Thermalization of a strongly interacting 1D quantum liquid

Michael Pustilnik
Georgia Tech

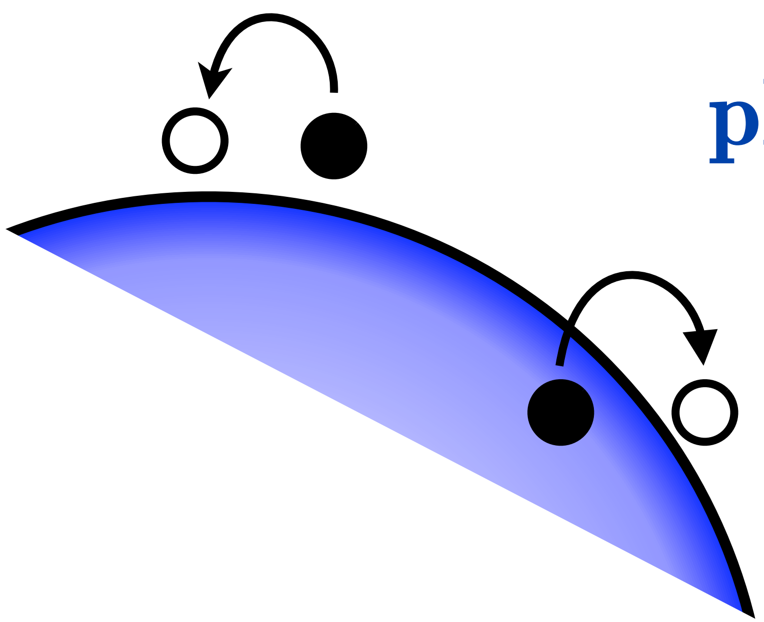
in collaboration with

Konstantin Matveev and **Jie Lin**
Argonne National Laboratory

Interacting fermions in 3D

Excitations: quasiparticles

- weakly interacting
- obey Fermi statistics



phase space restrictions

$$\Downarrow$$

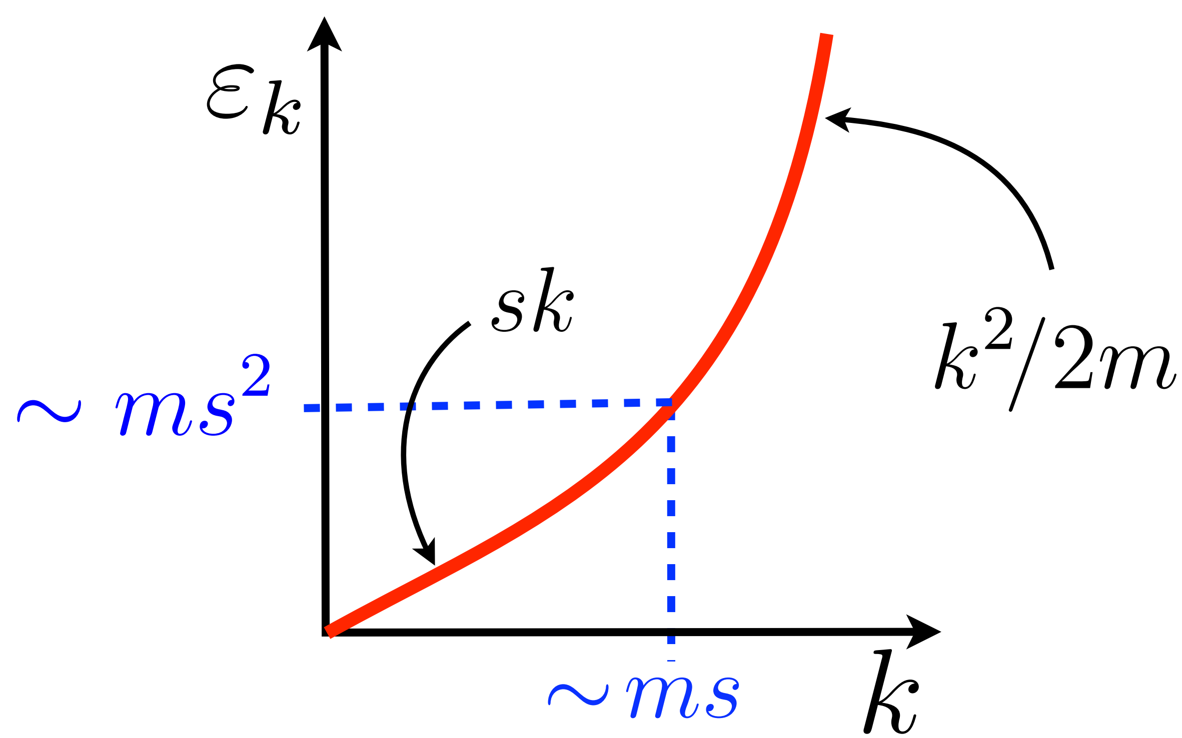
$$1/\tau_\varepsilon \propto \varepsilon^2$$

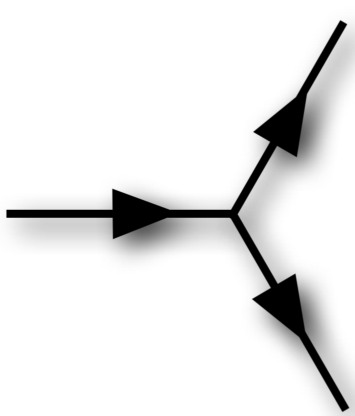
$$(\tau_\varepsilon \varepsilon)^{-1} \xrightarrow{\varepsilon \to 0} 0$$

Fermi liquid theory Landau (1956)

Interacting bosons in 3D

- **BEC transition at a finite temperature**
- **Excitations: Bogolubov quasiparticles (bosons!)**



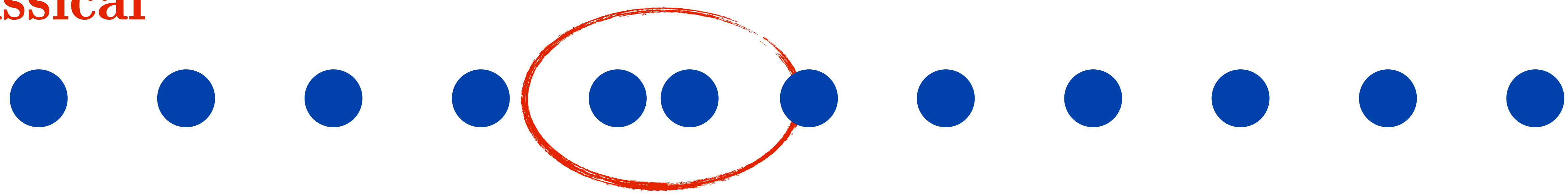


$$\Longrightarrow \quad 1/\tau_\varepsilon \propto \varepsilon^5 \text{ for } \varepsilon \ll ms^2$$

Beliaev (1958)

One-dimensional systems

classical



density fluctuations

propagating with sound velocity

quantum

regardless the statistics of constituent particles,
excitations are **bosons**, **the waves of density**

Tomonaga (1950)
Popov (1972)

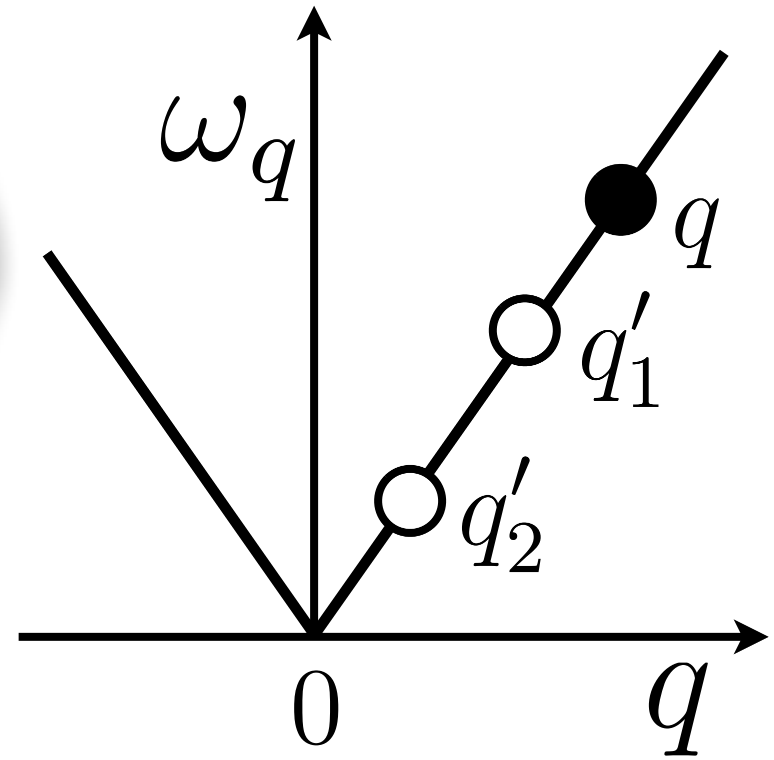
....

Haldane (1980, 1981)

Luttinger liquid theory

$$H_0 = \sum_q \hbar\omega_q b_q^\dagger b_q, \quad \omega_q = v|q|$$

Luttinger liquid: failure of perturbation theory



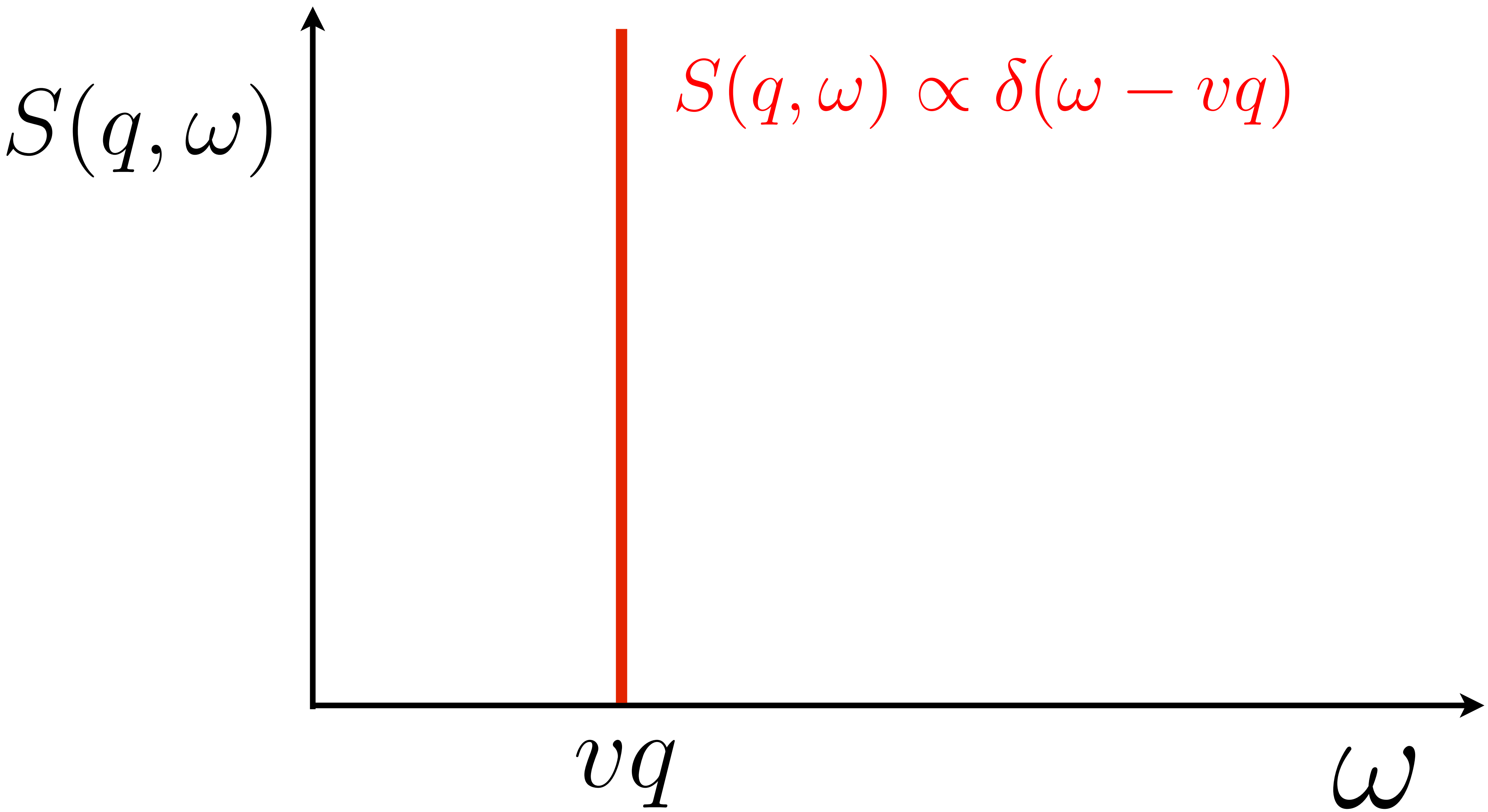
inelastic scattering rate

$$\tau_q^{-1} \propto \int dq_1' dq_2' \, [\ldots] \, \delta(q - q_1' - q_2') \, \underbrace{\delta\left(\omega_q - \omega_{q_1'} - \omega_{q_2'}\right)}_{\displaystyle \| \atop \displaystyle v^{-1} \delta(q - q_1' - q_2')} = \infty$$

Back to fermions: dynamic structure factor

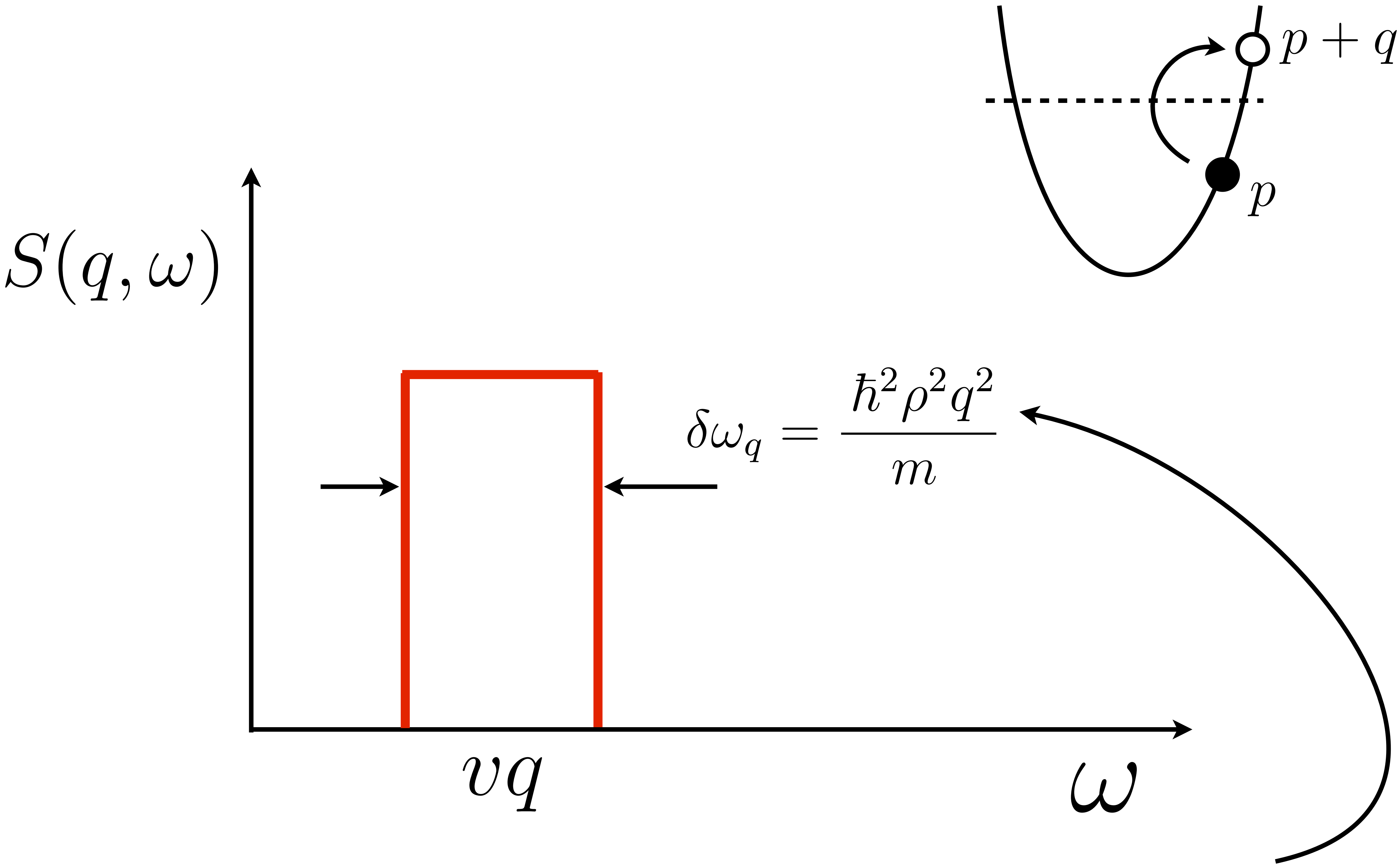
Luttinger model

density waves = exact eigenstates



Back to fermions: dynamic structure factor

free fermions

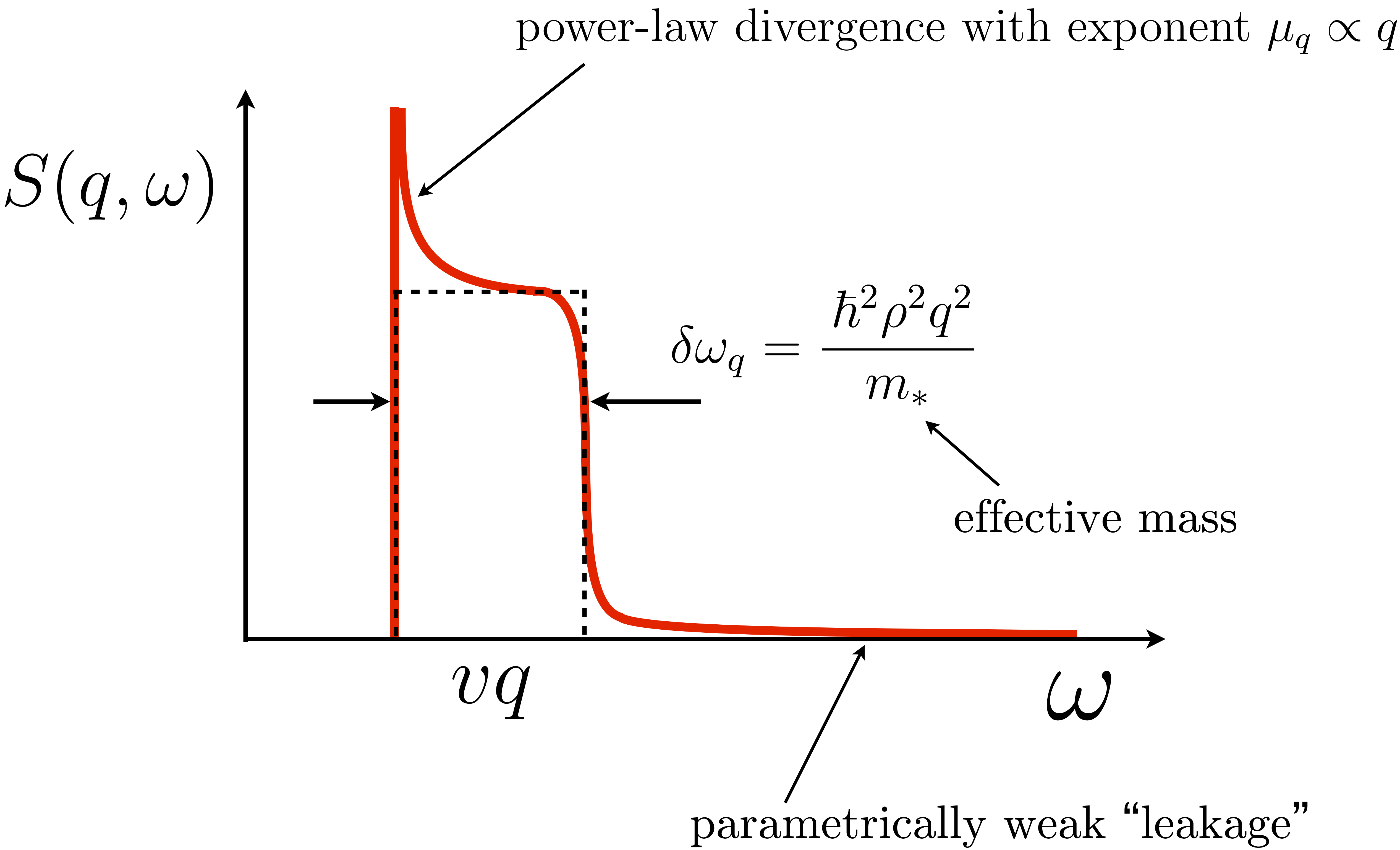


$$\delta\omega_q = \frac{\hbar^2 \rho^2 q^2}{m}$$

from now on, q is dimensionless, in units of density ρ

Back to fermions: dynamic structure factor

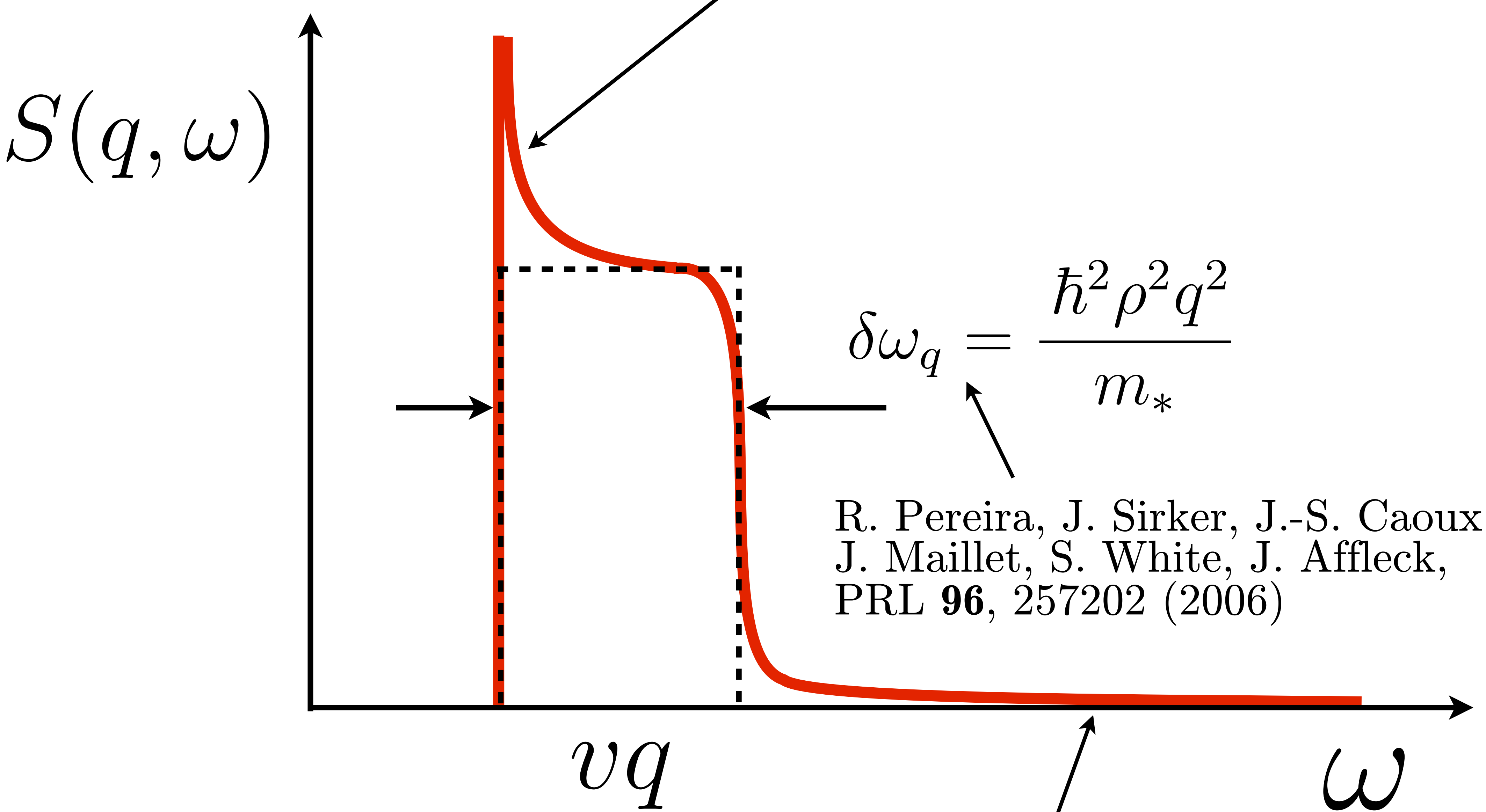
interacting fermions, small *q*



Back to fermions: dynamic structure factor

interacting fermions, small *q*

M.P., M. Khodas, A. Kamenev, L. Glazman, PRL **96**, 196405 (2006)
A. Imambekov and L. Glazman, PRL **100**, 206805 (2008)



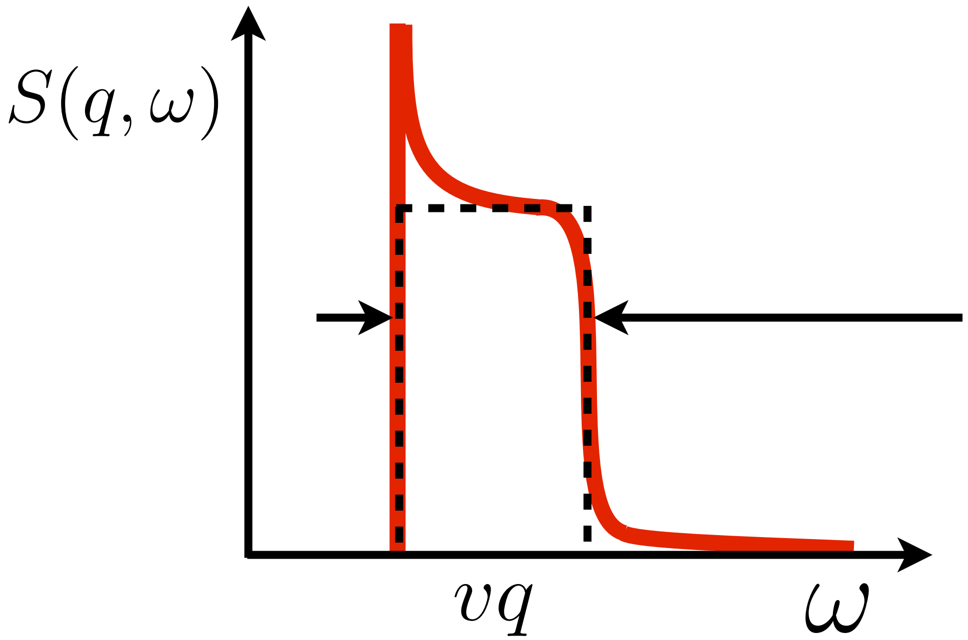
$$\delta\omega_q = \frac{\hbar^2\rho^2 q^2}{m_*}$$

R. Pereira, J. Sirker, J.-S. Caoux, R. Hagemans, J. Maillet, S. White, J. Affleck, PRL **96**, 257202 (2006)

M.P., E. Mishchenko, L. Glazman, A. Andreev, PRL **91**, 126805 (2003)

Back to fermions: dynamic structure factor

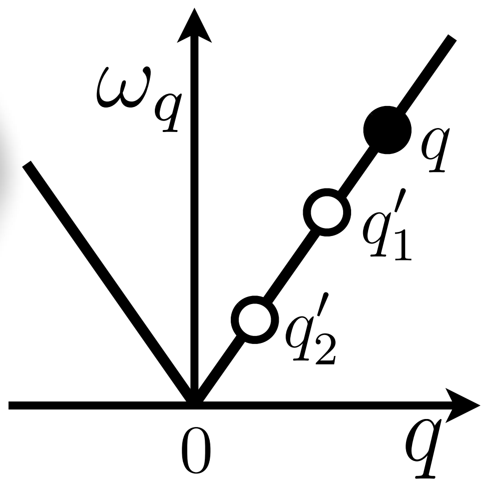
interacting fermions, small q



$$\delta\omega_q = \frac{\hbar^2 \rho^2 q^2}{m_*}$$

Uncertainty in the energy of the Luttinger liquid's boson (dephasing rate rather than decay rate)

Failure of perturbation theory

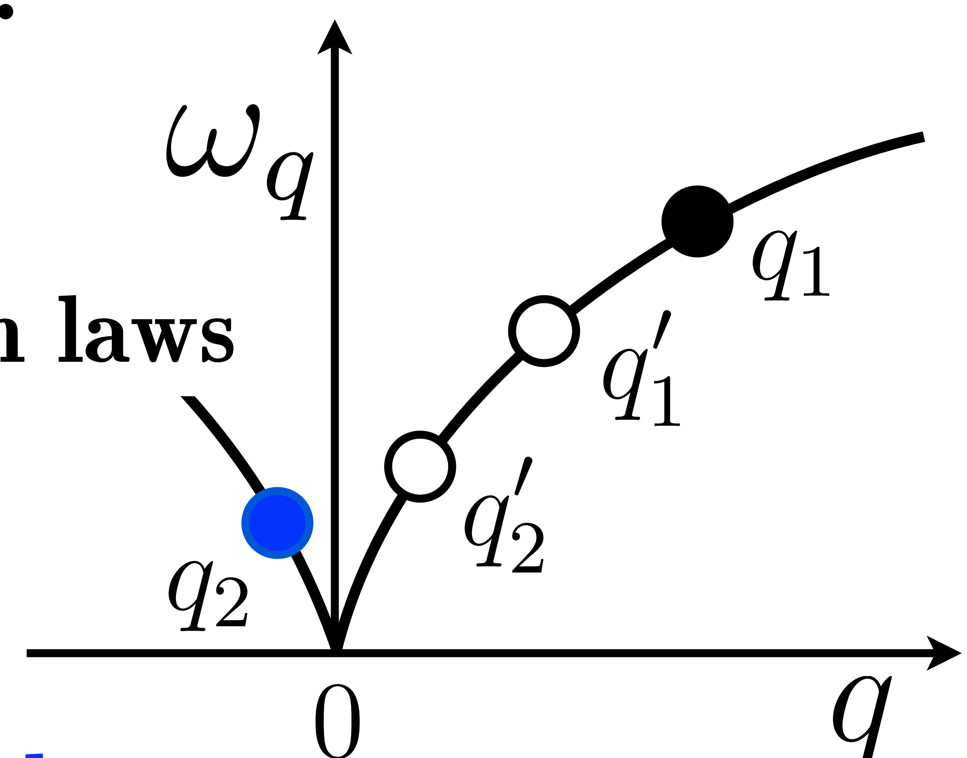


$$1/\tau_q \to \infty$$

The culprit: linear spectrum

What if the spectrum were nonlinear?

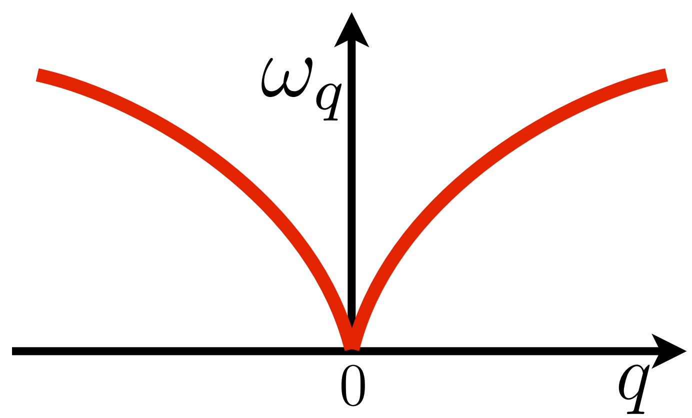
forbidden by conservation laws



we need at least one more boson

$\Rightarrow$ finite decay rate

Does nonlinear boson spectrum make sense?



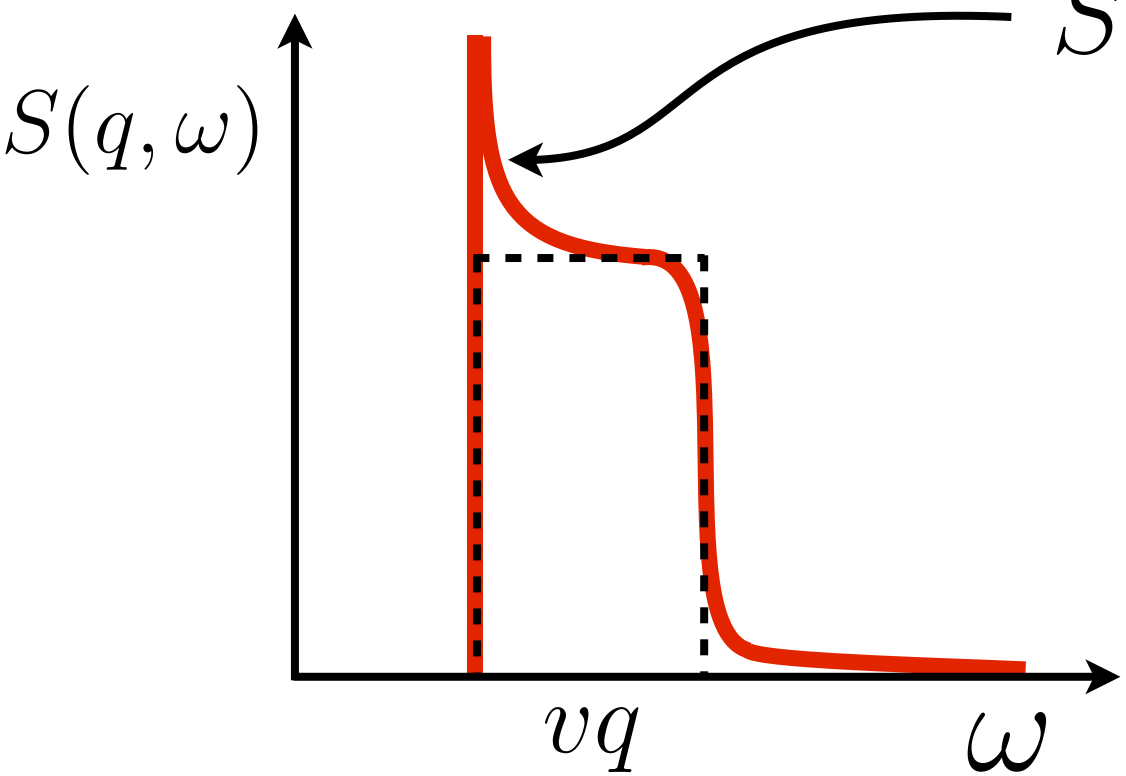
$$\omega_q = v|q|\left(1 - \xi q^2\right)$$

- $v\xi q^3 \gg \delta\omega_q$ (nonlinearity must exceed the uncertainty)
- $\xi q^2 \ll 1$ (we wish the spectrum to be almost linear)

$\Rightarrow$ $K = v_F/v \ll 1$ **(strong repulsion)**

$$\sqrt{K} \ll \xi q \ll \sqrt{\xi} \qquad (\xi \sim 1)$$

What happens with the "box" at a finite q?



$$S(q,\omega) \propto \frac{\theta(\omega - \omega_0)}{(\omega - \omega_0)^{\mu_q}}$$

$$\mu_q \propto q \ll 1 \text{ at } \xi q \ll \sqrt{K}$$

but

$$\mu_q \approx 1 - K \text{ at } \xi q \gg \sqrt{K}$$

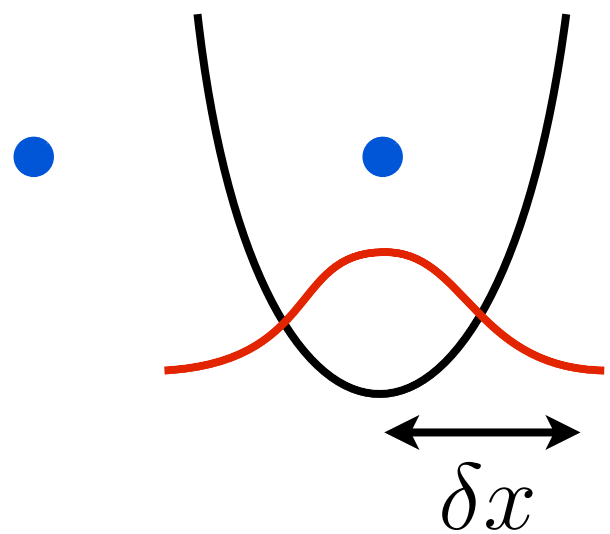
i.e., $S(q,\omega)$ resembles a $\delta-$function:

the spectral weight shifts towards $\omega = \omega_0$

One-dimensional Wigner crystal

$$H = \sum_l \frac{p_l^2}{2m} + \frac{1}{2}\sum_{l\neq l'} V(x_l - x_{l'}), \quad l = 1, \ldots, N$$

strong long-range repulsion



$$\delta x \ll 1/\rho \quad \Rightarrow \quad V''(1/\rho) \gg \hbar^2 \rho^4/m$$

$$K \sim \frac{\hbar \rho^2}{\sqrt{m V''(1/\rho)}} \ll 1$$

spectrum in the harmonic approximation:

$$\omega_q^2 = \frac{2}{m}\sum_{l=1}^{\infty} V''(l/\rho)\left[1 - \cos(ql)\right]$$

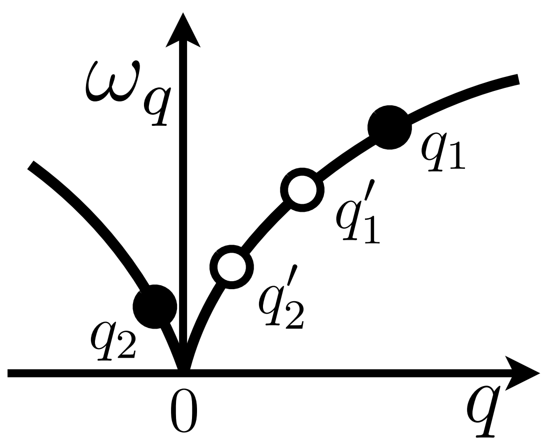
Anharmonisms: interaction between the bosons

Expansion in displacements $u_l = x_l - l/\rho$

Scattering amplitude:

- each boson brings a factor $(\hbar/\omega_q)^{1/2}|q| \propto (\hbar|q|)^{1/2}$
- the leading process has fewer $\hbar$'s

$$t_{q_1 q_2; q_1' q_2'} = \frac{\lambda}{N} \frac{\hbar^2 \rho^2}{m} \left| q_1 q_2 q_1' q_2' \right|^{1/2}$$



Scattering probability:

$$W_{q_1,q_2;q_1',q_2'} = \underbrace{\frac{2\pi}{\hbar^2} \left| t^2_{q_1 q_2; q_1' q_2'} \right|}_{\propto K^2} \delta_{q_1+q_2, q_1'+q_2'} \, \delta(\omega_{q_1} + \omega_{q_2} - \omega_{q_1'} - \omega_{q_2'})$$

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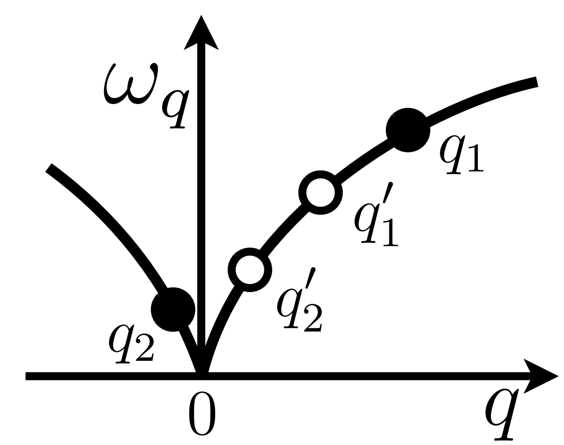
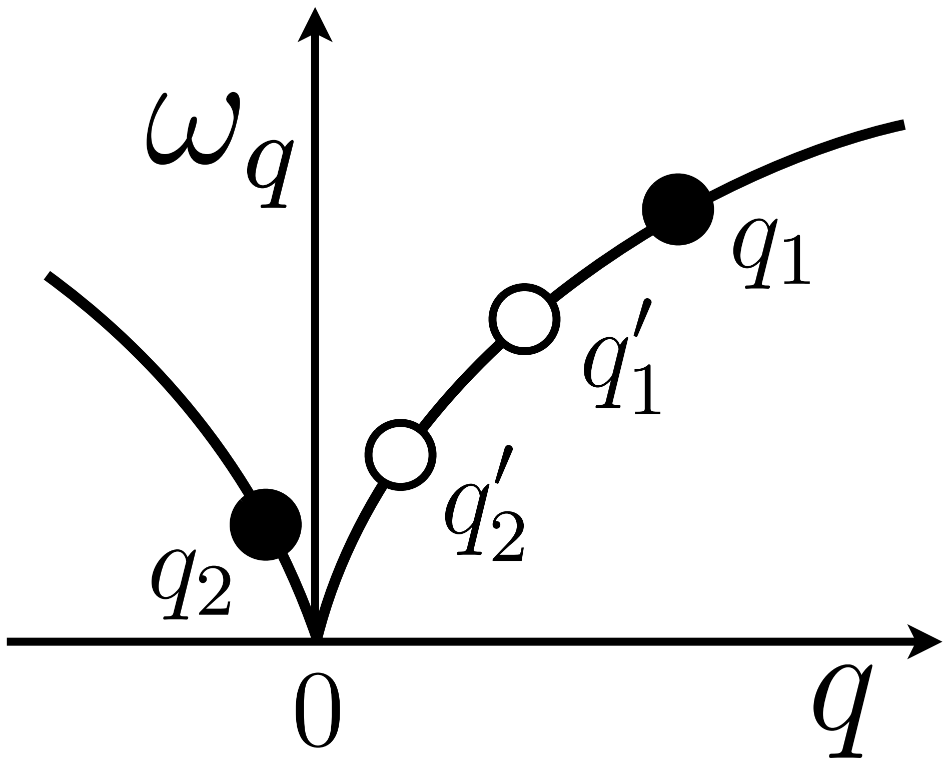


generic interaction: $\lambda \sim 1$

integrable models: $\lambda = 0$

Momentum and energy conservation



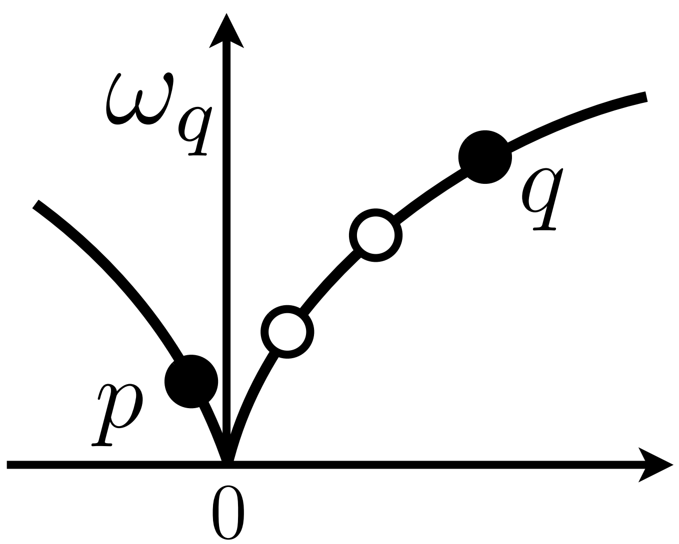
$$q_1 + q_2 = q_1' + q_2'$$

$$\omega_{q_1} + \omega_{q_2} = \omega_{q_1'} + \omega_{q_2'}$$

- $\{q_1, q_2\} \;\rightarrow\;$ unique set $\{q_1', q_2'\}$
- if q_1, q_2, q_1' are positive then $q_2 \approx -(3\xi/2) q_1 q_2 q_1'$

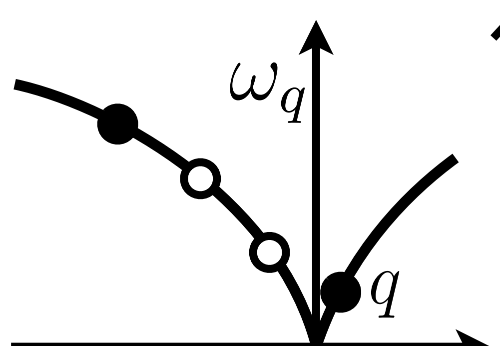
(parametrically small)

Linearized spectrum



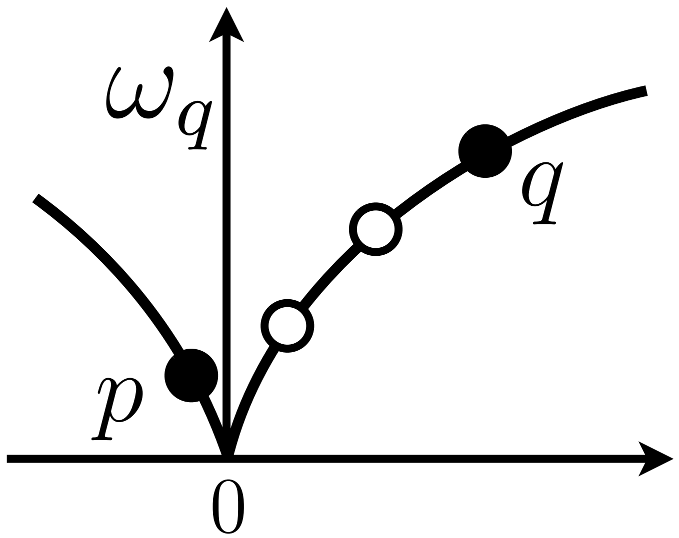
$$\xi(T/\hbar v)^3 \ll q \ll (T/\hbar v \xi)^{1/3}$$

ensures that $|p| \sim \xi q^3 \ll T/\hbar v$



exponentially small contribution

Linearized spectrum



$$\xi(T/\hbar v)^3 \ll q \ll (T/\hbar v\xi)^{1/3}$$

$$\Downarrow$$

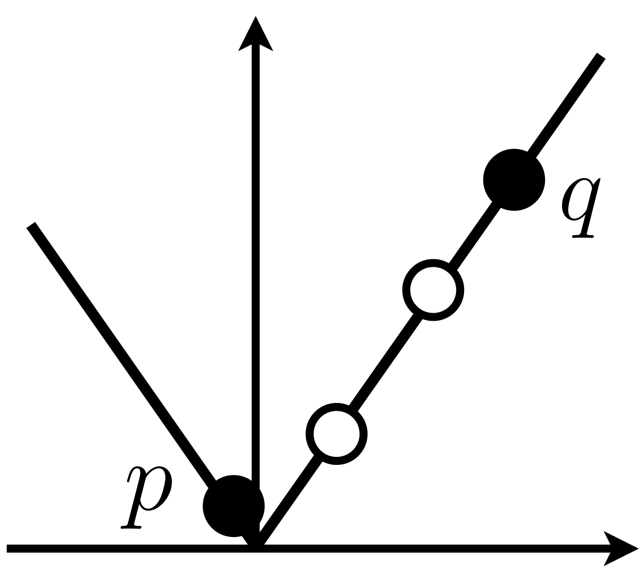
$$\xi(T/\hbar v)^2 \ll 1$$

for "thermal" bosons ($q \sim T/\hbar v$) nonlinearity is negligible

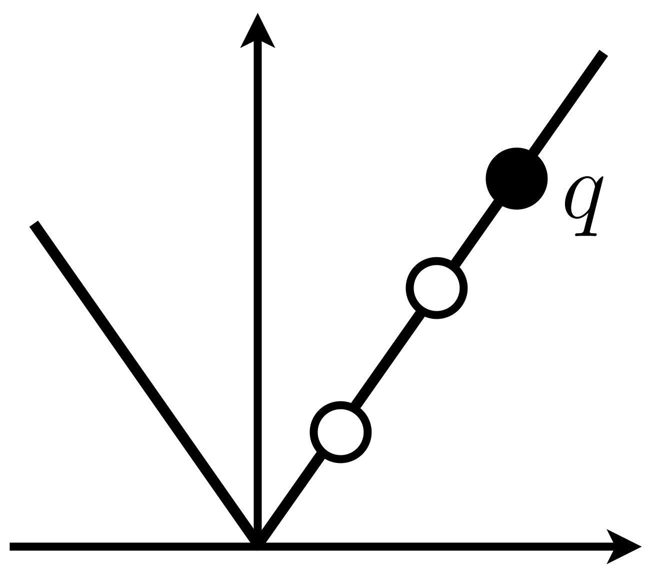
Any observable:

p enters as $|p|\underbrace{[n_p(+1)]}_{T/\hbar v|p|} = T/\hbar v$

from the amplitude

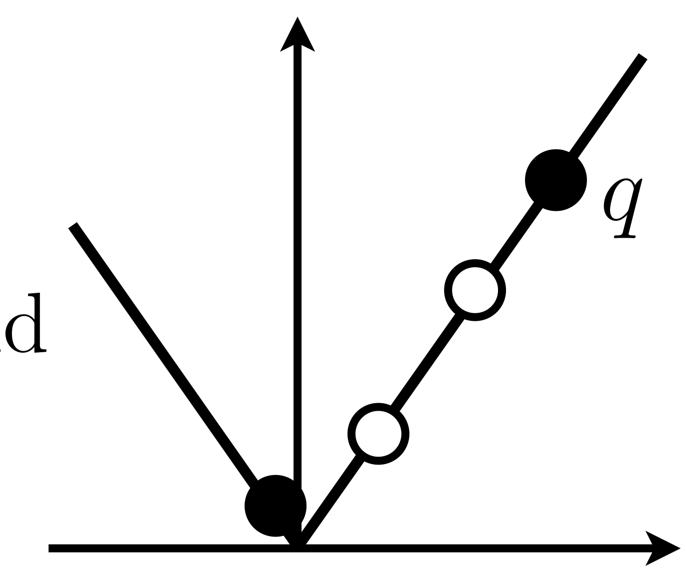


Linear spectrum vs. linearized spectrum



$(\delta - \text{function})^2 \quad \Rightarrow \quad \text{divergent rate}$

$\delta-$ function spent on left-movers, left $T/\hbar v$ behind



in both cases, left- and right- movers are decoupled

Kinetic equation

- derive kinetic equation
- substitute $N_q = n_q + \sqrt{n_q(n_q+1)}f_q$
- neglect all but linear in f_q terms
- linearize the spectrum

$$\frac{\partial}{\partial t} f(x,t) = -\tau_0^{-1} \int_0^\infty dy\, \mathcal{G}(x,y) f(y,t)$$

exactly solvable

$$\mathcal{G}(x,y) = \frac{1}{6}x^2(x^2+1)\delta(x-y) - \frac{xy(x+y)}{\sinh\left[\pi(x+y)\right]} - \frac{xy(x-y)}{\sinh\left[\pi(x-y)\right]}$$

$$q = (T/\hbar v)2\pi x$$

$$\tau_0^{-1} = 2\pi\lambda^2 K^2 v\,(T/\hbar v)^5$$

Thermalization of co-moving bosons

$$\tau_0^{-1} = 2\pi\lambda^2 K^2 v\,(T/\hbar v)^5$$

$t \ll \tau_0$: only states with $q \gg T/\hbar v$ are affected

$$N_q \propto e^{-t/\tau_q}, \quad 1/\tau_q \sim \lambda^2 K^2 v (T/\hbar v) q^4$$

$t \gg \tau_0$: thermal bosons equilibrated among themselves,
but T has not yet the equilibrium value

$$\delta T(t) \sim -\hbar v \int_0^\infty dq\, f_q(0)\, e^{-t/\tau'_q}, \quad 1/\tau'_q \sim \lambda^2 K^2 v\,(T/\hbar v)^3 q^2$$

Summary

- boson spectrum at $\sqrt{K} \ll q \ll 1$ is **not** linear

- **order of limits** matters:

$$\xi \to 0 \text{ first, } q \to 0 \text{ second} \quad \Rightarrow \quad 1/\tau = \infty$$

(strictly **linear** boson spectrum)

$$q \to 0 \text{ first, } \xi \to 0 \text{ second} \quad \Rightarrow \quad 1/\tau \propto T^5$$

(**linearized** boson spectrum)

- thermalization of co-moving acoustic bosons is only the **fastest** equilibration process