A detailed audit of the Gibbons Hawking dS entropy

Main content based on:

A. Lewkowycz, J. Liu, E. Silverstein and G. Torroba, "$T\bar{T}$ and EE, with implications for (A)dS subregion encodings," arXiv:1909.13808 [hep-th].

V. Gorbenko, E. Silverstein and G. Torroba, "dS/dS and $T\bar{T}$," arXiv:1811.07965 [hep-th].

X. Dong, E. Silverstein and G. Torroba, "De Sitter Holography and Entanglement Entropy," JHEP **1807**, 050 (2018) doi:10.1007/JHEP07(2018)050 [arXiv:1804.08623 [hep-th]].

M. Alishahiha, A. Karch, E. Silverstein, and D. Tong, "The dS/dS correspondence," *AIP Conf. Proc.* **743** (2005) 393–409, arXiv:hep-th/0407125 [hep-th]. [,393(2004)].

M. Alishahiha, A. Karch, and E. Silverstein, "Hologravity," *JHEP* **06** (2005) 028, arXiv:hep-th/0504056 [hep-th].

X. Dong, B. Horn, E. Silverstein, and G. Torroba, "Micromanaging de Sitter holography," *Class. Quant. Grav.* **27** (2010) 245020, arXiv:1005.5403 [hep-th].

X. Dong, B. Horn, S. Matsuura, E. Silverstein, and G. Torroba, "FRW solutions and holography from uplifted AdS/CFT," *Phys. Rev.* **D85** (2012) 104035, arXiv:1108.5732 [hep-th].

Other approaches (dS/CFT, FRW/CFT, cosmo within AdS,…) reviewed in e.g.

D. Anninos, "De Sitter Musings," *Int. J. Mod. Phys.* **A27** (2012) 1230013, arXiv:1205.3855 [hep-th].

D. Anninos and D. M. Hofman, "Infrared Realization of dS_2 in AdS_2," *Class. Quant. Grav.* **35** no. 8, (2018) 085003, arXiv:1703.04622 [hep-th].

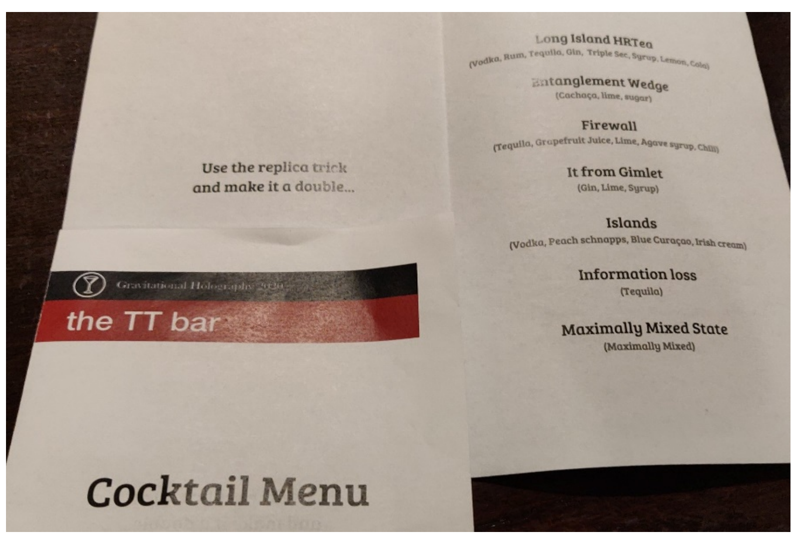
S. Cooper, M. Rozali, B. Swingle, M. Van Raamsdonk, C. Waddell, and D. Wakeham, "Black Hole Microstate Cosmology," *JHEP* **07** (2019) 065, arXiv:1810.10601 [hep-th].

I. Introduction

II. Finite dressed energy spectra and entanglement entropies of T T bar deformed theories, on both sides of (A)dS holographic duality. Role and form of non-locality of TTbar.

III. Accounting of Gibbons-Hawking dS entropy, via Renyi calculation and count of states, at the appropriate level of precision. Role of residual dynamical gravity & sources.

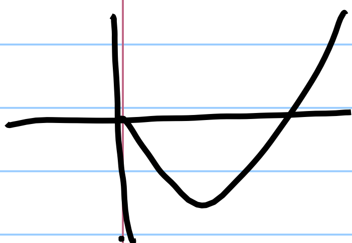
III. Later times and other future directions

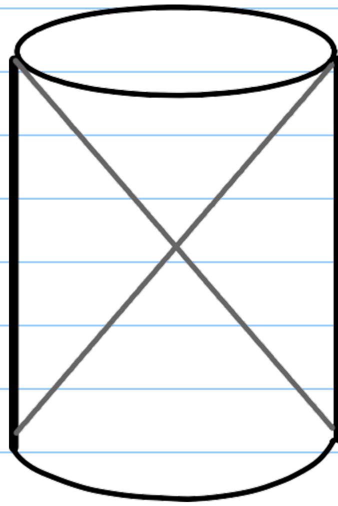


Motivations: Reality and quantum gravity/string theory.

We need a framework for the V>0 landscape.

$\Lambda < 0$



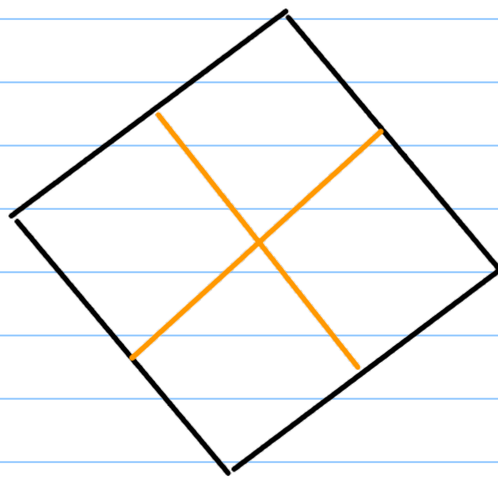


AdS:

observables are conformal field theory (CFT) correlation functions

Timelike boundary at infinity pins fields.

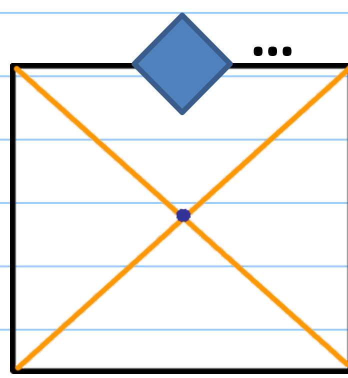
$\Lambda = 0$



Minkowski

Scattering matrix

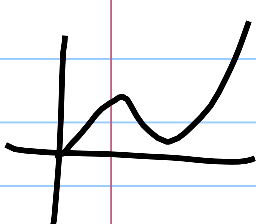
Asymptotics also very special.

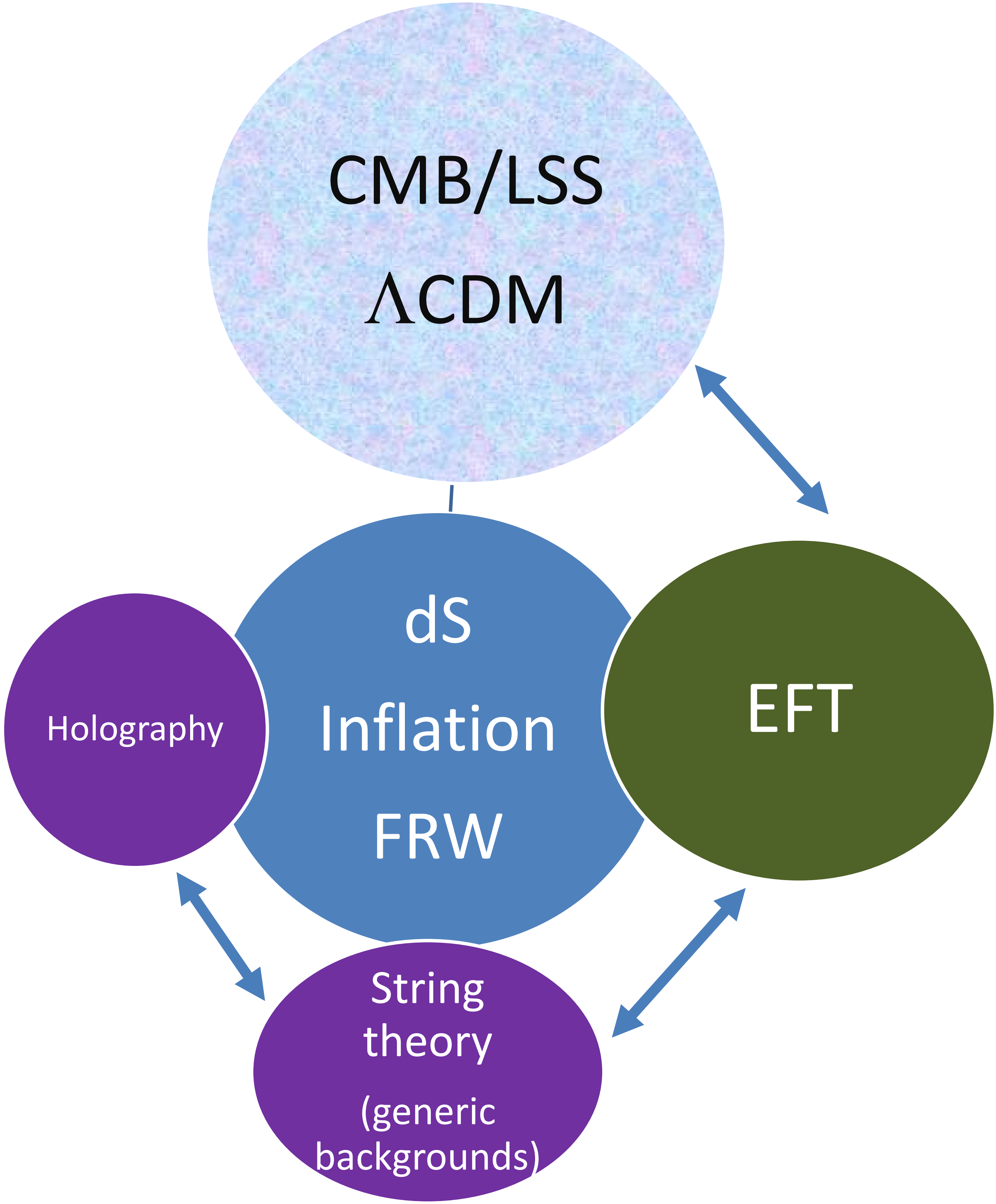


dS +decays

no boundary analogous to the AdS one. (Finite, fluctuating space. Eventually decays.)

$\Lambda > 0$




CMB/LSS
ΛCDM
dS
Inflation
FRW
Holography
EFT
String
theory
(generic
backgrounds)

AdS/CFT: Low energy QFT = highly redshifted region of D-brane construction. The near-horizon AdS x S has a timelike boundary at spatial infinity. Excellent QG case study, but this, along with the negative cosmological constant, is very special and unrealistic.

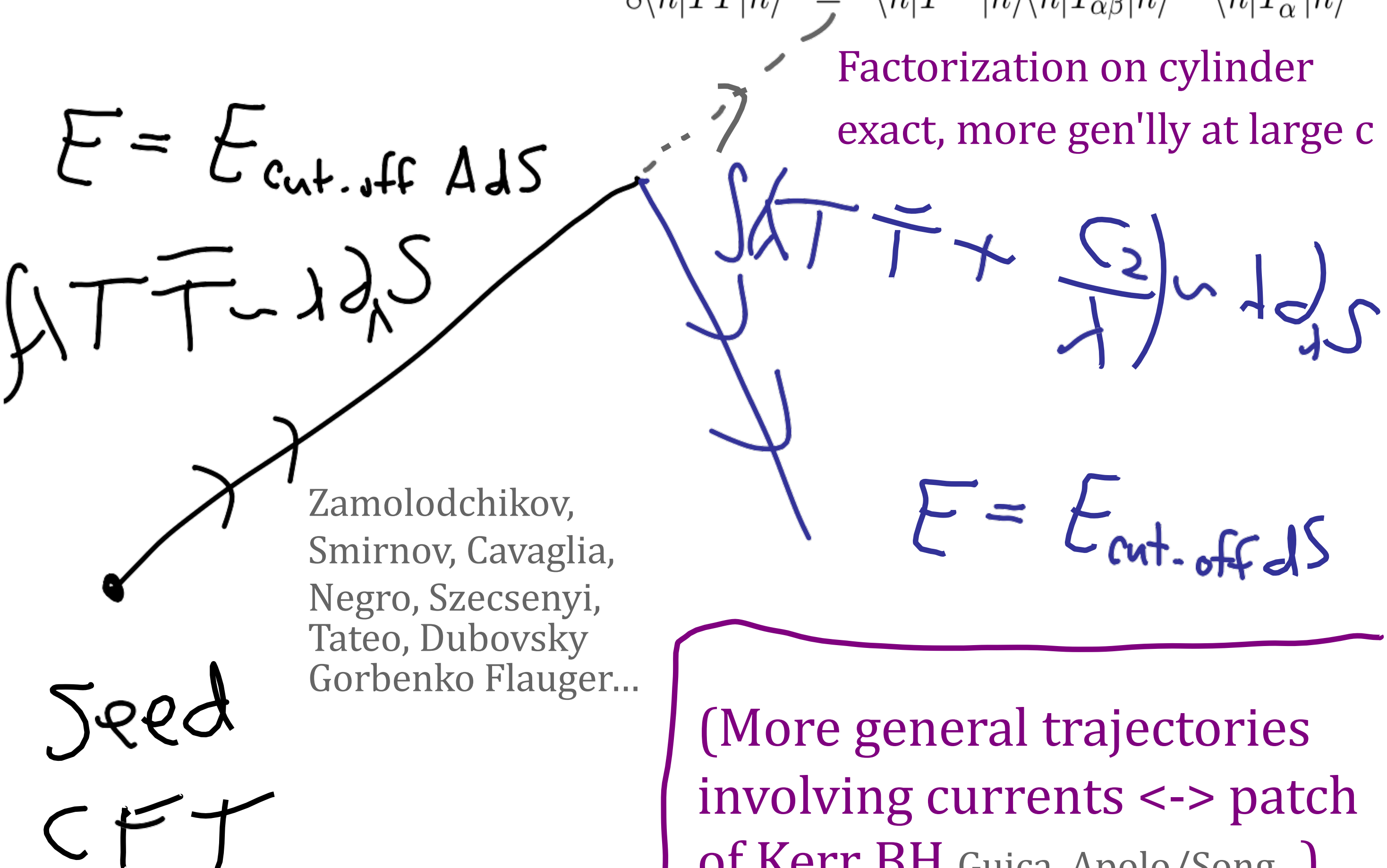
To progress toward more realistic QG, want to determine the dual of a finite patch of this and other spacetimes (e.g. a radial cutoff).

Recently, McGough, Mezei, Verlinde and followups proposed a specific answer to this question, isolating a radially cutoff region of AdS as the dual of a tractable `irrelevant' deformation of the dual CFT known as T-Tbar. We then (w/Gorbenko, Torroba) generalize this deformation to capture finite patches of bulk **dS** instead. cf Miyaji Takayanagi Sato, Nomura Rath Salzetta...

TTbar +... (A)dS Holography:

$$8\langle n|T\bar{T}|n\rangle = \langle n|T^{\alpha\beta}|n\rangle\langle n|T_{\alpha\beta}|n\rangle - \langle n|T^{\alpha}_{\alpha}|n\rangle^2$$

Factorization on cylinder exact, more gen'lly at large c



Zamolodchikov, Smirnov, Cavaglia, Negro, Szecsenyi, Tateo, Dubovsky Gorbenko Flauger…

(More general trajectories involving currents <-> patch of Kerr BH Guica, Apolo/Song...)

Both universal, solvable deformations whose real dressed energies match quasilocal Brown- York energy of a patch of (A)dS. The new extension enters nonlinearly in the integrated trajectory. There are generalizations with varying degrees of solvability, including for local bulk matter.

Hartman Kruthoff Shaghoulian Tajdini, Gross Kruthoff Rolph Shaghoulian; Giveon Kutasov et al, …

$$\frac{\partial}{\partial\lambda}\log Z = -2\pi\int d^2x\,\sqrt{g}\,\langle T\bar{T}\rangle + \frac{1-\eta}{2\pi\lambda^2}\int d^2x\,\sqrt{g}$$

Large λ, $_{tHooft}$, large c*: factorization

$$T^a_a = -\frac{c}{24\pi}\mathcal{R}^{(2)} - 4\pi\lambda T\bar{T} - \frac{\eta-1}{\pi\lambda}$$

$$\nabla^a T_{ab} = 0$$

=> Dressed energies, e.g. on dS_2

$$\langle T^\tau_\tau\rangle = \frac{1}{\pi\lambda}\left(1\mp\sqrt{\eta + c\frac{\mathcal{R}^{(2)}\lambda}{24} - \frac{C_1\lambda}{L(\tau)^2}}\right), \qquad L(\tau) = \ell\cosh\frac{\tau}{\ell}$$

$$\frac{E}{L} = \frac{\mathcal{E}}{L^2} = \frac{1}{\pi\lambda}\left(-1+\sqrt{\eta + c\frac{\mathcal{R}^{(2)}\lambda}{24} + \frac{(\mathcal{E}^{(0)}-\mathcal{E}^{(0)}_0)\lambda}{L^2}}\right)$$

=> Dressed entropies, on replicas

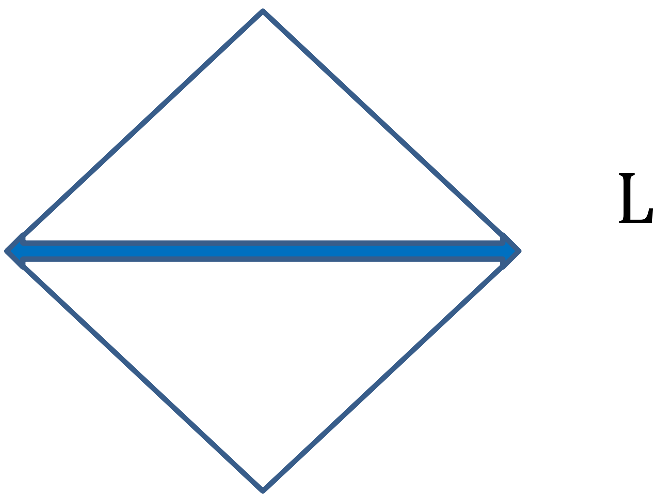
$$L\frac{d}{dL}\tilde{S}_n = -(1-n\partial_n)\int d^2x\,\sqrt{g}\,\langle \mathrm{tr}\,T\rangle$$

*For more, see Mazenc, Shyam, Soni '19 (talk)

Entropies: localization at end points of interval:

$$LS'(L) = 2 \times \lim_{n\to 1} (2\pi n) \int_0^{\rho_0 \ll L} \rho d\rho \, n\partial_n \langle \mathrm{tr}\, T(\rho) \rangle$$

e.g. interval in Poincare:

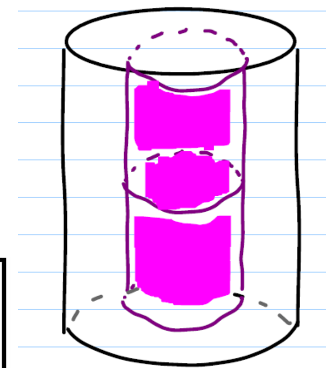


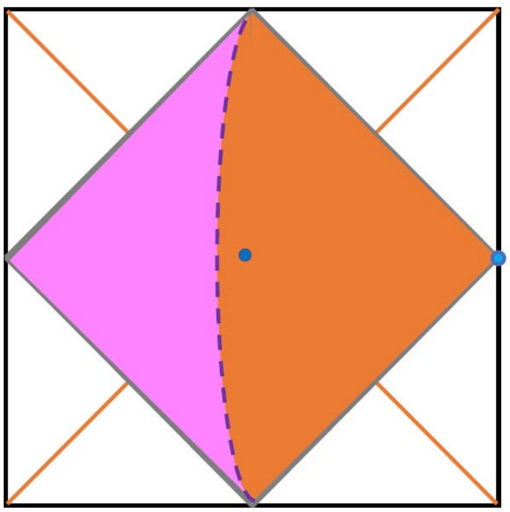
$$n\partial_n \langle \mathrm{tr}\, T(\rho) \rangle|_{n\to 1} = \epsilon \frac{c}{3} \frac{\lambda c}{24\pi} \frac{C(\frac{\lambda}{L^2})^2}{\rho^4 \left(1 + \epsilon \frac{\lambda c}{6\rho^2} C(\frac{\lambda}{L^2})\right)^{3/2}}$$

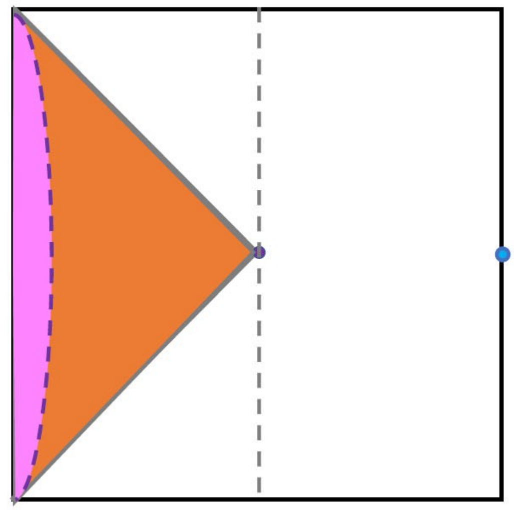
$$\epsilon \equiv 1 - n$$

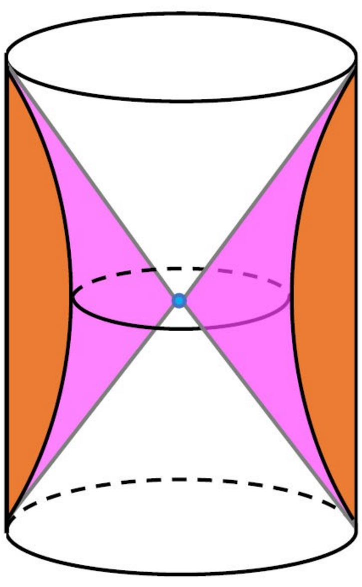
Calculation of $C(\lambda/L^2)$ below.

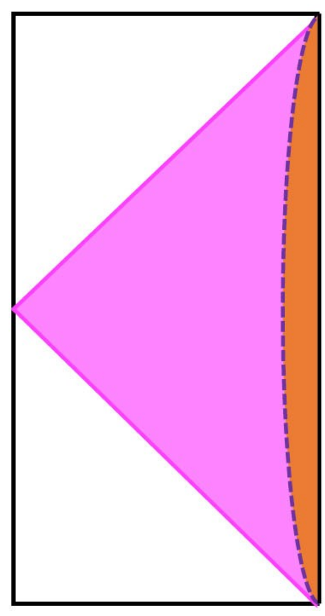
TTbar +... isolates a radially cut off subregion of (A)dS geometries:







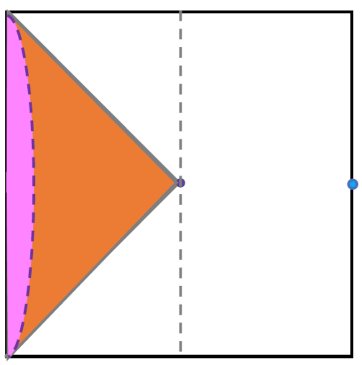




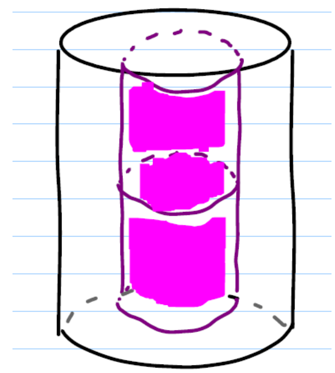
$$
\begin{aligned}
ds^2_{\text{(A)dS/dS}} &= dw^2 + \sin(h)^2 \frac{w}{\ell} \left(-d\tau^2 + \ell^2 \cosh^2 \frac{\tau}{\ell} d\phi^2 \right) \quad w \leq w_c \\
ds^2_{\text{(A)dS/cylinder}} &= -(1 \pm r^2/\ell^2) dt^2 + \frac{dr^2}{1 \pm r^2/\ell^2} + r^2 d\phi^2 \quad r \leq r_c \\
ds^2_{\text{AdS/Poincare}} &= \ell^2 \frac{-dt^2 + dx^2 + dz^2}{z^2} \quad z \geq z_c
\end{aligned}
$$

Dressed energies and additional dualities

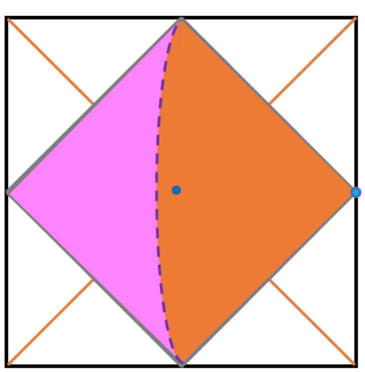
$$E = -2\pi L T_t^t = \frac{L}{\pi\lambda}\left(1 - \sqrt{\eta - \frac{4\pi^2\lambda(\Delta + \bar{\Delta} - c/12)}{L^2} + \frac{4\pi^4\lambda^2(\Delta - \bar{\Delta})^2}{L^4}}\right)$$



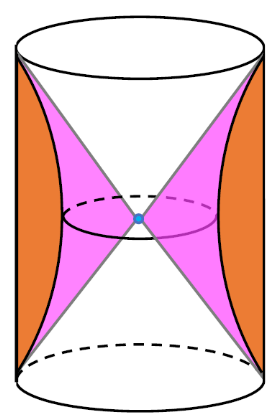
(A)dS/cylinder, formulates dS static patch. Unitarity => truncate to (finite) real Hilbert space.



$$\langle T_\tau^\tau \rangle = \frac{1}{\pi\lambda}\left(1 \mp \sqrt{\eta + c\frac{\mathcal{R}^{(2)}\lambda}{24} - \frac{C_1\lambda}{L^2}}\right)$$



(A)dS/dS, including full dS/dS patch with 2nd sign of square root



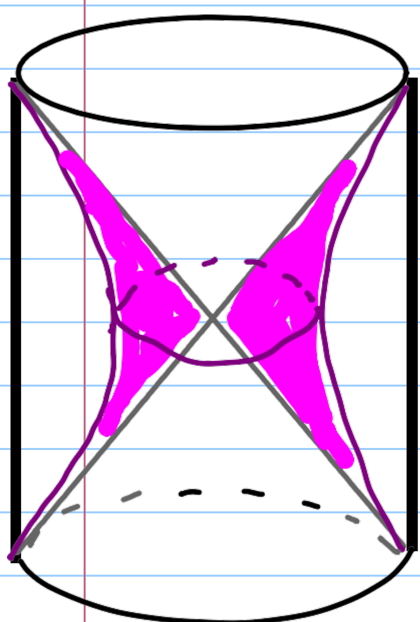
We'll focus on holographic patches closest to AdS/CFT:

- Radial direction emergent
- Boundary at the most UV radial slice
- Boundary causality (signals fastest along boundary)

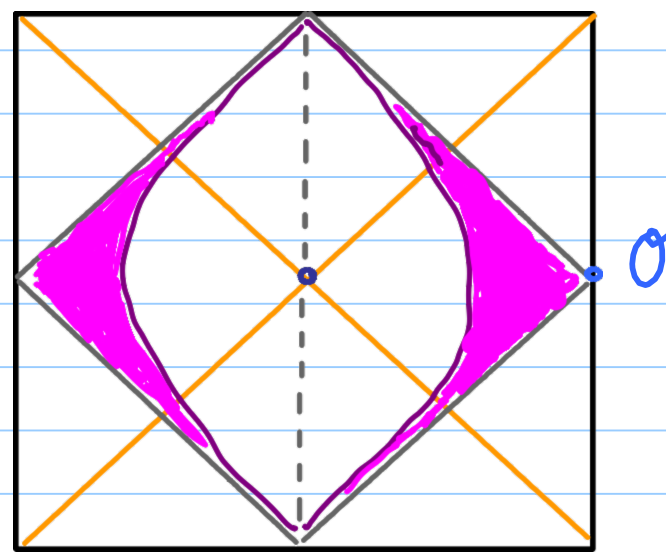
In this framework, find precise matching (as function of deformation parameter λ) between energies and entropies (beyond the symmetric cases), computed separately on the two sides.

Full dS/dS patch ultimately involves two TTbar+... deformed CFTs coupled via 2d gravity and other dynamical sources

$$
\begin{aligned}
ds^2_{(A)dS_{d+1}} &= dw^2 + \sin(h)^2(\frac{w}{\ell_{dS}})ds^2_{dS_d} \\
&= dw^2 + \sin(h)^2(\frac{w}{\ell_{dS}})\left[-d\tau^2 + \ell_{dS}^2\cosh^2\frac{\tau}{\ell_{dS}}d\Omega^2_{d-1}\right].
\end{aligned}
$$



AdS/dS



dS/dS (each point is (d-1)-sphere)

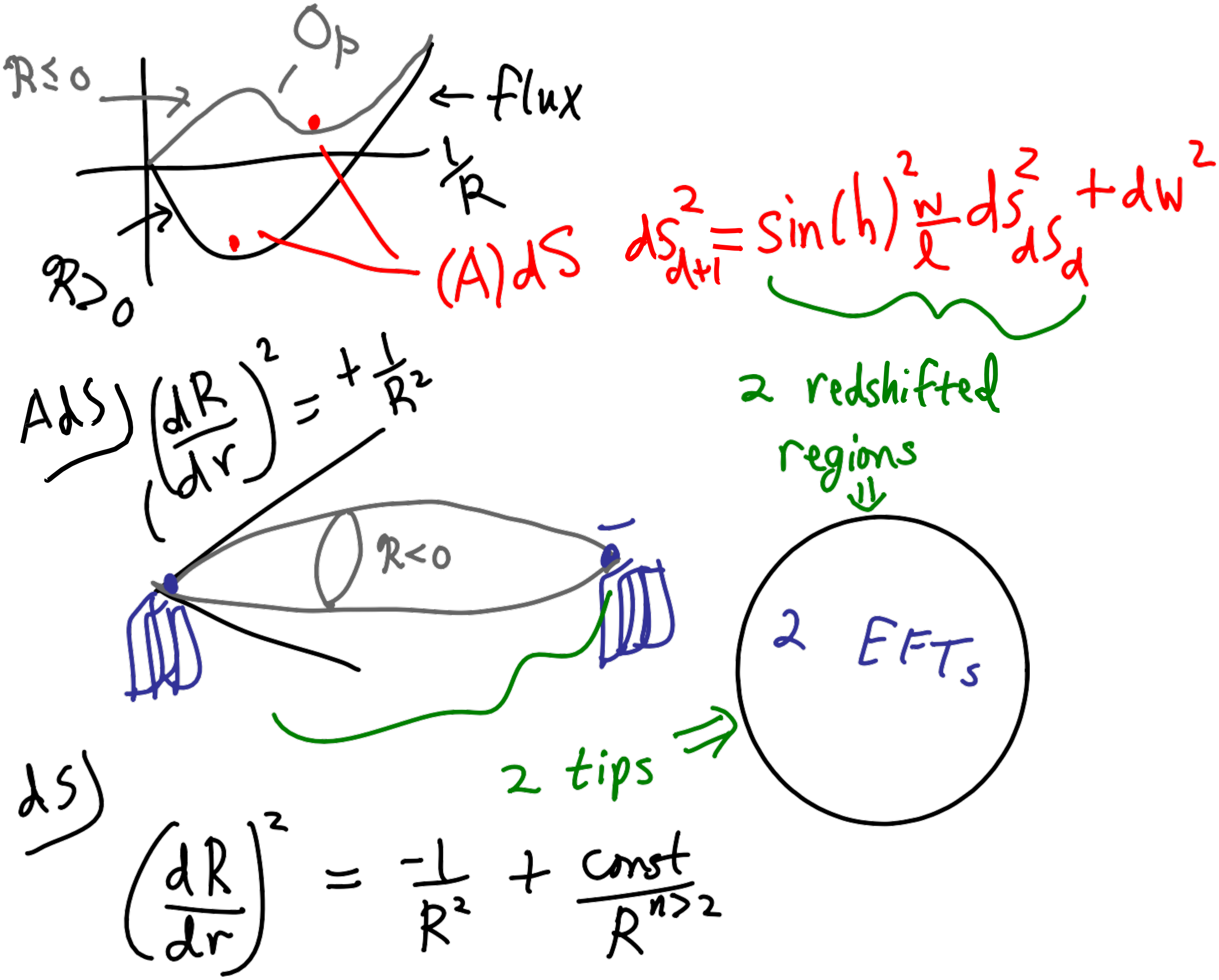
2 highly redshifted (IR) regions, each ~ IR region of AdS/dS

With T-Tbar+Λ_2 first formulate each warped throat, later join

Microscopic dS/dS:

Uplifting AdS/CFT => 2 sectors

Dong Horn ES Torroba '10



dS vs AdS brane construction:

independent derivation of the two sectors because of metastability.

Also true in dS/CFT

Repeat MMV, KLM et al calculation for (A)dS/dS. Solve for stress energy, matching AdS/dS and dS/dS near w=0.

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{-g}\left(\mathcal{R}^{(3)} + \frac{2\eta}{\ell^2}\right) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\sqrt{-g}\left(K - \frac{b_{CT}}{\ell}\right)$$

Taking radial slices

$$ds_3^2 = dw^2 + g_{ij}\,dx^i dx^j\,,$$

the quasilocal stress-tensor is

$$T_{ij} = \frac{2}{\sqrt{-g}}\frac{\delta S_{\text{on-shell}}}{\delta g^{ij}} = \frac{1}{8\pi G}\left(K_{ij} - K g_{ij} + \frac{b_{CT}}{\ell} g_{ij}\right)\,.$$

Result: 2 new terms in trace flow equation

$$\int d^2x\sqrt{-g}T^i_i \quad = \quad \int d^2x\sqrt{-g}\left(-4\pi\lambda T\bar{T} - \frac{c\mathcal{R}^{(2)}}{24\pi} + \frac{1-\eta}{\pi\lambda}\right)$$

On the gravity side, this equation is the radial analogue of the Hamiltonian constraint combined with the definition of the Brown-York stress tensor.

Starting from a seed CFT, holographically reconstruct a patch of de Sitter spacetime.

Beyond pure gravity, the matching the two parts of the trajectory for local bulk examples involves interpolation between the AdS and dS minima e.g. in the connected scalar sector of string theory.

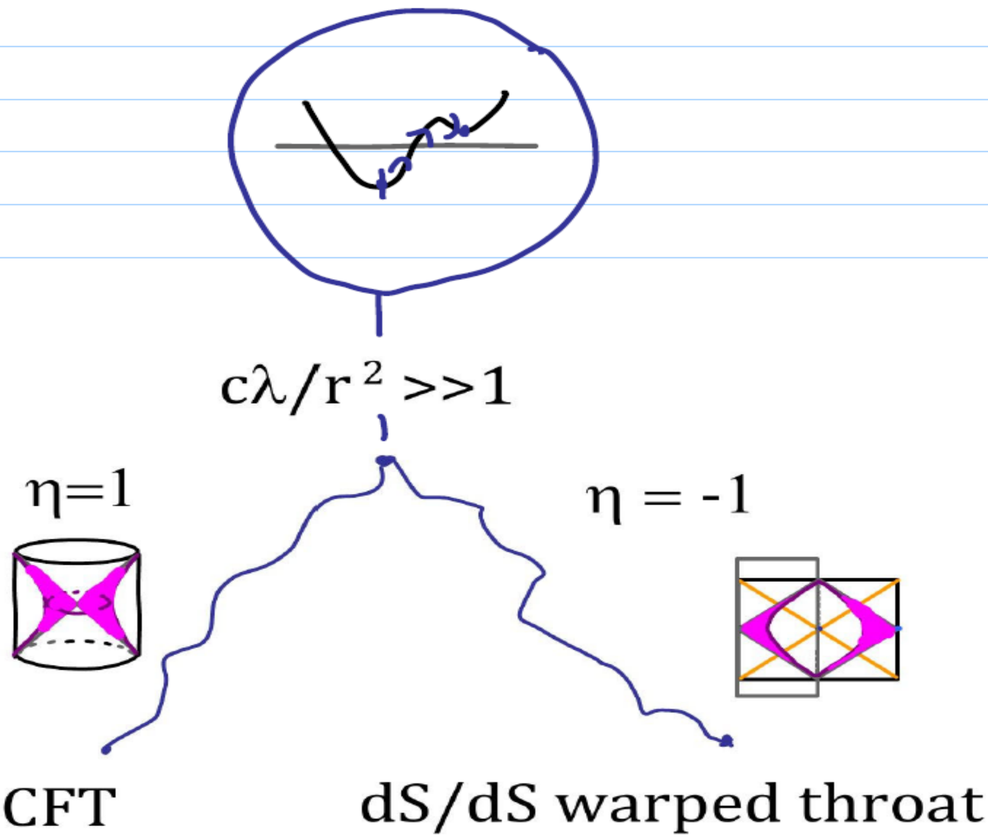


Figure 1: The reconstruction of the dS/dS throat from a seed CFT proceeds via two joined trajectories as described in the text. The first trajectory (on the left) evolves the system from a pure CFT, via a sequence of cutoff AdS/dS systems, to the limit where this cutoff scale goes to zero, indicated by the point at the top of the figure. That point is the start of a new trajectory incorporating $\Lambda_2 \propto \eta - 1$, with increasing cutoff scale, culminating in the full dS/dS warped throat.

For states that dominate the spectrum, (e.g. BTZ BHs), pure gravity suffices.

Many recent lessons in AdS/CFT, e.g.:

*Causal wedge reconstruction, Hamiltonian evolution (HKLL) **U** Bousso et al, Morrison, ...

*Entanglement entropy (RT) HRT,...

*Algebraic QFT and EE Casini Huerta...

*E. wedge reconstruction and modular Hamiltonian evolution **ρ** Dong Faulkner Lewkowyz Rangamani Maldacena... (+ entanglement shadows, islands, Almheiri et al, Pennington,...)

*Redundant encoding of bulk points (QEC) ADH

Question: fate of these properties in the (A)dS patches, via T Tbar +.... trajectories.

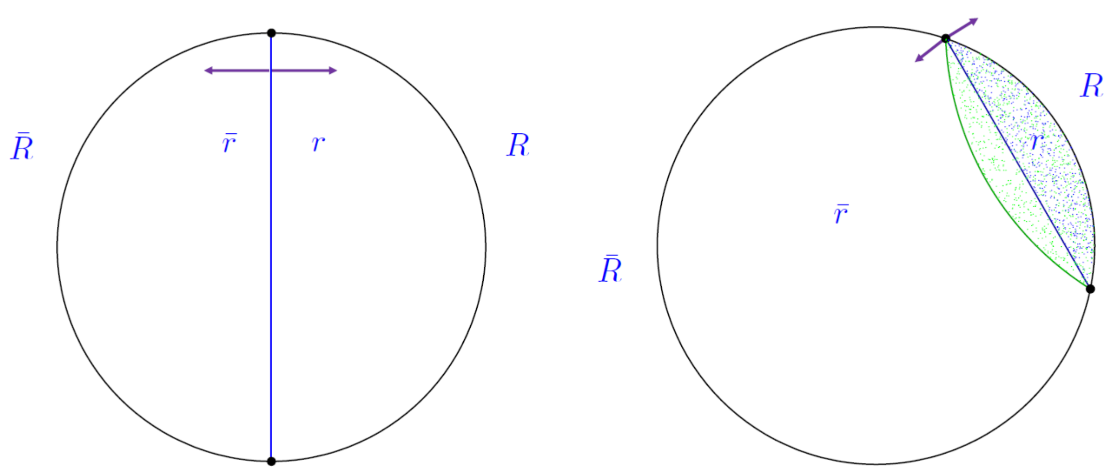
Summary: Ryu-Takayanagi works for cutoff (A)dS, precise agreement with deformed-CFT side non-perturbatively in TTbar coupling at large c. Technical reason: effect localizes to endpoints, order of limits crucial. Implications for subregion dualities: operator algebras and commutator structure reveal nature of non-locality, e.g. relative boost expands operator algebra. Some redundancy of bulk point encoding (cf quantum error correction).

Energy and Entropy, U and ρ:

The deformed theories are non-local, so no *a priori* guarantee of a division of the system (i.e. its operator algebra) into subregions of the boundary spacetime. Also no guarantee a priori of causality in time evolution. But we have emergent bulk locality down to string scale, and causal signaling along boundary for AdS/Poincare and (A)dS/dS.

Reproduce RT at leading order. For probe fields, Causal Wedge and Entanglement Wedge continue to exist, but CW can exceed EW unlike AdS/CFT.

$$[\mathcal{A}_{\bar{R}_s}, \mathcal{A}_{D[R]}] \neq 0\,.$$



$$\mathcal{A}_{\rm CW} = \mathcal{A}_{D[R]} = \{O(x),\ x \in D[R]\}\,.$$

Note: we have essentially local operators at large c, $\lambda_{'tHooft}$. On GR side these are the local radial field momentum at the Dirichlet wall, in contrast to the highly smeared HKLL operators from the asymptotic boundary.

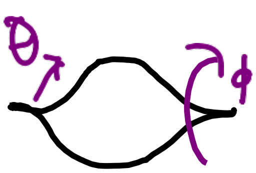
$$\mathcal{A}_{\rm EW} = \mathcal{A}_{R_s} = \{\rho_R^{is}\, O(x)\, \rho_R^{-is},\ x \in R,\ s \in \mathbb{R}\}\,.$$

As in Faulkner-Lewkowycz.

First, analyze leading entropy on deformed-CFT side. Equations for stress energy (quasilocal PDEs)

$$\nabla_a T^a_b = 0$$
$$T^a_a = -\frac{cR}{24\pi} + \frac{c_2}{\pi\lambda} - \frac{\pi\lambda}{2}\left(T^{ab}T_{ab} - (T^a_a)^2\right), \qquad c_2 = 1 - \eta$$

Donelly and Shyam used these to compute stress tensor on replicated geometry for half and half division of boundary dS_2 neck, in vacuum, here generalized with η:



$$T^\theta_\theta = \frac{1}{\pi\lambda}\left(1 - \sqrt{\eta + \frac{c\lambda}{12r^2}\left[1 + \left(\frac{1}{n^2} - 1\right)\frac{1}{\sin^2\theta + n^2\delta^2}\right]}\right)$$
$$T^\phi_\phi = \frac{1}{\pi\lambda}\left(1 - \frac{\eta + \frac{c\lambda}{12r^2}\left[1 - \delta^2\frac{n^2-1}{(\sin^2\theta + \delta^2 n^2)^2}\right]}{\sqrt{\eta + \frac{c\lambda}{12r^2}\left[1 + \left(\frac{1}{n^2} - 1\right)\frac{1}{\sin^2\theta + n^2\delta^2}\right]}}\right).$$

Symmetry in ϕ ⇒ ODE

$$L\frac{d}{dL}\tilde{S}_n = -(1 - n\partial_n)\int d^2x\,\sqrt{g}\,\mathrm{tr}\,T\,.$$ = RT result

Near one endpoint sin(θ)->0 :

$$\epsilon \equiv 1 - n \ll 1\,,$$

$$\partial_n \langle \mathrm{tr} T \rangle |_{n \to 1} = \epsilon \, \frac{c}{3} \, \frac{\lambda c}{24} \, \frac{1}{r^3 \sin^4 \theta} \, \frac{1}{\left(1 + \frac{\lambda c}{12 r^2} \left(1 + \frac{2\epsilon}{\sin^2 \theta}\right)\right)^{3/2}} \,.$$

Order of limits between n–>1 and λ–>0. Correct result from integrating –(1-nd_n)(Tr(T) first. Shift in denominator regulates endpoint singularity, no need for conical singularity cutoff: T Tb coupling regulates the endpoint itself.

Using the endpoint-dominant effect of λ enables exact (as function of λ at large c) calculation of entanglement entropy much more generally than 1/2-space. This agrees with RT.

Recall: interval L in Poincare AdS conformally maps to half of dS neck, L=2r_{ds}

ref

$$ds^2 = -(dx^0)^2 + (dx^1)^2 = \Omega(\theta,\tau)^2\,(-\sin^2\theta\,d\tau^2 + r^2\,d\theta^2)$$

$$\Omega(\theta,\tau) = \frac{1}{1+\sin\theta\,\cosh(\tau/r)}$$

$$L = 2r$$

Non-CFT couplings are dimensionful; this map would appear not useful for TTbar since the conformal factor multiplies the couplings, infecting the formulas.

AdS/Poincare case study

$$n\partial_n \langle \mathrm{tr}\, T\rangle|_{n\to 1} = \epsilon \frac{c}{3}\frac{\lambda c}{24\pi}\frac{C(\frac{\lambda}{L^2})^2}{\rho\left(\rho^2+\epsilon\frac{\lambda c}{6}C(\frac{\lambda}{L^2})\right)^{3/2}}\,.$$

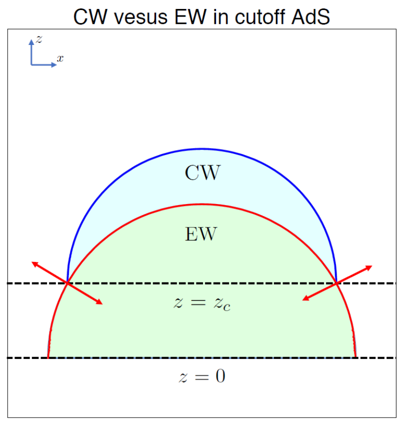
$$\begin{aligned} LS'(L) &= 2\times\lim_{n\to 1}(2\pi n)\int_0^{\rho_0\ll L}\rho d\rho\, n\partial_n\langle \mathrm{tr}\, T\rangle \\ &= \frac{c}{3}C(\frac{\lambda}{L^2})\frac{1}{\sqrt{1+\epsilon\frac{\lambda c}{6\rho_0^2}C(\frac{\lambda}{L^2})}}\Big|_{\epsilon\to 0} \\ &= \frac{c}{3}C(\frac{\lambda}{L^2})\,. \end{aligned}$$

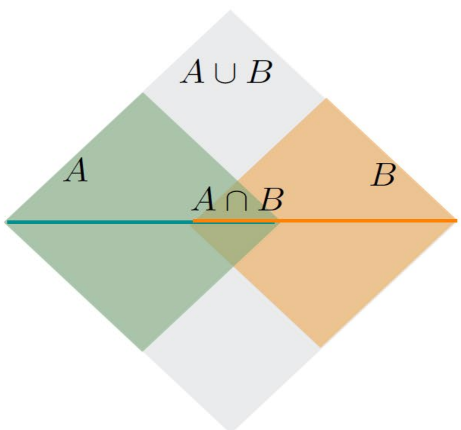
Finite interval in Poincare boundary can *still* be done via Weyl rescaling map to static patch in dS boundary cf Casini/Huerta/Myers because the Weyl factor Ω which normally complicates that map in a non-conformal theory reduces to 1 at the endpoints of the interval.

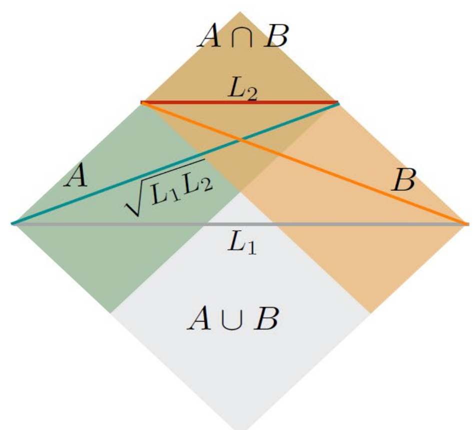
Leading (CFT) result from this order of limits; full non-perturbative (at large c) result from CHM Weyl map => precise match with RT.

$$LS'(L) = \frac{c}{3}\frac{1}{\sqrt{1+\frac{\lambda c}{3L^2}}}$$

Note O(λ/L^2) Contribution, cf Chakraborty et al





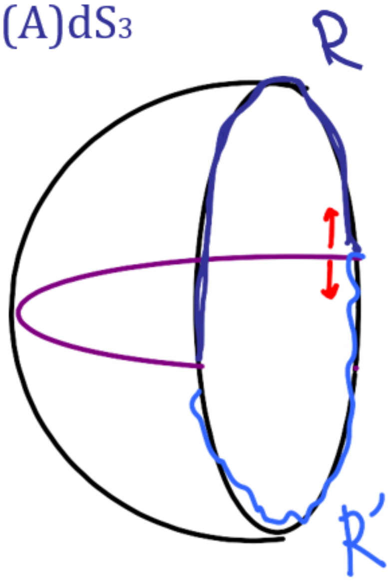


Boosted strong subadditivity of Casini And Huerta is violated. Interpretation: Relative boost => additional operators join the accessible operator algebra. The non-locality is similar to string theory

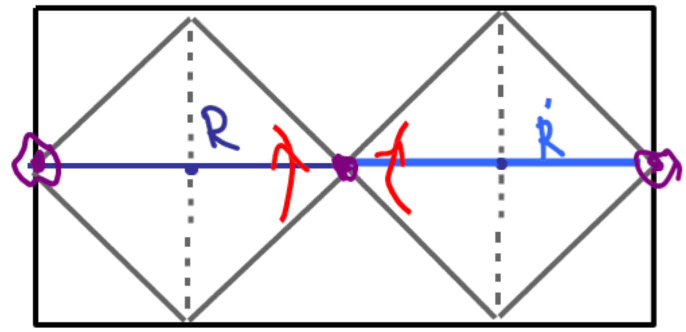
$$\mathcal{A}_{A_s} \not\subseteq \mathcal{A}_{(A\cup B)_s}$$

dS/dS case study: again sharp match

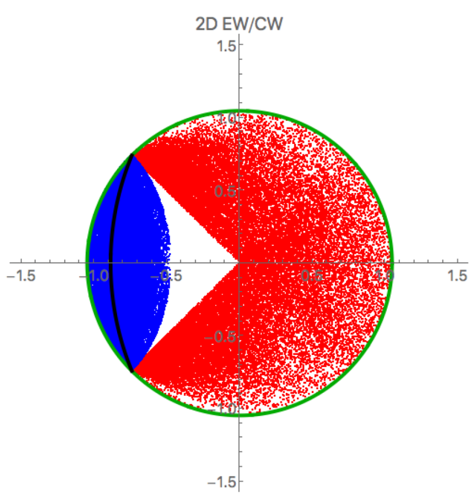
spatial slice of bulk (A)dS3

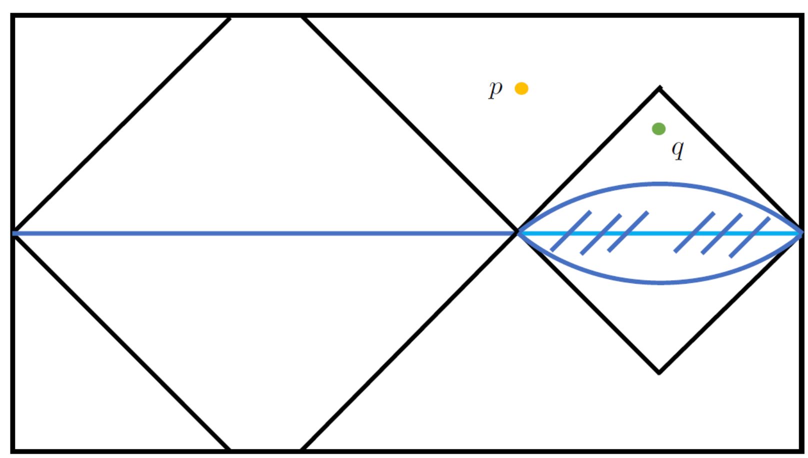


Boundary dS2



Modular Hamiltonian KR is local in this case (just Ttt) and modular evolution takes Ops(R)->Ops(D(R)).





$$\partial_r \log Z_n = -2\pi n r \int_0^{\pi} d\theta \, \sin\theta \, (T_\theta^\theta + T_\phi^\phi) \, .$$

maximal mixing:

$$S_0(r) = S_1(r) = \frac{\pi c}{6} \quad \text{for} \quad r = \sqrt{\frac{c\lambda}{12}} .$$

In gravity language, this corresponds to the central slice $w_c = \frac{\pi}{2}\ell$.

All cases: modular evolution gets stalled for R<1/2

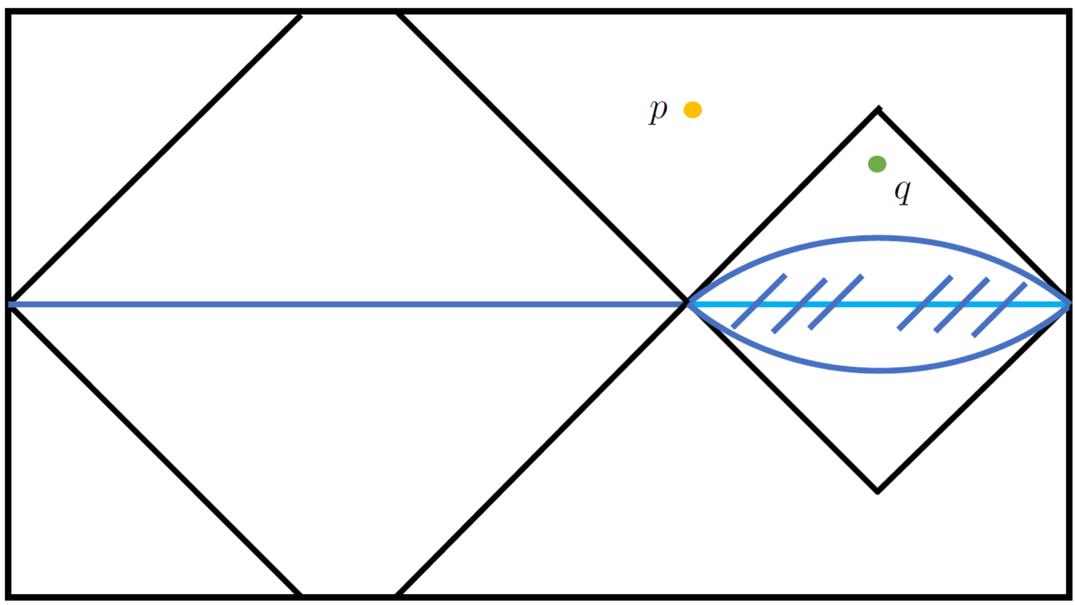
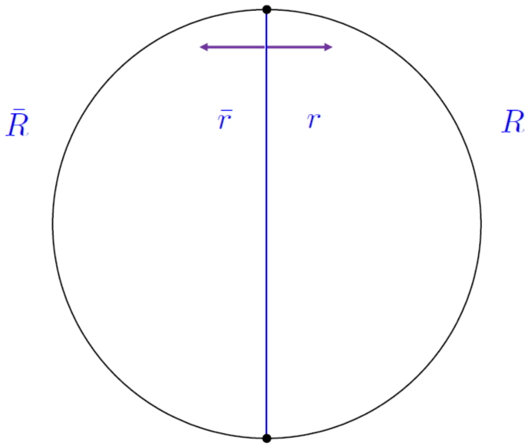
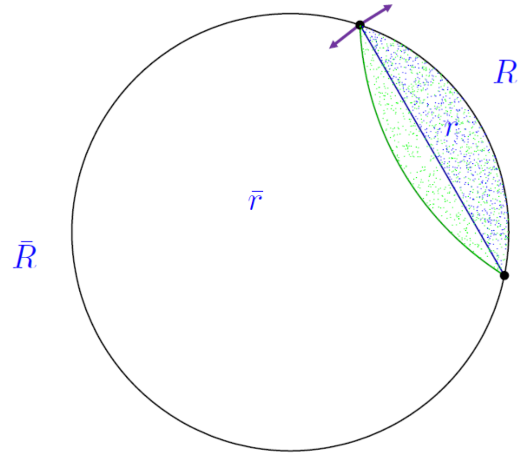


Figure 4: The regions $D[R]$ and $D[\bar{R}]$ for an asymmetric division of the system in the case of a boundary dS_2 spacetime, whose Penrose diagram we depict here. The region $E[R]$ for R the smaller subregion is indicated by the dark blue hatchmarks. Operators at the point p can be causally influenced from either region and are not in EW[R] or EW[$\bar{R}$]; this is familiar in asymptotic AdS/CFT. The point q illustrates a point in $D[R]$ which is not in $E[R]$. Operators at that point do not commute with those in the algebra $\mathcal{A}_{\bar{R}_s}$ corresponding to the complementary subsystem to R. We note that this point q on the boundary is also not part of the EW of either R or $\bar{R}$, since it can be reached causally from both.

The full dS/dS warped throat is a simple extreme example: max mixing means ρ ~ 1/dim(H) so modular Hamiltonian acts as a constant (**no** evolution).





$$\begin{aligned}\Phi(X \in \mathrm{EW}[\bar{R}] \cap \mathrm{CW}[R]) &= \int_{\bar{R}} dx_{\bar{R}} \int ds\, f^{\bar{R}}_{\Delta,s}\left(X, x_{\bar{R}}\right) \rho_{\bar{R}}^{-is/2\pi} O\left(x_{\bar{R}}\right) \rho_{\bar{R}}^{is/2\pi} \\ &= \int_{D[R]} \tilde{f}\left(X, x_{D[R]}\right) U^{\dagger}\left(t_{D[R]}, t_R\right) O\left(x_R\right) U\left(t_{D[R]}, t_R\right)\end{aligned}$$

Consider formulating a bulk field ϕ in the overlap region using the top line, and its conjugate π_ϕ at the same point using the bottom line of (4.7). First, we use the fact that $[O(x_R), O(x_{\bar{R}})] = 0$. Thus in order to obtain the required nonzero canonical commutator in the bulk, $[\pi(X), \phi(X')] = i\delta(X - X')$, it must happen that

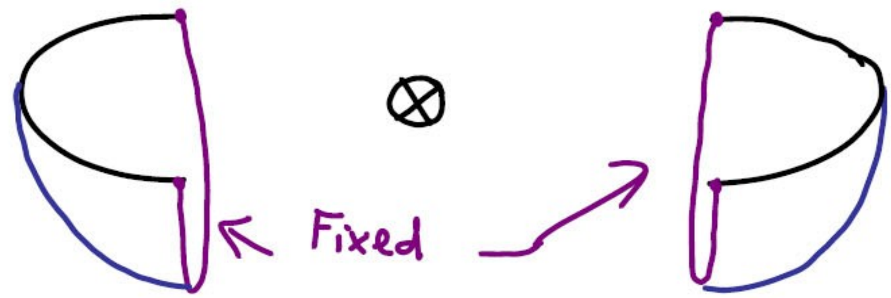
$$[\rho_{\bar{R}}, O(x_R)] \neq 0\,, \quad [U_{D[R]}, O(x_{\bar{R}})] \neq 0\,, \quad \text{or} \quad [\rho_{\bar{R}}, U_{D[R]}] \neq 0\,, \tag{4.8}$$

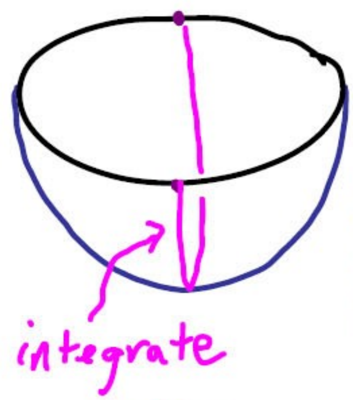
Causal wedge > Entanglement wedge for R<1/2 in cut-off patches consistent with novel commutator structure.

$$[\rho_{\bar{R}}, U_{D[R]}] \neq 0\,.$$

Joining throats: Given the duality for each throat, generalized to any fixed boundary metric and fields, we can construct the ground state of the joined system and compute the Renyi entropies.

$S_n = 0$

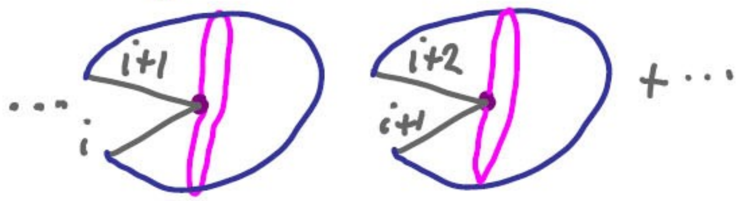




cf Dong; Dong-ES-Torroba; Soni et al, Akers-Rath/ Dong-Harlow-Marolf, Murdia Nomura Rath Salzetta, ...

$S_n = \frac{\hat{A}}{4G_N}$

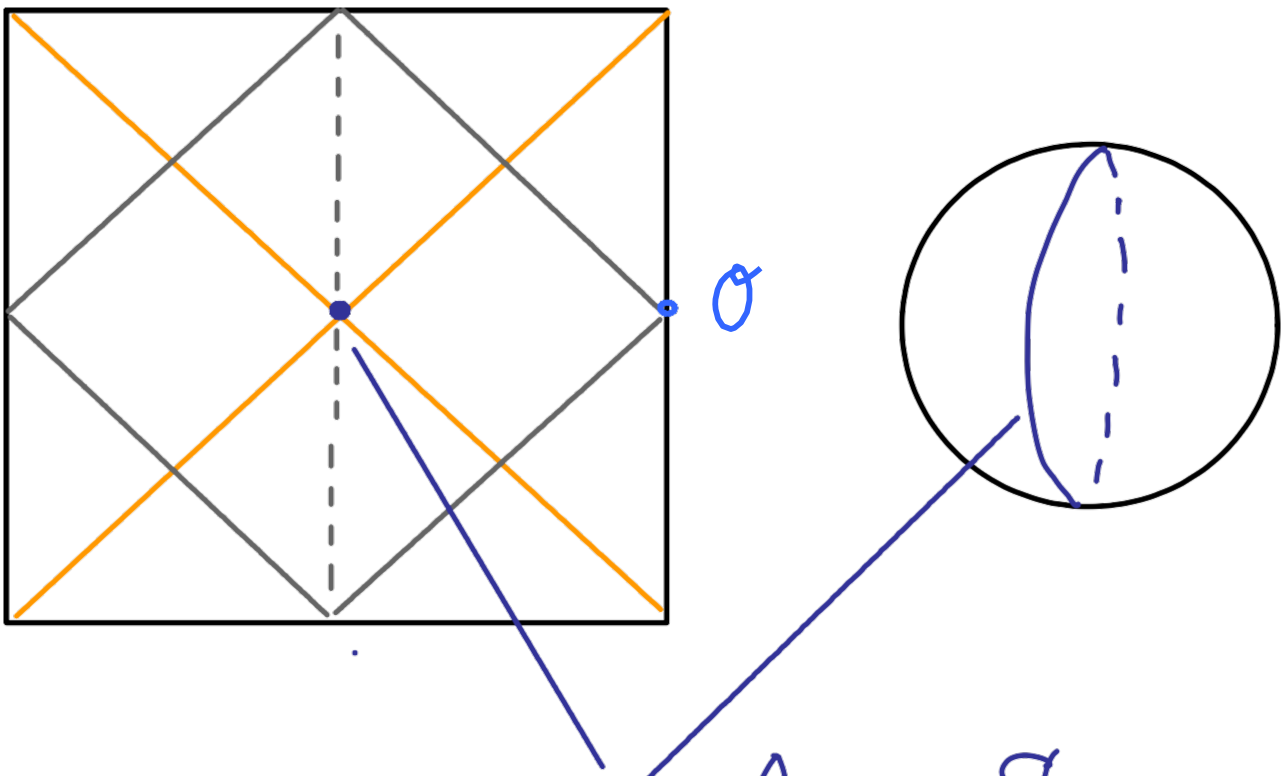
$\mathrm{Tr}\rho_1^n = \cdots$



In our case, the area of the shared locus has a saddle point independent of n, flat entanglement spectrum

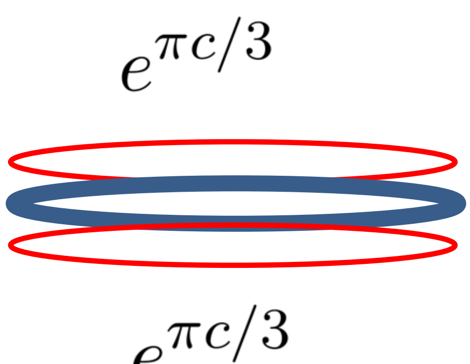
Interactions -> highly mixed grnd state

Interpretation of $S_{\text{Gibbons-Hawking}}$: trace out 1 of the 2 identical TTb+… deformed CFTs living on dS_2 saddle.



$$\frac{A}{4G_N} = S_{RT}$$

$$= S_n = S_{vN} = \log \dim H = S_{G-H}^{(\theta)}$$



Count of dressed energy states:

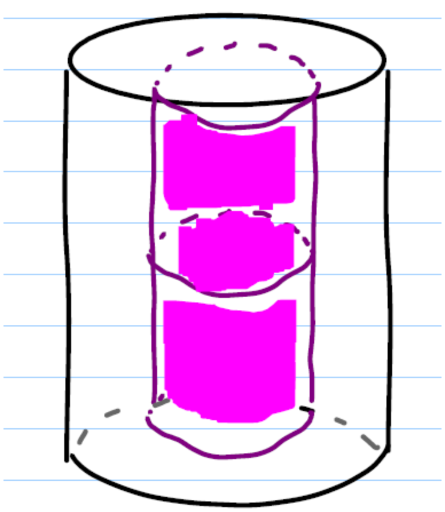
Dressed Energy spectrum and S_{GH} as a count of states :

CFTs under broad large-c sparsity conditions have Hartman Keller Stoica

$$\dim(H_{\Delta+\bar{\Delta}\leq c/6}) \simeq e^{\pi c/3}$$

Matches AdS BH entropy at HP transition

$$E=-2\pi L T_t^t=\frac{L}{\pi\lambda}\left(1-\sqrt{\eta-\frac{4\pi^2\lambda(\Delta+\bar{\Delta}-c/12)}{L^2}+\frac{4\pi^4\lambda^2(\Delta-\bar{\Delta})^2}{L^4}}\right)$$



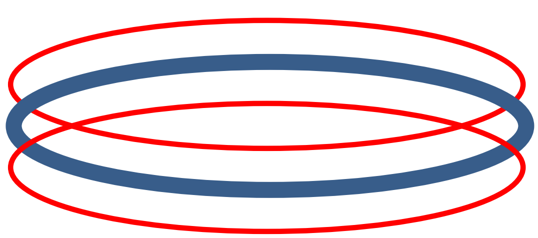
Real energy spectrum captures BHs that fit into cylinder cross section L. Identifying this with neck of dS_2, then turning on $R^{(2)}$and finally Λ_2 gives finite spectrum with correct count of S_{GH}

$S_{Gibbons-Hawking}$ is an approximate notion, obtained in a semiclassical (large c) limit.

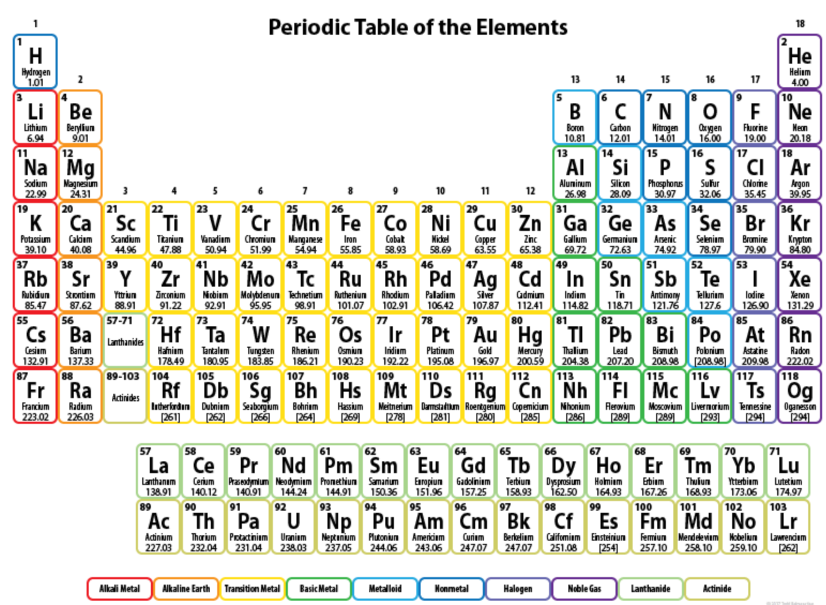
This should not concern us any more than the count of protons in the periodic table does.

- The proton and neutron may decay, but we would not teach high school chemistry by working back from the ultimate decay products. cf dS/CFT or FRW decay of dS
- The nuclei are partially UV completed by QCD, but (i) that is overkill and (ii) the full UV theory is larger and not yet complete. cf embedding in AdS.

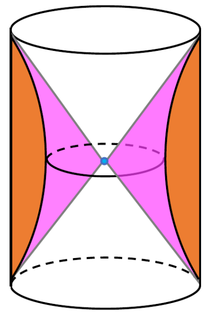
$e^{\pi c/3}$

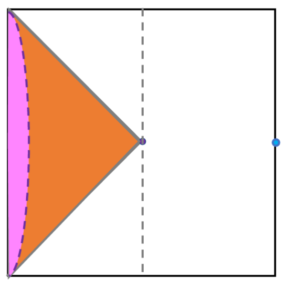


$e^{\pi c/3}$



Later times



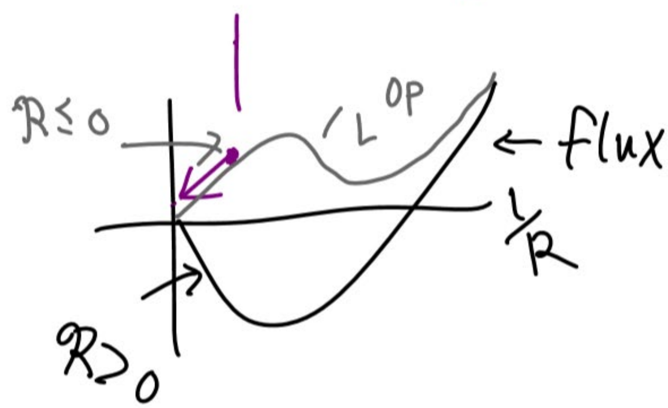


This formula by itself does not restrict energy levels: allows all that ever fit. => can use the theory to capture later time physics (beyond S_{GH} at neck).

$$\frac{E}{L} = \frac{\mathcal{E}}{L^2} = \frac{1}{\pi\lambda}\left(-1 + \sqrt{\eta + c\frac{\mathcal{R}^{(2)}\lambda}{24} + \frac{(\mathcal{E}^{(0)} - \mathcal{E}_0^{(0)})\lambda}{L^2}}\right).$$

dS/dS part of a larger theory with asymptotic unbounded growth of observer-accessible entropy => potentially arbitrary precision then

FRW decay



Again 2-throated warped metric

$$ds^2 = C^2(\tau^{2/C} - w^2)^{C-1}dw^2 + (1 - \frac{w^2}{\tau^{2/C}})^{C-1}(-d\tau^2 + C^2\tau^2 dH_{d-1})$$

Previously found (Dong, Horn, ES, Matsuura, Torroba '12) entropy bound and d.o.f. count grows like τ. GR_d also decouples at late times.

Future directions

*T Tbar (+Λ_2) flow of the modular Hamiltonian from other points of view, e.g. generalization (Mazenc, Shyam, Soni) of the 2d gravity formulation w/Parrikar, Kruthoff,…

*Fully specify algebra of observables? finite S.

*String theory description of the Λ_2 deformation with bulk dS, including in the single-trace TTbar with its repulsion-like singularity to resolve W/Anderson, Coleman, Aguilera-Damia

*Entanglement islands (cf Pennington; Almheiri et al,…) in T T bar+… deformed theories. Z. Yang: transitions in Euclidean gravity calculationss of late-time dS+BH density matrix. Note: cosmology demands extra integral (cf ensemble issue in replica wormholes?)

*2d/1d case of dS/dS: e.g. integral over shared sources replaces SYK disorder average w/Kruthoff, Z. Yang, …

Backup slides

Back to individual patches: First, original AdS/cylinder case:

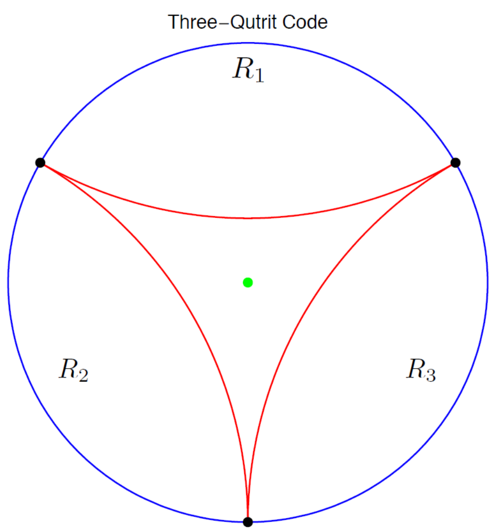
$$
\begin{aligned}
E &= \frac{r_c}{4G}\left(1-\sqrt{1-\frac{8GM}{r_c^2}+\frac{16G^2J^2}{r_c^4}}\right)\\
r_c &= \frac{L_{proper}}{2\pi\ell_{AdS}}, \quad \lambda = 8G\ell, \quad c=\frac{3\ell}{2G}\\
M\ell &= \Delta+\bar{\Delta}-\frac{c}{12}, \quad J\ell = \Delta-\bar{\Delta}\\
E &= \frac{L}{\pi\lambda}\left(1-\sqrt{1-\frac{4\pi^2\lambda(\Delta+\bar{\Delta}-c/12)}{L^2}+\frac{4\pi^4\lambda^2(\Delta-\bar{\Delta})^2}{L^4}}\right)
\end{aligned}
$$

BH energies go complex above cutoff scale. They are absent in the unitary theory obtained by truncating to real levels => finite entropy. To match GR side, we should have emergent locality down to the bulk string scale.

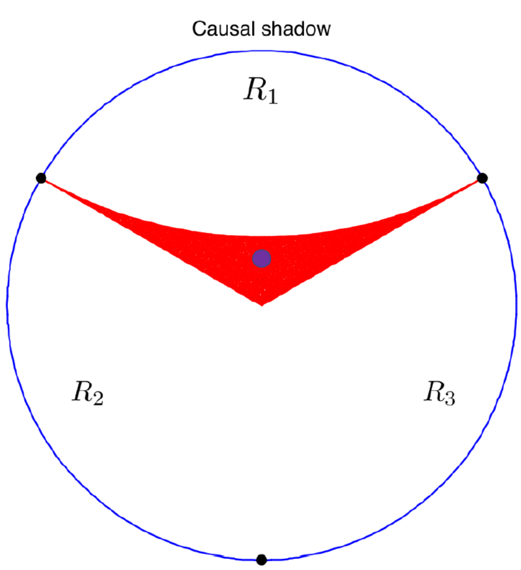
Various subtle aspects addressed in recent and ongoing work, including bulk matter beyond pure gravity, higher dimensions, classifying well-defined observables, ... To binge-watch T-Tbar, see talks at Simons Center workshop April 2019...

Redundant encoding (QEC)

Almheiri Dong Harlow…The point in the middle is redundantly encoded, reconstructible by any of the complements of R1, R2, and R3.



For CW (HKLL) reconstruction, the causal shadow in our case limiting, but still exhibits Redundancy:



HKLL-Reconstructible via the complements of R2 and R3 (but not by the complement of R1).