1d $T\bar{T}$ and Hamiltonian deformations in quantum mechanics

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KITP: Geometry from the Quantum
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[Gross, Kruthoff, Rolph, ES 1912.06132]
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Why quantum mechanics?

- Interested in emergent spacetime, black holes, quantum gravity, etc.; field theory is inessential, QM is enough! e.g. SYK in context of AdS_2/CFT_1, or D0-brane QM/BFSS.

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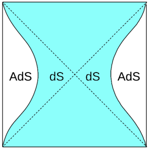

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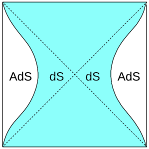

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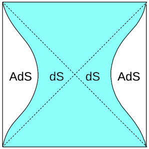

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- Any "composite" operator built out of T is well-defined; 1d $T\overline{T}$ is one example of infinite class of *integrable* deformations $H \to f(H)$.

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Finite-temperature correlators obtained as integral transform. Consider $Z(\beta)$:

$$K(\beta, \beta') = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dE e^{-\beta f(E) + \beta' E} \implies e^{-\beta f(E)} = \int_0^\infty d\beta' e^{-\beta' E} K(\beta, \beta')$$

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Since eigenfunctions unchanged, correlation functions treated similarly:

$$\langle O(\tau_1)\dots O(\tau_n)\rangle = \int\left(\prod_{i=1}^{n-1} dE_i\right)\langle 0|O|E_1\rangle\cdots\langle E_{n-1}|O|0\rangle e^{-\sum_{i=1}^{n-1}(\tau_i-\tau_{i+1})E_i}$$

AdS_2 JT gravity at finite cutoff: 1d $T\bar{T}$

Consider s-wave sector of AdS_3 pure gravity:

$$S_{JT} = -\frac{1}{16\pi G}\int d^2x\sqrt{g}\,\Phi\left(R+\frac{2}{\ell^2}\right) - \frac{1}{8\pi G}\int d\tau\sqrt{h}\,\Phi\left(K-\frac{1}{\ell}\right).$$

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Flow $\partial S^{(2d)}/\partial\lambda = \int d^2x\, T\bar{T}$ in CFT_2 is supposed to implement finite cutoff in AdS_3 [McGough, Mezei, Verlinde]. Dimensionally reduce flow to get

$$\frac{\partial S^{(1d)}}{\partial\lambda} = \int d\tau\,\frac{T^2}{1/2-2\lambda T}\,.$$

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Energy levels of deformed theory given as

$$\frac{\partial E}{\partial\lambda} = \frac{E^2}{1/2 - 2\lambda E} \implies H = \frac{1}{4\lambda}\left(1 - \sqrt{1 - 8H_0\lambda}\right).$$

This $f(H)$ leads to a computable kernel for $\lambda < 0$:

$$\boxed{K(\beta,\beta') = \frac{\beta}{\sqrt{-8\pi\lambda\beta'^3}}\exp\left(\frac{(\beta-\beta')^2}{8\beta'\lambda}\right)}$$

General dilaton gravity (needed for exotic interiors!) must be analyzed directly using method of [Hartman, Kruthoff, ES, Tajdini]

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$$S_E = \int d\tau \, \left(\frac{1}{2} \dot{q_i}^2 + V(q_i) \right) , \qquad i = 1, \ldots, N$$

under our flow. Deformed action found by using $T = L_E - \frac{\partial L_E}{\partial \dot{q}} \dot{q}$ to write a flow equation which is solved by

$$S_E = \frac{1}{4\lambda} \int d\tau \left(1 - \sqrt{(1 - 4\lambda \dot{q}_i^2)(1 - 8\lambda V(q_i))} \right) .$$

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For $\lambda < 0$ this is a worldline action with cosmological constant and mass $m = 1$ in a *curved* target space metric $g_{\mu\nu} = \delta_{\mu\nu}(1 - 8\lambda V(q_i))$:

$$S_E = \frac{1}{4\lambda} \int d\tau \left(1 - \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} \right) , \qquad \mu = 1, \ldots, N+1$$

Pick static gauge $x^0(\tau) = \tau$, $x^i(\tau) = 2\sqrt{-\lambda} q_i(\tau)$. Sharp worldline interpretation for $\lambda < 0$ (wrong-sign kinetic terms otherwise).

1d $T\bar{T}$ as coupling to worldline gravity

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Worldline actions for $\lambda < 0$ suggest similar connection. Proposal:

$$Z_\lambda(\beta) = \int \frac{\mathcal{D}e\mathcal{D}X\mathcal{D}\Phi}{\text{Vol(Diff)}} e^{-S_0[e,\Phi]-S[e,X;\lambda]}$$

for $S_0[e,\Phi]$ the undeformed theory with fields $\Phi(\tau)$ on einbein e, $\tau \sim \tau + \beta'$,

$$S[e,X] = -\frac{1}{8\lambda}\int_0^{\beta'} e\, d\tau \left(e^{-1}\dot{X} - 1\right)^2.$$

$X(\tau + \beta') = X(\tau) + m\beta$ compact scalar with winding m.

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$X(\tau+\beta') = X(\tau) + m\beta$ compact scalar with winding m. Gauge fixing $e = 1$ reduces the path integral over einbeins to an integral over β':

$$Z_\lambda(\beta) = \frac{\beta}{\sqrt{-8\pi\lambda}}\int_0^\infty \frac{d\beta'}{\beta'^{3/2}} \sum_{m\in\mathbb{Z}} \exp\left(\frac{1}{8\beta'\lambda}(m\beta-\beta')^2\right) Z(\beta') .$$

Unit winding sector is the integral transform for $Z(\beta')$!

Applications: Schwarzian theory

Consider Schwarzian action

$$S = -C \int d\tau \, \{e^{i\theta(\tau)}, \tau\} = -C \int d\tau \left(\left(\frac{\theta''}{\theta'} \right)' - \frac{1}{2} \left(\frac{\theta''}{\theta'} \right)^2 + \frac{\theta'^2}{2} \right) .$$

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$$q_1 = \theta, \qquad q_2 = \theta', \qquad p_1 = \frac{\partial L}{\partial \theta'} - \frac{d}{du} \left(\frac{\partial L}{\partial \theta''} \right), \qquad p_2 = \frac{\partial L}{\partial \theta''} .$$

The undeformed and deformed Hamiltonian are

$$H_0 = p_2^2 q_2^2 + \frac{C}{2} q_2^2 + p_1 q_2 \, , \qquad H(\lambda) = f(H_0).$$

Euclidean Lagrangian becomes

$$L_E(\lambda) = \frac{C}{2} \frac{e^{\phi}}{\theta'} (\phi'^2 - \theta'^2) + f(\dot{f}^{-1}(e^{-\phi}\theta')) - e^{-\phi}\theta' \dot{f}^{-1}(e^{-\phi}\theta'),$$

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OTOC: linearize around saddle $\theta = \tau + \varepsilon(\tau)$, $e^{\phi} = c_f e^{\eta(\tau)}$, compute $\langle \varepsilon(\tau)\varepsilon(0) \rangle$ which feeds into 4-pt function. Lyapunov exponent unaffected.

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This theory has a one-loop exact partition function [Stanford, Witten]

$$Z(\beta) = \frac{\alpha}{\beta^{3/2}} \exp\left(\frac{\pi^2}{\beta}\right), \qquad \rho(E) = \frac{\alpha}{\pi^{3/2}} \sinh\left(2\pi\sqrt{E}\right).$$

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One cut matrix model description? Deformed partition function can be computed exactly by integral transform for $\lambda < 0$:

$$Z_\lambda(\beta) = \frac{\alpha\, \beta\, e^{-\frac{\beta}{4\lambda}}}{\sqrt{-2\pi\lambda}(\beta^2 + 8\pi^2\lambda)} K_2\left(-\frac{1}{4\lambda}\sqrt{\beta^2 + 8\pi^2\lambda}\right).$$

Hagedorn divergence! Can be continued to $\lambda > 0$. Bulk calculation would be a check of $T\overline{T}$-ology at subleading order in $1/N$.

Applications: SYK

$$H = i^{q/2} \sum_{i_j} J_{i_1 \cdots i_q} \psi_{i_1} \cdots \psi_{i_q}, \quad \langle Z \rangle_J \sim \int dJ_{i_1 \cdots i_q} \exp\left(-\frac{J^2_{i_1 \cdots i_q}}{2 \langle J^2_{i_1 \cdots i_q} \rangle} \right) Z(J_{i_1 \cdots i_q})$$

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$$S_{E,\lambda} = N \left(-\log \mathrm{Pf}(\partial_\tau - \Sigma) + \frac{1}{2} \int d\tau \left[\int d\tau' \Sigma(\tau, \tau') G(\tau, \tau') + 2f(H/N) \right] \right)$$

where

$$H = -i^q \frac{J^2 N}{2q} \int d\tau' G(\tau, \tau')^q - E_0 \,,$$

with E_0 a constant shift. Deforming microscopic SYK $H + \lambda H^2$ then disorder averaging leads to a particular $f(H)$.

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$$\int d\tau' G(\tau, \tau') \Sigma(\tau', \tau'') - \partial_\tau G(\tau, \tau'') = -\delta(\tau - \tau'') \,,$$

$$\Sigma(\tau, \tau') - i^q f'(H/N) J^2 G^{q-1}(\tau, \tau') = 0 \,.$$

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Our proposed solution to the Schwinger-Dyson equations is

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Integral transforms work for $H + \lambda H^2$ with $\lambda \sim O(1)$, but no effective action understanding.

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- Questionable tangent to entertain my friends and connect directly to "Geometry from the Quantum": how do higher-form symmetries / Eguchi-Kawai fit into the 1d framework, if at all? [ES]