EFT for Quantum Black Hole Horizons

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Based on work w/ Rothstein,
arXiv:1912.13435
+ in progress

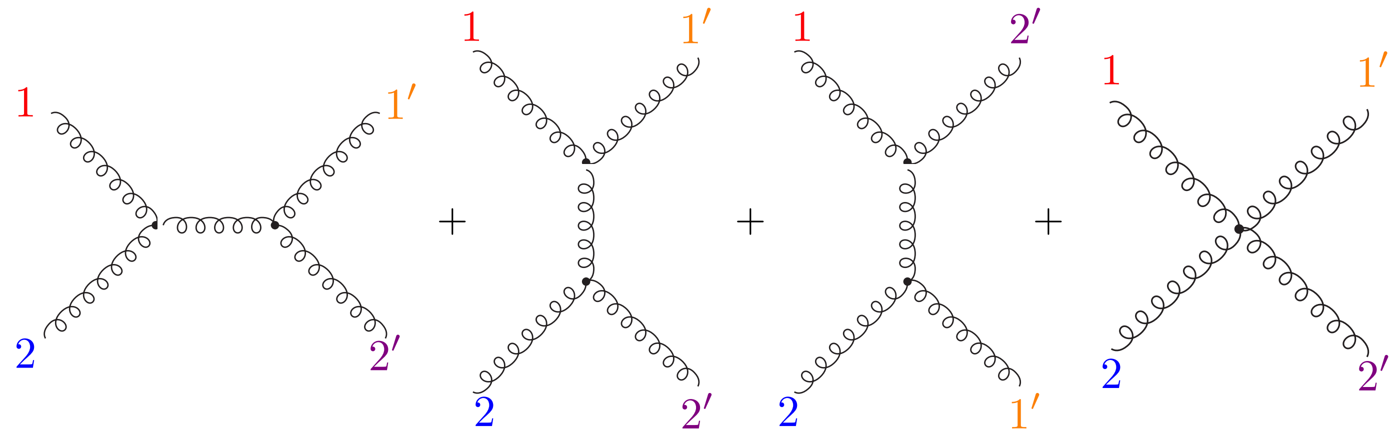
It is well known that quantum gravity at $E \ll M_{Pl}$ can be treated as an EFT of interacting gravitons coupled to matter.

This EFT can be used to obtain well defined predictions for the long distance part of quantum gravity S-matrix,
e.g:

Graviton scattering:

DeWitt, 1967.

$$\mathcal{A} =$$

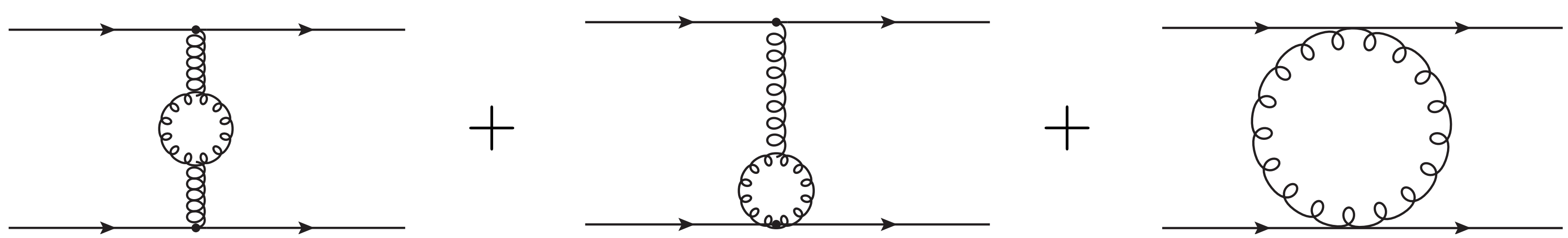


$$\sim \frac{E^2}{M_{Pl}^2}$$

$\mathcal{O}(\hbar)$ corrections to Newton potential:

Donoghue et al 2002, Khriplovich et al 2002

$$V(r)_{\hbar^1} =$$



$$= -\frac{41}{10\pi}\frac{G_N \hbar}{r^2 c^3} \cdot \frac{G_N m_1 m_2}{r}$$

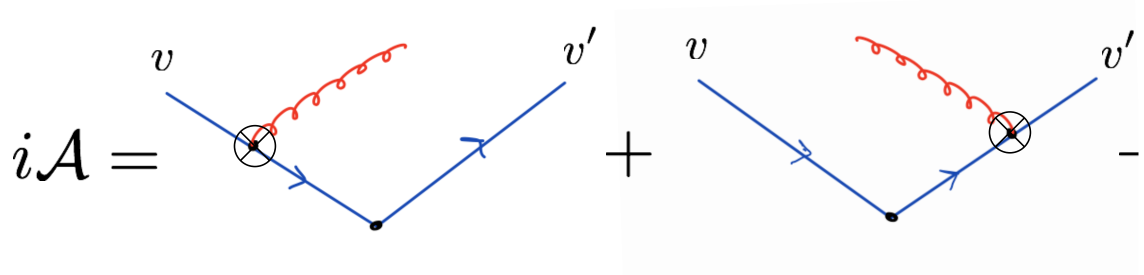
See also: cosmological correlation functions.

While this EFT can be used to study low energy processes involving elementary particles, it is not known how to use it to describe the S-matrix in the case where black holes appear as the "asymptotic states".

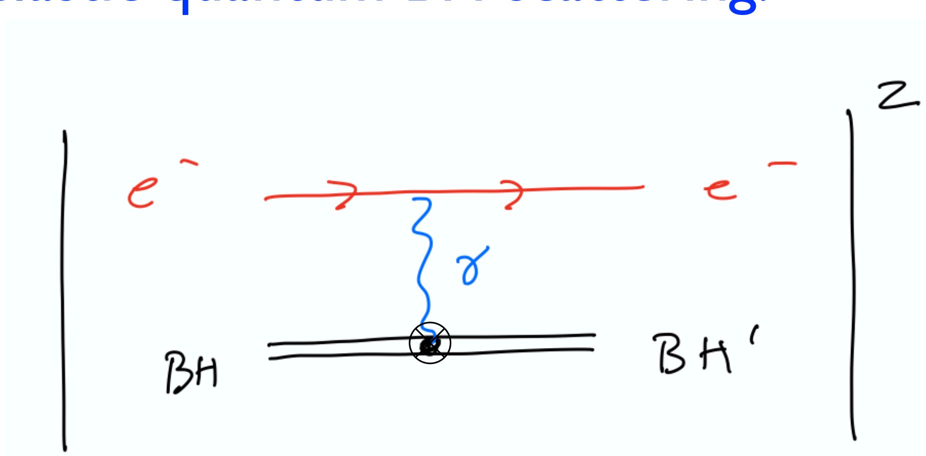
In particular, how do perturbative quantum effects associated with black hole horizons, Hawking radiation, appear in low energy scattering amplitudes involving black holes?

E.g

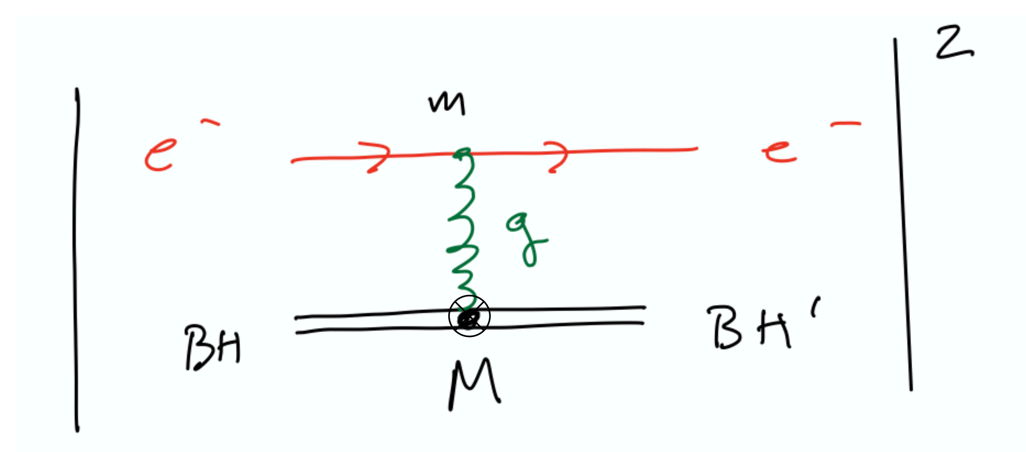
Soft Hawking photon theorem for electrically neutral BHs



Inelastic quantum BH scattering:



(electromagnetic)



(gravitational)

The goal of this talk is to setup an EFT that is able to calculate Hawking radiation corrections to such processes. The EFT that I will present will be valid for length/time scales Δ such that:

$$\tau_{BH} \gg \Delta \gg G_N M \gg 1/m_{Pl},$$

$$\tau_{BH} \sim G_N^2 M^3$$ =BH lifetime

i.e. we assume the BH's are semiclassical. Thus the EFT necessarily breaks down at time scales of order the BH lifetime , where $M_{BH} \sim M_{Pl}$ or $r_s = 2G_N M \sim \ell_{Pl}$ and non-perturbative quantum gravity effects play an important role.

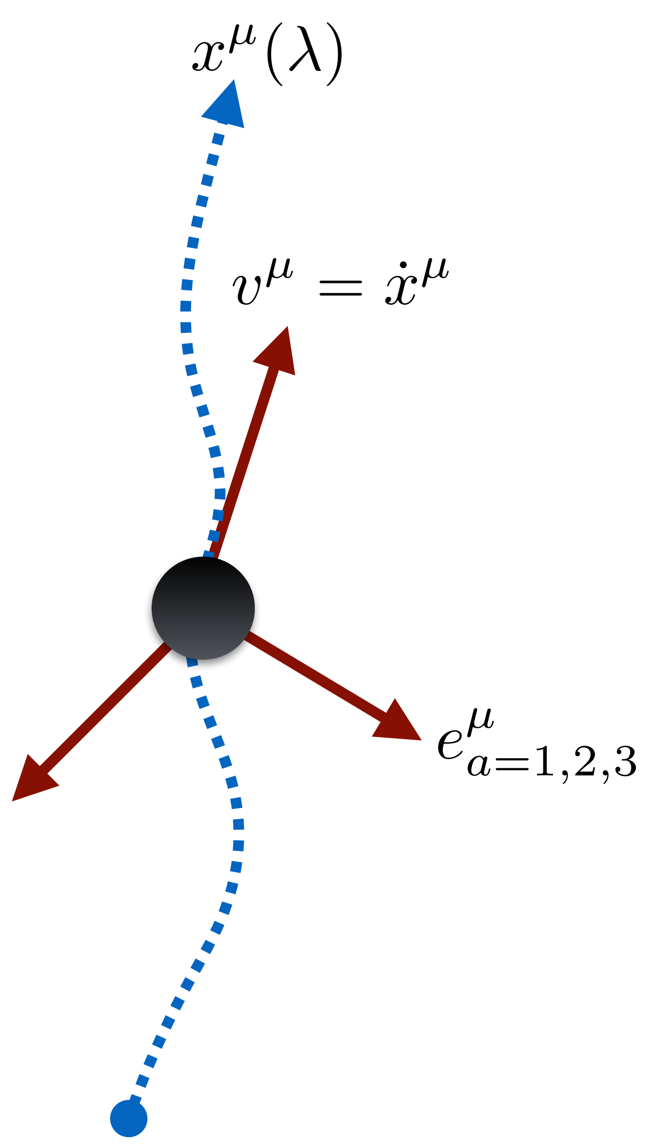
Worldline EFT for BHs

In the range of scales defined above, we can still treat the BH as a (dynamical) point source, i.e. the BH is described by a worldline EFT coupled to gravity

DOFs:

$x^\mu(\lambda) =$ worldline CM coordinate

$e^\mu_{a=1,2,3}(\lambda) =$ local frame (SPIN)



Symmetries:

Diff. invariance $x^\mu \mapsto x^\mu + \xi^\mu(x)$

Worldline RPI $\lambda \mapsto \lambda'(\lambda)$

Local $SO(3)$ rotations acting on e^μ_a (for BH only)

EFT for gravity coupled to BH, in the point particle limit:

$(\hbar = c = 1)$

$(m_{Pl}^2 = 1/(32\pi G_N))$

$S_{EH} = -2m_{Pl}^2 \int d^4x \sqrt{g} R(x)$

$$S = S_{EH} + S_{pp}$$

The most general (mod. e.o.m's) point particle Lagrangian consistent with symmetries (ignoring spin, assume parity invariance), organized in a derivative expansion:

$$S_{pp} = \underbrace{-m \int d\tau}_{\mathcal{O}(\partial^0 g)} + \underbrace{c_E \int d\tau E_{\mu\nu} E^{\mu\nu} + c_B \int d\tau B_{\mu\nu} B^{\mu\nu}}_{\mathcal{O}(\partial^4 g)} + \cdots$$

w/ $E_{\mu\nu} = R_{\mu\alpha\nu\beta} v^\alpha v^\beta$ = "electric" curvature tensor

$B_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\rho\sigma\lambda} v^\rho v^\alpha R_{\nu\alpha}{}^{\sigma\lambda}$ = "magnetic" curvature tensor

(Note: $\mathcal{O}(\partial^2 g)$ terms, eg $\int d\tau g^{\mu\nu} R_{\mu\nu}$ are redundant due to source free eom $R_{\mu\nu} = 0$).

In the point particle EFT, the quadrupole moment induced by an external field:

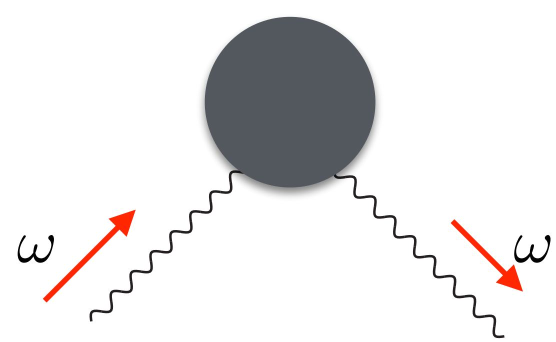
$$\delta Q_{ij} = -\frac{\delta}{\delta E_{ij}} S_{pp} = -2 c_E E_{ij} \qquad \text{vs.} \qquad \delta Q_{ij} = -\frac{2}{3} k \, \frac{R^5}{G_N} E_{ij}$$

We expect on dimensional grounds:

$$c_E \sim R^5 / G_N$$

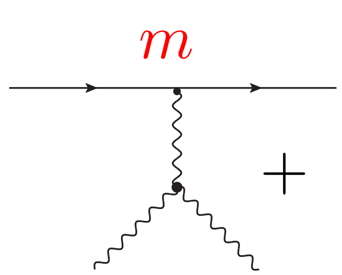
Same scaling also holds for relativistic (compact) objects. Eg elastic graviton+BH scattering amplitude: $(r_s \omega \ll 1)$

Full theory: $i\mathcal{A} =$

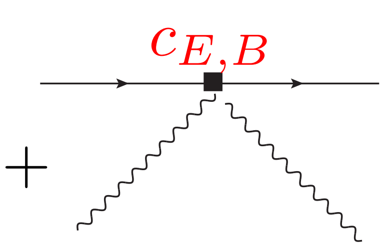


$= r_s f(r_s \omega)$

EFT: $i\mathcal{A} =$



$+ \quad \cdots \quad +$



$+ \quad \cdots$

$$\sim \frac{c_{E,B}}{m_{Pl}^2} \omega^4$$

$\Rightarrow \quad c_{E,B} \sim r_s^5 / G_N$

from matching (WG, '06)

For neutron stars, c_E depends on the EOS and has been calculated numerically in (Flanagan+Hinderer, 2007)

For the case of Schwarzschild black holes in $d=4$, the tidal response in the full theory $R_{\mu\nu}=0$ has been computed analytically by

Damour+Nagar, 2009

Binnington+Poisson, 2009

Kol+Smolkin, 2011

Steinhoff et al 2013

while the EFT side corrections were shown to vanish in Kol+Smolkin, 2011. The result for BH's in $d=4$

$$c_E^{BH} = c_B^{BH} = 0$$

so no (static) tidal response at $\ell = 2$ (and likely also for $\ell > 2$ ...)

BH absorption and Emission

The results on tidal coefficient suggests that finite size effects are absent for black holes.

But formalism outlined so far neglects dissipation, ie absorption of energy and angular momentum by the compact objects themselves.

On general grounds, dissipation implies the existence of low frequency modes (eg NS: hydro modes,…. BH: horizon absorption) not captured by the point particle EFT

$$S_{pp} = -m \int d\tau + c_E \int d\tau E_{\mu\nu} E^{\mu\nu} + c_B \int d\tau B_{\mu\nu} B^{\mu\nu} + \cdots$$

Even though the form of the internal spectrum depends on the details of the internal structure, can incorporate the effects of dissipation in a model independent w/o the need to explicitly track the light DOFs

The idea is to treat the compact object as $R \to 0$ as an "atom", i.e a worldline with local operators coupled to gravitons. For a spherical symmetric object, the leading interactions with gravitons take the form

$$S_{int} = -\int d\tau(\lambda) Q^E_{ab}(\lambda) E^{ab}(x) - \int d\tau(\lambda) Q^B_{ab}(\lambda) B^{ab}(x).$$

With operators $Q_E^{ab}(\lambda),\ Q_B^{ab}(\lambda)\quad \ldots$ acting on the Hilbert space of internal states. These are gravitational analogs of the EM dipole interaction

$$H_{em} = -\hat{\vec{p}} \cdot \vec{E}$$

Microscopic properties are then encoded in the correlation functions

$$\langle Q^{E,B} \cdots Q^{E,B} \rangle$$

which can be related to observable quantities of the compact object.

Example: Classical graviton absorption and power dissipation in binary systems

Consider an compact object of mass M . Graviton absorption amplitude in the object's rest frame:

$$i\mathcal{A}(g_h(k) + M \to X) = \langle X|Te^{-i\int dt H_{int}}|k,h;M\rangle$$

$$\approx -\int dt \langle X|Q^E_{ij}(t)|M\rangle\langle 0|E_{ij}(t,0)|k,h\rangle + (E \leftrightarrow B)$$

absorption cross section is

$$\sigma_{abs}(\omega) = \lim_{T\to\infty} \frac{1}{T} \cdot \frac{1}{2\omega} \sum_X |\mathcal{A}(g(k) + M \to X)|^2$$

then, assuming unitarity (even for BHs!):

$$\sum_X |X\rangle\langle X| = \mathbb{I}$$

→

$$\sigma_{abs}(\omega) = \frac{\omega^3}{8m_{Pl}^2} \int dt e^{-i\omega t} \epsilon_{ij}(k)\epsilon^*_{rs}(k) \left[\langle M|Q^E_{rs}(0)Q^E_{ij}(t)|M\rangle + \langle M|Q^B_{rs}(0)Q^B_{ij}(t)|M\rangle\right],$$

where the 2-pt. correlators are in the initial state of the compact object

$$\langle Q^E(0)Q^E(x^0)\rangle = \langle M|Q^E(0)Q^E(x^0)|M\rangle$$

(alternatively, initial state could be mixed/thermal)

These absorption coefficients were computed by Page (1975) for massless particles of arbitrary spin in the case of Kerr black holes:

$$\sigma_s(\omega) = \pi\omega^{-2} \sum_{l,m} \Gamma_{s\omega lmp} \underset{\omega \to 0}{\sim} \begin{cases} A, & s = 0 \\ 2\pi M^2, & s = \frac{1}{2} \\ \frac{4}{9} A (3M^2 - a^2)\omega^2, & s = 1 \\ \frac{16}{225} A (5M^2 + \frac{5}{2} M^2 a^2 + a^4)\omega^4, & s = 2. \end{cases}$$

Using his result we can match the two-point functions in the case $s = 2$

$$\int dt e^{i\omega t} \langle M | Q_{ij}^{E,B}(t) Q_{kl}^{E,B}(0) | M \rangle = \frac{1}{2} A_+(\omega) \left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl} \right),$$

and

$$\sigma_{abs}^{\ell=2}(\omega) = \frac{\omega^3}{4m_{Pl}^2} A_+(\omega) \approx 4\pi r_s^2 \left[\frac{(r_s\omega)^4}{45} + \mathcal{O}(r_s\omega)^6 \right]$$

$$A_+(\omega > 0) = \frac{1}{2G_N} \frac{r_s^6 \omega}{45} + \mathcal{O}(r_s^8 \omega^3).$$

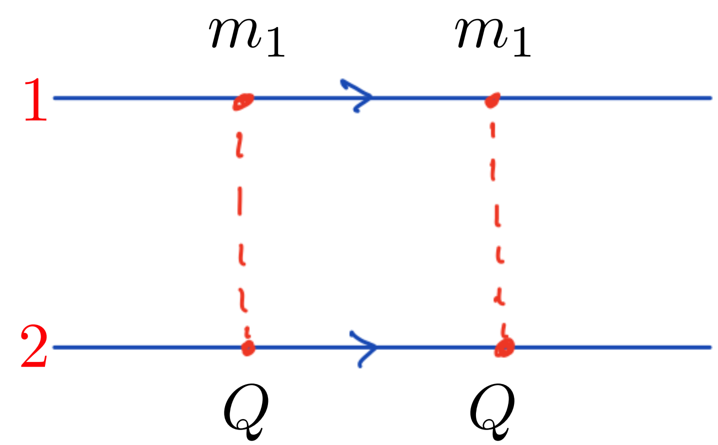
$$A_+(\omega < 0) = 0$$

IF NO EMISSION FROM BH

"Boulware state"

The same two-pt. function that controls BH absorption also shows up in BH-BH binary dynamics. Absporptive corrections to two-body eqns of motion from IN-IN effective action induced by integrating out internal DOFs of BH:

$$i\Gamma_{eff}[x_1, x_2] = \cdots +$$



$$+(1 \leftrightarrow 2) + \cdots$$

By causality, the dissipative part of the classical EOM, $\delta_{x_{1,2}}\Gamma_{eff}[x_1, x_2] = 0$ only depends on the retarded Green's function

$$G^R_{ij,rs}(t) = -i\theta(t)\langle M|[Q^E_{ij}(t), Q^E_{rs}(0)]|M\rangle \equiv \frac{1}{2}G^R(t)\left(\delta_{ir}\delta_{js} + \delta_{is}\delta_{jr} - \frac{2}{3}\delta_{ij}\delta_r\right).$$

e.g tidally induced quadrupole moment:

$$\langle Q_{ij}(t)\rangle = -2c_E\bar{E}_{ij}(t,0) + \int_{-\infty}^{\infty} dt' G^R_{ij,rs}(t-t')\bar{E}_{rs}(t',0),$$

given in freq. space by a dispersion relation involving Wightman 2-pt fns:

$$\langle Q_{ij}(\omega)\rangle = -2c_E(\omega)\bar{E}_{ij}(\omega,0),$$

$$c_E(\omega) = c_E - \frac{1}{2}G^R(\omega) = c_E - \frac{1}{2}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{A_+(\omega') - A_+(-\omega')}{\omega - \omega' + i\epsilon}.$$

"dynamical Love number"

$$c_E(\omega)|_{classical} = \frac{ir_s^6\omega}{360G_N} + \mathcal{O}(r_s^8\omega^2).$$

(tune $\mathrm{Re}c_E(\omega \to 0) = 0$ to ensure vanishing static Love number)

Applications of the BH response function I:

Horizon friction in non-relativistic BH/BH binary system:

Variation of the IN-IN action gives dissipative force:

$$\vec{F}_1(t) = \frac{\delta}{\delta \vec{x}_1(t)} \Gamma[\vec{x}, \tilde{\vec{x}}]\Big|_{\vec{x}=\tilde{\vec{x}}} = -G_N m_1 m_2 \left\langle \frac{Q^{jk}_{E,1}(t)}{m_1^2} + \frac{Q^{jk}_{E,2}(t)}{m_2^2} \right\rangle \nabla \partial_j \partial_k \frac{1}{|\vec{x}(t)|} = -\vec{F}_2(t),$$

From the retarded response function, in the $|\vec{v}| \ll 1$ limit:

$$\vec{F}_1(t) = -\vec{F}_2(t) = -\frac{32}{5} \frac{G_N^7 (m_1 m_2)^2 (m_1^4 + m_2^4)}{|\vec{x}|^8} \left[\vec{v} + \frac{2\vec{v} \cdot \vec{x}_{12}}{|\vec{x}|^2} \vec{x} \right],$$

Dissipated mechanical energy and angular momentum:

$$\frac{d}{dt} E_h = \sum_{\alpha=1}^{2} \vec{v}_\alpha \cdot \vec{F}_\alpha = -\frac{32}{5} G_N^{-1} \left(\frac{G_N M}{|\vec{x}|} \right)^8 \left(\frac{\mu}{M} \right)^2 \left(1 - \frac{2\mu}{M} + \frac{2\mu^2}{M^2} \right) \left[\vec{v}^2 + \frac{2(\vec{x} \cdot \vec{x})^2}{|\vec{x}|^2} \right]$$

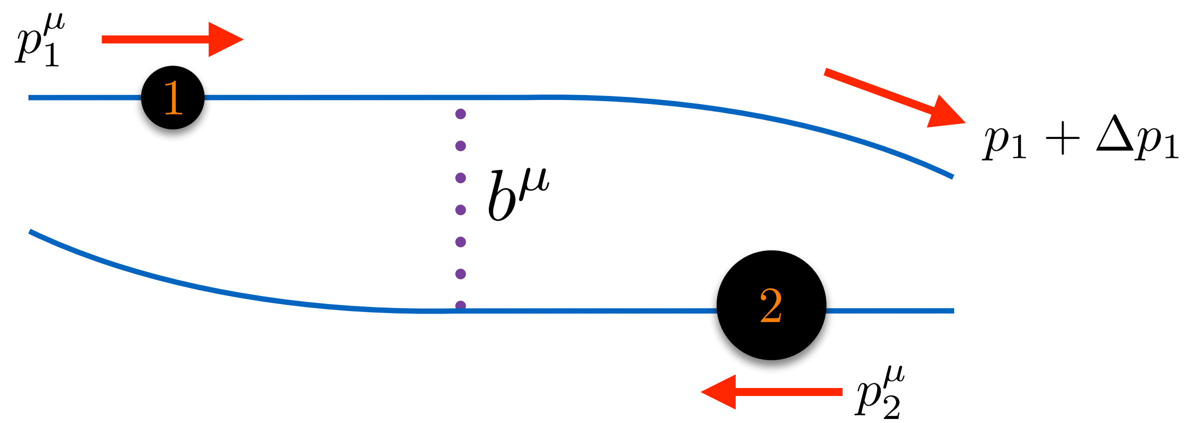
$$\frac{d}{dt} \vec{L}_{CM} = \sum_{\alpha=1}^{2} \vec{v}_\alpha \times \vec{F}_\alpha = \frac{64}{5} G_N^7 \mu M^6 \left(1 - \frac{2\mu}{M} + \frac{2\mu^2}{M^2} \right) \frac{\vec{v} \cdot \vec{x}}{|\vec{x}|^{10}} \vec{L}_{CM}$$

(WG+Rothstein, 2006; agrees w/ Poisson ’95 at $\mu \ll M$)

Note: $P_{abs}/P_{quad} \sim v^8$ is a (small) 4PN effect. Absorption enhanced to v^5 for rotating black holes (see Porto; Endlich+Pencco for EFT description)

Applications of the BH response function II: (WG+Rothstein, to appear)

Inelastic BH-BH scattering: First finite size effects at $\mathcal{O}(G_N^6)$ beyond linearized Einstein ("6PM")



LO momentum in large impact parameter/ relativistic scattering of two BH's:

$$\Delta p_1^\mu = \int_{-\infty}^{\infty} \frac{dp_1^\mu}{ds} = -\Delta p_2^\mu = \frac{4G_N m_1 m_2}{b^2} \frac{(v_1\cdot v_2)^2 - \frac{1}{2}}{\sqrt{(v_1\cdot v_2)^2 - 1}} b^\mu$$

Including dissipative effects due to $G^R_{\mu\nu;\rho\sigma}(s-s')$ in the EOM,

$$\Delta p^\mu_{1,in} = -\Delta p^\mu_{2,in} =$$

$$-\frac{5\pi}{96}\frac{G_N^7 m_1^6 m_2^2}{(-b^2)^{7/2}} \frac{20\gamma^8 + 63\gamma^6 - 73\gamma^4 + 21\gamma^2 + 17}{\sqrt{\gamma^2-1}} (v_1 - (v_1\cdot v_2)v_2)^\mu - (1 \leftrightarrow 2)$$

check:

$$\Delta m_1^2 = \frac{5\pi}{48}\frac{G_N^7 m_1^6 m_2^2}{(-b^2)^{7/2}} \sqrt{\gamma^2-1}\left[20\gamma^8 + 63\gamma^6 - 73\gamma^4 + 21\gamma^2 + 17\right] > 0$$

as required by Hawking's area theorem.

Hawking emission and EFT

In principle, same methods can be applied to capture long distance effects of emission by black hole horizon. Decay amplitude:

$$\mathcal{A}(M \to X + g(k)) \approx -\frac{\omega^2}{2m_{Pl}} \int dt e^{i\omega t} \epsilon^*_{ij}(k) \langle X | Q^E_{ij}(t) \mp i Q^B_{ij}(t) | M \rangle.$$

Graviton emission rate:

$$d\Gamma(\omega) = \lim_{T\to\infty} \frac{1}{T} \sum_X |\mathcal{A}(M \to X + g(k))|^2 \frac{d^3\vec{k}}{(2\pi)^3 2\omega},$$

yields an isotropic graviton emission spectrum

$$\frac{d\Gamma}{d\omega} = \frac{4G_N}{\pi} \omega^5 A_+(-\omega) \qquad (\omega > 0)$$

vs. Hawking's 1975 result

$$\frac{d\Gamma}{d\omega} = \frac{\omega^2}{2\pi^2} \frac{\sigma_{abs}(\omega)}{e^{\hbar\omega/kT_H} - 1}, \qquad kT_H = \frac{\hbar}{4\pi r_s}$$

Comparing EFT to Hawking's spectrum then suggests:

$$A_+(-|\omega|) = \frac{8\pi}{G_N} \frac{\sigma_{abs}(\omega)/\omega^3}{e^{\hbar\omega/kT_H} - 1},$$

BUT NOTICE:

$$\hbar\omega/kT_H = \frac{\not{\hbar}\omega}{\not{\hbar}/4\pi r_s} = 4\pi r_s \omega$$

So expect emission effects of order

$$A_+(-|\omega|) \sim \frac{r_s^5}{G_N} \qquad (r_s\omega \ll 1)$$

w/no $\omega/M_{Pl} \ll 1$ suppression!

Quantum (Hawking) effects are of the same order as classical Love number. (A consequence of DETAILED BALANCE between absorption and emission).

Systematic matching:

To extract the quantum EFT, we compare to two predictions of QFT in the Schwarzschild background:

In-In two point function: (Candelas, 1980)

$$\langle\Psi|\phi(x)\phi(x')|\Psi\rangle$$

Transition probabilities: (Bekenstein+Meisels; Pananganden+Wald, 1977)

$$p(m \to n)$$

We will work in a toy model of a scalar field propagating in the BH background. Generalization to higher bosonic spins (almost) immediate. The EFT is then (in the BH rest frame)

$$S_{BH} = -M\int dt\sqrt{1+h_{00}} - \int dt Q(t)\phi$$

$$S_{bulk} = \frac{1}{2}\int d^4x\sqrt{g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Two-point function: Full theory

Consider spatial coincidence limit of curved space Wightman 2-pt function

$$\langle \Psi_{in} | \phi(x) \phi(x') | \Psi_{in} \rangle$$

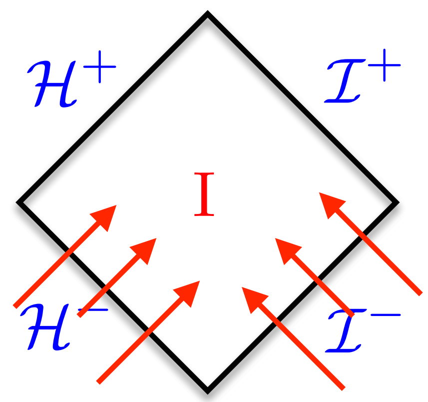
there are three physically reasonable choices for the initial vacuum state $|\Psi_{in}\rangle$:

$$|\Psi_{in}\rangle = |B\rangle$$

("Boulware state"):   Annihilated by + energy w.r.t Schwarzschild time
No Hawking radiation.

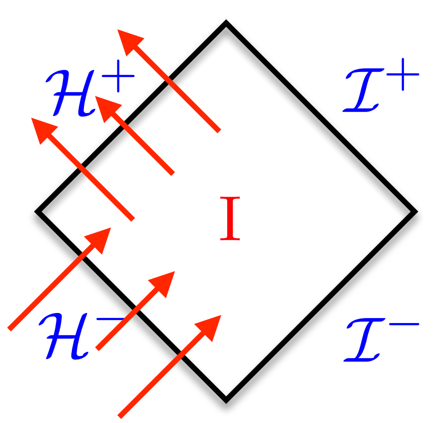
$$|\Psi_{in}\rangle = |U\rangle$$

("Unruh state"): Approx. description of BH formed in realistic gravitational collapse, followed by (eventual) evaporation due to Hawking emission.   Annihilated by + energy modes w.r.t that generate $\mathcal{H}^- \cup \mathcal{I}^-$



$$|\Psi_{in}\rangle = |H\rangle$$

("Hartle-Hawking state"):   BH in (unstable) thermal equilibrium with radiation bath at temp. T_H



$$G_{HH}(x + i\beta_H, x') = G_{HH}(x, x')$$

Taking the spatial coincidence limit, with $r \to \infty$

$$\langle\Psi|\phi(t)\phi(t')|\Psi\rangle = \langle 0|\phi(t)\phi(t')|0\rangle + \frac{1}{4\pi r^2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\sum_\ell (2\ell+1)F_\ell^\Psi(\omega) \;+\cdots$$

flat space
Wightman fn.

for $\Psi = B, U$, with:

$$F_\ell^B(\omega) = \frac{1}{2\omega}\theta(\omega)|B_\ell(\omega)|^2$$
$$F_\ell^U(\omega) = \frac{1}{2\omega}\frac{|B_\ell(\omega)|^2}{1-e^{-\beta_H\omega}}$$

Response function of "Unruh detector" at $r \to \infty$

For the Hartle-Hawking state:

$$\langle H|\phi(t)\phi(t')|H\rangle = \langle\phi(t)\phi(t')\rangle_\beta + \frac{1}{4\pi r^2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\sum_\ell (2\ell+1)F_\ell^H(\omega), \;+\cdots$$

flat space thermal
Wightman fn.

$$F_\ell^H(\omega) = F_\ell^U(\omega) = \frac{1}{2\omega}\frac{|B_\ell(\omega)|^2}{1-e^{-\beta_H\omega}}.$$

note that. $\Psi = U, H$ obeys the KMS condition $F_\ell^{U,H}(-\omega) = e^{-\beta_H\omega}F_\ell^{U,H}(\omega)$ required for detailed balance.

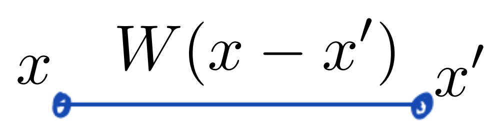
Two-point function: EFT

Focus on the case where full theory is $\Psi = B, U$.   From:

$$S_{BH} = -M \int dt \sqrt{1 + h_{00}} - \int dt Q(t)\phi$$

the perturbative calculation of the two-point function has the diagrammatic expansion:

$$\langle in|\phi(x)\phi(x')|in\rangle =$$



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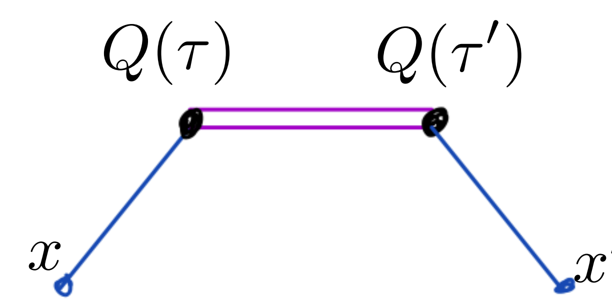

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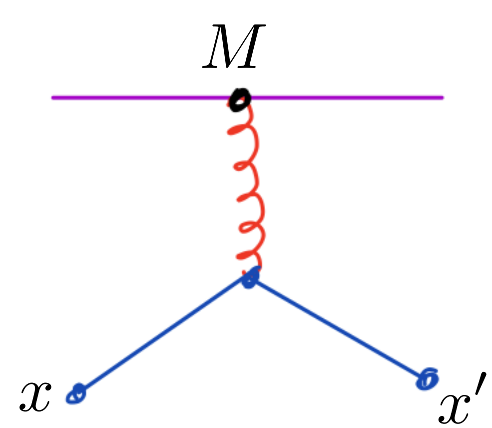


$+ \cdots$ (higher multipoles, more monopole insertions ....)

Absent in the spatial coincidence limit, since scattering phases cancel out (full theory independent of $\mathrm{Arg} A_\ell$)

The full theory observable is an expectation value in an initial state $|\Psi_{in}\rangle$ so must work in the IN-IN or "closed time path" formalism on the EFT side. DOFs in the path integral are doubled

$$\langle in|\phi(x)\phi(x')|in\rangle = \int DXD\tilde{X}\; D\phi(x)D\tilde{\phi}(x)\; e^{iS[\phi,X]}e^{-iS[\tilde{\phi},\tilde{X}]}\Psi_{in}[\tilde{\phi},\tilde{X}]^*\Psi_{in}[\phi,X]\;\tilde{\phi}(x)\phi(x')$$

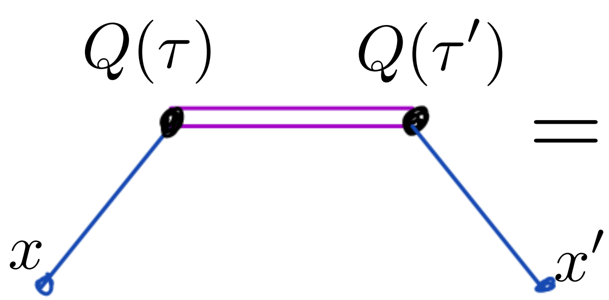
BH worldline DOFs

in order to keep track of insertions on either side of the Schwinger-Keldysh contour.

For the initial state we choose a factorized ansatz

$$\rho_{in} = |0\rangle\langle 0| \otimes \rho_{BH}$$

which is suggested by the $r\to\infty$ limit of the full theory result:



$$= \int d\tau d\tau' G^{2a}(x,x(\tau))\langle Q_a(\tau)Q_b(\tau')\rangle G^{b1}(x(\tau'),x')$$

$$G^{a,b}(x,x') = \begin{pmatrix} D_F(x-x') & -W(x'-x) \\ -W(x-x') & D_D(x-x') \end{pmatrix}$$

$$D_F(x-x') = -\frac{1}{4\pi^2}\frac{1}{(x-x')^2 - i\epsilon},$$

$$W(x-x') = -\frac{1}{4\pi^2}\frac{1}{(x_0-x_0'-i\epsilon)^2-(\vec{x}-\vec{x}')^2}.$$

$$\langle Q_a(\tau)Q_b(\tau')\rangle = \begin{pmatrix} \langle TQ(\tau)Q(\tau')\rangle & \langle Q(\tau')Q(\tau)\rangle \\ \langle Q(\tau)Q(\tau')\rangle & \langle T^*Q(\tau)Q(\tau')\rangle \end{pmatrix}$$

So in the spatial coincidence limit, the EFT 2-point function is

$$\langle in|\phi(x)\phi(x')|in\rangle = \langle\phi(t)\phi(t')\rangle + \frac{1}{(4\pi r)^2}\int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\left[A_+(\omega)+\theta(\omega)(A_+(\omega)-A_+(-\omega))\right]$$

$$A_+(\omega) = \int dt e^{i\omega t}\langle Q(t)Q(0)\rangle$$

vs. full theory, $\Psi = B, U$:

$$\langle\Psi|\phi(t)\phi(t')|\Psi\rangle = \langle 0|\phi(t)\phi(t')|0\rangle + \frac{1}{4\pi r^2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\sum_\ell(2\ell+1)F_\ell^\Psi(\omega)$$

The spatial dependence of EFT result is consistent with the full theory. Matching yields the EFT 2-point Wightman function at LO:

$$A_+^B(\omega) = \theta(\omega)\omega\sigma^{abs}_{\ell=0}(\omega)$$

$$A_+^U(\omega) = \frac{\omega\sigma_{abs}(|\omega|)}{e^{\beta_H\omega}-1}\left[2e^{\beta_H\omega}\theta(-\omega)+\theta(\omega)(1+e^{\beta_H\omega})\right] \approx 2\beta_H^{-1}\sigma_{abs}(|\omega|) = 2r_s + \mathcal{O}(r_s\omega)^2$$

$$A_+^U/A_+^B \sim r_s\omega$$

Check:

Energy flux at infinity:

$$\langle in|T^{rt}(x)|in\rangle = -\frac{1}{2}\lim_{x'\to x}\left(\partial_r\partial_{t'} + \partial_t\partial_{r'}\right)G(x,x'). \quad = \quad \frac{1}{8\pi^2 r^2}\int\frac{d\omega}{2\pi}\theta(\omega)\omega^2 A_+(-\omega)$$

$$\frac{dM}{dt} = \lim_{r\to\infty} 4\pi r^2\langle in|T^{rt}(x)|in\rangle = \int_0^\infty d\omega\hbar\omega\frac{d\Gamma}{d\omega}$$

$$\frac{d\Gamma}{d\omega} = \frac{\omega}{4\pi^2}\theta(\omega)A_+^{\Psi}(-\omega)$$

No radiation at infinity if full theory is in the Boulware state. For Unruh, we get the Rayleigh-Jeans (ie classical) distribution for a greybody at $T = T_H$

$$\frac{d\Gamma}{d\omega} \approx \frac{\omega}{2\pi^2}kT_H\sigma^{abs}_{\ell=0}(\omega) \approx \frac{1}{2\pi^2}r_s\omega, \qquad \text{(Page, 1974)}$$

Transition probabilities: Full Theory

Probability to emit n particles in normalized wavepacket, starting with m particles (same wavepacket) in the initial state:

(Bekenstein+Meisels; Pananganden+Wald, 1977)

$$p_\ell(m \to n) = \frac{(1-x)x^n(1-|R_\ell|^2)^{n+m}}{(1-x|R_\ell|^2)^{n+m+1}} \sum_{k=0}^{\min(n,m)} \frac{(n+m-k)!}{k!(n-k)!(m-k)!} \left[\frac{(|R_\ell|^2-x)(1-x|R_\ell|^2)}{x(1-|R_\ell|^2)^2}\right]^k$$

Boltzmann factor $x = e^{-\beta_H \omega}$

$$|R_\ell(\omega)|^2 = 1 - |B_\ell(\omega)|^2$$

Transition probs. obey detailed balance condition:

$$e^{m\beta_H\omega} p(m \to n) = e^{n\beta_H\omega} p(n \to m)$$

necessary such that BH can come to equilibrium if placed in a thermal bath at Hawking temp. (Hartle-Hawking state).

Transition probabilities: EFT

Dyson's formula for the amplitudes

$$i\mathcal{A}(m+M\to n+X) = \langle X;m|T\exp\left[-i\int dt Q(t)\phi(x)\right]|M;n\rangle$$

Multi-particle initial/final states in normalizable s-wave state (peaked at $|\vec{k}|\approx\omega$)

$$|n\rangle = \frac{1}{\sqrt{n!}}\left[\int\frac{d^3\vec{k}}{(2\pi)^3 2|\vec{k}|}\psi(\vec{k})a^\dagger(k)\right]^n|0\rangle \qquad \int\frac{d^3\vec{k}}{(2\pi)^3 2|\vec{k}|}|\psi(\vec{k})|^2 = 1$$

Sum over final BH states:

$$p(m\to n) = \sum_X |\mathcal{A}(m+M\to n+X)|^2$$

Multi-particle probs in terms of higher pt. correlators:

$$m \neq n$$

$$\frac{p(m\to n)}{p(0\to 0)} \approx \frac{n!}{k!^2 m!}\left[\prod_{j=1}^{k}\int d\tilde{t}_j dt_j \psi(\tilde{t}_j)\psi^*(t_j)\right] G^{(n,n)}(\tilde{t}_1,\cdots \tilde{t}_k; t_1,\cdots t_k), \quad (n>m)$$

$$\frac{p(m\to n)}{p(0\to 0)} \approx \frac{m!}{k!^2 n!}\left[\prod_{j=1}^{k}\int d\tilde{t}_j dt_j \psi^*(\tilde{t}_j)\psi(t_j)\right] G^{(n,n)}(\tilde{t}_1,\cdots \tilde{t}_k; t_1,\cdots t_k), \quad (n<m)$$

$$\psi(t) = \sqrt{\tfrac{\omega}{2\pi}}\int_0^\infty \tfrac{dk}{2\pi} e^{-ikt}\psi_0(k)$$

$$k = |n-n|$$

depends on Wightman functions

$$G^{(n,m)}(\tilde{t}_1,\cdots,\tilde{t}_n; t_1,\cdots,t_m) = \langle M|\tilde{T}[O(\tilde{t}_1)\cdots O(\tilde{t}_n)]T[O(t_1)O(t_m)]|M\rangle$$

e.g:

$$p(1\to 0) \approx \sum_X \left|\int dt \langle X;0|Q(t)|M\rangle\langle 0|\phi(x)|1\rangle\right|^2 = \frac{\omega}{2\pi}A_+(\omega)$$

$$p(0\to 1) = \tfrac{\omega}{2\pi}A_+(-\omega)$$

$$A_+(\omega) \approx A_-(\omega) = 2\beta_H^{-1}\sigma^{abs}_{\ell=0}(\omega) \approx 2r_s + \cdots$$

consistent w/ $\langle in|\phi(x)\phi(x')|in\rangle$

matching

For $m \neq n$

$$\frac{p(m \to n)}{p(0 \to 0)} \approx \frac{(n+m-k_*)!}{k_*!(n-k_*)!(m-k_*)!} \left[\frac{|B_0(\omega)|^2}{\beta_H \omega}\right]^{n+m-k_*}$$

$k_* = \min(m,n)$

The combinatorics of this formula suggests that the EFT correlators are Gaussian, ie. they factorize according to the IN-IN version of Wick's theorem. For example, the four-point funtion:

$$\begin{aligned} G^{(2,2)}(\tilde{t}_1, \tilde{t}_2, t_1, t_2) &= \langle Q(\tilde{t}_1)Q(t_1)\rangle\langle Q(\tilde{t}_2)Q(t_2)\rangle + \langle Q(\tilde{t}_1)Q(t_2)\rangle\langle Q(\tilde{t}_2)Q(t_1)\rangle \\ &+ \langle \tilde{T}Q(\tilde{t}_1)Q(\tilde{t}_2)\rangle\langle TQ(t_1)Q(t_2)\rangle \end{aligned}$$

or more generally, $G^{(n,m)}$ has generating fn.

$$Z[J] = \exp\left[\frac{1}{2}\int d\tau d\tau' J_a(\tau)\langle O_a(\tau)O_b(\tau')\rangle J_b(\tau')\right]$$

Making this ansatz, the EFT result becomes

$$\frac{p(m \to n)}{p(0 \to 0)} \approx \frac{(n+m-k_*)!}{k_*!(n-k_*)!(m-k_*)!} \left[\frac{\omega}{2\pi}A_+(-\omega)\right]^{n-k_*} \left[\frac{\omega}{2\pi}A_+(\omega)\right]^{m-k_*}$$

consistent with the full theory,.

Perhaps not too surprising that the EFT is Gaussian, given that we are matching to a free field propagating in a curved background.

Interpretation of result:

We have found that, for a classical BH the response in the Unruh state, corresponding to a BH decaying via Hawking radiation, the worldline Wightman 2-pt fns are to LO:

$$A_+^U(\omega) \approx A_+^U(\omega) = 2r_s + \cdots$$

while in the Boulware state, where the response is purely absorptive (i.e "classical")

$$A_+^B(\omega > 0) = 4\pi r_s^2 \omega + \cdots \qquad\qquad A_+^B(\omega < 0) = 0$$

(classical absorption) (emission)

Rather than being suppressed by powers of $\hbar$ and/or $1/M_{Pl}$, the Hawking contribution to the BH's response function is enhanced by a power of $1/\beta_H \omega = (4\pi r_s \omega)^{-1}$.

The enhancement $A_+^U/A_+^B \sim 1/r_s\omega$ is just the usual Bose enhancement of black body radiation at low temperature, $T_H \to 0$, which is the same limit $r_s\omega \ll 1$ where the EFT is valid.

However, the lack of suppression does not mean that the response induced by Hawking radiation is somehow observable in macroscopic ("classical") experiments, which are only sensitive to the retarded response fn

$$G_{ret}(x,x') = i\theta(t-t')\langle\Psi|[\phi(x),\phi(x')]|\Psi\rangle$$

since, ignoring interactions in the bulk, canonical quantization in a fixed background implies that

$$[\phi(x),\phi(x')] = \text{c-number}$$

So G_{ret} is independent of the state $|\Psi\rangle$. In the EFT,

$$\langle[\phi(x),\phi(x')]\rangle = \frac{1}{8\pi r^2}\int\frac{d\omega}{2\pi}e^{-i\omega t}\left[A_+^\Psi(\omega) - A_+^\Psi(-\omega)\right]$$

$$A_+^B(\omega) - A_+^B(-\omega) = A_+^U(\omega) - A_+^U(-\omega) = \omega\sigma_{abs}(|\omega|)$$

Hawking radiation cannot be detected in classical observables, which by causality, only depend on .

$$G_R(\omega) = -i\int\frac{d\omega'}{2\pi}\frac{A_+(\omega') - A_-(\omega')}{\omega-\omega'-i\epsilon}$$

Quantum effect of Hawking radiation are not Planck suppressed at the level of the Wightman functions, but suppressed by $\omega^2/M_{Pl}^2 \ll 1$ in G_{ret} due to non-linearities in field eqns.

Check:

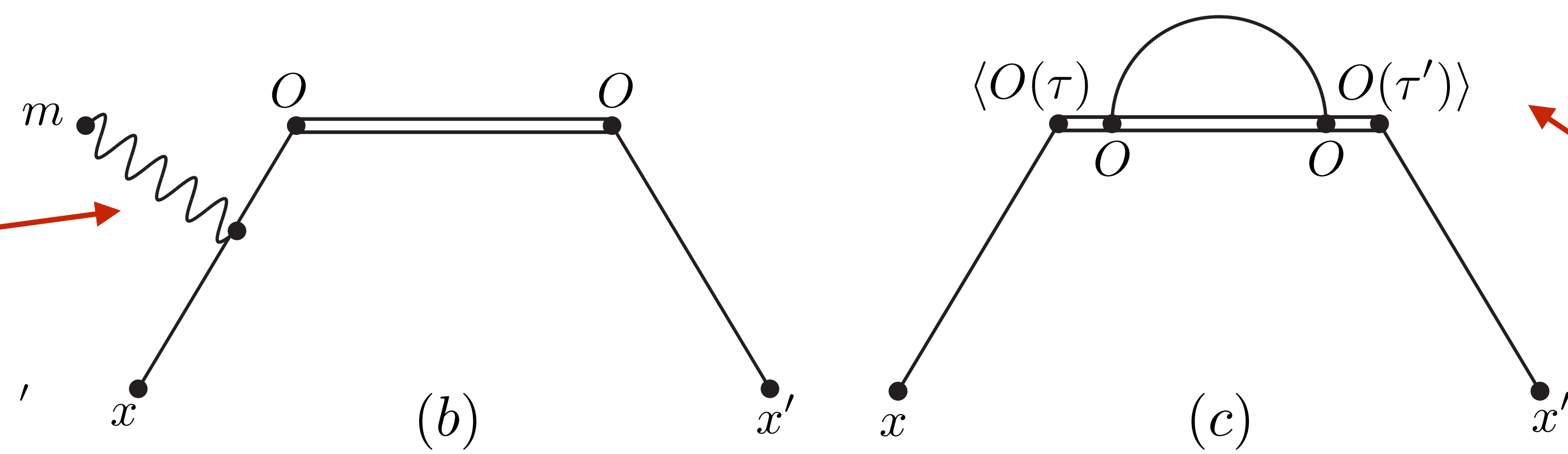
LO:
$$A_+^B(\omega) - A_+^B(-\omega) \approx 4\pi r_s^2 \omega,$$

$$A_+^U(\omega) - A_+^U(-\omega) = 0 + \mathcal{O}(r_s\omega)$$

Due to Bose enhancement/detailed balance, must go to NLO in $r_s\omega$ to check state independence…

NLO:

(pot. graviton exchange with source)



$\langle O_a O_b O_c O_d \rangle$

(LO IN-IN 4pt.)

$$A_+^U(\omega) = 2\beta_H^{-1}\sigma_{abs}(|\omega|)\left[\theta(\omega)\left(1-\frac{1}{4}\beta_H\omega\right)+\theta(-\omega)\left(1+\frac{3}{4}\beta_H\omega\right)\right]+\mathcal{O}(r_s\omega)^3$$

$$A_+^U(\omega) - A_+^U(-\omega) = \omega\sigma_{abs}(|\omega|) + \mathcal{O}(r_s\omega)^3 \approx 4\pi r_s^2\omega$$

In agreement with Boulware state response.

Application: Inelastic gravitational scattering off BH.

WG+Rothstein, in progress

Bekenstein/Wald-Panaganden formula generalizes to gravitons, which allows us to match the Wightman functions of the grav. $\ell = 2$ operators in the EFT,

$$S_{int} = -\int d\tau \left(Q^E_{\mu\nu} E^{\mu\nu} + Q^B_{\mu\nu} B^{\mu\nu}\right) + \cdots$$

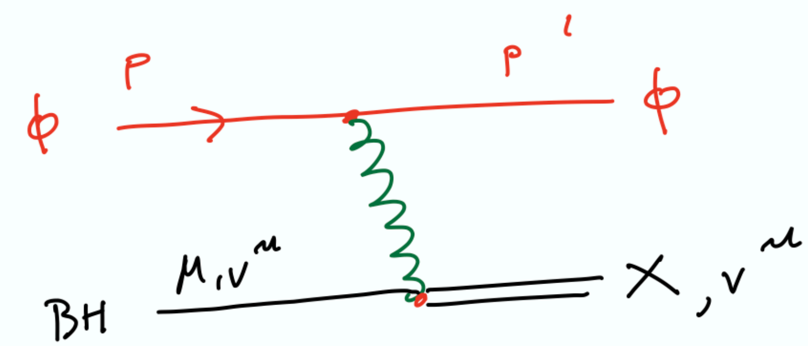
$$p(1 \to 0) \approx p(0 \to 1)$$

$$\approx \frac{|B^h_{\ell=2,m}(\omega)|^2}{\beta_H \omega} \approx \frac{(r_s \omega)^6}{225\pi} \quad \Rightarrow \quad A^E_+(\omega) = A^B_+(\omega) \approx A^E_+(-\omega) = \frac{\pi}{45} \frac{r_s^5}{G_N}$$

Wightman functions can be used to predict inclusive scattering cross section $\phi + BH \to \phi + BH'$:

$$\sum_X |\mathcal{A}(\phi(p) + BH \to \phi(p') + X)|^2$$

$$= \sum_X \Bigg|$$



$$\Bigg|^2 \quad \Rightarrow$$

$$\frac{d^2\sigma}{dq^2 d(q\cdot v)} \approx \frac{m r_s^5}{2880 m_{Pl}^2} \frac{\left[(v\cdot p)^2 + m^2 \frac{(v\cdot q)^2}{q^2}\right] \cdot \left[\frac{4}{3}\left((v\cdot p)^2 + m^2 \frac{(v\cdot q)^2}{q^2}\right) - m^2\right]}{(v\cdot p)^2 \sqrt{(p\cdot v)^2 - m^2}}$$

(In BH rest frame $v^\mu = (1, 0)$)

Suppressed over elastic scattering by a factor of $\sim \frac{q^2}{M_{Pl}^2}(r_s q)^3$ relative to LO scattering off BH's Newtonian gravitational field.

This is same order in q^2/M_{Pl}^2 as the correction to elastic scattering from one-loop graviton vacuum polarization, a new calculable perturbative quantum gravity effect…

Conclusions

Constructed a consistent EFT to describe BH horizon dynamics interacting with soft radiation, with applications to classical and quantum multi-body processes. There is no hierarchy of scales between classical and quantum (Hawking) effects at the level of Wightman correlators in this EFT.

Despite this, the effects of Hawking radiation drop out of "classical observables," ie those that depend only on

$$G_{ret}(x,x') = i\theta(t-t')\langle\Psi|[\phi(x),\phi(x')]|\Psi\rangle$$

at least up to (calculable) corrections down by powers of $\hbar\omega/M_{Pl} \ll 1$

Classically, this EFT has implication for dissipative effects in BH-BH or BH-NS binaries. Quantum mechanically, expect applications to BH-BH scattering amplitudes in quantum gravity, soft factorization theorems for Hawking emission in BH collisions, bound states…