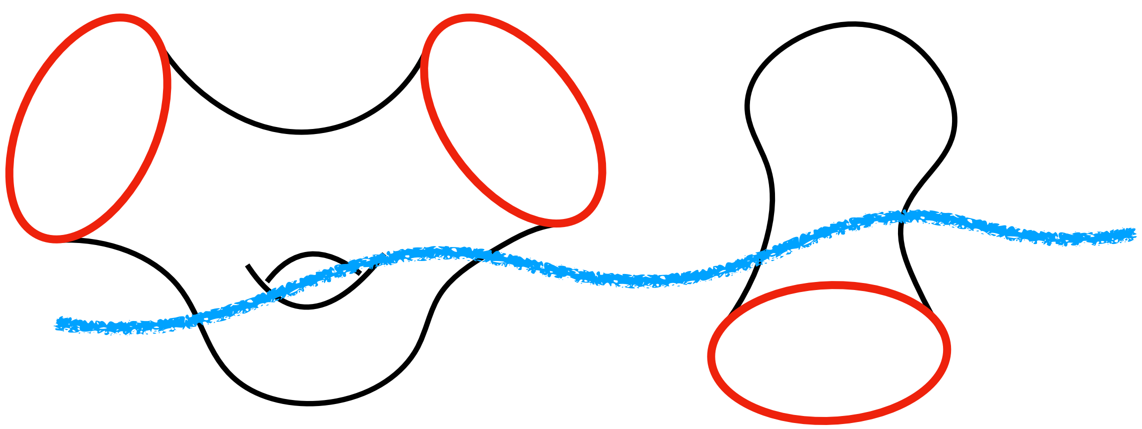
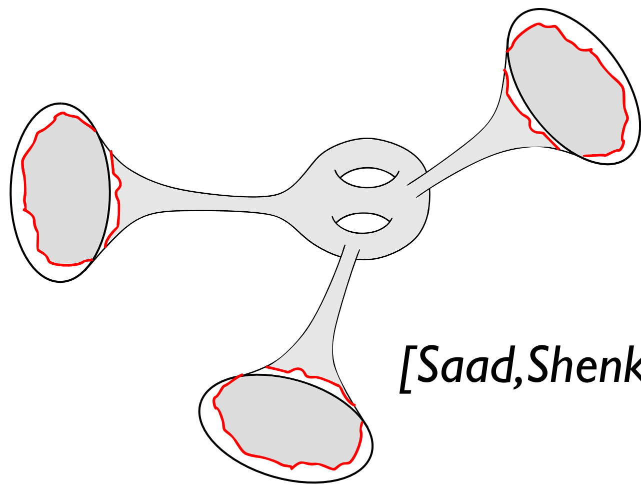
Return of the Baby Universes



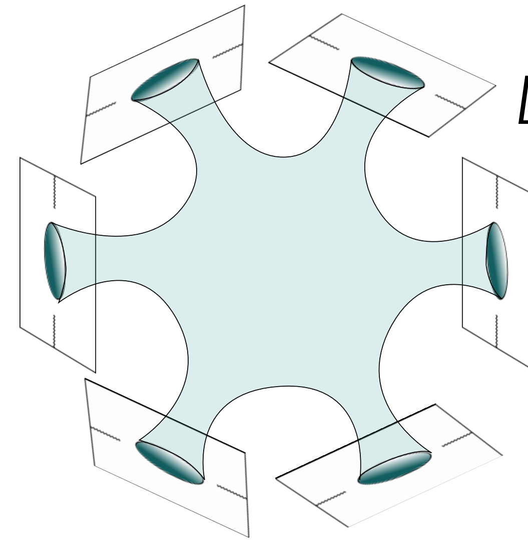
Henry Maxfield, UCSB

Work in progress with Don Marolf



[Saad,Shenker,Stanford]

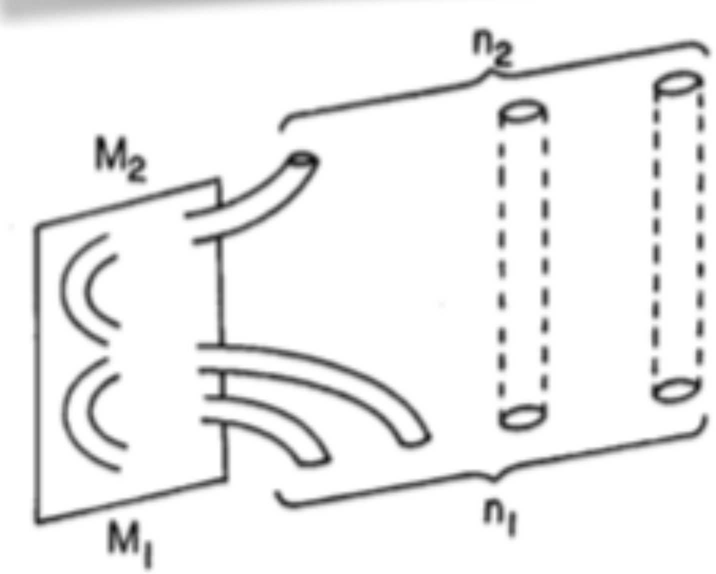
Ensembles



[Almheiri,Hartman,Maldacena, Shaghoulian,Tajdini]

Page curve: replica wormholes

Spacetime wormholes & baby universes

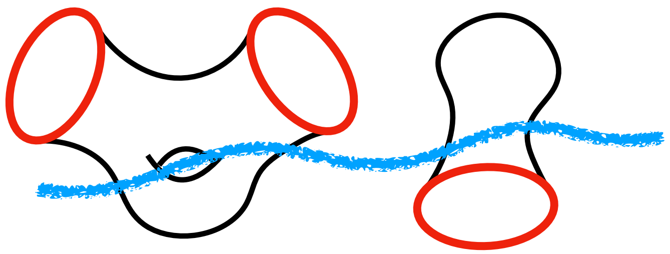


[Coleman]

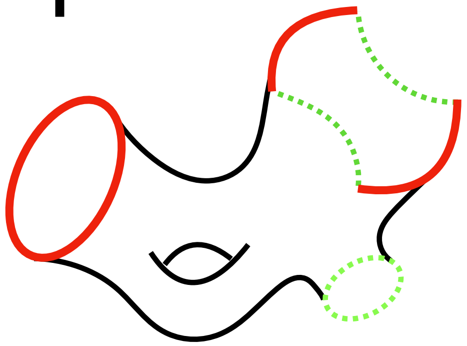
[Giddings,Strominger]

Figure 3.2. A topologically nontrivial process involving joining and splitting universes.

1. Spacetime wormholes and baby universes



2. A solvable example



3. Page curve

Spacetime wormholes and baby universes

AdS gravity with wormholes

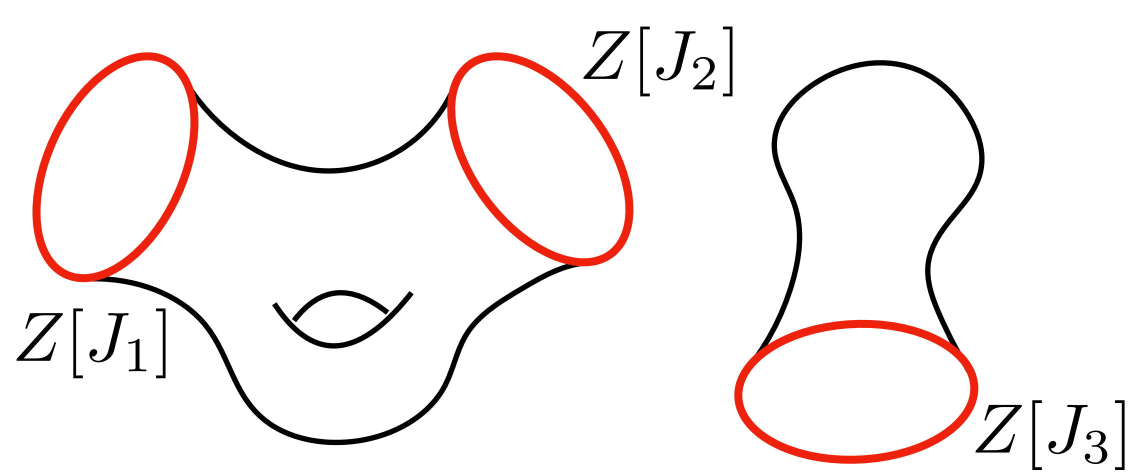
Gravitational EFT, fields Φ, action $S[\Phi]$

A set of aAdS boundary conditions: $\Phi \sim J$

$\Phi \supseteq g$, $J \supseteq g_\partial$ **connected** boundary metric

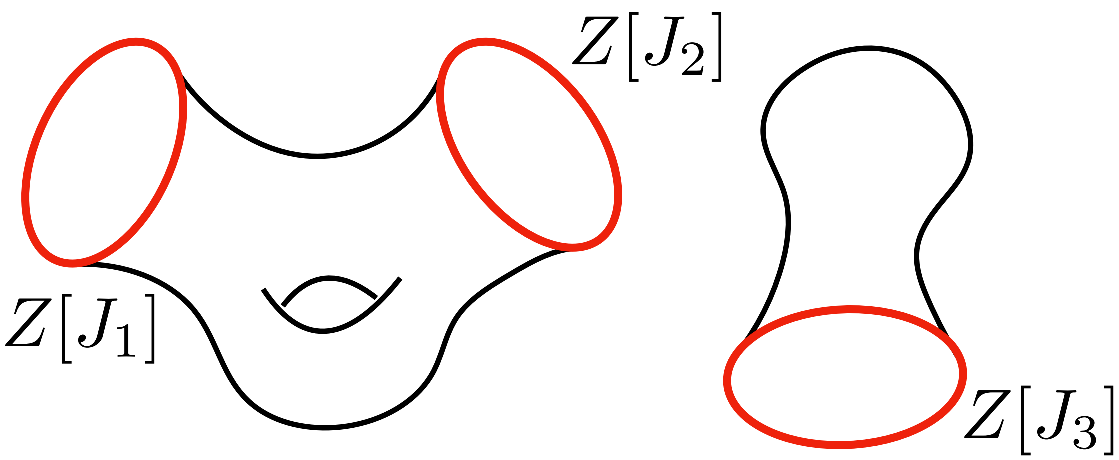
Notation for path integral:

$$\left\langle Z[J_1] \cdots Z[J_n] \right\rangle = \int_{\Phi \sim J} \mathcal{D}\Phi \, e^{-S[\Phi]}$$



A dual CFT ensemble?

$$\left\langle Z[J_1]\cdots Z[J_n]\right\rangle = \int_{\Phi\sim J} \mathcal{D}\Phi\, e^{-S[\Phi]}$$



Dual interpretation:

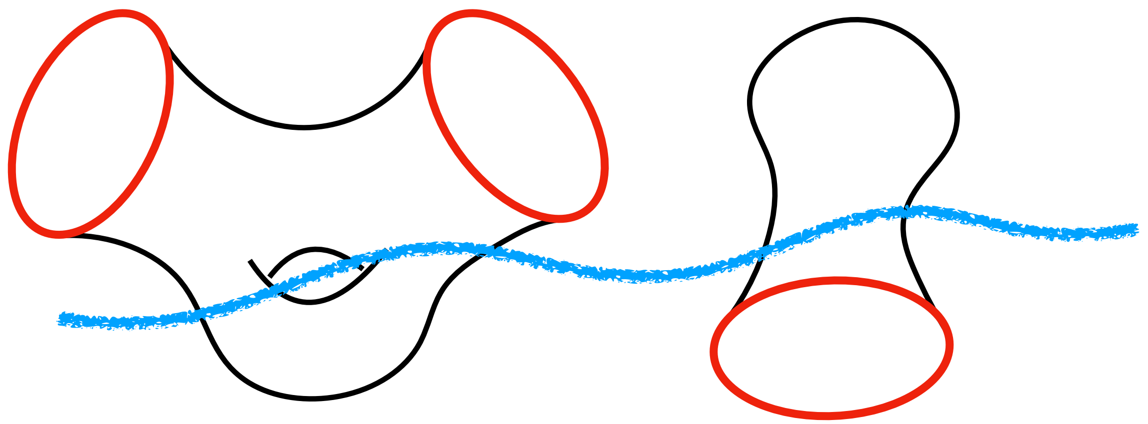
- $Z[J]$ is a CFT partition function
- $\langle \cdot \rangle$ is an average over theories

Example: JT gravity *[SaadShenkerStanford]*

3D pure gravity? *[HM,Turiaci, in progress]*

Bulk description of a single dual?

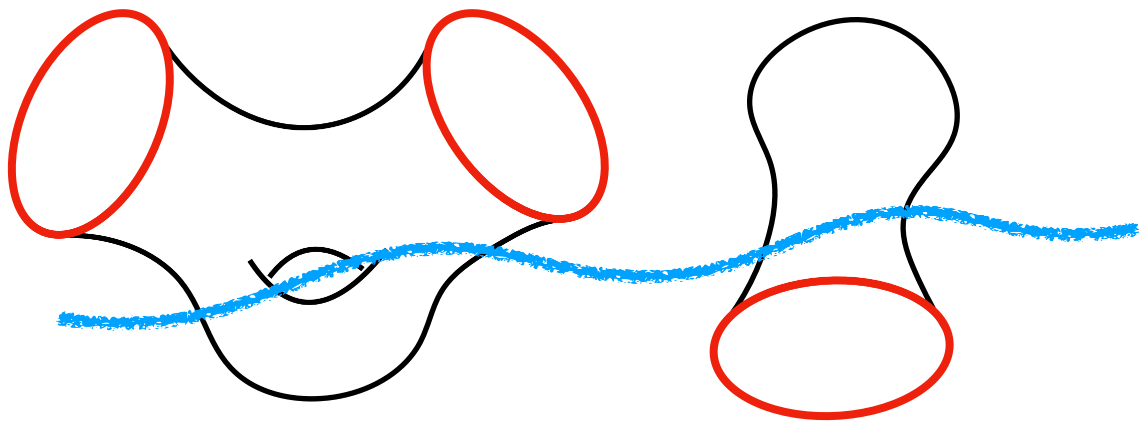
Hilbert space interpretation



Sum over intermediate states of closed "baby" universes

Challenge: diff invariance, interacting WdW equation

Hilbert space interpretation



Sum over intermediate states of closed “baby" universes

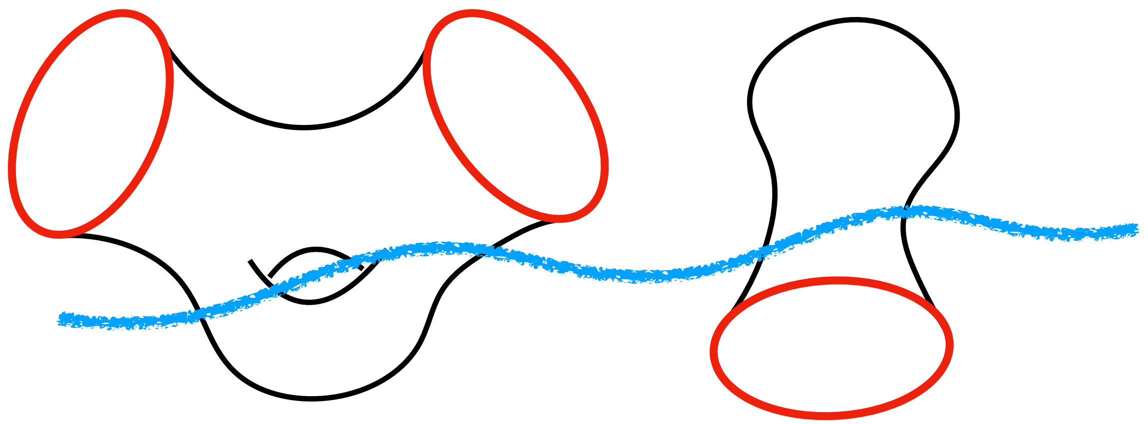
Challenge: diff invariance, interacting WdW equation

Solution: use asymptotic boundaries to define states

$$\Big| Z[J_1] \cdots Z[J_m] \Big\rangle \in \mathcal{H}_{\mathrm{BU}}$$

“Past” boundary conditions

Hilbert space interpretation



$$\left| Z[J_1] \cdots Z[J_m] \right\rangle \in \mathcal{H}_{\mathrm{BU}}$$

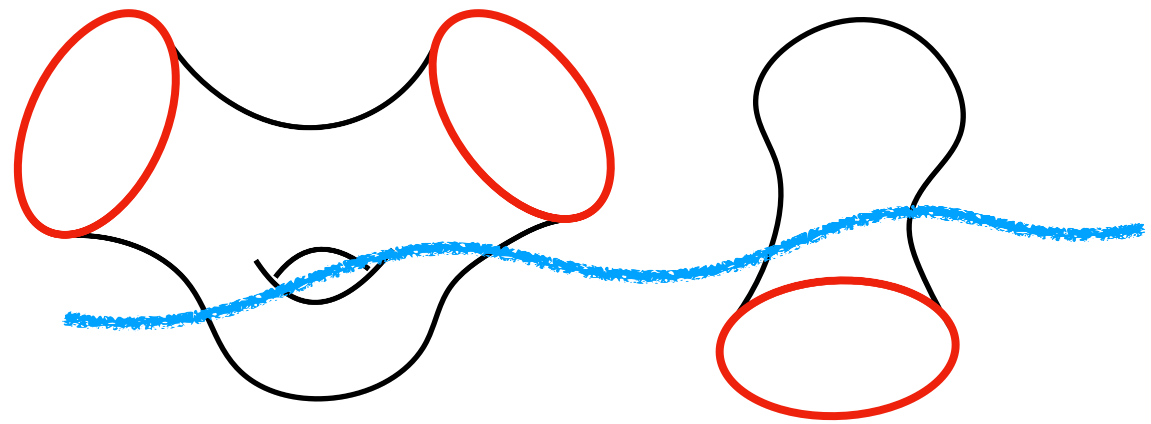
Inner product: *(* = CPT)*

$$\left\langle Z[\tilde{J}_1] \cdots Z[\tilde{J}_n] \middle| Z[J_1] \cdots Z[J_m] \right\rangle = \left\langle Z[\tilde{J}_1^*] \cdots Z[\tilde{J}_n^*] Z[J_1] \cdots Z[J_m] \right\rangle$$

Dual interpretation: inner product = covariance matrix

Assumption: reflection positivity. This IP is positive semidefinite

Hilbert space interpretation



$$\Big\langle Z[\tilde{J}_1]\cdots Z[\tilde{J}_n]\Big| Z[J_1]\cdots Z[J_m]\Big\rangle = \Big\langle Z[\tilde{J}_1^*]\cdots Z[\tilde{J}_n^*] Z[J_1]\cdots Z[J_m]\Big\rangle$$

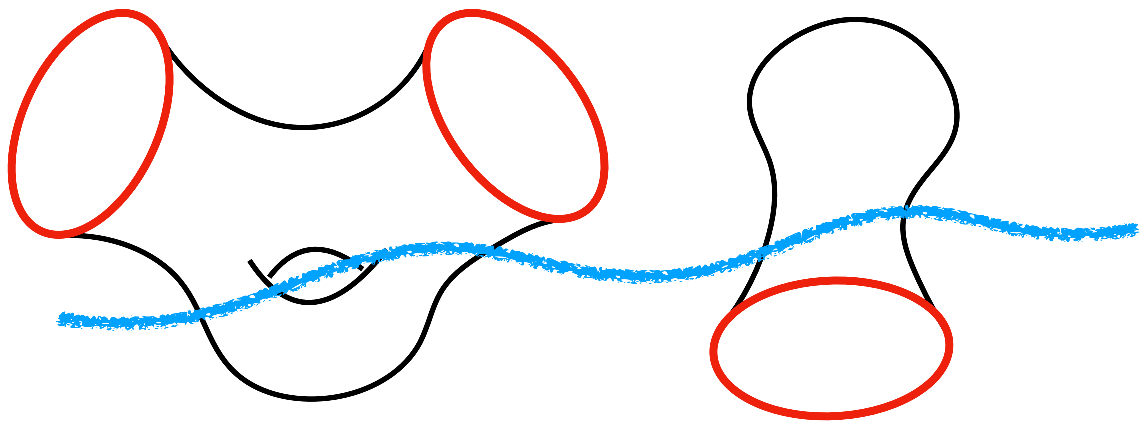
Use this to *define* the Hilbert space $\mathcal{H}_{\rm BU}$:

$\mathcal{H}_{\rm BU} =$ completion of space spanned by states $\Big| Z[J_1]\cdots Z[J_m]\Big\rangle$

C.f. Osterwalder-Schrader construction in QFT

AdS is useful: initial states "expand" in Euclidean time to aAdS boundary

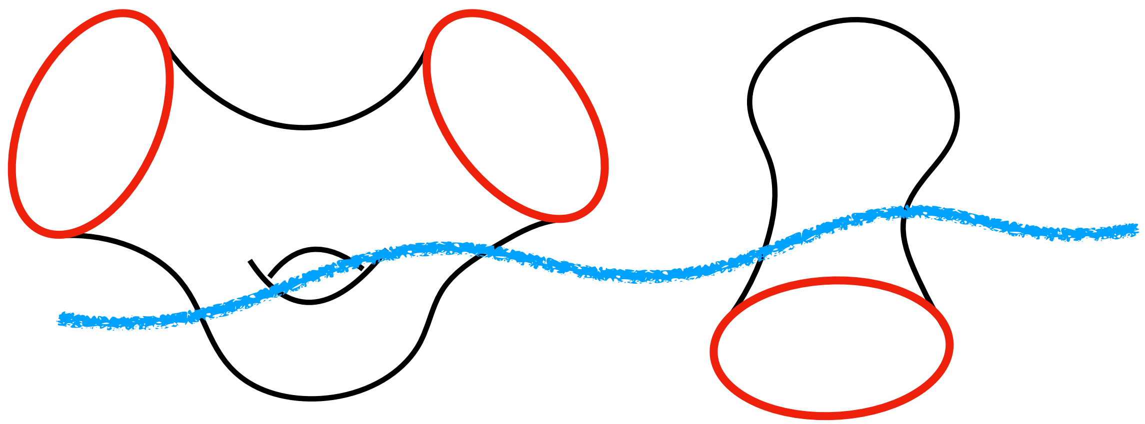
Hilbert space interpretation



$$\mathcal{H}_{\rm BU} = \text{completion of formal polynomials in } \Big| Z[J_1] \cdots Z[J_m] \Big\rangle$$

Projects out null states:
nonperturbative effect of gauging diffs

Hilbert space interpretation



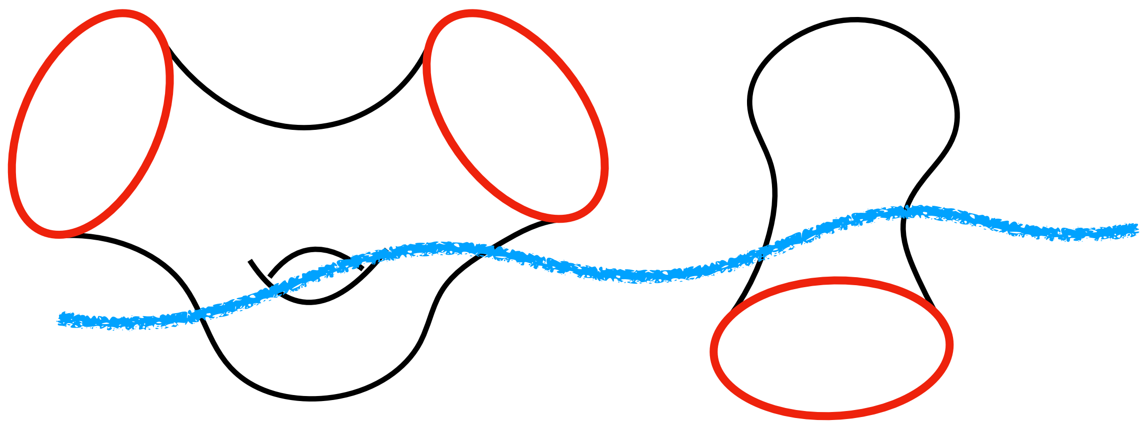
$\mathcal{H}_{\mathrm{BU}} =$ completion of formal polynomials in $\left| Z[J_1] \cdots Z[J_m] \right\rangle$

Projects out null states:
nonperturbative effect of gauging diffs

Perturbative Hilbert space truncated by splitting/joining universes

Example: $$|\mathrm{ER}\rangle_{LR} = \sum_i e^{-\beta E_i/2} |E_i\rangle_L \, |E_i\rangle_R$$

Hilbert space interpretation



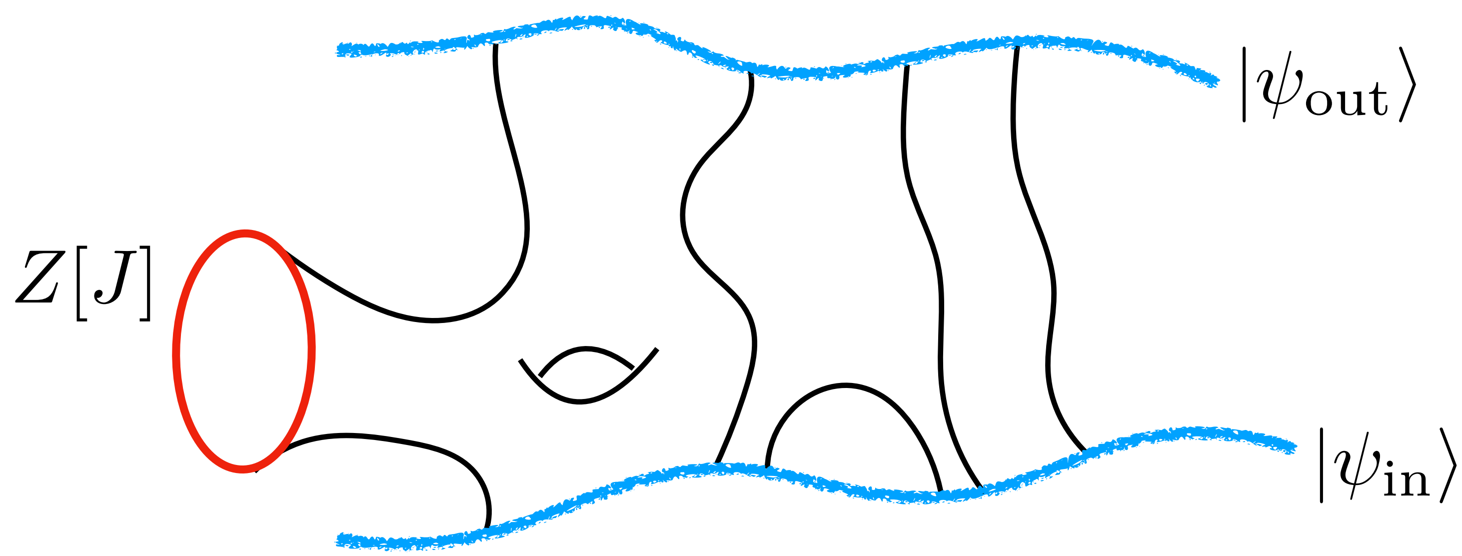
$\mathcal{H}_{\mathrm{BU}} =$ completion of formal polynomials in $\left|Z[J_1]\cdots Z[J_m]\right\rangle$

Projects out null states:
nonperturbative effect of gauging diffs

Similar discussion of bulk observables: ***[Jafferis]***

CFT objects as operators

Any aAdS boundary is an operator,
defined by insertion in the path integral:



$$\widehat{Z[J]}\Big|Z[J_1]\cdots Z[J_m]\Big\rangle = \Big|Z[J]Z[J_1]\cdots Z[J_m]\Big\rangle$$

CFT objects as operators

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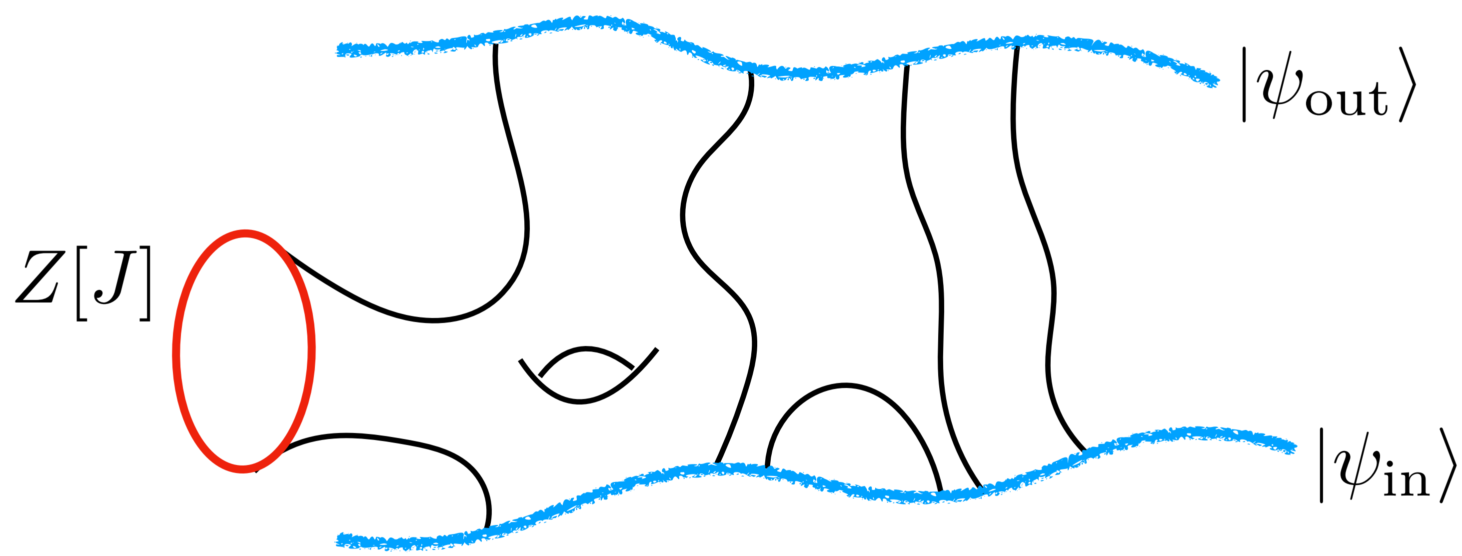
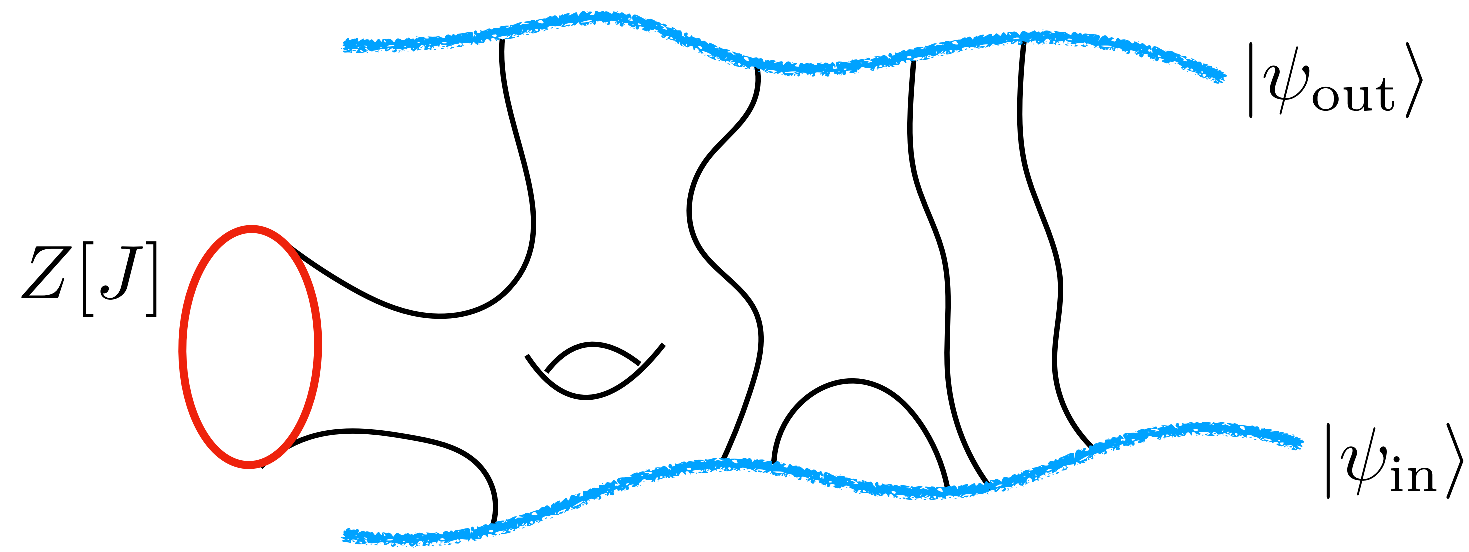


$$\widehat{Z[J]}\Big|Z[J_1]\cdots Z[J_m]\Big\rangle = \Big|Z[J]Z[J_1]\cdots Z[J_m]\Big\rangle$$

Commuting: $\left[\widehat{Z[J]}, \widehat{Z[J']}\right] = 0$

CFT objects as operators



Simultaneous eigenstates of all $\widehat{Z[J]}$**:**

$$\widehat{Z[J]}|\alpha\rangle = Z_\alpha[J]|\alpha\rangle \quad \forall J$$

States $|\alpha\rangle$ **are:**

- **Unique for given eigenvalues**
- **Mutually orthogonal**
- **Complete in** $\mathcal{H}_{\rm BU}$
- **Overlapping with no-boundary state** $\langle {\rm HH}|\alpha\rangle \neq 0$

BU states as dual CFTs

Orthonormal basis $|\alpha\rangle$ **for** $\mathcal{H}_{\rm BU}$ $\longleftrightarrow$ **CFTs** $\mathcal{C}_\alpha$ **in the ensemble**

Ensemble classified by spectrum of $\widehat{Z[J]}$

Probability of each theory is $p_\alpha = |\langle \mathrm{HH}|\alpha\rangle|^2$

$$\Big\langle Z[J_1]\cdots Z[J_n]\Big\rangle = \sum_\alpha p_\alpha Z_\alpha[J_1]\cdots Z_\alpha[J_n]$$

Classical ensemble interpretation guaranteed

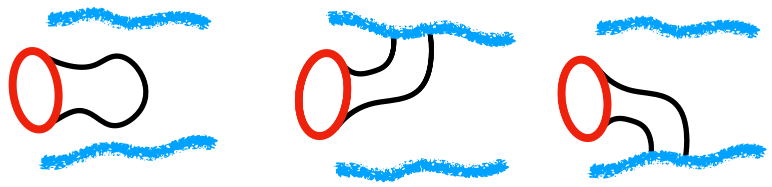
EFT description of each member of the ensemble

Failure of free approximation

Approximation: small amplitude for universes to split/join

$\mathcal{H}_{\rm BU} =$ Fock space of single universe states

$$\widehat{Z[J]} = \langle Z[J] \rangle + a_J^\dagger + a_{J^*} + \cdots$$



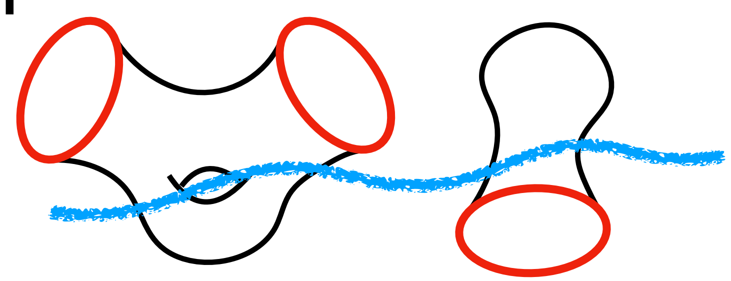
"Position" operator: continuous spectrum?

This is a bad approximation!

Fails for large "universe number"

Summary

- Construct baby universe Hilbert space from asymptotic boundaries
- Boundaries give commuting operators $\widehat{Z[J]}$



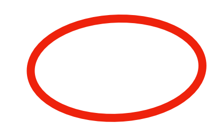
- Diagonalise: basis $|\alpha\rangle$ corresponding to members of dual ensemble
- Nonperturbative splitting/joining processes truncate Hilbert space

A solvable example

A topological theory of gravity

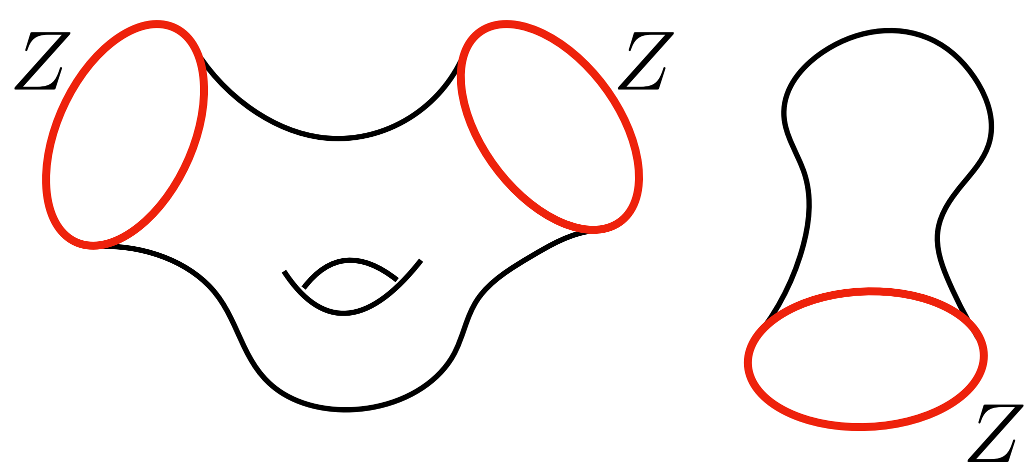
A topological theory of 2D gravity (no metric!):

$$\int \mathcal{D}\Phi\, e^{-S[\Phi]} = \sum_{\substack{\text{Topological} \\ \text{surfaces}}} e^{\frac{S_0}{2}\chi}$$

Boundary conditions: circular boundaries $Z =$ 

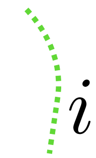
Dual interpretation: $Z = \mathrm{Tr}_{\mathcal{H}_{\mathrm{CFT}}} 1 = \dim \mathcal{H}_{\mathrm{CFT}}$

Fudge: **do not count contributions to** χ **from boundaries:** $\chi \to 2 - 2g$

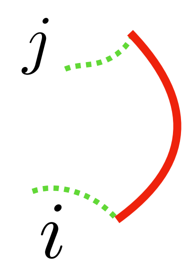
$\langle Z^3 \rangle \supseteq$ 

$= 1 \times e^{S_0}$

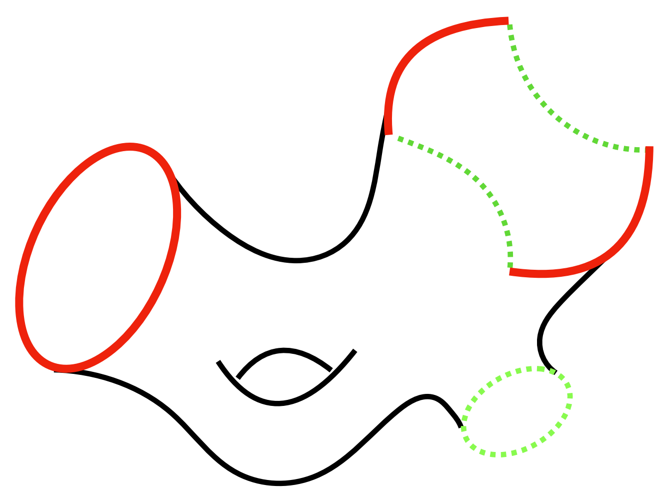
End-of-the-world branes

Include dynamical boundaries (EOW brane)  $i = 1, 2, \ldots, k$

Extra boundary condition: intervals between EWBs

$$(\psi_j, \psi_i) =$$ 

Dual interpretation: overlap of EOW states

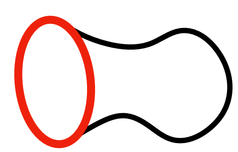
$$\left\langle Z(\psi_{j_1}, \psi_{i_1})(\psi_{j_2}, \psi_{i_2}) \right\rangle \supset$$  $$= k\delta_{i_1 j_2}\delta_{i_2 j_1}$$

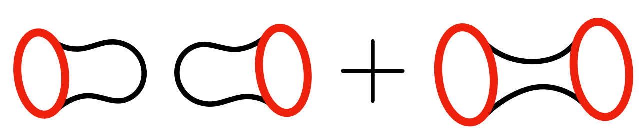
Solving for amplitudes

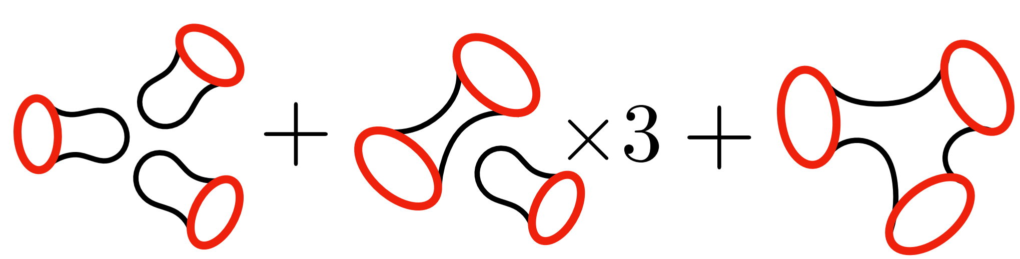
$$\sum_{\substack{\text{Connected surfaces}\\ \text{Fixed boundaries}}} e^{\frac{S_0}{2}\chi} = \lambda$$

Classical limit: $\lambda \gg 1$

No boundaries: $\mathfrak{Z} = \langle 1 \rangle = e^{\lambda}$

$\mathfrak{Z}^{-1}\langle Z \rangle =$  $= \lambda$

$\mathfrak{Z}^{-1}\langle Z^2 \rangle =$  $= \lambda^2 + \lambda$

$\mathfrak{Z}^{-1}\langle Z^3 \rangle =$  $= \lambda^3 + 3\lambda^2 + \lambda$

Solving for amplitudes

$$\sum_{\substack{\text{Connected surfaces}\\ \text{Fixed boundaries}}} e^{\frac{S_0}{2}\chi} = \lambda$$

Classical limit: $\lambda \gg 1$

Generating function for all $\langle Z^n \rangle$ amplitudes:

$$\left\langle e^{uZ} \right\rangle = \exp\left[\lambda e^u\right]$$

Solving for amplitudes

$$\sum_{\substack{\text{Connected surfaces}\\ \text{Fixed boundaries}}} e^{\frac{S_0}{2}\chi} = \lambda$$

Classical limit: $\lambda \gg 1$

Generating function for all $\langle Z^n \rangle$ amplitudes:

$$\left\langle e^{uZ} \right\rangle = \exp\left[\lambda e^u\right] = e^\lambda \sum_{d=0}^{\infty} p_d e^{ud}$$

Moment generating function of Poisson random variable

$$p_d = e^{-\lambda} \frac{\lambda^d}{d!} \qquad Z \sim \mathrm{Poisson}(\lambda)$$

Integer-valued distribution! $Z = \dim \mathcal{H}_{\mathrm{CFT}} \in \mathbb{N}$

Solving for amplitudes

Include EWB boundaries:

$$\left\langle e^{uZ+\sum_{i,j=1}^{k} t_{ij}(\psi_j,\psi_i)} \right\rangle = \exp\left[\lambda \frac{e^u}{\det(1-t)}\right]$$

Sum of boundaries on each connected component:

$$\exp\left[u + \sum_{n=1}^{\infty} \frac{1}{n} \operatorname{Tr} t^n\right] = \frac{e^u}{\det(1-t)}$$

Solving for amplitudes

Include EOW brane boundaries:

$$\left\langle e^{uZ+\sum_{i,j=1}^{k} t_{ij}(\psi_j,\psi_i)} \right\rangle = \exp\left[\lambda \frac{e^u}{\det(1-t)}\right]$$

$$= e^{\lambda} \sum_{d=0}^{\infty} p_d \, e^{ud} \left\langle e^{\sum_{i,j=1}^{k} t_{ij}(\psi_j,\psi_i)} \right\rangle_{Z=d}$$

Amplitude conditioned on $Z = \dim \mathcal{H}_{\mathrm{CFT}} = d$:

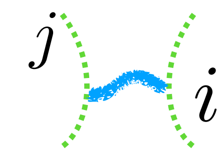
$$\left\langle e^{\sum_{i,j=1}^{k} t_{ij}(\psi_j,\psi_i)} \right\rangle_{Z=d} = \det(1-t)^{-d}$$

Statistics of states ψ_i chosen independently at random from d-dimensional Hilbert space

$$\mathrm{rank}\,(\psi_j,\psi_i) = \min\{Z,k\}$$

Hilbert space

$$|\alpha\rangle = \Big|Z, \{(\psi_j, \psi_i)\}\Big\rangle$$

Free approximation: Fock space built on  and 

Suggests that $Z,\ (\psi_j, \psi_i)$ are arbitrary

Exact result: $Z \in \mathbb{N}, \quad \mathrm{rank}\,(\psi_j, \psi_i) = \min\{Z, k\}$

Example of gauge equivalence: $\left|e^{2\pi i Z}\right\rangle = |\mathrm{HH}\rangle$

Page curve

The Page curve

Toy model: $\mathrm{rank}\,(\psi_j, \psi_i) \leq Z$ in α states

Number of independent states is bounded by thermal entropy

$\Longrightarrow$ external `radiation' follows Page curve

The Page curve

Toy model: $\mathrm{rank}\,(\psi_j, \psi_i) \leq Z$ in α states

Number of independent states is bounded by thermal entropy
$\implies$ external `radiation' follows Page curve

Follows in general from reflection positivity:

$$S(\rho_R) \leq S_\alpha(E)$$

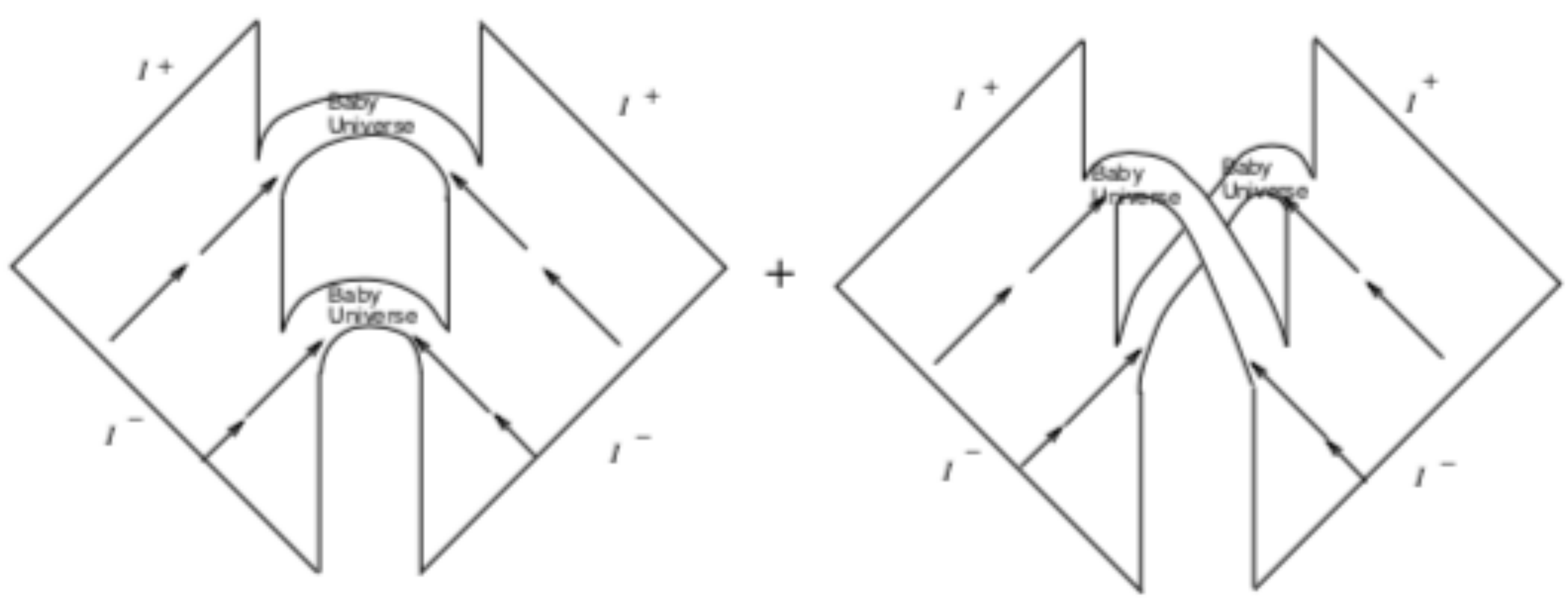
Trace out single-sided state with average energy *E*

Entropy computed from Legendre transform of $Z_\alpha(\beta)$.

$$\left\| |\mathrm{TFD};\alpha\rangle - \sum_{i=1}^{n} |\psi_i, \psi_i^*;\alpha\rangle \right\|^2 \geq 0$$

Information loss?

- Hawking computes average state of radiation $\overline{\rho_R}$ in HH state
- Mixed: entangled with state of closed universes
- RT computes $\overline{S(\rho_R)} \neq S(\overline{\rho_R})$
- Decohere onto branches with definite α.
- Repeated experiments: many copies of the same pure state



[Polchinski Strominger '94]

Questions to ponder

- Can we describe real-time dynamics for pure Hawking radiation?
- How does this work for N=4, etc?
- Cosmology? dS?
- Is reflection positivity enough to guarantee unitary duals?