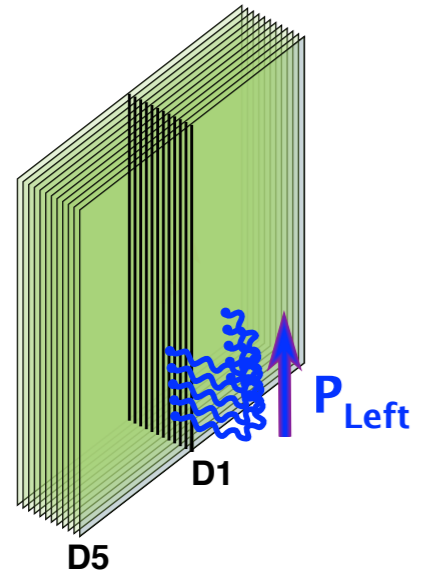
Beautifully controlled supersymmetry breaking.

All of this was done in the CFT and with $g_s \rightarrow 0$

Can we find the gravity dual with finite g_s ?

The BPS story gave us a precise holographic dictionary ...

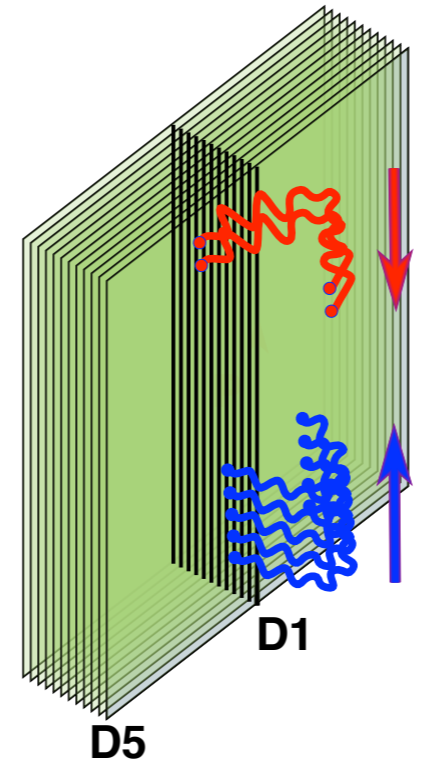
Setting up the scattering problem in gravity



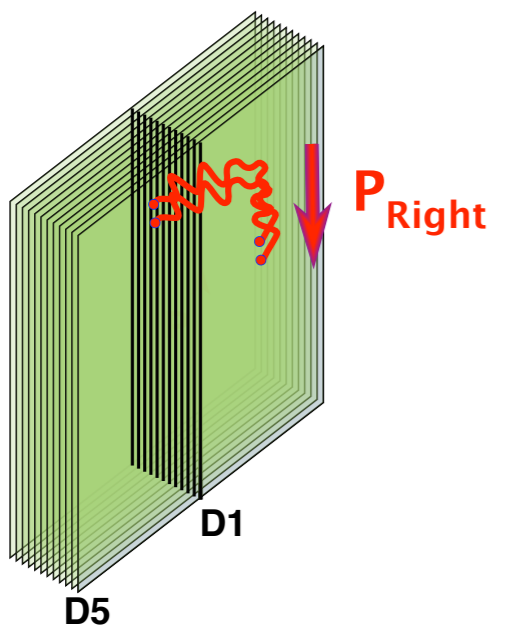
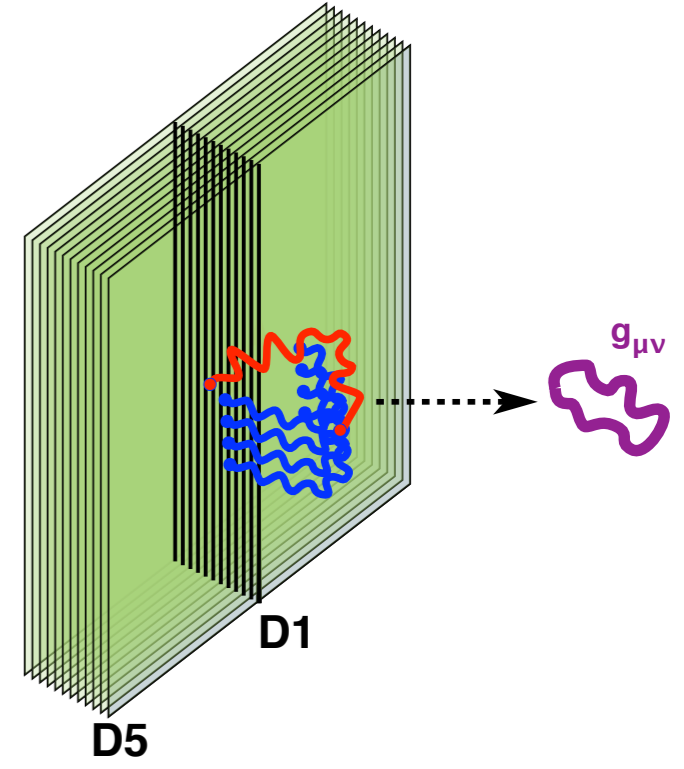
(0,4) supersymmetry

These BPS solutions are exactly known - *localizable* momentum waves
Holographic dictionary is also known

Holomorphic waves in superstrata: Accurate approximations to solutions with **localized regions** with (0,4) and (4,0) supersymmetry



The really hard part:
The waves collide.



(4,0) supersymmetry

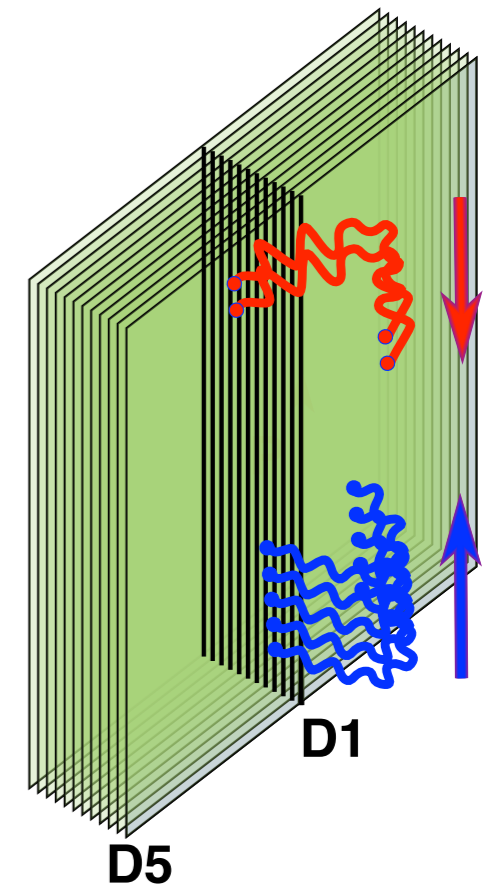
BPS elements → Precisely-posed initial value problem
Still depends on all coordinates: $(u, v, r, \theta, \psi, \phi)$

Colliding superstratum states:

Options:

- ★ Treat the anti-BPS, right-moving states perturbatively ...
in the exactly-known, left-moving superstratum background
- ★ Numerical simulations
 - *Precisely-posed initial value problem for explicitly known D1-D5 CFT microstates*
 - *One can do the dual computation in the CFT*

Perturbative calculations: Mathur; Bombini

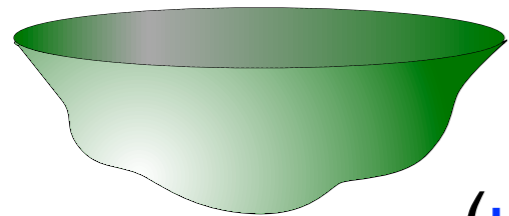


Gravity side:

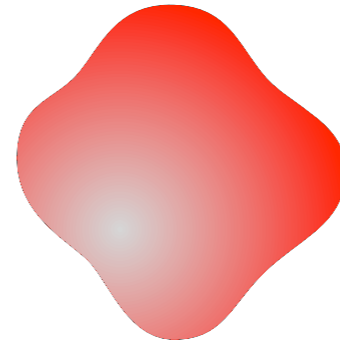
In principle, the solution depends on all six coordinates: $(u, v, r, \theta, \psi, \phi)$

However, exact results on superstrata have led to a huge bonus ...

In the underlying six-dimensional supergravity theory, *colliding superstrata* can be described in terms of harmonic deformations of $\text{AdS}_3 \times S^3$



(u, v, r)



(θ, ψ, ϕ)

- ◆ There are still very rich families of both purely left-moving and purely right-moving superstrata that *only involve the lowest harmonics of S^3* .
- ◆ Restricting to these modes **seems** to be a *consistent truncation* of six-dimensional supergravity to a **three-dimensional gauged supergravity**

D. Mayerson, R. Walker and NPW

Such superstrata, *and their collisions* **could** be described entirely in a gauged supergravity in three-dimensions (u, v, r) :

All degrees of freedom described by a gauged non-linear σ -model ($SO(5,4)$) depending on only three space-time variables

Conclusions

- ★ To understand black-hole microstructure we need to apply all that we have learned from holographic field theory \Rightarrow *Microstate geometries*
- ★ *Microstate geometries*: a laboratory for studying horizon-scale microstructure
- ★ Superstrata: holographic dual of “supergraviton gas”
 - ★ Capped BTZ geometries
 - ★ Supergravity solutions that depend on arbitrary holomorphic functions of three variables
- ★ Infall and tidal scrambling
 - ★ Infall from large distances \Rightarrow scrambling half-way down the throat
 - ★ Viscosity/hydrodynamics of the microstructure: *Supergravity is good at this!*
 - ★ CFT dual of tidal disruption?

Important open problems

- ★ Systematics of twisted sector states; *W-branes*;
Magnon condensates $?\leftrightarrow?$ Holography of multi-centered geometries
- ★ Non-BPS/Near-BPS ★ *Perturbation theory* ★ *Numerical simulations*
 - ★ We are in a position to address this problem because the study of BPS microstate geometries has given us the *holographic dictionary* and the ability to *localize waves*
 - ★ non-BPS scattering can be compared to CFT computations
 - ★ Gauged supergravity in three dimensions might lead to huge simplifications ...