Localization of information in quantum gravity

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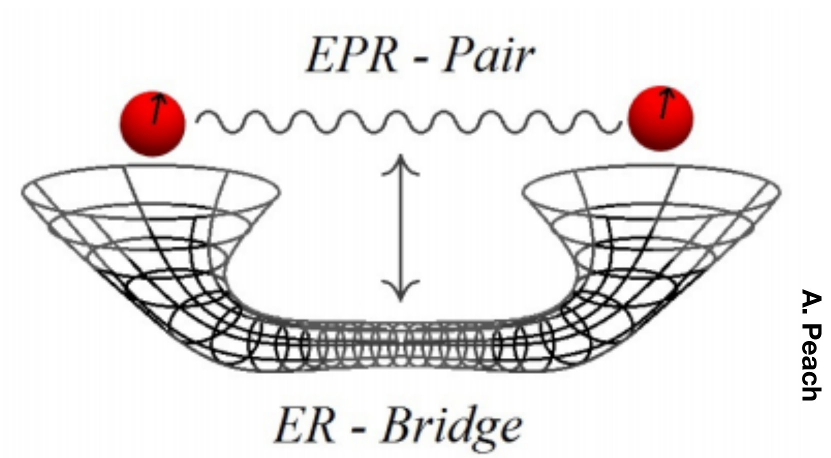
Common Theme: *emergent spacetime*

Emergent from what?

Entanglement?

ER=EPR



E.g.

CFT x CFT

+ entanglement (thermofield double)

= two-sided AdS BH

Alternate possibility: from other *hardwired mathematical structure*

On what?

Examples?

(Then entanglement = useful property, but dependent on other structure ~ epiphenomenon)

Starting point: suppose that there *is* a quantum-mechanical theory of gravity

Expect implies basic mathematical structure

$\mathscr{H}$ **Linear space of states ("Hilbert space")**

$\mathscr{A}$ **Linear operators: quantum observables**

Unitarity (in appropriate circumstances, e.g. S-matrix)

These are, apparently, the universal elements of a QM theory.

(discussion: 0711.0757)

What *additional* structure is required for a physical theory of gravity?

On $\mathscr{H}$: "Quantum-first" approach

1803.04973;1805.06900
also Carroll + collabs

(Possibly from AdS/CFT, or possibly other structure)

Guides:

Sufficient mathematical structure to describe physics, with gravity

Correspondence w/ local QFT + gravity, in "weak-field" limit

Key basic notion: localization of information (definition of subsystems)

Einstein separability:

"… it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things 'lie in different parts of space.' Without such an assumption of the mutually independent existence (the 'being-thus') of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible. Nor does one see how physical laws could be formulated and tested without such a clean separation."

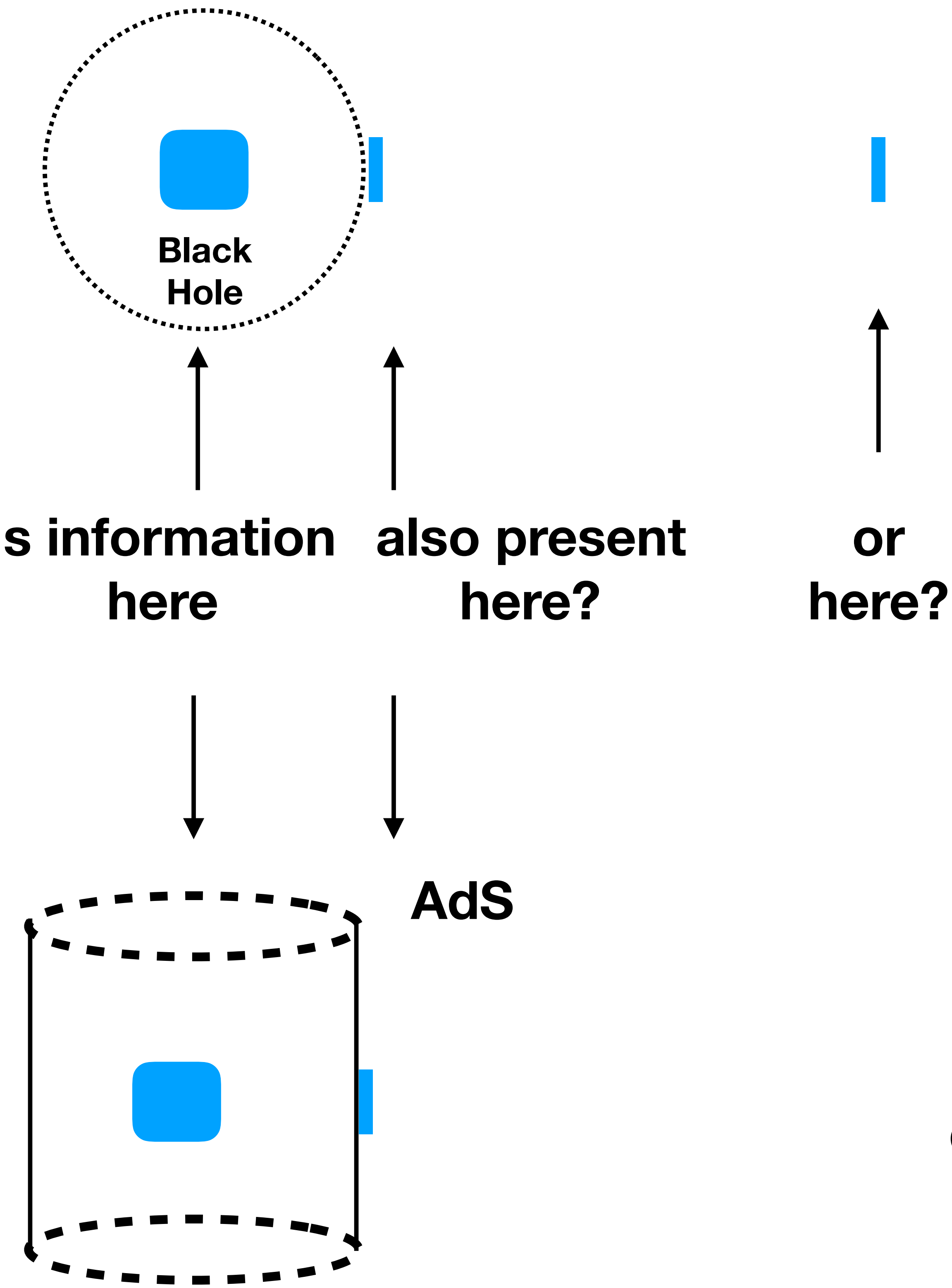
-A. Einstein

Prerequisite **for entanglement (+ entropy), information transfer, complexity, …**

Often taken for granted, but particularly nontrivial in gravity

Subtleties of information localization illustrated by:

soft hair:
(pert.)



**How to describe/
understand?**

**"is a black hole a
quantum subsystem?"**

holography:
(nonpert.)

**How to reconcile/explain
different notions of localization?**

What mathematical structure on $\mathscr{H}$ describes localization of information?

Finite quantum systems:

$$\mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$$

e.g. information localized here ***not here***

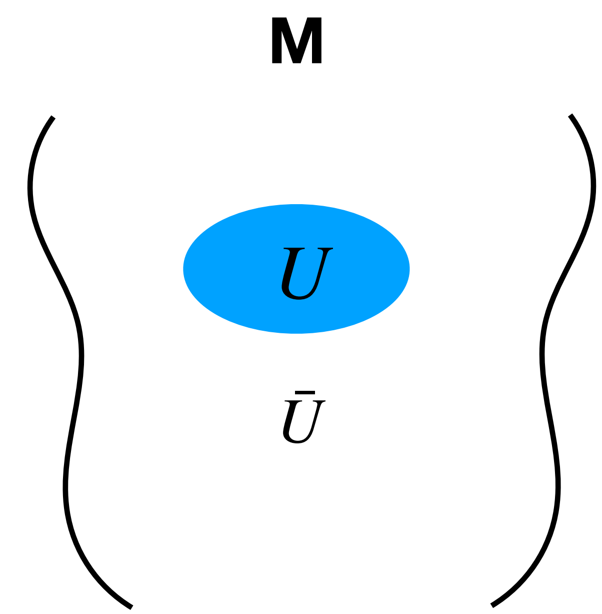
LQFT:

$$\mathscr{H} \neq \mathscr{H}_U \otimes \mathscr{H}_{\bar{U}}$$

(vN type III: "infinite enganglement")

Instead, commuting subalgebras of $\mathscr{A}$, associated with open regions

$U \leftrightarrow \mathscr{A}_U$ **e.g.** $\phi_f = \int d^4x\, f(x)\phi(x)$



If U and U' are spacelike separated:

$$[A_U, A_{U'}] = 0$$

... locality

Subalgebras define "subsystems," localization of information

Mathematical structure: **(see, e.g., Haag)**

"net" of subalgebras $\xrightarrow{\text{emerges}}$ ***<u>topological structure of spacetime</u>***

Back to gravity:

If there is a Hilbert space in gravitational physics, is there an analogous structure on it, corresponding to "quantum spacetime," which reduces to this QFT structure in the weak gravity correspondence limit?

and *what is it*?

Explore in perturbative limit

$$\mathscr{L} = \frac{2}{\kappa^2} R - \frac{1}{2}\left[(\nabla \phi)^2 + m^2 \phi^2\right] \qquad \kappa = \sqrt{32\pi G}$$

(match onto QFT localization at small G: correspondence)

$\phi(x)$ **not gauge invariant** $\quad \delta_\xi \phi(x) = -\kappa \xi^\mu \partial_\mu \phi(x) \neq 0 \quad \longleftrightarrow$

$[C_\mu(x), \phi(y)] \neq 0$ **w/** $C_\mu(x) = G_{0\mu}(x) - 8\pi G\, T_{0\mu}(x)$

constraints

EM analog $\quad \delta\phi(x) = -iq\lambda(x)\ \phi(x)$

Gauge invariant observables via *dressing*

$$\Phi(x) = \phi(x) e^{i\Lambda(x)} \qquad [C(x), \Phi(y)] = 0$$

E.g. Faraday line

[Dirac, 1955]

$$\Lambda(x) = q \int_x^\infty A$$

x

Dressed observables in gravity: basic points

Let $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$

$\phi(x) \to \Phi(x) = \phi(x + V(x))$ **where** $V^\mu[h, x]$ **transforms as** $\delta V^\mu(x) = \kappa \xi^\mu(x)$

dressed observable

1507.07921 and 1607.01025 w/ Donnelly; 1805.06900

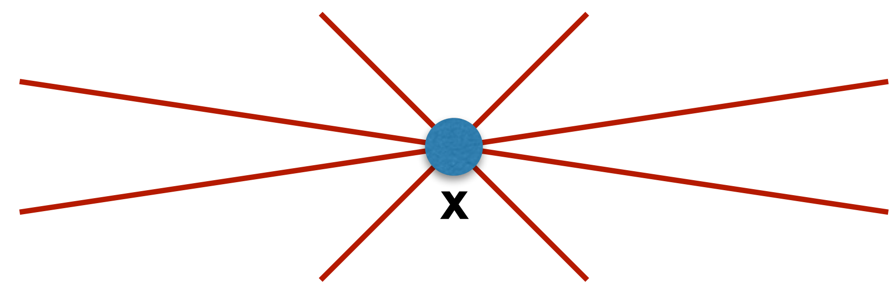
See also previous work: Heemskerk; Kabat and Lifschytz

- There are many choices of V^μ:

e.g.

gravitational line

$$V^\Gamma_\mu(x) = \frac{\kappa}{2}\int_x^\infty dx'^\nu \left\{ h_{\mu\nu}(x') + \int_{x'}^\infty dx''^\lambda \left[\partial_\mu h_{\nu\lambda}(x'') - \partial_\nu h_{\mu\lambda}(x'')\right]\right\}$$



Coulomb dressing

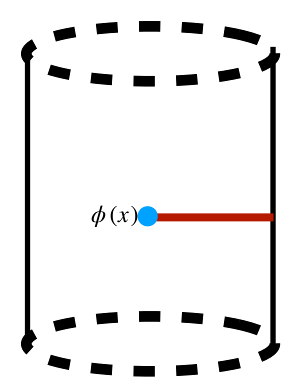
($\mathcal{O}(\kappa)$ constructions)

- These create different physical states

Different radiation fields

- Constructions extend to AdS

1802.01602 w/ Kinsella



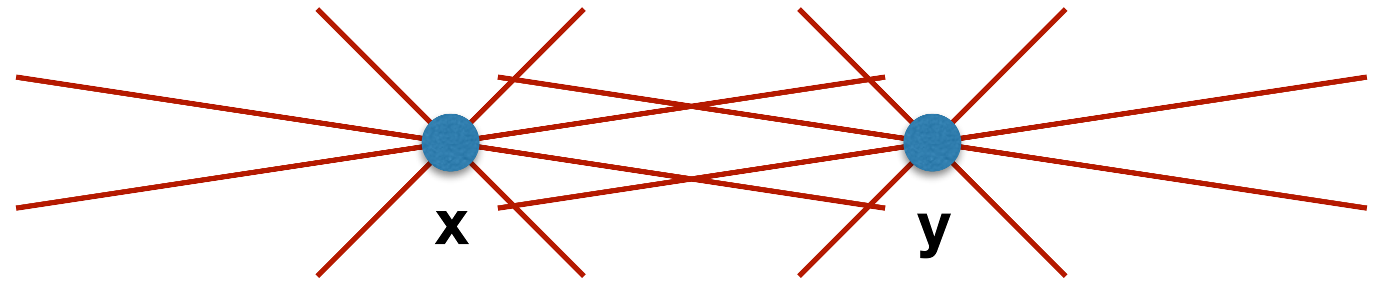
- Constructions extend to BH backgrounds

Including operators deep in BH interior

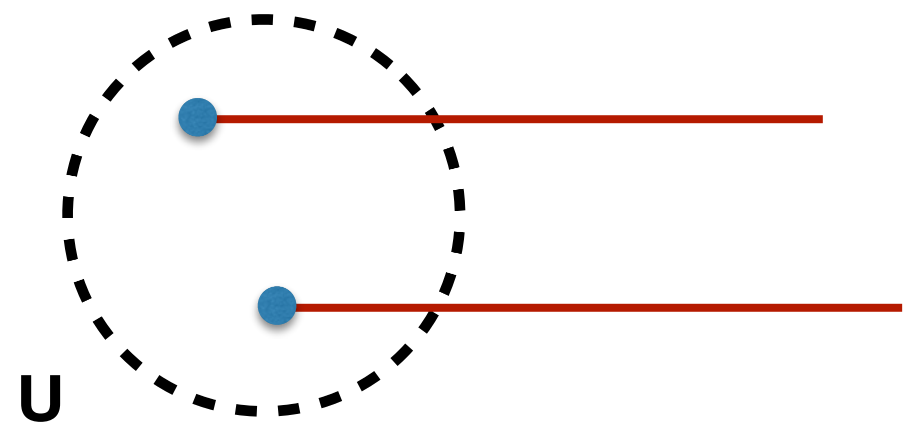
1911.09115 w/ S. Weinberg

Generically, don't commute at spacelike separation

$$[\Phi(x), \Phi(y)] \neq 0 \qquad (x-y)^2 > 0$$



Distant gravitational field config. depends on matter config.



These are a challenge to localization of information.

And: connect to soft hair

holography

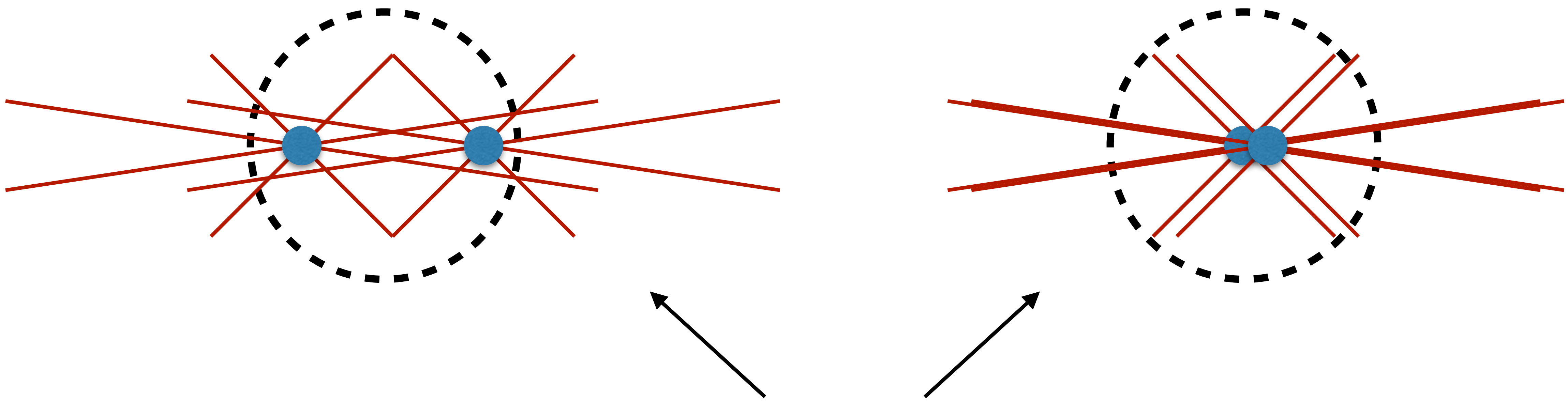
One asymptotic characteristic of gravitational field: soft charges

$$Q_\epsilon \sim \int_{S^2} d\Omega\, \epsilon(\theta) \lim_{r\to\infty} (r h_{..})$$

Strominger
Hawking, Perry, Strominger
...

$$[Q_\epsilon, \Phi(x)] = i \int d^3y\, [Q_\epsilon, V^\mu(y)]\, [T_{0\mu}(y), \phi(x)]$$

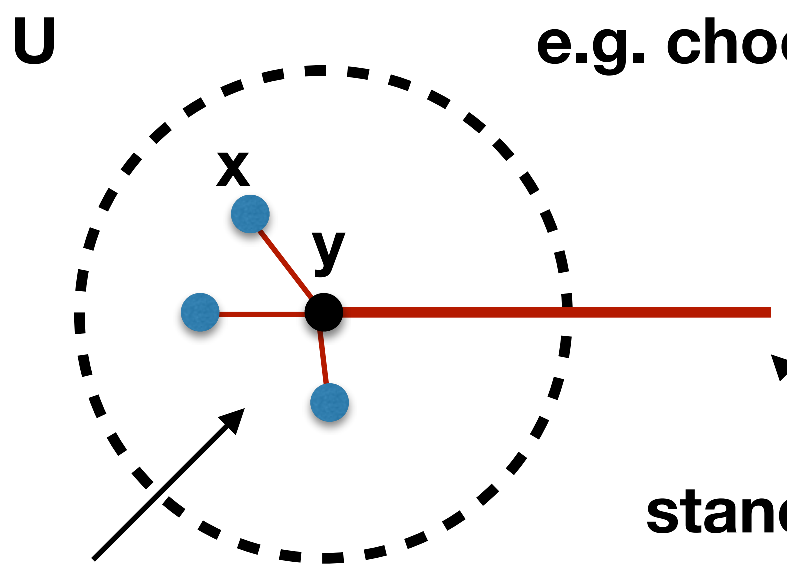
Does this allow determination of matter config?



different soft charges

No! Flexibility in dressing

In fact, can choose a "standard dressing" for all matter configurations



e.g. choose fiducial point y

1805.11095 w/ Donnelly, 1903.06160

standard dressing of y (*choice*)

$$\delta V^\mu_S(y) = \kappa \xi^\mu(y)$$

composite operator A

Can show:

$$[Q_\epsilon, \hat{A}] = -\, i q^\mu_{\epsilon,S}(y)\,[P_\mu, A] - \frac{i}{2}\partial^\mu q^\nu_{\epsilon,S}(y)\,[M_{\mu\nu}, A]$$

dressed composite operator

soft charges of standard dressing

Only depends on A through

P_μ **and** $M_{\mu\nu}$

I.e. *total* Poincare charges

Likewise for dressed state: $|\Psi\rangle \to |\,\widehat{\Psi}\,\rangle = e^{i\int d^3x\, V^\mu(x)\, T_{0\mu}(x)}|\Psi\rangle \; + \mathcal{O}(\kappa^2)$

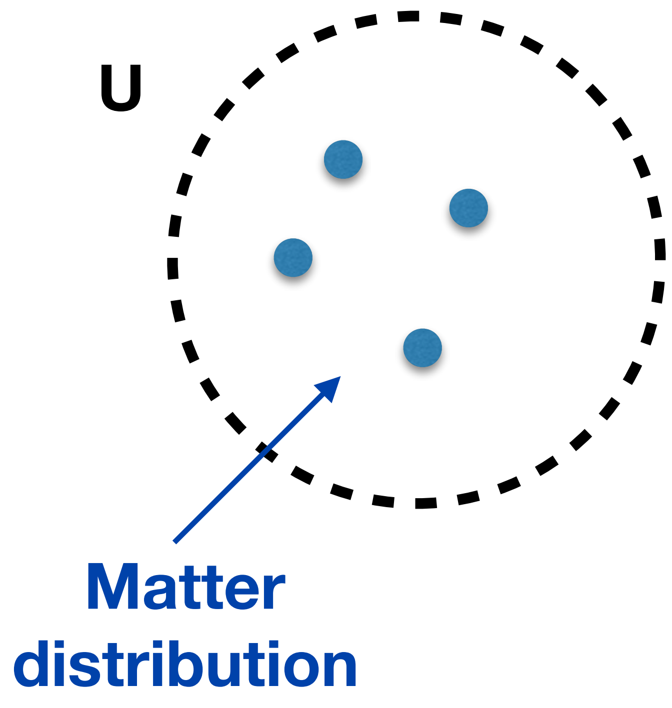
$$\langle \hat{\Psi}'|Q_1 \cdots Q_N|\hat{\Psi}\rangle = i\langle \Psi'|\left(q^{\mu_1}_{1,S}P_{\mu_1} + \frac{1}{2}\partial^{\mu_1}q^{\nu_1}_{1,S}M_{\mu_1\nu_1}\right)\cdots\left(q^{\mu_N}_{N,S}P_{\mu_N} + \frac{1}{2}\partial^{\mu_N}q^{\nu_N}_{N,S}M_{\mu_N\nu_N}\right)|\Psi\rangle + \cdots$$

Implication:

Information registered by soft charges is:

1) Poincare charges of matter distribution

2) Soft structure of superposed radiation field

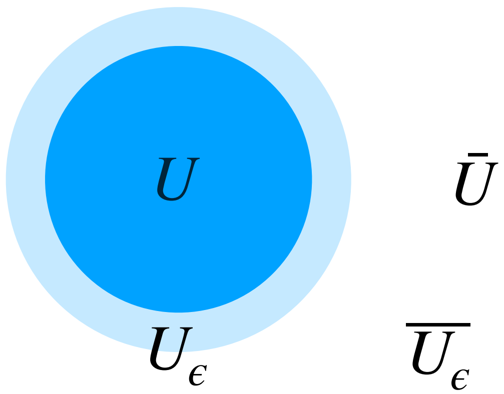


I.e. don't register other features of matter distribution
(at this leading order in κ - plausibly extends)

(related comments Bousso/Porrati)

Suggests an approach to localize information, perturbatively

First, back to QFT and another way to think about localization



Said: $\mathscr{H} \neq \mathscr{H}_U \otimes \mathscr{H}_{\bar{U}}$

But, split vacuum: $|U_\epsilon\rangle$ **[Haag, and refs. therein]**

Key property: **For** $A \in \mathscr{A}_U$ **,** $A' \in \mathscr{A}_{\overline{U}_\epsilon}$ **:** $\langle U_\epsilon | AA' | U_\epsilon\rangle = \langle 0|A|0\rangle\langle 0|A'|0\rangle$

"disentangles" degrees of freedom

$A_I|U_\epsilon\rangle$ **,** $A_J|U_\epsilon\rangle$ **indistinguishable via measurements in** $\overline{U_\epsilon}$

$A_I, A_J \in \mathscr{A}_U$

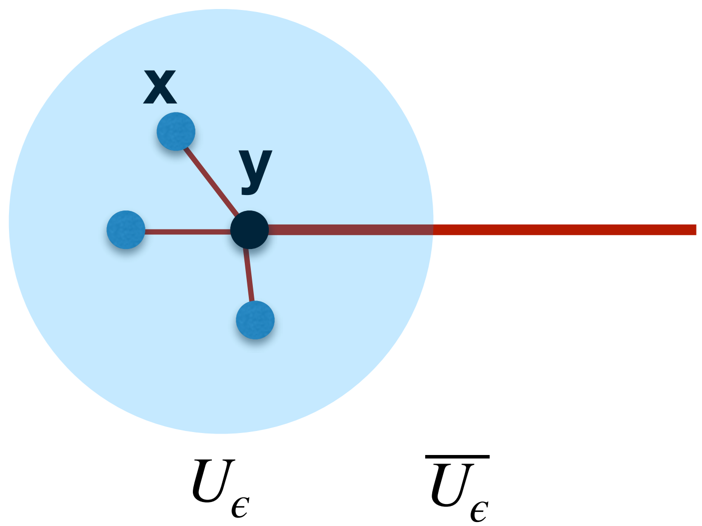
~ "localized qubit"

So, have localized quantum information.

Mathematical structure: $\mathscr{H}_U \otimes \mathscr{H}_{\overline{U}_\epsilon} \hookrightarrow \mathscr{H}$

Previous discussion suggests analog in gravity

The idea: build states on split vacuum, with a standard dressing



"Expresses" P, M

Observables here only "see" P and M

I.e. *localization of information* (perturbative)

localized "gravitating qubits"

Candidate mathematical structure

$$\sim \bigoplus_{P_\mu, s, s_z} \mathscr{H}_{U, P_\mu, s, s_z} \otimes \mathscr{H}_{\overline{U}_\epsilon, P_\mu, s, s_z} \hookrightarrow \mathscr{H}$$

"Gravitational splitting"

Network of Hilbert space embeddings

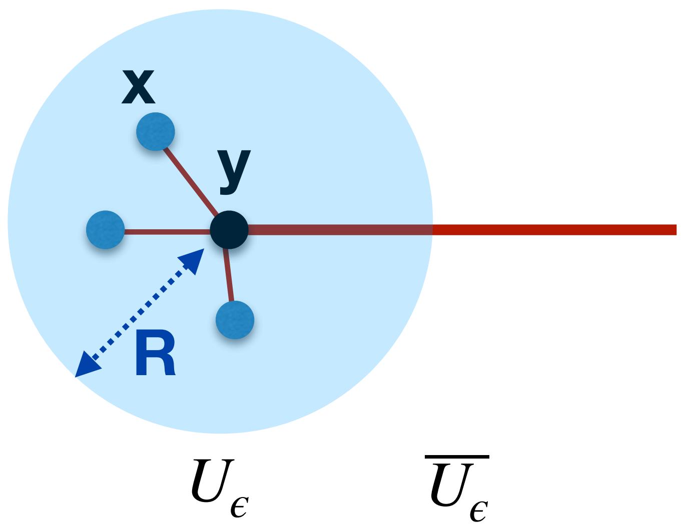
[1903.06160]

What role does this have in full theory of quantum gravity?

(At minimum, it needs to reproduce this in weak limit)

Nonperturbative questions

1. Strong field behavior



E.g. $R_s(\langle E\rangle) > R$: strong field extends outside;

Not all naive QFT states allowed

Related restriction on inclusion maps

Smallest Hilbert space? $R \gtrsim \kappa$

(some more discussion: 1803.04973)

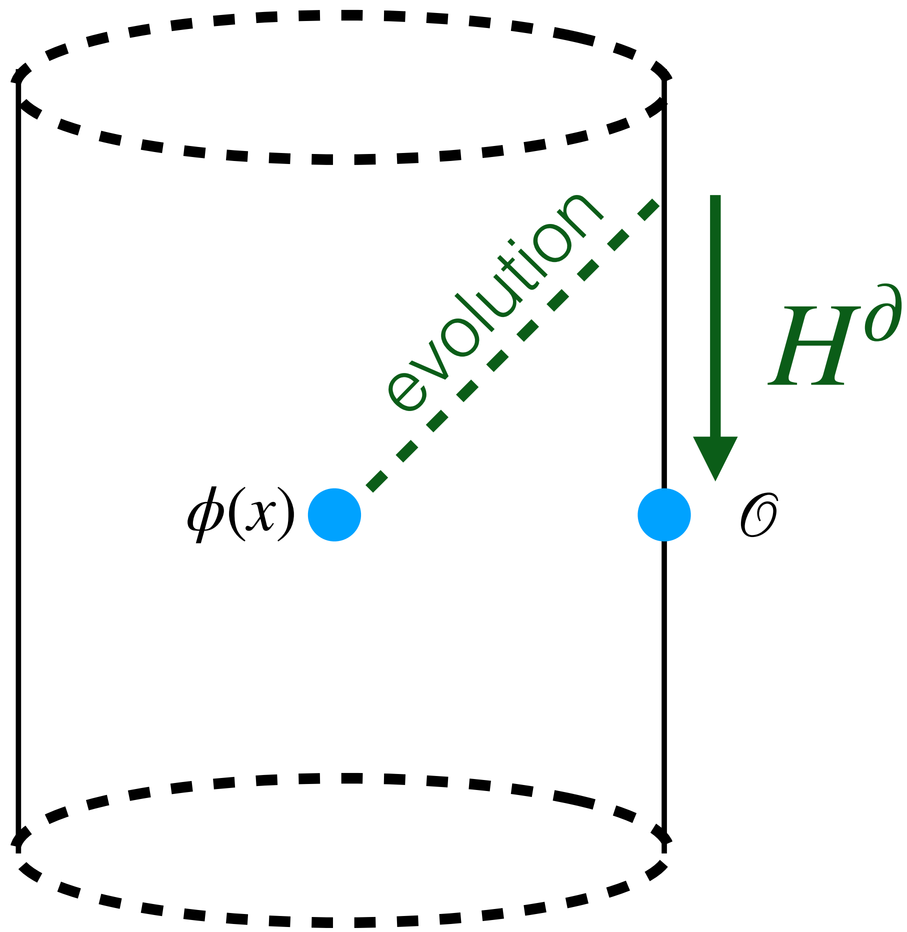
2. Holography and dressing

Holography: candidate explanation/derivation of holographic map

Marolf [0808.2842, 1308.1977] (+ Jacobson 1212.6944, …):

By virtue of gravitational constraints.

AdS:



In gravity, H is a surface term: H^∂

if the constraints are solved

$$C_\mu(x) = G_{0\mu}(x) - 8\pi G\, T_{0\mu}(x) = 0$$

Recall dressing $\longleftrightarrow$ $[C_\mu(x), \Phi(y)] = 0$

Alternate construction: $\lim_{a\to\infty} e^{ia^\mu P^\partial_\mu}\, \Phi(x)\, e^{-ia^\mu P^\partial_\mu}$

In either case:

Apparently need large translation

$\longleftrightarrow$ **need to solve grav. constraints to *all orders* in κ**

i.e. nonperturbative dressing

$\longleftrightarrow$ **Specifying unitary bulk evolution**

So suggests: **Need to know unitary bulk evolution**
in order to construct this holographic map

apparently important conclusion

[Donnelly & SBG, 1706.03104; SBG & Kinsella 1802.01602; see also Jacobson and Nguyen]

(Similarly ent wedge reconstruction.
need "$H_{\rm bulk} = H_\partial$" : constraints)

- if *do* have, appears to change story of localization?

(e.g. preceding translation argument)

how to extend definition of splitting, nonperturbatively?

Summary

Question of defining subsystems appears to be a very basic one

Prerequisite for entanglement, information transport, complexity, entropy, …

Gravity (perturbative) appears to have distinctive mathematical structure

~ “Gravitational splitting”

Questions surrounding nonperturbative extension

Dressing plays key role in candidate holographic map

Need nonperturbative solution of constraints = unitary evolution

Similarly EWR? (How to *define*, nonperturbatively?)

Can explore counterpoints:

Is there a way to define holographic map without solving constraints?

Does gravitational splitting only emerge in weak field limit?

(e.g. from some story involving entanglement of other subsystems, in CFT?)

But:

Possibly $\mathscr{H}$, plus such structure = fundamental ingredient in nonperturbative quantum gravity

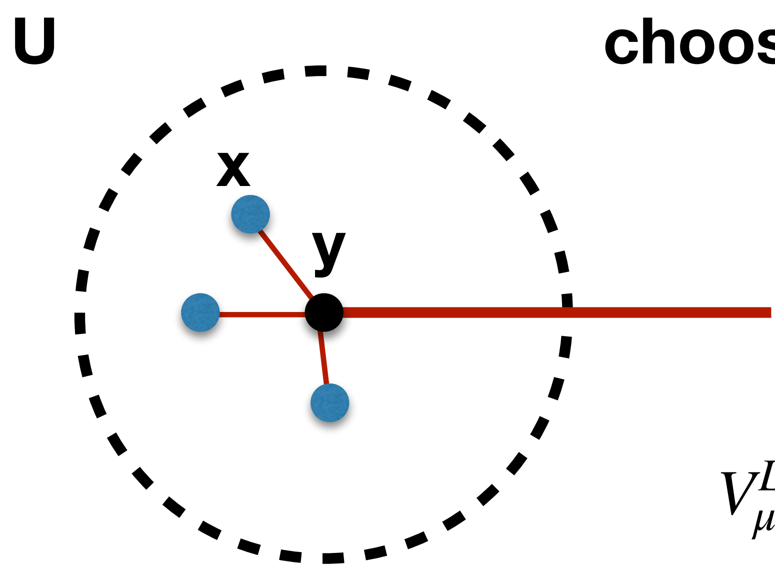
"Quantum first"

"Geometry from the quantum" —> Geometry from quantum structure (on $\mathscr{H}$)

Backups

$\mathcal{O}(\kappa)$ construction for gravity:

[1805.06900, 1805.11095 w/Donnelly]



choose fiducial point y

$$V^L_\mu(x,y) = -\frac{\kappa}{2}\int_y^x dx'^\nu \left\{ h_{\mu\nu}(x') - \int_y^{x'} dx''^\lambda \left[\partial_\mu h_{\nu\lambda}(x'') - \partial_\nu h_{\mu\lambda}(x'')\right] \right\}$$

$$V^L(y) = V^L_\mu(y,\infty)$$

Then:

$$V^\mu(x) = V^\mu_L(x,y) + V^\mu_L(y) + \frac{1}{2}(x-y)_\nu[\partial^\nu V^\mu_L(y) - \partial^\mu V^\nu_L(y)]$$ **satisfies** $$\delta V^\mu(x) = \kappa\xi^\mu(x)$$

“key relation”

- **Creates “standard” line grav. fields outside U:** $\tilde{h}^\mu_{\lambda\sigma}(x) \sim [h_{\lambda\sigma}(x), V^\mu_L(y)]$

- **Works for *any* “standard”** $V^\mu_S(y)$ **satisfying** $\delta V^\mu_S(y) = \kappa\xi^\mu(y)$

So, argument ~ EM:

$$[Q_\epsilon, \hat{A}] = i\int d^3x\, [Q_\epsilon, V^\mu(x)]\, [T_{0\mu}(x), A]$$

E.g consider dressing a localized A

$$V^\mu(x) = V^\mu_L(x,y) + V^\mu_S(y) + \frac{1}{2}(x-y)_\nu[\partial^\nu V^\mu_S(y) - \partial^\mu V^\nu_S(y)]$$

$$[Q_\epsilon, V^\mu_L(x,y)] = 0$$

(since $V^\mu_L(x,y)$ localized to U)

Let $$[Q_\epsilon, V^\mu_S(y)] = q^\mu_{\epsilon,S}(y)$$

"soft charges" of standard dressing

$$[Q_\epsilon, \hat{A}] = -iq^\mu_{\epsilon,S}(y)\,[P_\mu, A] - \frac{i}{2}\partial^\mu q^\nu_{\epsilon,S}(y)\,[M_{\mu\nu}, A]$$

Only depends on A through P_μ and $M_{\mu\nu}$

I.e. total Poincare charges

(compare EM)

Likewise

$$\langle\hat{\Psi}'|\, Q_1 \cdots Q_N \,|\hat{\Psi}\rangle = i\langle\Psi'|\left(q^{\mu_1}_{1,S}P_{\mu_1} + \frac{1}{2}\partial^{\mu_1}q^{\nu_1}_{1,S}M_{\mu_1\nu_1}\right)\cdots\left(q^{\mu_N}_{N,S}P_{\mu_N} + \frac{1}{2}\partial^{\mu_N}q^{\nu_N}_{N,S}M_{\mu_N\nu_N}\right)|\Psi\rangle + \cdots$$

This depends on the moments of the total Poincare charges, and on the soft charges of the standard dressing