Charge and spin fractionalization beyond the Luttinger liquid paradigm

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Quantum Dynamics in Far from Equilibrium Thermally Isolated Systems
September 27 2012
KITP - Santa Barbara

Introduction and motivation

Fermion fractionalization in one dimension

Luttinger-liquid and spin-charge separation

F. D. M. Haldane, J. Phys. C 14, 2585 (1981)
T. Giamarchi, *Quantum Physics in One Dimension. Clarendon, Oxford (2006)*

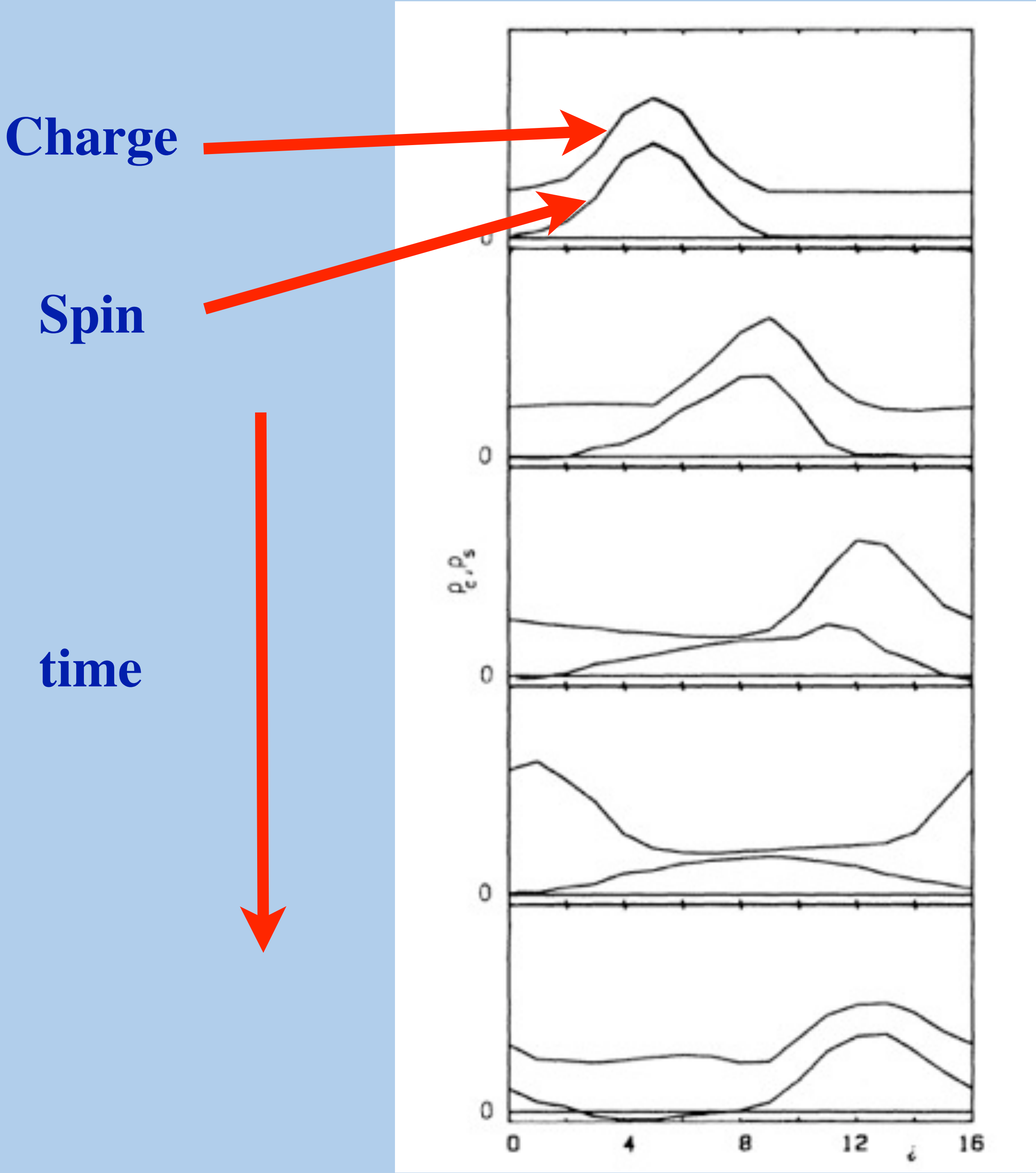
$$H = -t \sum_{i,\sigma} \left(c^{\dagger}_{i,\sigma} c_{i+1,\sigma} + \text{h.c.} \right) + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow}$$

↓ Low energy, long wavelength limit

$$H = \frac{1}{2\pi} \sum_{\mu=\rho,\sigma} \int dx \left\{ u_\mu K_\mu \left[\pi \Pi_\mu(x) \right]^2 + \frac{u_\mu}{K_\mu} \left[\nabla \phi_\mu(x) \right]^2 \right\}$$

Fermion fractionalization in one dimension

Spin-charge separation



Hubbard model - U/t = 10

Beyond the asymptotic low energy, long wavelength limit

$$\frac{v_c}{v_s} \simeq 3$$

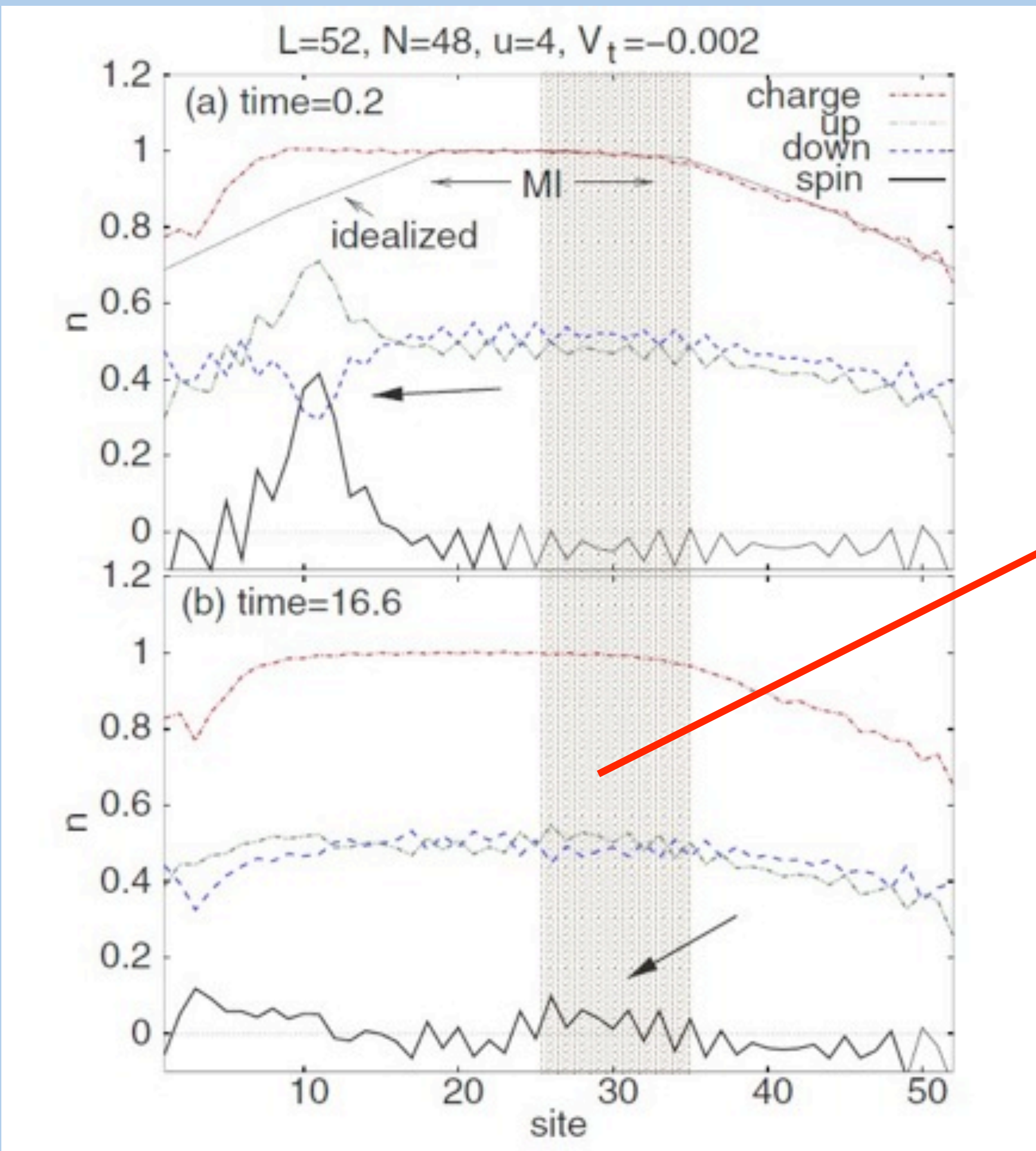
Time evolution with Lanczos approximation

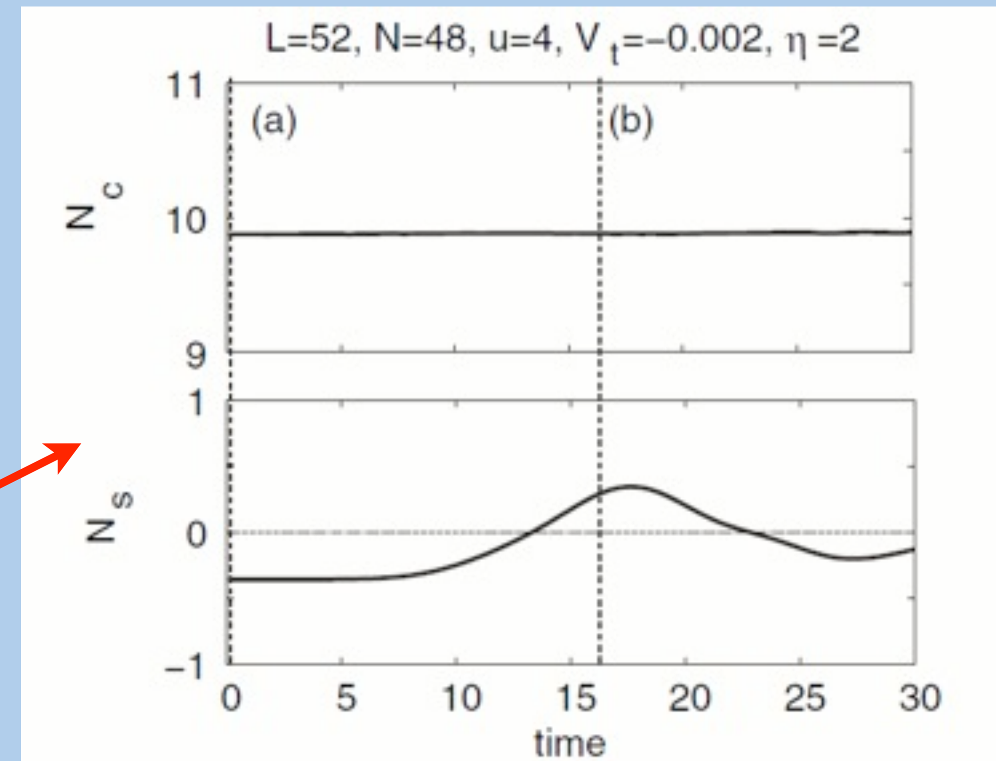
E. A. Jagla, K. Hallberg, and C. A. Balseiro
Phys. Rev. B 47, 5849 (1993)

Fermion fractionalization in one dimension

Charge-spin separation in trapped ultra-cold atoms

C. Kollath, U. Schollwöck, and W. Zwerger, Phys. Rev. Lett. 95, 176401 (2005)





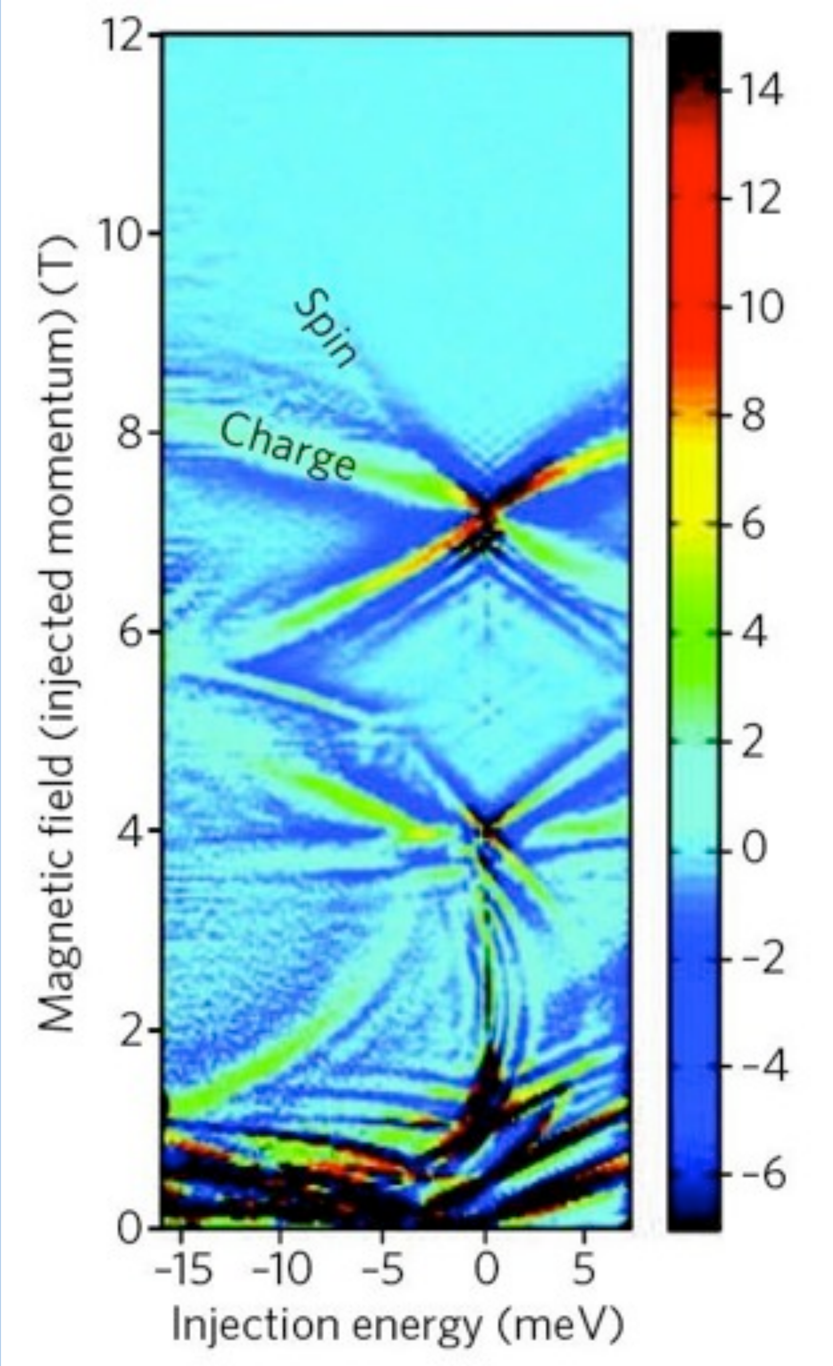
Hubbard model in a parabolic trap

Time evolution with t-DMRG

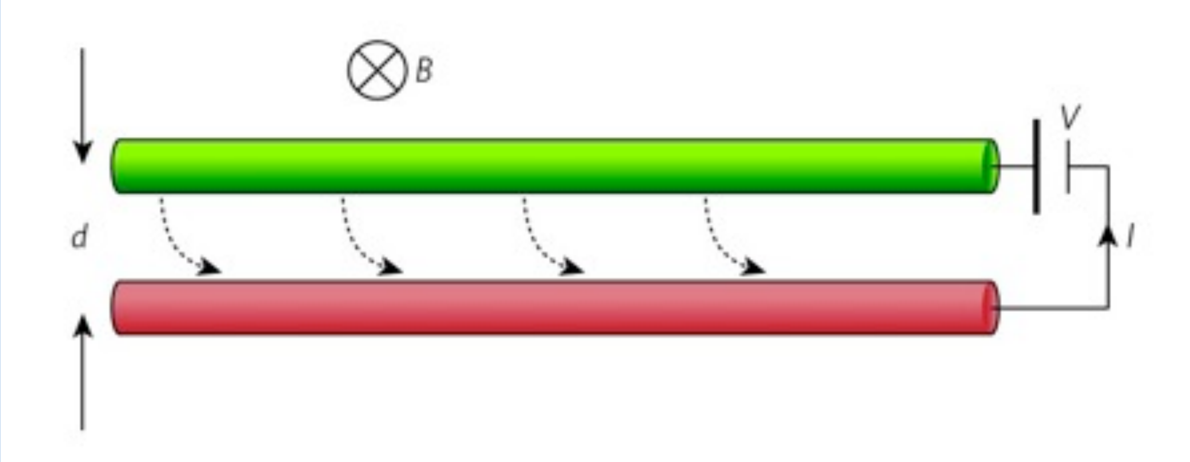
Fermion fractionalization in one dimension

Charge-spin separation in quantum wires

O. M. Auslaender *et al.*, Science 308, 88 (2005)
V. V. Deshpande, M. Bockrath, L. I. Glazman, and A. Yacobi
Nature 464, 209 (2010)



Tunneling injection of carriers



B: control of momentum

V: control of energy

Fermion fractionalization in one dimension

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Charge fractionalization in one dimension

K.-V. Pham, M. Gabay, and P. Lederer, Phys. Rev. B 61, 16397 (2000)

Independent chiral modes in a Luttinger liquid

$$H_{LL} = H_R + H_L$$

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- **Inject one fermion to the right branch**

$$Q_R = \frac{1 + K_\mu}{2}$$

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Experiment: **H. Steinberg *et al.*, Nat. Phys. 4, 116 (2008)**

Fermion fractionalization in one dimension

- **Further fractionalization?**

Fermion fractionalization in one dimension

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  - **Fractionalization of an electron in the $1/r^2$ supersymmetric t-J model**

Z. N. C. Ha and F. D. M. Haldane, Phys. Rev. Lett. 73, 2887 (1994)

$$H = -\sum_{i\neq j,\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + \sum_{i<j} \left(J_{ij} \vec{S}_i \cdot \vec{S}_j + V_{ij} n_i n_j \right)$$

$$J_{ij} = 2t_{ij} = -4V_{ij} = \frac{\pi^2}{N^2 \sin^2 \left[\pi \left(i-j\right)/N\right]}$$

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spinon: Q=0, S=1/2, semion

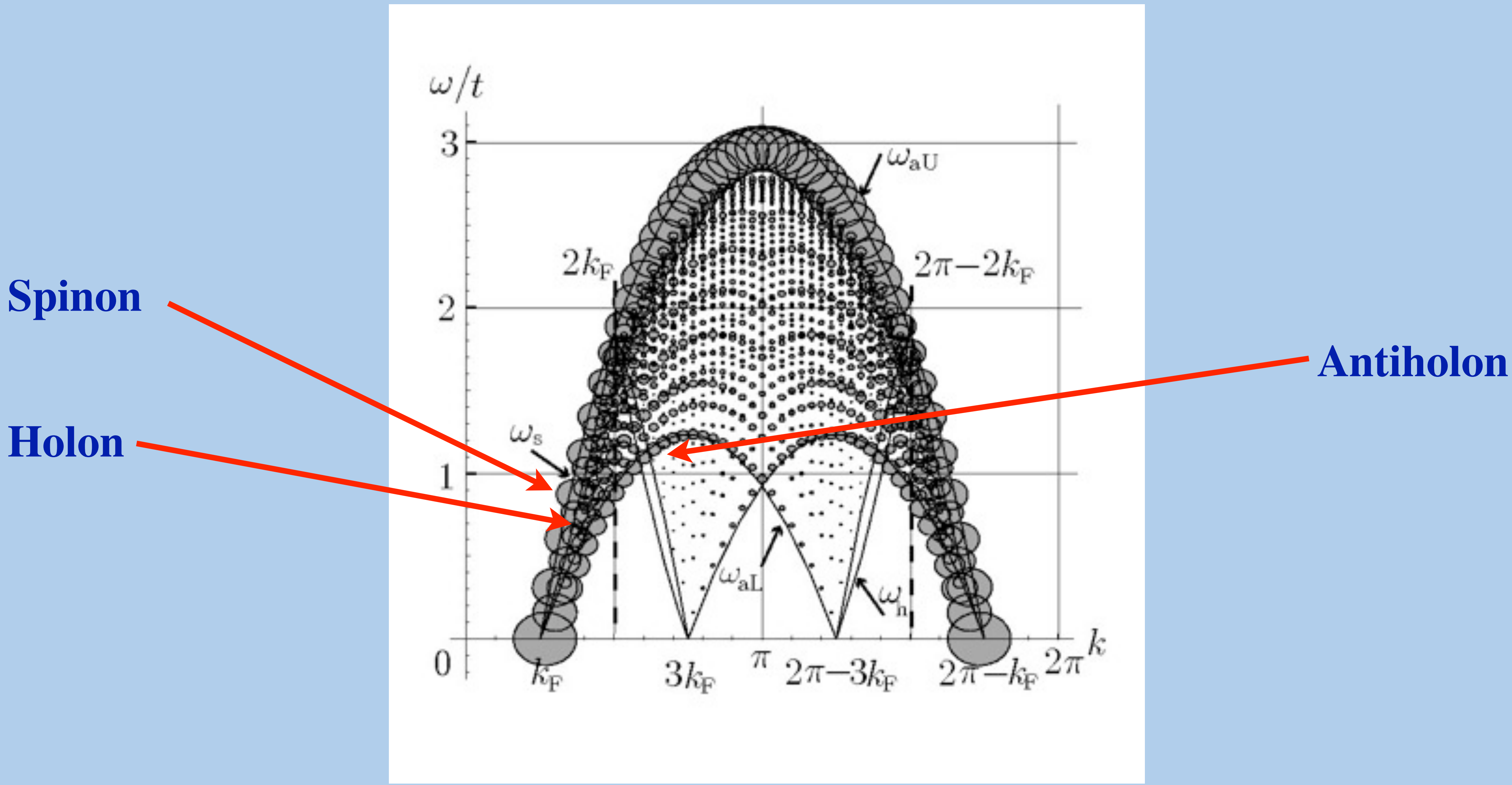
holon: Q=-e, S=0, semion

antiholon: Q=2e, S=0, boson

Fermion fractionalization in one dimension

- **Fractionalization in the $1/r^2$ SUSY t-J model**
  - **Spinon, holons, and antiholons in the electron addition spectrum**

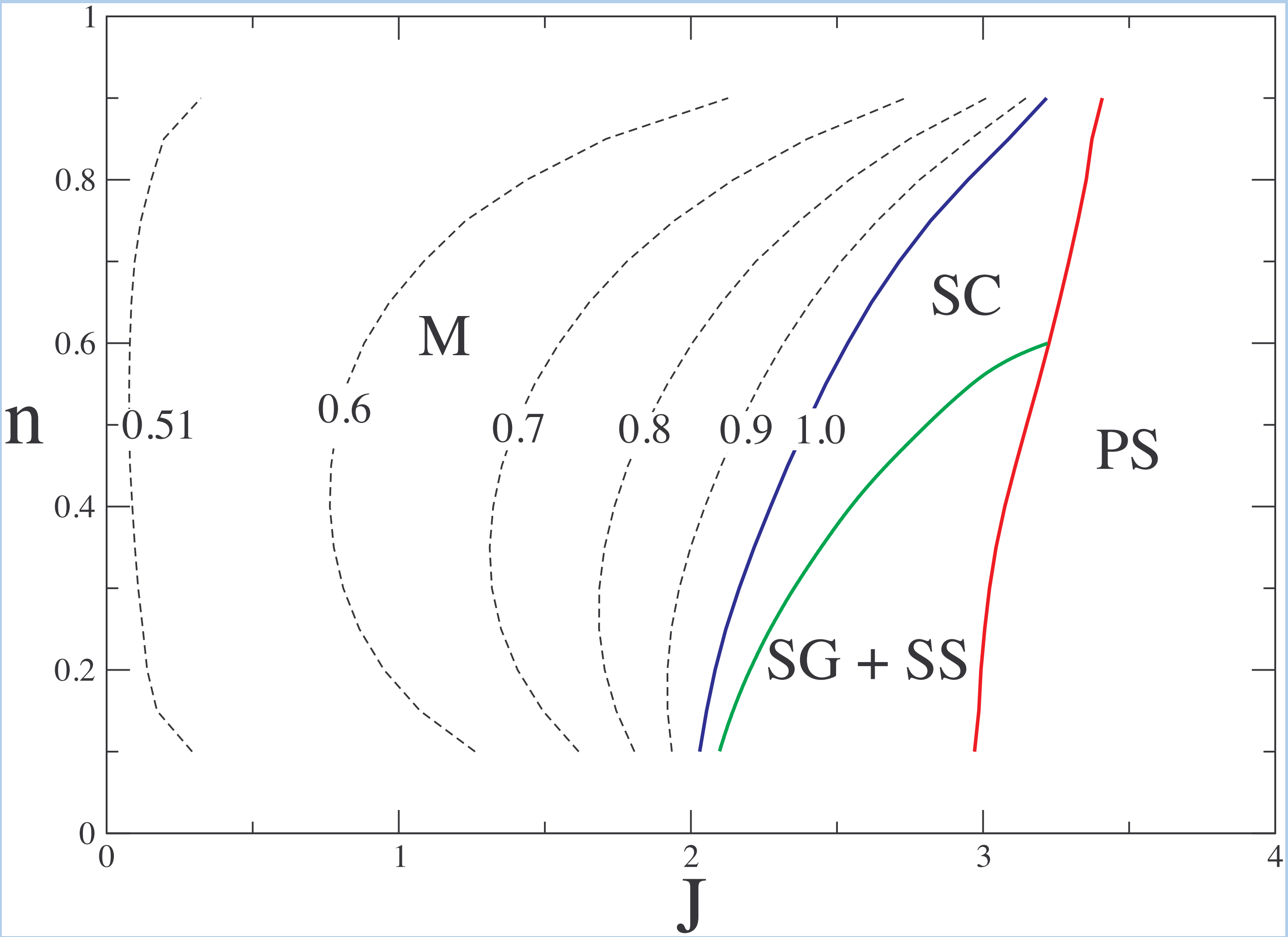
M. Arikawa, Y. Saiga, and Y. Kuramoto, Phys. Rev. Lett. 86, 3096 (2001)

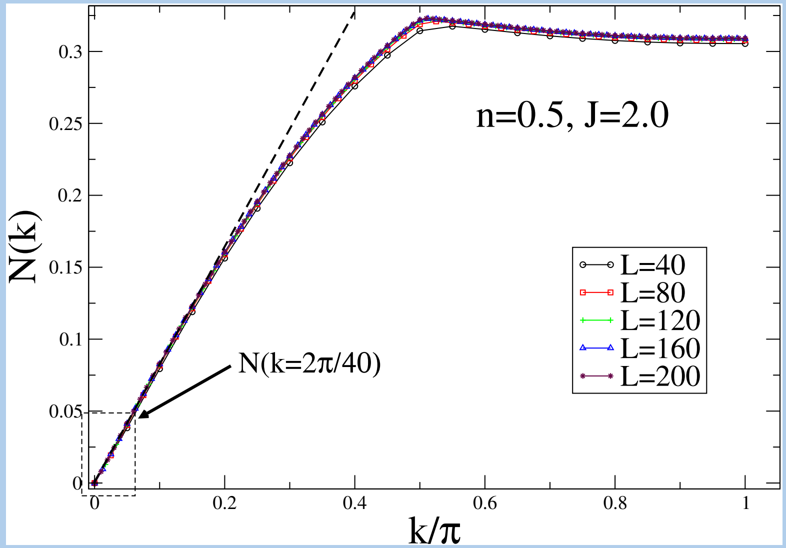


Fermion fractionalization in one dimension

- **Nearest neighbor t-J model**

$$H = -\tilde{t} \sum_{\substack{\langle i,j \rangle \\ \sigma}} c_{i\sigma}^\dagger c_{j\sigma} + J \sum_{\langle i,j \rangle} \left(\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} n_i n_j \right)$$





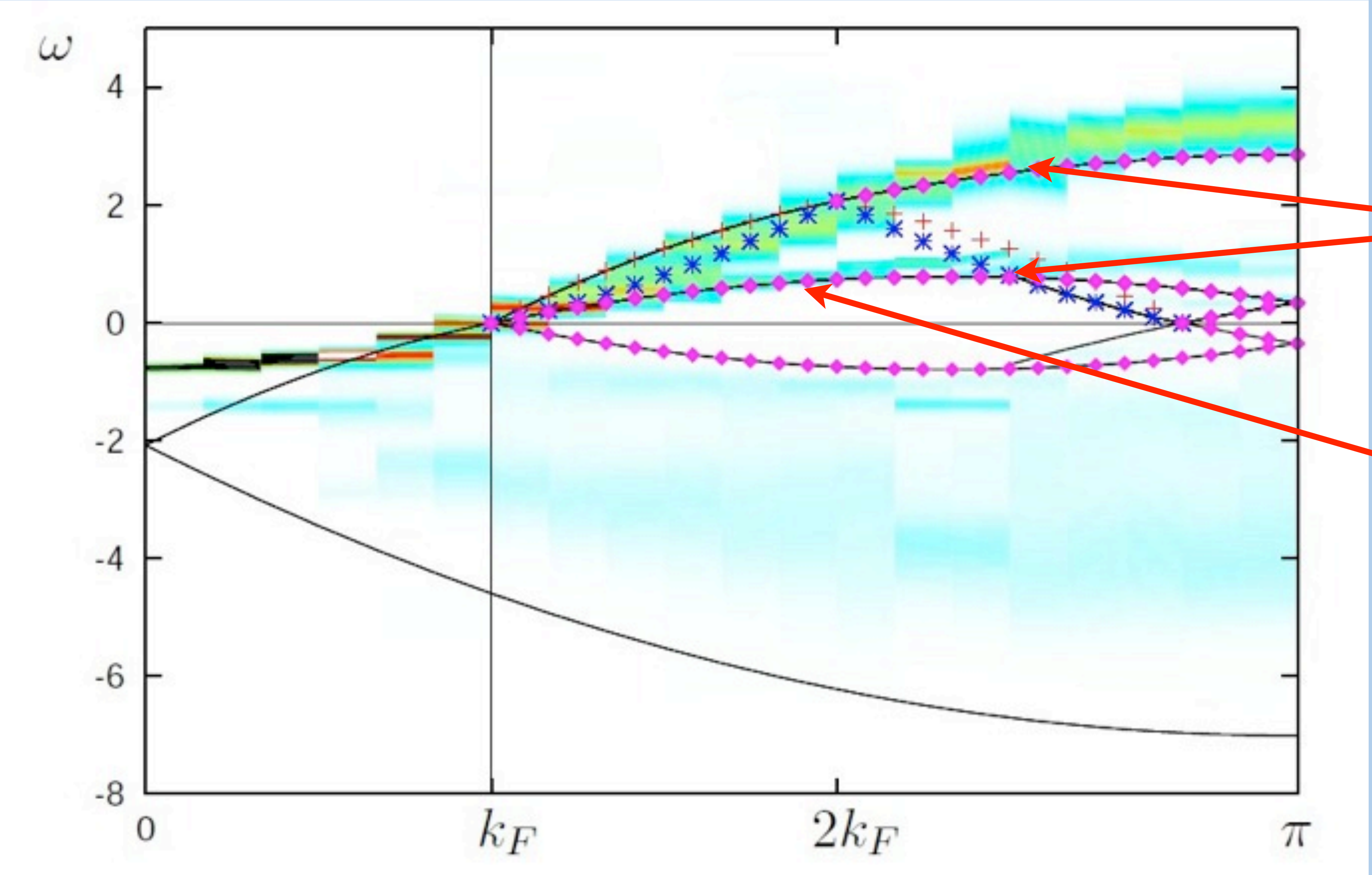
$$N(k \to 0) = K_\rho |k| a / \pi$$

A. Moreno, A.M., and S. R. Manmana, Phys. Rev. B 83, 205113 (2011)

Fermion fractionalization in one dimension

- Nearest neighbor t-J model
  - One-particle spectrum from quantum Monte Carlo simulations

C. Lavalle, M. Arikawa, S. Capponi, F. F. Assaad, and A.M., Phys. Rev. Lett. 90, 216401 (2003)



$J = 2\tilde{t}$

Splitting of the one particle addition spectrum

Antiholon in $1/r^2$ t-J model

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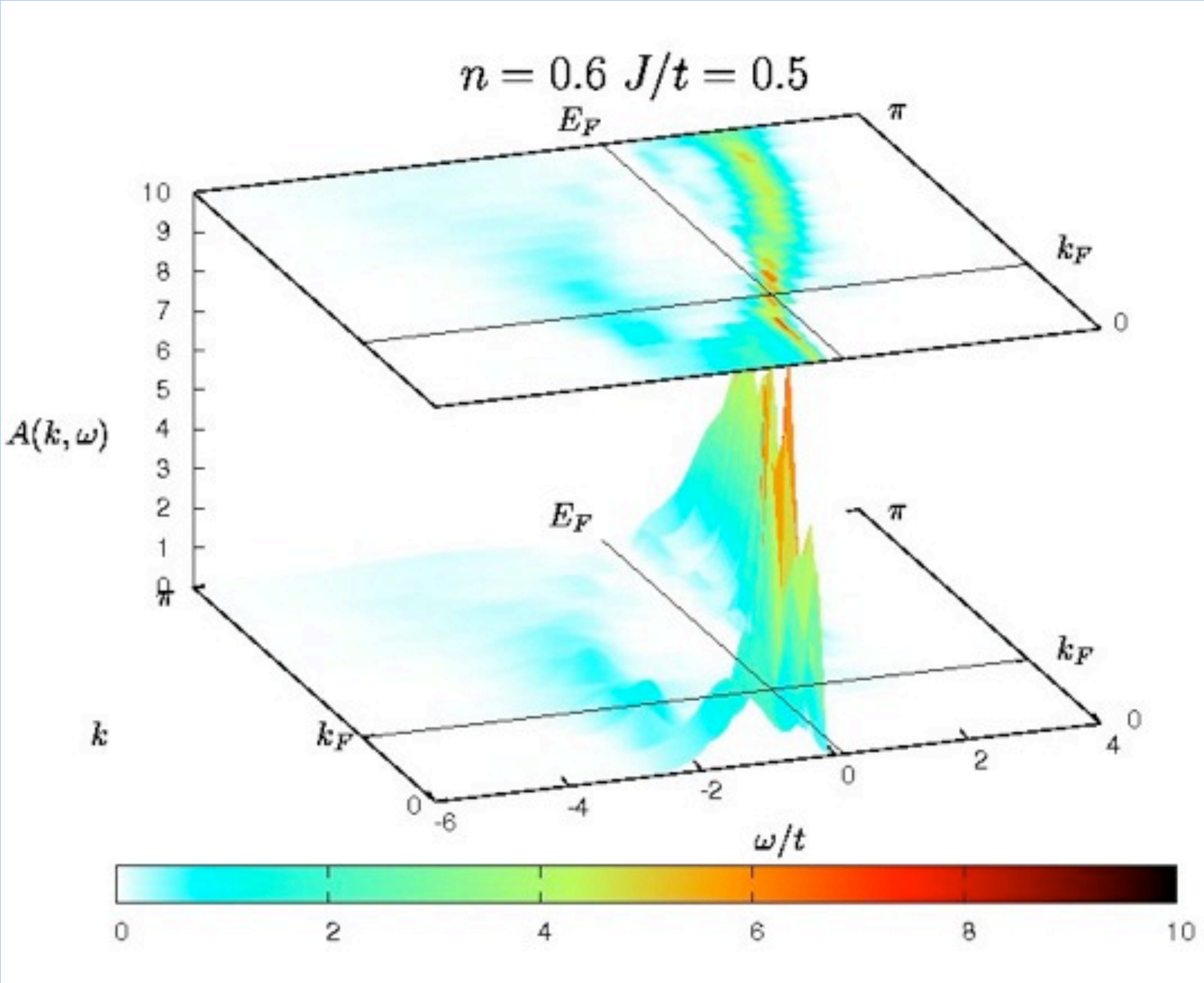

Time dependent DMRG

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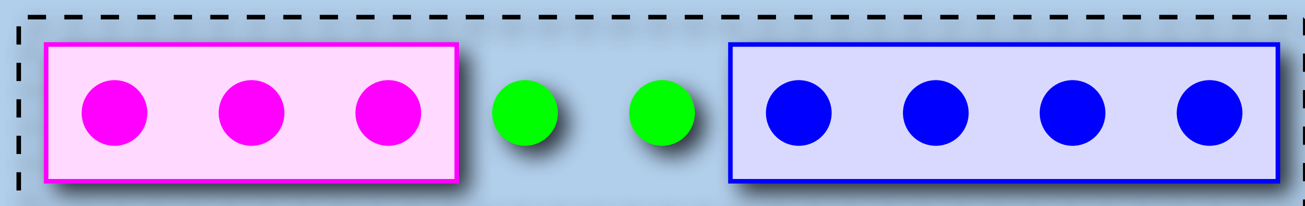
S. R. White and A. E. Feiguin, Phys. Rev. Lett. 93, 076401 (2004)
A. J. Daley, C. Kollath, U. Schollwöck, and G. Vidal, J. Stat. Mech.: Theor. Exp P04005 (2004)

Trotter approximation → $H = \sum_i H_{i,i+1}$

$$\mathrm{e}^{-iH\Delta t} = \mathrm{e}^{-iH_{even}\Delta t}\mathrm{e}^{-iH_{odd}\Delta t} + \mathcal{O}\left(\Delta t^2\right)$$



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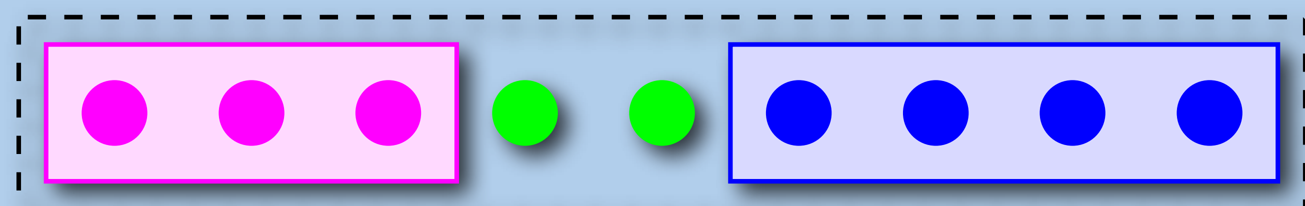

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Exact error bound M. Hochbruck and Ch. Lubich, SIAM J. Numer. Anal. 34, 1911 (1997)

$$||\mathrm{e}^{-iH\Delta t} \mid \psi\rangle - \mid \tilde{\psi}\rangle|| \leq 12\mathrm{e}^{(W\Delta t)^2/16m}\left(\frac{eW\Delta t}{4m}\right)^m$$ **almost exponential convergence**

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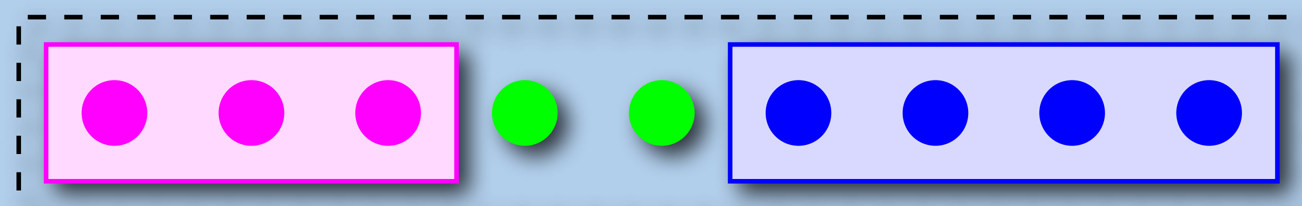

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S. R. Manmana, S. Wessel, R. M. Noack, and A.M., Phys. Rev. Lett. 98, 210405 (2007)

Time dependent charge and spin dynamics in the 1D t-J model

Fractionalization of charge in real time

- **Nearest neighbor t-J model: Luttinger-liquid phase**

$J = 2t$ $n = 0.8$ $k = 0.45\pi$

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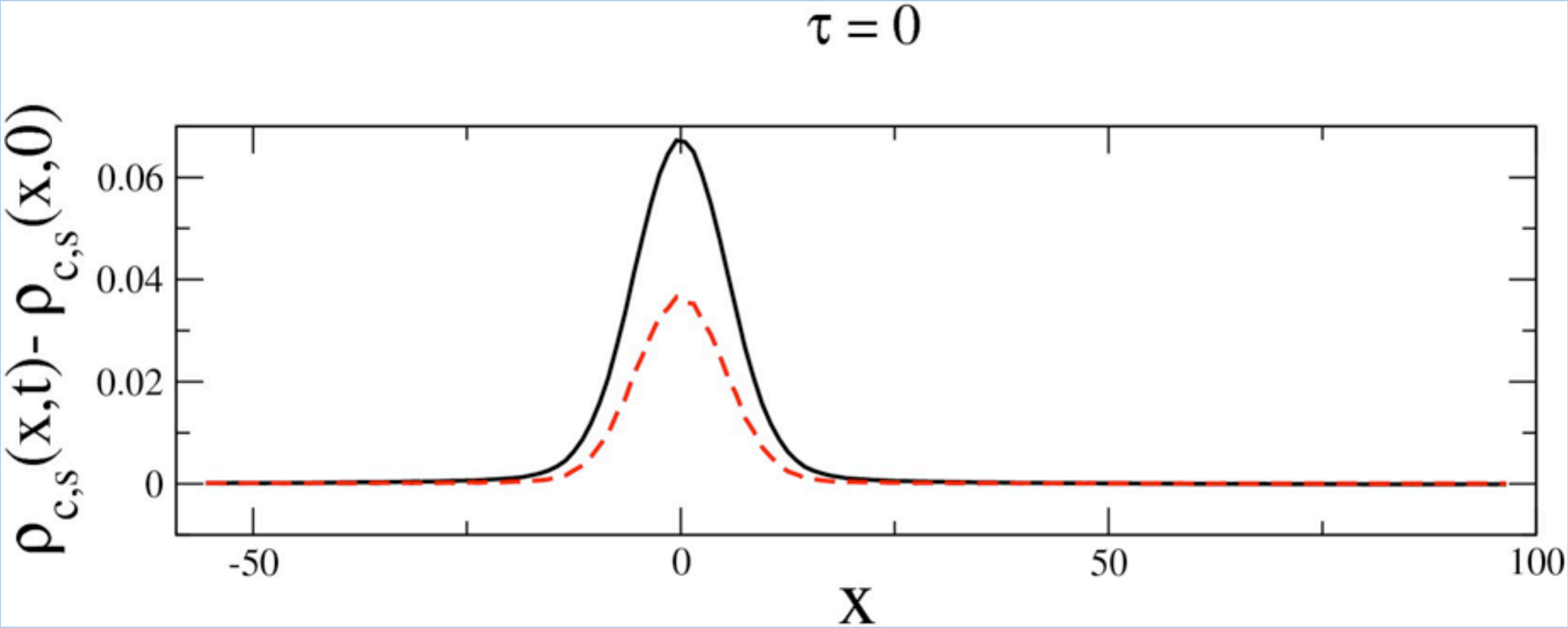
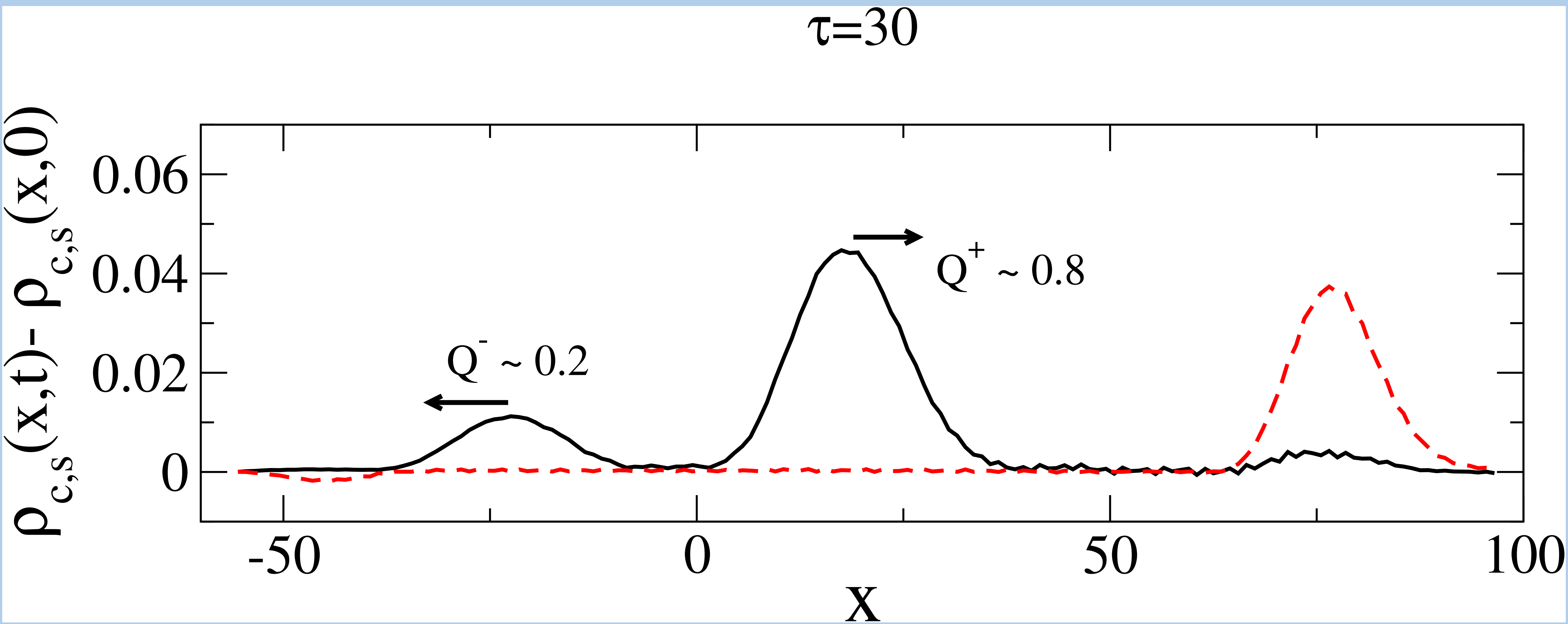


Fractionalization of charge in real time

- Nearest neighbor t-J model: Luttinger-liquid phase

$$K_\rho \simeq 0.67$$



Fractionalization of fractionalized charge

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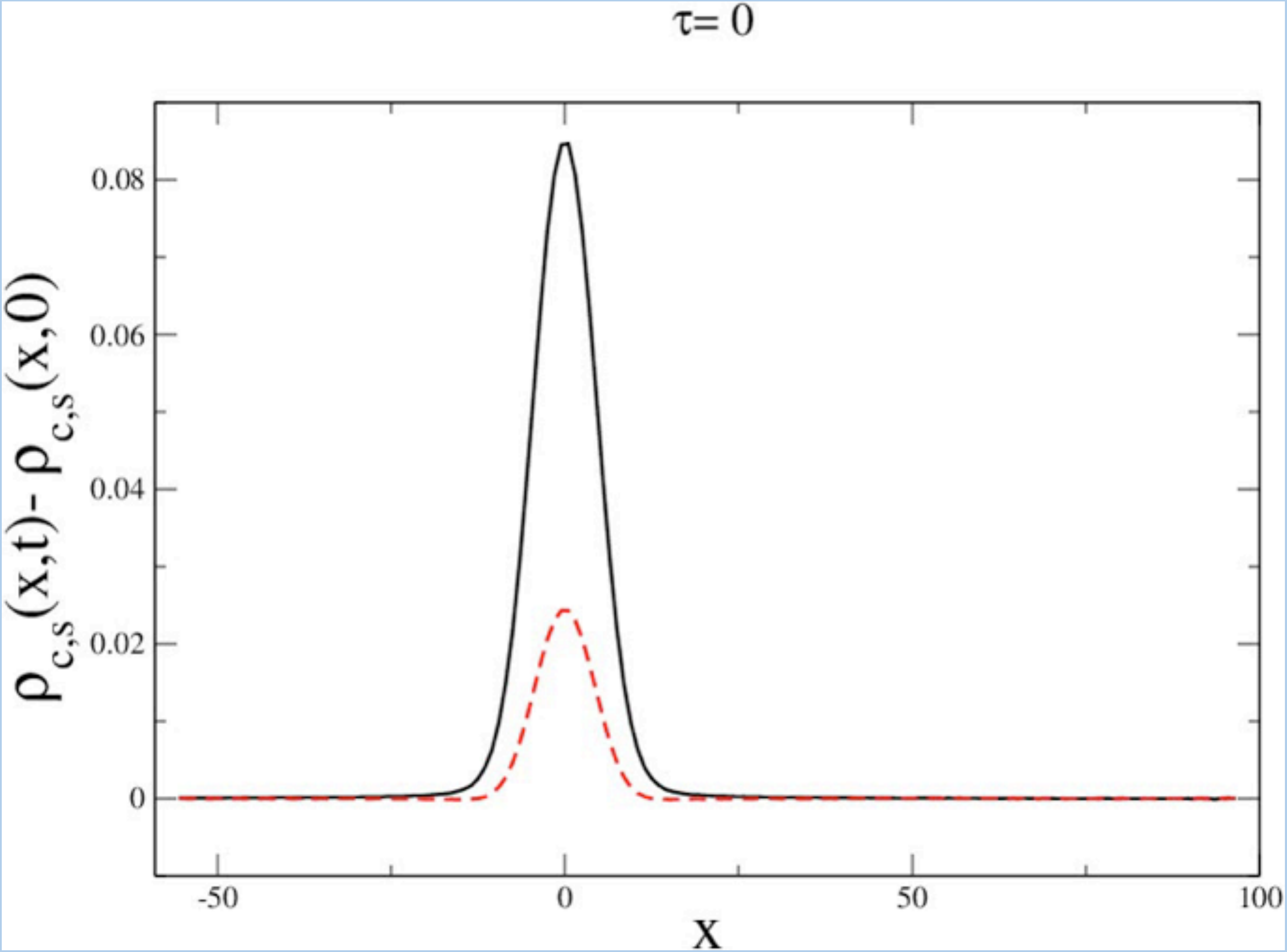

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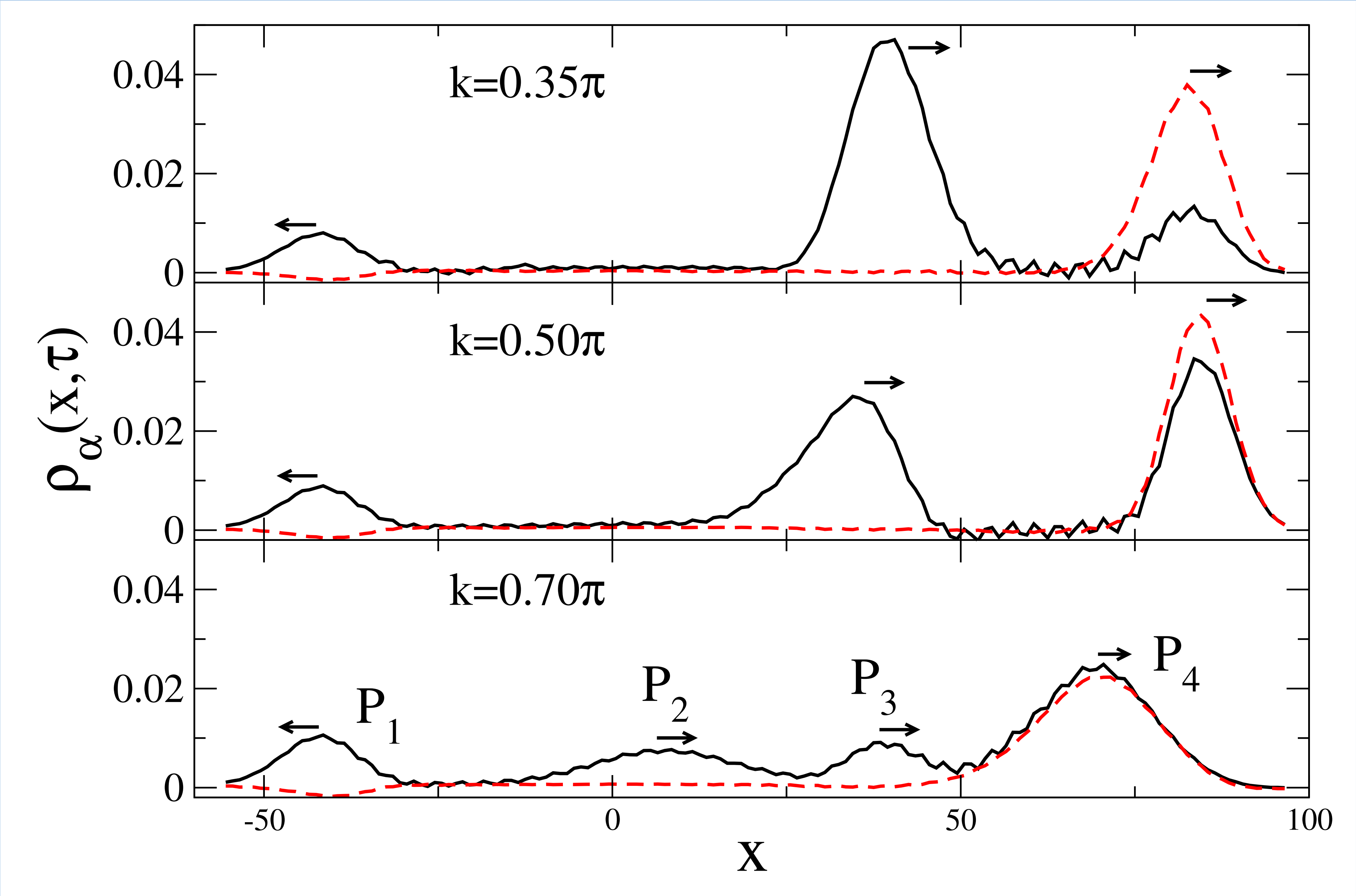

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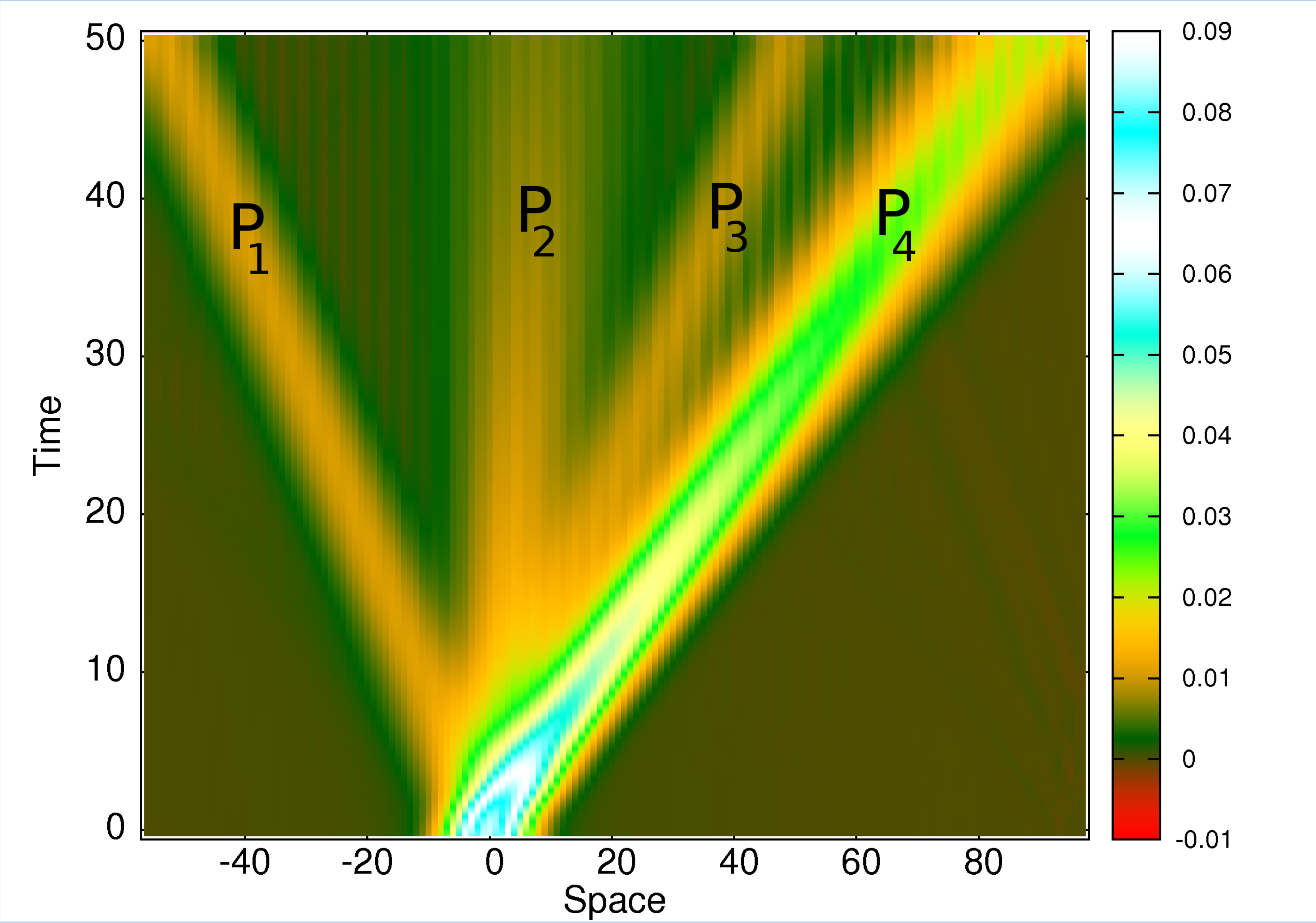

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Excitations at the SUSY point from Bethe-Ansatz

P. A. Bares and G. Blatter, Phys. Rev. Lett. 64, 2567 (1990)
P. A. Bares, G. Blatter, and M. Ogata, Phys. Rev. B 44, 130 (1991)
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- **Excitations associated with spin and charge**

$$\epsilon_s(q_s), \qquad q_s \in [-(\pi - k_F), (\pi - k_F)]$$

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- **Energy and momenta for electron addition**

$$\omega(k) = -\epsilon_c(q_c) - \epsilon_s(q_s)$$

$$k = \pm 2k_F - q_c - q_s$$

$$q_{Fs} = \pi - k_F \ , \ \ q_{Fc} = \pi - 2k_F$$

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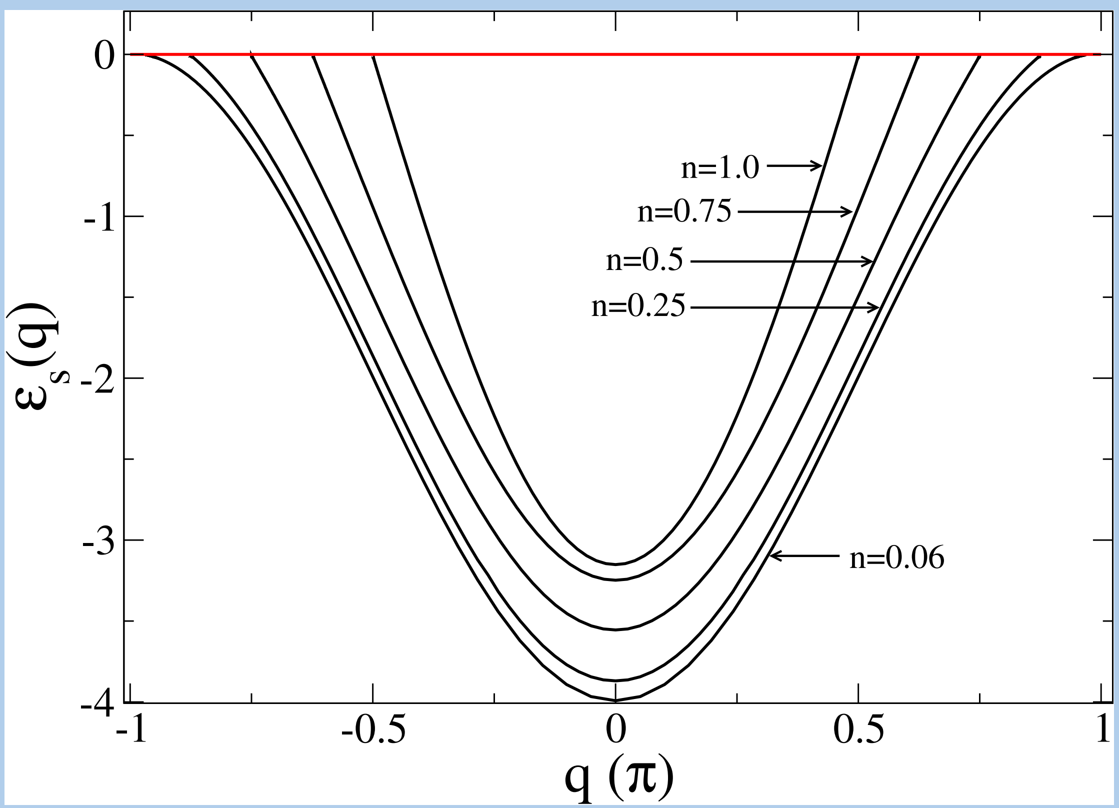

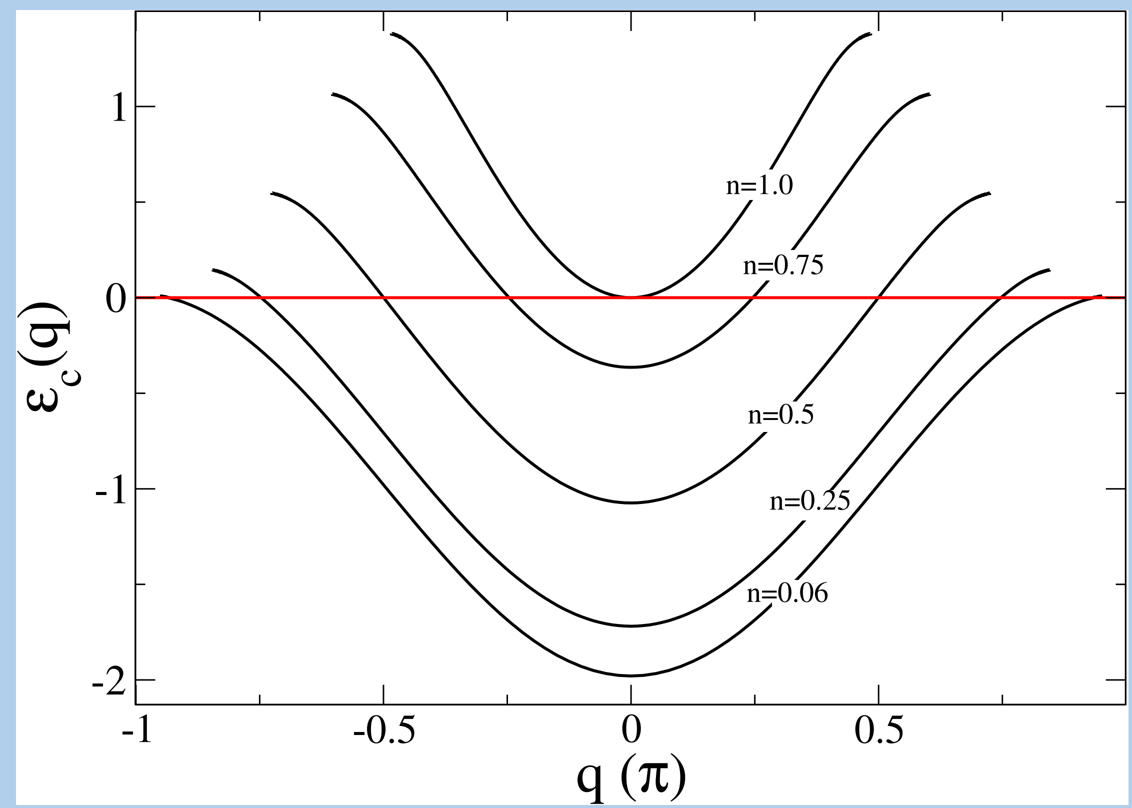

Excitations at the SUSY point: BA *vs.* t-DMRG

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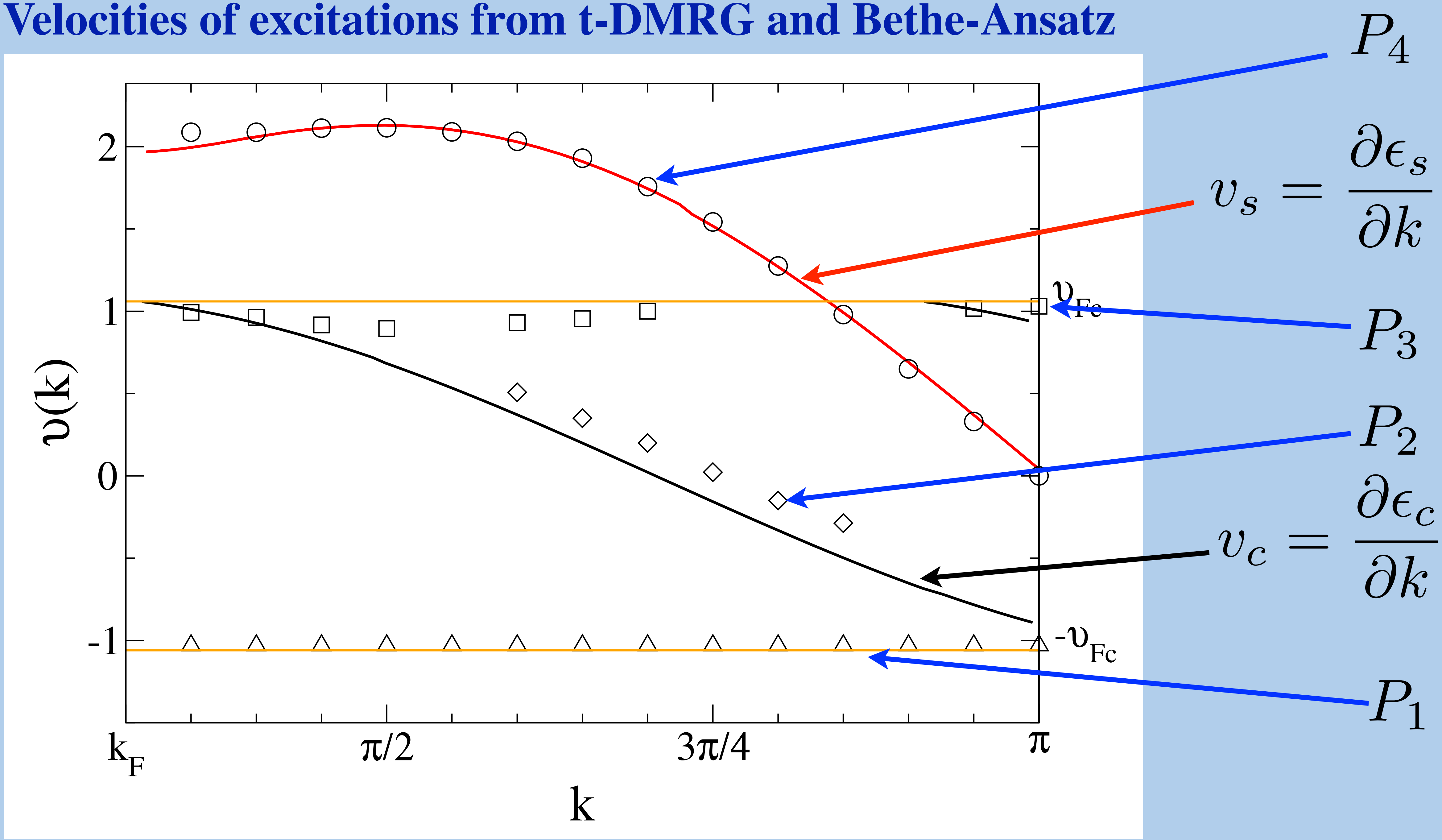
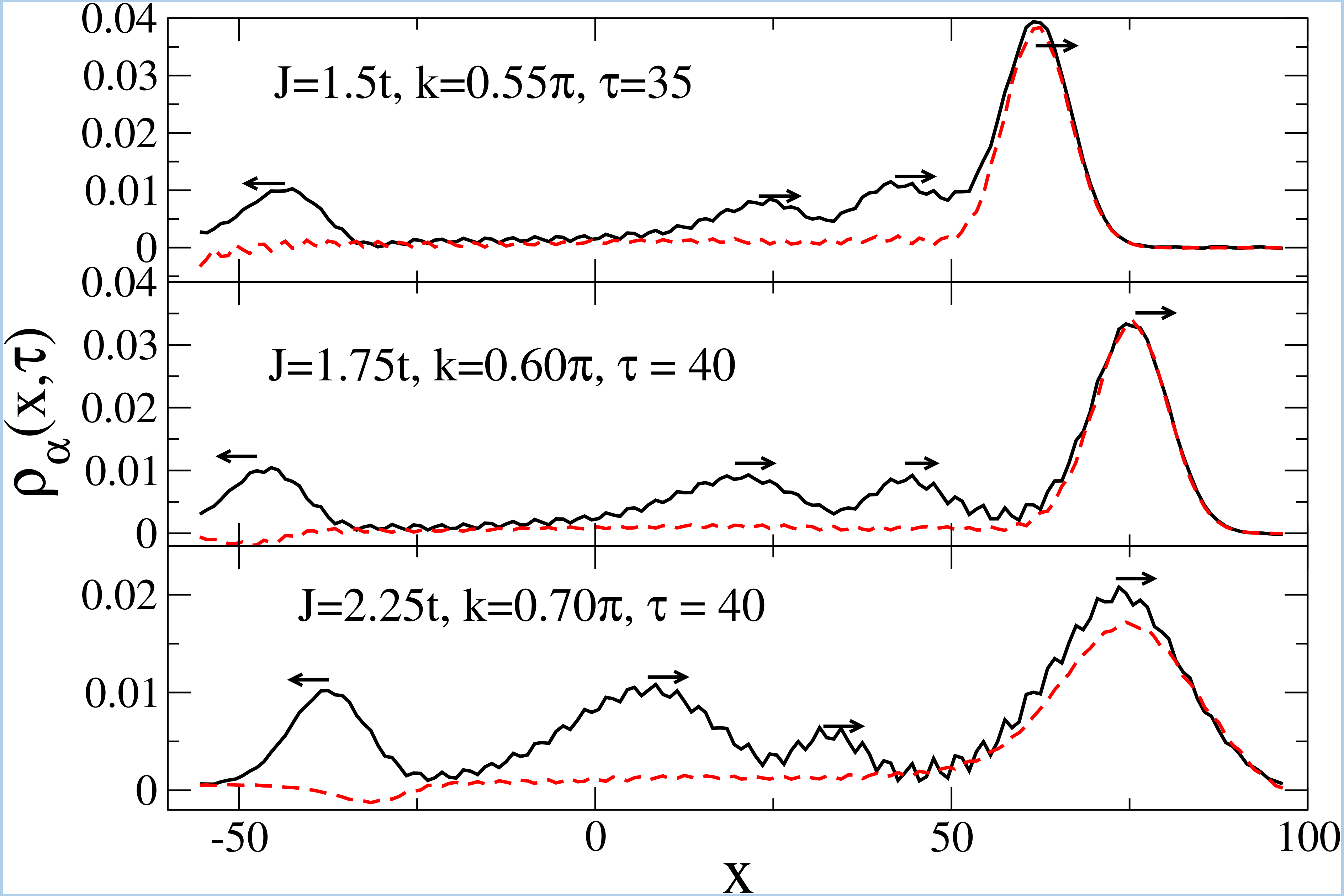
- Velocities of excitations from t-DMRG and Bethe-Ansatz



Fractionalization away from the SUSY point

- $K_\rho < 1 \,, \quad v_s > v_c$



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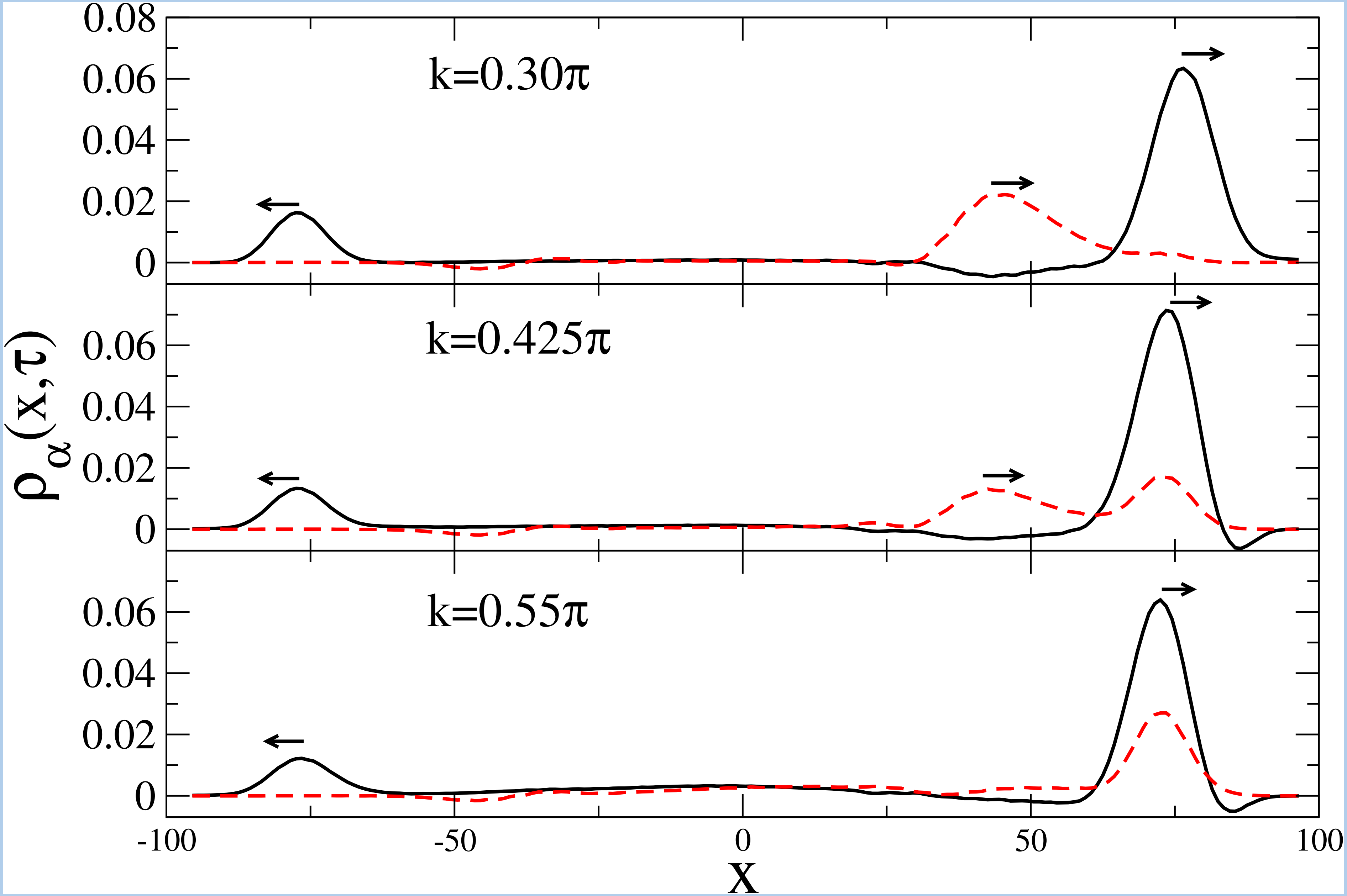

- **Fractionalization of spin**

Fermion fractionalization in one dimension

- Possible experimental realization?

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  - **Generalized t-J model with ultracold polar molecules**

A. Gorshkov *et al.*, Phys. Rev. Lett. 107, 115301 (2011)

$$H = -\tilde{t} \sum_{\substack{\langle i,j \rangle \\ m}} c_{im}^{\dagger} c_{jm} + \sum_{i \neq j} \frac{\left(\frac{J}{2} \vec{S}_i \cdot \vec{S}_j + \frac{V}{2} n_i n_j \right)}{|\vec{r}_i - \vec{r}_j|^3}$$

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Control of interactions with electric and microwave fields beyond strong coupling expansion of Hubbard-like models

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