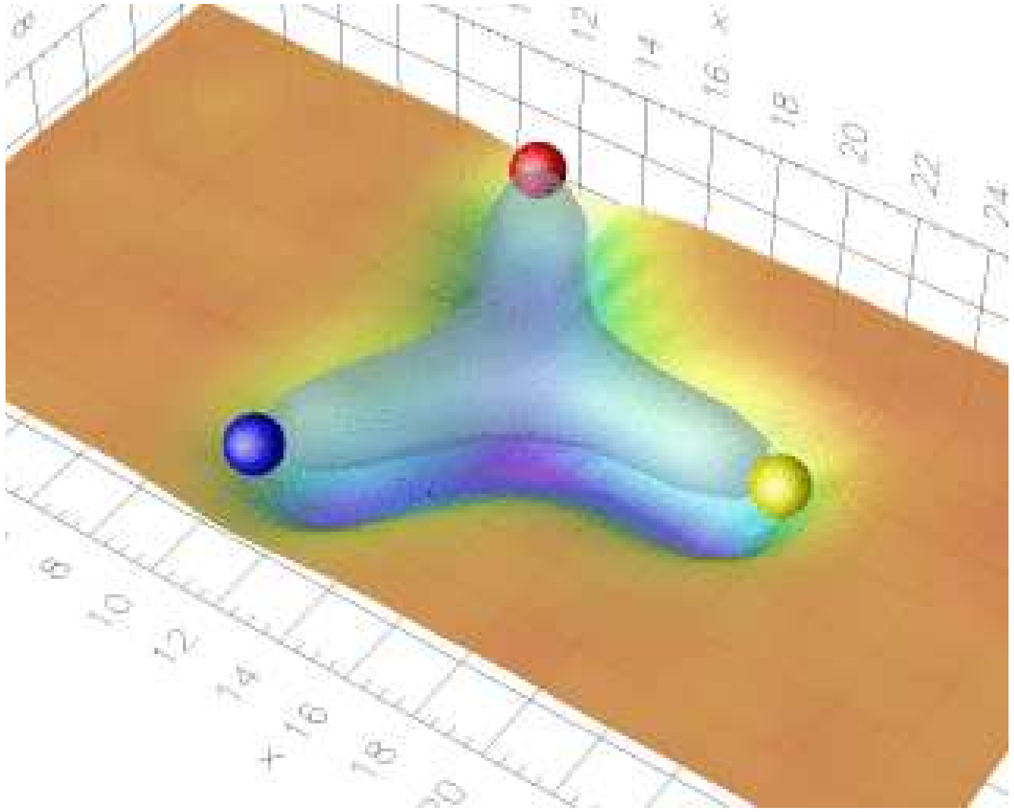
Geometry from Confinement

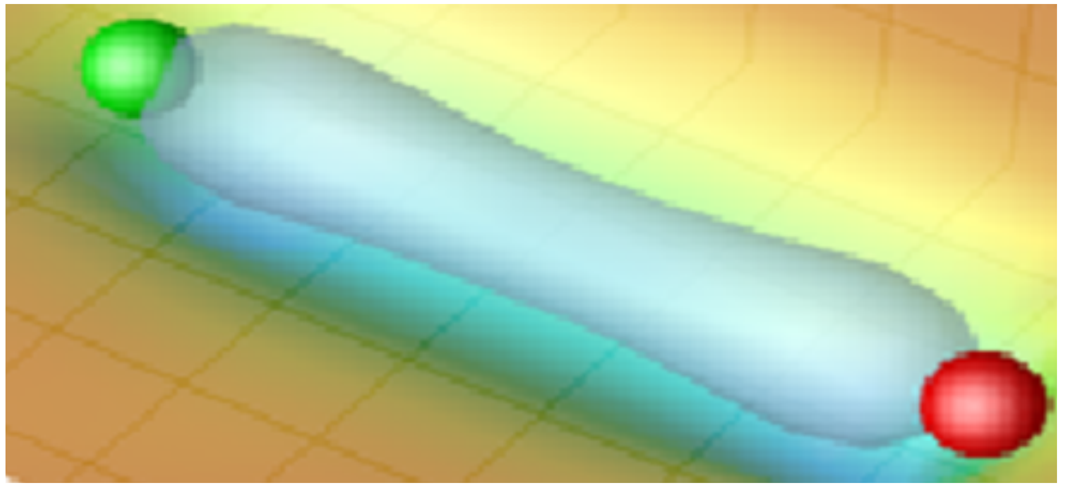
1907.07799, 1912.08885+ work in progress
Donahue, SD



One of the surprising lessons of AdS/CFT:

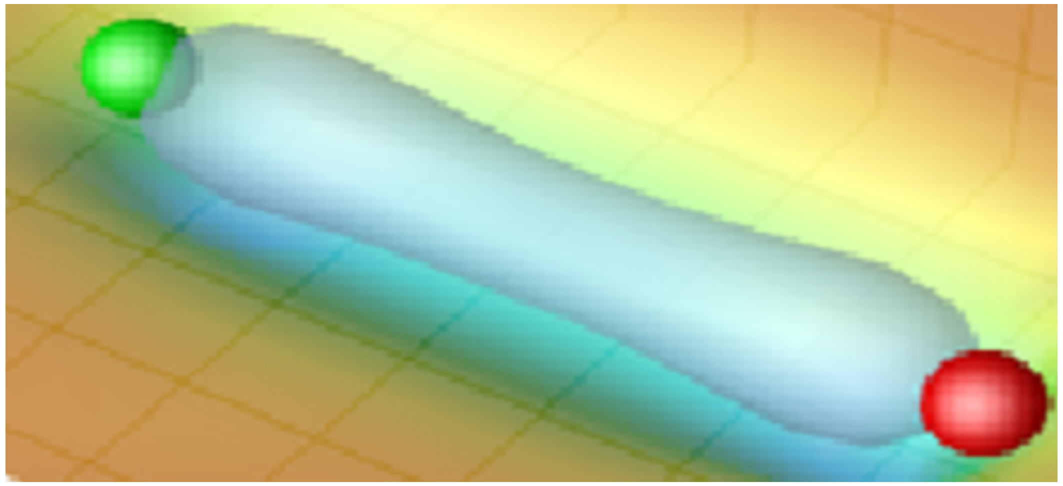
Conventional gauge theory may be used to describe quantum gravity (dynamical geometry) in higher dimensions

The focus of this talk is another (related) aspect of gauge /gravity relation. Confinement gives rise to an obvious lower dimensional dynamical geometrical object:



Am I cheating?

Why



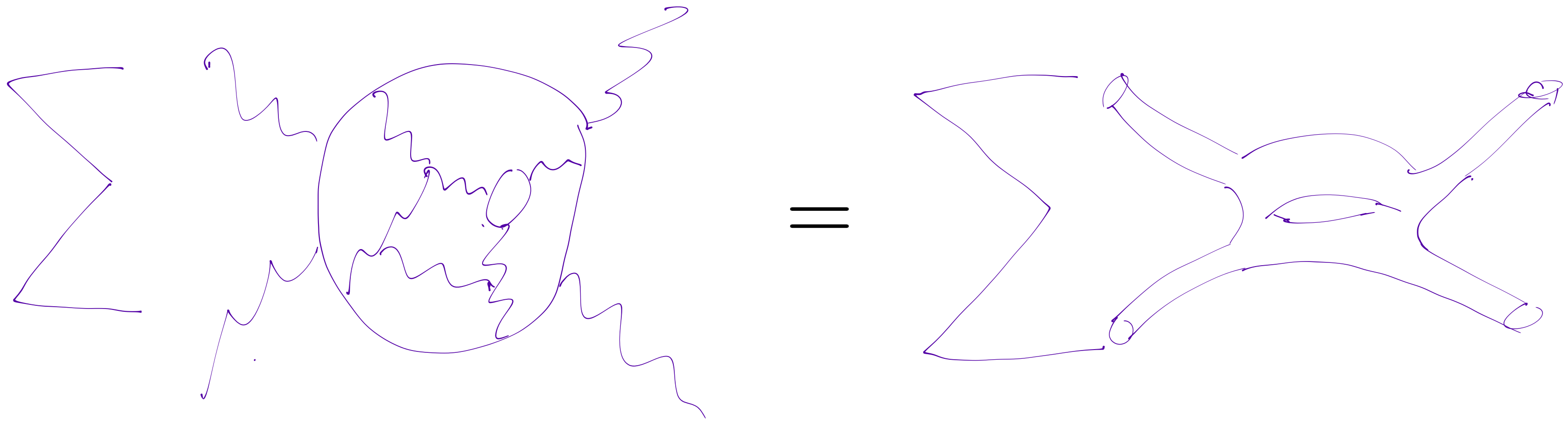
is more interesting than

or

???

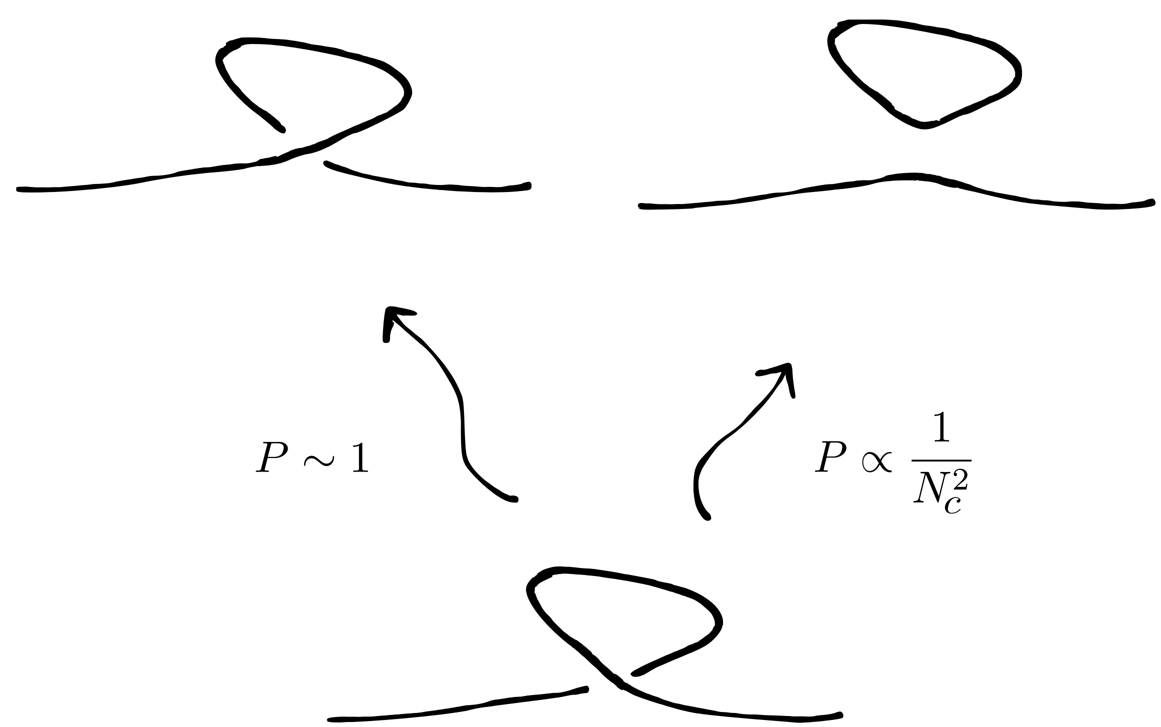
Large N

't Hooft'74



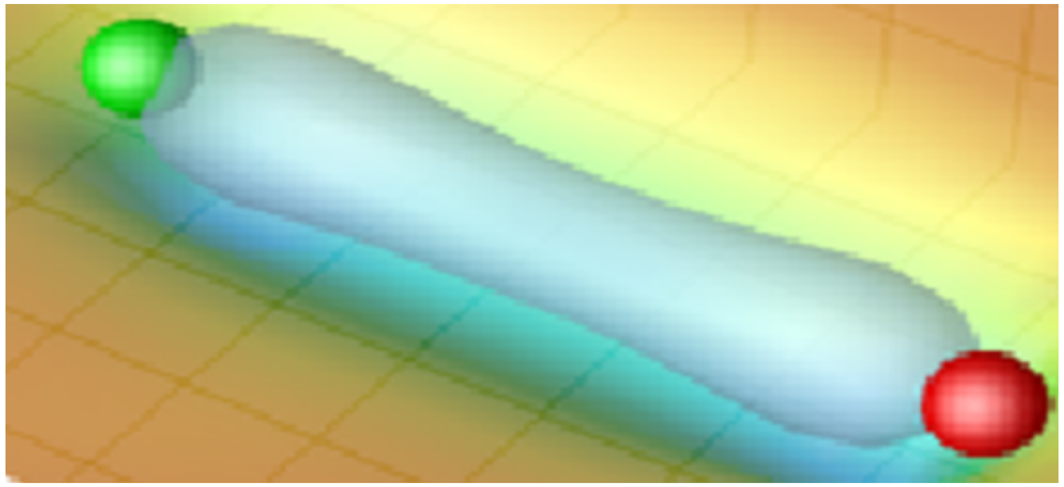
$$= \sum_g N^{2-2g} F_g(\lambda)$$

or to put it differently



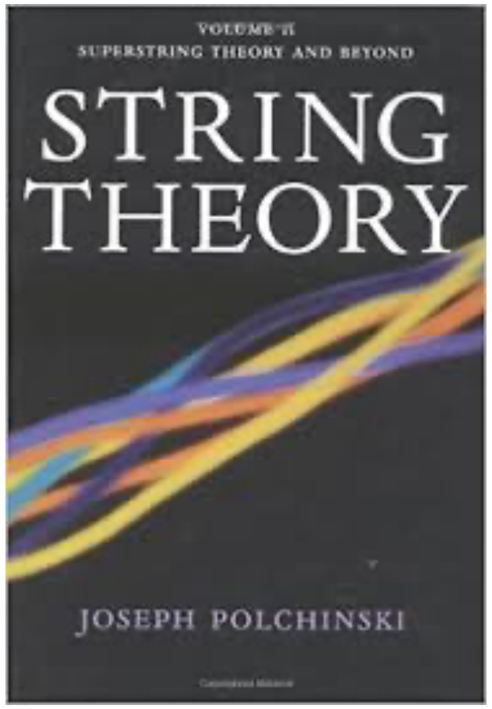
In the ’t Hooft limit

confining strings



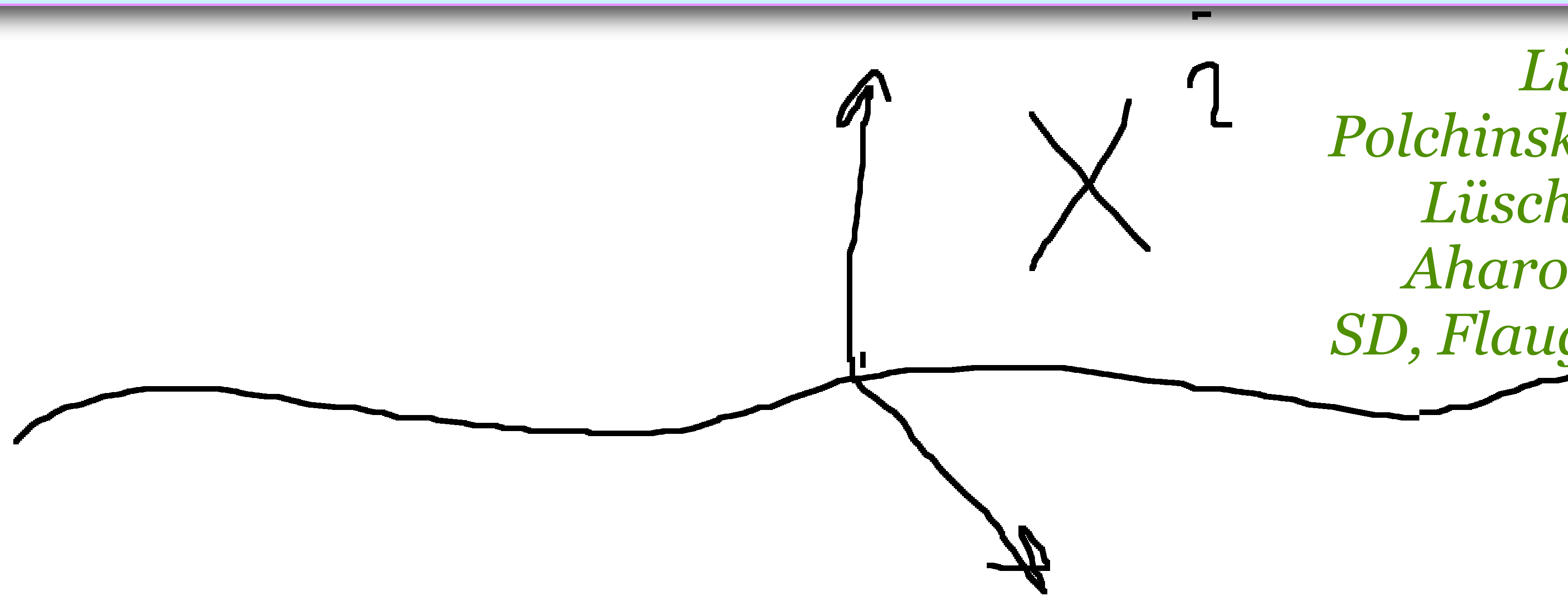
are as “fundamental” as

critical strings



at least as a 2d geometrical object

(Long) String as seen by an Effective Field Theorist



Lüscher '81
Polchinski, Strominger'91
Lüscher, Weisz '04
Aharony et al '07-11
SD, Flauger, Gorbenko'12

Theory of Goldstone Bosons

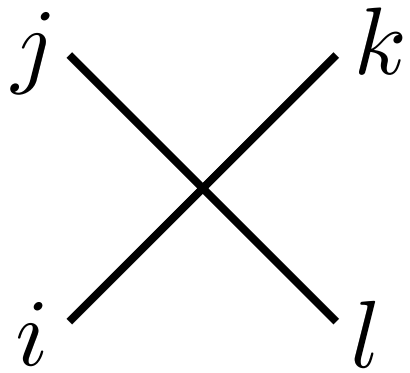
$$ISO(1, D-1) \to ISO(1,1) \times SO(D-2)$$

$$\delta_{\epsilon}^{\alpha i} X^j = -\epsilon(\delta^{ij}\sigma^{\alpha} + X^i \partial^{\alpha} X^j)$$

$$S_{string} = -\ell_s^{-2} \int d^2\sigma \sqrt{-\det h_{\alpha\beta}} + \dots$$

Large N QCD is hard because the worldsheet theory is

✴not free



$$= \delta_{ik}\delta_{jl}\frac{i\ell_s^2 s}{4}$$

tree level

Large N QCD is hard because the worldsheet theory is

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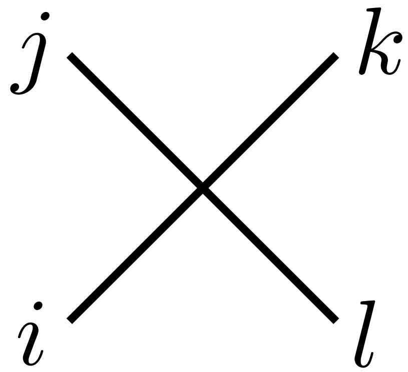
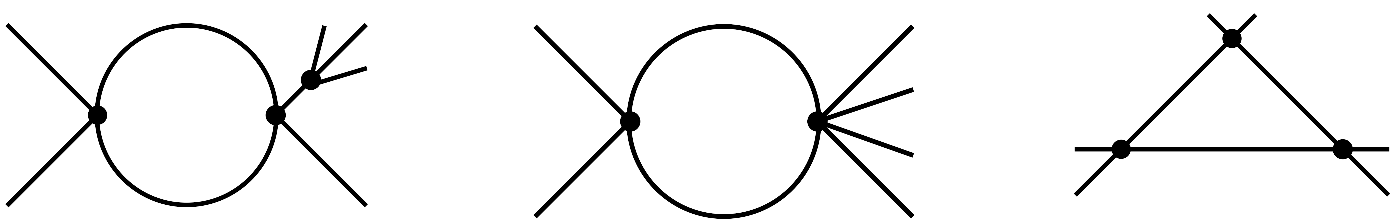


$$= \delta_{ik}\delta_{jl}\frac{i\ell_s^2 s}{4}$$

tree level

✴not integrable

$$X^i(p_+ + q_+)X^i(p_- + q_-) \to X^j(p_+)X^j(q_+)X^k(p_-)X^k(q_-)$$



one loop

$$-\ell_s^6\frac{D-26}{24\pi}p_+q_+p_-q_-(p_+ + q_+)^2(p_- + q_-)^2$$

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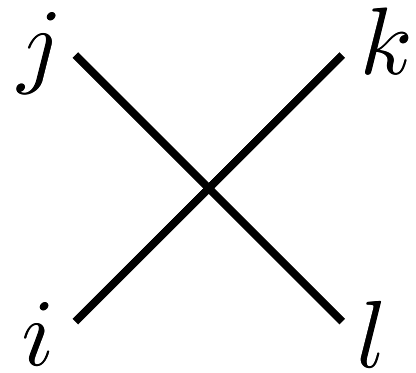

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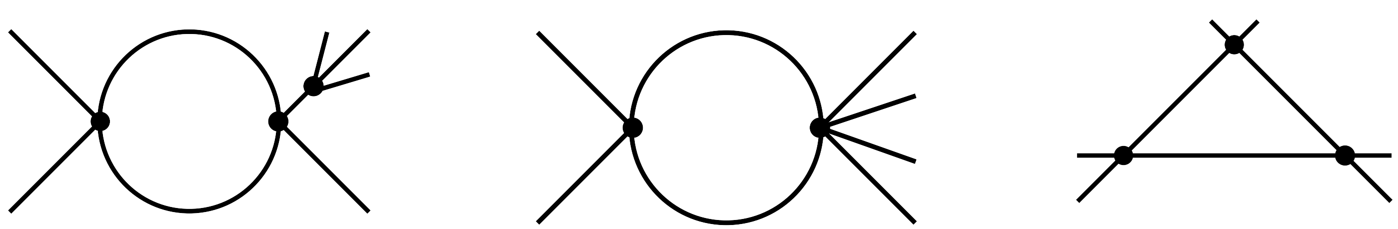


one loop

$$-\ell_s^6\frac{D-26}{24\pi}p_+q_+p_-q_-(p_+ + q_+)^2(p_- + q_-)^2$$

✶strongly coupled in the UV: 2d gravity rather than a QFT

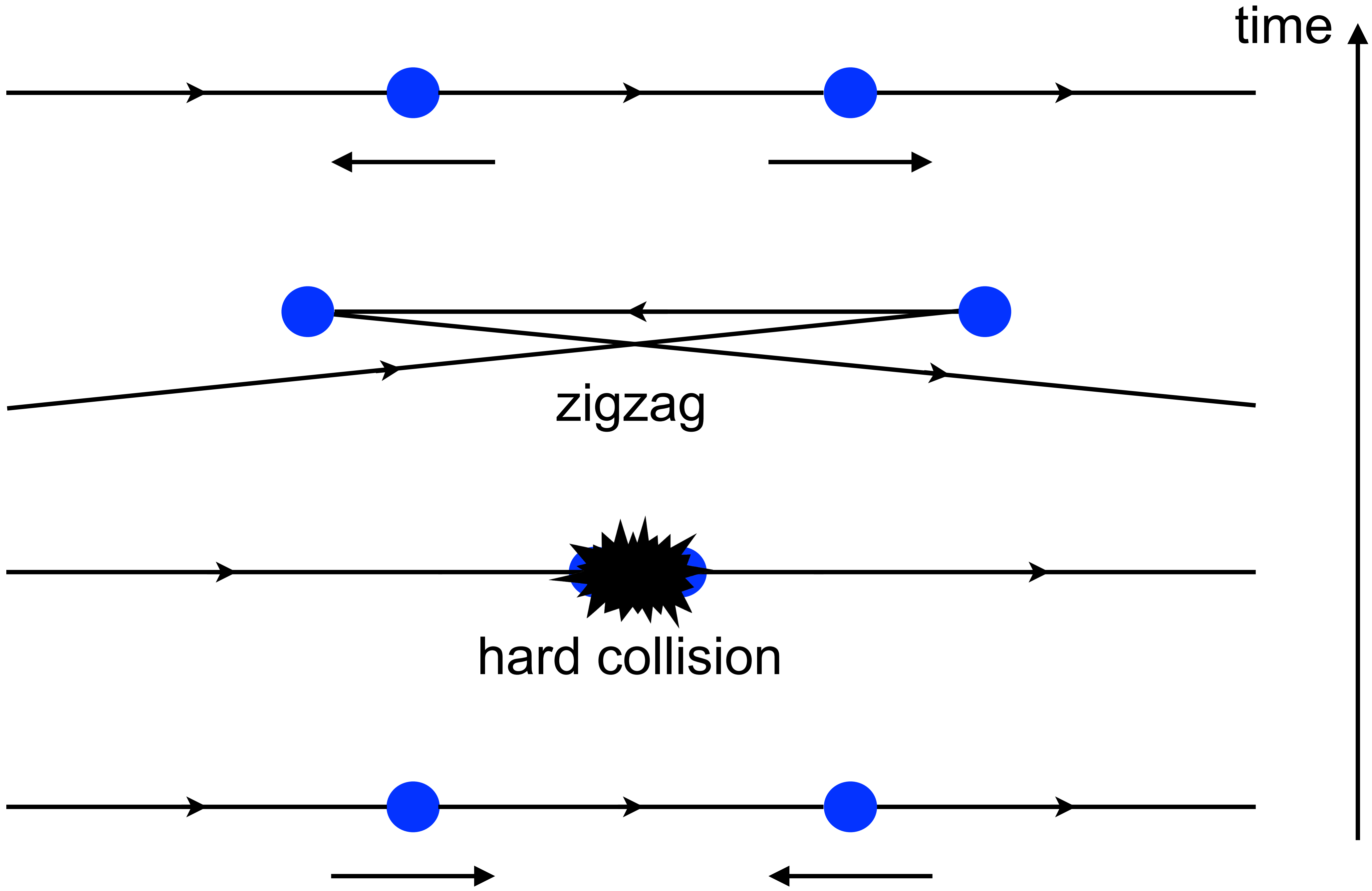
The best hope to understand the UV behavior is to find a "close by" integrable model and perturb around it.

This is not hopeless: thanks to the asymptotic freedom we know "atoms" of the (2d) space-time in this case. These are quarks and gluons of perturbative QCD.

An alternative complementary approach would be the S-matrix bootstrap: we are after an isolated 2d theory here.

Miro, Guerrieri, Hebbar, Penedones, Vieira' 19

High-Energy Worldsheet Scattering



Asymptotic Freedom+Confinement=Integrability

Worldsheet theory lives in d=2 independently of D

Good chances to draw useful lessons from analytically tractable lower dimensional models (D=2)

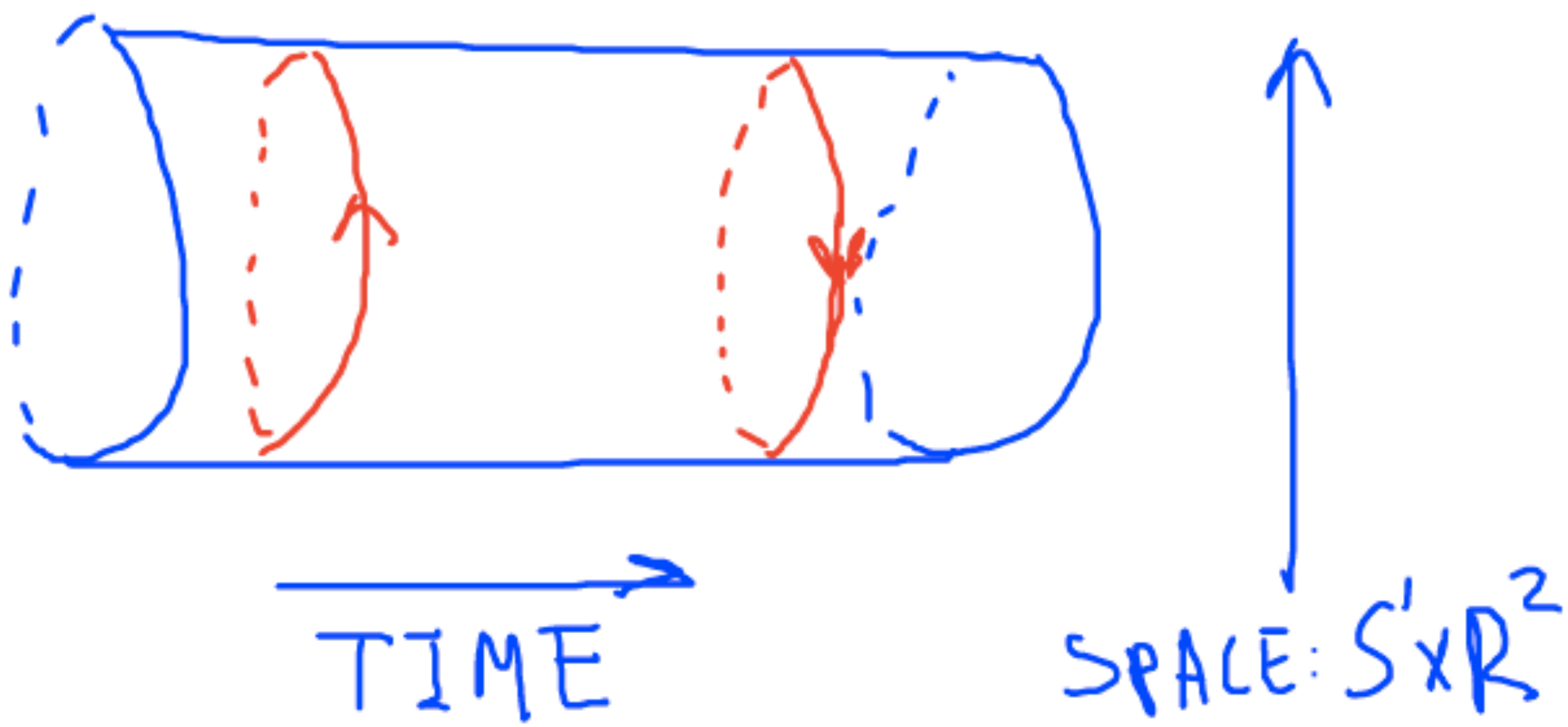
$$S = \int d^2\sigma \mathrm{Tr}\left\{ -\frac{1}{2g^2} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}(i\gamma^\mu \nabla_\mu^{(ad)} - m)\psi \right\}$$

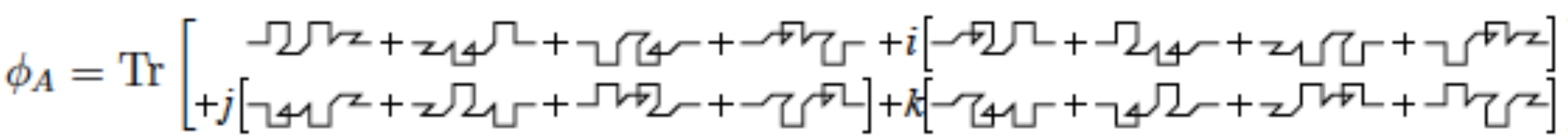
perturbative in heavy mass regime $m^2 \gg g^2 N$

But is there a worldsheet geometry to talk about at D=2?

Worldsheet theory at D=4
(as seen on a lattice)

$$\mathcal{O}_P = TrPe^{i\oint A}$$

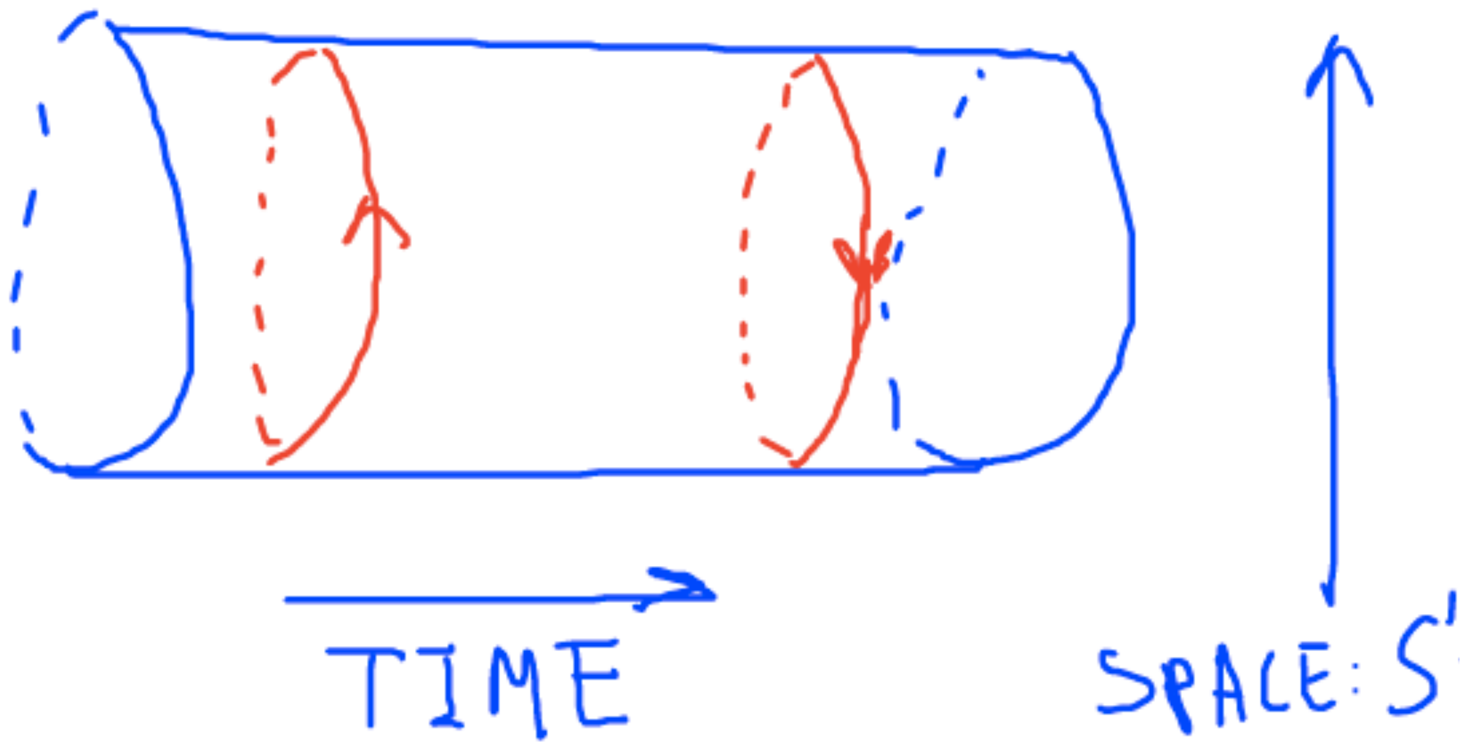




$$\int \mathcal{D}A e^{-S_{YM}} \mathcal{O}(0)\mathcal{O}^{\dagger}(t) \rightarrow e^{-E_{\mathcal{O}}t} + \ldots$$

Worldsheet Theory at D=2

$\mathcal{O}_P = TrPe^{i\oint A}$



✳pure YM is an interesting theory of non-critical strings solved at all N

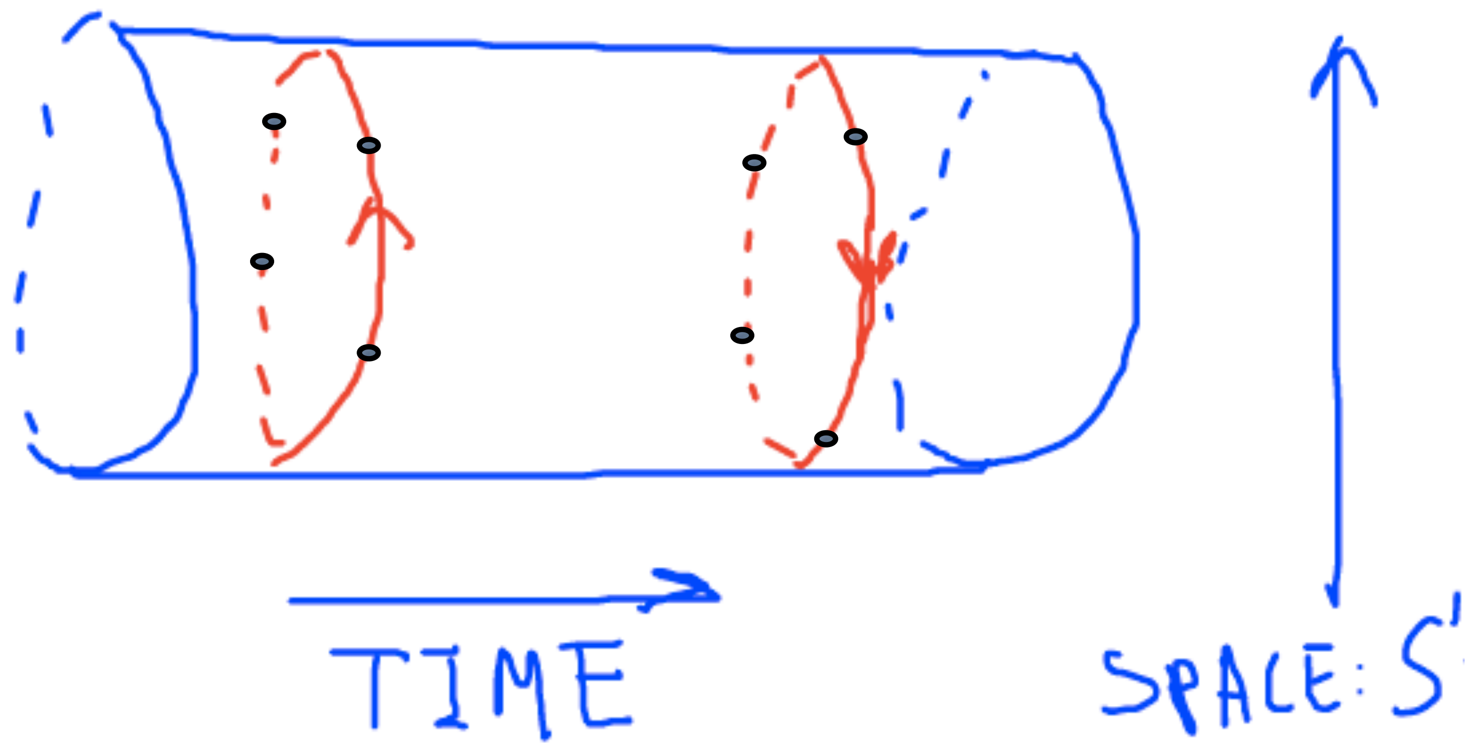
Migdal'75, Kazakov, Kostov'80, Rusakov'90, Witten'91, Gross,Taylor'93,...

✳shows that there is a path to non-critical strings without a Liouville mode

✳a bit too minimalistic—no local d.o.f. whatsoever

Worldsheet Theory at D=2

$$\mathcal{O}_P = TrPe^{i\oint A}\psi_1\psi_2\ldots\psi_n$$



✶local d.o.f. appear in the presence of adjoint matter
✶interesting results on the spectrum

Daley, Klebanov'92, Kutasov'93, Katz et al'13, Cherman et al'19,...

✶worldsheet dynamics was not explored until recently technical analysis is a linear superposition of

| More about the Massive Schwinger Model*

SIDNEY COLEMAN

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Cambridge, Massachusetts 02138

Received April 2, 1976 ⟩

+

| **θ-Vacua in Two-Dimensional Quantum Chromodynamics** (*).

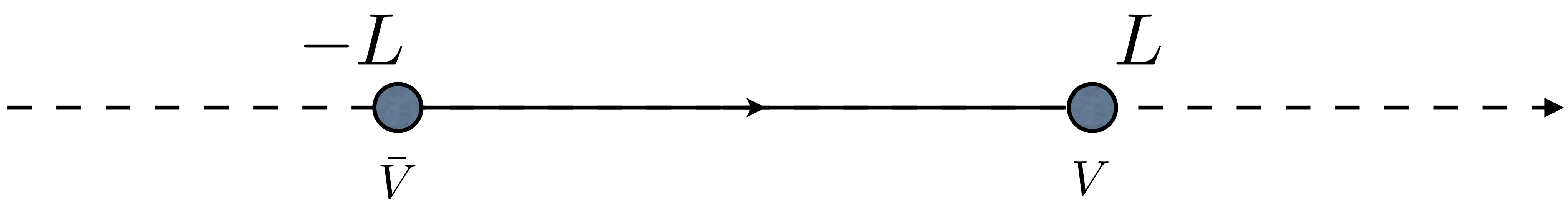
E. WITTEN (**)

Lyman Laboratory of Physics, Harvard University - Cambridge, Mass. 02138

(ricevuto il 23 Febbraio 1979) ⟩

Hamiltonian Formalism on the Worldsheet

✴Introduce heavy "end-of-the-world" fundamental quarks



✴Fix $A_x = 0$ gauge, and solve for A_0 from the Gauss' law

$$H = \int d\sigma \left(\frac{1}{2g^2}(E^a)^2 + \frac{1}{2}\bar{\psi}^a(-i\gamma^1\partial_\sigma + m)\psi^a \right)$$

$$\int d\sigma \frac{(E^a)^2}{2g^2} = \frac{g^2 NL}{2} - \frac{g^2}{4}\int d\sigma d\sigma' |\sigma - \sigma'| \rho^a(\sigma)\rho^a(\sigma') + \frac{g^2}{2}(\bar{T}^a - T^a)\int d\sigma \sigma \rho^a(\sigma)$$

string tension

Coulomb force

interaction with the background electric field

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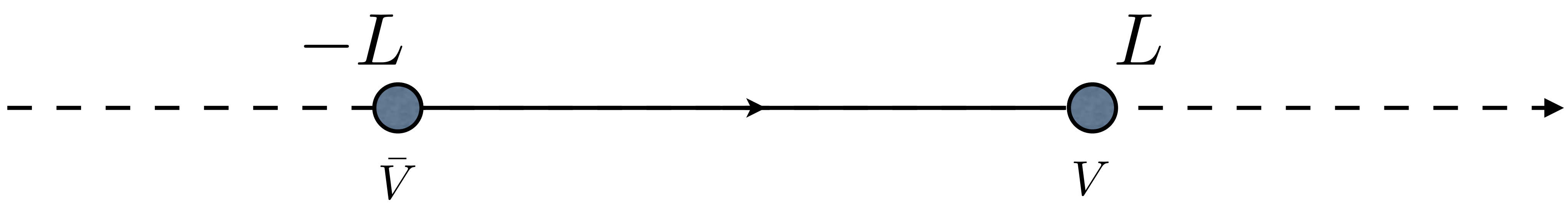

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string tension

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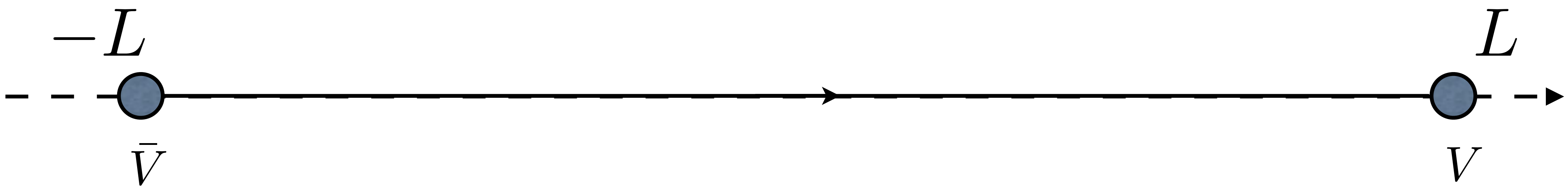
interaction with the background electric field

quantum mechanical operators



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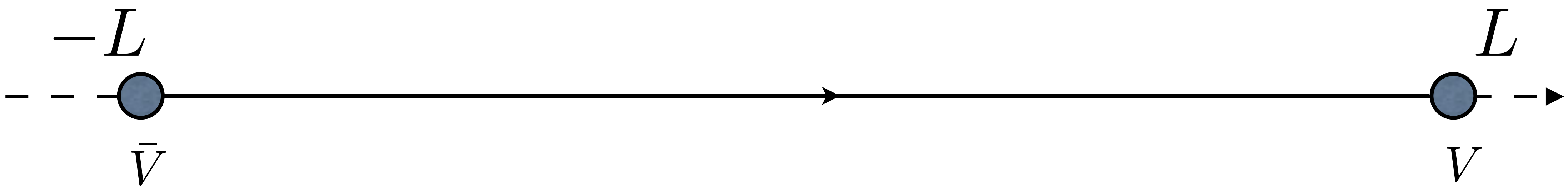
Extended Hilbert space

$$\mathcal{H}_{ext}=\mathcal{H}_{free}\otimes\bar{V}\otimes V$$

Physical states

$$Q^a|ph\rangle=\left(T^a+\bar{T}^a+\int\rho^a\right)|ph\rangle=0$$

* required for the energy to be finite
* ensures translational invariance



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operators in V

Physical states

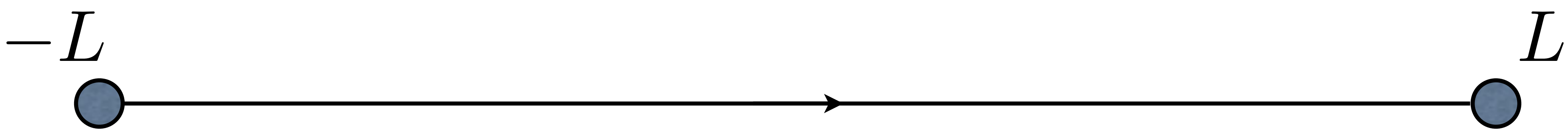
$$Q^a|ph\rangle = \left(T^a + \bar{T}^a + \int \rho^a \right)|ph\rangle = 0$$

✴required for the energy to be finite
✴ensures translational invariance

At $m^2 \gg g^2 N$ states can be characterized by a number of fermions ("partons")

✴Vacuum

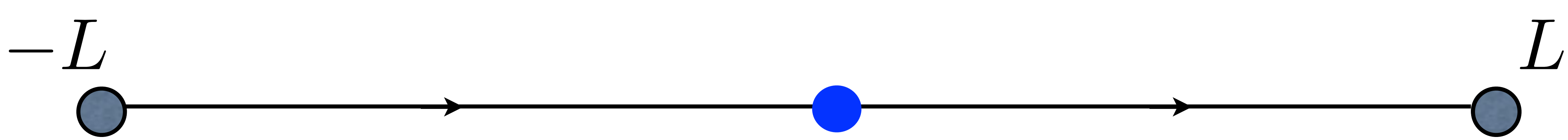
$$|0\rangle = |0\rangle_f \otimes \bar{e}_i \otimes e^i \equiv |0\rangle_f \otimes \mathbf{1}$$



$$E_0 \approx \frac{g^2 N}{2} L \equiv 2L/\ell_s^2$$

✴ One-particle state ("free quark")

$$|k\rangle = |k, a\rangle \otimes T^a$$

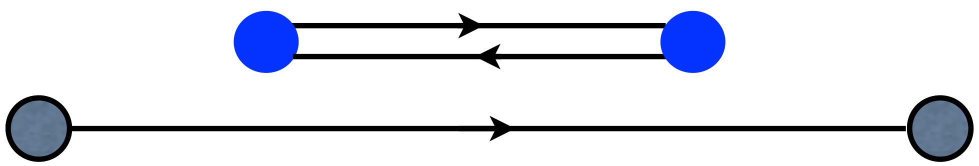


$$\text{mass} \approx m$$

✴ Two-particle states

mesons

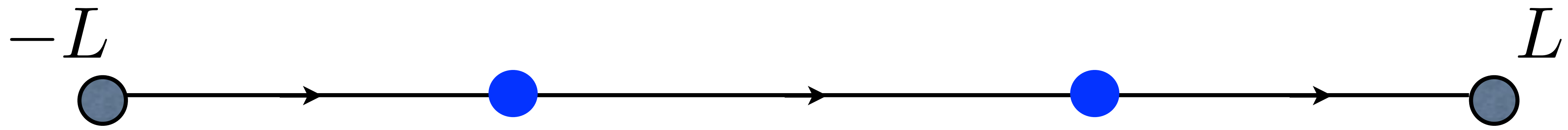
$$|k_1, k_2\rangle_{mes} = \sum_a |k_1, k_2, a, a\rangle \otimes \mathbf{1}$$



decouple from the worldsheet
in the planar limit

✳ Two-particle states

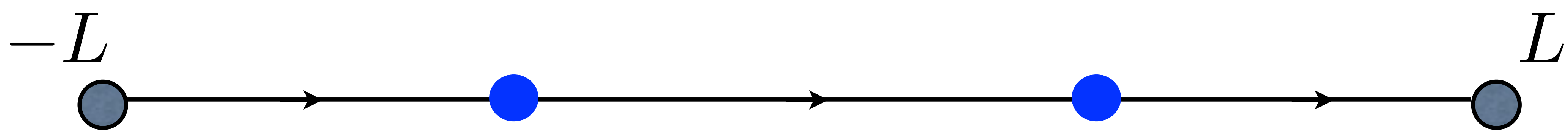
scattering states on the worldsheet



$$|k_1, k_2\rangle_{sc} = \sum_{a_1, a_2} |k_1, k_2, a_1, a_2\rangle \otimes T^{a_1} T^{a_2}$$

✳ Two-particle states

scattering states on the worldsheet



$$|k_1, k_2\rangle_{sc} = \sum_{a_1, a_2} |k_1, k_2, a_1, a_2\rangle \otimes T^{a_1} T^{a_2}$$

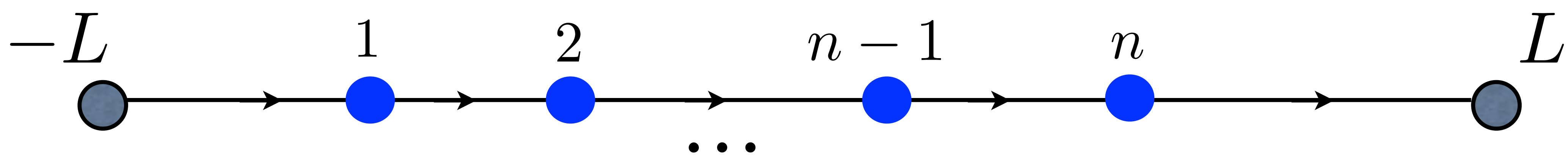
Too many states for a pair of identical particles ?!

- Momenta take values on a plane, rather than on a half-plane
- Exchange term is missing in the inner product

$$\langle k_1', k_2' | k_1, k_2\rangle = \delta(k_1 - k_1')\delta(k_2 - k_2')$$

✳ n-particle states

scattering states on the worldsheet



$$|k_1, \dots, k_n\rangle = \sum_{a_i} |k_1, a_1; \dots ; k_n, a_n\rangle T_1^a \dots T_n^a$$

$$\langle k_1, \dots, k_n | k_1', \dots, k_n'\rangle = \delta(k_1 - k_1') \dots \delta(k_n - k_n')$$

Infinite (quantum Boltzman) Statistics

Need to learn how to do perturbation theory in this space

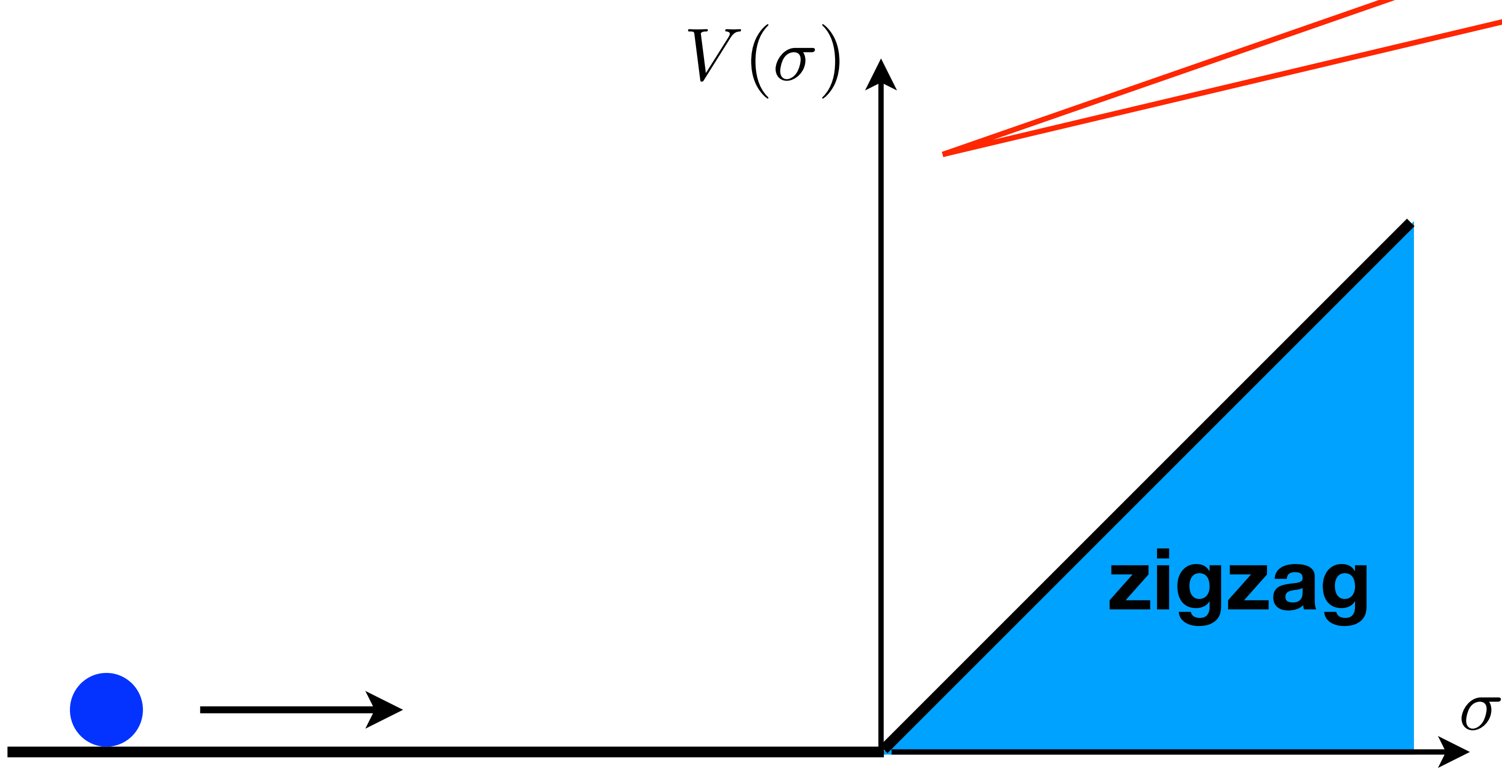
Dynamics

Let's calculate the Hamiltonian

$$\langle k_1', k_2' | H | k, -k \rangle = \delta(k_1' + k_2') \langle k' | H_{eff} | k \rangle$$

One finds

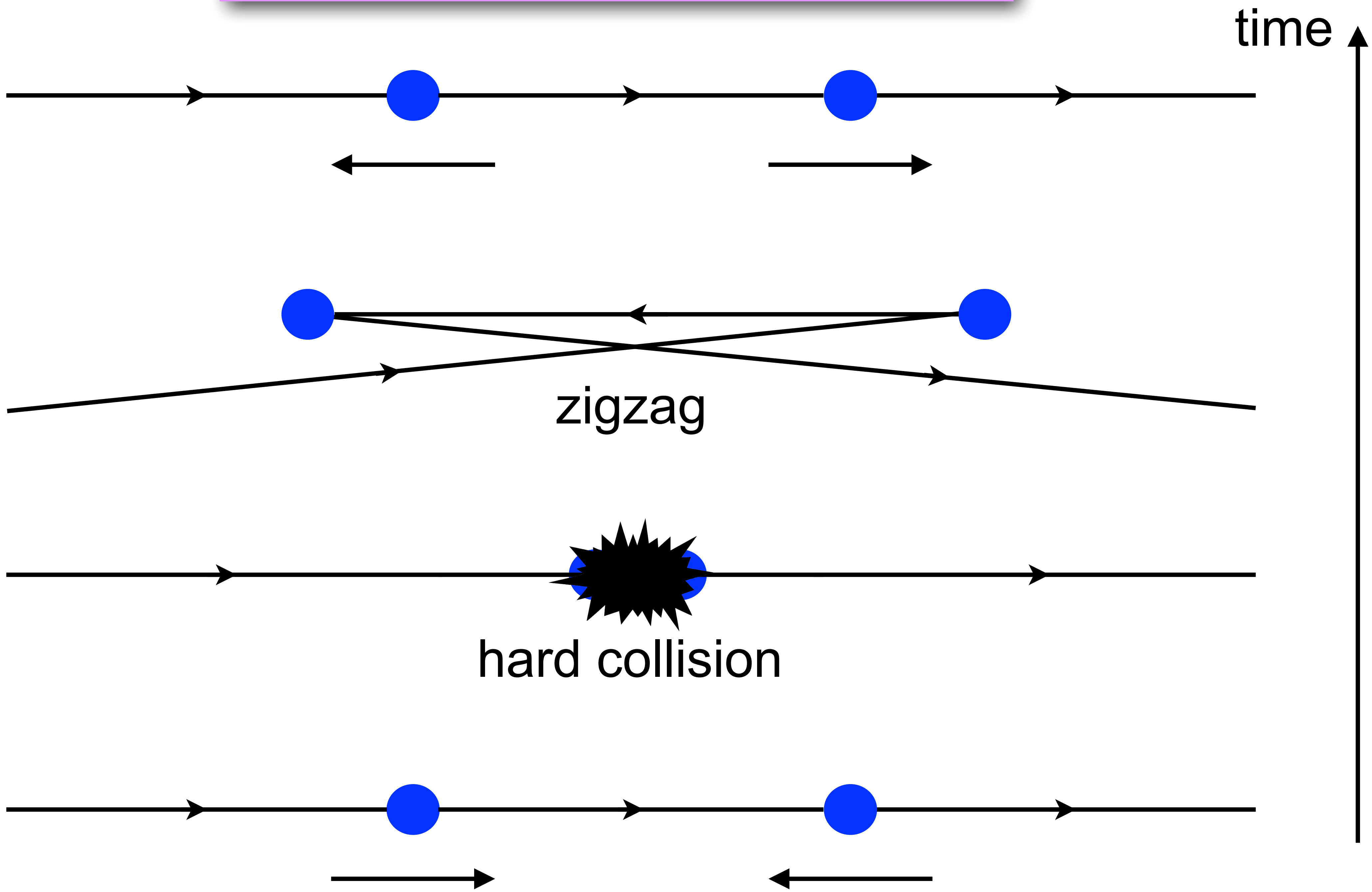
$$\langle k' | H_{eff}^{L(R)} | k \rangle = \delta(k - k') 2\omega_k - \frac{g^2 N}{4\pi} \left(\mathcal{U}(k, k') \frac{\mathcal{P}}{(k - k')^2} \pm i\pi\delta'(k - k') \right)$$



conventional statistics
is restored on-shell

long time delay without a resonance

Worldsheet Scattering



Zigzags restore much of the D>2 stringy dynamics

Leading N-particle Hamiltonian

$$H_N = \sum_{i=1}^{n} \sqrt{p_i^2 + m^2} + \sum_{i=1}^{N-1} (q_i - q_{i+1} - |q_i - q_{i+1}|)$$

- ✴Poincare invariant at the level of classical Poison brackets
- ✴Integrable in the massless limit

c.f. "folded strings"   Bardeen, Bars, Hanson, Peccei '76

The Zigzag Model

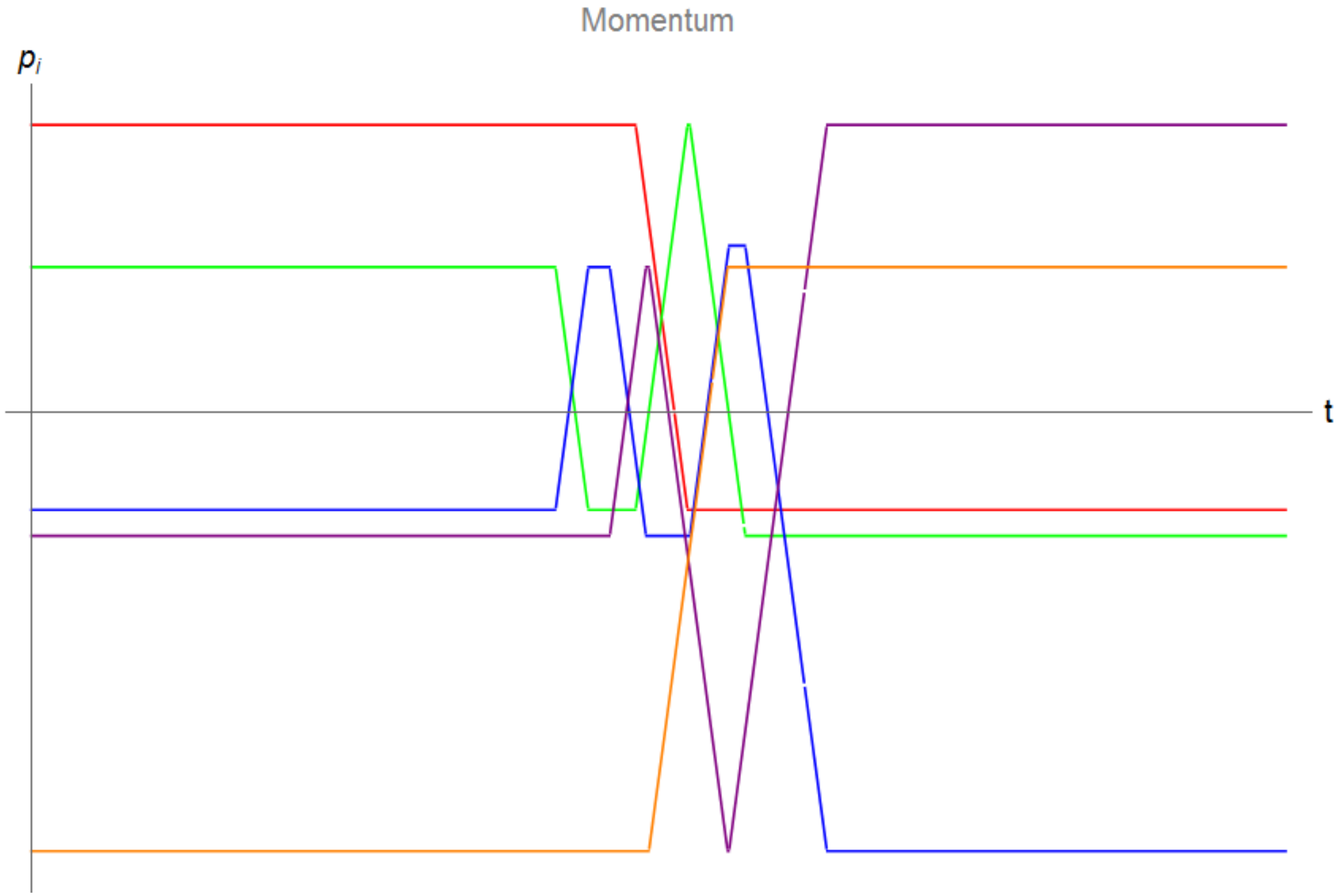
$$H = \sum_{i=1}^{N} |p_i| + \sum_{i=1}^{N-1} (q_{i,i+1} + |q_{i,i+1}|)$$

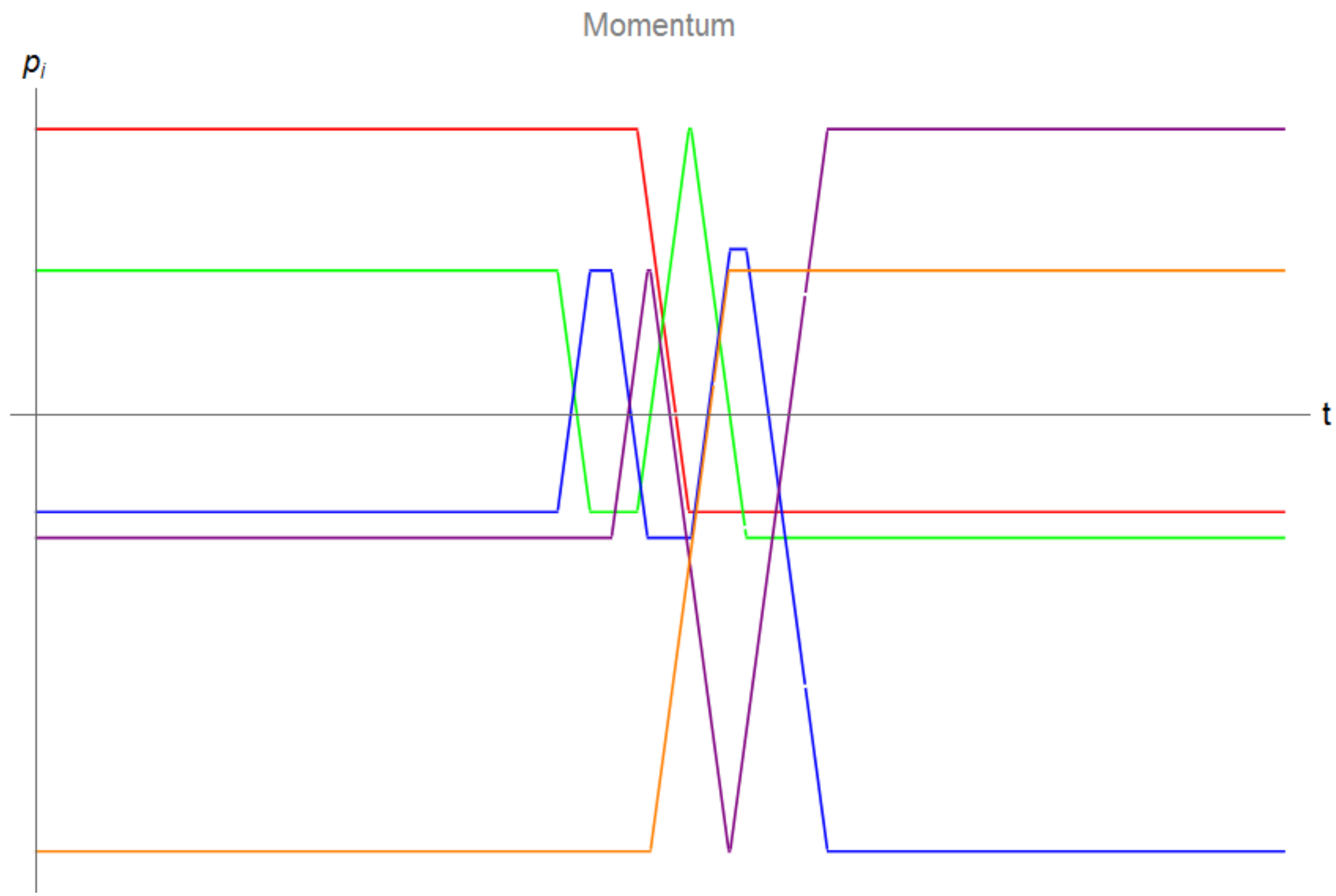
where $q_{i,i+1} = q_i - q_{i+1}$

Poincare algebra

$$P = \sum_{i=1}^{N} p_i \qquad J = \sum_{i=1}^{N} q_i |p_i| + \frac{1}{2} \sum_{i=1}^{N-1} (q_i + q_{i+1})(q_{i,i+1} + |q_{i,i+1}|)$$

$$\{H, P\} = 0 \quad \{J, P\} = H \quad \{J, H\} = P$$


Momentum
p_i
t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p_i
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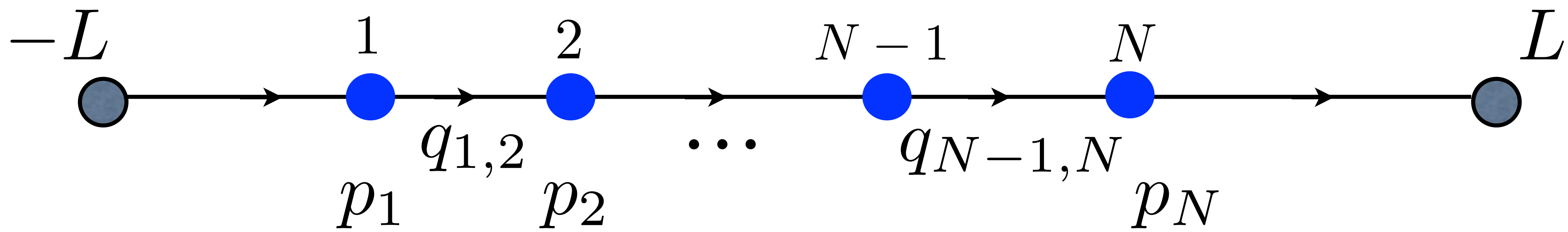


$$Q_a = (p_1, q_{1,2}, p_2, \ldots, q_{N-1,N}, p_N)$$

$$S_a = (s_1, s_{1,2}, s_2, \ldots, s_{N-1,N}, s_N)$$

where $\quad s_i = \text{sign}(p_i)\ , \quad s_{i,i+1} = \text{sign}(q_{i,i+1})$

Equations of motion

$$\dot{Q}_a = S_{a-1} - S_{a+1}$$

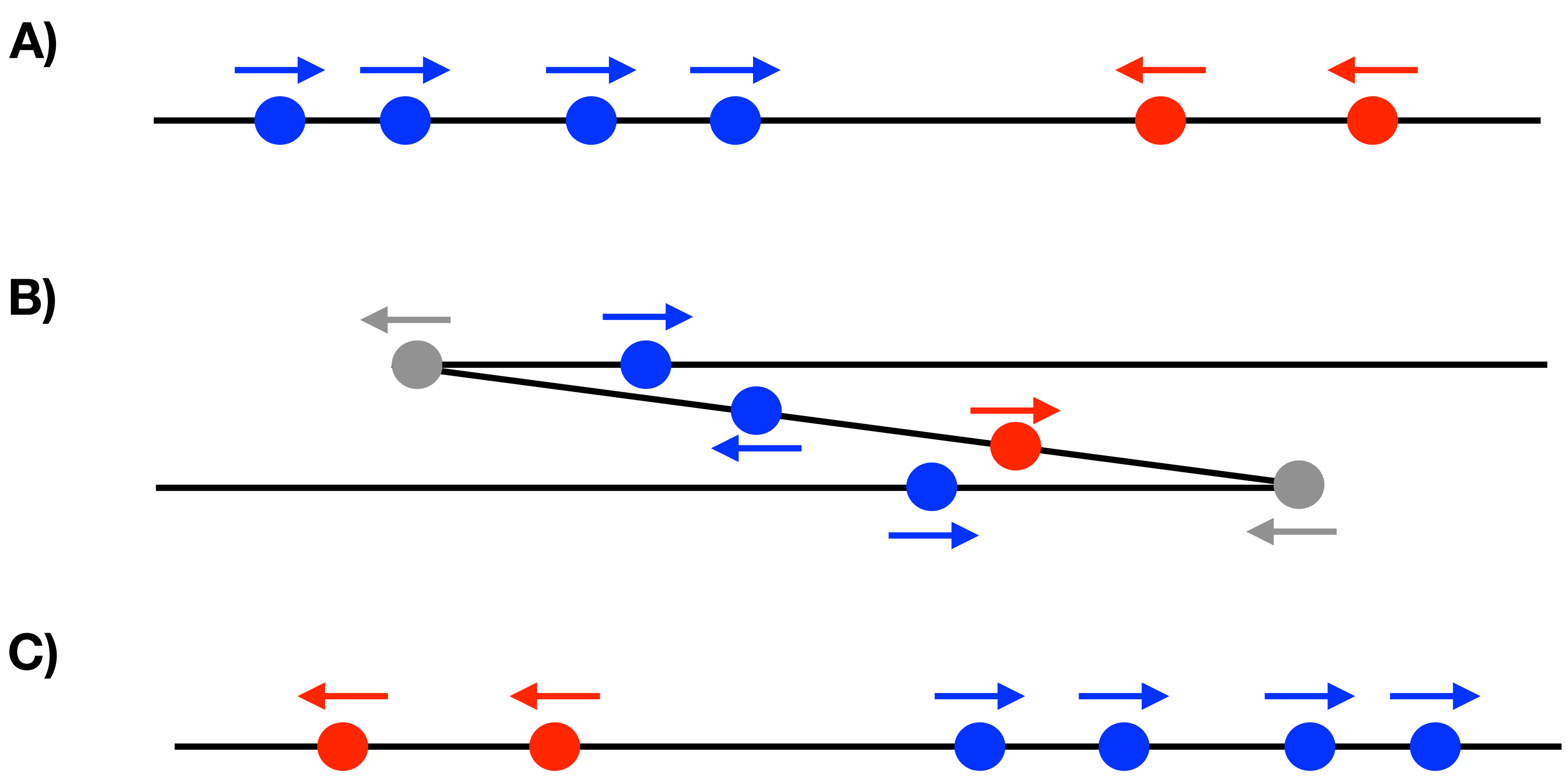
where

$$S_0 = S_{2N} = -1$$

Topological invariant: "Ising Hamiltonian"

$$T_{2N} = \frac{1}{2}\sum_{i=0}^{2N} S_a S_{a+1} \xrightarrow[s_{i,i+1}=-1]{\text{asymptotic regions}} N_L - N_R$$

at intermediate times counts $N_L - N_R$ along the worldsheet



Dynamical Charges

$$I = \sum_{a=1}^{2N-1} F_a(S) Q_a$$

coefficient functions satisfy

$$\partial_b F_a(S)(S_{b-1} - S_{b+1}) = 0 \text{ for } a \neq b$$

which is solved by

$$F_a(S) = F_a(T_a)$$

where

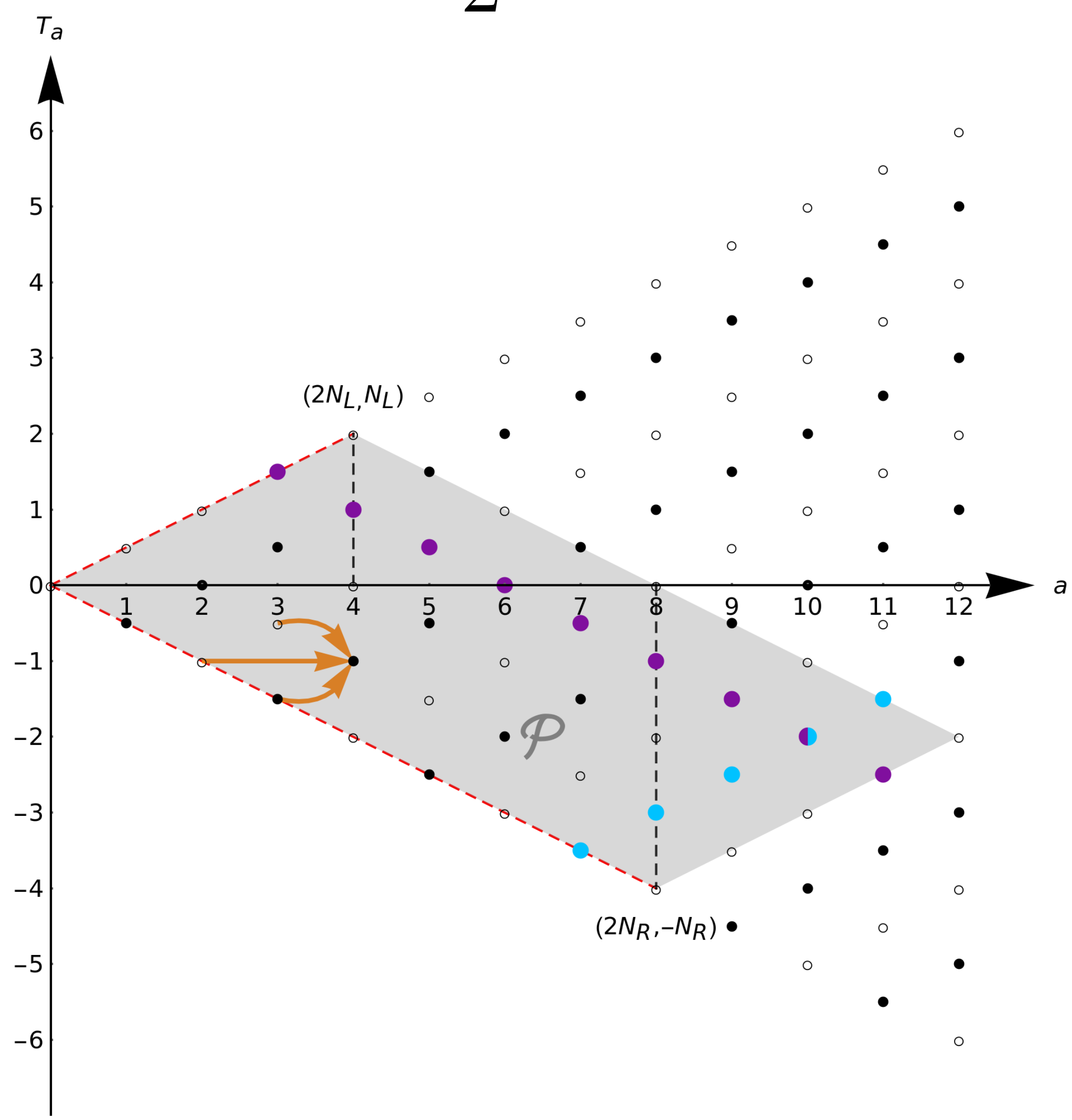
$$T_a = \frac{1}{2} \sum_{b=0}^{a-1} S_b S_{b+1} = \frac{a}{2} - \# \text{ sign flips}(0, a)$$

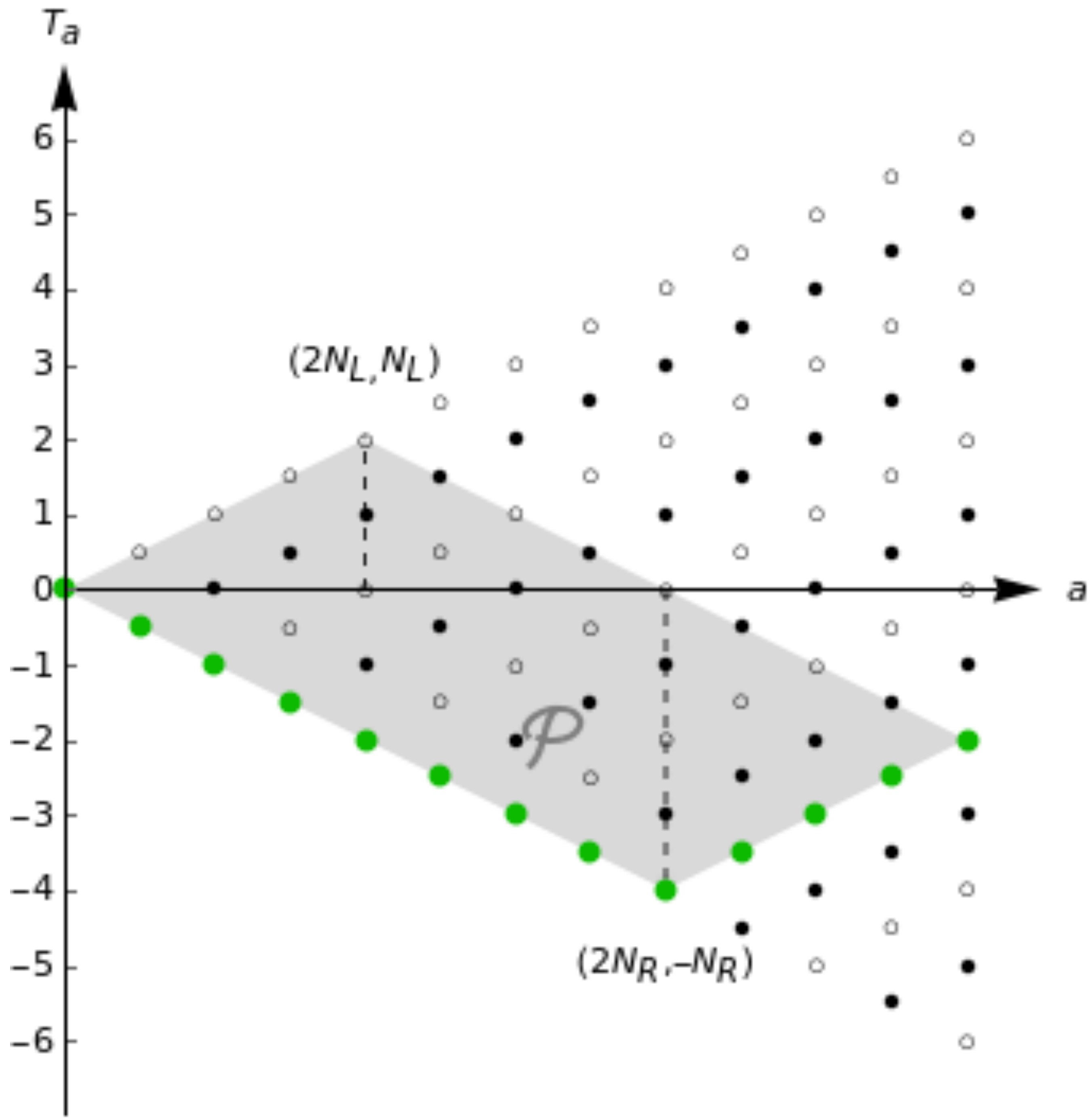
the only remaining equation

$$\sum_{a=1}^{2N-1} F_a(S)(S^{a-1} - S^{a+1}) = 0$$

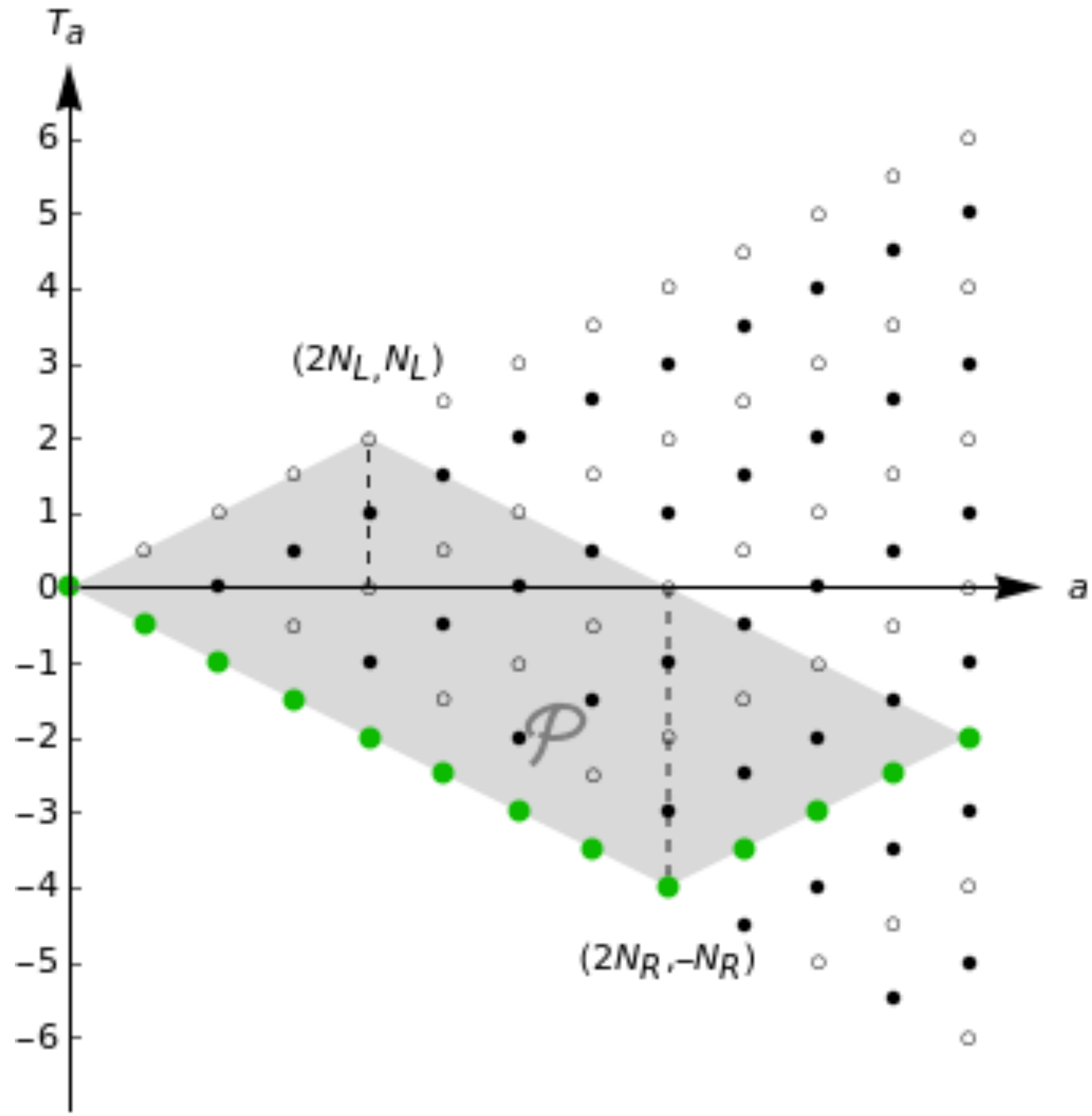
can be reduced to the set of linear recursion relations

$$F_a(T_{a-1} - \frac{S_{a-1}}{2}) - F_a(T_{a-1} + \frac{S_{a-1}}{2}) = S_{a-1}(F_{a-1}(T_{a-1}) - F_{a+1}(T_{a-1}))$$





T_a
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
a
$(2N_L,N_L)$
$\mathcal{P}$
$(2N_R,-N_R)$



T_a
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
a
$(2N_L, N_L)$
$(2N_R, -N_R)$
$\mathcal{P}$

this produces 2N-2 translationally invariant charges

which asymptotically reduce to $p_i,\ q^L_{i,i+1},\ q^R_{i,i+1}$

e.g., for N=3

with 2 left-movers

$$I_{1,2} = \frac{1+s_1}{2} p_1 + \frac{1+s_{1,2}}{2} q_{1,2} + \frac{1+s_2}{2}\left(\frac{3}{2} + T_3\right) p_2$$

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One more charge comes from the boundary

$$\tilde{P} = \begin{cases} Q_L + Q_R - P_L \; , & \text{at } t \to -\infty \\ Q_L + Q_R - P_R \; , & \text{at } t \to +\infty \end{cases}$$

This allows to calculate the time delay

$$e^{2i\delta(s)} = e^{is\ell_s^2/4}$$

which reproduces $T\bar{T}$-deformed massless fermion

Shadow Poincare Symmetry

$$H = \sum_{i=1}^{N} |p_i| + \sum_{i=1}^{N-1} (q_{i,i+1} + |q_{i,i+1}|) \qquad P = \sum_{i=1}^{n} p_i$$

$$J = \sum_{i=1}^{N} q_i |p_i| + \frac{1}{2} \sum_{i=1}^{N-1} (q_i + q_{i+1})(q_{i,i+1} + |q_{i,i+1}|)$$

$$\tilde{H} = \text{quasilinear} \qquad \tilde{P} = \text{quasilinear}$$

$$\{H, P\} = 0 \quad \{J, P\} = H \quad \{J, H\} = P$$ *physical Poincare*

$$\{\tilde{H}, \tilde{P}\} = 0 \quad \{J, \tilde{P}\} = \tilde{H} \quad \{J, \tilde{H}\} = \tilde{P}$$ *shadow Poincare*

$$\{H, \tilde{H}\} = \{P, \tilde{P}\} = 4 N_L N_R \quad \{P, \tilde{H}\} = \{H, \tilde{P}\} = 0$$

This was a classical analysis. However, the resulting classical time delay coincides with that of a one-loop exact integrable quantum model (“flat space JT gravity”)

$$S_{T\bar{T}} = \frac{\Lambda}{2}\int \epsilon^{\alpha\beta}\epsilon_{ab}\left(\partial_\alpha X^a - e^a_\alpha\right)\left(\partial_\beta X^b - e^b_\beta\right) + S_0(\psi, g_{\alpha\beta})$$

Strongly suggests that it should be possible to quantize the zigzag model preserving integrability and Poincare.

May lead to a construction of off-shell observables for TT-deformed theories (not in a conventional Fock space)

Some Questions

- ✴ "Flat space JT gravity" arises as an integrable UV asymptotics on the worldsheet. What is the full (non-integrable) gravitational theory on the worldsheet?

- ✴ More generally one expects a map

$$S = \int Tr\left(\Phi \epsilon^{\alpha\beta} F_{\alpha\beta} + V(\Phi)\right) + matter$$

$\Big\Downarrow$ worldsheet

"Dilaton gravities"+matter

What are the details of this map?