On QFT in dS

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Part I: w. Leonardo Senatore 1911.00022

Part II: w. Lorenzo Di Pietro, Shota Komatsu

Why study QFT in dS?

- Applications to inflationary cosmology, e.g. (Panagopoulos, Silverstein '19)

- Some features of QFT correlators should be preserved when gravity is made dynamical.

- Understand relation between the wave function and correlators.

- For fun!

Part I: EFT approach and light fields.

e.g. Starobinski '84

$\mathcal{L} = (\partial\varphi)^2 - V(\varphi)$ in rigid dS_{d+1}

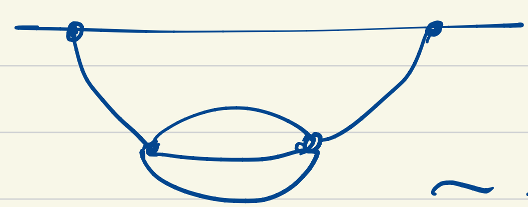
$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$

e.g. $V(\varphi) \simeq m^2\varphi^2 + \lambda\varphi^4$

focus on $m^2 \ll H^2$, $\lambda \ll 1$

Compute $\langle \varphi(\vec{x}_1, t) \ldots \varphi(\vec{x}_n, t) \rangle$, $a(t)\, x_{ij} \to \infty$

Diagrammatic calculation is hard, especially for $m \to 0$:



$\sim \frac{\lambda^2}{m^6}$

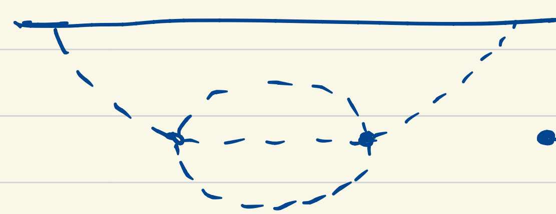
$\int e^{i\vec{k}\vec{x}}$  $\sim k^{-d+m^2}$, $k \to 0$

Instead, let us first compute the wave-function, IR divergences are understood:

$$\Psi_{BD}[\varphi,\eta] = Z_{EAdS}[\varphi, z]\Big|_{\substack{z=i\eta \\ L_{AdS}=iH^{-1}}}$$

$$ds^2 = \frac{-d\eta^2 + d\vec{x}^{\,2}}{H^2\eta^2}$$

Perturbatively:



 $\sim$ const,

$k \to 0$

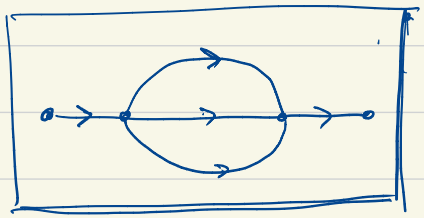
$$\log \Psi_{BD} \underset{\eta\to 0}{\sim} \frac{i}{\eta^3}\left(V(\varphi) + V'(\varphi)^2\right) + \dots +$$

$$+ \underbrace{\varphi(x)\varphi(y)\langle O_x O_y\rangle + \varphi(x)\varphi(y)\varphi(z)\langle O_x O_y O_z\rangle}_{\log Z_{CFT}} + \dots$$

Correlators are integrals over sources:

$$\langle \varphi(\vec{x}_1,\eta)\ldots\varphi(\vec{x}_n,\eta)\rangle = \int D\varphi(\vec{x})\,\Psi\Psi^*\varphi(\vec{x}_1)\ldots\varphi(\vec{x}_n)$$

This d-dimensional path int. is still IR div.:

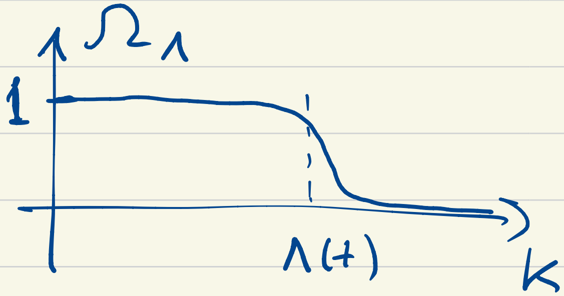


$$\sim \int \frac{d^d k}{k^{d-m^2}} \sim \frac{1}{m^2}$$

However, let us split $\varphi = \varphi_\ell + \varphi_s$

$$\varphi_\ell = \int dk\, e^{i\vec{k}\cdot\vec{x}}\,\Omega_\Lambda(k)\,\varphi_{\vec{k}}\,, \qquad \Lambda(t) = \varepsilon a(t) H\,,$$

$$\varepsilon \ll 1$$



$$\varphi_\ell \sim \frac{H}{\sqrt{m}} + \frac{H}{\lambda^{1/4}} \gg \varphi_s \sim H$$

Next, define n-point distributions:

$$P_n(\varphi_1 \ldots \varphi_n, t) = \int D\varphi(\vec{x}) \prod_i \delta(\varphi_i - \varphi_e(\vec{x}_i)) \Psi\Psi^* =$$

$$= \left\langle \prod_i \delta(\varphi_i - \hat{\varphi}_e(\vec{x}_i, t)) \right\rangle$$

P_n's generate correlators of φ_e.

We still cannot compute them directly, but we can derive an equation they satisfy:

$$\partial_t P_n(\varphi_1 \ldots \varphi_n, t) = \text{"Drift"} + \text{"Diffusion"}$$

$$\text{"Drift"} \downarrow \; \partial_t \Psi\Psi^* \qquad \text{"Diffusion"} \downarrow \; \delta(\varphi_i - \partial_t \varphi_e(\vec{x}_i))$$

Drift (Let us focus on P_1):

$$\partial_t \Psi\Psi^* = i a^{-3} \frac{\delta}{\delta\varphi}\left(\Psi^* \frac{\delta}{\delta\varphi}\Psi\right) + \text{c.c.}$$

$$i a^{-3} \frac{\delta}{\delta\varphi}\Psi_{BD} \equiv \Pi(\varphi)\Psi_{BD}, \quad \Pi(\varphi, x) = V'(\varphi(x)) + \dots$$

$$\int D\varphi\, \delta(\varphi_1 - \varphi_e(x_1)) \frac{\delta}{\delta\varphi}\left[\Pi(\varphi)\Psi\Psi^*\right] =$$

$$= \frac{\partial}{\partial\varphi_1}\left\langle \Pi(\varphi, x_1)\right\rangle_{\varphi_1}$$

We never need to path-integrate over long fields:

$$\sim \lambda^k \int d\varphi_2 \dots d\varphi_k \left\langle \Pi(\varphi)\right\rangle_{\varphi_1 \dots \varphi_k}$$

After some path-integral manipulations we get a set of PDE's:

$$\Gamma_i = \overbrace{\frac{\partial^2}{\partial \varphi_i^2}}^{\text{Diffusion}} + \overbrace{\frac{\partial}{\partial \varphi_i} V'(\varphi_i)}^{\text{Drift}} + O(\lambda)$$

$$\Gamma_{ij} = \frac{\sin \varepsilon a x_{ij}}{\varepsilon a x_{ij}} \frac{\partial^2}{\partial \varphi_i \partial \varphi_j} + O(\lambda)$$

$$\partial_t P_1 = \underline{\Gamma_1 P_1} + \lambda D_{12} P_2 + \ldots$$

$$\partial_t P_2 = \underline{\left(\Gamma_1 + \Gamma_2 + \Gamma_{12}\right) P_2} + \lambda D_{23} P_3 + \ldots$$

...

$$P_2(\varphi_1, \varphi_2, t)\Big|_{t \to -\infty} = P_1(\varphi_1) \cdot \delta(\varphi_1 - \varphi_2)$$

...

leading eqn. agrees w. Starobinsky

Solving at the leading order amounts to finding Eigenvalues and Eigenfunctions of Γ:

$$\Gamma \Phi_n = \frac{\partial^2}{\partial \varphi^2} \Phi_n + \frac{\partial}{\partial \varphi}\left(V' \Phi_n\right) = -\lambda_n \Phi_n$$

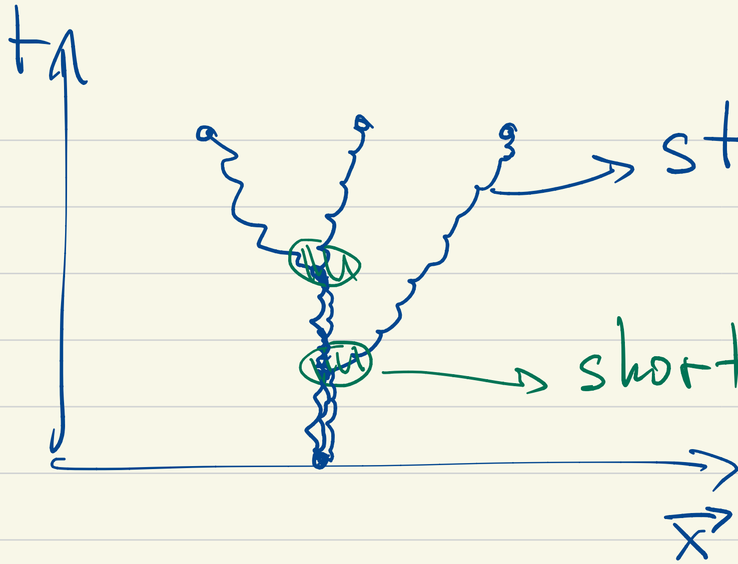
$\lambda\varphi^4$: $\lambda_n \sim \sqrt{\lambda}$

$m^2\varphi^2$: $\lambda_n \sim m^2/H^2$

$\lambda_0 = 0 \quad \lambda_{n\geq 1} > 0$

- $P_1^{eq} = \Phi_0 = e^{-\frac{V(\varphi_1)}{H^4}}$
- $\langle \varphi(x_1)\varphi(x_2)\rangle \sim (a x_{12})^{-\lambda_1}$
- $\langle \varphi(x_1)\varphi(x_2)\varphi^2(x_3)\rangle \sim \dfrac{C_{112}}{a x_{12}^{2\lambda_1-\lambda_2}\; a x_{13}^{\lambda_2}\; a x_{23}^{\lambda_2}}$

$$C_{112} = \int d\varphi\, \Phi_1^2 \Phi_2$$



strongly coupled one-particle dynamics, Γ_i

short period of interaction through Γ_{ij}, D_{ij}

$$\partial_t P_1 = \Gamma_1 P_1 + \lambda \underline{D_{12}} P_2 + \ldots$$

$$\partial_t P_2 = \left(\Gamma_1 + \Gamma_2 + \underline{\Gamma_{12}}\right) P_2 + \lambda \underline{D_{23}} P_3 + \ldots$$

- No more secular growth $\quad \Gamma_{ij} \sim \dfrac{\sin \varepsilon a x_{ij}}{\varepsilon a x_{ij}} \xrightarrow[t\to\infty]{} 0$
- $\langle \varphi_s \ldots \varphi_s \rangle_{\varphi_e}$ can be computed in P.T.,
  $e^{-\frac{1}{\sqrt{\lambda}}} \ll \varepsilon \ll \sqrt{\lambda}$, ε dependence cancels.
- For $m^4 \gg \lambda$ we get 1d harmonic oscillator + pert. in λ corrections

Comments on gravitational backreaction

We assume $V(\varphi)$ does not dominate expansion, there is some other clock $\chi \equiv$ inflaton.

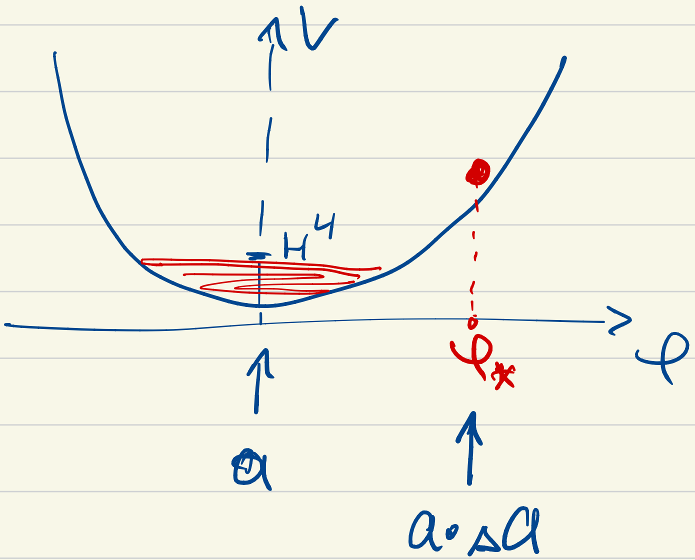
This allows for $\sim \frac{M_{pl}^2}{H^2}$ e-foldings

Arkani-Hamed, Dubovsky et. al. '07, '08

Take $V(\varphi) = \lambda \varphi^4$,

$\varphi_* \gg H \lambda^{-\frac{1}{4}}$

$P \sim e^{-\frac{\lambda \varphi_*^4}{H^4}}$, but also



$$\frac{\Delta H}{H} \sim \frac{\lambda \varphi_*^4}{H^2 M_{pl}^2} \Rightarrow \Delta a \sim e^{\Delta H t_*}$$

and $t_* \sim \frac{1}{\lambda \varphi_*^2} \Rightarrow \Delta a \sim e^{\frac{\varphi_*^2}{M_{pl}^2}}$

so for $\varphi_* \gg M_{pl}$ $\Delta a \gg 1$

(still $\Delta a / a \ll 1$)

We can find the maximum of volume-weighted distribution:

$$P_{\Delta a} \sim e^{-\frac{\lambda \varphi_*^4}{H^4} + \frac{\varphi_*^2}{M_{pl}^2}}$$

$$\text{at } \varphi_* \simeq \frac{H^2}{\sqrt{\lambda} M_{pl}} \gg \lambda^{-\frac{1}{4}} H \quad \text{if } \lambda \ll \left(\frac{H}{M_{pl}}\right)^4$$

Due to gravitational backreaction field distribution is very different.

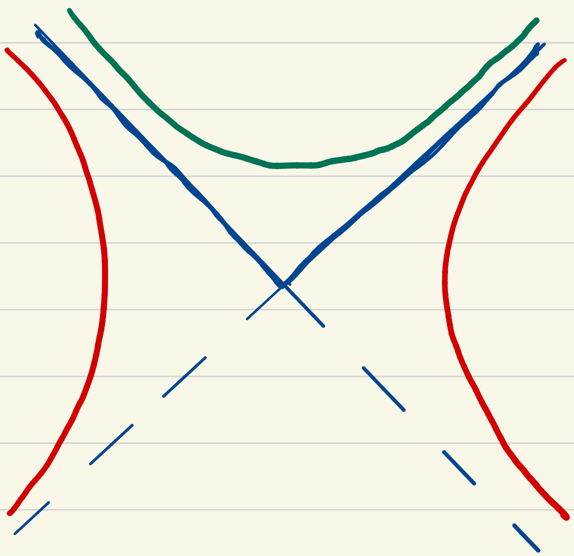
- The effect is not there for $V = m^2 \varphi^2$
- Formalism can be extended to include $\Delta a \gg 1$

- e.g. Goncharov, Linde, Mukhanov '87

$$P \sim e^{\frac{M_{pl}^4}{V(\varphi)}}$$

Part II: Diagrammatic approach

It is convenint to use embedding coordinates:



$$X \in R^{1,d+1} \quad (-1, 1 \ldots 1)$$

$$X^2_{EAdS} = -1$$

$$X^2_{dS} = +1$$

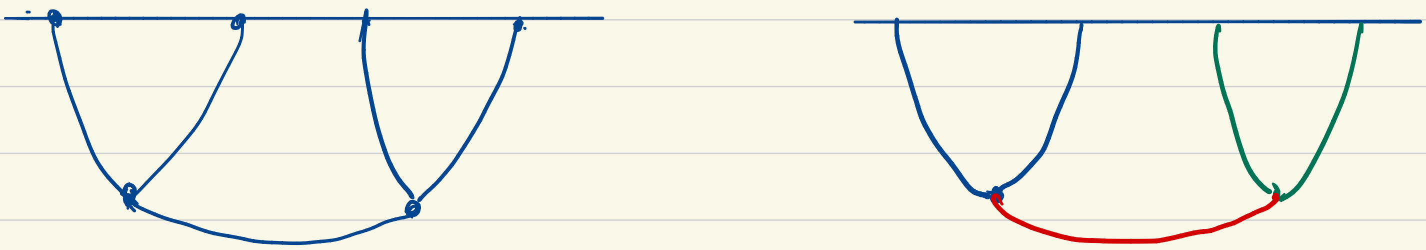
$$P^2 = 0\,, \quad \lambda P \equiv P \to \text{boundary}$$

$SO(1,d+1)$ sym.

To compute correlators use "in-in" formalism

$$\langle 0 | e^{iHt} \varphi(t, x_1) \ldots \varphi(t, x_n) e^{-iHt} | 0 \rangle$$

Let's study Feynman rules for $g\varphi^3$ theory



internal lines: $G_{++} = \langle T \varphi\varphi \rangle$

$G_{--} = \langle \bar{T} \varphi\varphi \rangle$ $G_{+-} = \langle \varphi\varphi \rangle$

Bulk to Boundary propagator: $\left(\nu = \sqrt{m^2 - \left(\frac{d}{2}\right)^2}\right)$

$$(P \cdot X_+)^{-\frac{d}{2} + i\nu} \quad , \quad X_\pm = X(1 \pm i\varepsilon)$$

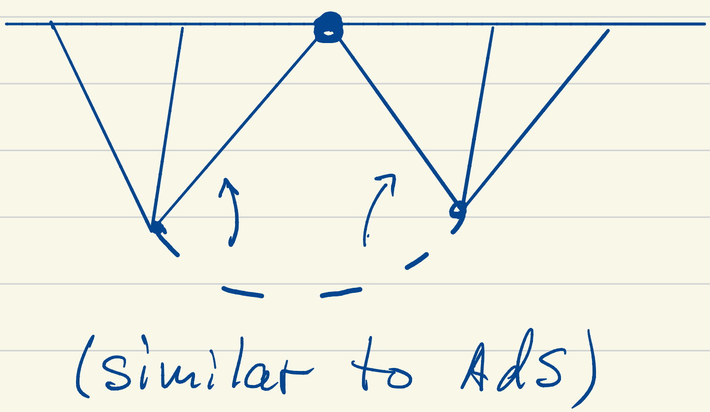
Direct calculation even of these diagrams

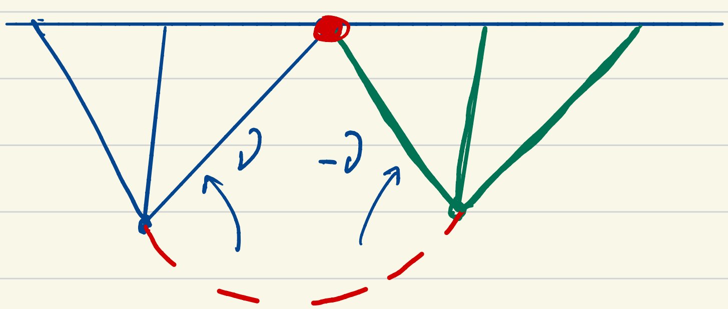
is quite complicated.

However, one can use split representation:

$$G^{\nu}_{+-}(X_+, Y_-) = \int dP \, (P\cdot X_+)^{-\frac{d}{2}+i\nu} (P\cdot Y_-)^{-\frac{d}{2}-i\nu}$$

(It is slightly more complicated for G_{++} and G_{--})

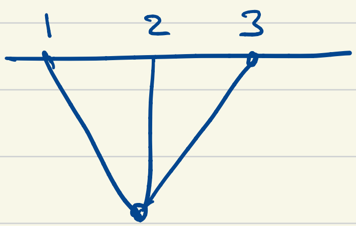




Diagrams factorize into + and − parts "glued" only along the boundary.

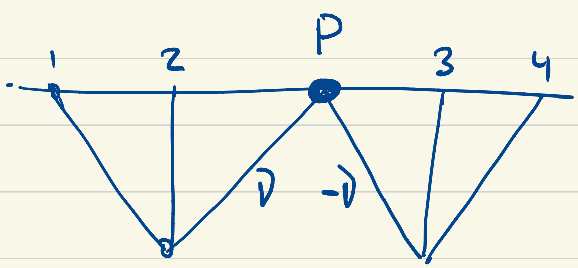
This is so because we could first compute the wave function:

$$\langle \varphi \dots \varphi \rangle = \int D\varphi \, \Psi \, \Psi^{\dagger} \varphi \dots \varphi$$



$$\sim \langle O_1 O_2 O_3 \rangle$$

Shadow operator, $\Delta_{\tilde{O}} = d - \Delta_O$



$$\sim \int dP \, \langle O_1 O_2 \tilde{O}(P) \rangle \langle O(P) O_3 O_4 \rangle \sim$$

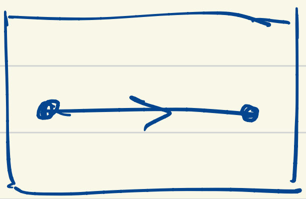
$\sim$ conformal blocks.

So also in dS exchange diagrams can be expressed in terms of conformal blocks.

c.f. Sleight, Taronna '19

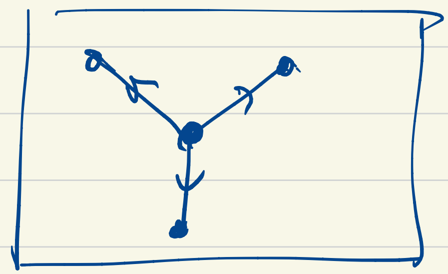
Let us look closer at the boundary path integral:

$$\log \Psi_{BD} \underset{\eta \to 0}{\sim} \frac{i}{\eta^3}(\dots) + \underbrace{\varphi(x)\varphi(y)\langle O_x O_y \rangle + \varphi(x)\varphi(y)\varphi(z)\langle O_x O_y O_z \rangle}_{\log Z_{CFT}} + \dots$$



$$\simeq \langle \tilde{O}_x \tilde{O}_y \rangle$$

$$\Delta_{\tilde{O}} = d - \Delta_O$$



$$=$$

$$= \langle \varphi(x_1)\varphi(x_2)\varphi(x_3) \rangle = \int D\varphi(\vec{x})\, \Psi \Psi^{*} \varphi(x_1)\varphi(x_2)\varphi(x_3) =$$

$$= \int dy_1 dy_2 dy_3 \langle O_{y_1} O_{y_2} O_{y_3} \rangle \langle \tilde{O}_{y_1} \tilde{O}_{x_1} \rangle \langle \tilde{O}_{y_2} \tilde{O}_{x_2} \rangle \cdot$$

$$\cdot \langle \tilde{O}_{y_3} \tilde{O}_{x_3} \rangle \sim \langle \tilde{O}_{x_1} \tilde{O}_{x_2} \tilde{O}_{x_3} \rangle$$

It appears that pert. theory is just doing conformal integrals!

Karateev, Kravchuk, Simmons-Duffin '18

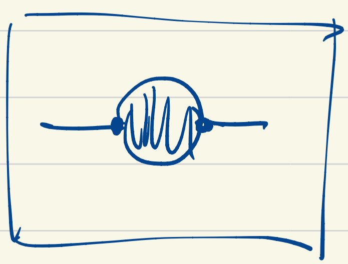
We know, however, that for $\lambda\varphi^4$ theory

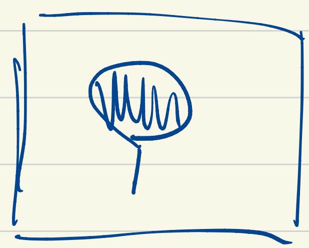
$$\langle O_x O_y \rangle \sim (x-y)^{-2d+m^2}, \quad \text{while}$$

$$\langle \varphi(x)\varphi(y) \rangle \sim (x-y)^{-\lambda_1}, \quad \lambda_1 \sim \sqrt{\lambda} >> m^2$$

So it cannot be just a shadow transform.

The catch is that one- and two-point conformal integrals are divergent:



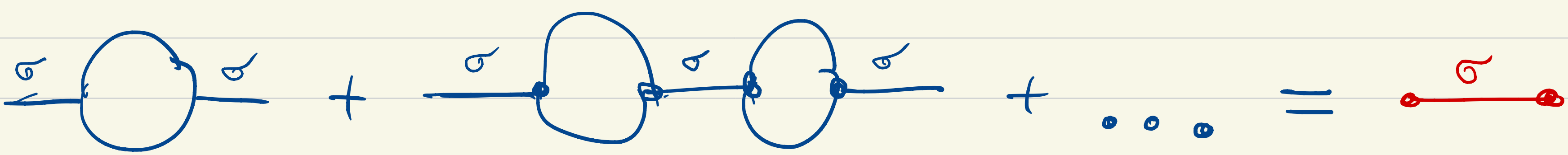


But, presumably, these are the only ones.

This, actually, resonates with the EFT approach where equation for the 2-point function determined everything.

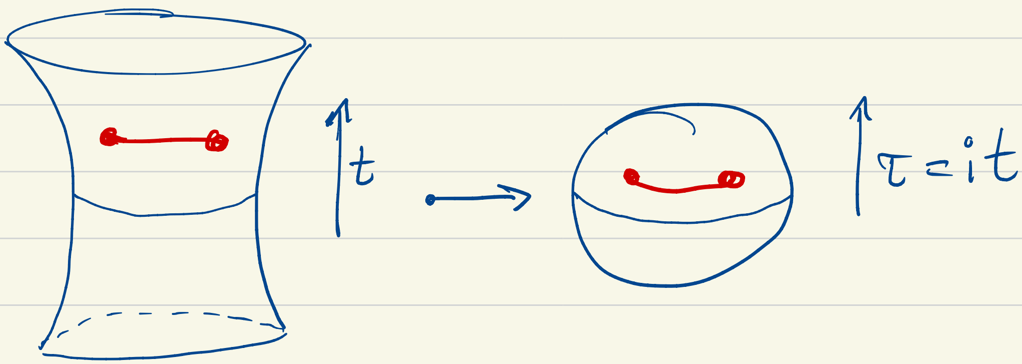
Next, we want to compute the 2-point function in the diagrammatic approach and compare. We did it at large-N so far:

$$\mathcal{L} = \partial\varphi^i\partial\varphi^i - m^2\varphi^{i\,2} + \frac{\lambda}{N}(\varphi^i\varphi^i)^2 \,, \quad i = 1\ldots N,$$

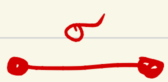


dominates at large-N, any λ.

To compute the "Bubble" it is convenient to Wick rotate $dS \to S$



On the sphere it can be computed as a sum over spherical harmonics:



$$= \sum_{\ell=0}^{\infty} \frac{\Omega_3(\ell,3)}{\lambda^{-1} + \frac{4\pi}{\ell+1} B(\ell)}$$

Marolf, Morrison '10

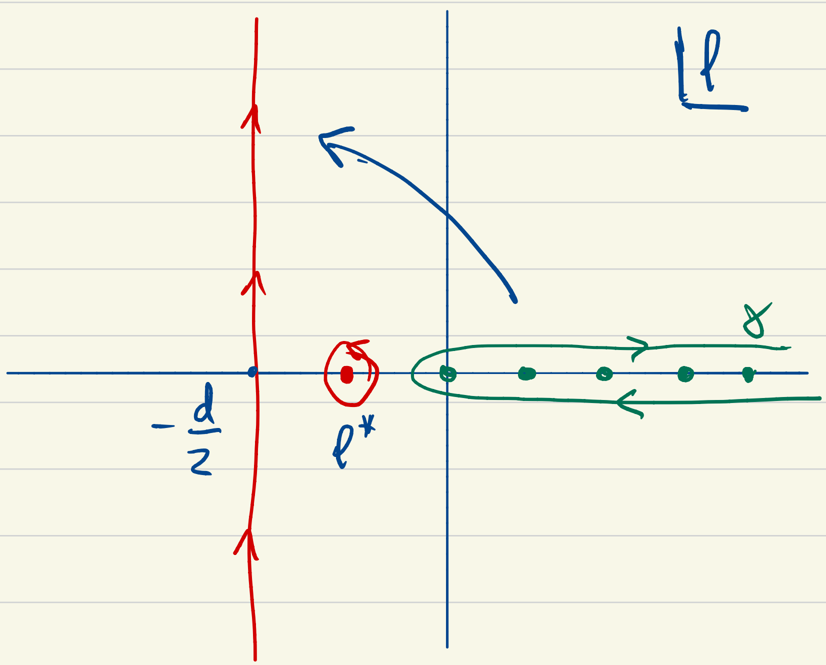
on S_3,

$$\boxed{B(\ell) = \frac{1}{16\pi^2} + \cot(\pi\sigma)\frac{\psi(\frac{\ell}{2}-\sigma) - \psi(\frac{\ell}{2}+\sigma+2)}{16\pi^3}}$$

Watson–Sommerfeld transform:

$$\langle \sigma(x)\sigma(y)\rangle = \oint_\gamma \frac{d\ell}{2\pi i}\, g(\ell)\, \frac{\pi}{\sin \pi \ell}(-1)^\ell\, \Omega_S(\ell,\xi) =$$

$$= \sum_{\ell^*} \mathrm{res}\, \frac{\pi\, g(\ell)}{\sin\pi\ell}\, \Omega_{dS}(\ell,\xi) + \int_{-\infty}^{\infty} d\nu\, g\!\left(-\frac{d}{2}+i\nu\right) \Omega_{dS}(\nu,\xi).$$



In this form the answer can be continued to dS. Poles determine the leading long-distance behavior.

For $m^2 \ll \sqrt{\lambda}$, $\lambda \ll 1$ we get

$$\Delta_* = 4\left(\frac{\lambda}{2}\right)^{\frac{1}{2}} + 3\lambda + O(\lambda^{3/2})$$

$$\langle \sigma\sigma\rangle \approx \xi^{-\Delta_*}$$

This matches the leading Eigenvalue of the operator

$$\Gamma^{(N)} = \frac{\partial^2}{\partial\varphi_i\partial\varphi_i} + N\,W\left(\frac{\varphi^i\varphi^i}{N}\right), \qquad \Gamma^{(N)}\Phi_1 = \lambda_1\Phi_1$$

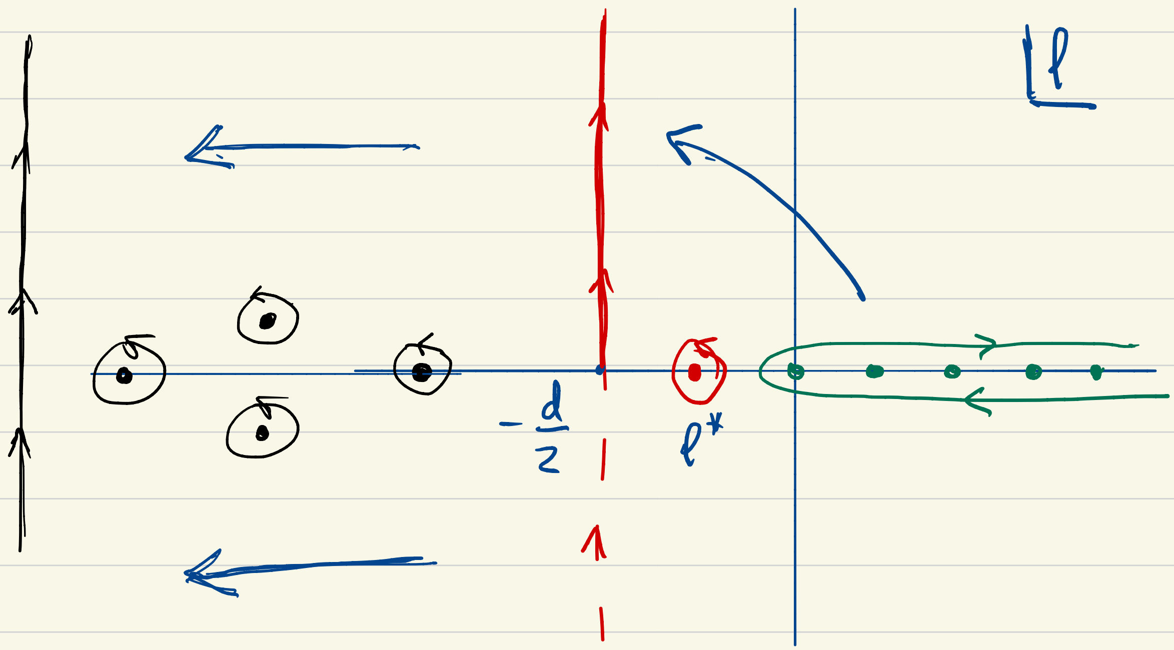
$$W(\rho) = \frac{\lambda^2}{8d^2}\rho^3 - \frac{\lambda\rho}{4d} - \frac{\bar{m}^2}{4d} + \frac{\lambda\bar{m}^2}{4d^2}\rho^2 + \frac{3\lambda^3\rho^4}{4d^2} + O(\lambda^{3/2}),$$

which we obtained in the EFT approach (at large N)

Technically, this match appears quite non-trivial

On unitarity in dS and AdS.

There are two more manipulations we can do to the contour: rewrite it over half-line $\mathrm{Im}\,\ell > 0$, and push it further to the left:



$\langle \sigma \sigma \rangle =$

$= \int d\ell \, g(\ell) \, \Omega(\ell, \xi)$

In AdS sum over poles on the left would correspond to OPE expansion. In dS, however, "Operators" generically come in complex pairs. Instead, the integral over principal series must have a positive density, as a consequence of bulk unitarity.

Summary

- We developed an EFT formalism for correlators in QFTs in dS space; computation of correlators reduces to a system of PDE's; result agrees with diagrammatic calculation.
- Boundary correlators define a Euclidean CFT, unitarity is encoded through $SO(1,d+1)$ partial wave decomposition
- It will be interesting to see how bulk causality is encoded.
- The wave function also defines a CFT, which is very different, but is related via conformal integrals
- Inclusion of (perturbative) gravity appears possible.