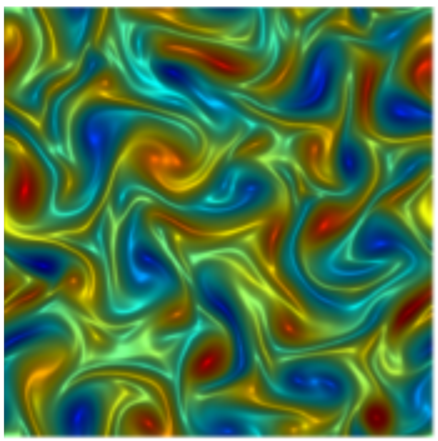
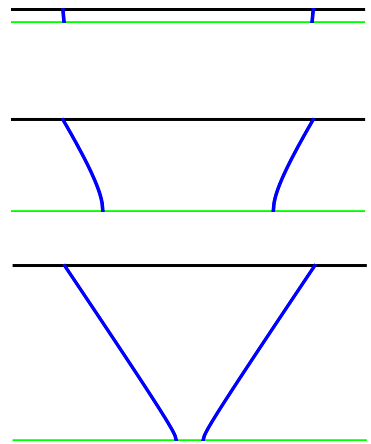
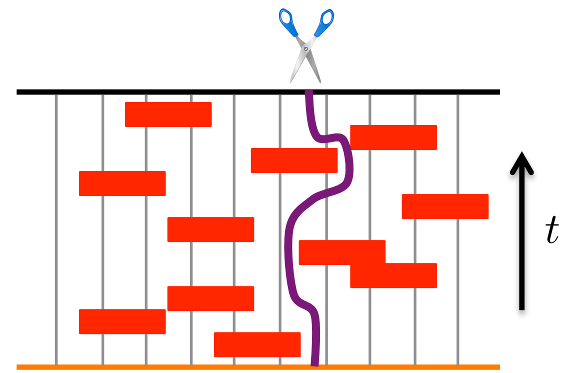
Entanglement hydrodynamics

and comments on tensor networks







Márk Mezei (SCGP, Stony Brook)

In collaboration with: Agón, Bao, Casini, Cotler, Hertzberg, Liu, Mueller, Stanford, van der Schee, Virrueta

Geometry from the Quantum, KITP, 01/13/2020

Quantum chaotic dynamics

Phenomena associated with chaotic dynamics:

Transport

Thermalization

Butterfly effect

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Thermalization

Butterfly effect

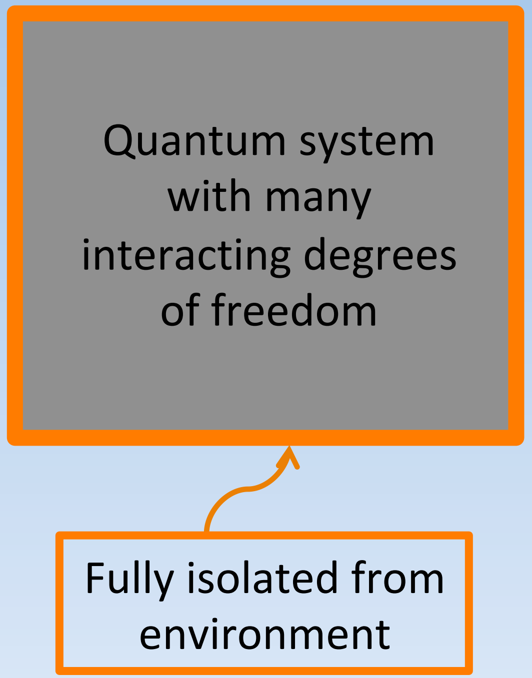
Ultimate goal: understand these phenomena and their relation in quantum systems, relate them to gravity through AdS/CFT

Goal of talk:
- Develop an effective theory of entanglement dynamics in the hydrodynamic limit
- Study its interplay with other chaotic phenomena
- Comment on relations to tensor network approaches to AdS/CFT

Quantum chaotic dynamics

Setup:

- Degrees of freedom interacting strongly through local **chaotic** Hamiltonian.
- In highly excited state, out of equilibrium at $t = 0$, in equilibrium for $t \to \infty$.
- Foundational question in statistical physics. Subject of intense current activity in HEP, CMT, QI, and AMO experiments.



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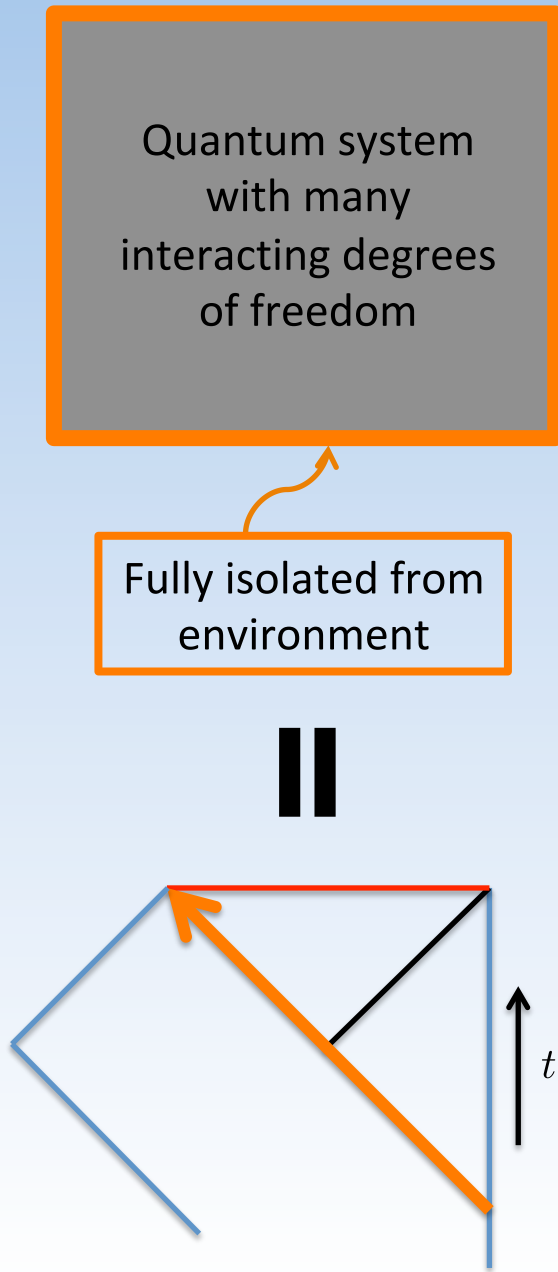
Study the setup using holographic duality:

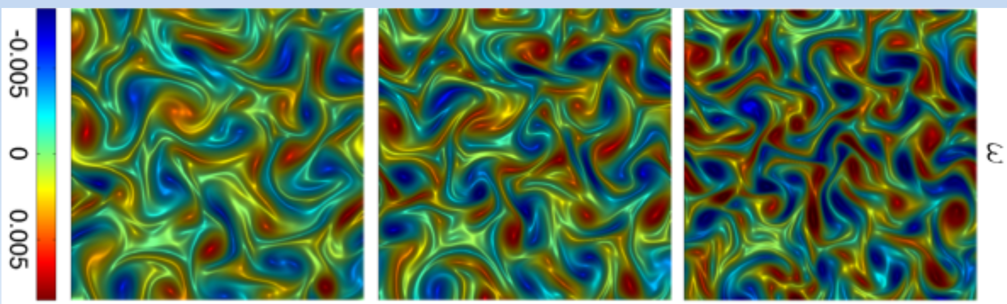
- A QFT settling to thermal equilibrium is dual to a collapsing black hole.
- No small parameters, holography is indispensible in understanding real time quantum dynamics.
- Entanglement plays a crucial role in thermalization.



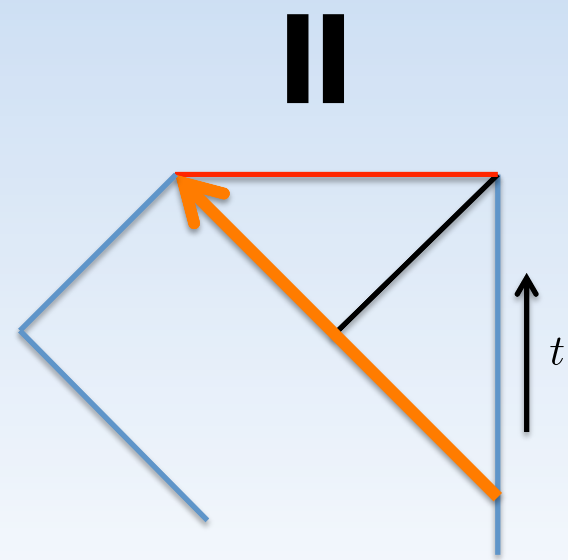
Hydrodynamics

We have an effective theory for describing conserved densities.

- Hydrodynamics applies universally for all chaotic systems. Generalized hydrodynamics for integrable systems.
- Navier-Stokes equations: $\partial_t v + (v \cdot \nabla) v - \nu \nabla^2 v = -\nabla p$



Quantum system with many interacting degrees of freedom



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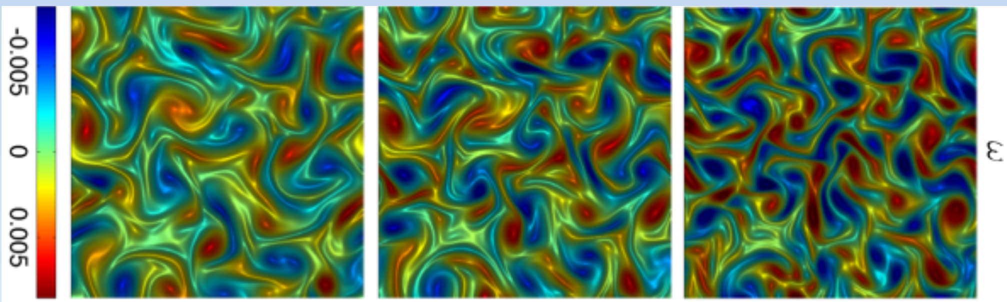

- Relativistic hydro from hep-th POV is an EFT based on systematic long distance, late time expansion. Fluid variables:

$$T_{ab} = (\rho + p) u_a u_b + p\, \eta_{ab} + \Pi_{ab}$$

- Hydrodynamics follows from the conservation of T_{ab} . Solution determines $\langle T_{ab} \rangle$ out of equilibrium.

Quantum system with many interacting degrees of freedom

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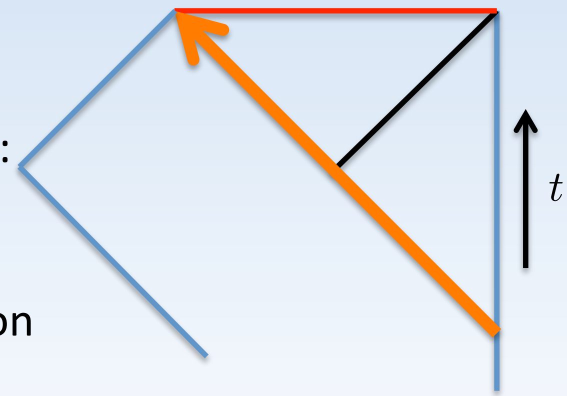

Hydrodynamics

We have an effective theory for describing conserved densities.

- Fluid/gravity constructs black holes with bumpy horizons from fluid flows. [Bhattacharyya et al.]

$$ds^2 = \frac{1}{z^2}\left[2u_a(x)dx^a dz + \left[\eta_{ab} + \left(1 - a\left(\frac{d\,z}{4\pi T(x)}\right)\right)u_a(x)u_b(x)\right]dx^a dx^b\right] + (\text{gradients})$$

- Alternative history: String theorists discover hydrodynamics by studying AdS black holes.

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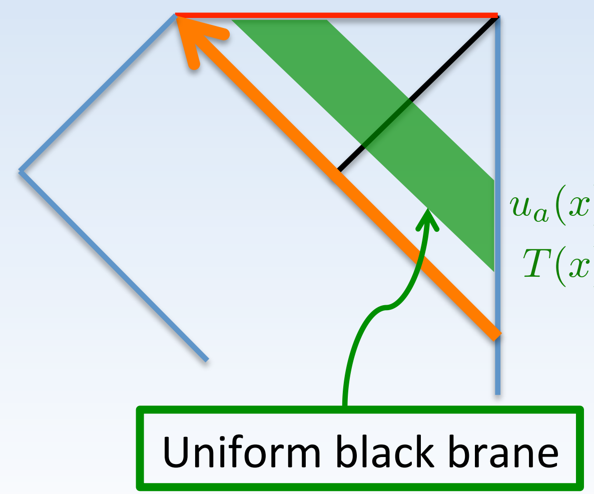

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$$+ \text{(gradients)}$$

- Alternative history: String theorists discover hydrodynamics by studying AdS black holes.

- Interested in more data than $\langle T_{ab}\rangle$: entanglement entropy, butterfly effect, etc.

- I want to follow the “alternative history” path to discover a hydrodynamic effective theory of entanglement dynamics (and operator growth).

- Hydrodynamics is universal, there is evidence for the universality of the effective theory of entanglement hydrodynamics.

Quantum system with many interacting degrees of freedom

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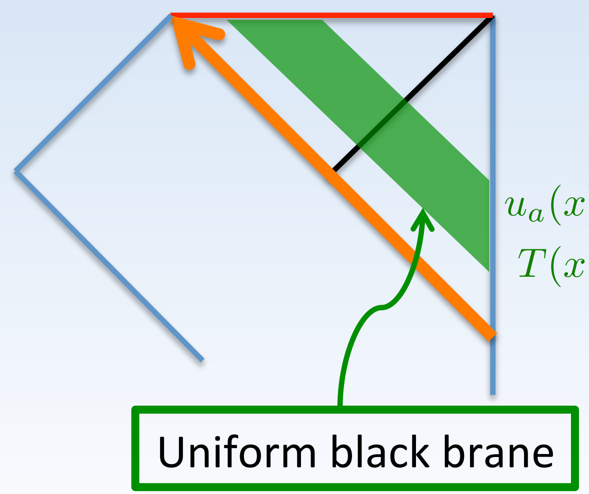
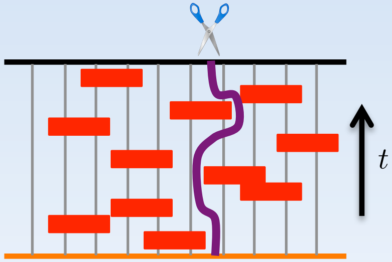


Outline

Transport
- Hydro as an EFT
- Holography for real time dynamics

Thermalization
- Entanglement entropy as a probe
- Membrane theory is the EFT
- Interplay with hydro



Comments on tensor networks
- Membrane theory from random circuits
- Interplay with operator growth

Conclusions and open questions

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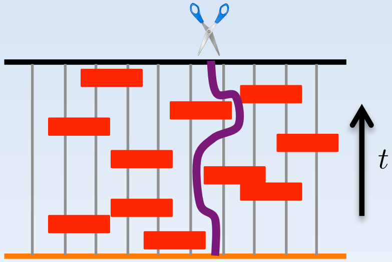

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Conclusions and open questions

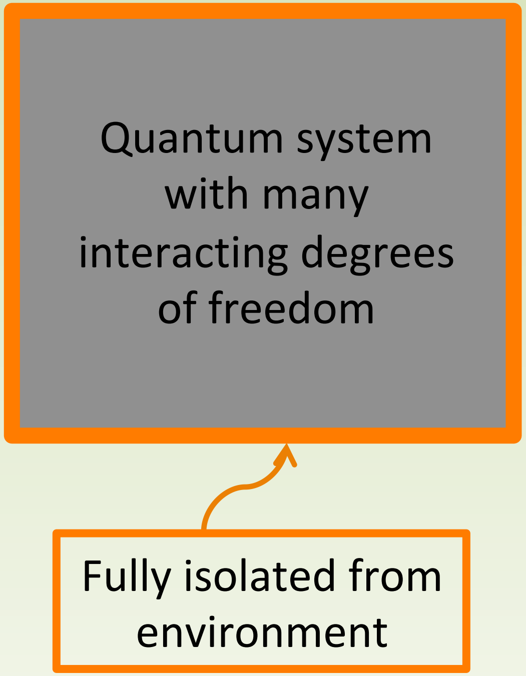
Quantum thermalization and subsystems

Quantum thermalization

- Pure state with nonzero energy density: $|\psi(0)\rangle$
  Unitary time evolution: $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

- $\rho(t) \equiv |\psi(t)\rangle\langle\psi(t)| \not\to \dfrac{e^{-\beta H}}{Z}$ cannot mean thermalization.

  $\rho(t)$ encodes all the information in $|\psi(0)\rangle$, but at late times in a very nonlocal way.



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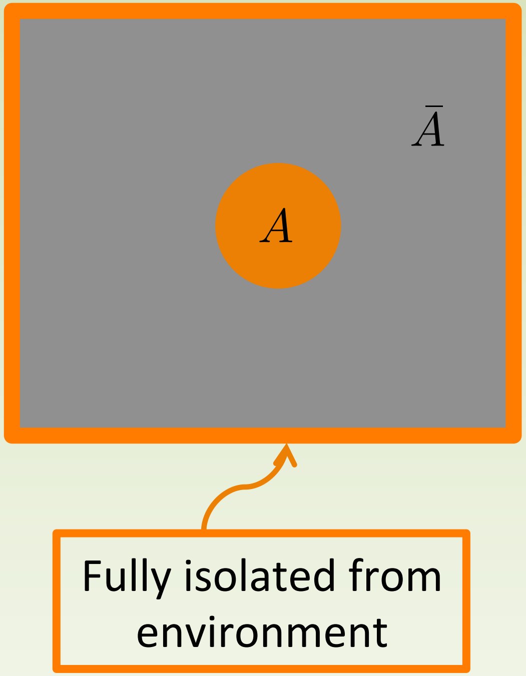
- Consider subsystems, reduced density matrix:
  $\rho_A = \mathrm{Tr}_{\bar{A}}\, |\psi\rangle\, \langle\psi|$

- Thermalization: $\rho_A(t) \;\to\; \rho_A^{(\mathrm{eq})}(\beta) = \mathrm{Tr}_{\bar{A}}\, \frac{e^{-\beta H}}{Z}$

  For $t \to \infty$, in the thermodynamic limit $\bar{A} \to \infty$, with β determined by the energy density. **Entanglement is crucial in making this possible.**



Entanglement entropy

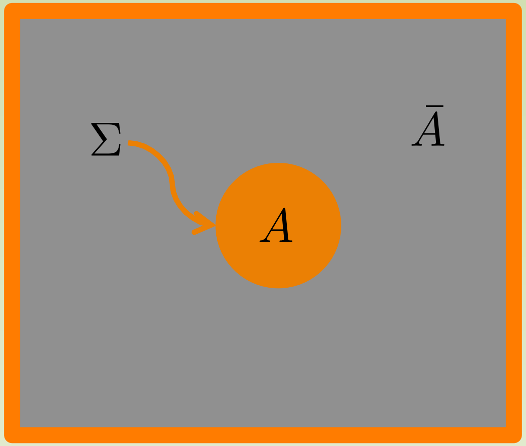
Entanglement entropy is a good diagnostic of thermalization, we **focus on this quantity**.

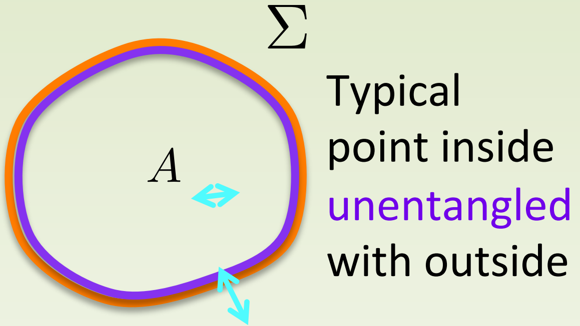
- In ground states of local Hamiltonians the entropy scales with the area:

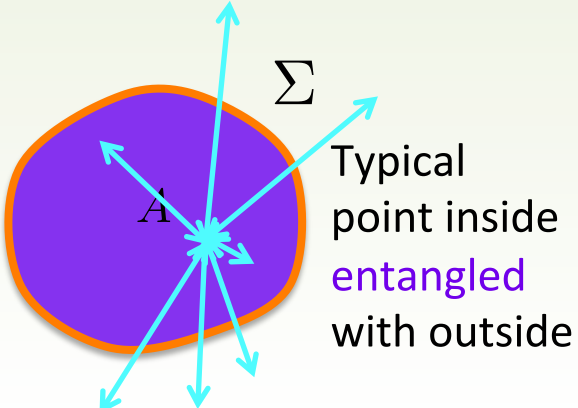
$$S_A = \# \frac{\text{area}(\Sigma)}{\delta^{d-2}} + \dots$$

- A generic state in the Hilbert space shows volume scaling:

$$S_A = s_{\text{th}} \, \text{vol}(A) + \dots$$







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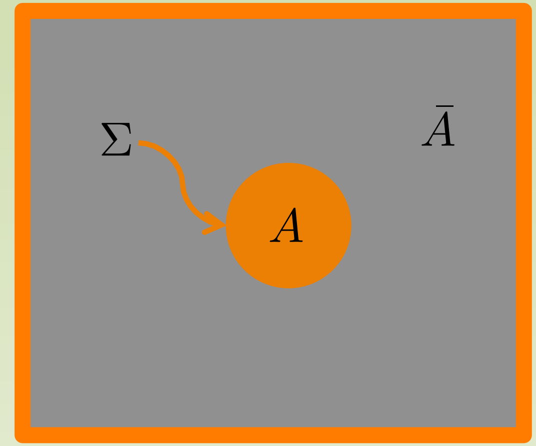

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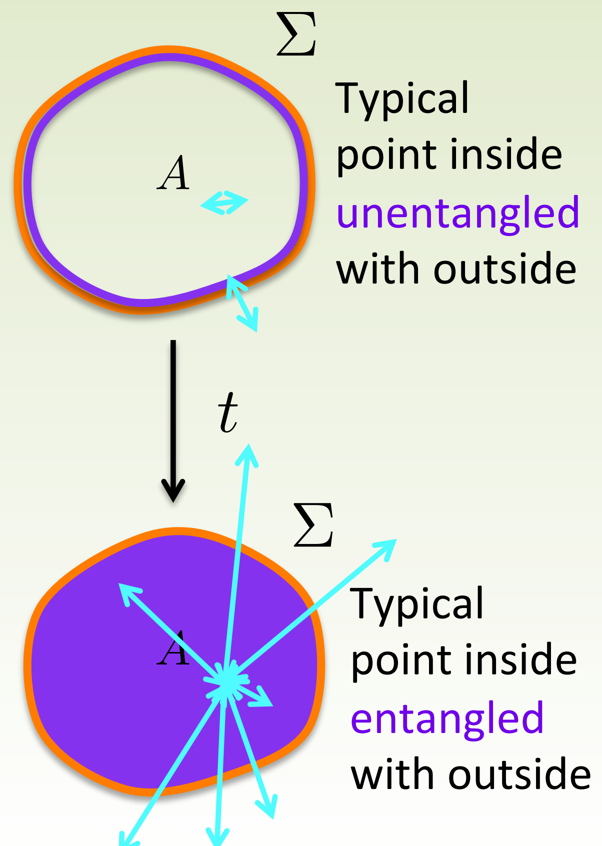
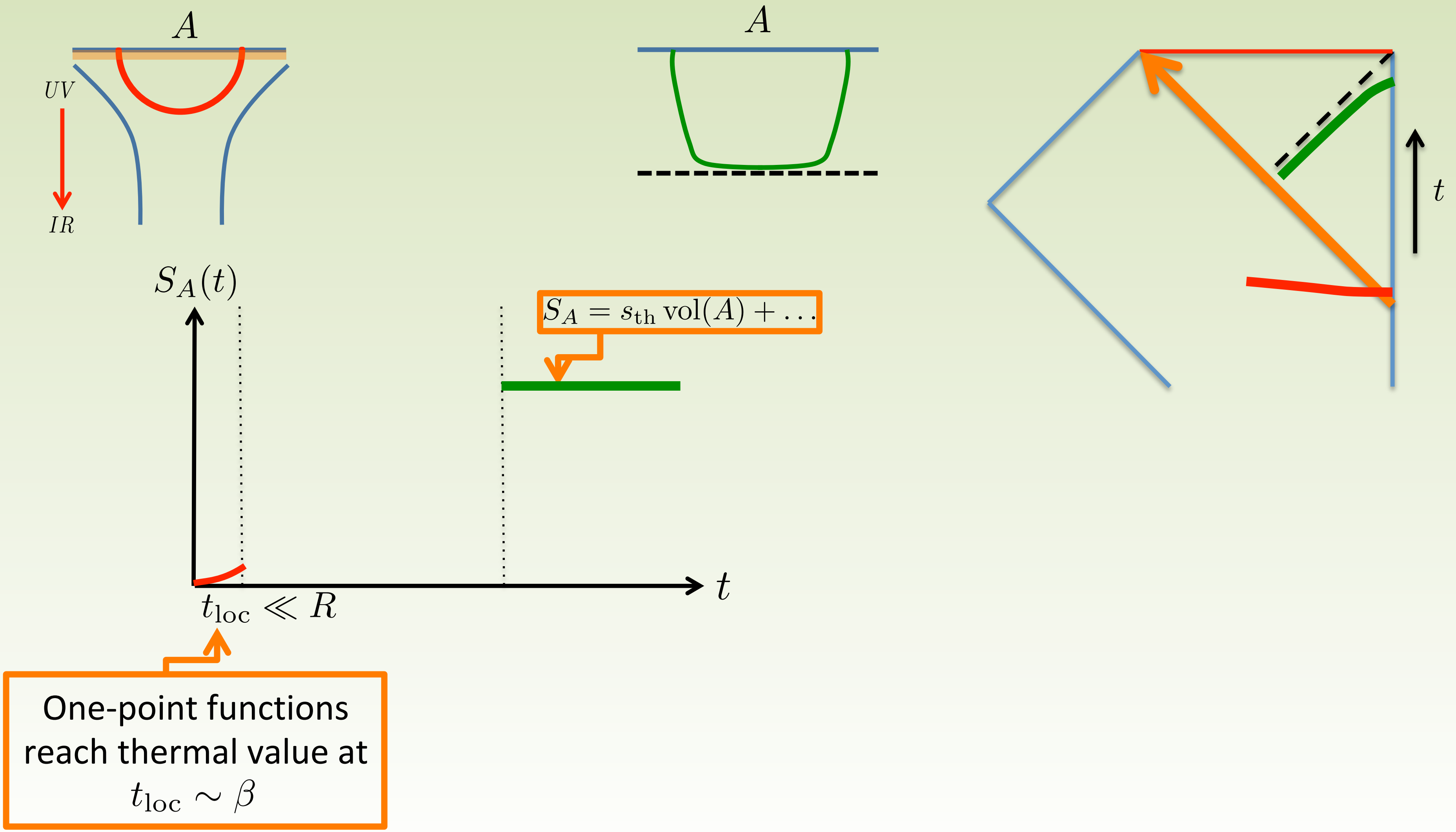
- Purest setup is a quench: start with ground state of a local Hamiltonian, change the Hamiltonian suddenly, and let the system evolve. (No transport.)



Entropy in the hydrodynamic limit

Qualitative picture of entanglement entropy at time t of a region of characteristic size R, $R, t \gg t_{\text{loc}}$ [Cardy, Calabrese; Hartman, Maldacena; Liu, Suh]



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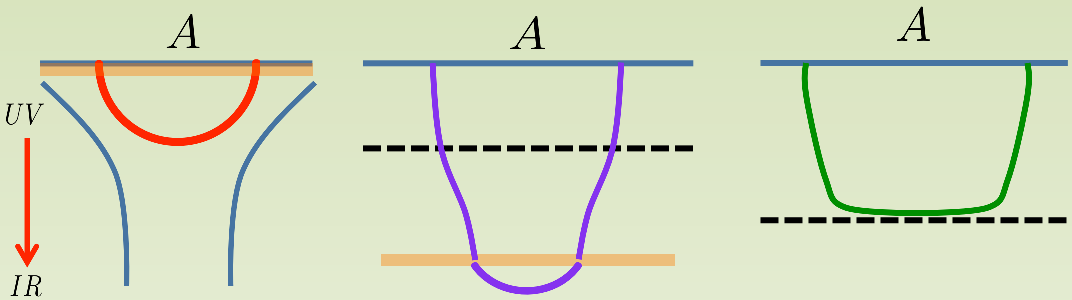

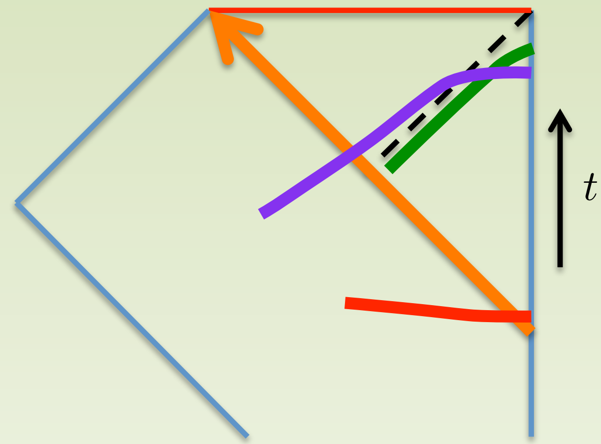

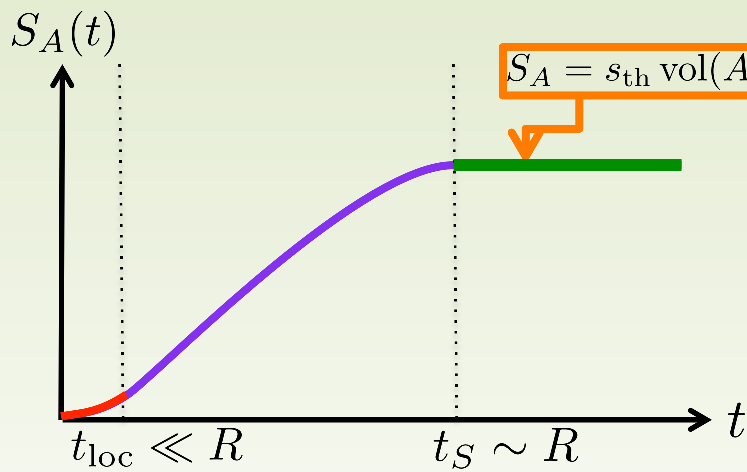

One-point functions reach thermal value at $t_{\mathrm{loc}} \sim \beta$

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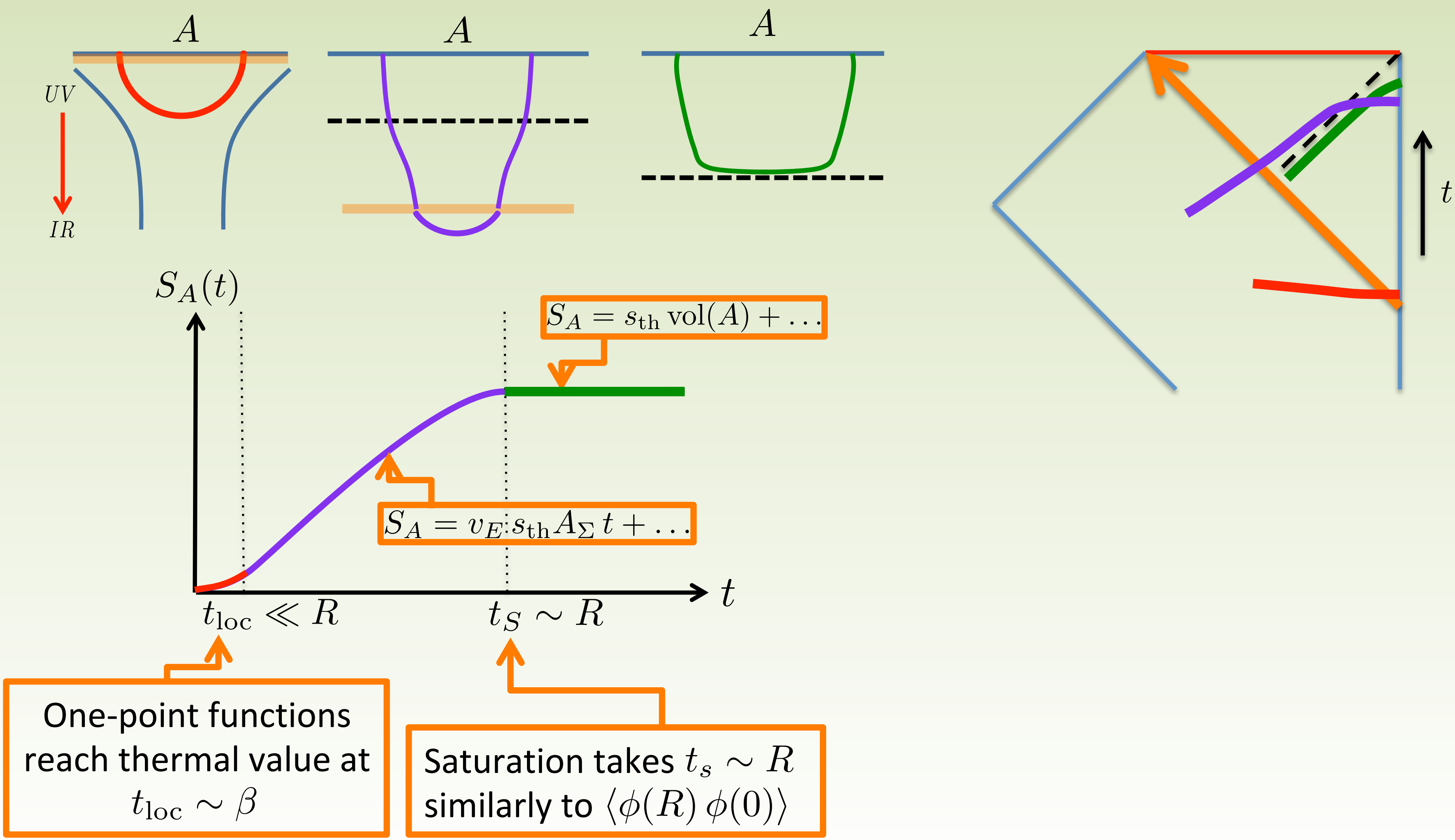
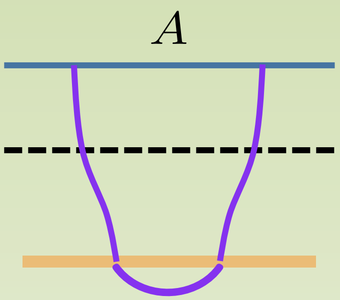
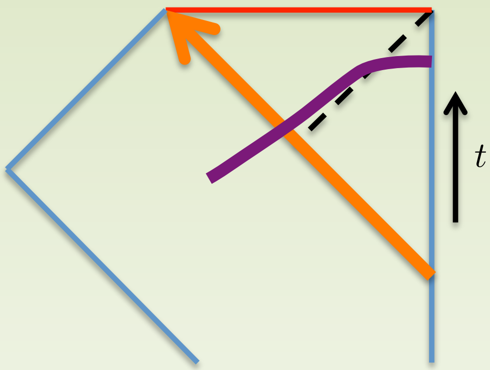


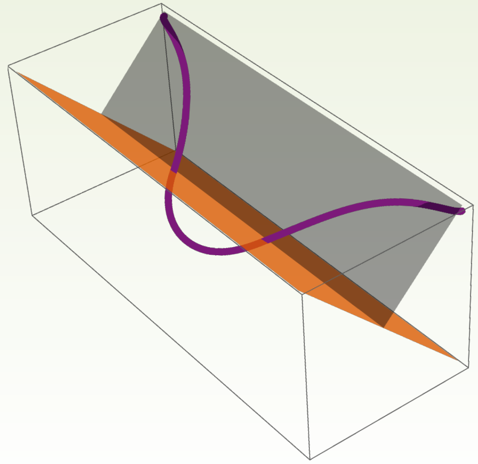
Membrane theory of entanglement dynamics

Can reformulate holographic surface extremization in d+1 dimensions as membrane minimization in d dimensions in the limit $R, t \gg t_{\text{loc}}$. [MM$_2$]

- Detailed understanding of HRT surfaces. The surface has three parts: [MM$_1$]
  1. Outside the horizon part gives (divergent) area law.
  2. Behind the horizon region.
  3. Behind the shell part gives entropy in the vacuum.







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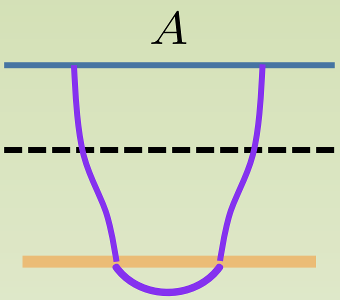

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  1. Outside the horizon part gives (divergent) area law.
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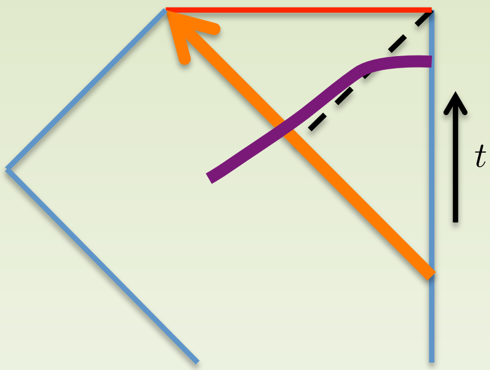
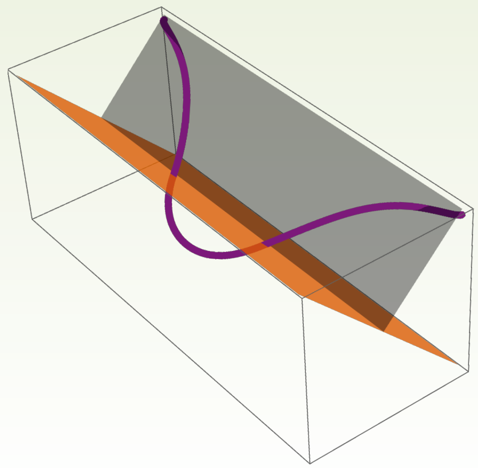
- Only the **2. part** contributes to the extensive part of the entropy.

$$S(t) = s_{\rm th} R^{d-1} \mathcal{S}_{\rm ext}\left(\frac{t}{R}\right) + \dots$$





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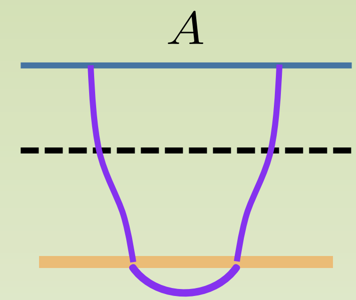

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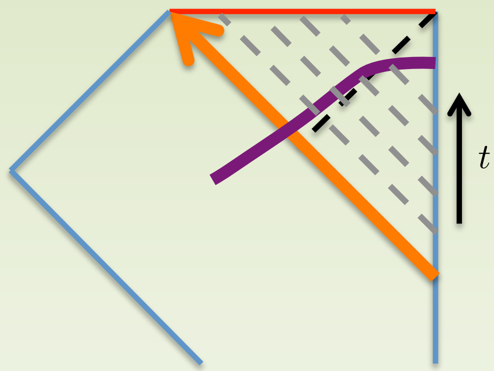
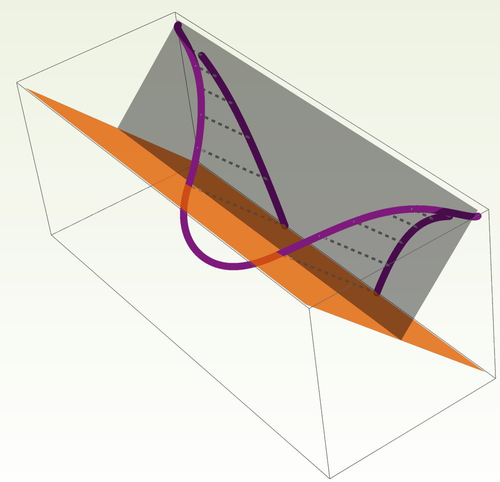
- Push HRT surface to the boundary along constant infalling time.





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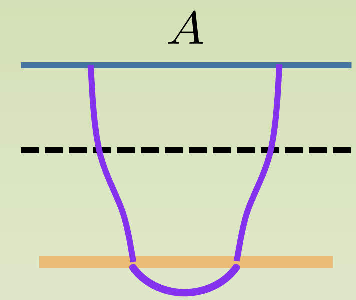

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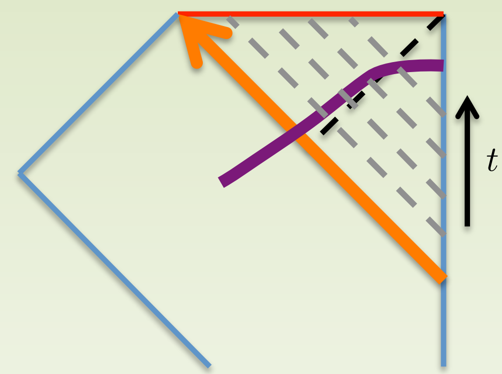

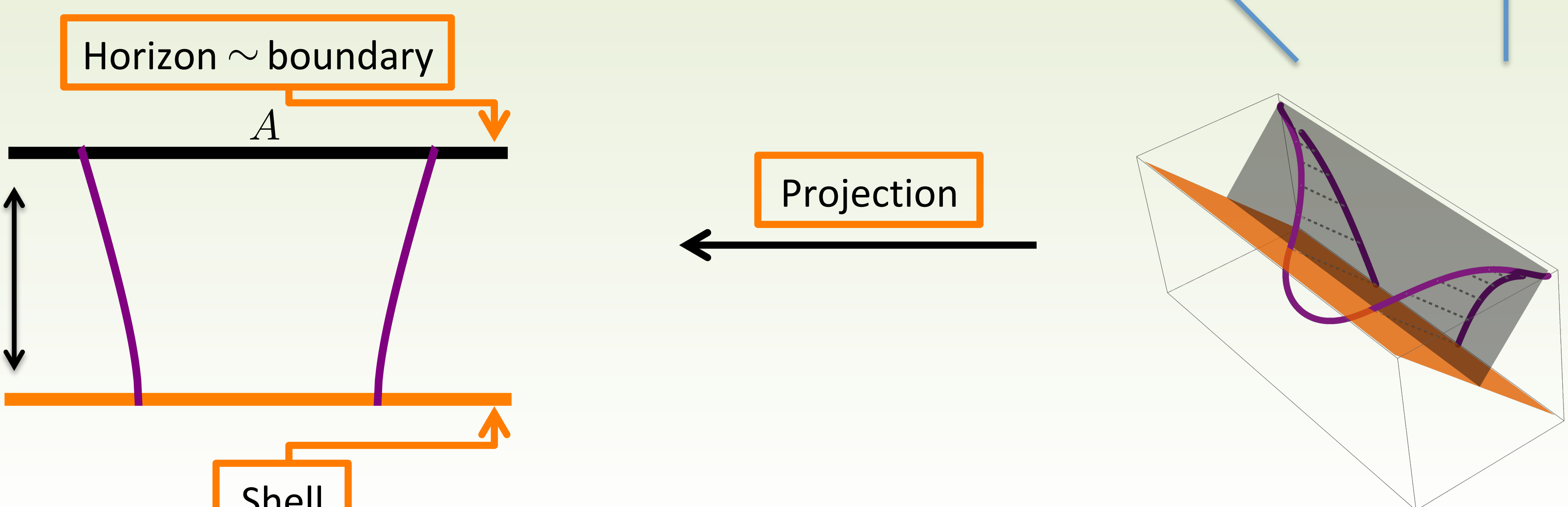

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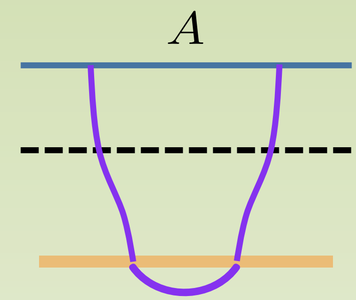

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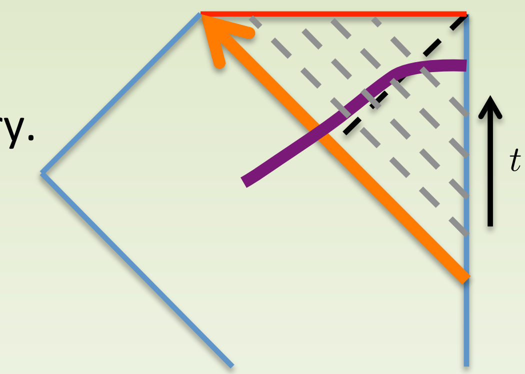
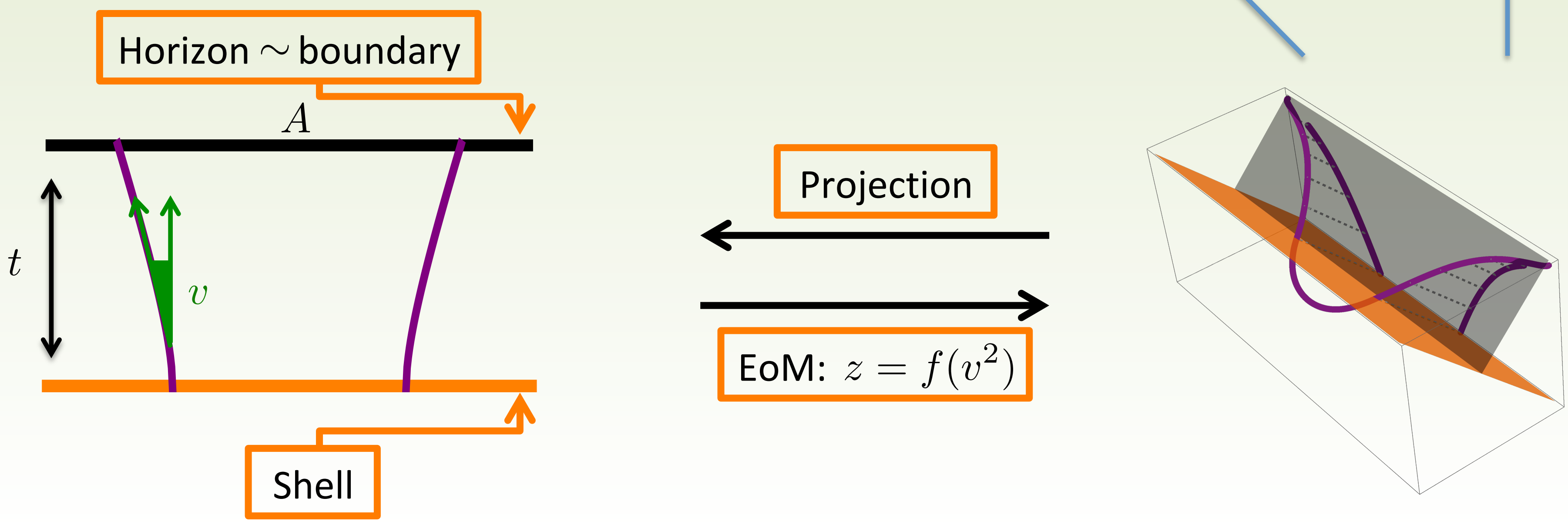
- Push HRT surface to the boundary along constant infalling time.
- Scaling limit: $x^\mu \to R\, x^\mu\,, \quad z \to z$
  Area functional independent of the derivatives of z. Solve algebraic EoM, plug back into action to derive membrane theory.



$$S[A] = s_{\mathrm{th}} \int d^{d-1}\xi \; \sqrt{\gamma} \, \frac{\mathcal{E}\,(v)}{\sqrt{1-v^2}}$$



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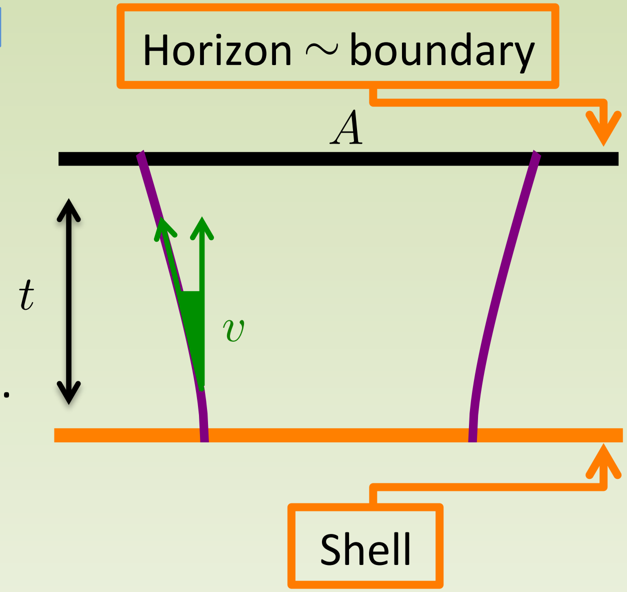
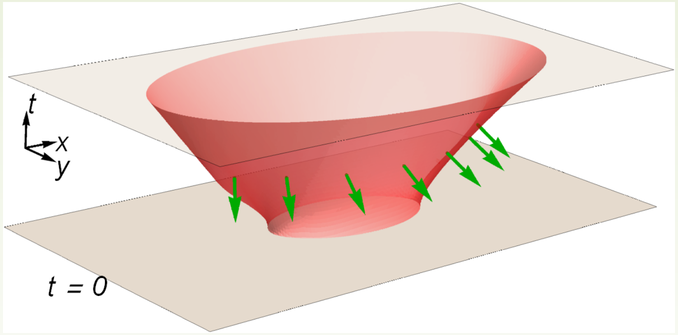
Membrane is projection of HRT to boundary. $\mathcal{E}(v)$ is repackaging of geometry, independent of quench details.





$$v = \frac{(n \cdot \hat{t})}{\sqrt{1 + (n \cdot \hat{t})^2}}$$

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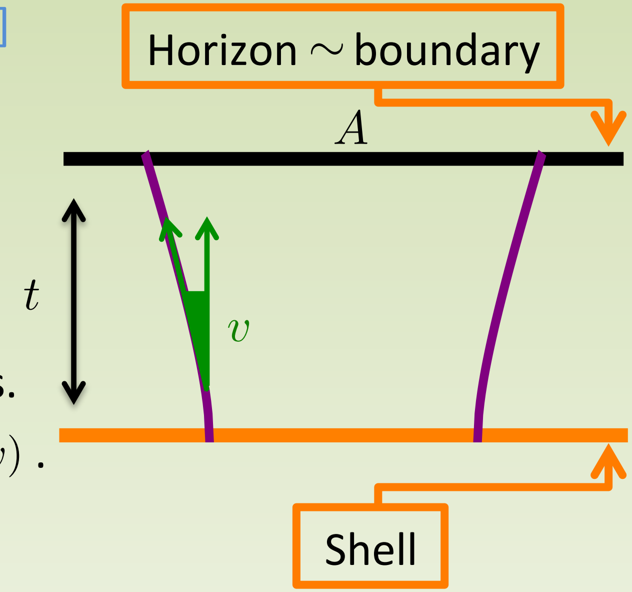
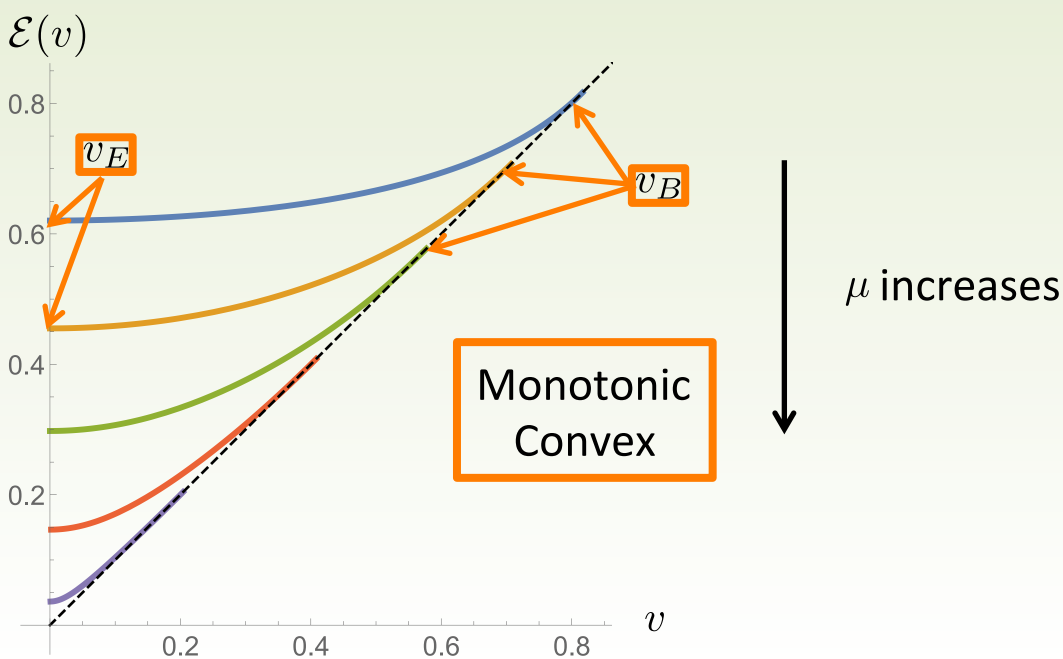
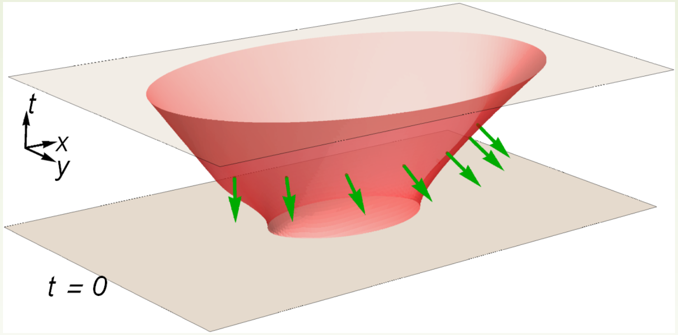
  Membrane is projection of HRT to boundary. $\mathcal{E}(v)$ is repackaging of geometry, independent of quench details.

- Using the NEC, can prove the following properties of $\mathcal{E}(v)$. $\mathcal{E}(v)$ can be thought of as a transport coefficient.







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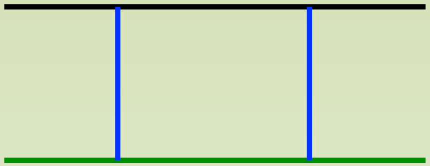
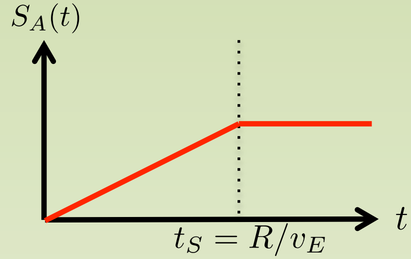
Applications

EE for strip, sphere, cylinder regions in the hydro limit is analytically solvable. [MM$_1$; MM$_2$]

- Strip:





Applications

EE for strip, sphere, cylinder regions in the hydro limit is analytically solvable. [MM_1; MM_2]

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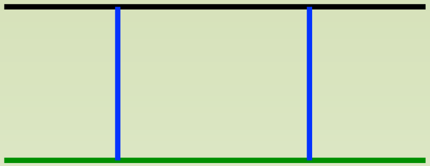

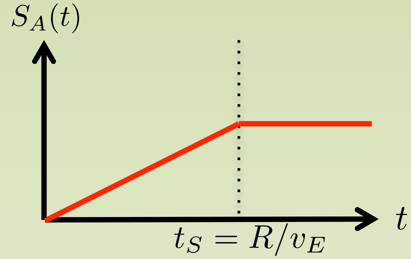

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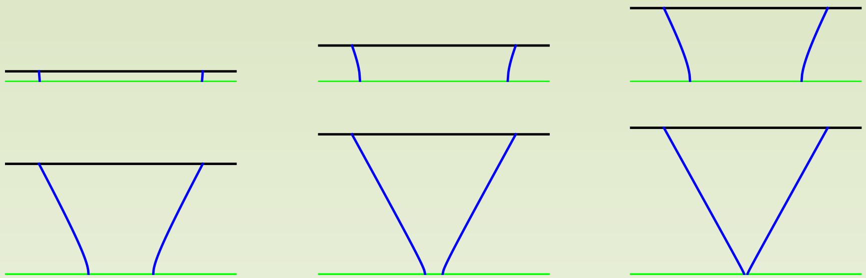

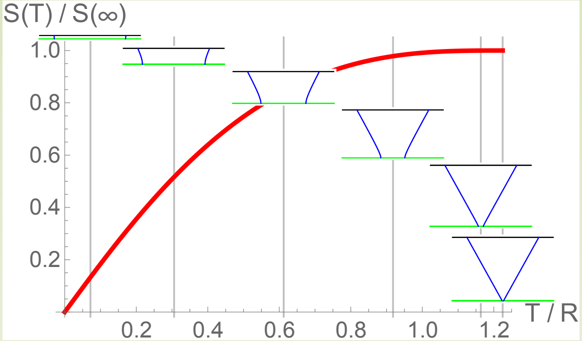

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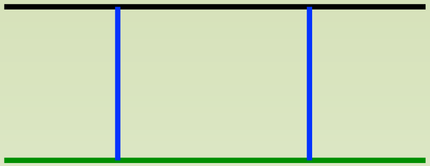

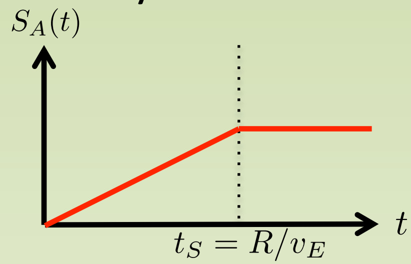

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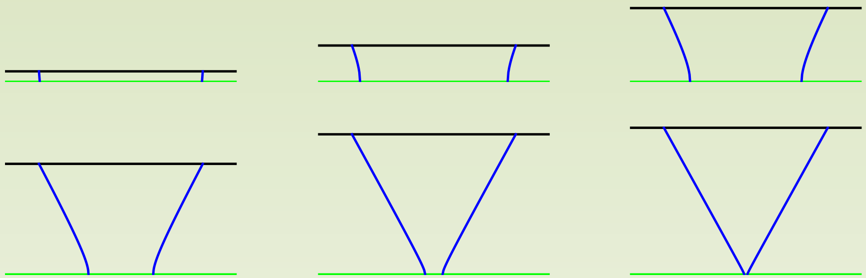
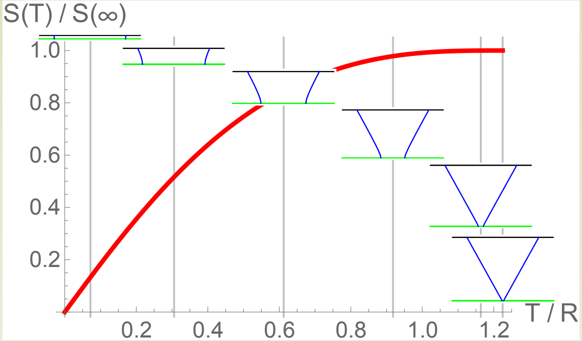
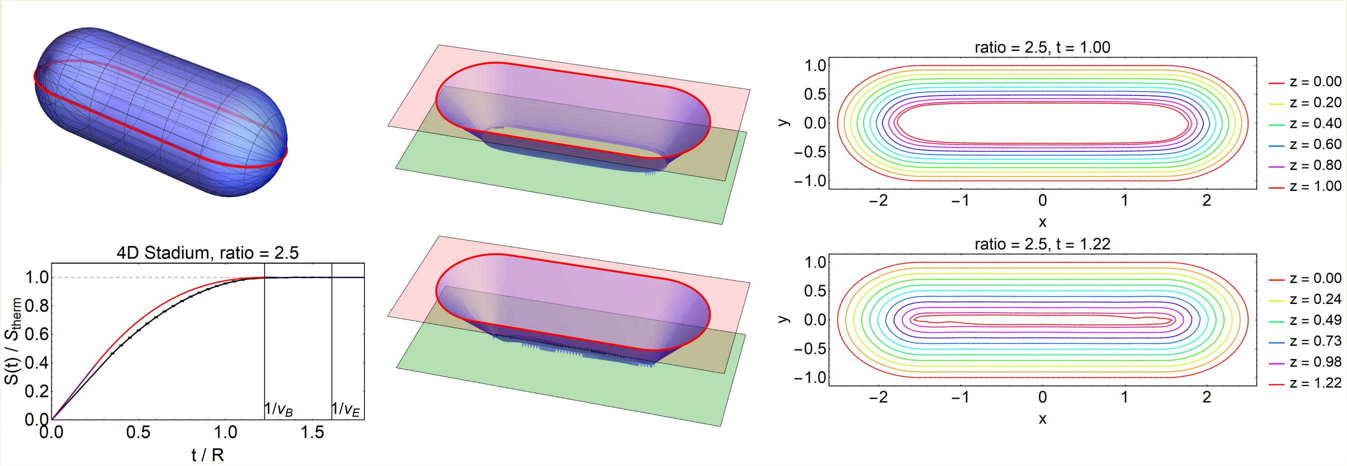




- Stadium shape: [MM, van der Schee]



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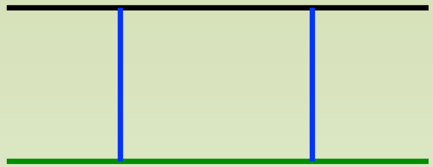

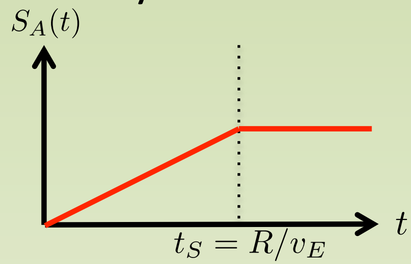

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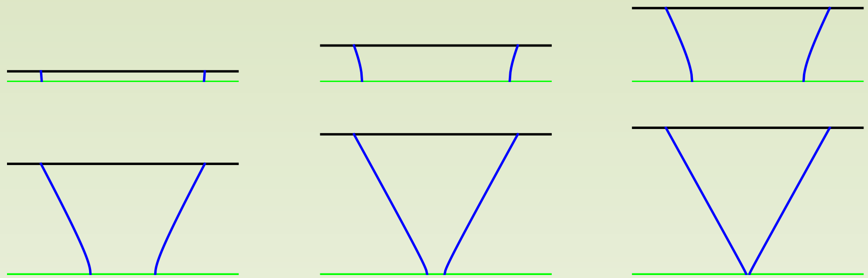

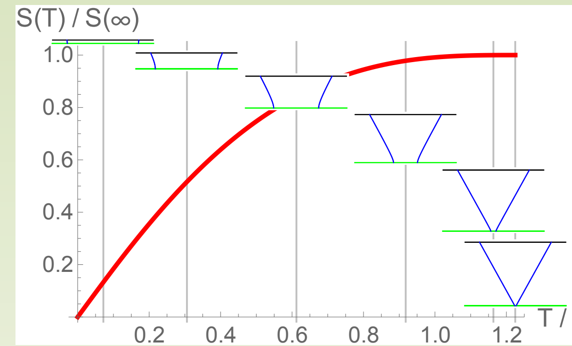

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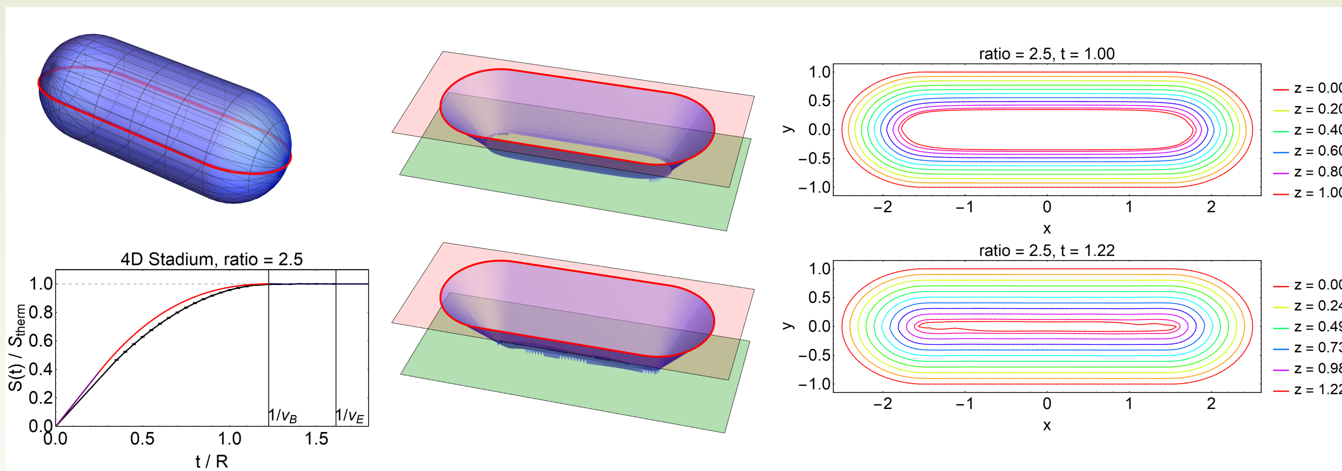

- Simple bound on saturation time from operator growth: [MM, Stanford] $t_S \geq R/v_B$
  For elongated shapes in 4D we find: $t_S = R/v_B$
  Black holes often saturate entanglement entropy the fastest.

Interplay with hydro

The membrane theory is robust, can be generalized away from global quenches.
[MM, Virrueta]

- Fluid/gravity black brane dual to an inhomogenous state in local thermal equilibrium. To subleading order, we get the membrane coupled to hydrodynamics:

$$S = \int d^{d-1}\xi \, \sqrt{\gamma}\, s_{\rm th}(x) \frac{\mathcal{E}(v)}{\sqrt{1-v^2}} + \ldots\,, \quad v(x) \equiv \frac{(n\cdot u(x))}{\sqrt{1+(n\cdot u(x))^2}}$$

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- Adaptable to other inhomogenous setups, can incorporate β/R and $1/\lambda$ corrections without change in the structure of the membrane theory. $1/N$ corrections would be most interesting.

- **Membrane theory is versatile, has connections to operator growth and hydrodynamics, and has all the features to be a universal theory.**

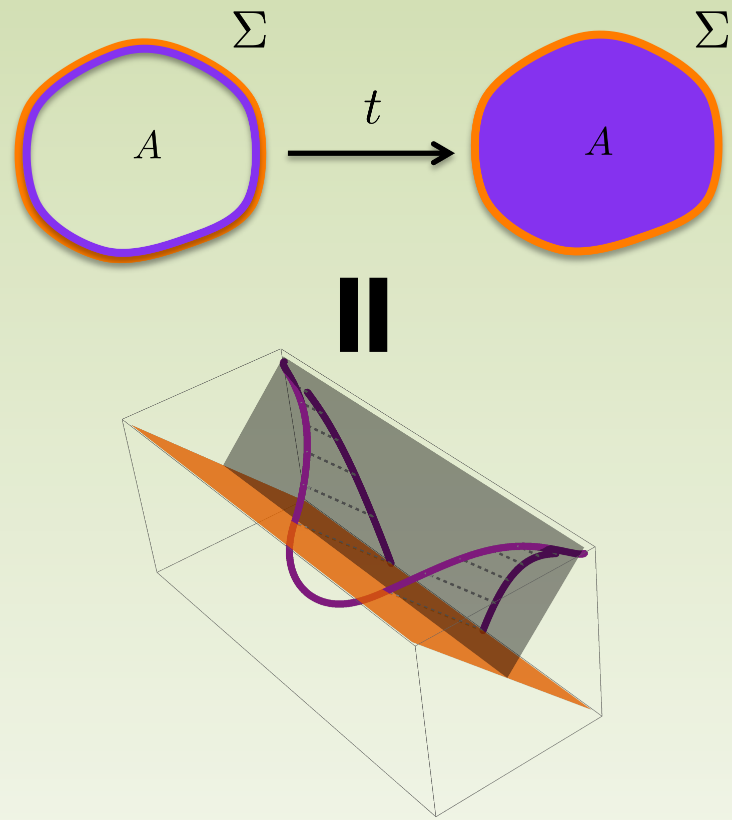
Summary

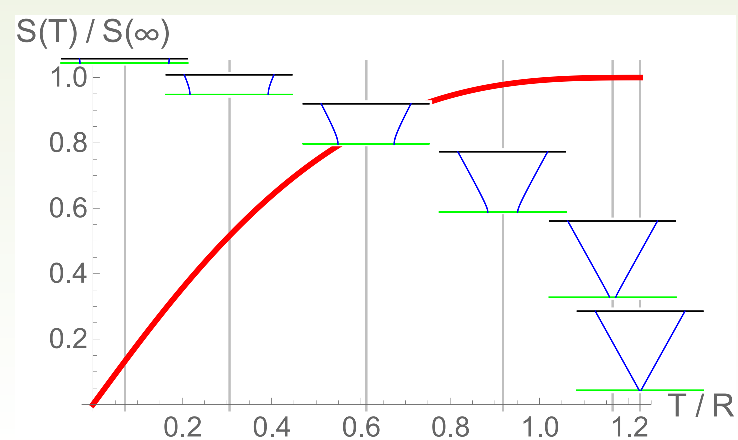
Features of the thermalization:

- Conserved densities described by hydro.
- State of the entire system cannot become thermal. Small subsystem thermalize by becoming entangled with the rest of the system.
  $S_A(t) \rightarrow S_A^{(\mathrm{eq})}(\beta) = s_{\mathrm{th}}(\beta)\,\mathrm{vol}(A)$
  Captures the essence of thermalization.

Goal: Find effective theory (akin to hydro) of entanglement dynamics.

- Alternative history method: Discovered membrane theory by studying AdS black holes, has structure applicable to all chaotic theories.
- In the following conduct further tests. Elucidate connections to other manifestations of chaotic dynamics.





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- Entanglement entropy as a probe
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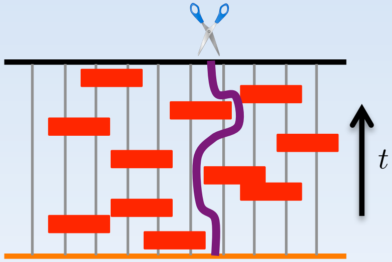

Comments on tensor networks
- Membrane theory from random circuits
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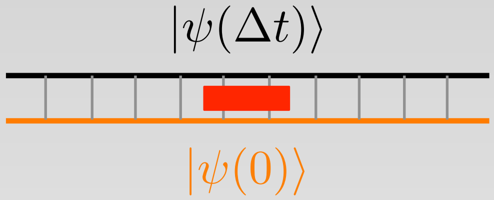
Conclusions and open questions

Membrane theory from random circuits

The same description of entanglement dynamics arises in CMT.

- Random quantum circuit model for the evolving wave function.



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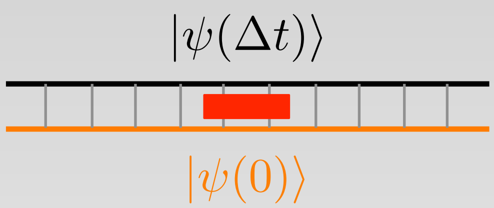

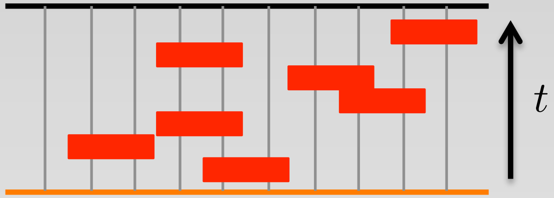

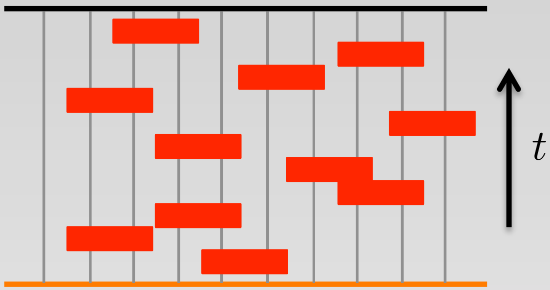

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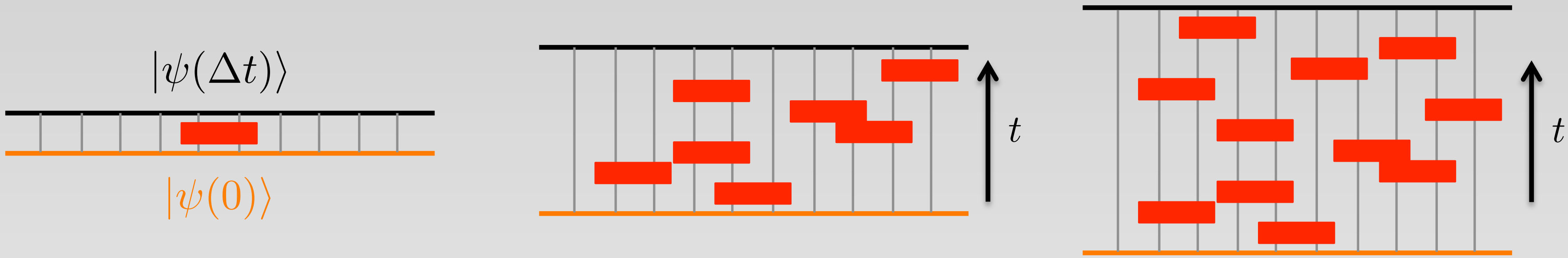
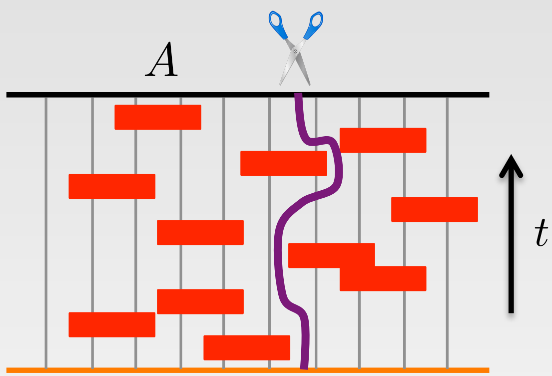
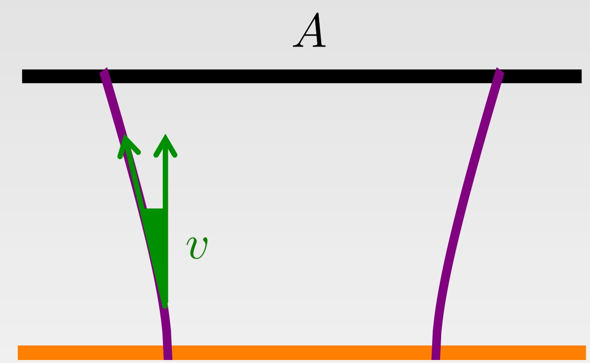


- Minimal cut computes the entropy. [Nahum, Ruhman, Vijay, Haah]



Coarse graining

$$S = s_{\text{th}} \int_0^t dt' \;\; \mathcal{E}(v)$$



Minimal membrane phenomenology of entropy dynamics. [Jonay, Huse, Nahum]

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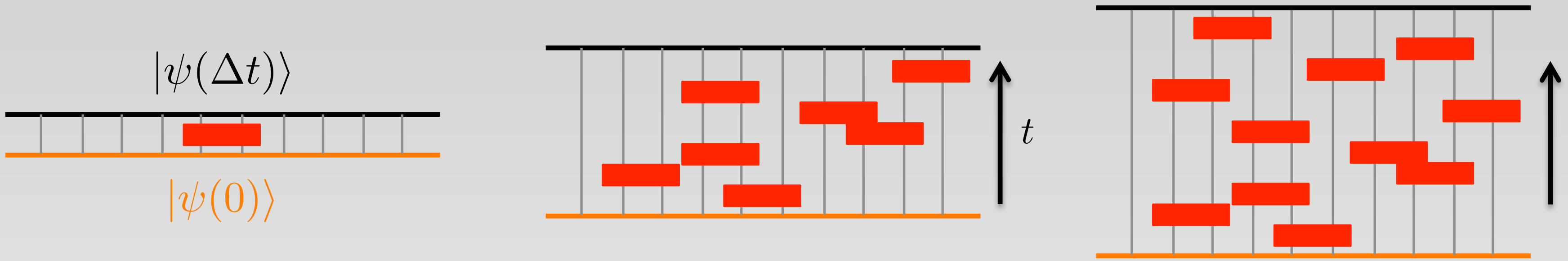

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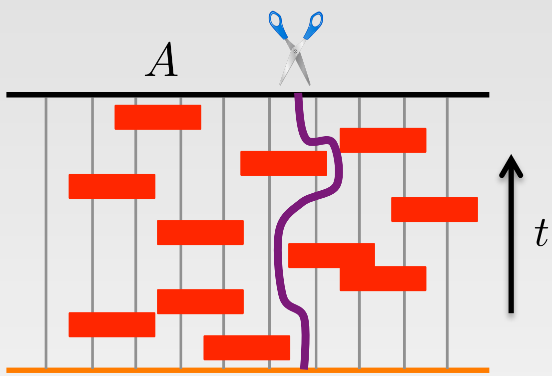

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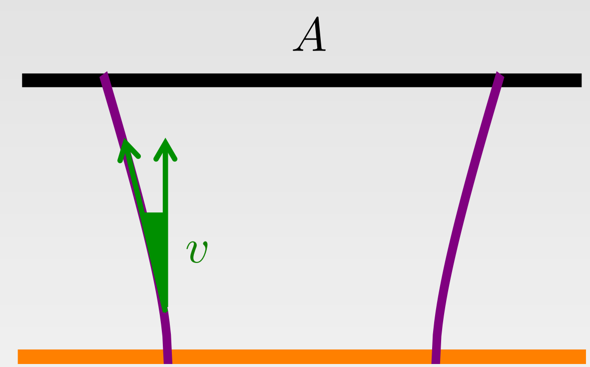

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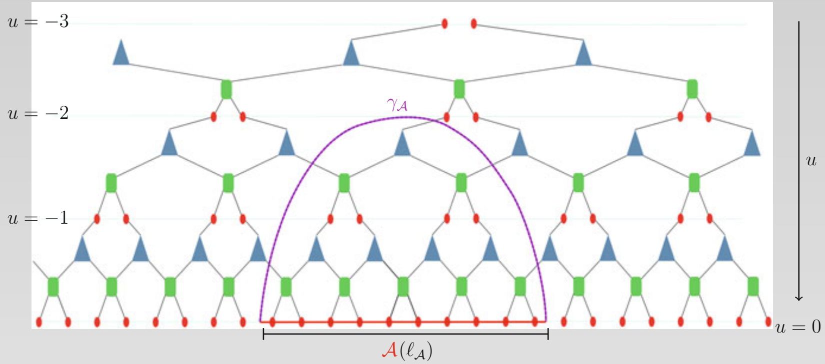
- Analytic arguments in Floquet systems. [Nahum, Zhou] Evidence in chaotic spin chains. [Jonay, Huse, Nahum]
- Remarkable unification of CMT and HEP approaches: **Membrane description of EE growth in quenches.**

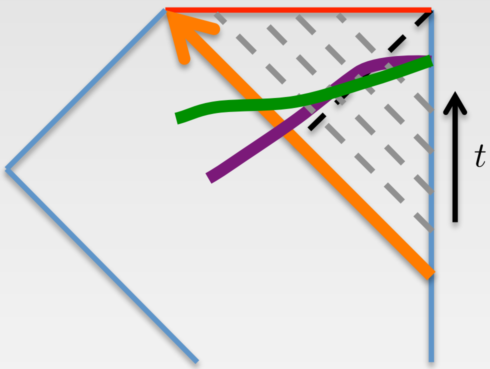
Tensor networks and holography

The analogy between minimal cuts and the RT surface computing entanglement entropy has inspired toy models of holography.

- AdS/MERA analogy, [Swingle] perfect and random tensor networks [Pastawski et al; Hayden et al.]



- Suggestive results for maximal volume slice. [Hartman, Maldacena; Roberts, Stanford, Susskind] But HRT surfaces for different shapes do not lie on same Cauchy slice.



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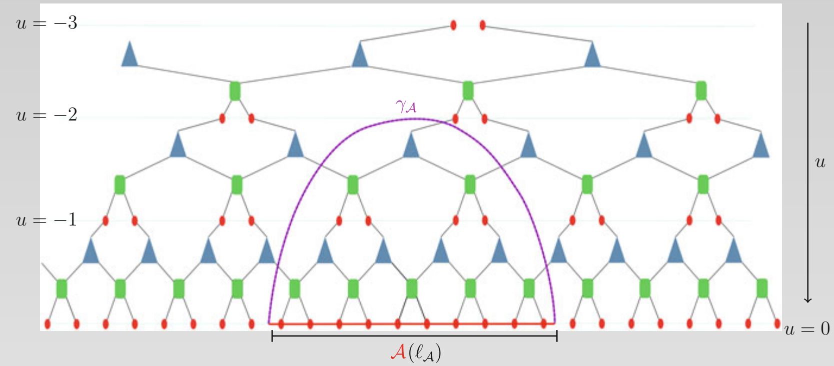

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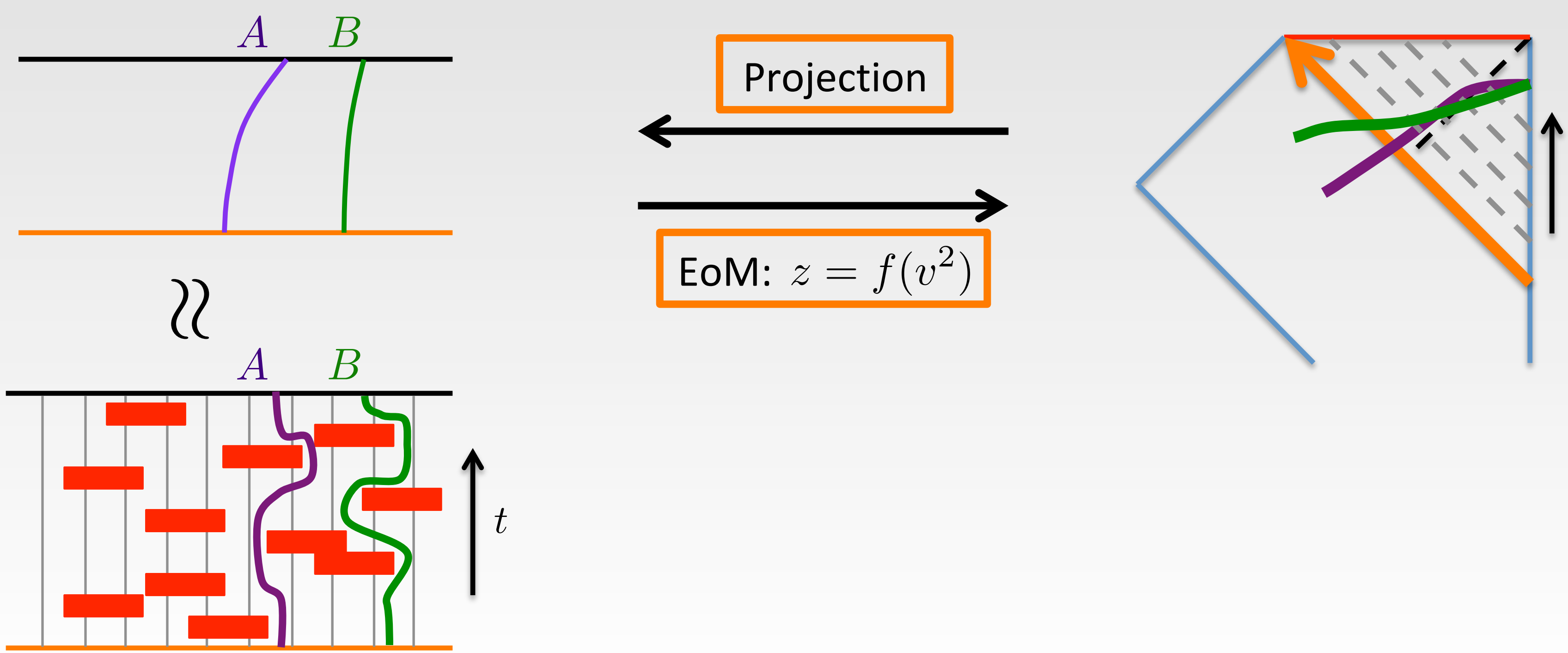

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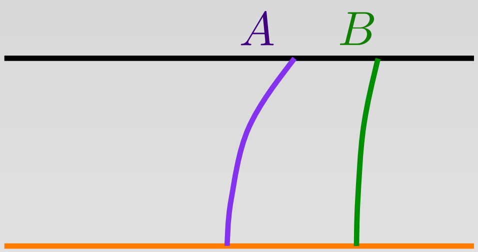
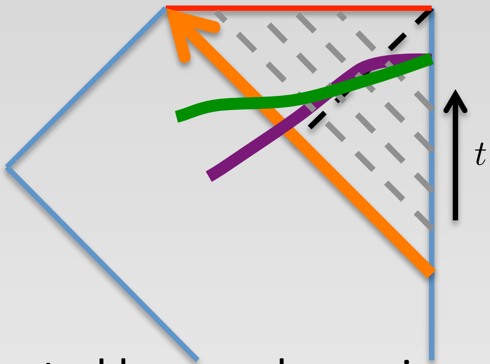
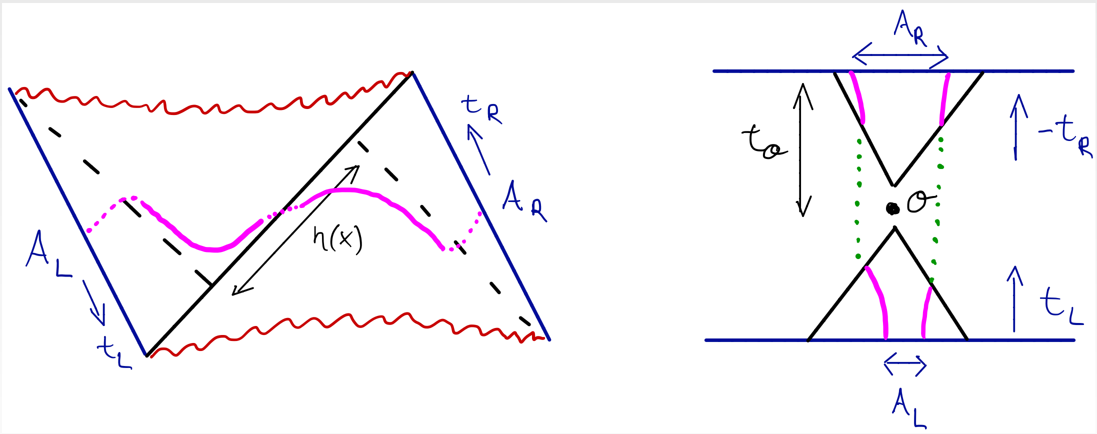


Projection

EoM: $z = f(v^2)$



- Entanglement of local operator with growing footprint is computed by membrane in time fold geometry. [Roberts, Stanford, Susskind; Jonay, Huse, Nahum; MM, Virrueta]



- Quantitative connection to TNs through EoM, bulk geometry encoded in $\mathcal{E}(v)$.

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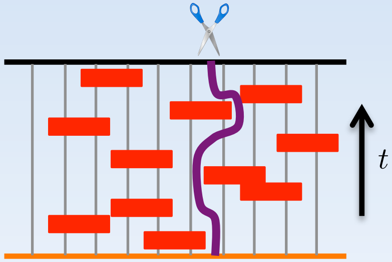

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Conclusions and open questions

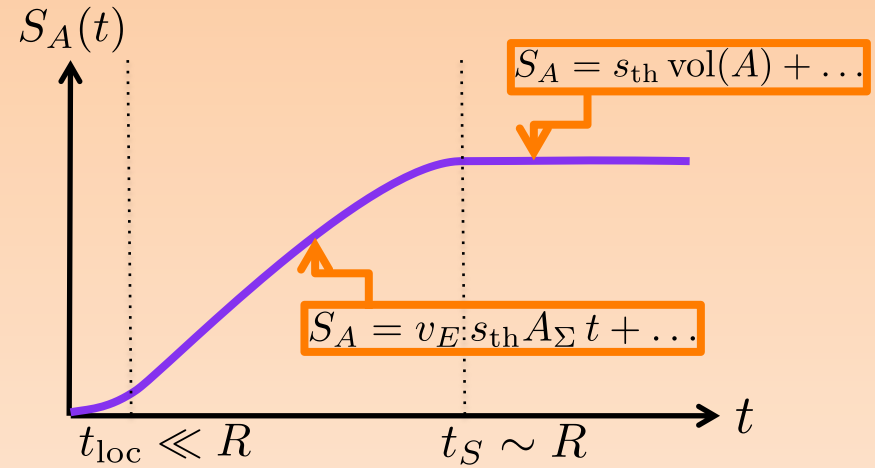
Universality classes of entropy dynamics

I propose that there are two universality classes of entropy dynamics at long distances and late times (in translationally invariant systems).

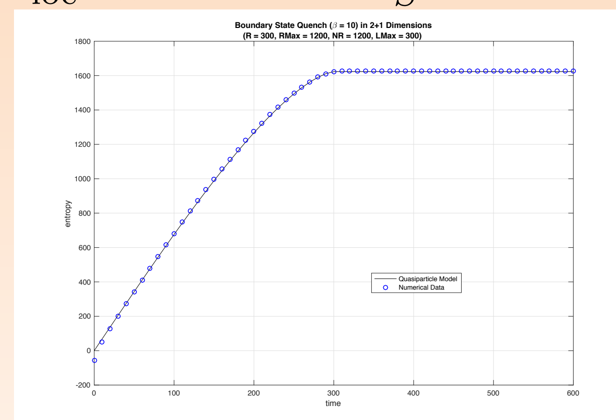
- 2d integrable models, RCFTs, d>2 free theories are described by the **quasiparticle theory**.
- The holographic results can be reformulated in terms of a **membrane theory**, which then can be adopted to any chaotic system. Applies to holographic theories, random circuits, evidence for chaotic spin chains. [Jonay et al., MM_2]
- Is there something in between?
- Analogous to the dichotomy between generalized hydrodynamics applicable to integrable systems (giving ballistic transport) and hydrodynamics (describing diffusive transport).

Entropy in the hydrodynamic limit

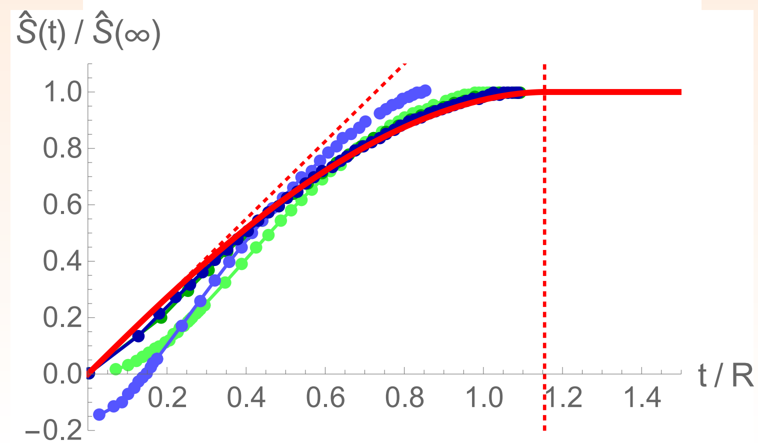
- Qualitative picture of entanglement entropy at time t of a region of characteristic size R, $R, t \gg t_{\text{loc}}$. [Cardy, Calabrese; Hartman, Maldacena; Liu, Suh]



- EE in free scalar theory for a disk, dots are data points, line is quasiparticle theory [Cotler, Hertzberg, MM, Mueller]



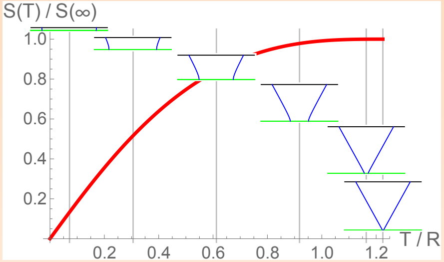
- EE in holographic theories for a disk, data collapse, solid line is membrane theory, deviation is controlled by 1/R [MM_1]

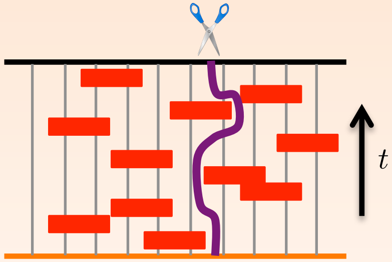


Summary

Phenomena associated with chaotic dynamics:

- Hydrodynamics is the EFT for transport, serves as target
- Universality classes of thermalization:
  Quasiparticle theory vs Membrane theory
- Derived the membrane theory of entanglement dynamics from holography. Evidence for universality from CMT





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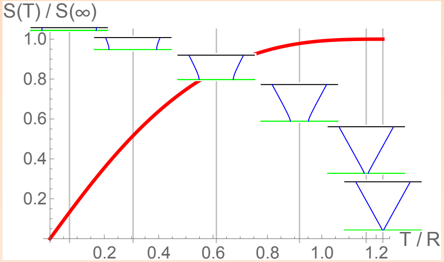

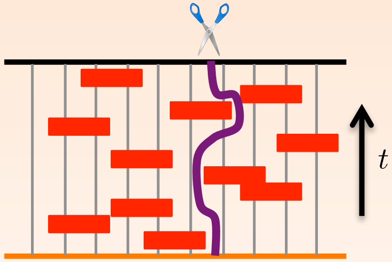

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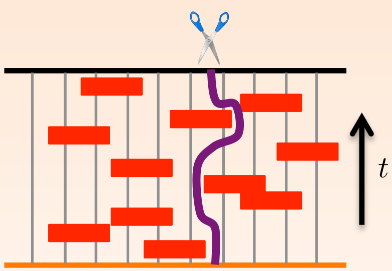

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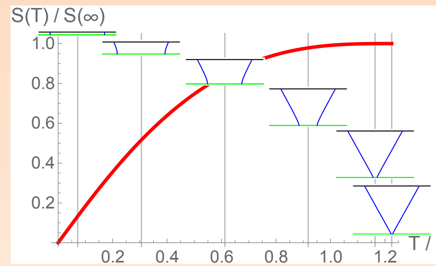

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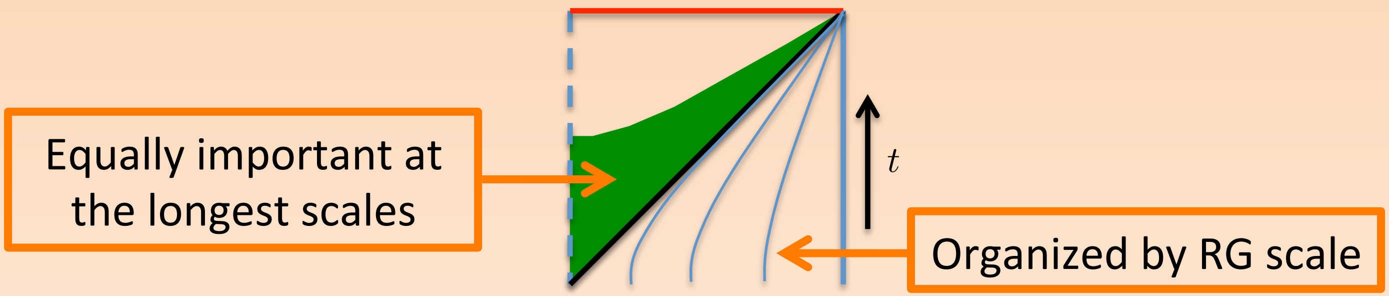
- Rich applications
  - ➢ Entropy cone inequalities generalized to time dependent settings. [Hayden, Headrick, Maloney; Bao et al.; Bao, MM]
  - ➢ Bit threads reformulation. [Freedman, Headrick; Agon, MM]
- **Membrane theory has all the features to be a universal theory.**

Open questions and outlook for gravity

Open questions and some hints

- What does the membrane theory imply for holographic RG?
  Hint: The metric inside the horizon does not seems to be organized by scale.



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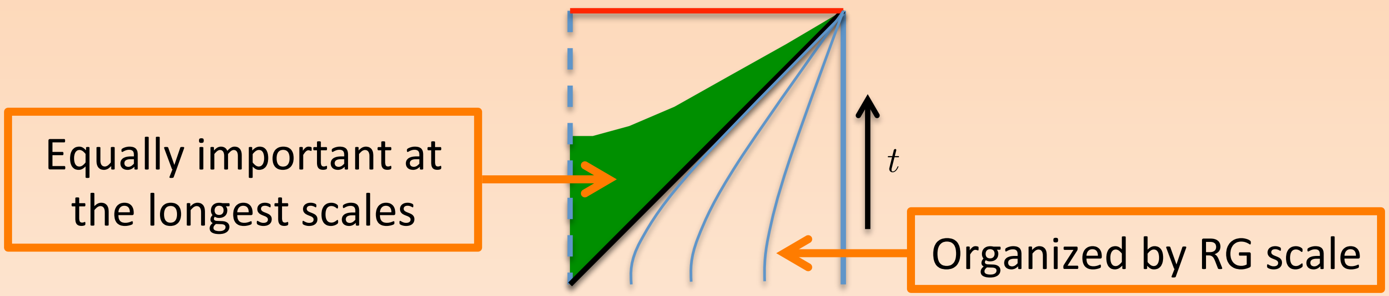

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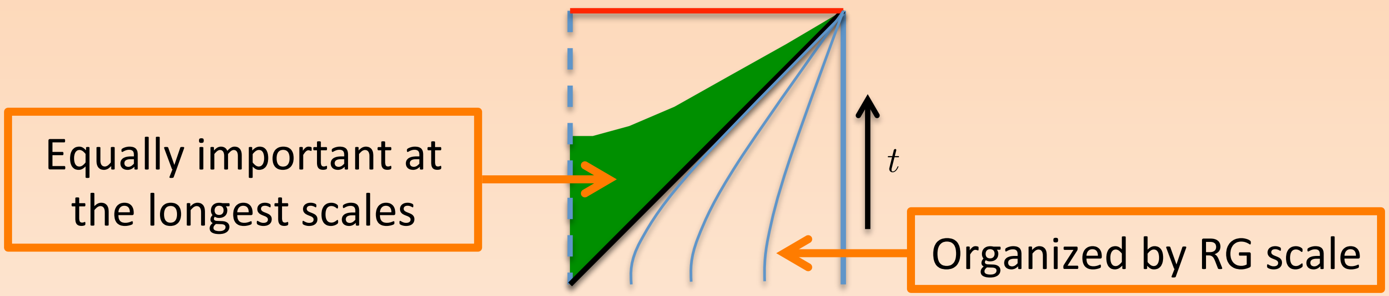

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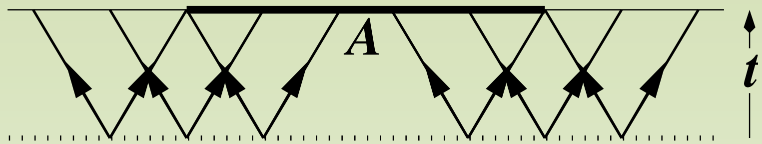
- Is the membrane theory a good starting point to getting gravitational dynamics out of entanglement?
  Hint: Slogan: "Gravity is the hydrodynamics of entanglement."May have to go to shorter times and distances in CFT to see dynamical geometry.

Backup slides

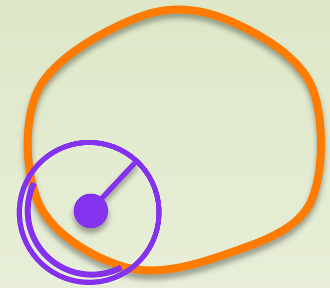
Quasiparticle model

Calabrese-Cardy model: energy injection from quench creates a finite density of EPR pairs, subsequently travel freely at the speed of light isotropically.

- Leads to linear growth with $v_E = 1$ in 2d.



- Higher dimensions: entanglement spreading depends on entanglement pattern on the light cone $\mu[L_\Sigma]$. Contribution from each light cone has to be added. [Casini, Liu, MM]



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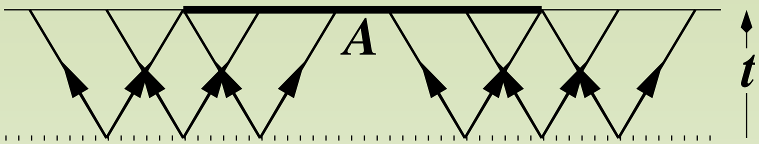

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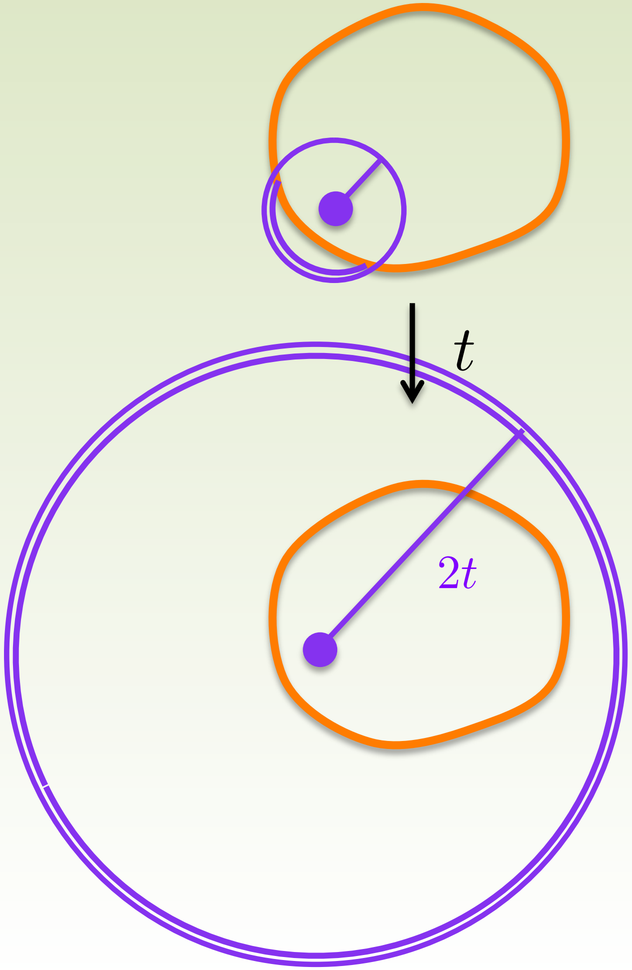

Quasiparticle model

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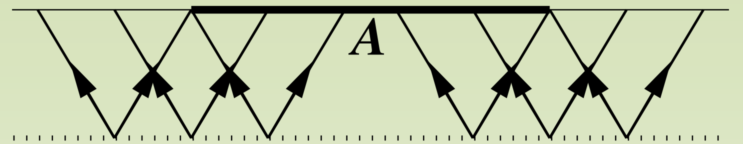

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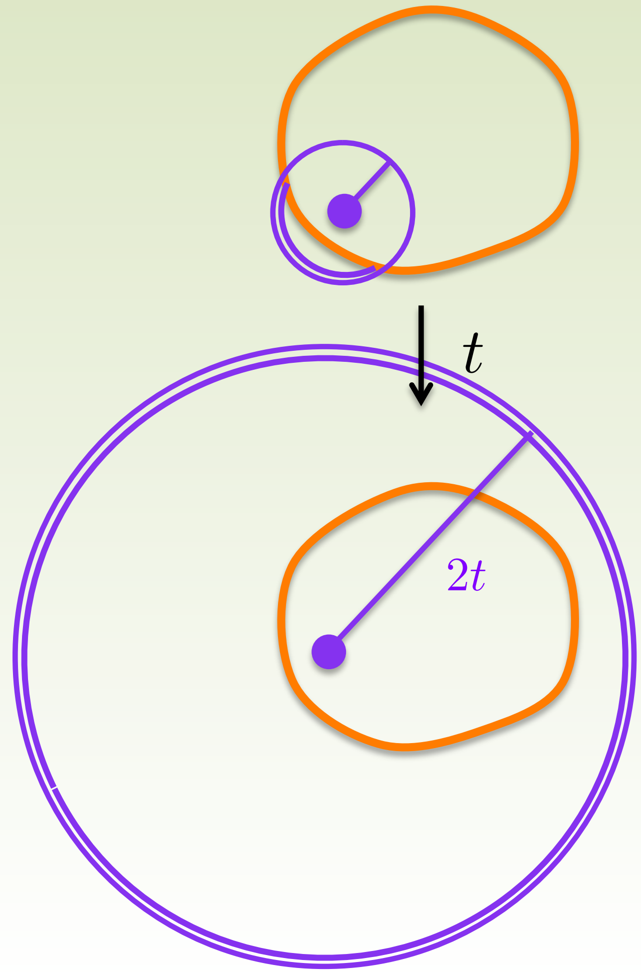

Bound on the entanglement speed from SSA:

$$v_E \leq v_E^{(\mathrm{EPR})} = \frac{\Gamma(\frac{d-1}{2})}{\sqrt{\pi}\,\Gamma(\frac{d}{2})} < v_E^{(\mathrm{SBH})}$$

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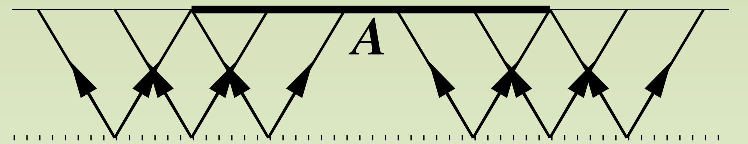

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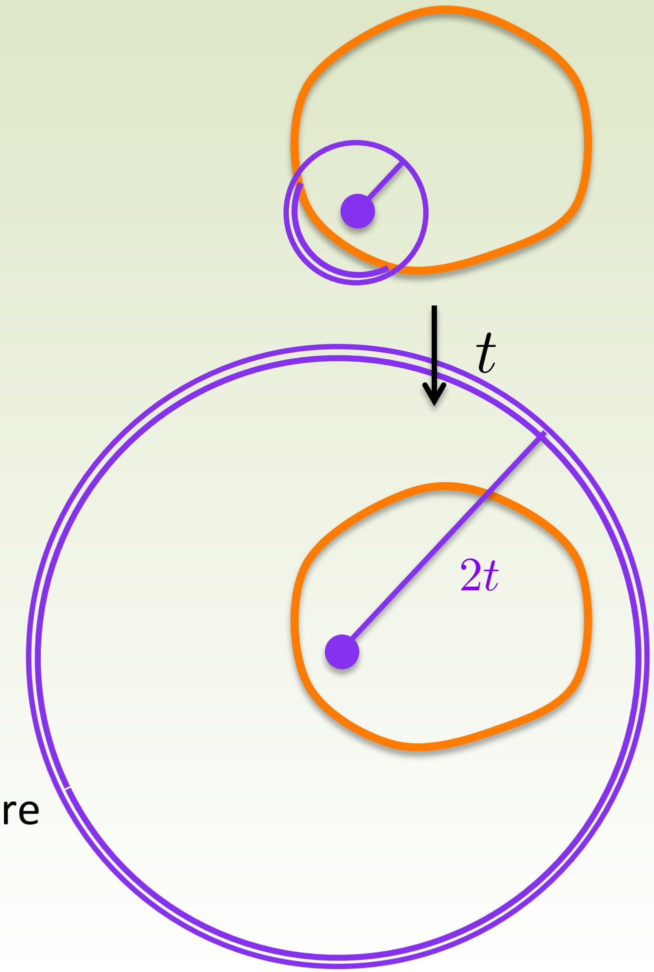

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- In strongly coupled systems, entanglement grows faster than what's possible for free particles streaming at the speed of light!
- Consider the effect of interactions: tensor network picture emerging from scattering particles is natural. [Hartman, Maldacena; Casini, Liu, MM]

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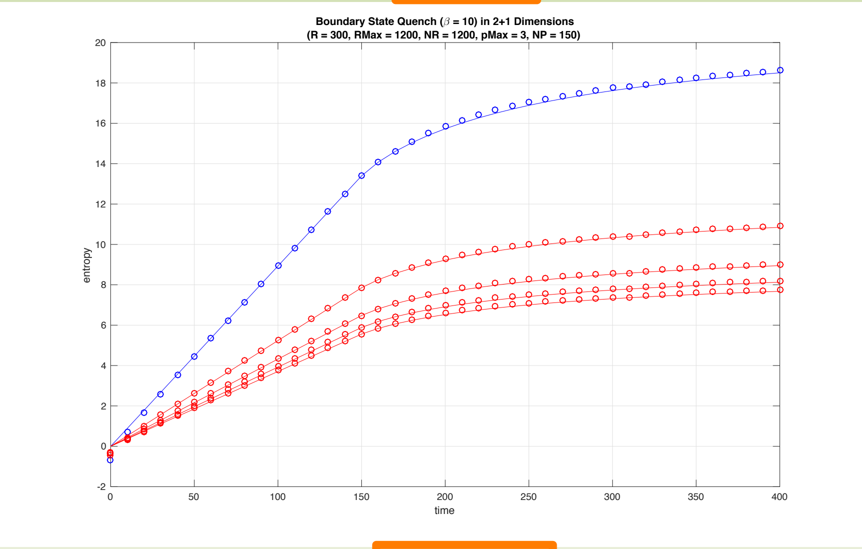
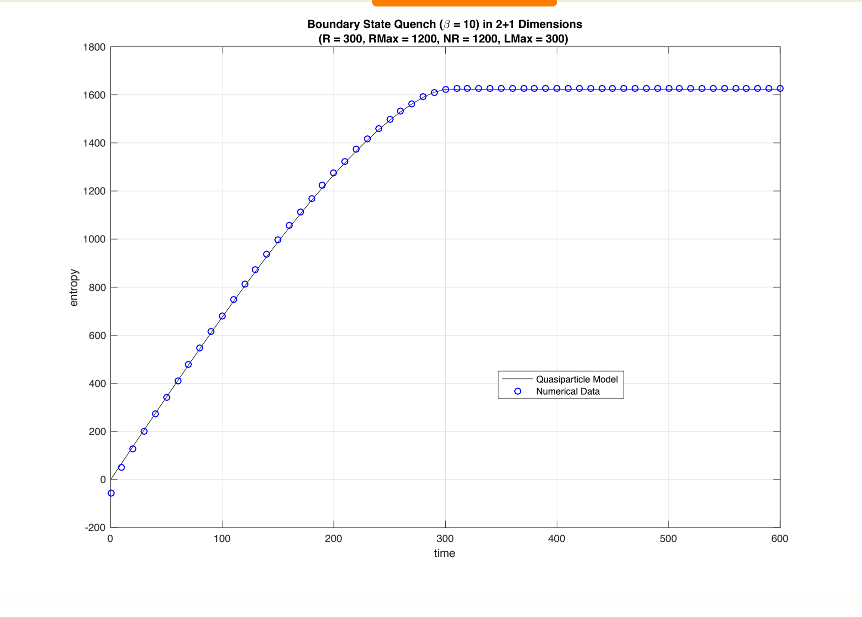
- Numerical results for 3d boundary state quench for scalar field. [Cotler, Hertzberg, MM, Mueller]

Strip



Sphere

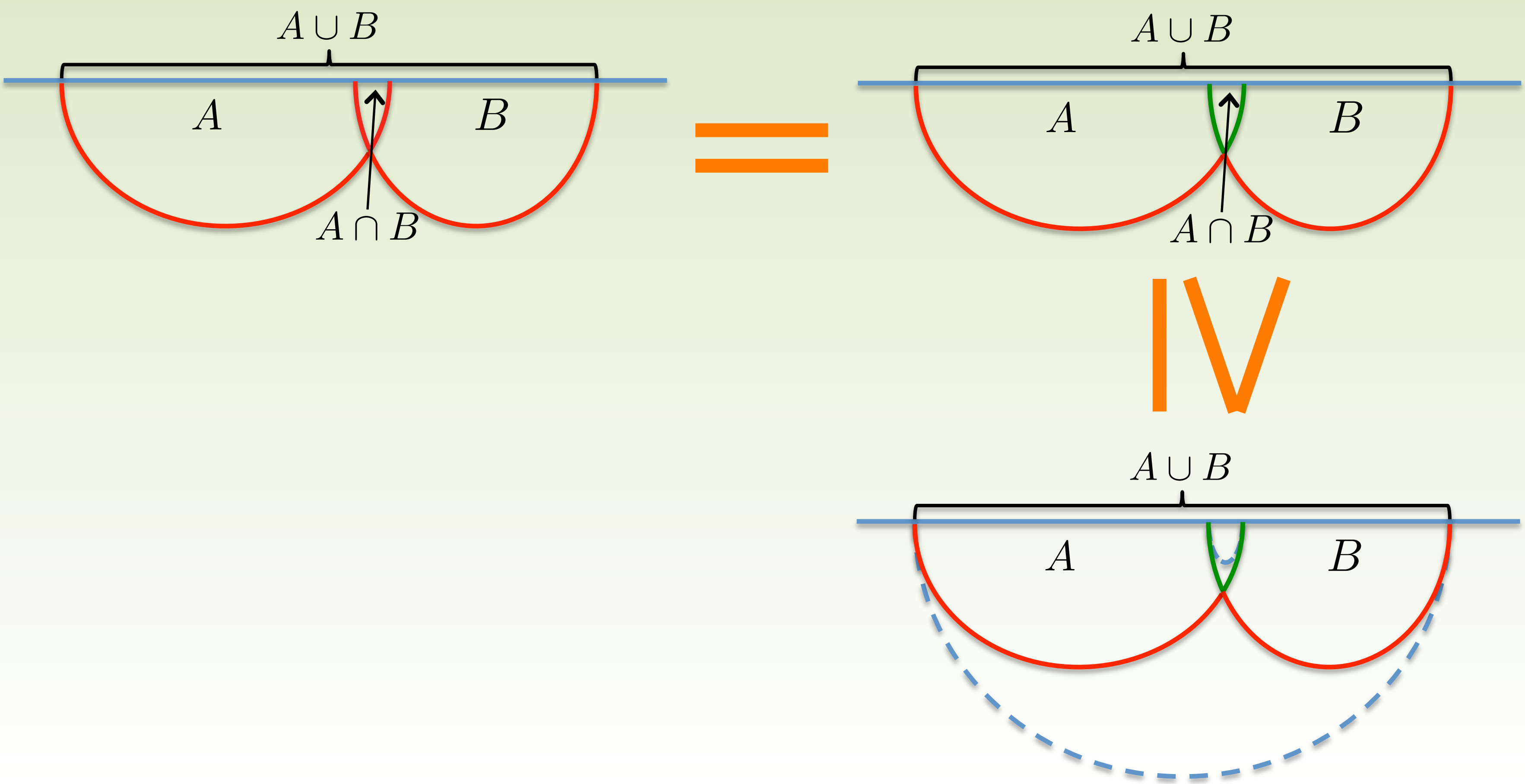


Entropy cone

Entanglement entropy in static holographic states obeys inequalities, that are not true in general in QM.

- The best known one is the monogamy of mutual information. [Hayden, Headrick, Maloney] It can be proven using the same steps as in the proof of SSA.

$$S(AB) + S(BC) + S(AC) \geq S(A) + S(B) + S(C) + S(ABC)$$



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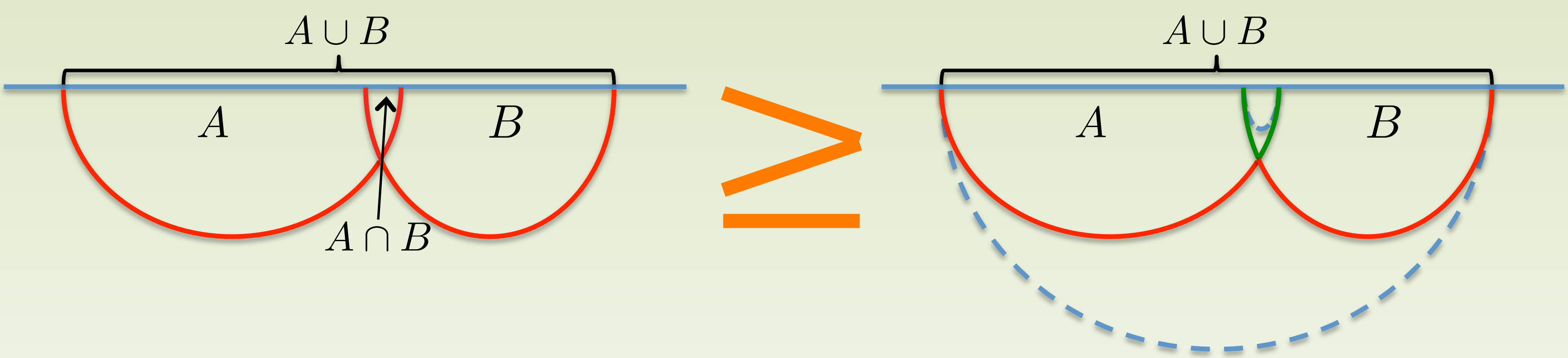

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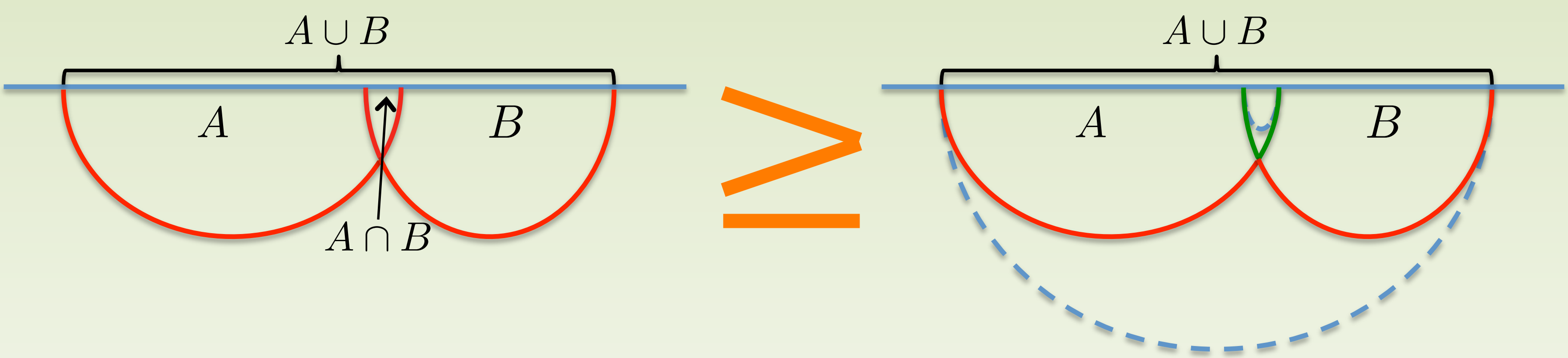

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- HRT is an extremization of codimension-2 surface, no proof (or counterexample) is known for many-party inequalities. Inclusion-exclusion applies to the membrane theory, hence proof for time dependent states (large regions, late times). [Bao, MM]

Bit threads

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- Maximize $\int_A \sqrt{h}\, n_\mu w^\mu$

  Constraints: $\nabla_\mu w^\mu = 0\,, \quad 1 - |w^\mu| \geq 0$

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- The map that reconstructs the HRT surface from the minimal membrane can be used to push the membrane theory bit thread into the bulk.
- **Membrane theory is versatile, has connections to operator growth and hydrodynamics, and has all the features to be a universal theory.**

Bounds

Entanglement entropy obeys inequalities, natural to consider bounds in the quench setup.

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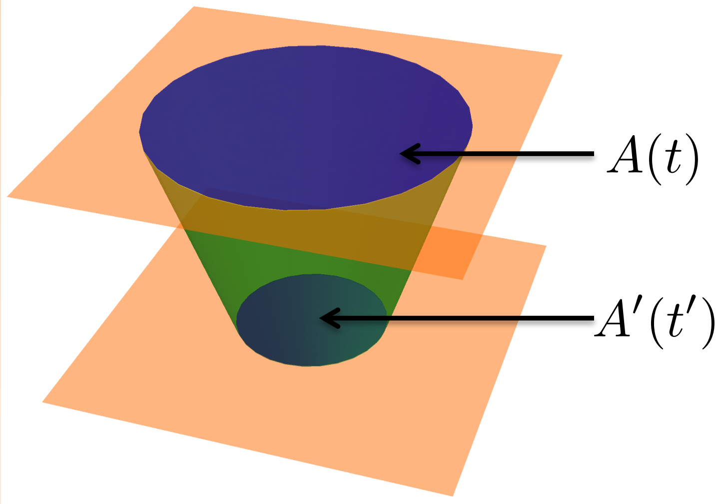
  Gives bound for all times. Can be combined with another proposed inequality. [MM, Stanford]



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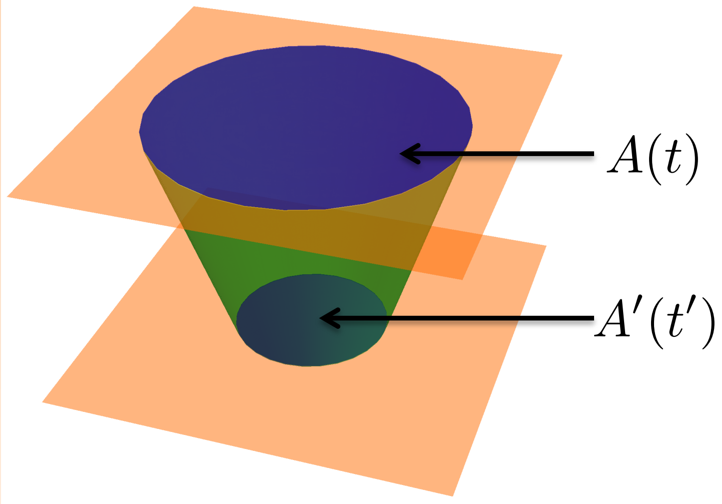

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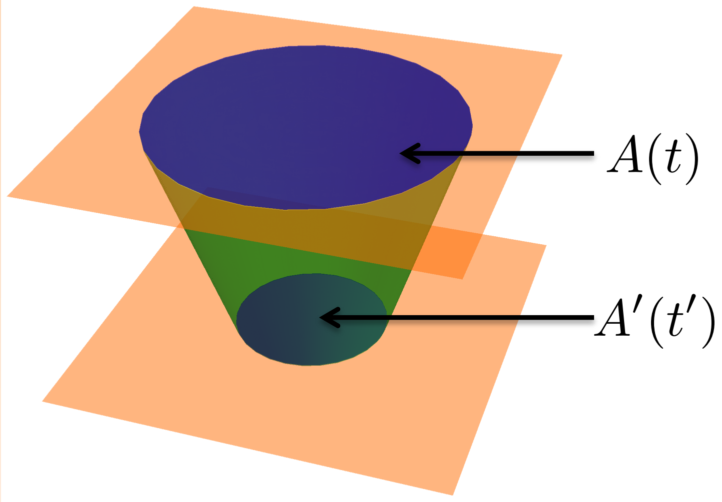

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- Membrane theory proof: there exists a maximal membrane tension compatible with the general properties discussed before.

$$\mathcal{E}_{\max}(v) = v_E + \left(1 - \frac{v_E}{v_B}\right)|v| \qquad (|v| \leq v_B)$$

The resulting minimal membrane is a combination of a cylinder and the cone saturating the combined inequalities. [MM_2]