Classical to Quantum Maps and their consequences

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KITP - September 20, 2012

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Conclusions

Using a classical to quantum mapping and, in particular, a new general relation regarding dynamical correlations, it is possible to prove, as a matter of principle, the existence of

Glassy dynamics, quantum dynamical heterogeneities, quantum critical jamming, quantum turbulence, and many transitions

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The idea in a nutshell:

Classical viscous system $\longleftrightarrow$ Quantum many body system

Old idea mentioned in statistical field theory textbooks by G. Parisi, J. Zinn-Justin. More recently looked anew by, e.g., G. Biroli, C. Chamon, and F. Zamponi, Phys. Rev. B 78, 224306 (2008) and others. Mostly treated as a mathematical curiosity.

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Viscous classical system

$$\boxed{\gamma_i \frac{dx_i}{dt} = -\frac{\partial}{\partial \vec{x}_i} V_{\mathsf{N}}(\vec{x}_1, ..., \vec{x}_N) + \vec{\eta}_i(t)}$$

$$\langle \eta_i^\alpha(t) \eta_j^\beta(t') \rangle = 2T_{cl} \gamma_i \delta_{ij} \delta_{\alpha\beta} \delta(t-t')$$

Fokker-Planck Equation

$$\mathcal{P}(x, t; x_0, t_0) = \langle \prod_{i=1}^{N} \delta[x_i(t) - x_i] \rangle_{\eta, x_0}$$

$$\boxed{\frac{\partial \mathcal{P}}{\partial t} = -H_{FP} \mathcal{P}}$$

$$H_{FP} = -\sum_i \frac{1}{\gamma_i} \frac{\partial}{\partial \vec{x}_i} \left[\nabla_i V_{\mathsf{N}} + T \frac{\partial}{\partial \vec{x}_i} \right]$$

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Classical first order differential equations of motion

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First order Schrodinger like equation

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Transform to a bona fide many body quantum problem

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$$\equiv \sum_i \frac{p_i^2}{2m_i} + \mathcal{V}_{\mathsf{Quantum}}(\{\vec{x}\})$$

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Temperature $T_{cl} \geq 0$

Viscosity γ_i Noise η_i

Potential energy:

$$V_{\mathsf{N}}(\vec{x}_1, \ldots, \vec{x}_N)$$

Zero temperature quantum system:

Effective mass: $m_i = \dfrac{\gamma_i}{2T_{cl}}$

Potential energy:

$$\mathcal{V}_{\mathsf{Quantum}}(\{\vec{x}\}) = \sum_i \frac{1}{\gamma_i}\left[+\frac{1}{4T}(\nabla_i V_{\mathsf{N}})^2 - \frac{1}{2}\nabla_i^2 V_{\mathsf{N}}\right]$$

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For a symmetric $V_{\mathsf{N}}(\{\vec{x}\})$
this is a bosonic wave-function

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Zero temperature quantum system:

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Quantum response function

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$$G_{\text{Quantum}}(t) = \frac{1}{2}(G_{\text{classical}}(it) + G_{\text{classical}}(-it)).$$

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Physical consequences for quantum systems:

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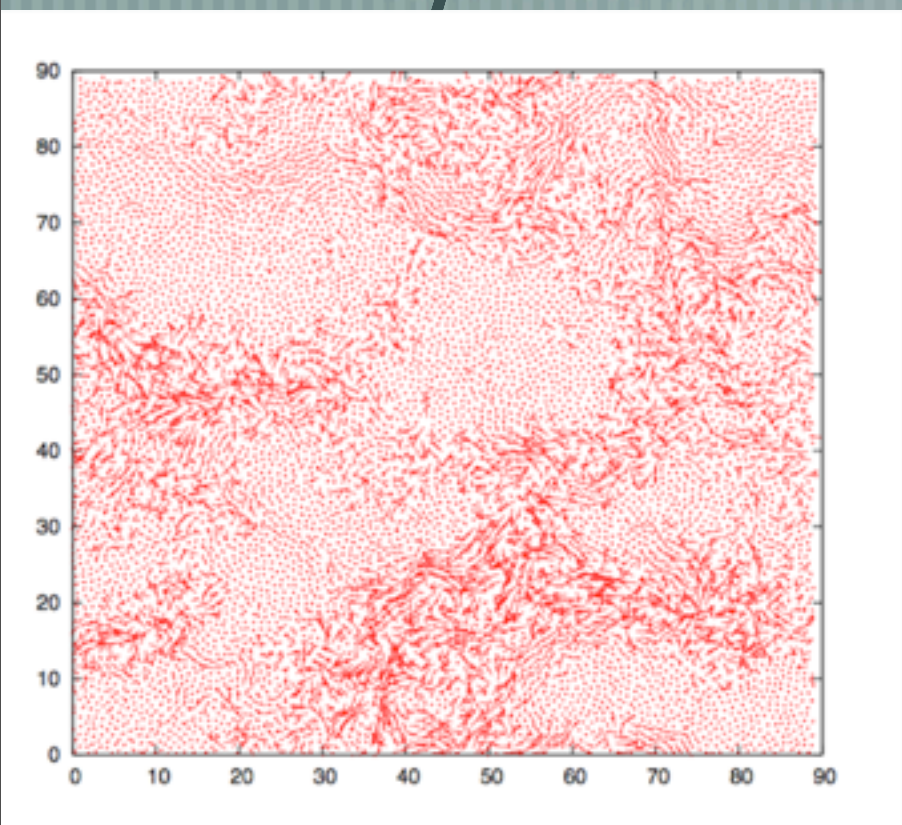

$$C(t) = \langle \delta\phi(x,0)\delta\phi(x,t)\rangle$$

$$\begin{aligned} G_4(x-y,t) &= \langle \delta\phi(x,t)\delta\phi(x,0)\delta\phi(y,t)\delta\phi(y,0)\rangle \\ &- C^2(t) \end{aligned}$$

$$S_4^{\text{classical}}(\vec{q},t) = \frac{\chi_4(t)}{1+q^2\xi_4(t)^2}$$

L. Berthier and G. Biroli, RMP 83, 587 (2011)

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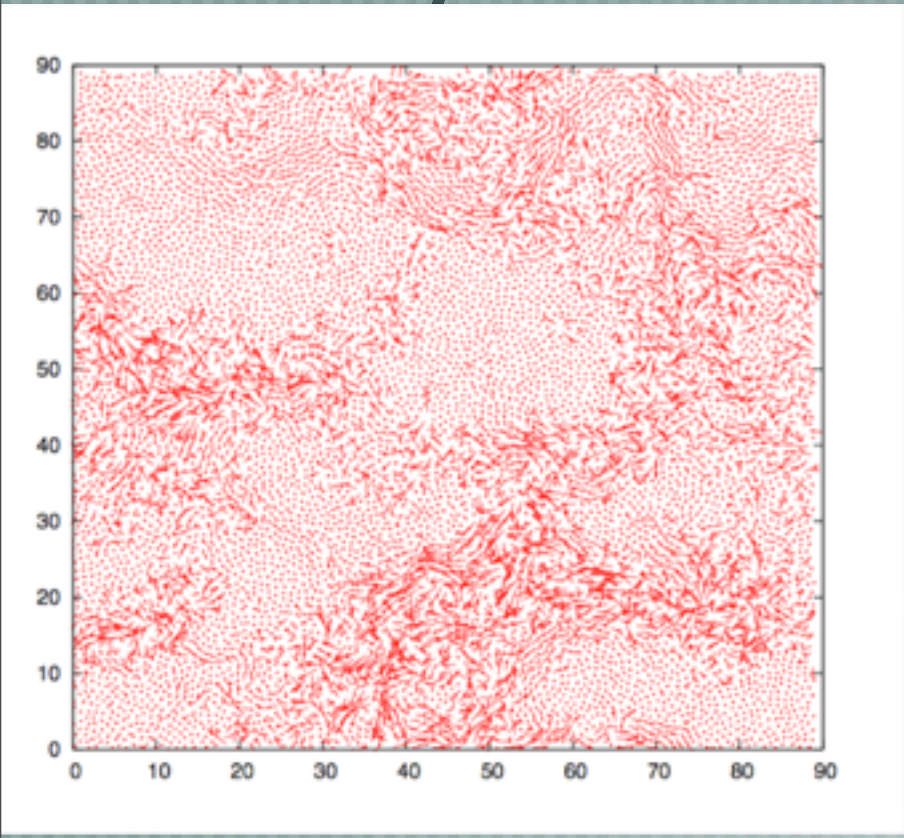

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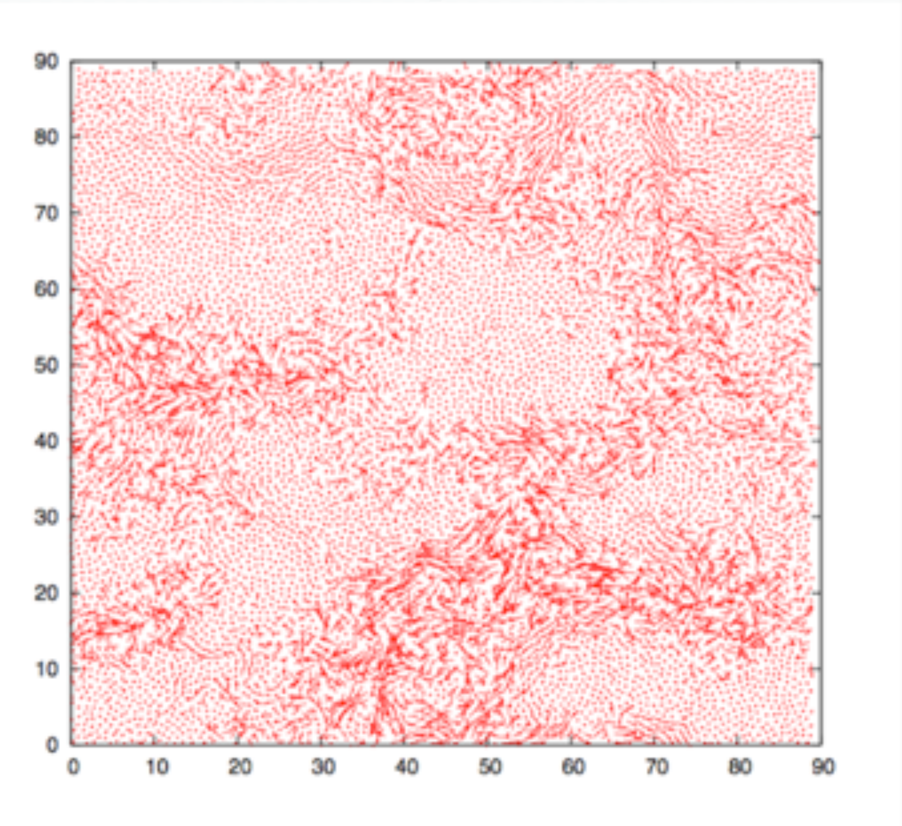

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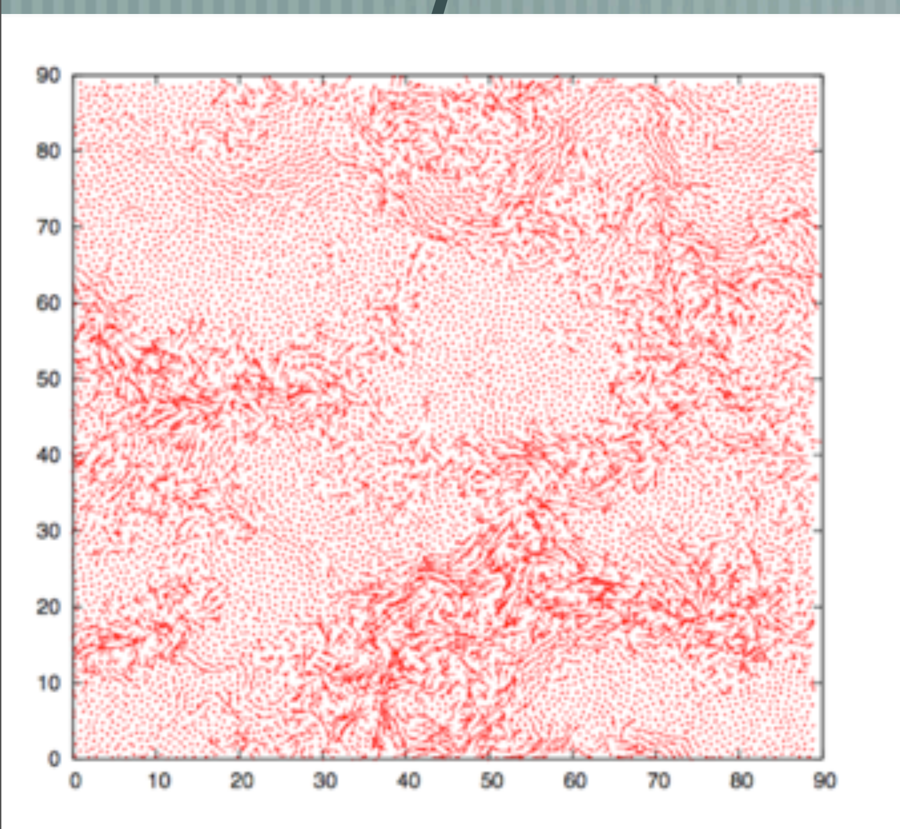

$$\xi_4(\tau) \sim \tau^{1/z}$$

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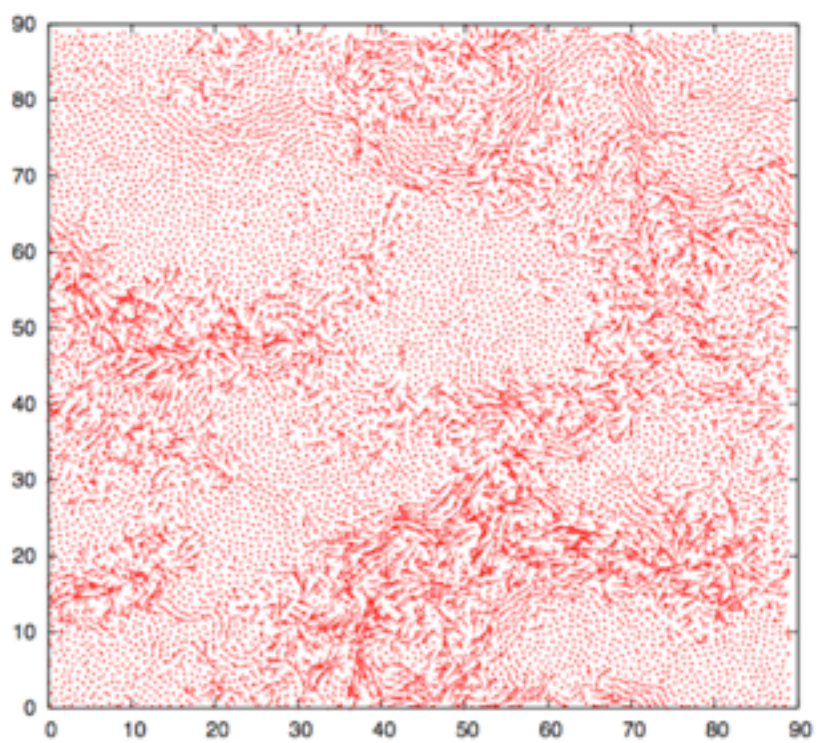

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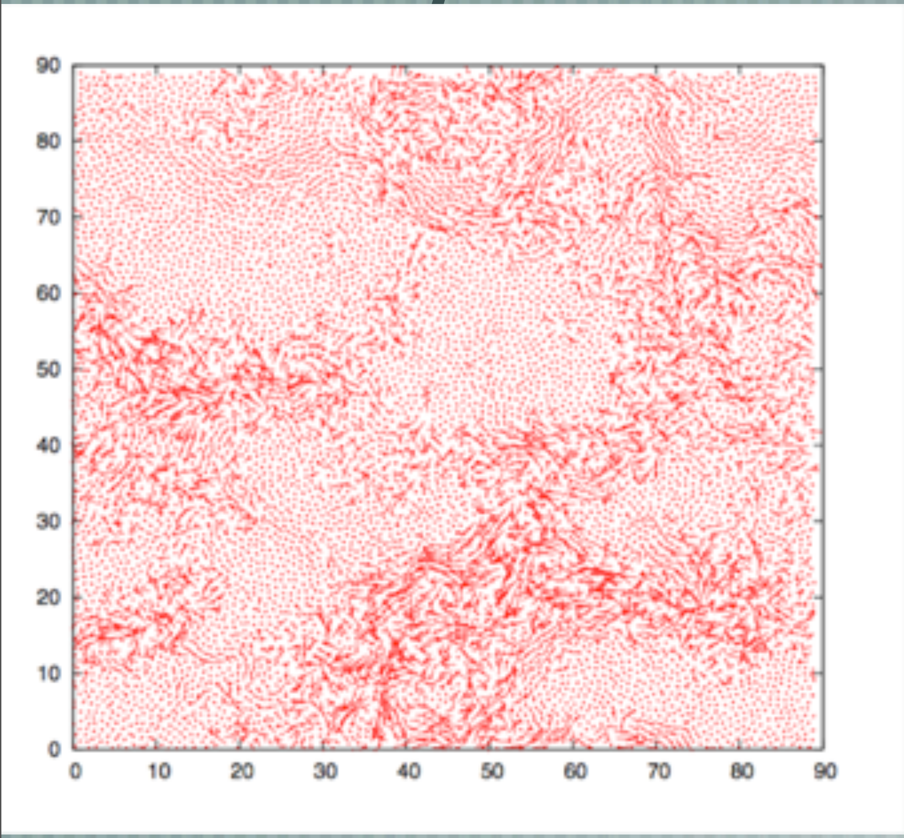

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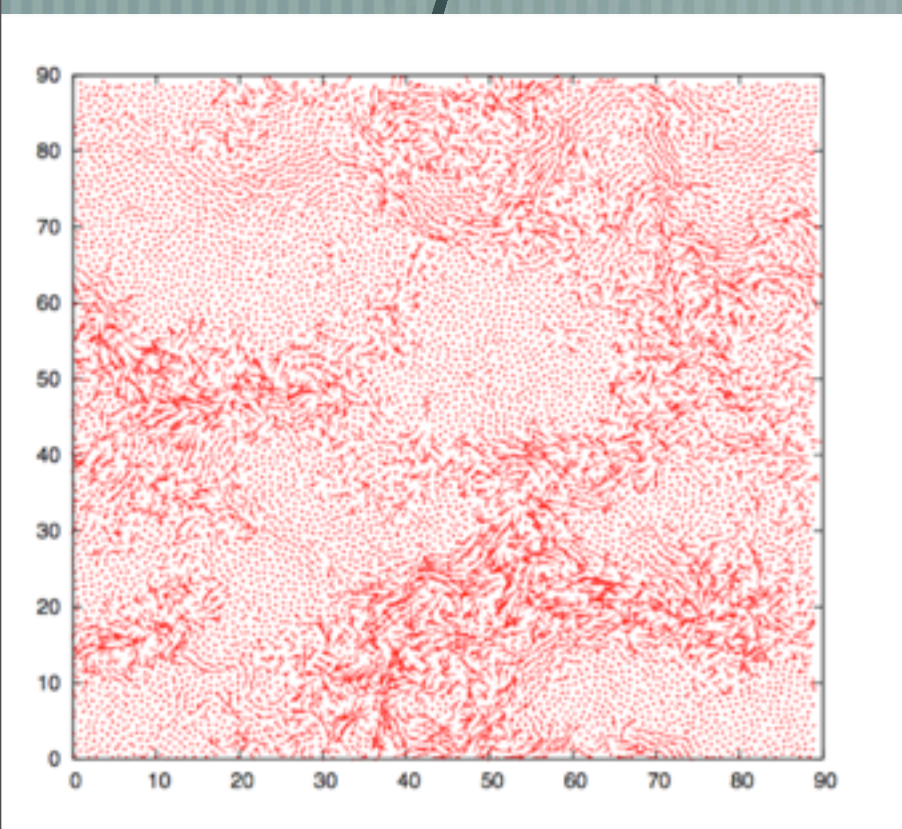

Set

$$\mathcal{O}(t) = \phi_q(t)\phi_{-q}(0) - \langle\phi_q(t)\rangle\langle\phi_{-q}(0)$$

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$$G_4(q,t) = \langle\mathcal{O}(t)\mathcal{O}(0)\rangle$$

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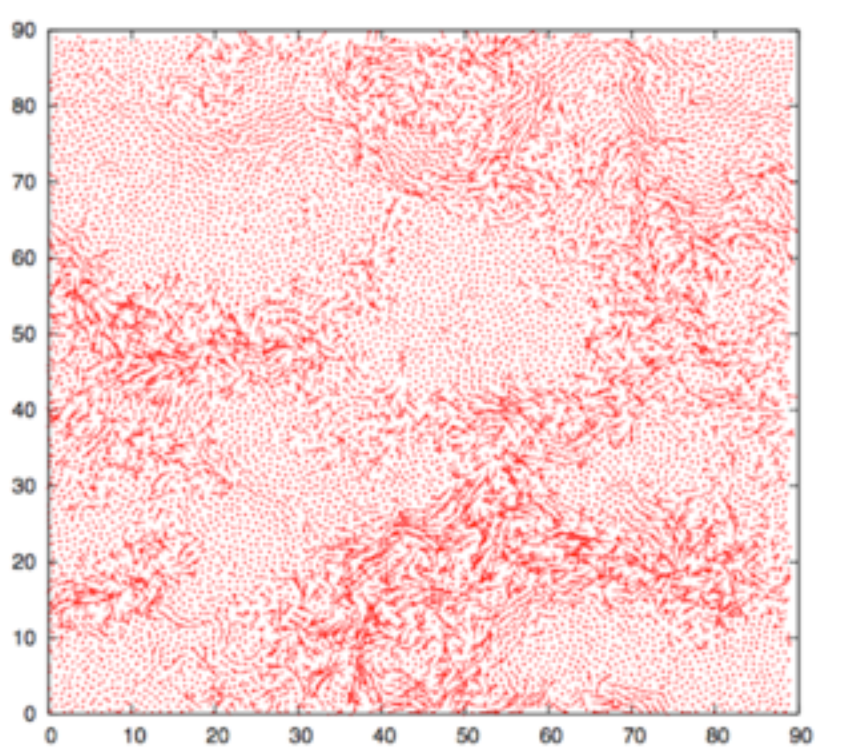

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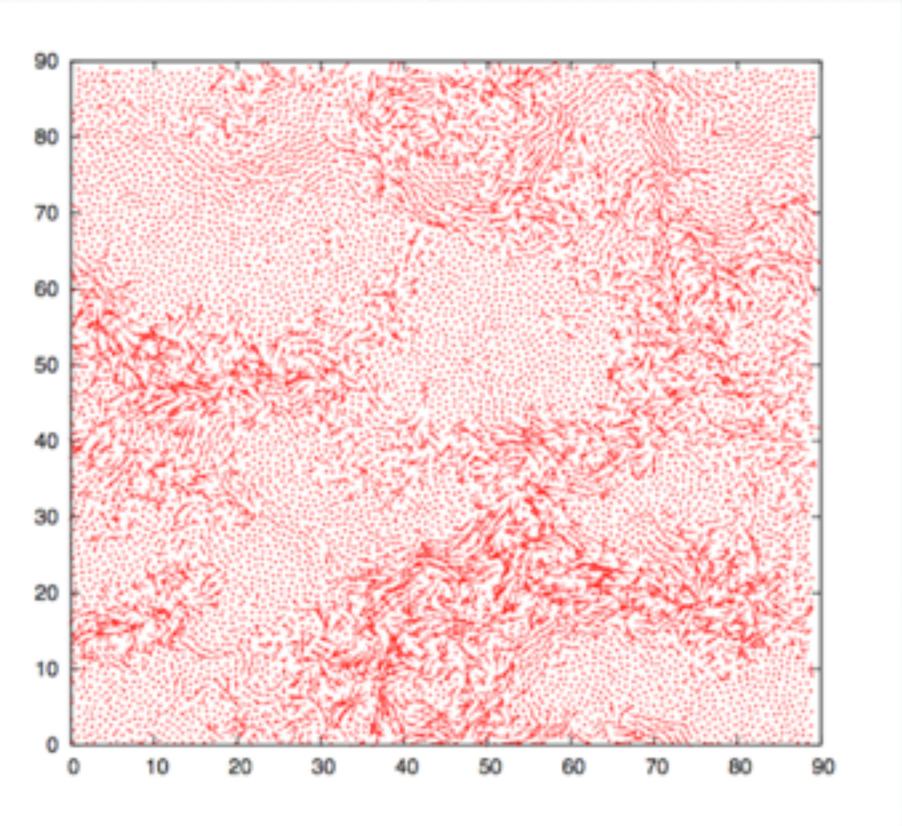

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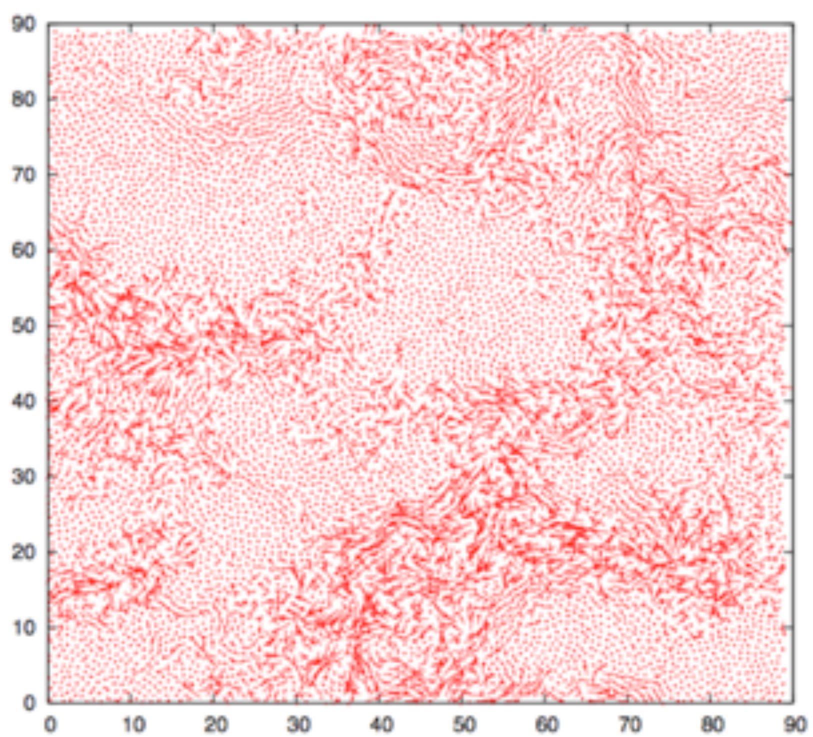

Set

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$$G_{4\ \text{Quantum}}(t) = \Re G_{4\ \text{classical}}(it)$$

$$e^{-r/\xi_4} \sim e^{-r/\tau^{1/z}} \to \cos\left(\frac{r}{\tau^{1/z}}\sin\frac{\pi}{2z}\right)e^{-\frac{r}{\tau^{1/z}}\cos\frac{\pi}{2z}}$$

Dynamical heterogeneity density wave

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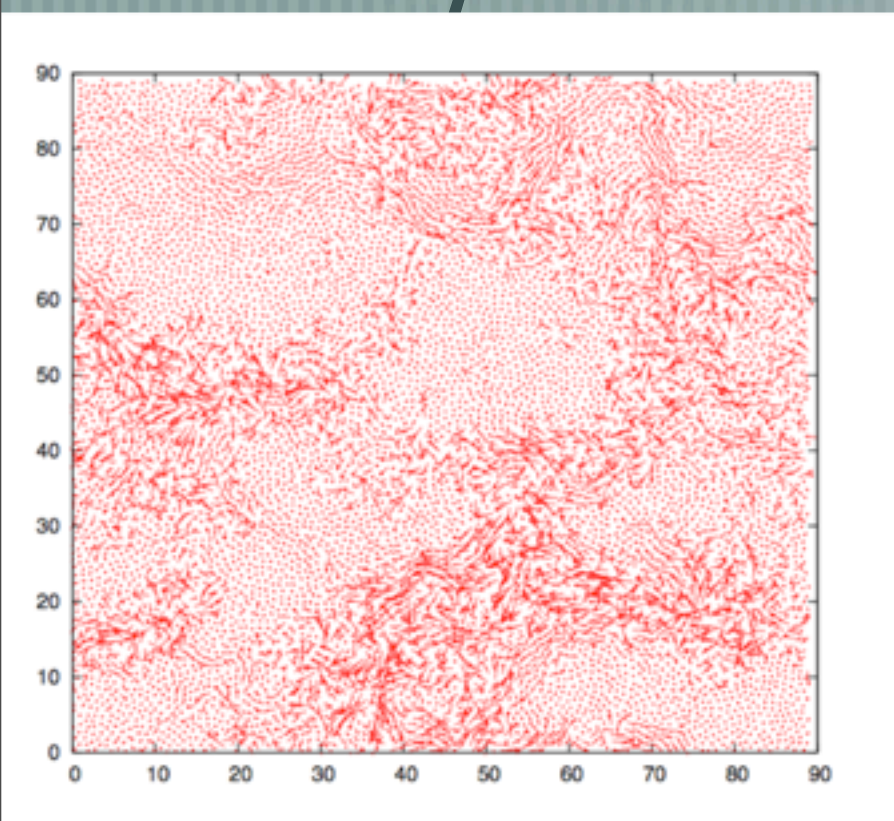

Set

$$\mathcal{O}(t) = \phi_q(t)\phi_{-q}(0) - \langle \phi_q(t)\rangle\langle \phi_{-q}(0)\rangle$$

$$G_{4\ \mathsf{Quantum}}(t) = \Re G_{4\ \mathsf{classical}}(it)$$

$$e^{-r/\xi_4} \sim e^{-r/\tau^{1/z}} \rightarrow \cos\left(\frac{r}{\tau^{1/z}} \sin \frac{\pi}{2z}\right) e^{-\frac{r}{\tau^{1/z}} \cos \frac{\pi}{2z}}$$

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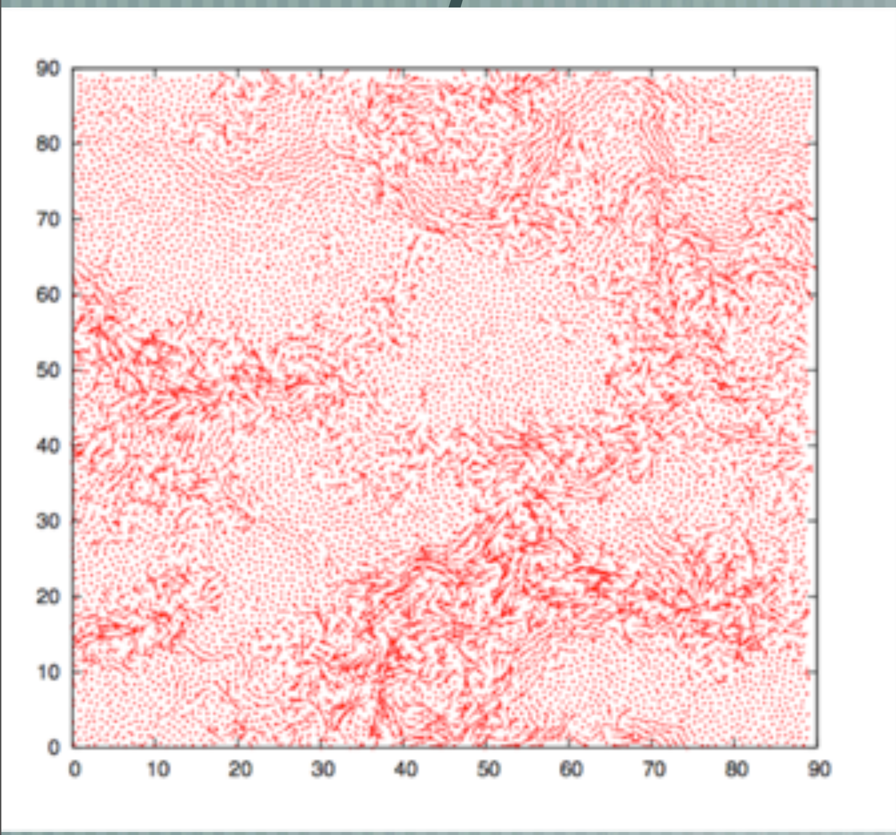

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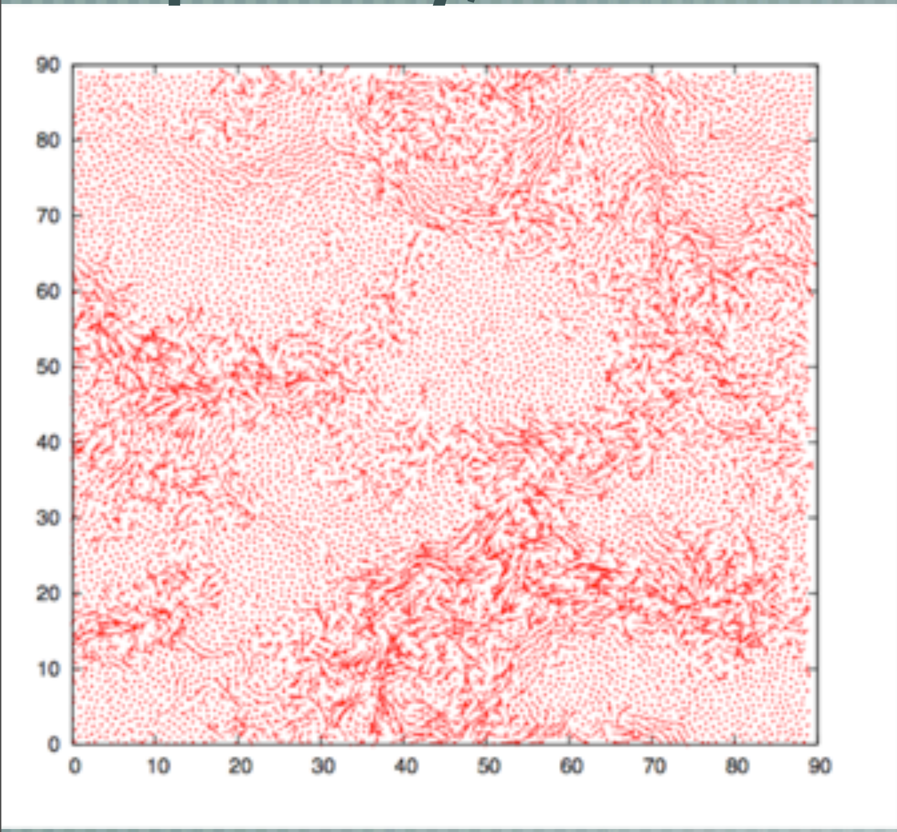
Physical consequences for quantum systems:

Empirically, in models of classical glass formers: $\tau \sim \exp(k\xi^{\theta})$

$$\theta \simeq 1.3$$



Alternate empirical fit: $\tau \sim \xi_4^z$

$$z \simeq 10.8$$

K. Mizuno and R. Yamamoto PRE 84, 011506 (2011)

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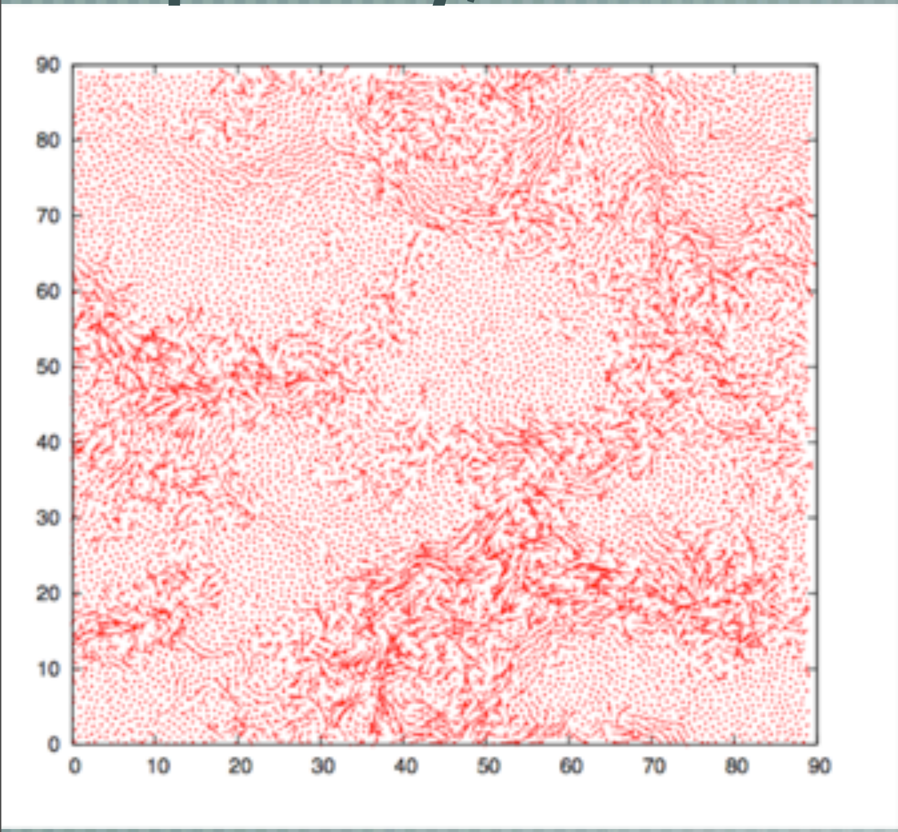

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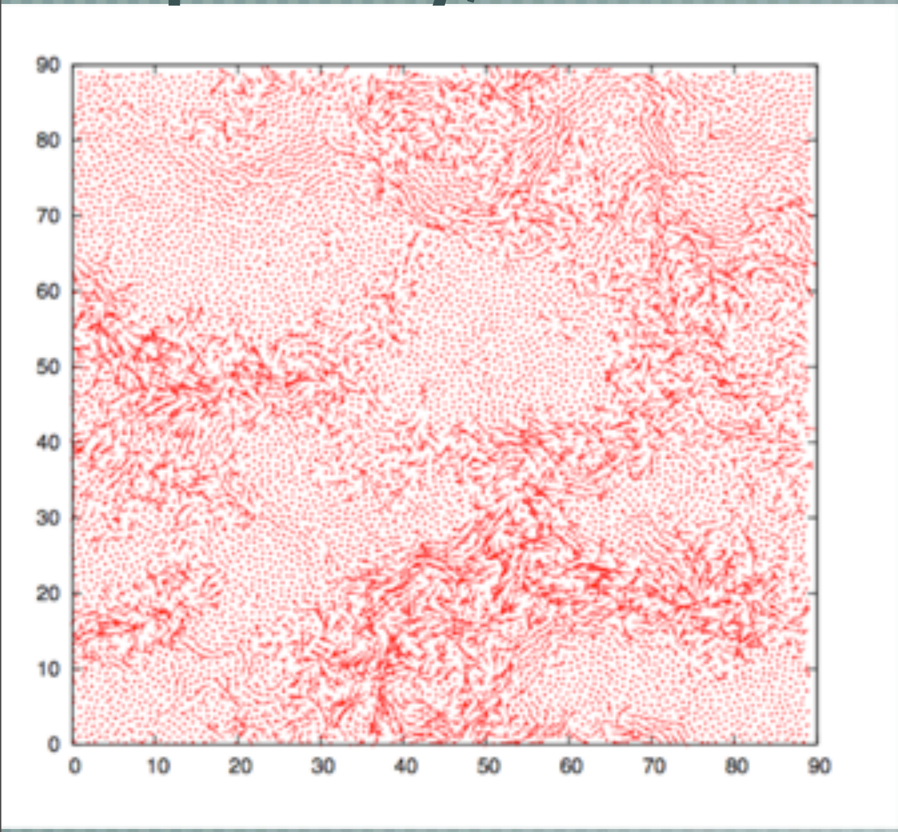

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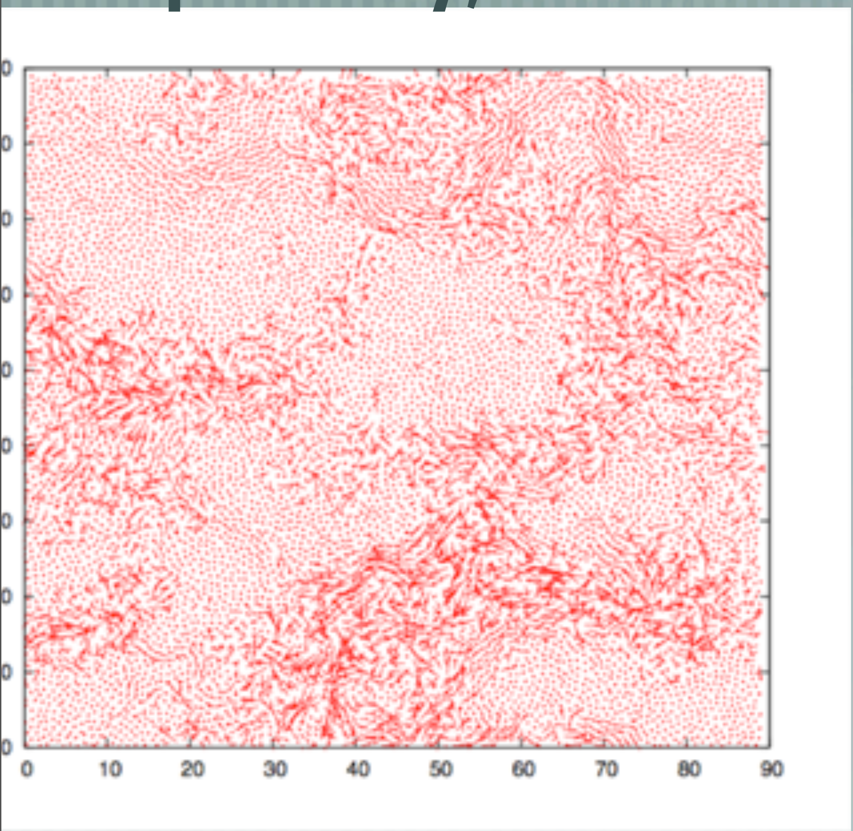

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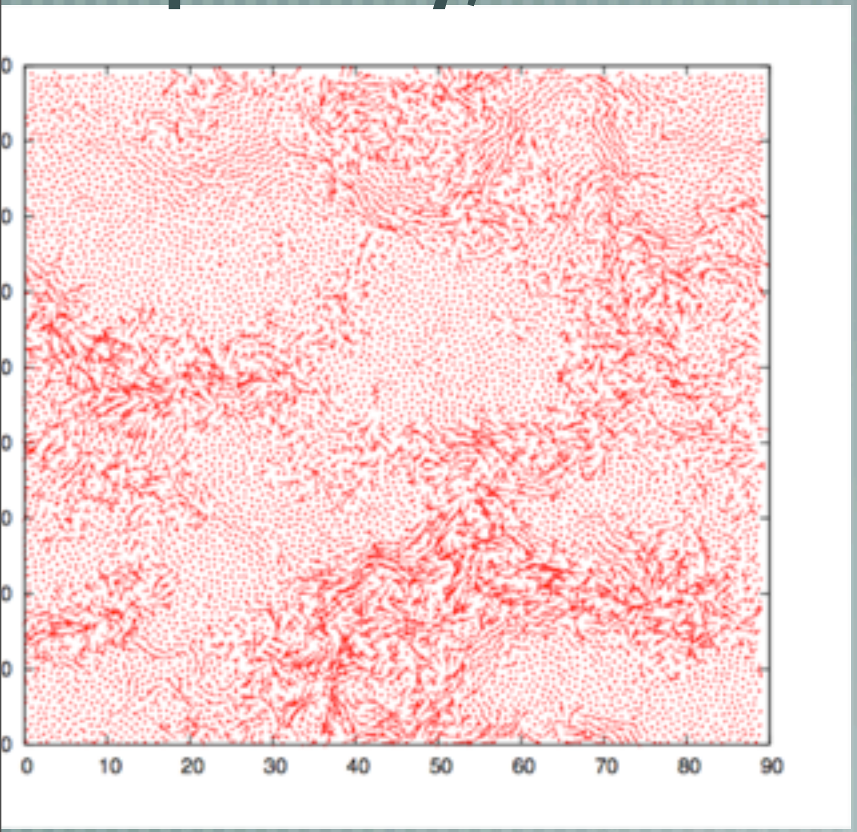

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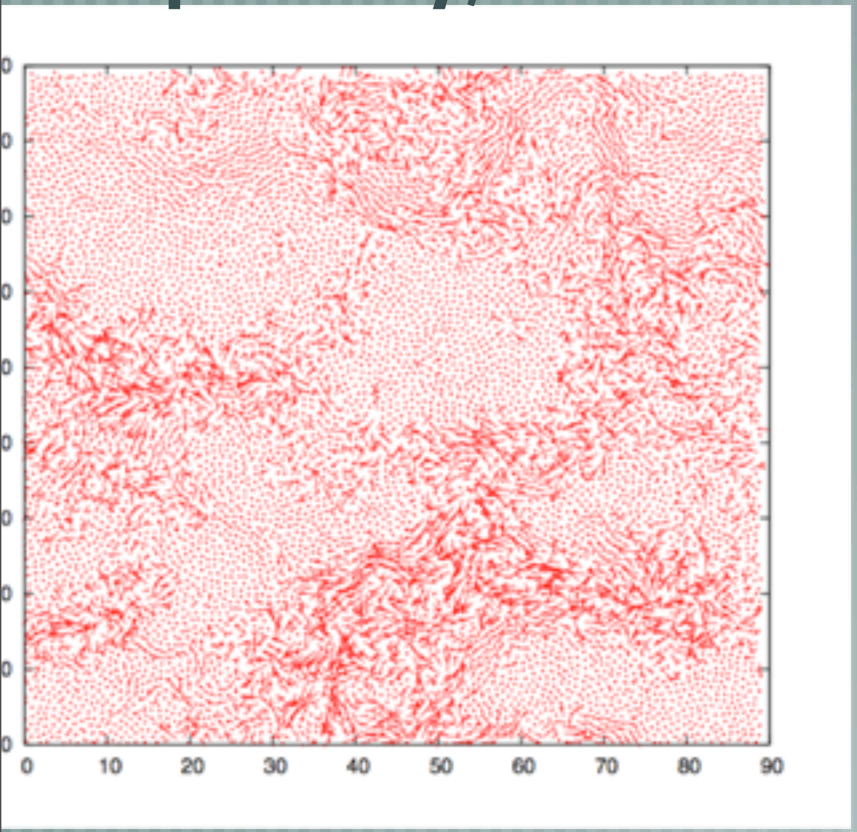

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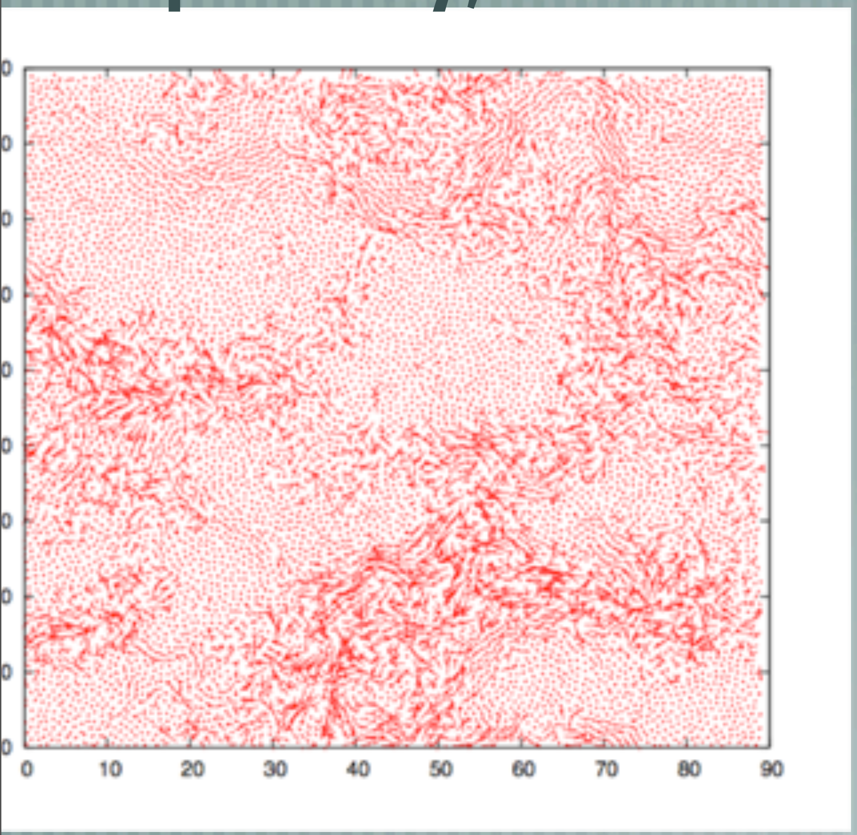

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Bottom line: a much more dramatic increase in time scales than length scales

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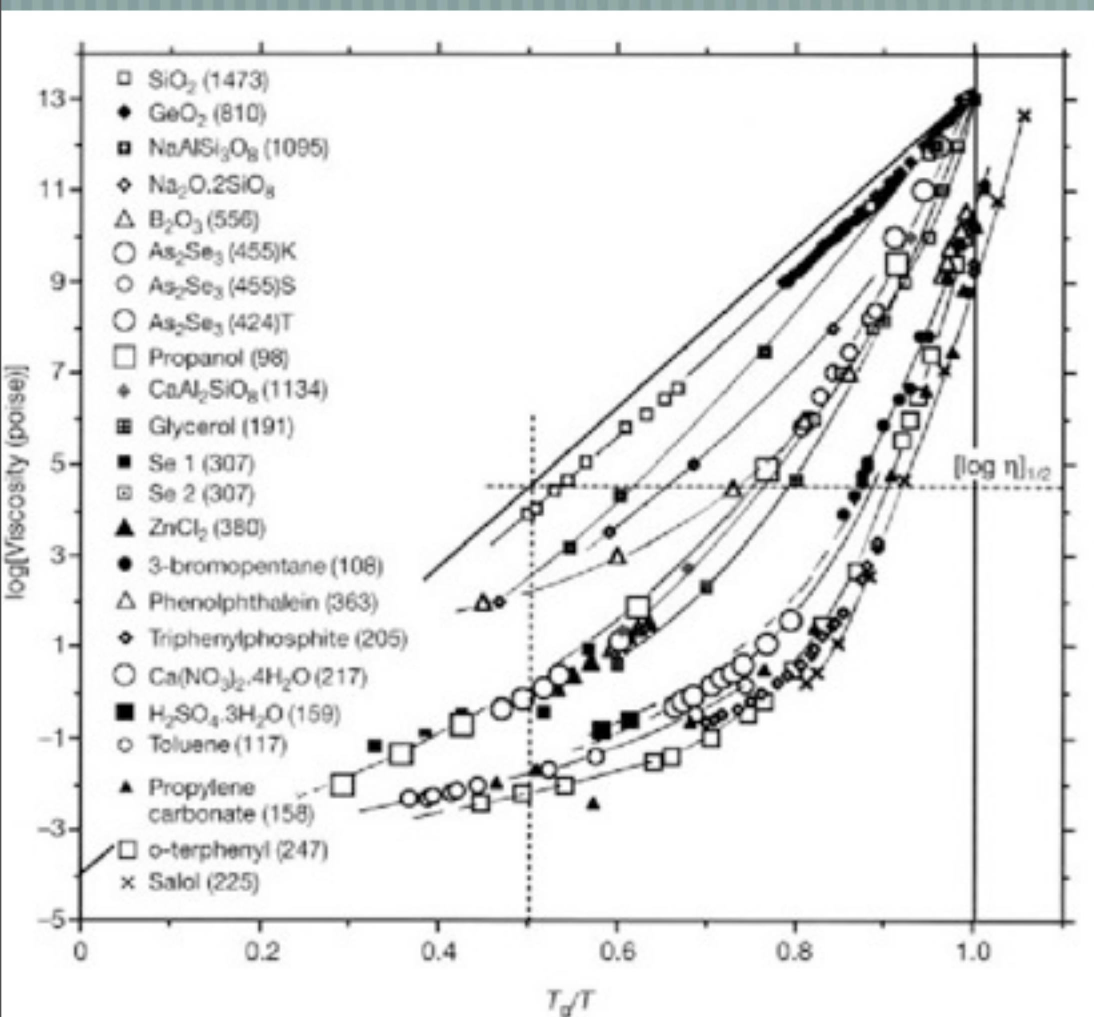

Classical glass formers:

Empirical Vogel-Fulcher-Tamman law for relaxation times

$$\tau(T) = \tau_0\, e^{\Delta/(T-T_0)}$$

$$G_{\text{classical}} = A \exp[-(t/\tau)^a]$$

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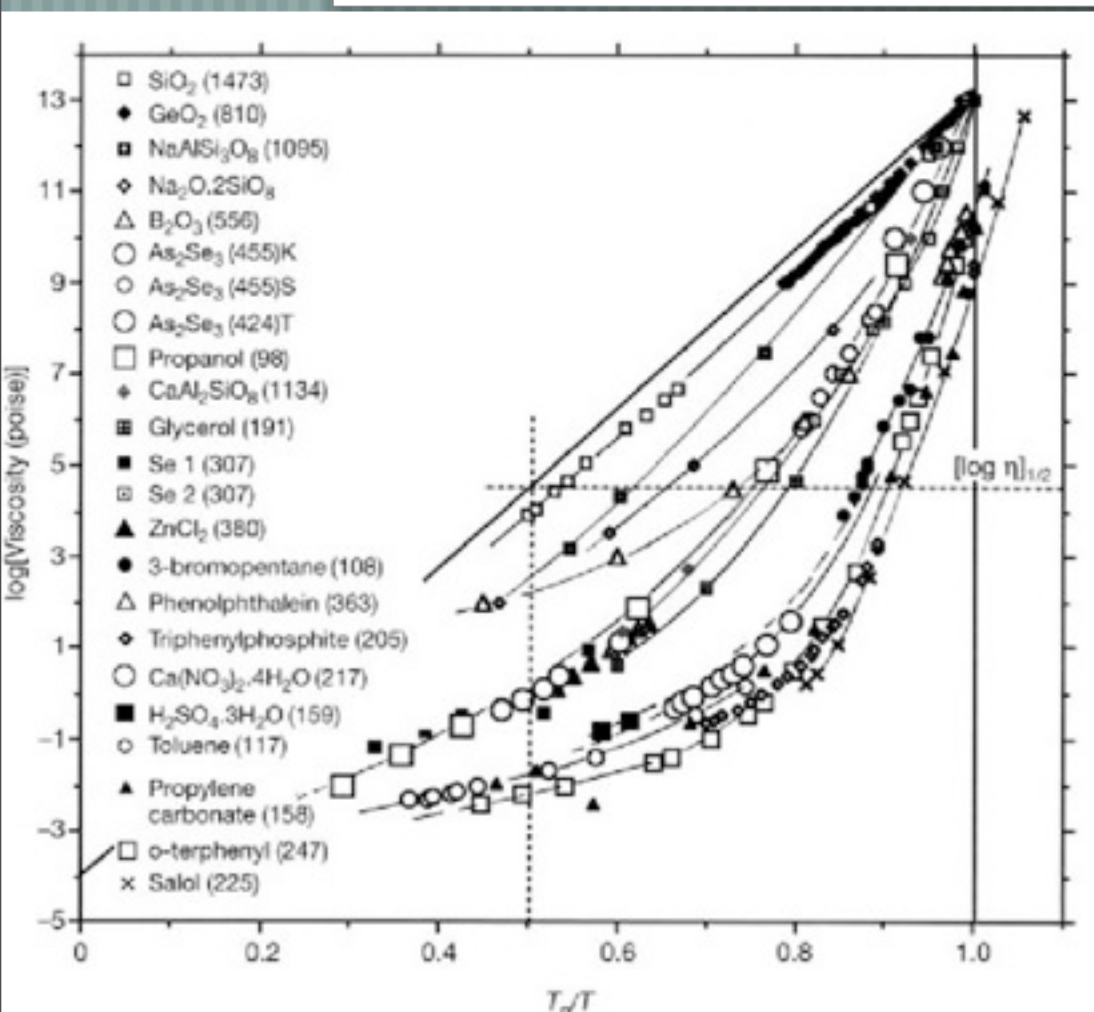

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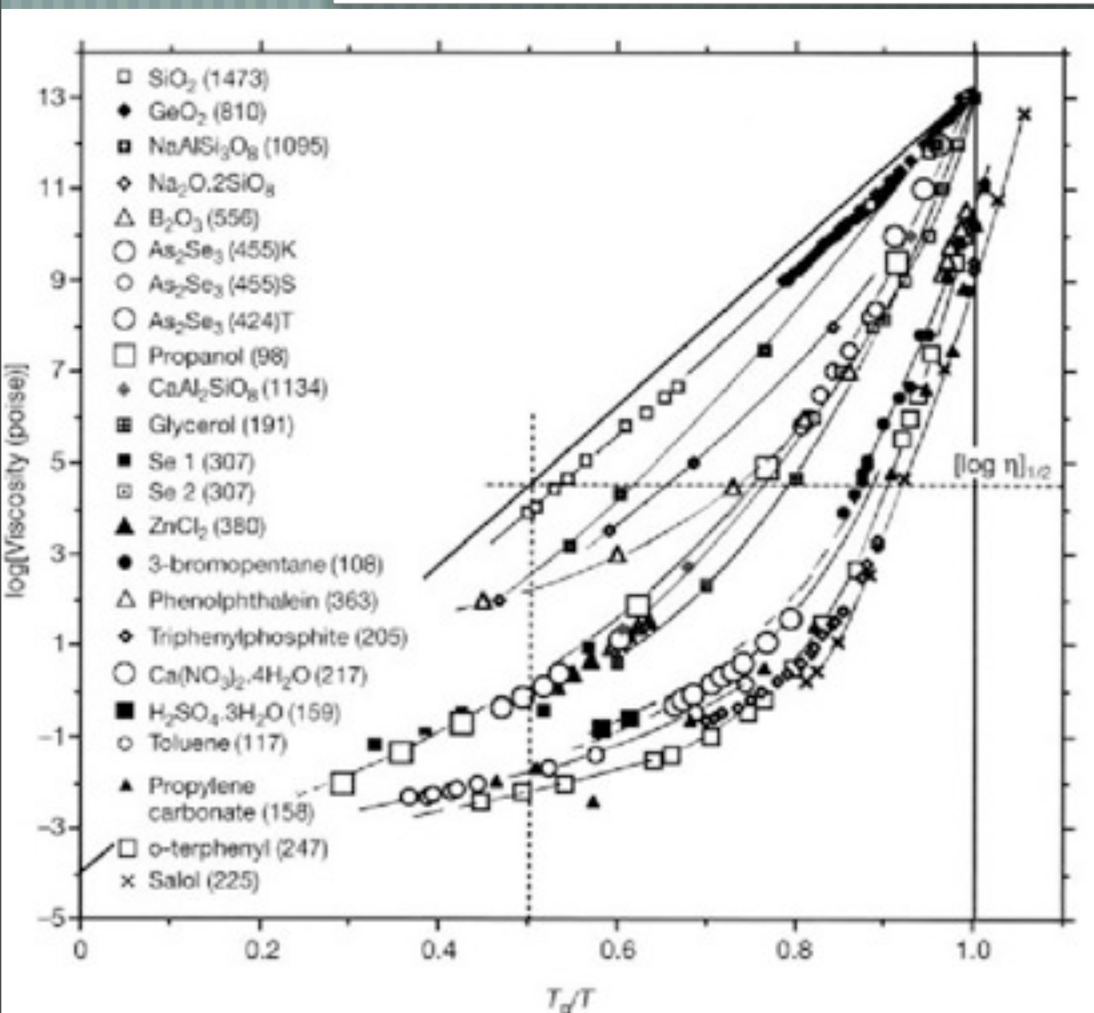

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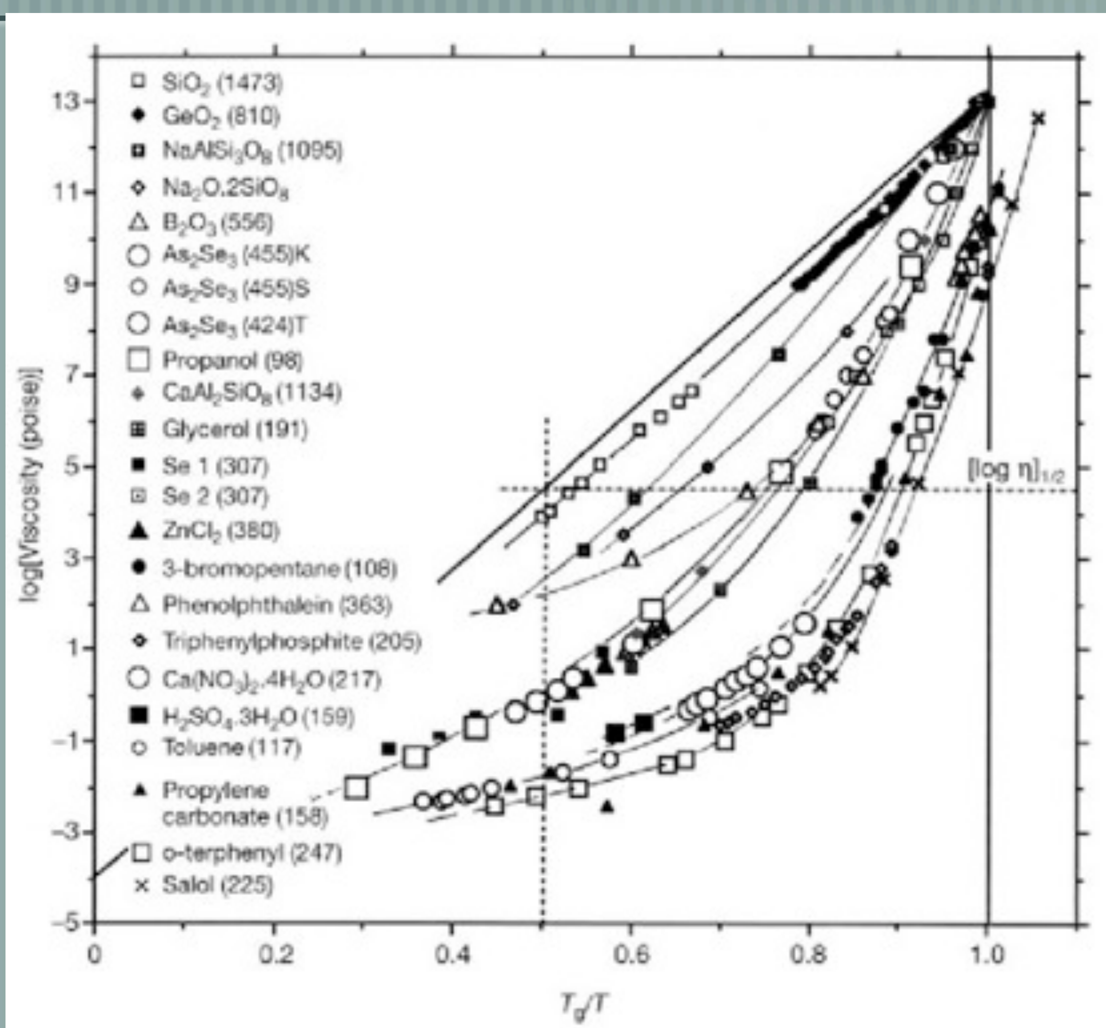

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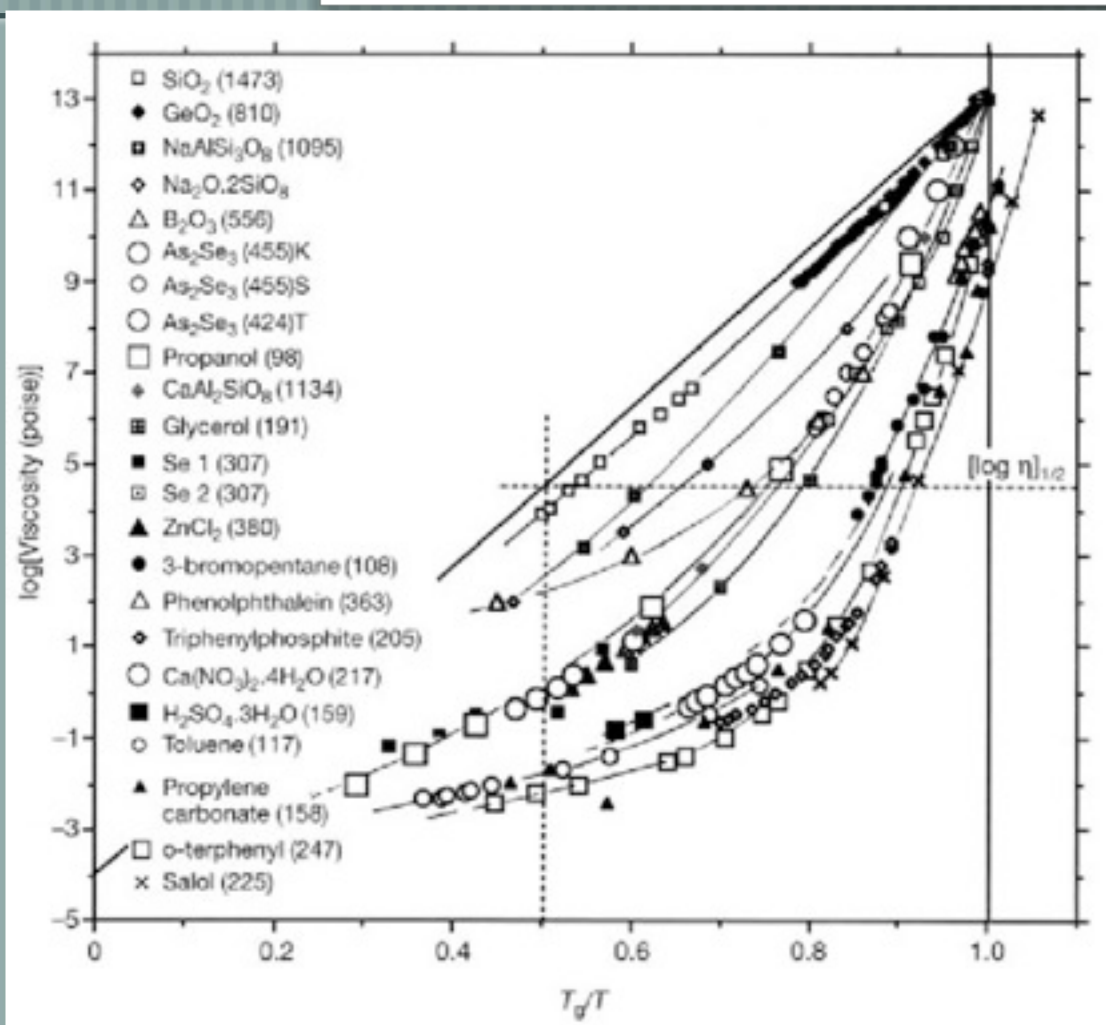

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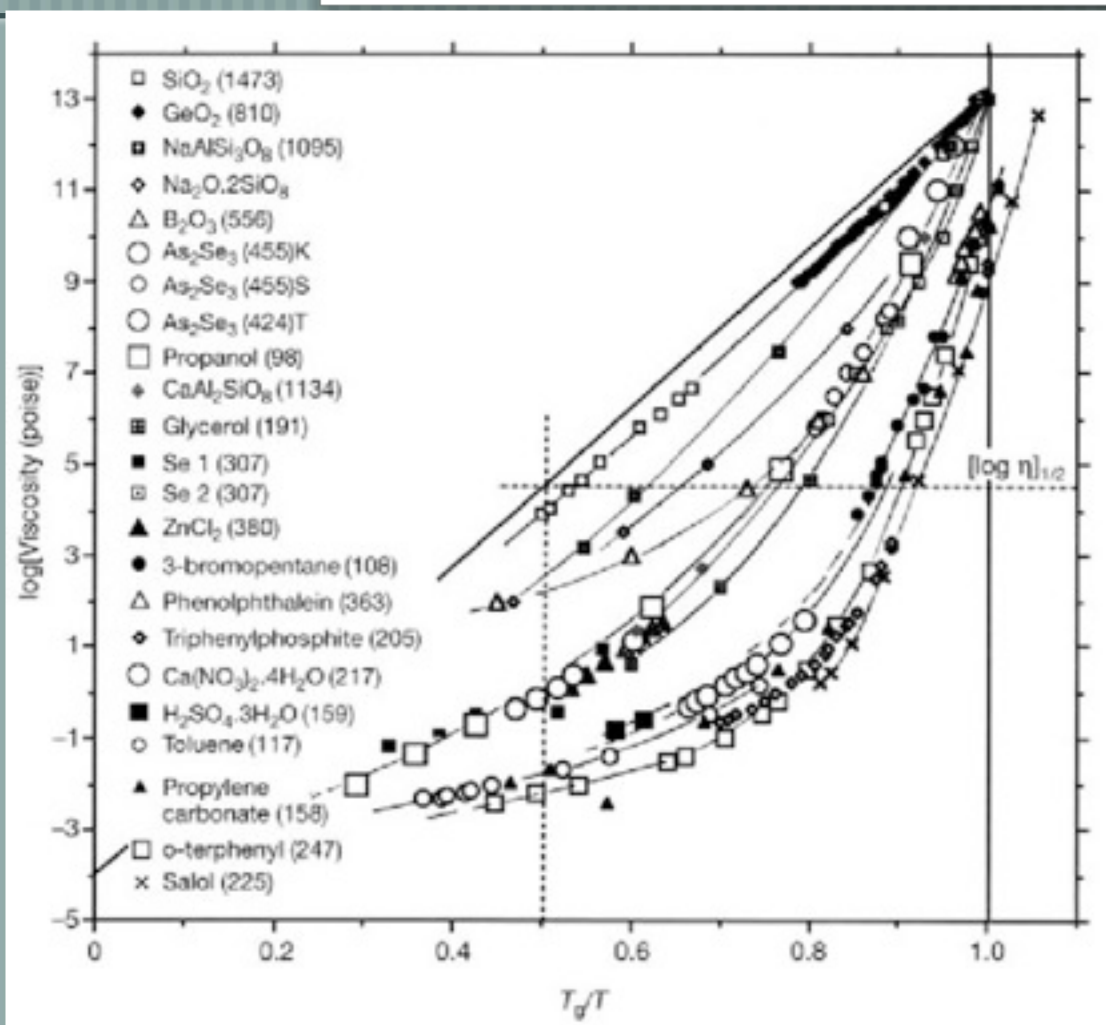

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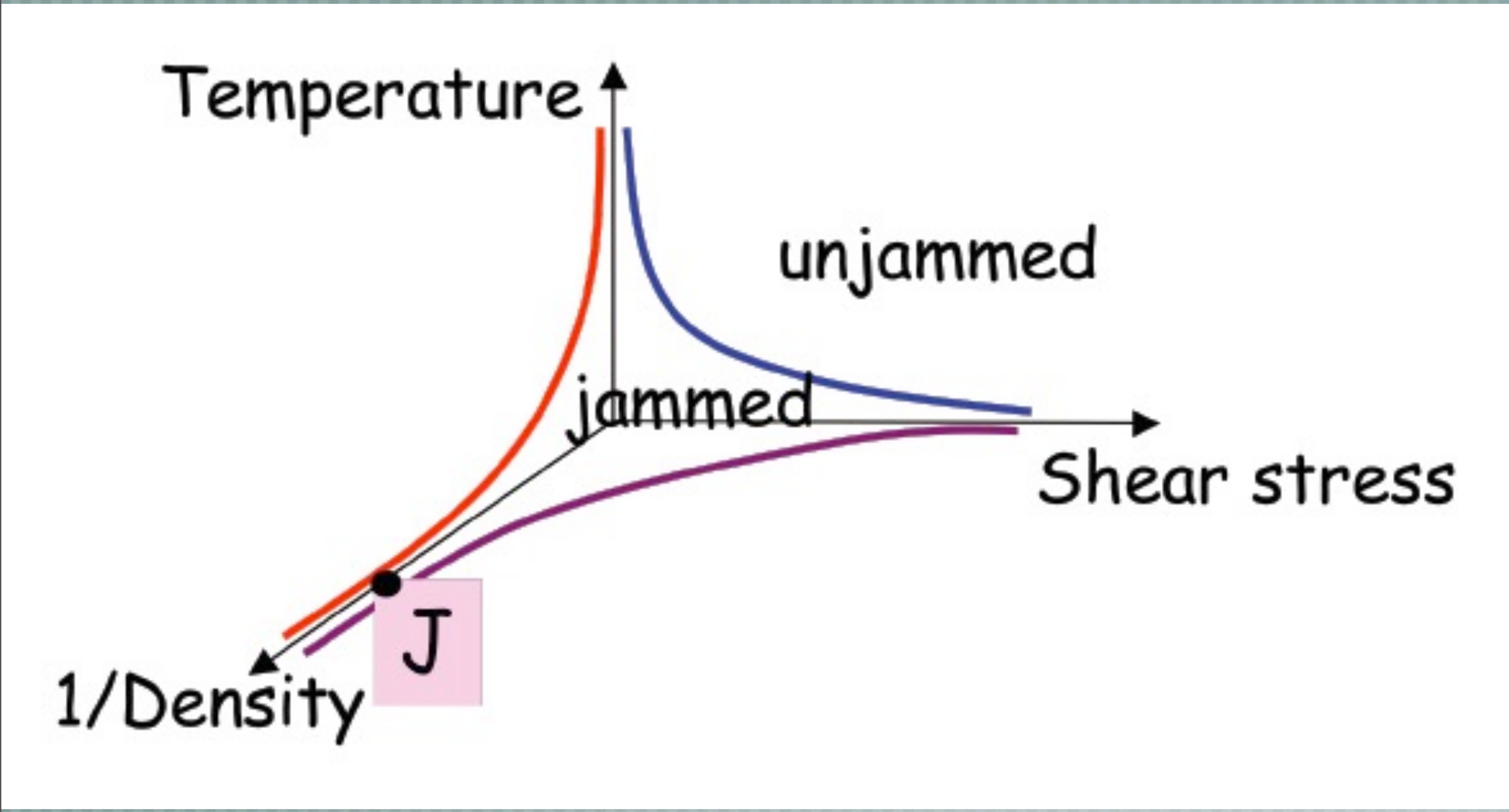

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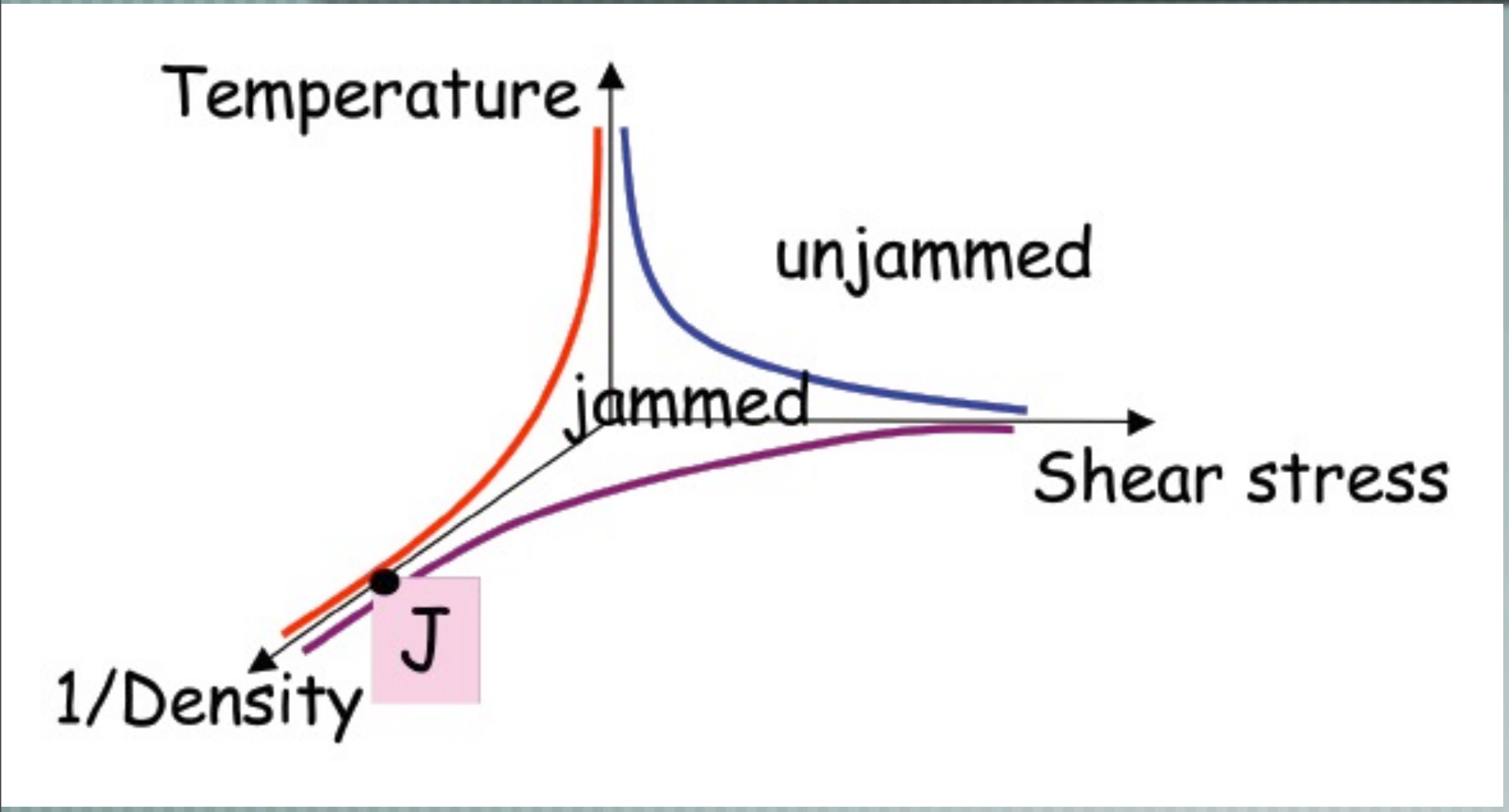

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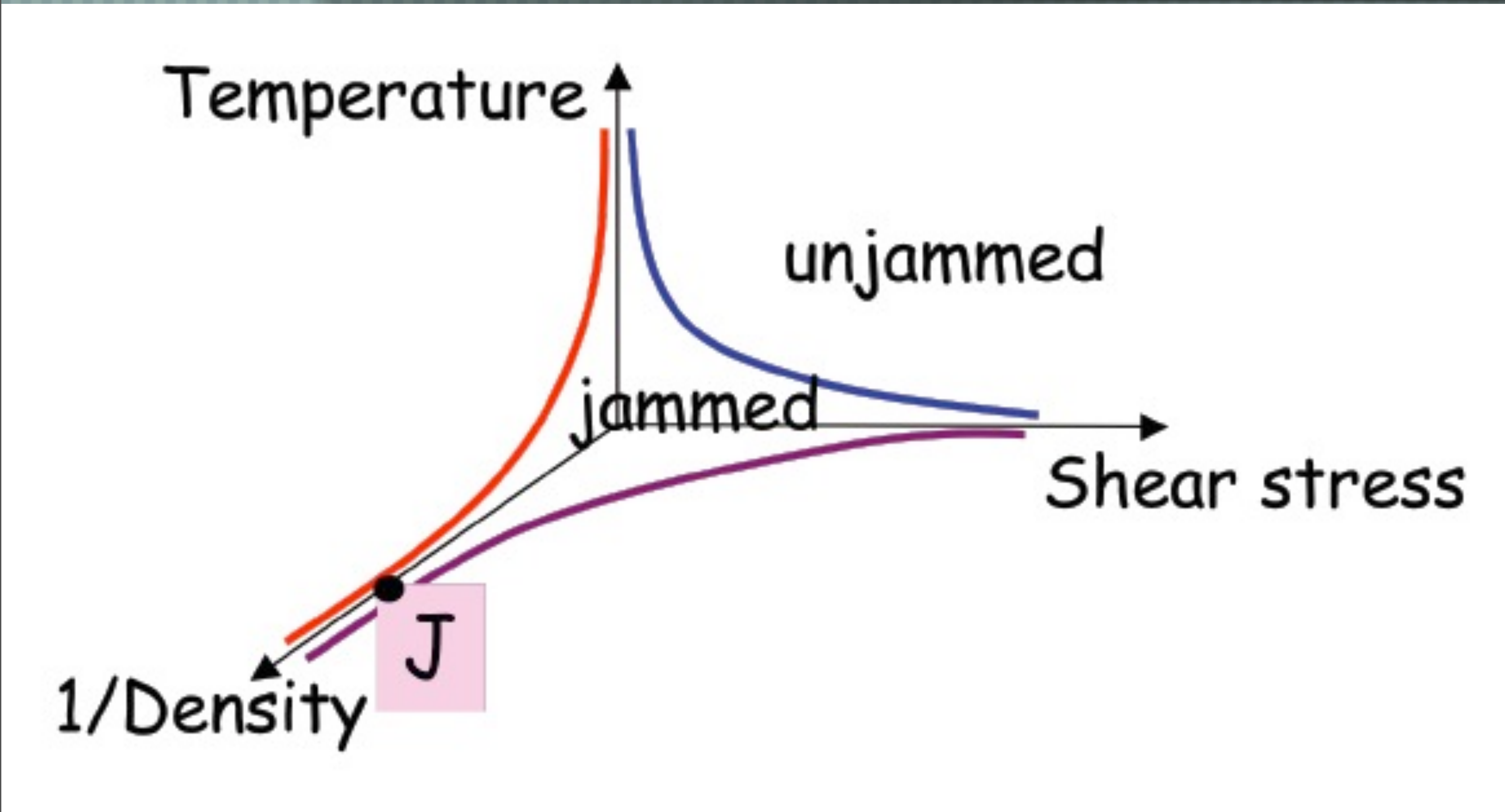

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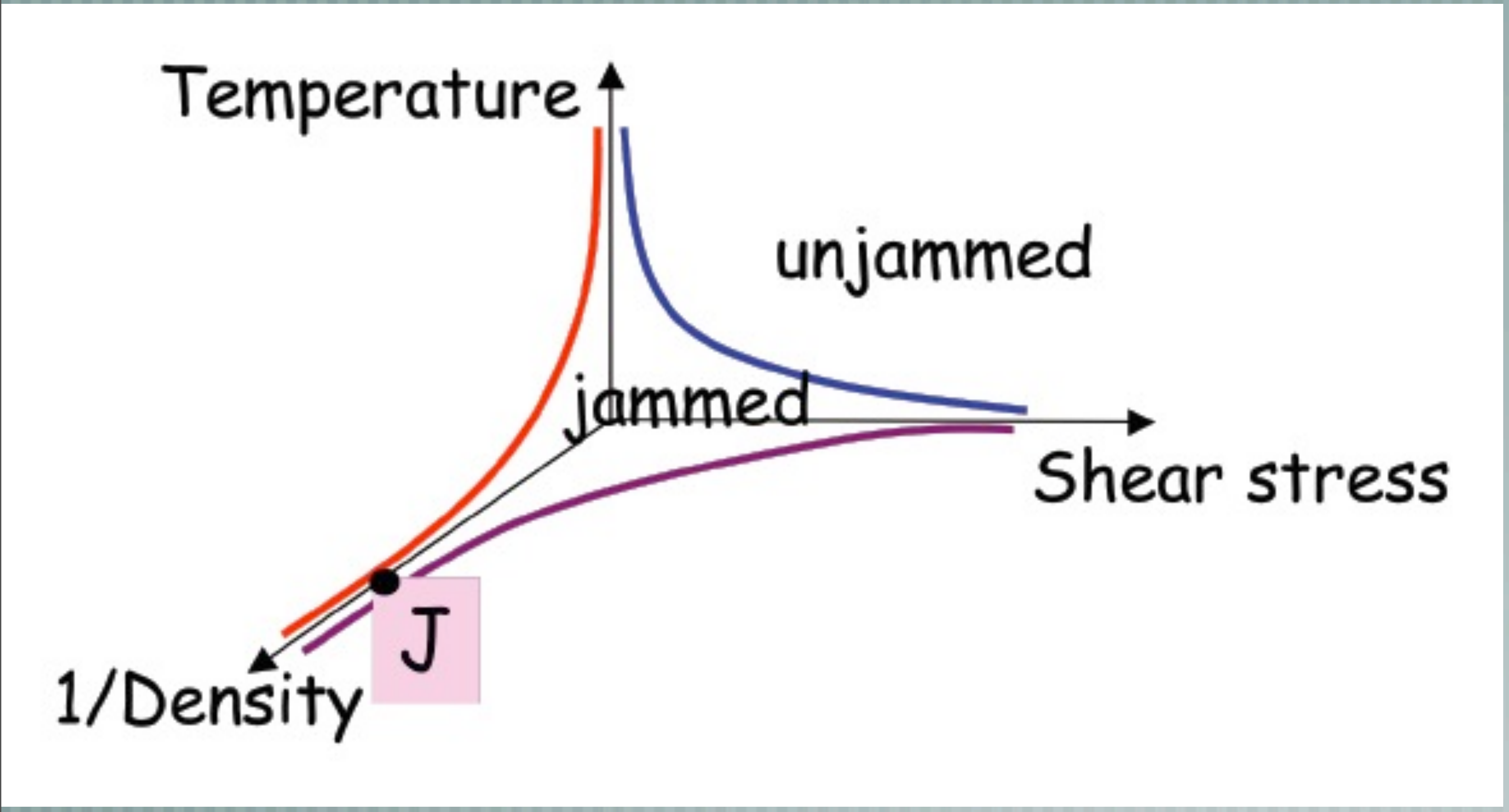

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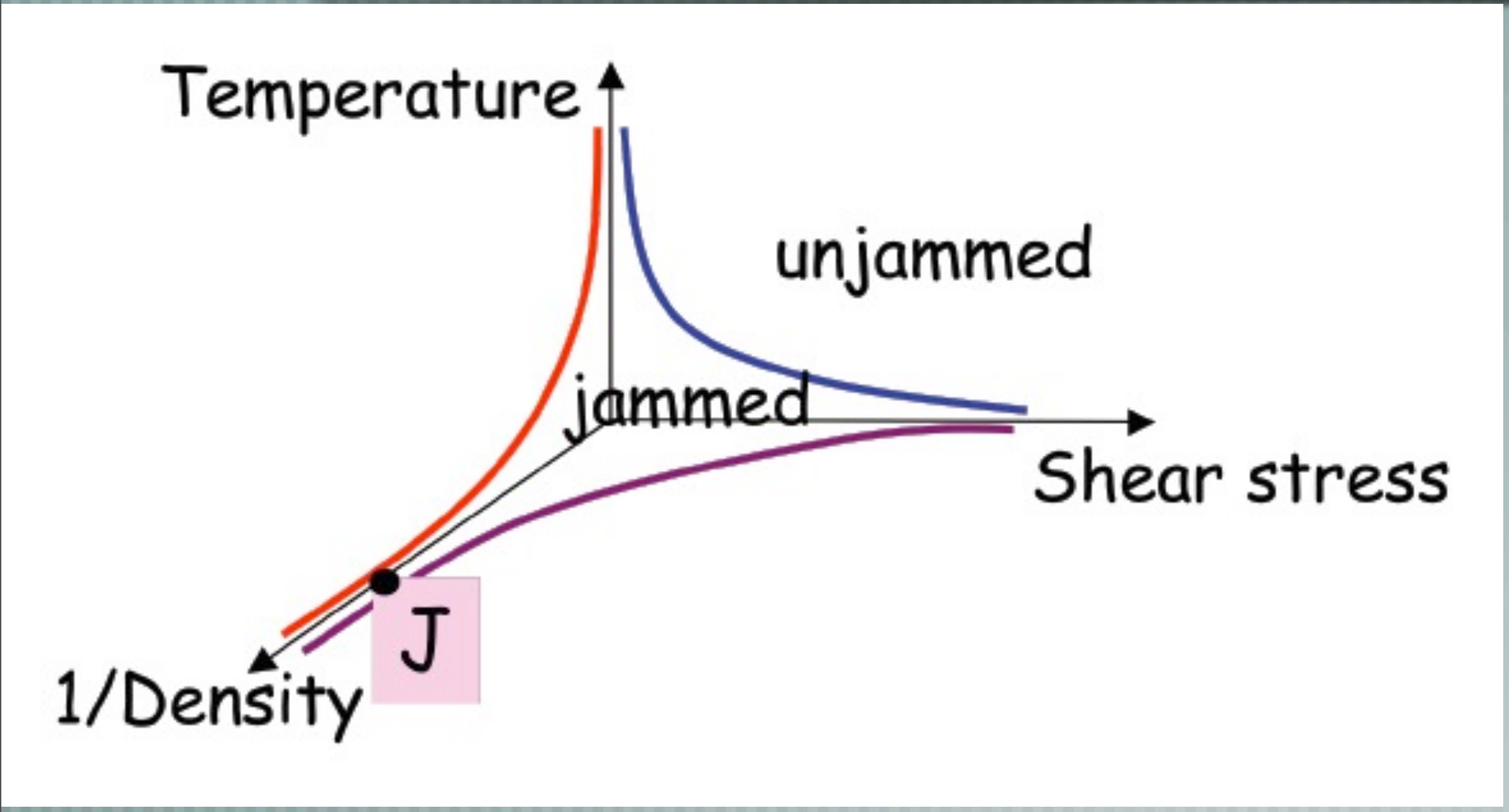

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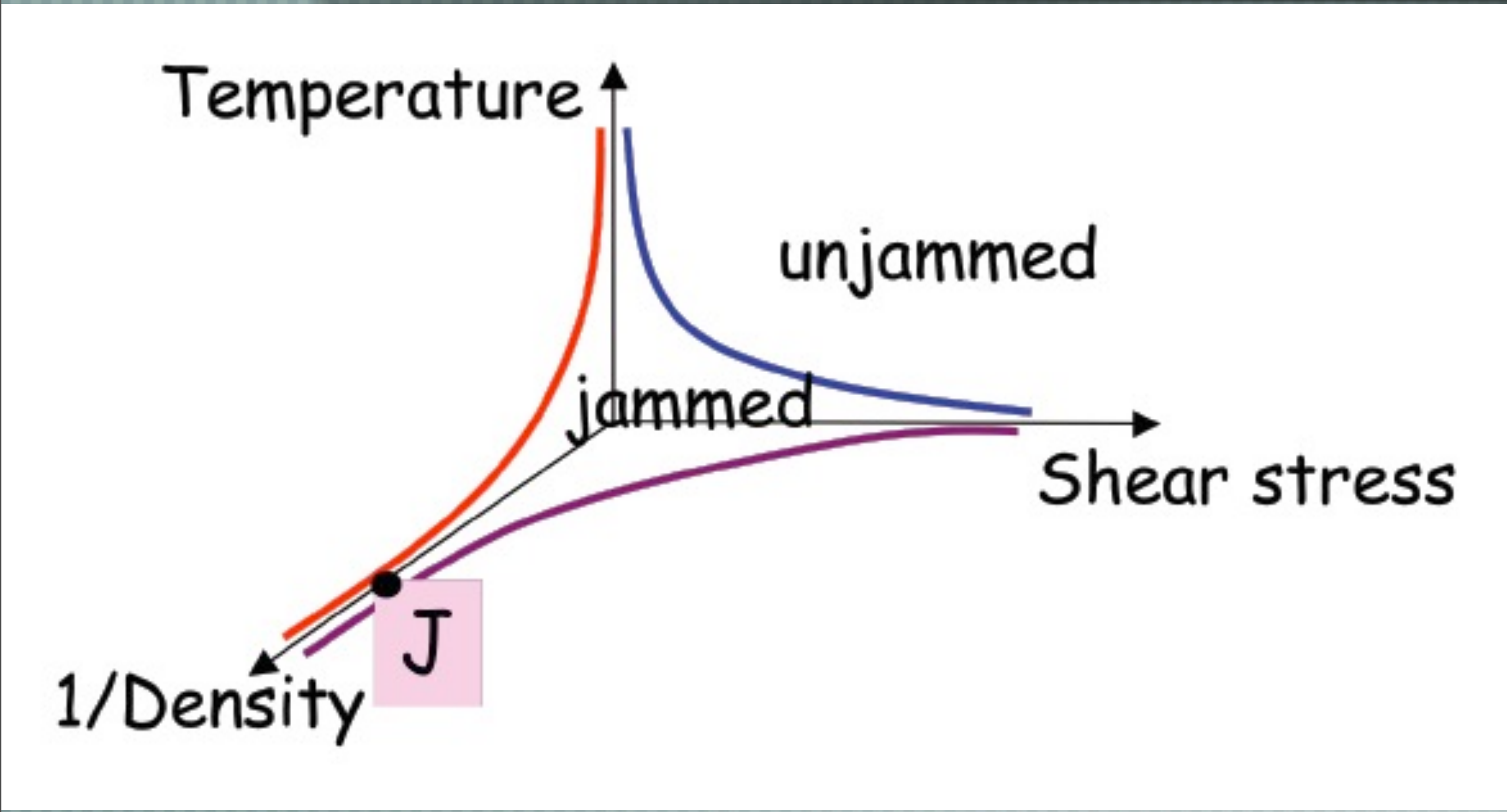

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Bose Hubbard type model with no external disorder

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$$H = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + h.c.) + U \sum_i n_i(n_i - 1) + \sum_{ij} V_{ij} n_i n_j$$

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