Localization, condensation & the role of quantum statistics in strongly interacting Bose glasses

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Outline

- Intro: Anderson localization in interacting systems

- Strong localization of disordered bosons?
  Locator expansion for bosons

- Magnetoresistance of fermionic versus bosonic insulators?
  Strong, opposite effect due to quantum statistics
  Structure of localized wavefunctions

- Localization and superfluid transition?
  Mobility edges in Bose insulators?

“Dirty bosons”

- Superconductors with preformed pairs

  * Exp. systems: InOx, PbTe, and others

  * Models: - negative U Hubbard model

              - Ma&Lee/Anderson pseudospin model

- Granular superconductors / Josephson junction arrays

- Cold bosonic atoms (+disorder potential)

- Helium in disordered media (e.g. porous silica)

- Disordered quantum spin systems

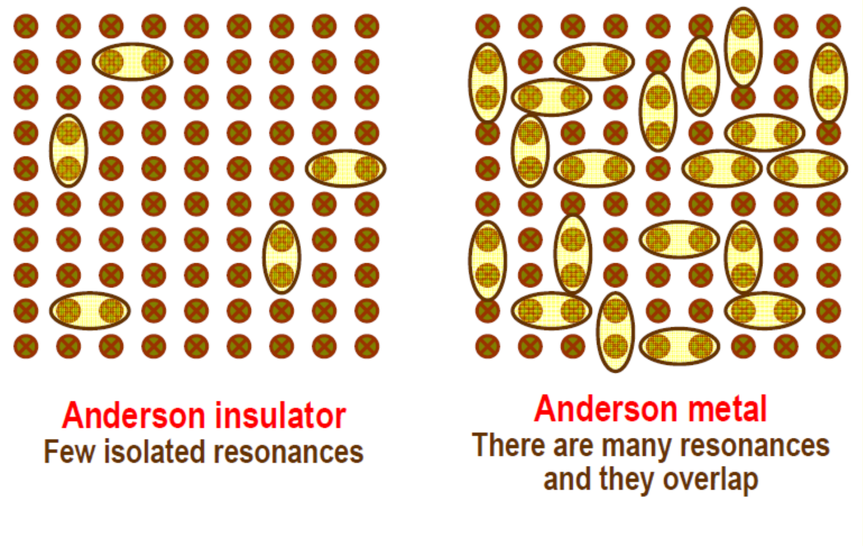
Localization: single/many particle

Anderson localization (1958) [single particle]

$$H = \sum_i \epsilon_i n_i - t \sum_{\langle i,j \rangle} \left(c_i^{+} c_j + \text{h.c.} \right)$$

Resonance = Δε < hopping t

Delocalization transition
(insulator → metal)
= Percolation of resonances



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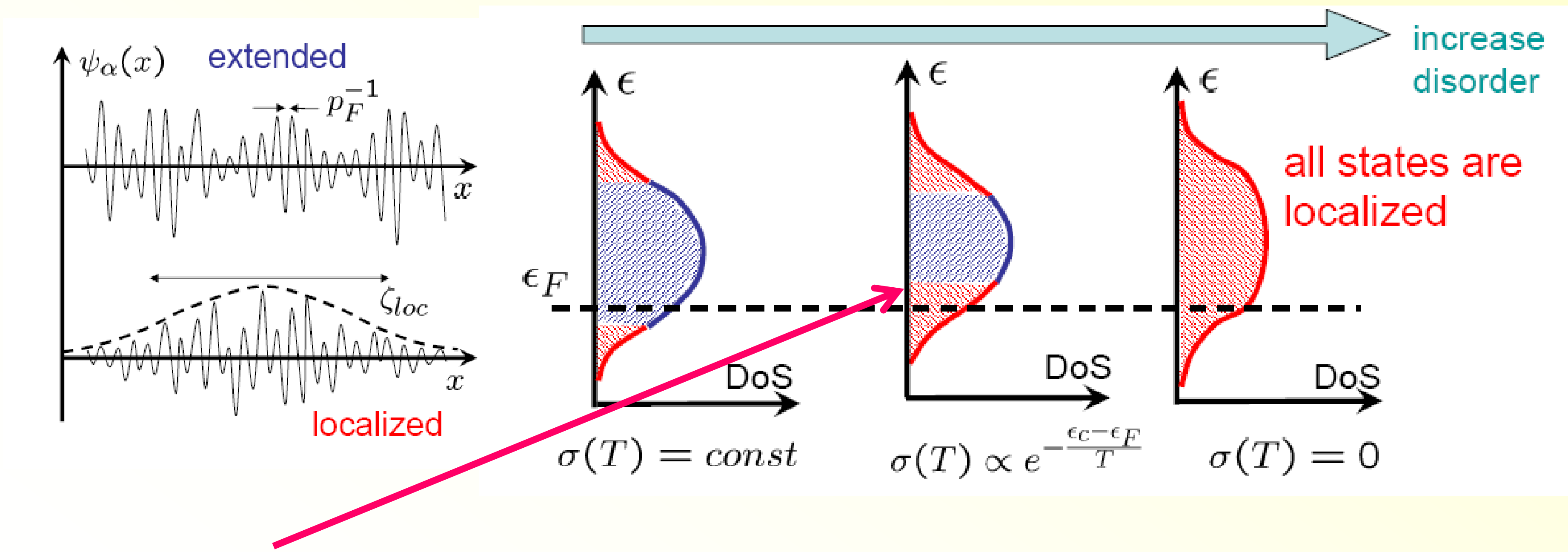


Mobility edge: separates delocalized (higher DOS) from localized states (low DOS)

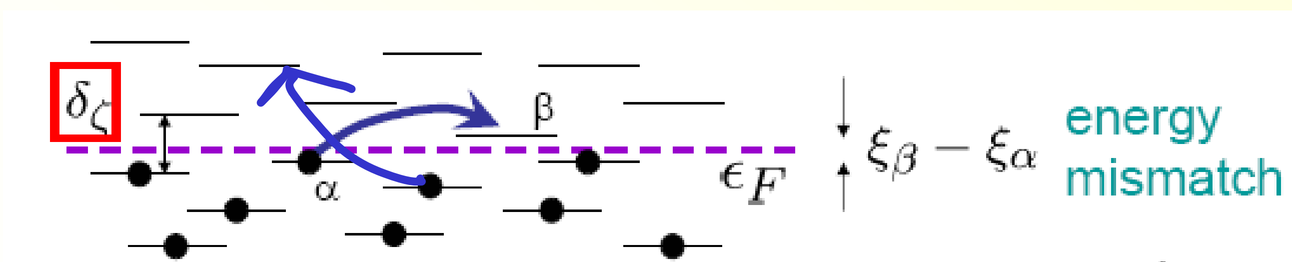
Localization: single/many particle

Anderson localization [many particle] *(Anderson, Fleishman 80's, Altshuler, Gefen, Kamenev, Levitov 90's Aleiner, Basko, Mirlin, Gornyi… 2005)*

$$H = \sum_{\alpha} \varepsilon_\alpha n_\alpha - \sum_{\alpha,\beta,\gamma,\delta} V_{\alpha\beta\gamma\delta}\left(c^+_\alpha c^+_\beta c_\gamma c_\delta + \mathrm{h.c.}\right)$$

Disorder-localized single particle levels

Interaction (short range) – → hopping in Fock space



Phases and phenomena I

Non-interacting fermions + NOTHING ELSE
(no bath of any sort: no phonons, no EM fields)

Transport as a function of
temperature and disorder?

Phases and phenomena I

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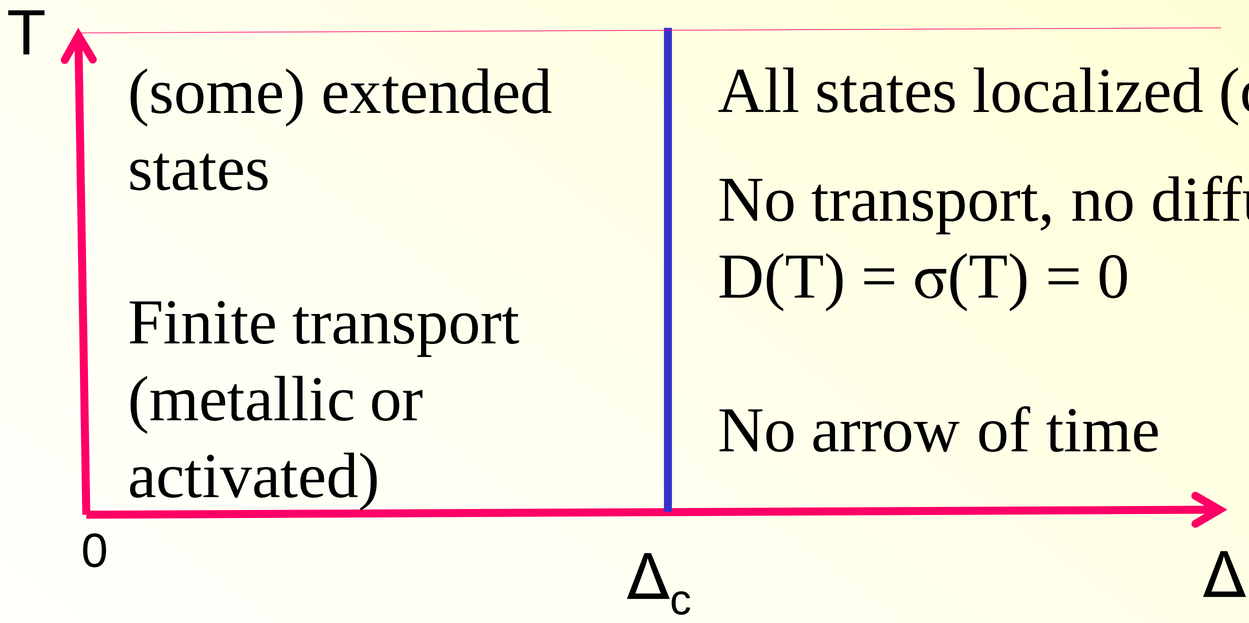

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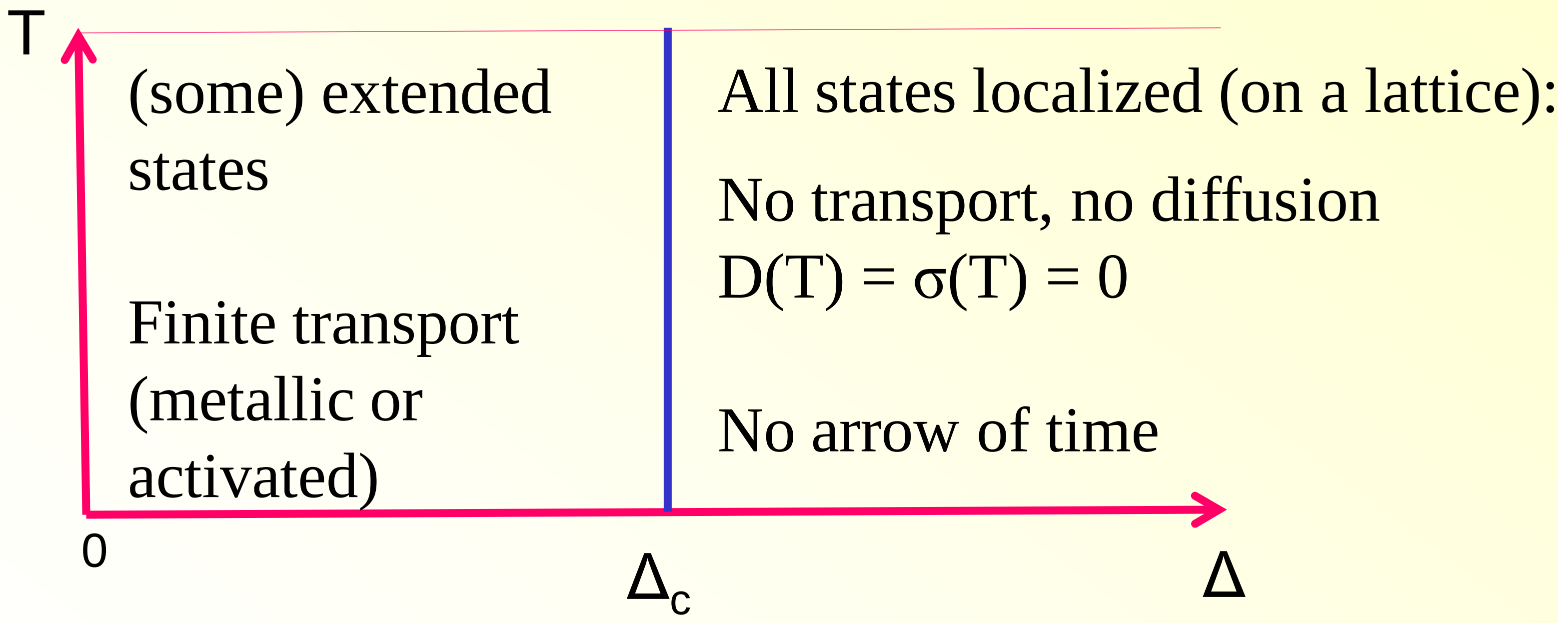


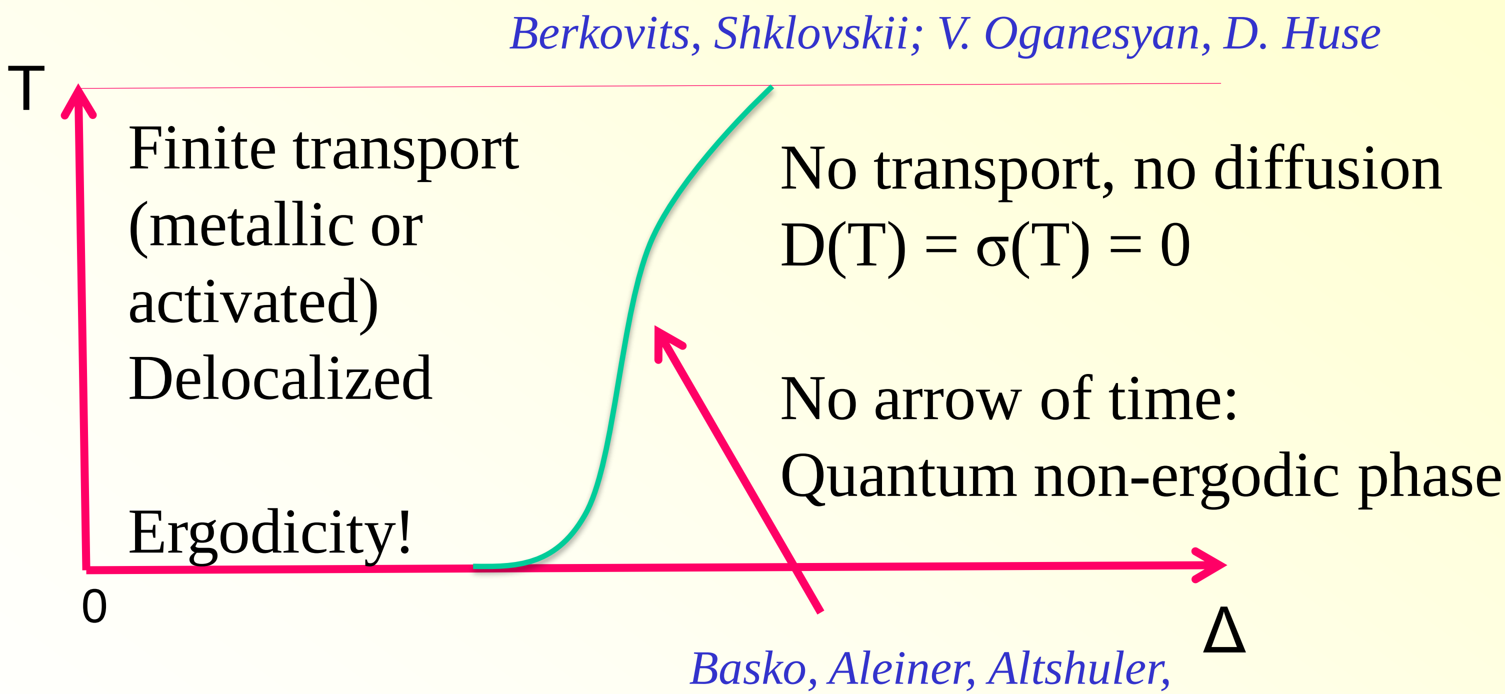
Role of dimension: $\Delta_c = 0$ in d=1,2 (without special symmetries)

FULLY UNDERSTOOD! (for physicists)

Phases and phenomena II

Interacting particles of finite density + NOTHING ELSE

(no bath of any sort: no phonons, no EM fields)



HARDLY ANY ASPECT IS FULLY UNDERSTOOD!

Phases and phenomena II

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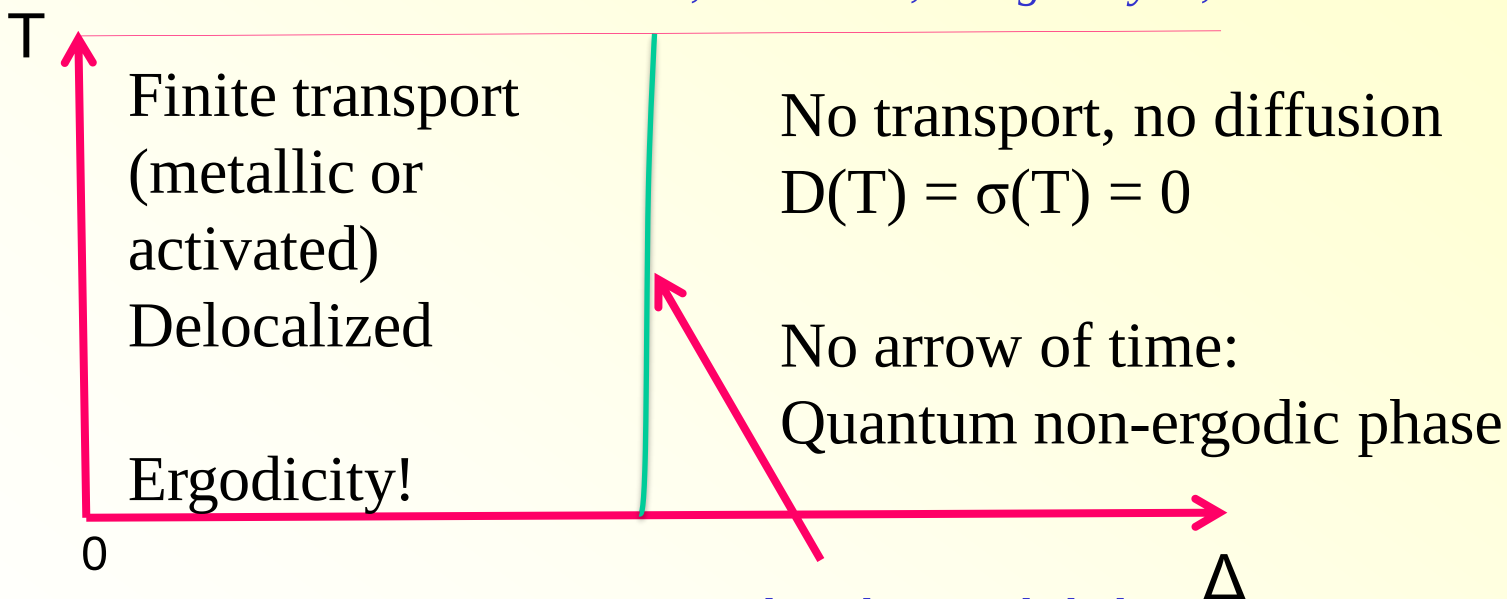
Berkovits, Shklovskii; V. Oganesyan, D. Huse



Basko, Aleiner, Altshuler,

Role of dimension? Can a finite T transition occur in high d??

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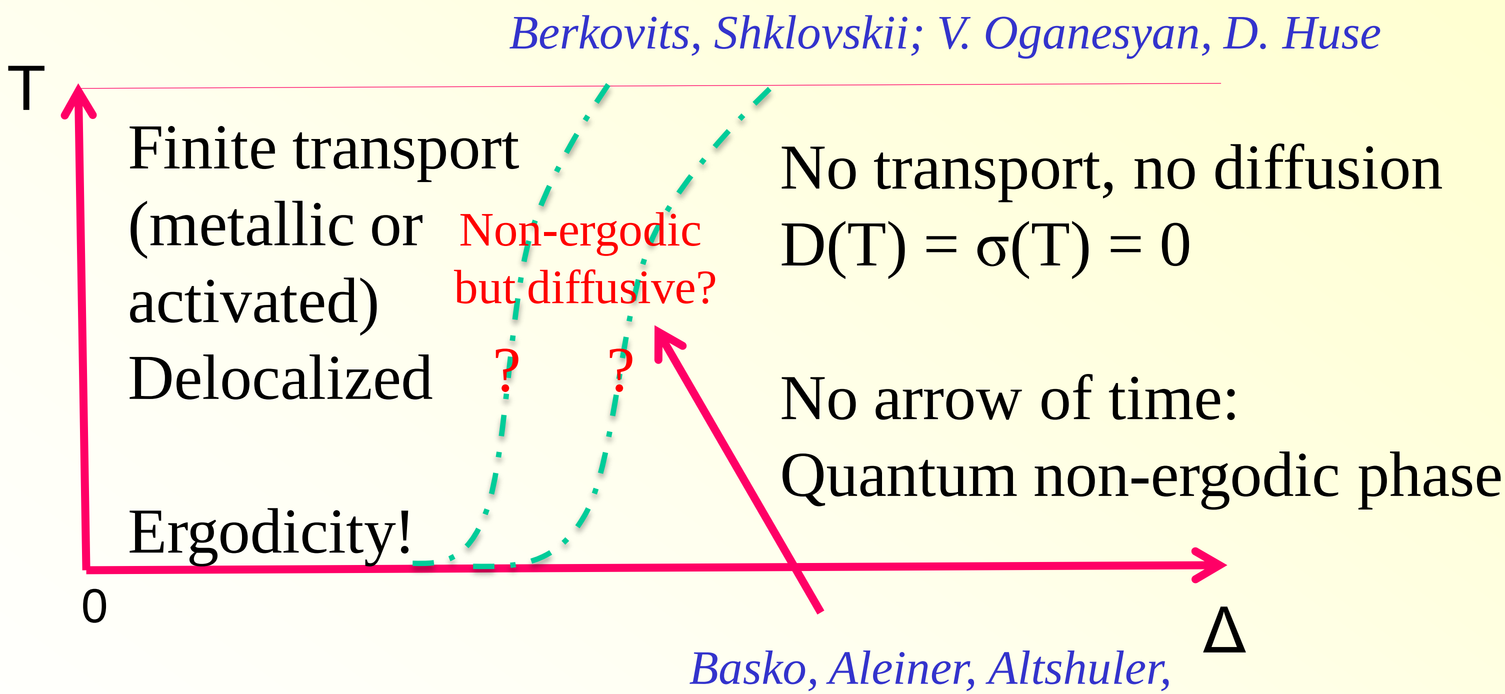

Number and nature of the transitions??

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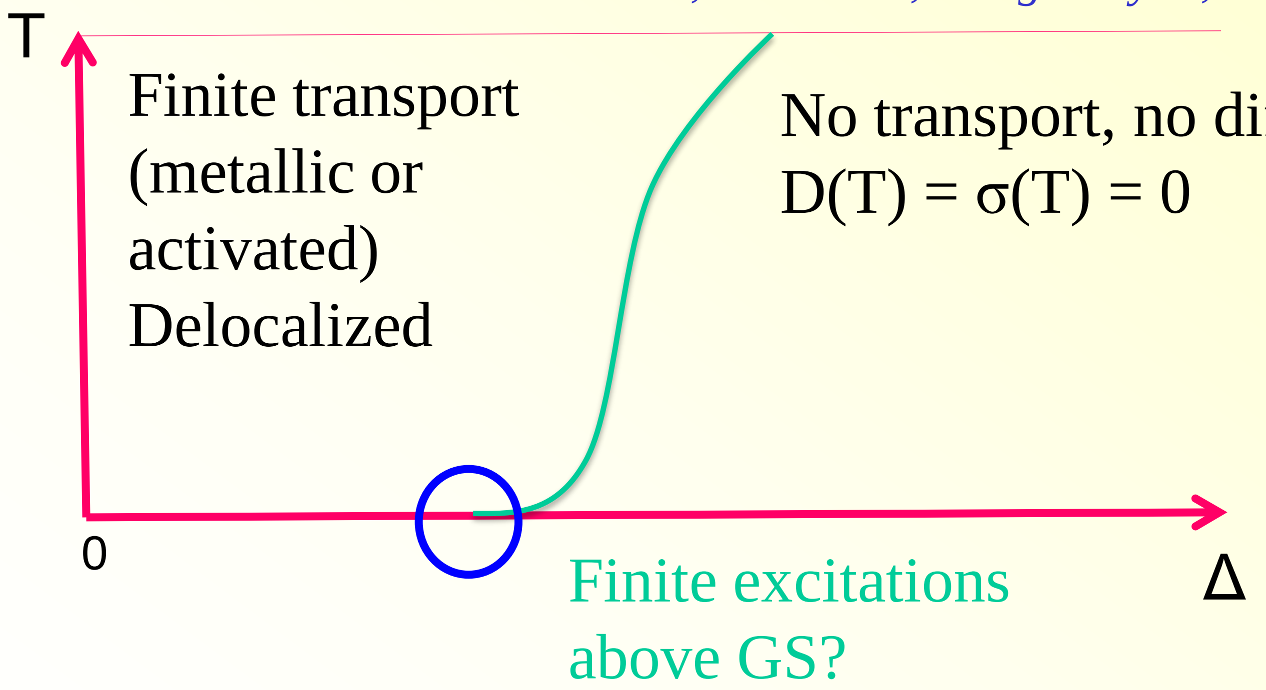

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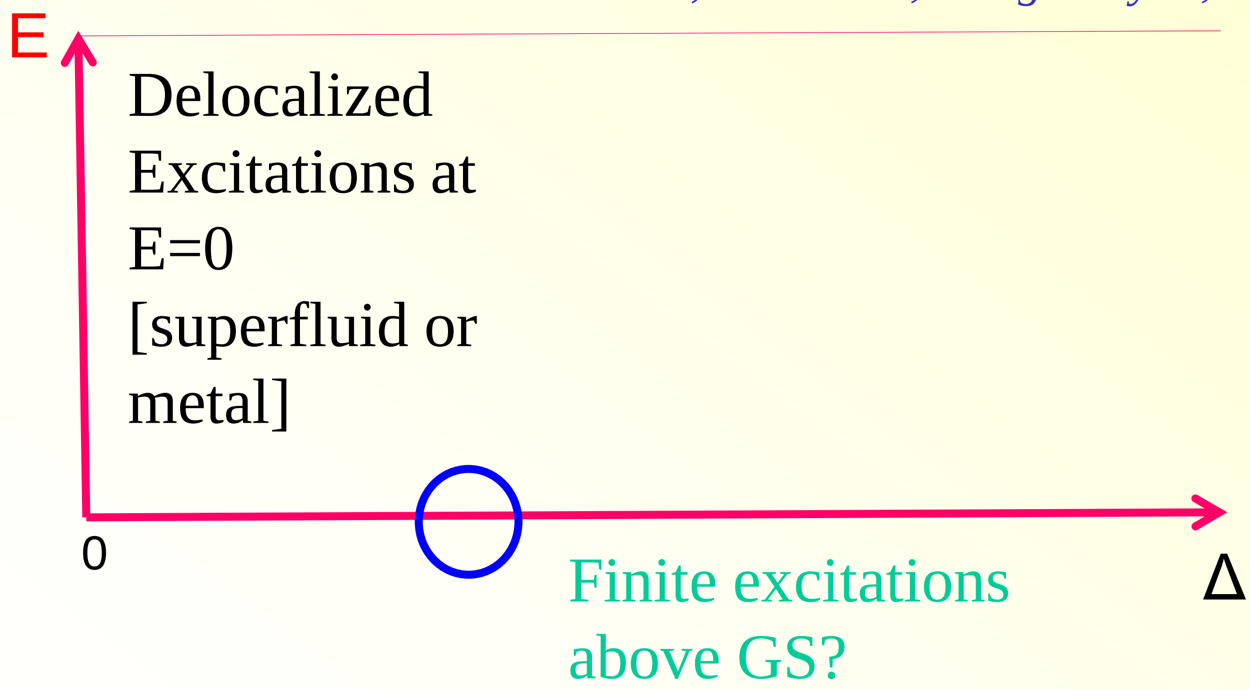

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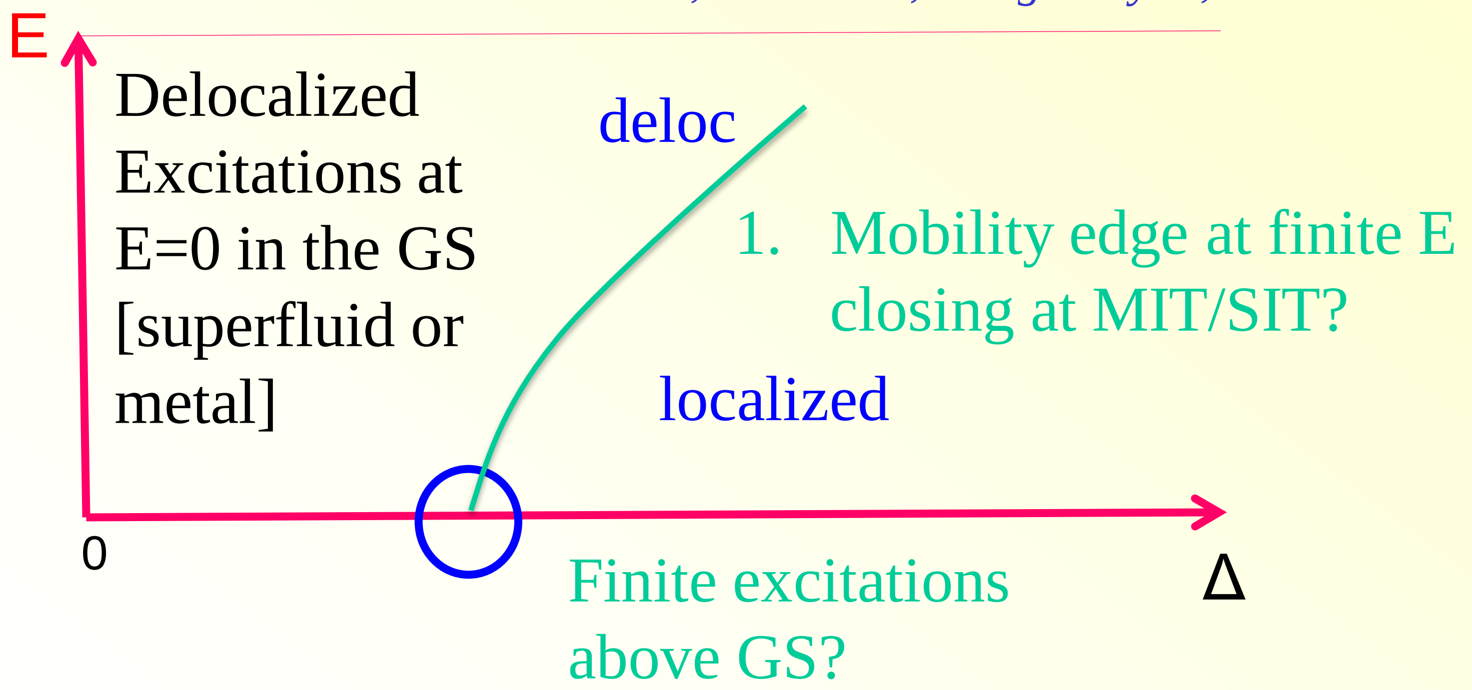

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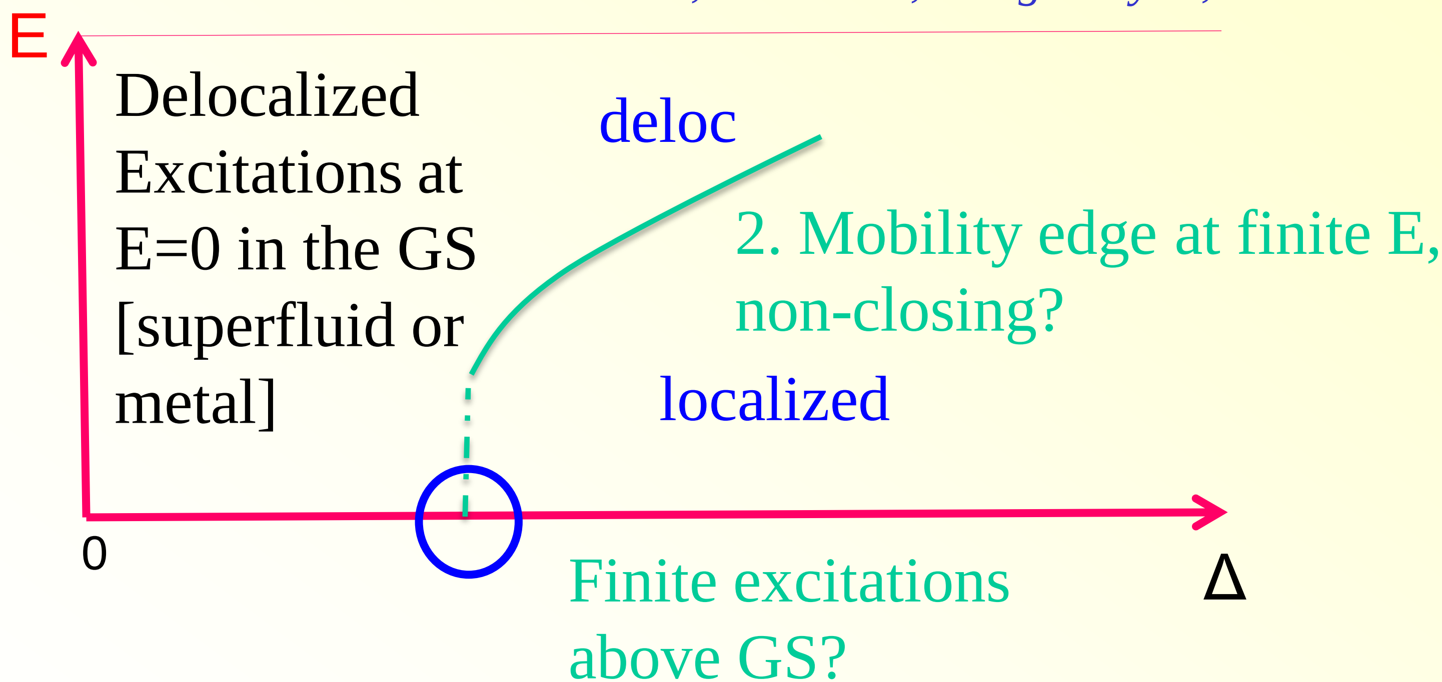

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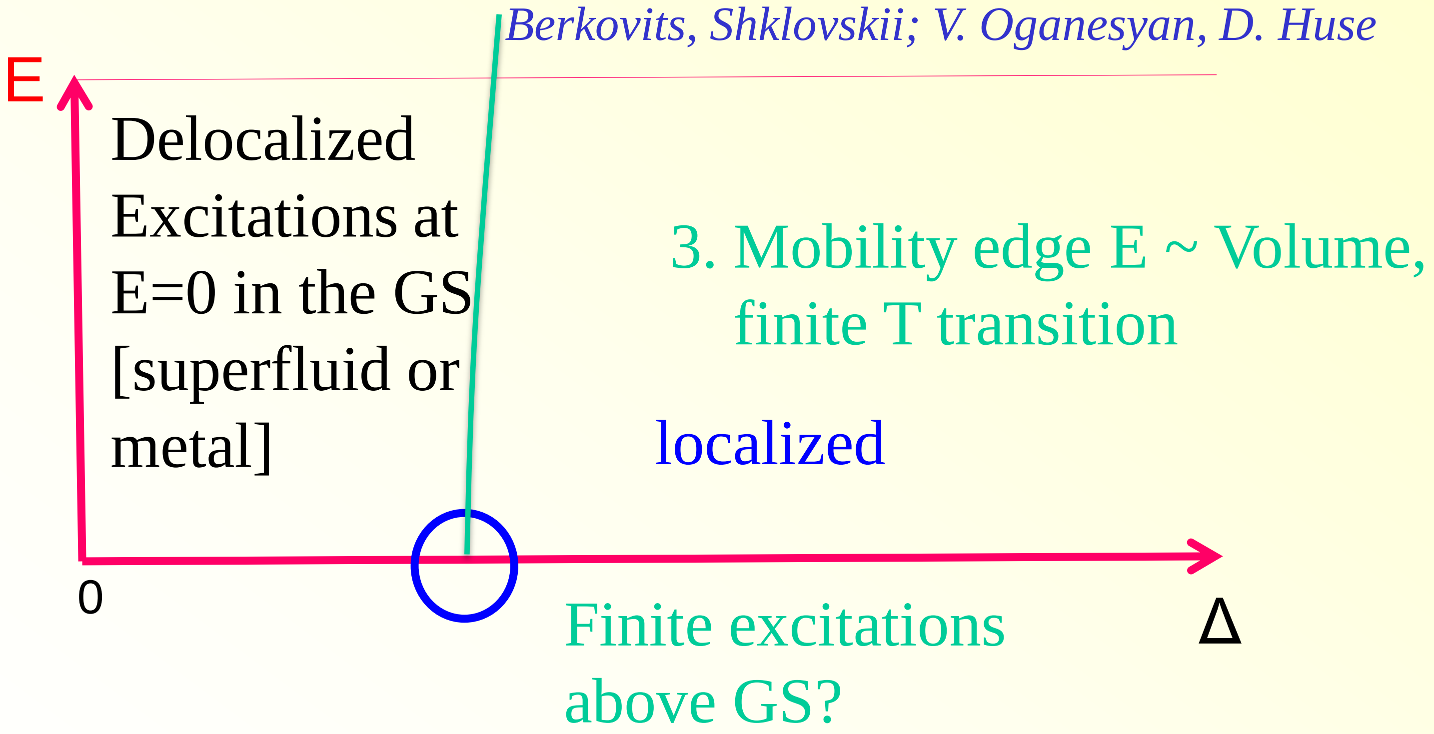

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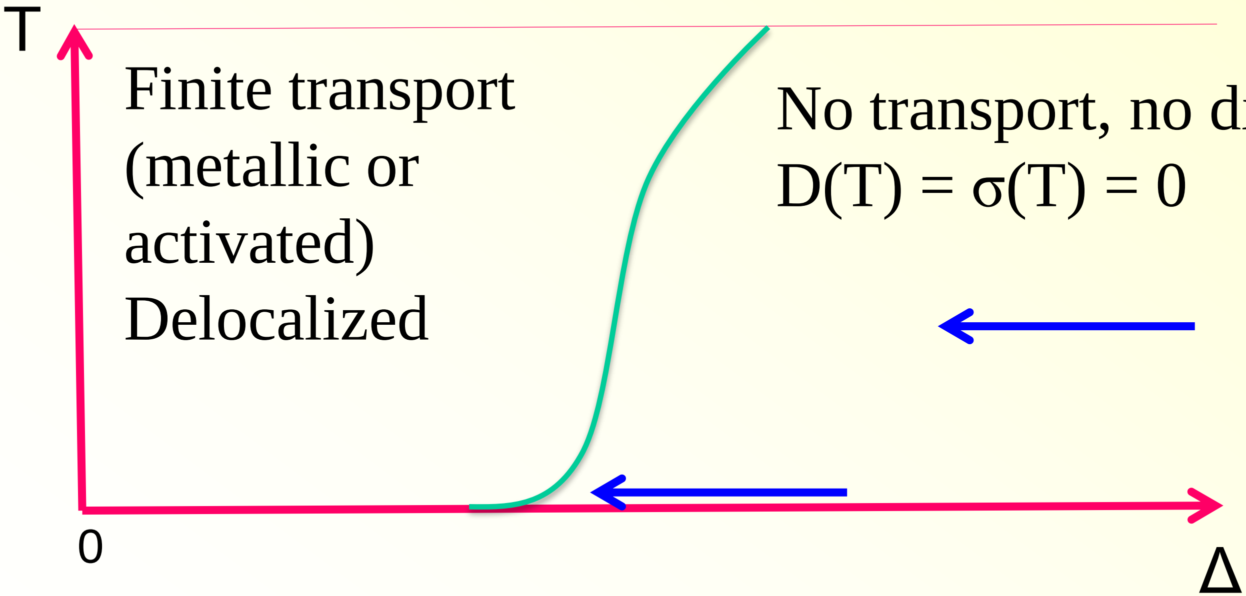
This talk: approach from deep insulator at $T = 0$
Result for bosons: scenario 2 or 3 are found!



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Questions

- Effects of quantum statistics in insulators?
- Strong localization of interacting disordered systems (especially: dirty bosons)?
  Locator expansion (applicability to many other systems)
- Approach to delocalization (superfluid transition)?

Disordered insulators: Simplest model: hopping+disorder

Model

$$H = \sum_i \varepsilon_i n_i - \sum_{\langle i,j \rangle} t_{ij} (b_j^\dagger b_i + b_i^\dagger b_j), \qquad n_i = b_i^\dagger b_i.$$

Fermions

$$\{b_i, b_j\} = 0, \qquad \{b_i^\dagger, b_j\} = \delta_{ij}$$

P. W. Anderson (1958)

.....

Disordered insulators: Simplest model: hopping+disorder

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P. W. Anderson (1958)
.....

Hard core bosons

M. Ma and P. A. Lee (1985), Kapitulnik and Kotliar (1985)

$$[b_i, b_j] = 0, \quad [b_i^\dagger, b_j] = \delta_{ij}(2n_i - 1)$$

Krauth, Trivedi, Randeria;
Feigelman, Ioffe, Kravtsov
Ioffe, Mézard, Feigelman
Syzranov, Moor, Efetov

[Anyons (in 2d): interpolate smoothly F ↔B]

Localization length

Strong insulators: Hopping transport! - Localization length ξ?

Localization length

Strong insulators: Hopping transport! - Localization length ξ?

Fermions

$$G^R_{i,0}(t-t') = -i\Theta(t-t')\langle\{b_i(t), b_0^\dagger(t')\}\rangle$$

Bosons

$$G^R_{i,0}(t-t') = -i\Theta(t-t')\langle[b_i(t), b_0^\dagger(t')]\rangle$$

Localization length

Strong insulators: Hopping transport! - Localization length ξ?

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Bosons $$G^R_{i,0}(t-t') = -i\Theta(t-t')\langle[b_i(t), b_0^\dagger(t')]\rangle$$

Generalized localization length (also interacting)

$$\xi(\omega)^{-1} = -\lim_{\vec{r}_i\to\infty} \overline{\ln[|G^R_{i,0}(\omega)/G^R_{0,0}(\omega)|]/|\vec{r}_i - \vec{r}_0|}$$

Free fermions: no features near E_F, ξ(ω) ~ const. - What about bosons?

Locator expansion and forward scattering

Fermions

J. Hubbard (1963): Equation of motion for Green's function!

$$i\dot{b}_i(t)$$

$$\left(i\frac{d}{dt}-\varepsilon_i\right)G^R_{i,0}(t)$$

$$=\delta(t)\delta_{i,0}+i\Theta(t-t')\left\langle\left\{\sum_{j\in\partial i}t_{ij}b_j(t), b_0^\dagger(t')\right\}\right\rangle$$

$$=\delta(t)\delta_{i,0}-\sum_{j\in\partial i}t_{ij}G^R_{j,0}(t)$$

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$$=\delta(t)\delta_{i,0}-\sum_{j\in\partial i}t_{ij}G^R_{j,0}(t)$$

Fourier transform → Anderson's sum over paths

Anderson (1958)

$$\frac{G^R_{i,0}(\omega)}{G^R_{0,0}(\omega)}=\sum_{\mathcal{P}=\{j_0=0,\ldots,j_\ell=i\}}\prod_{p=1}^{\ell}t_{j_{p-1},j_p}\frac{1}{\varepsilon_{j_p}-\omega}$$

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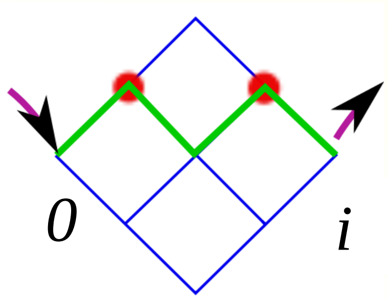
Fourier transform → Anderson's sum over paths *Anderson (1958)*

Forward scattering approximation: sum over shortest paths!

Spivak, Shklovskii, Nguyen (1983)



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Locator expansion and forward scattering

Fermions

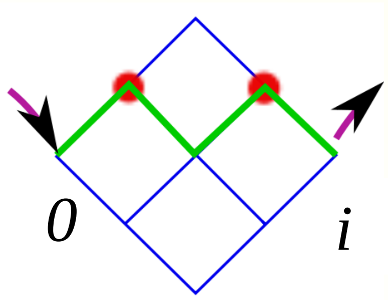
Magnetoresistance: **negative** *(Nguyen, Spivak, Shklovskii)*

Path amplitudes: real with random signs!

B-field: $t_{ij} \longrightarrow t e^{-i\varphi_{ij}}$ makes destructive interference less likely → ξ and 1/R increase.

Forward scattering approximation: sum over shortest paths!

Spivak, Shklovskii, Nguyen (1983)



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Locator expansion and forward scattering

Bosons
(hard core)

$$\left(i\frac{d}{dt} - \varepsilon_i\right) G^R_{i,0}(t) = \delta(t)\delta_{i,0}(1 - 2\langle n_0\rangle)$$

$$+ i\Theta(t - t')\left\langle \left[(-1)^{n_i(t)} \sum_{j\in \partial i} t_{ij} b_j(t), b_0^\dagger(t') \right] \right\rangle$$

$$\approx \delta(t)\delta_{i,0}(1 - 2\langle n_0\rangle) - \mathrm{sgn}(\varepsilon_i) \sum_{j\in \partial i} t_{ij} G^R_{j,0}(t)$$

Locator expansion and forward scattering

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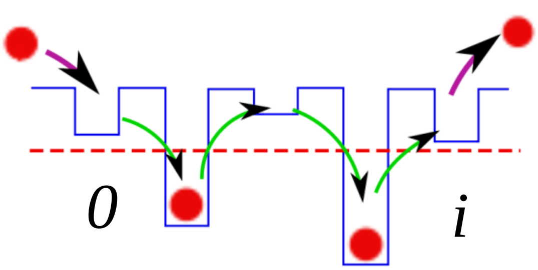
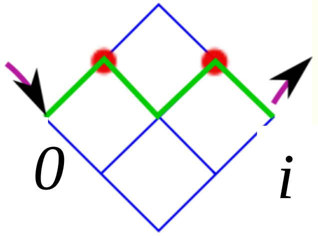
Forward scattering: Sum over shortest paths, lowest order in t! *MM (2011)*





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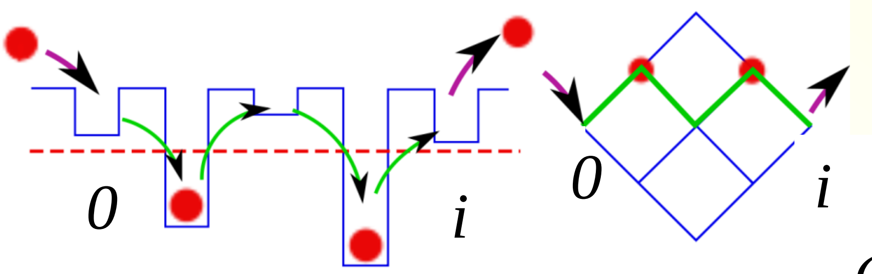

Sign difference Bosons/Fermions:
Loop of two paths:
Ring exchange of particles

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Locator expansion and forward scattering

Bosons (hard core)

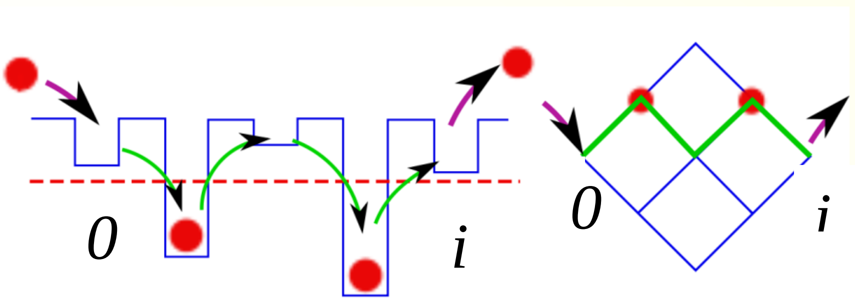
Magnetoresistance: **positive**

cf also Zhou, Spivak (1991)Syzranov et al (2012)

Path amplitudes: **all** positive at ($\omega \rightarrow 0$) !

B-field: $t_{ij} \longrightarrow t e^{-i\varphi_{ij}}$ destroys constructive interference, ξ and 1/R decrease.

Forward scattering: Sum over shortest paths, lowest order in t! *MM (2011)*



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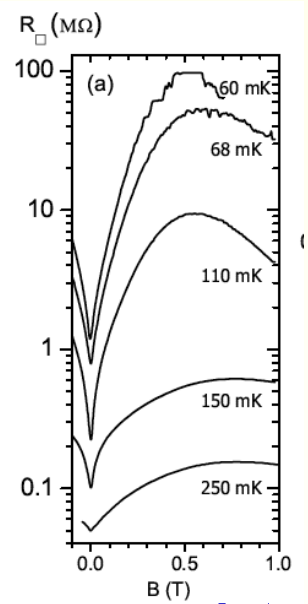
Magnetoresistance peak

Hebard+Palaanen,
Gantmakher et al.,
Shahar et al,
Baturina et al, W. Wu,
Valles et al., Goldman et al.

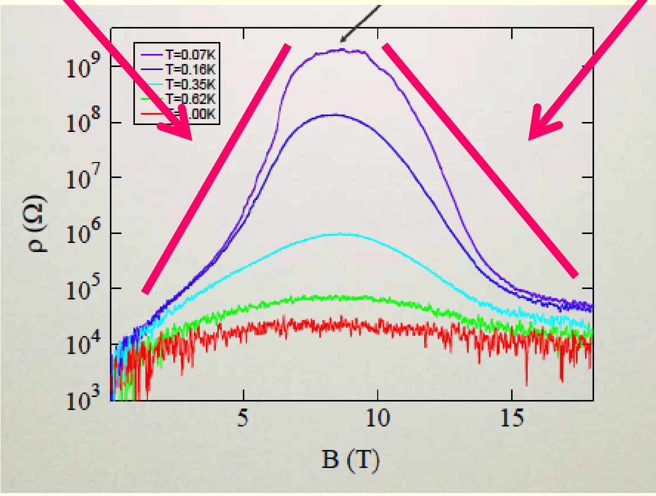
A key ingredient to the MR peak:

Local pairs = bosons
→ exponentially positive MR

Unpaired fermions
→ exponentially negative MR



Baturina et al. (2007) TiN



Sambandamurthy, Shahar et al. (2005) - InO_x

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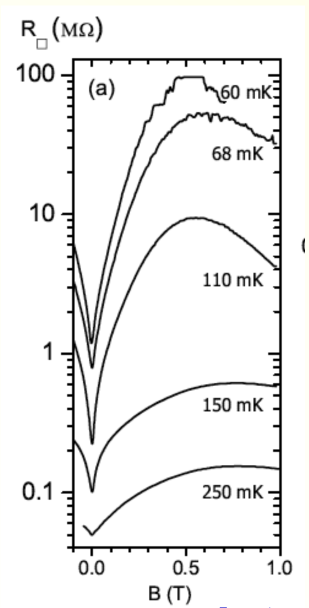

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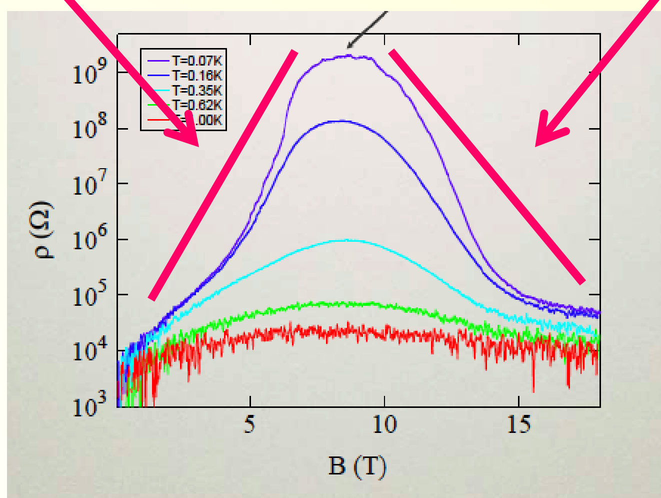


Sambandamurthy, Shahar et al. (2005) - InO_x

Magnetoresistance more quantitatively? Fermions vs. bosons? ξ(B)?

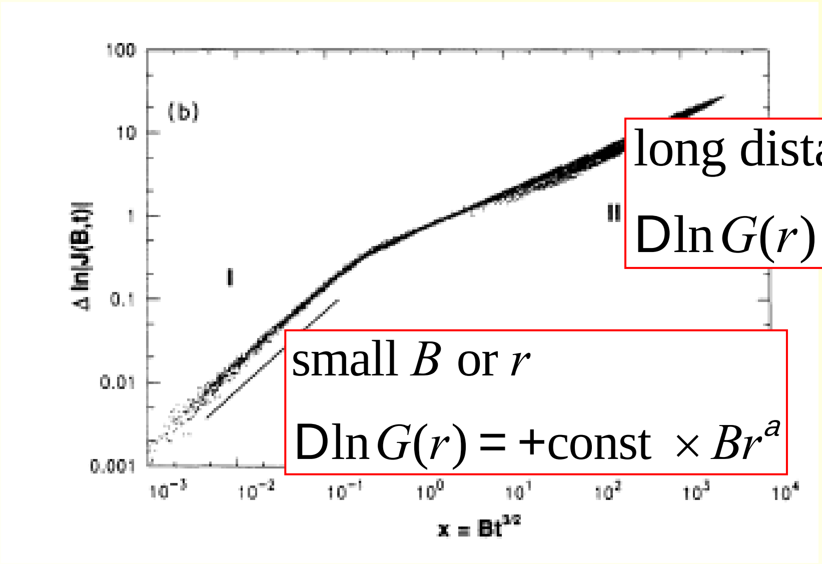
Magnetoresistance quantitaively

Past studies:

- Mostly numerics (fermions) *Medina+Kardar, Spivak et al*

  Directed paths in random media: *Kardar's book: Stat. Physics of Fields*

Data fitting:



long distance r

$$\Delta \ln G(r) = +\text{const} \times B^{\gamma} r$$

small B or r

$$\Delta \ln G(r) = +\text{const} \times B r^{\alpha}$$

α,γ = ??

(Kardar:
$\alpha = 3/2$,
$\gamma = 1/2$)

Magnetoresistance quantitaively

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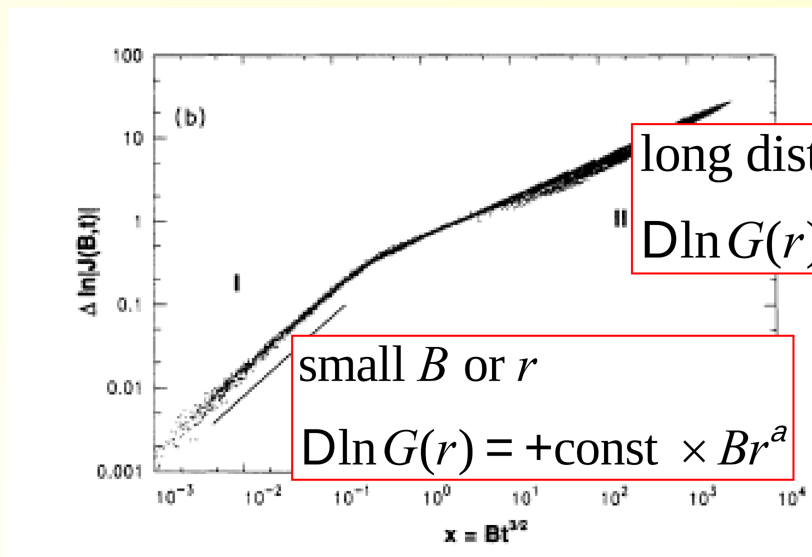


long distance r

$D\ln G(r) = +\text{const} \times B^{g} r$

small B or r

$D\ln G(r) = +\text{const} \times B r^{a}$

α,γ = ??
(Kardar:
α = 3/2,
γ =1/2)

- Analytical studies of phases of *complex interference sums* in simplified models (Bethe/hierarchical lattices): *Derrida, Cook, Spohn*

Magnetoresistance quantitaively

Our numerical studies:

- *Apparent* different scalings of bosons and fermions
  (γ appeared bigger for bosons than for fermions – why?)

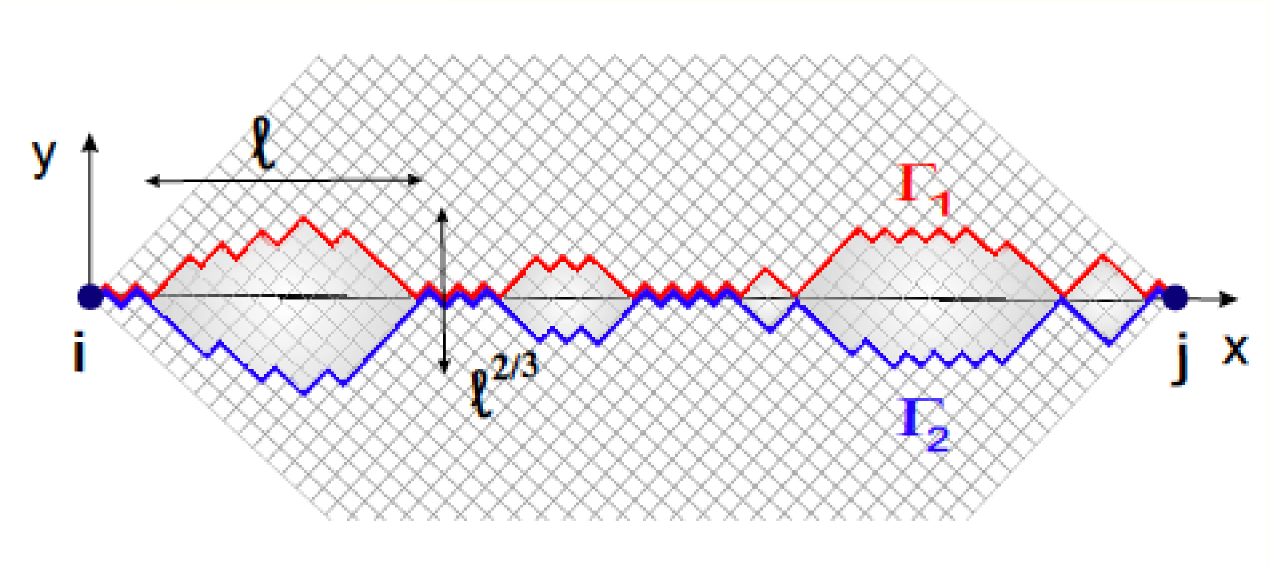
long distance r

$$\Delta \ln G(r) = +\text{const} \times B^{g} r$$

- No satisfactory scaling collapse for values in the fermion literature

Magnetoresistance quantitaively

Typical relevant paths form droplets:

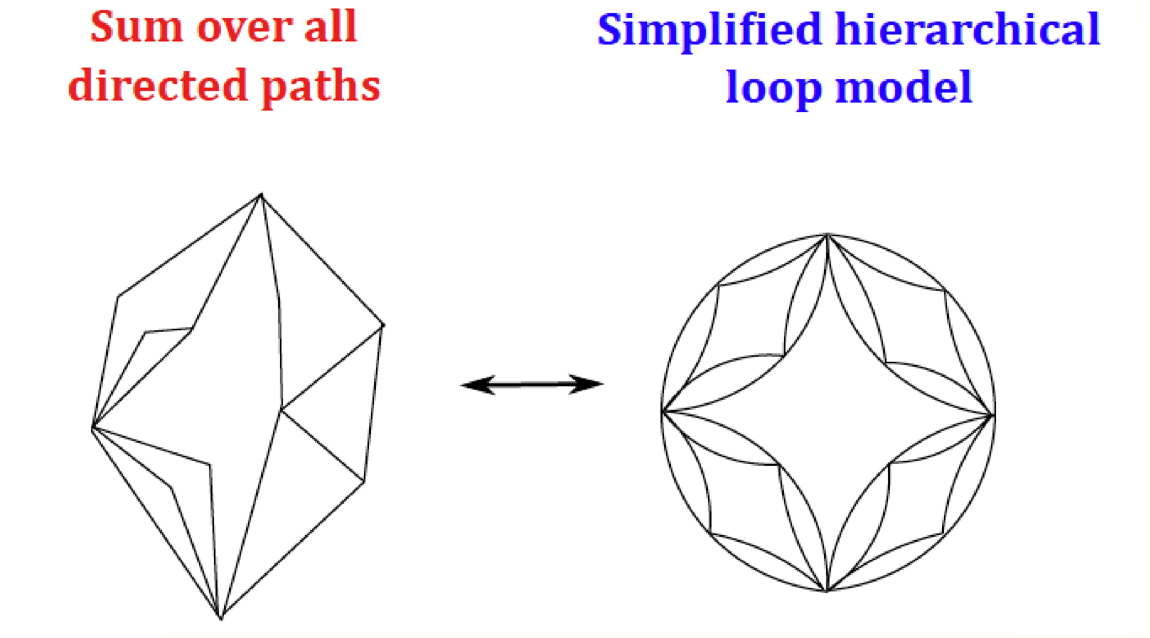


Exactly like directed polymers in random media!

(Monthus, Garel; Ortuno, Prior, Somoza)

Simplified hierarchical model

A. Gangopadhyay, V. Galitski, MM (in prep)



(cf. *Hwa, Fisher+Huse's* droplet theory for directed polymers, 1994)

Sum over directed positive weight paths = Partition function of directed polymer

Simplified hierarchical model

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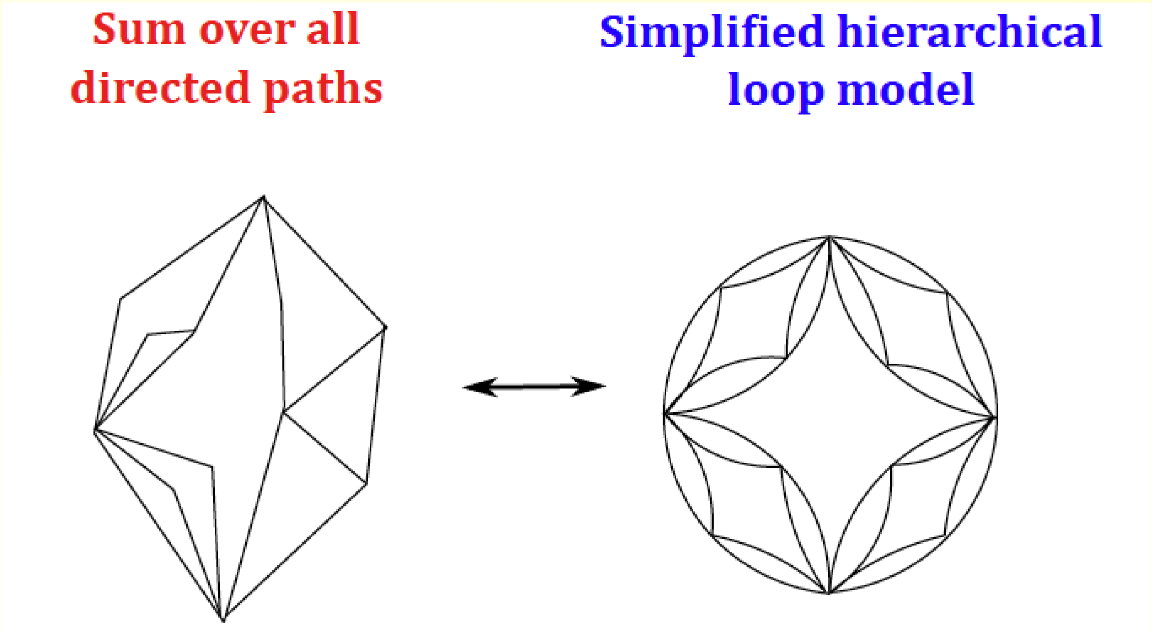
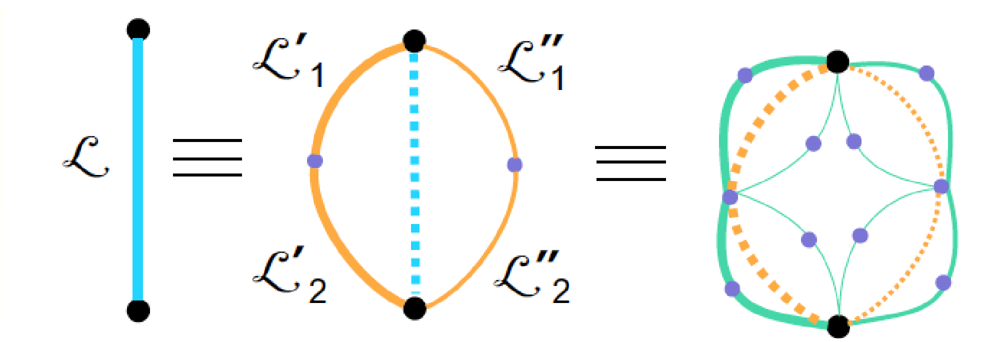


(cf. *Hwa, Fisher+Huse's* droplet theory for directed polymers, 1994)

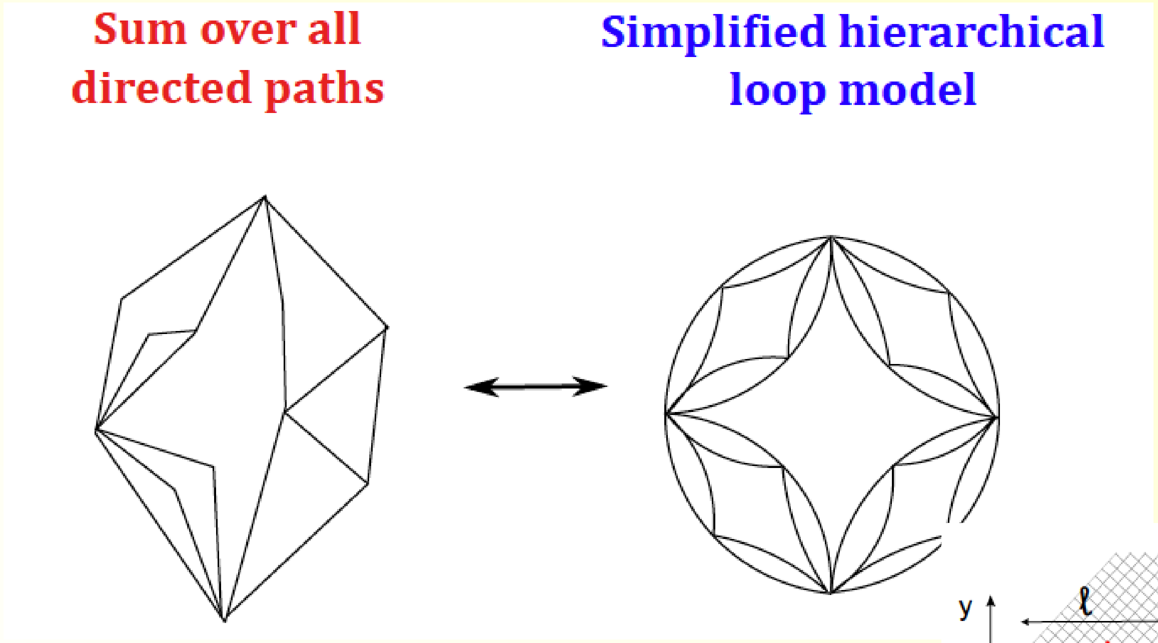


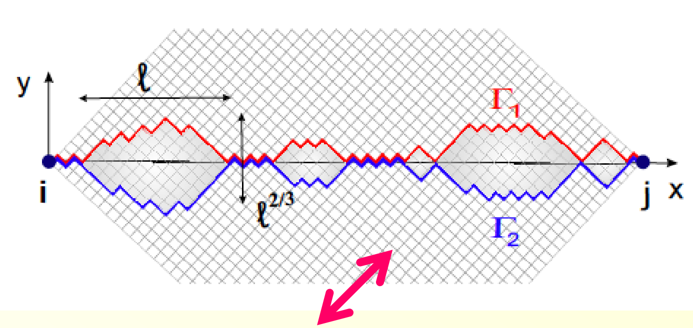
$$S^k_{\mathcal{L}} = S^{k+1}_{\mathcal{L}'_1} S^{k+1}_{\mathcal{L}'_2} + e^{-f_{\mathcal{L}} L_k^{\theta}} e^{i a_{\mathcal{L}} B L_k^{1+\zeta}} S^{k+1}_{\mathcal{L}''_1} S^{k+1}_{\mathcal{L}''_2}$$

Interference sum S recursively defined

Simplified hierarchical model

A. Gangopadhyay, V. Galitski, MM (in prep)





Advantage: Analytically tractable model:
Virial expansion: small B ⟷ low density of interfering loops

Numerics: exponents (in finite size) are very similar to full model

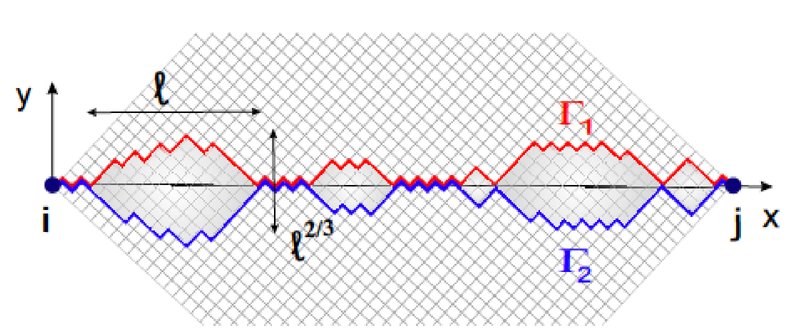
Virial expansion for droplet model

A. Gangopadhyay, V. Galitski, MM (in prep)

Disorder is strong!

Larkin scale (disorder dominates entropy) $L_c \gg a_{lattice} = 1$

→ interfering loops are NOT random walks!



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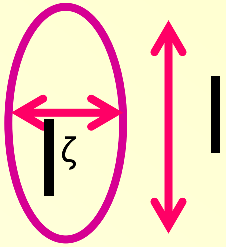
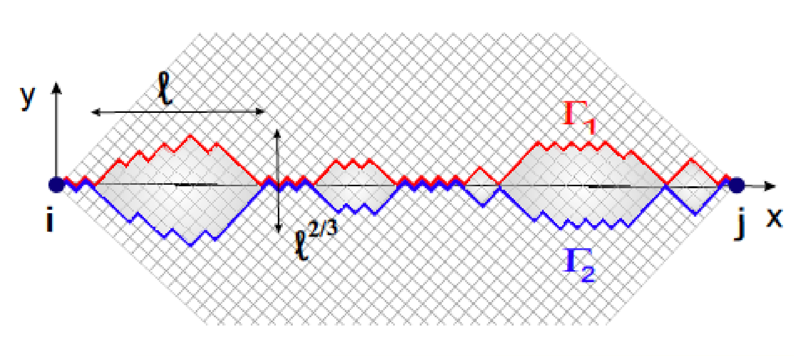


Size of interfering regions ("magnetic length")

$$B\ell_B \ell_B^z = 1 \rightarrow \ell_B = B^{-1/1+z} \qquad \zeta = 2/3$$



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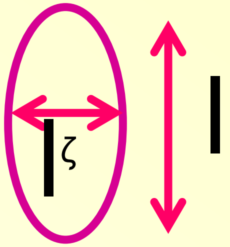

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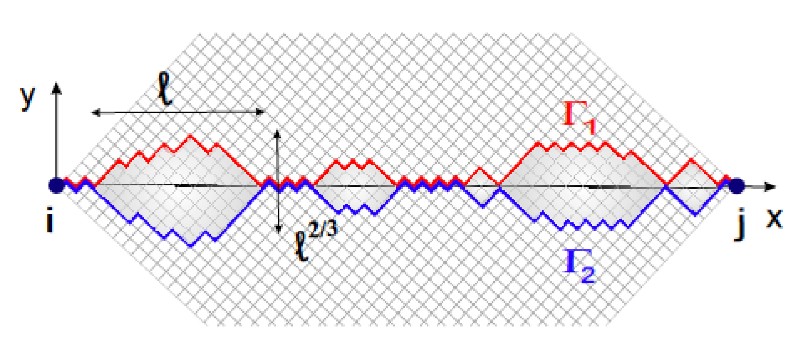
Probability of significant interference

$$P_{\text{interf}} \propto \ell_B^{-\theta} \qquad \theta = 1/3$$



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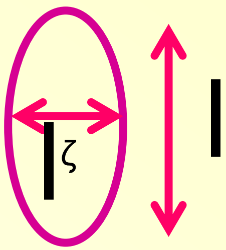

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Probability of significant interference

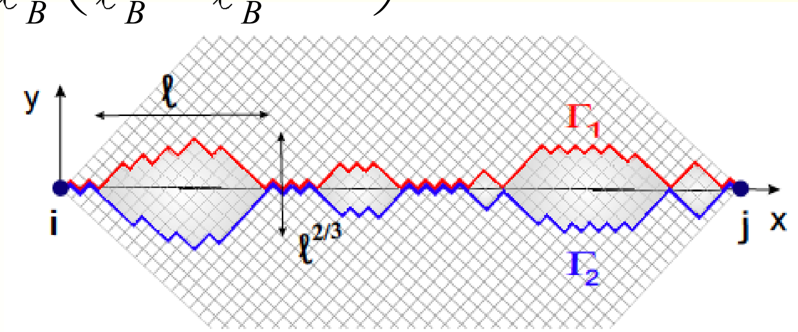
$$P_{\text{interf}} \;µ\; \ell_B^{-q} \qquad \theta = 1/3$$

Virial expansion

$$D\left(\frac{L}{x}\right) \sim \frac{L}{\ell_B}\left(\frac{1}{\ell_B^q} + \frac{1}{\ell_B^{2q}} + \ldots\right) \sim B^c\left(1 + B^a + \ldots\right)$$

$$c = \frac{1+q}{1+z} = \frac{1+1/3}{1+2/3} = \frac{4}{5}$$

$$a = \frac{q}{1+z} = \frac{1}{5}$$



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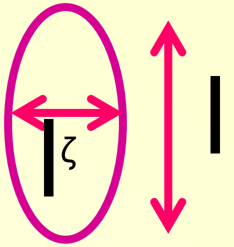

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Virial expansion

$$\Delta\left(\frac{L}{x}\right) \sim \frac{L}{\ell_B}\left(\frac{1}{\ell_B^q} + \frac{1}{\ell_B^{2q}} + \dots\right) \sim B^c\left(1 + B^a + \dots\right)$$

$$c = \frac{1+q}{1+z} = \frac{1+1/3}{1+2/3} = \frac{4}{5}$$

$$a = \frac{q}{1+z} = \frac{1}{5}$$

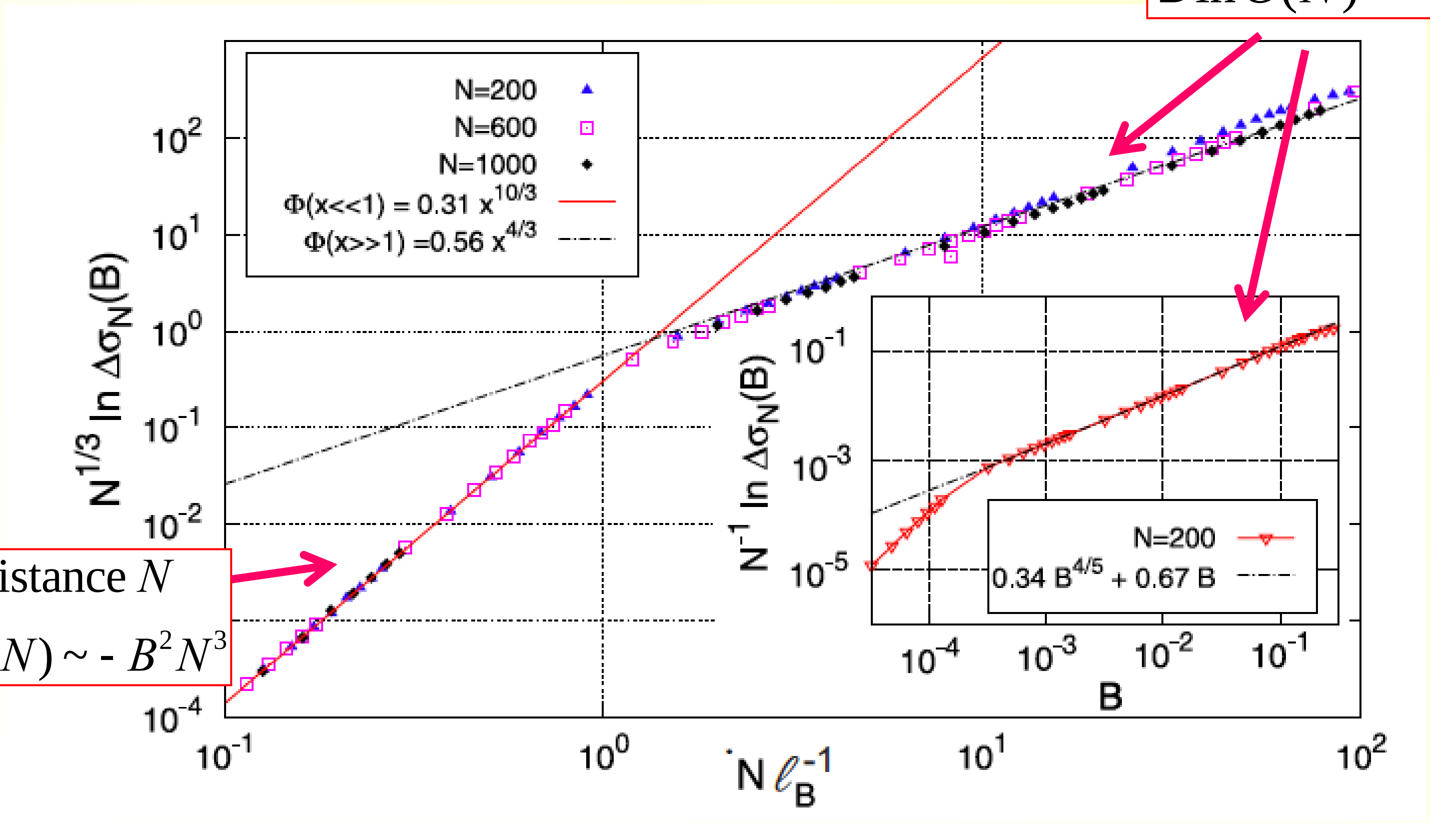
$$\Delta[1/x]_{boson} = -c_b B^c\left[1 + a_b B^a + O\left(B^{2a}\right)\right]$$

Larger apparent exponent!

Num. confirmation: Full lattice model

long distance N

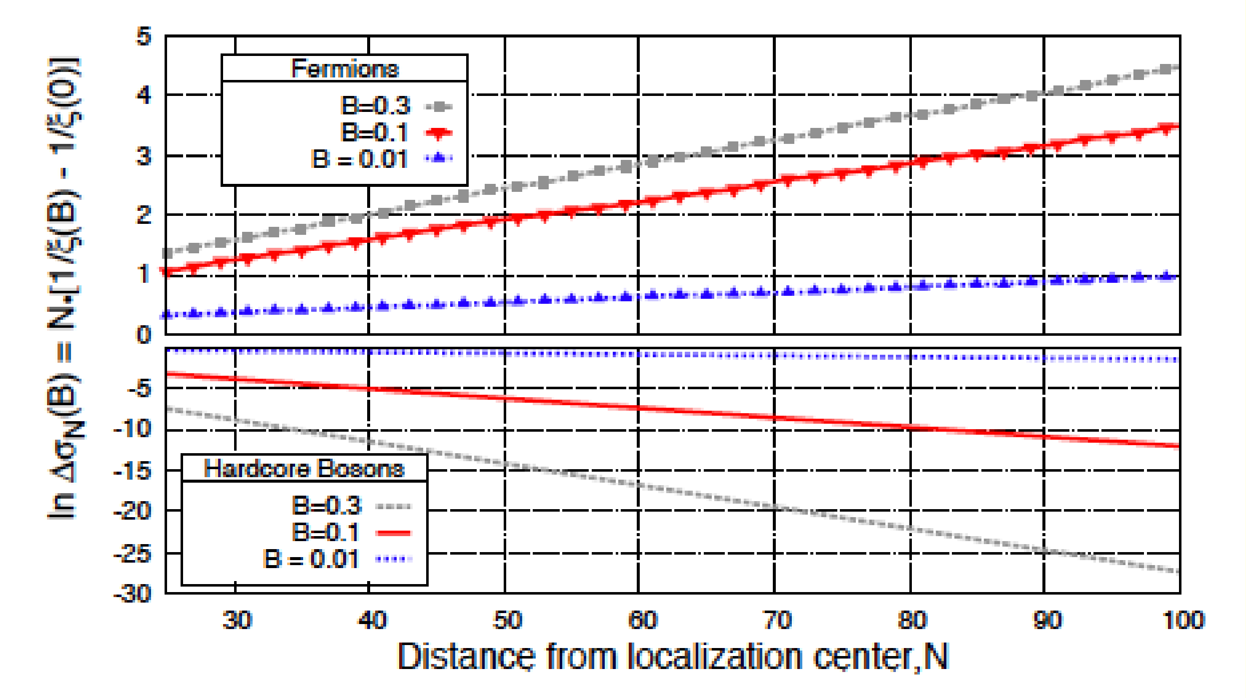
$\mathrm{D}\ln G(N) = -\, B^{0.8} N$



short distance N

$\mathrm{D}\ln G(N) \sim -\, B^2 N^3$

Quantum Statistics : modified localization length



Bosons: localization much more strongly enhanced than it is diminished for fermions

Predict: R(B)/R(0) ~ O(100) in strong insulators and fields

Back to $B = 0$

Approach to delocalization ?

Boseglass-to-superfluid transition

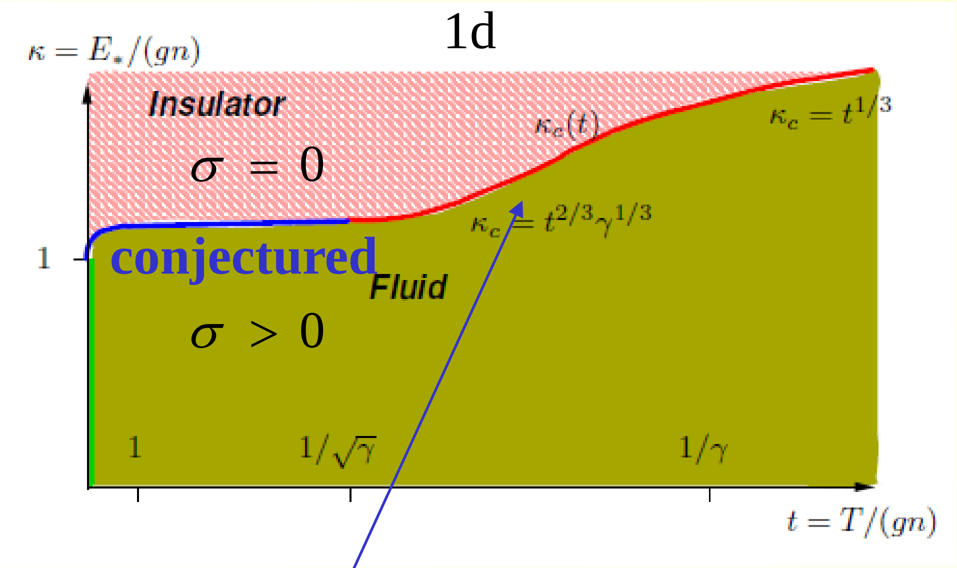
? = ?

boson delocalization + condensation

1d and 2d case

(Aleiner, Altshuler, Shlyapnikov 2009)

Calculations and conjectures about the phase diagram of
soft core bosons in 1d and 2d:



Genuine finite T phase transition in 1d!

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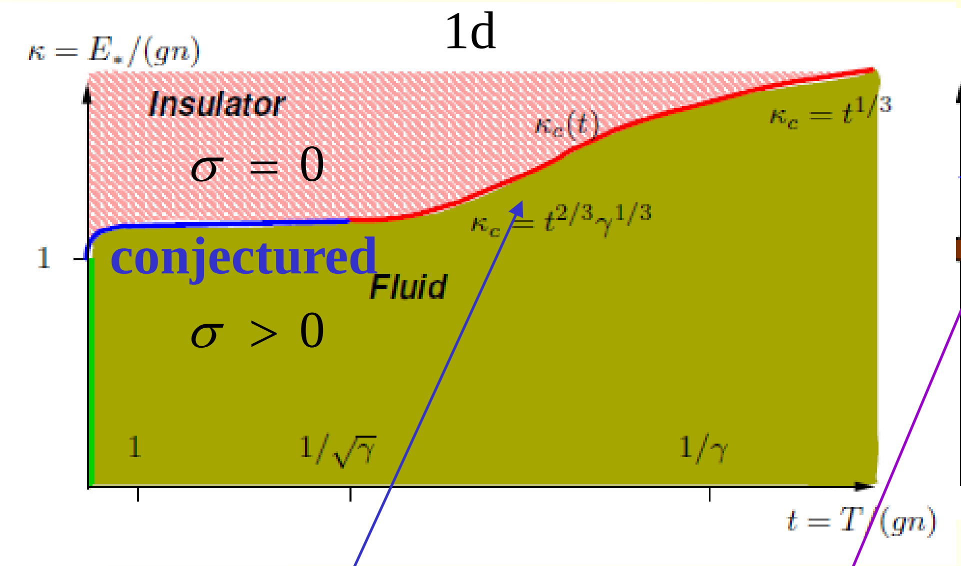
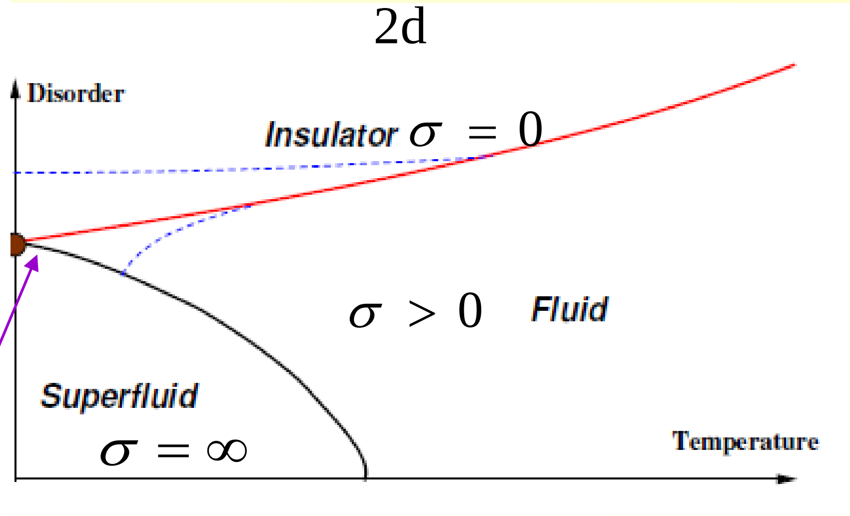




Genuine finite T phase transition in 1d!

Conjecture for 2d: Direct transition from superfluid to a many body localized phase, with full localization up to finite T

Higher dimensions?

Closing of a (many body) mobility gap?

Hertz, Anderson, Fleishman (1979) "Marginal bose glass"

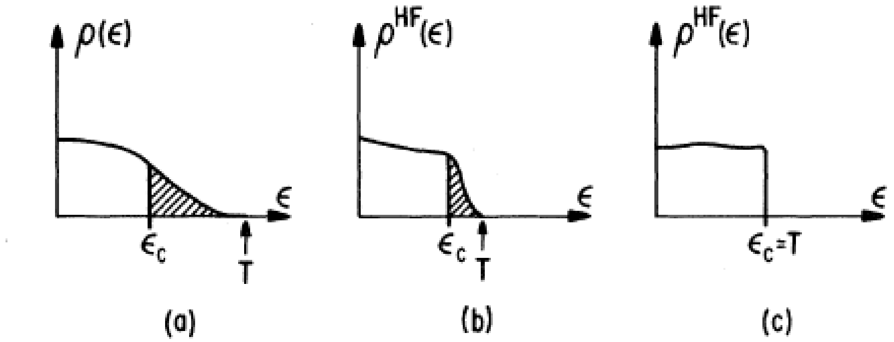


FIG. 1. Hartree-Fock density of states at three different temperatures (schematic): (a) For high T, $\rho^{HF} \approx \rho$ = density of eigenvalues of $\underline{J}$; (b) for intermediate T, tail of localized states moves to keep to the left of T; (c) for T reaching the mobility edge, no localized states remain.

Scenario for the ordering transitions in

- Disordered magnets
- Spin glasses
- Dirty superfluids (SI transition)

→ Idea: transition when extended Hartree-Fock state reaches chemical potential → condensation

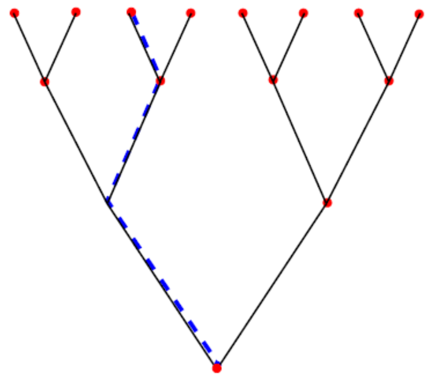
Higher dimensions?

Closing of a (many body) mobility gap?

Ioffe, Mézard; &Feigelman ('09, '11);

Hard core bosons on a Bethe lattice ("d = ∞")

$$H_{XY} = \sum_i \epsilon_i n_i - t \sum_{\langle i,j \rangle} \left(b_i^\dagger b_j + b_j^\dagger b_i \right)$$



N = 2

Bethe lattice of **large** connectivity N [→ approach close to transition possible]
(like Abou-Chacra-Anderson-Thouless (1973) for fermions)

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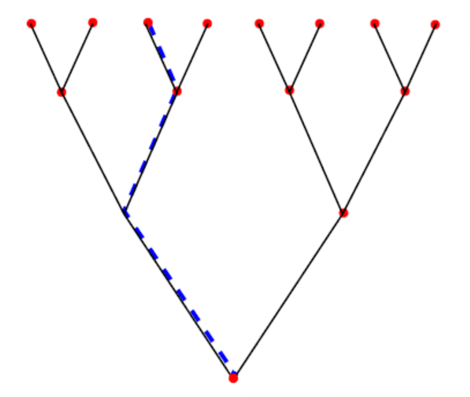
Reported phase diagram:



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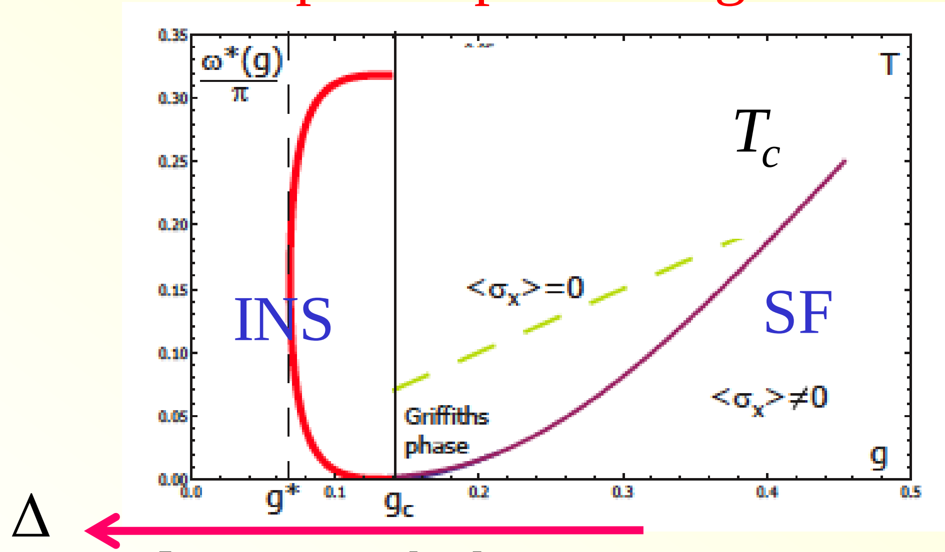

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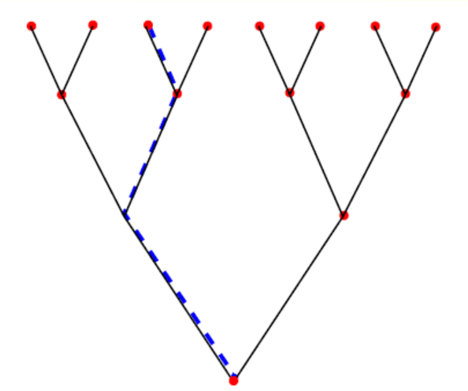

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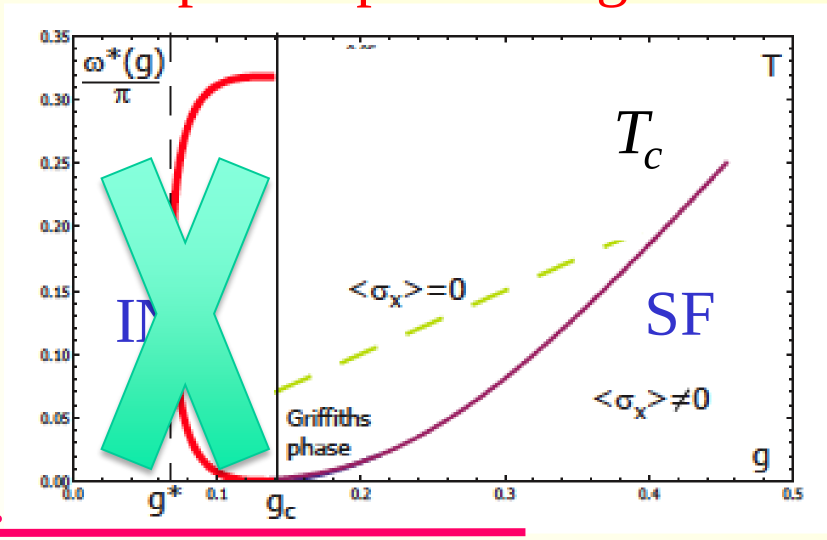

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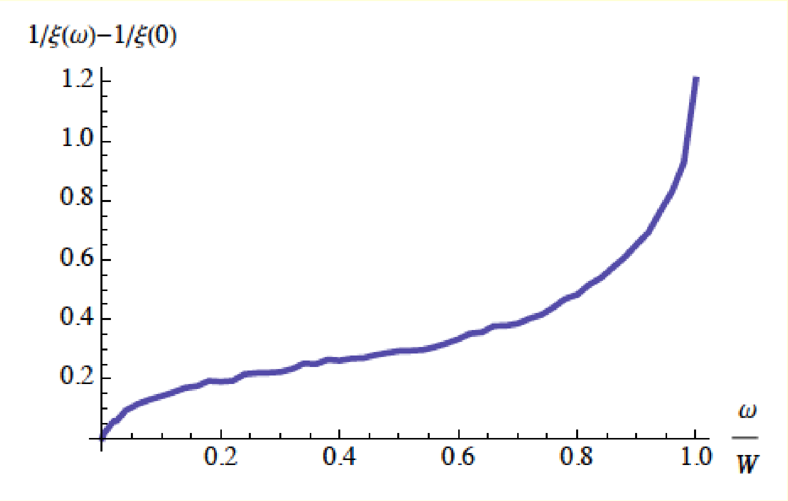
(like Abou-Chacra-Anderson-Thouless (1973) for fermions)

Higher dimensions?

Interference terms in finite dimensions give opposite trend!

$$\frac{G^R_{i,0}(\omega)}{G^R_{0,0}(\omega)} = \sum_{\mathcal{P}=\{j_0=0,\dots,j_\ell=i\}} \prod_{p=1}^{\ell} t_{j_{p-1},j_p} \frac{\mathrm{sgn}(\varepsilon_{j_p})}{\varepsilon_{j_p}-\omega}$$



Delocalization **strongest** at lowest energies: ξ(0) > ξ(ω)!

→ Bosons delocalize first at zero energy! No closing mobility edge!

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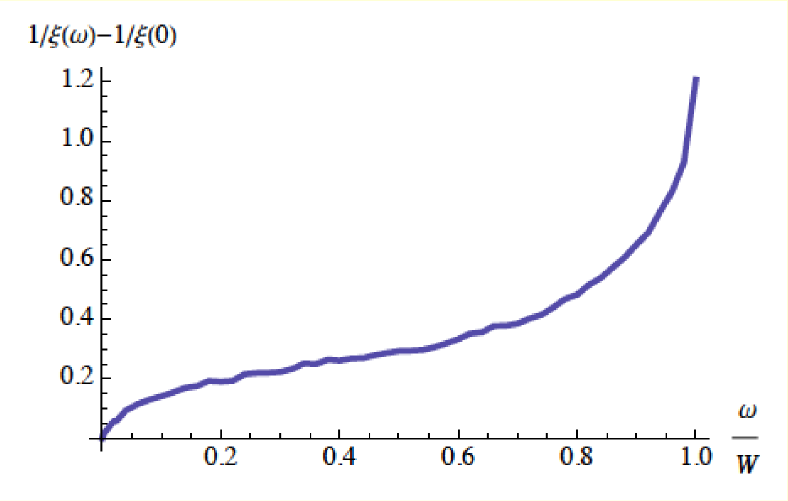

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Similar as related exact results in 1d!

Random transverse field Ising chain:

Map to free fermions [class BDI]: most delocalized at ω = 0!

Is there never a mobility edge
in bose insulators?
Are “bose glasses” always “many body localized”?

Not necessarily!
Trivial case: DOS increases with energy above chemical potential
Less trivial: interaction-frustrated (glassy) bosons

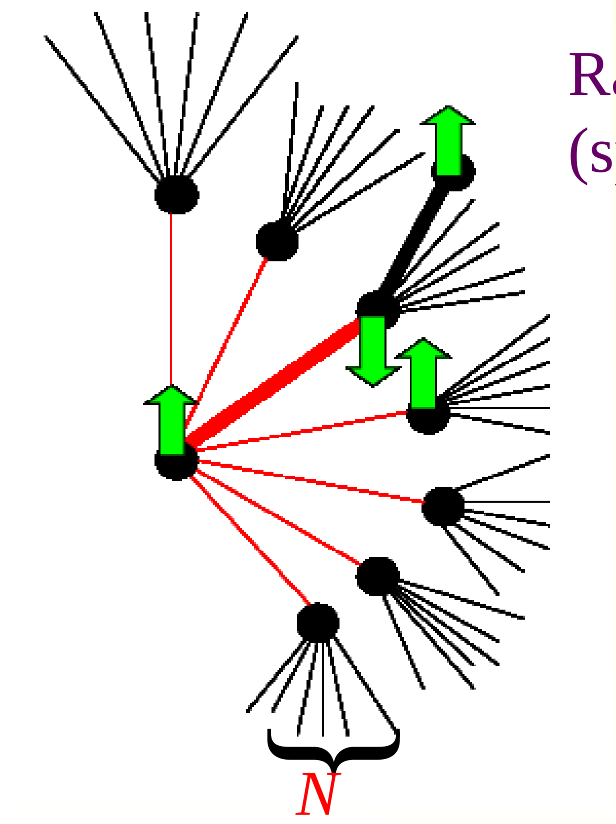
An exactly solvable model of a glassy SIT

X. Yu, MM in prep

$$H = -\sum_{\langle i,j\rangle} \frac{J_{ij}}{\sqrt{N}} n_i n_j - \frac{t}{N} \sum_{\langle i,j\rangle} (b_j^\dagger b_i + b_i^\dagger b_j)$$

Random n.n.-interactions (spin glass like)

Unfrustrated hopping of hard core bosons



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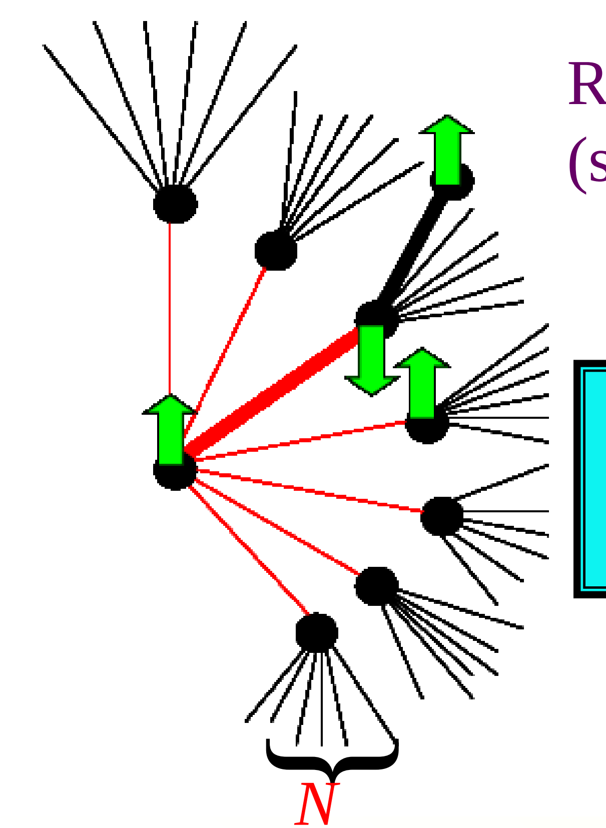

Exactly solvable model on large N Bethe lattice, $N \rightarrow \infty$!

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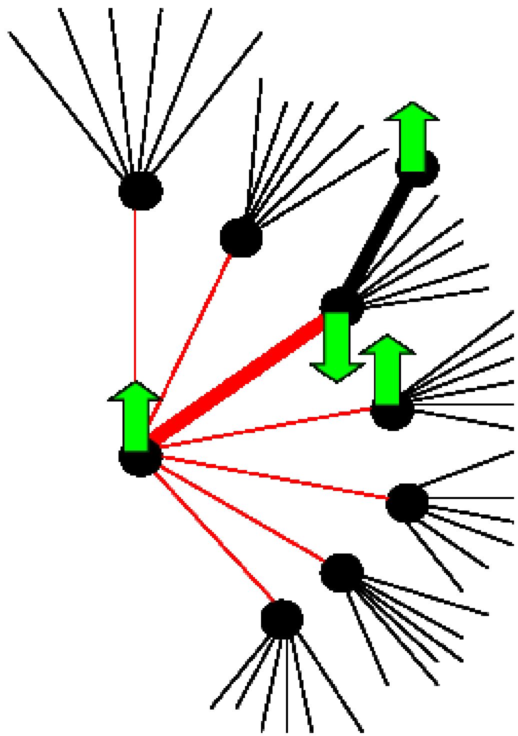

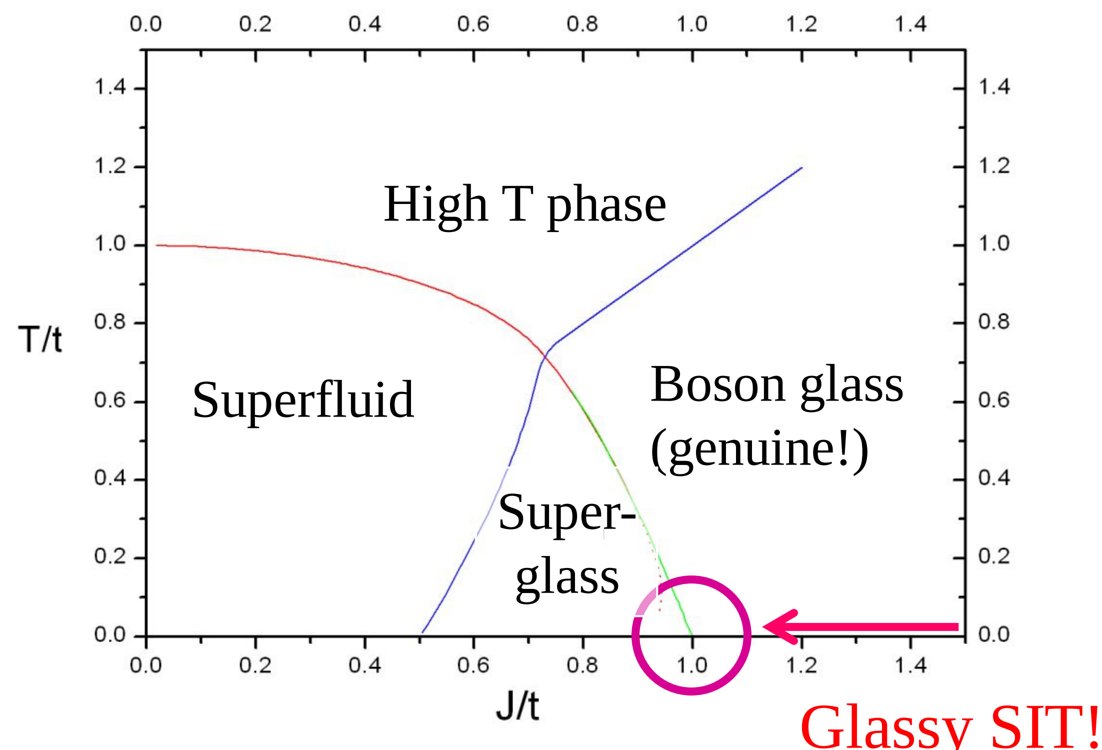

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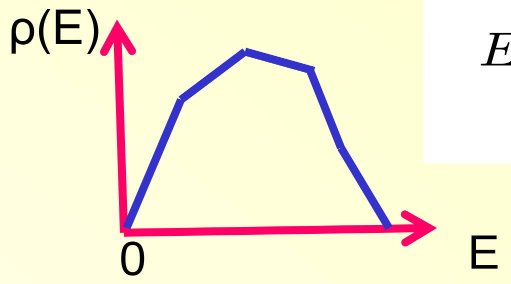
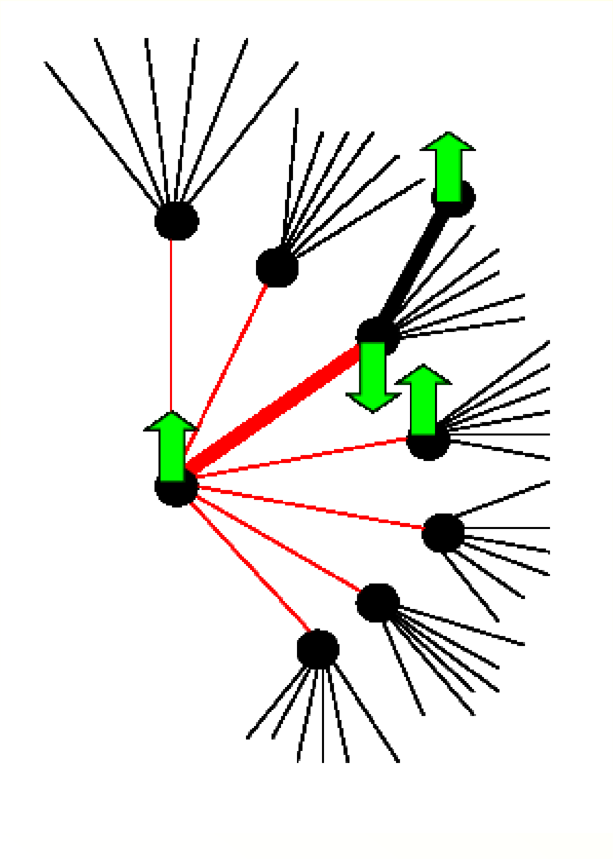


$$E_i = -\sum_{j\in\partial i} \frac{J_{ij}}{\sqrt{N}} n_j$$

Boson glass: "Coulomb gap" (linear) in the local DOS!



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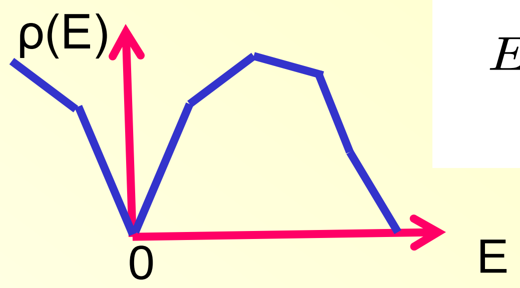

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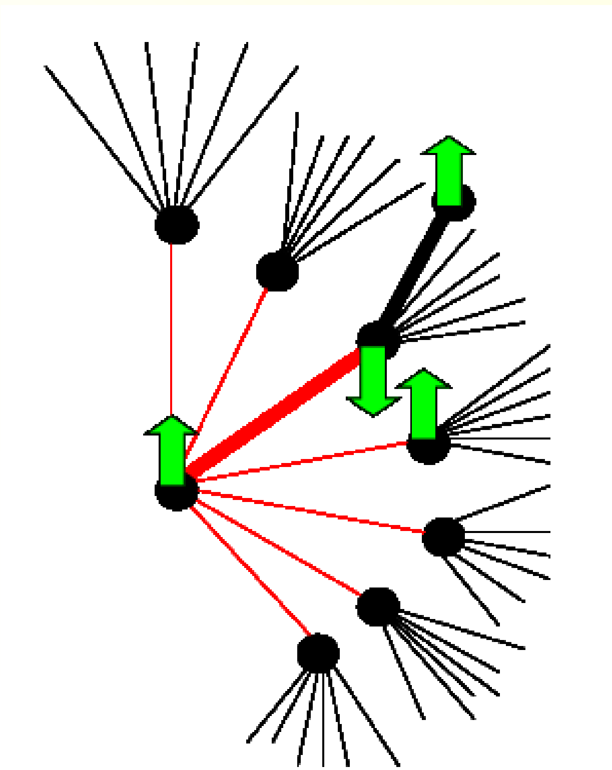
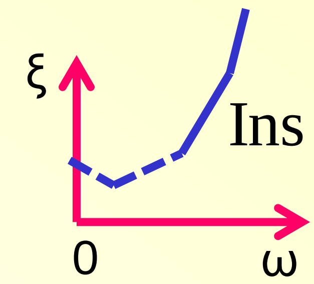


→ Suppresses superfluidity (at ω = 0)

→ Higher ω modes remain delocalized in insulator!



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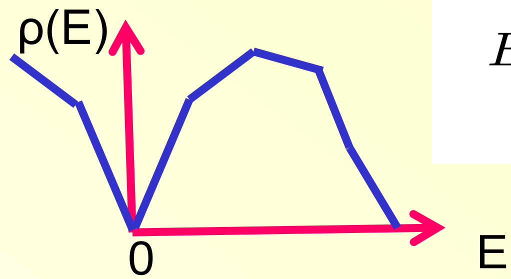

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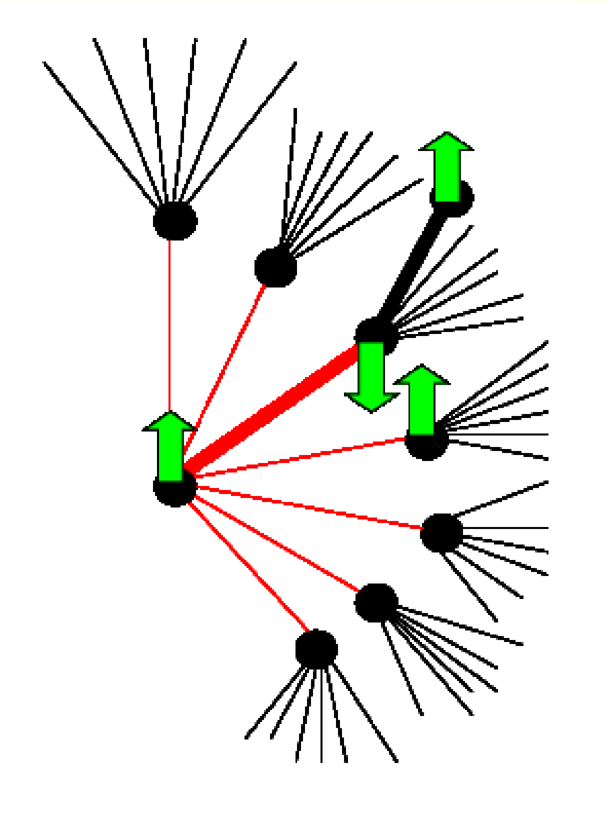

→ Suppresses superfluidity (at ω = 0)

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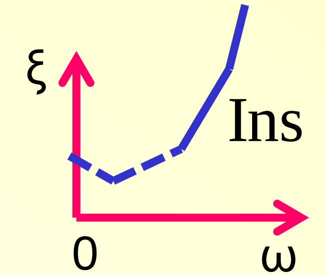
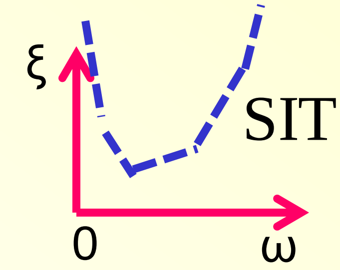
→ **Finite, but non-critical mobility edge at ω ~ 1/log(N)!**





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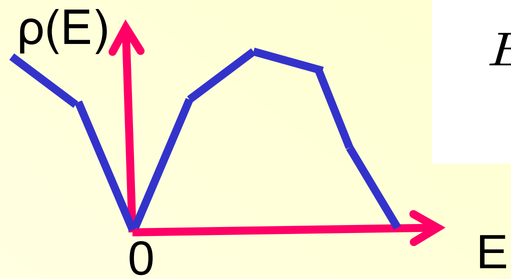

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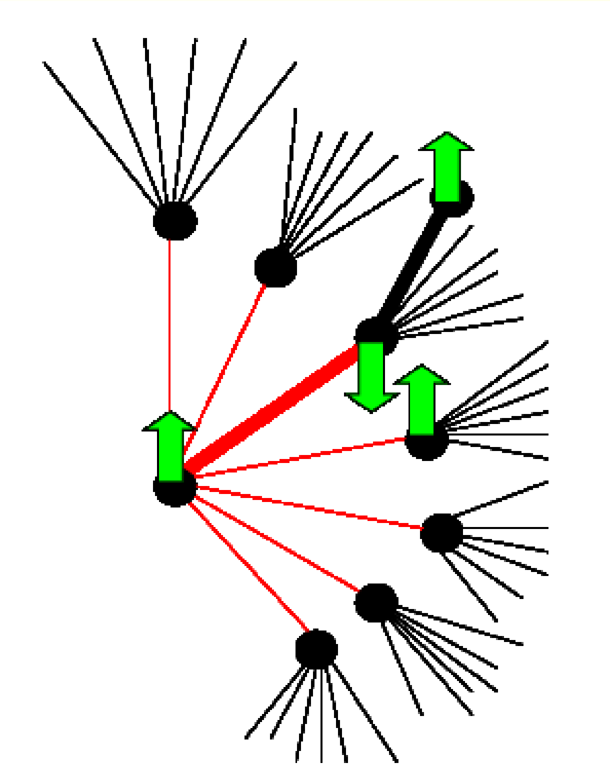

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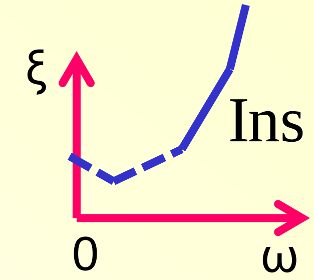
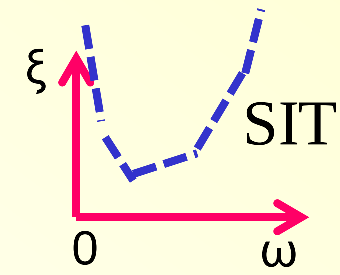


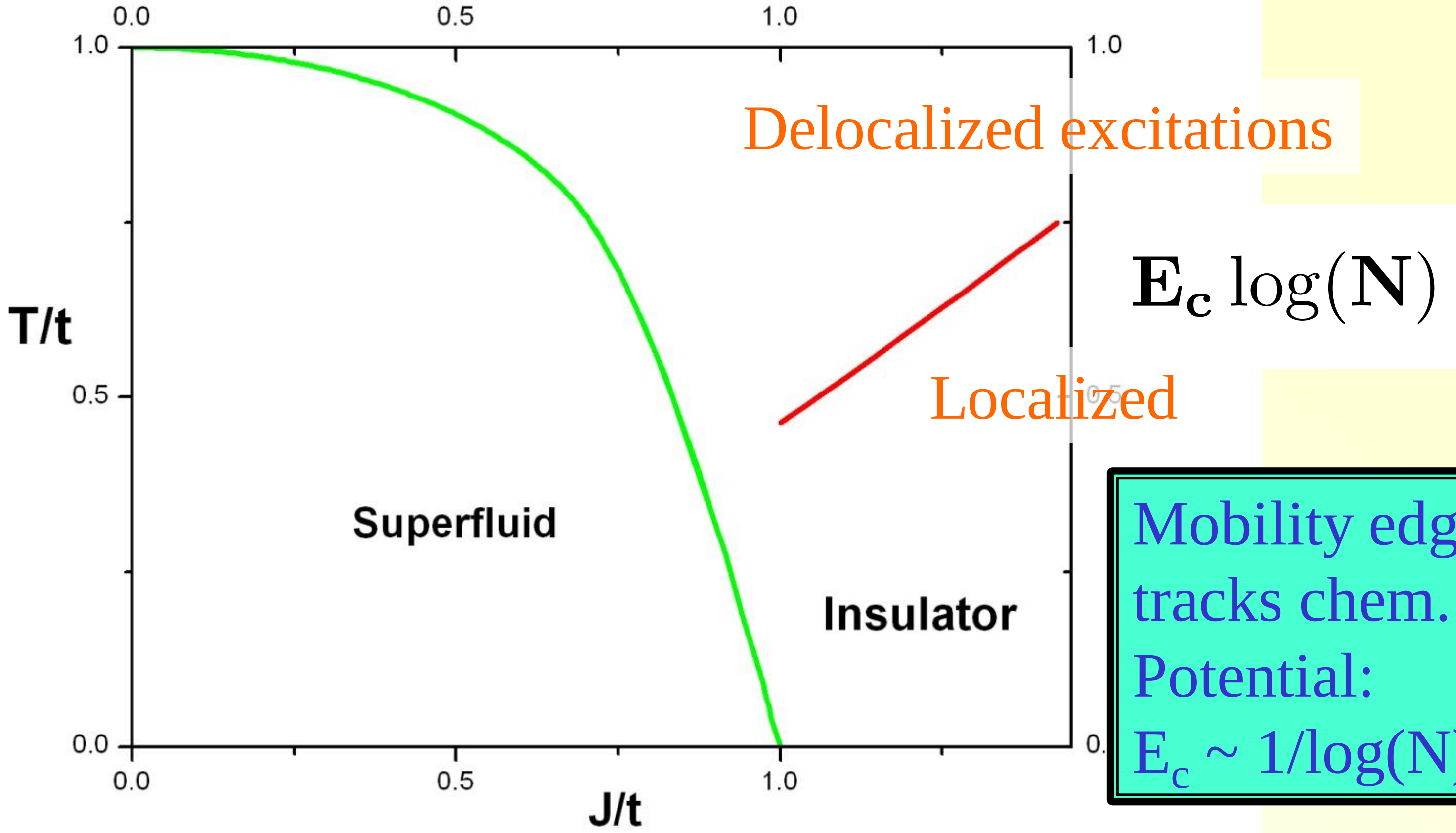
→ Finite, but non-critical mobility edge at ω ~ 1/log(N)!



→ Coulomb gap: mobility edge tracks the chem. potential!

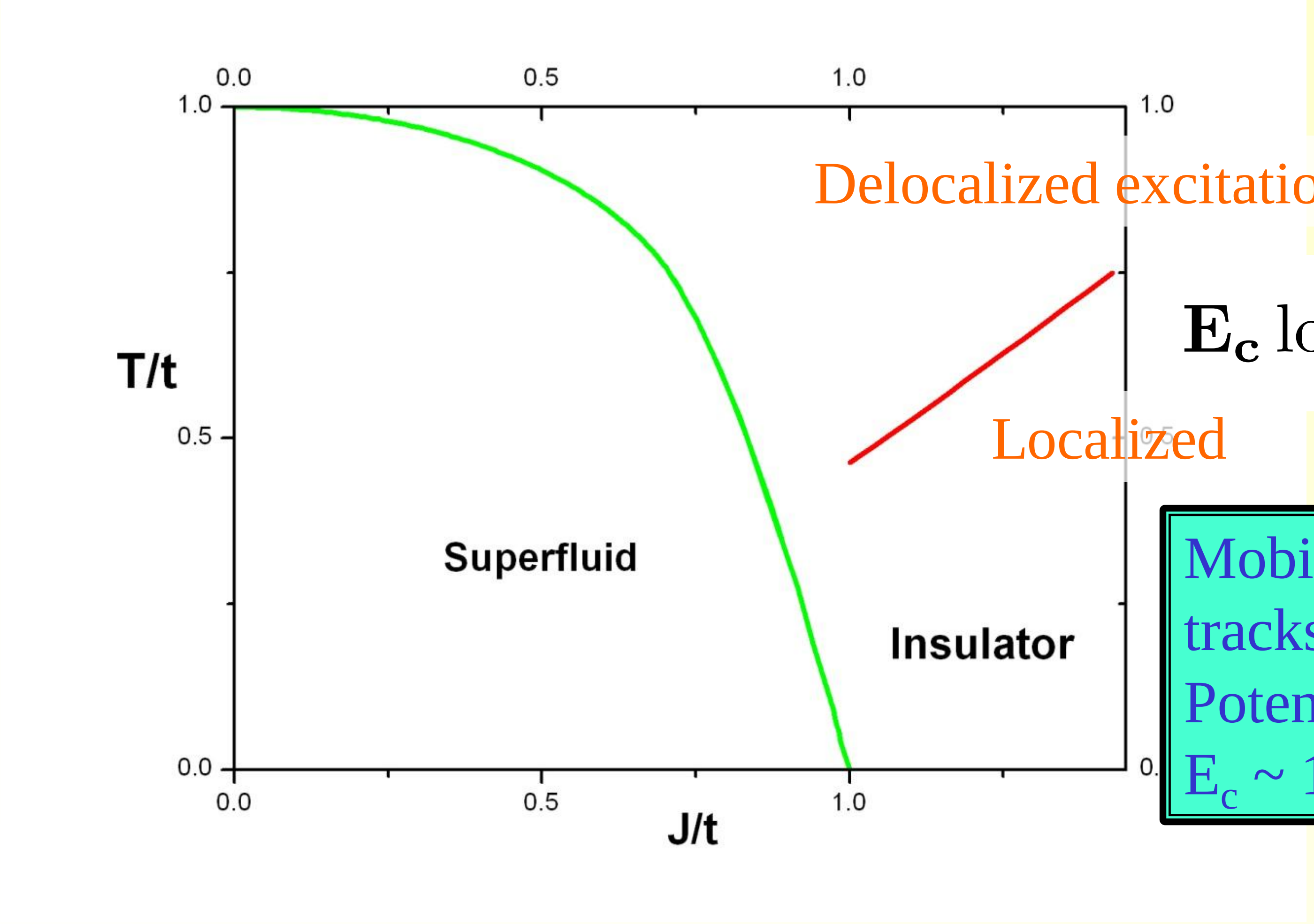
(cf. exp by Yazdani et al)

Mobility edge at glassy SIT



Mobility edge tracks chem. Potential: $E_c \sim 1/\log(N)$!

Mobility edge at glassy SIT



$\mathbf{E_c} \log(\mathbf{N})$

Mobility edge tracks chem. Potential: $E_c \sim 1/\log(N)$!

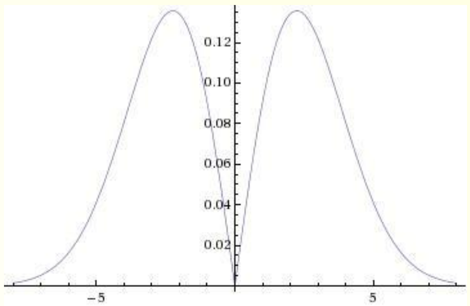
Solvable model of a glassy SI transition:

Superfluid emerges without closing mobility gap E_c!

Actual calculations

Quantum transport at the glassy SI transition

P(y)



y

$$H = -\frac{t}{N}\sum_{\langle i,j\rangle}\left(\sigma_i^+\sigma_j^- + \text{h.c.}\right) - \sum_{\langle i,j\rangle}\sigma_i^z J_{ij}\sigma_j^z \quad \text{(J – model)}$$

↔ non-glassy model:

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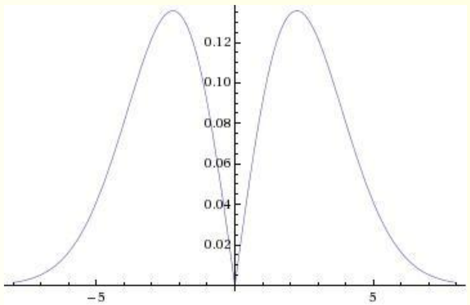
Localization? Level width with weak coupling to bath?

$$G_{0,0}(t) \equiv -i\Theta(t)_b\langle \text{GS}|\sigma_0^+(t)\sigma_0^-|\text{GS}\rangle_b$$

Quantum transport at the glassy SI transition

P(y)



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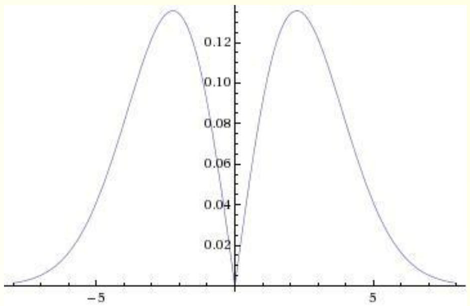

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Quantum transport at the glassy SI transition

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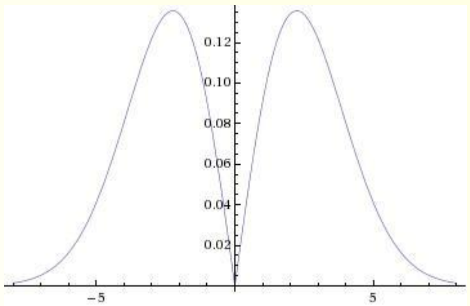

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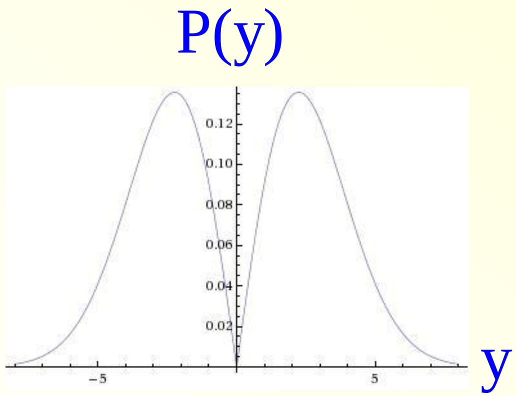
Superfluidity (response to boundary field):

Needed: $$\chi_\omega = \sum_{l\in\partial M}\int_0^\infty dt e^{i\omega t}\langle GS|\sigma_0^x(t)\sigma_l^x(0)|GS\rangle_{(\omega\to 0)}$$

Quantum transport at the glassy SI transition



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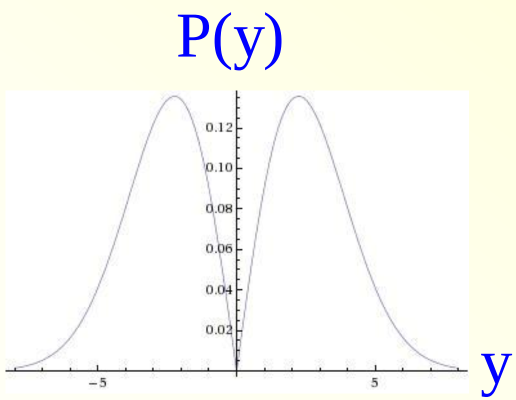
Superfluidity (response to boundary field):

Needed: $$\chi_\omega = \prod_{p=1}^{l}\frac{t}{N}\frac{\text{sign}(\epsilon_p)}{\epsilon_p - \omega}$$

$$\left(\leftrightarrow \prod_{p=1}^{l}\frac{t}{N}\frac{1}{|\epsilon_p| - \omega}\right.$$ *Ioffe-Mézard*)

Quantum transport at the glassy SI transition



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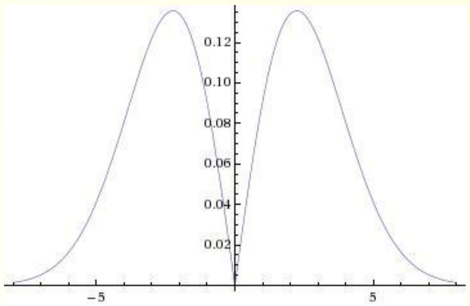
1. Superfluid transition? → $\left\langle \sigma_i^+ \right\rangle \neq 0$

(J): $t_c = J$ (ε): $t_c = c/\log(N)$

Quantum transport at the glassy SI transition

P(y)



y

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1. Superfluid transition? → $\langle \sigma_i^+ \rangle \neq 0$

(J): $t_c = J$ (ε): $t_c = c/\log(N)$

Same value as for free fermions (in ”upper limit” approximation: neglecting self-energies)

(Abou Chacra et al)

Quantum transport at the glassy SI transition

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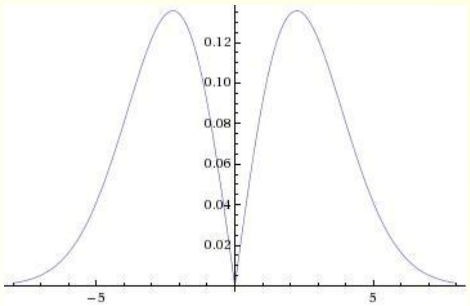

y

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1. Superfluid transition? → $\langle \sigma_i^+ \rangle \neq 0$

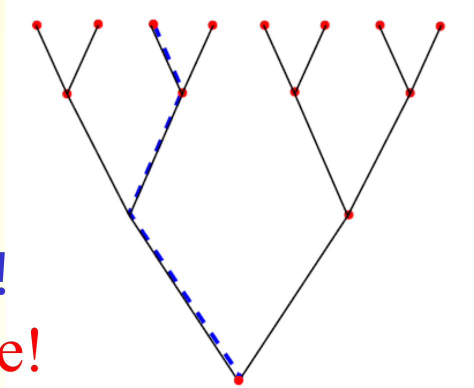
Condensate: propagation of transverse fields!

Large N: like directed polymer! (Sol: Derrida+Spohn)

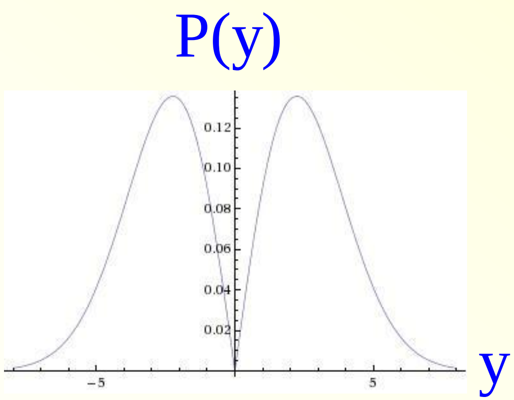
(ε): $$\chi_{bdy} = \sum_{\text{paths } P}\prod_{i \in P}\frac{t}{N\epsilon_i} = Z_{pol}^{eff}$$

Rare paths dominate!

↔ fractal condensate!



Quantum transport at the glassy SI transition



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Condensate: propagation of transverse fields!
Large N: like directed polymer problem!

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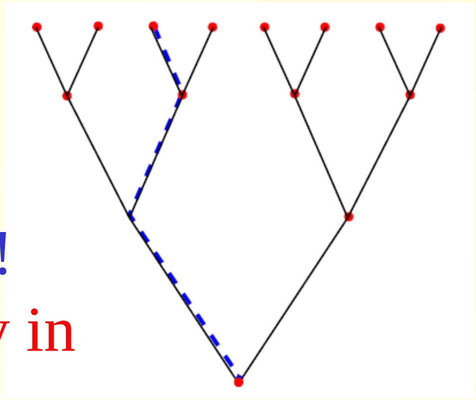
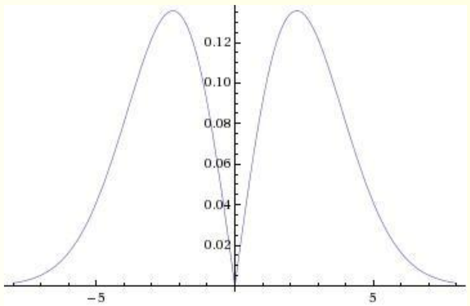
NOT only rare paths!
↔ reduced fractality in finite dimensions?



Quantum transport at the glassy SI transition



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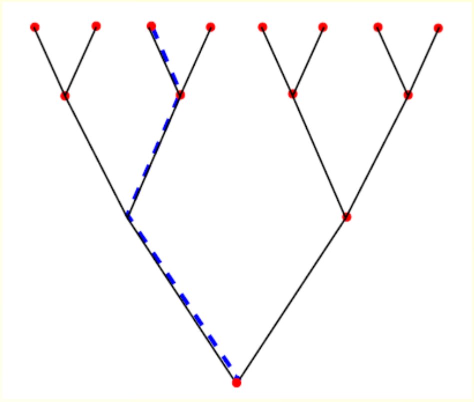
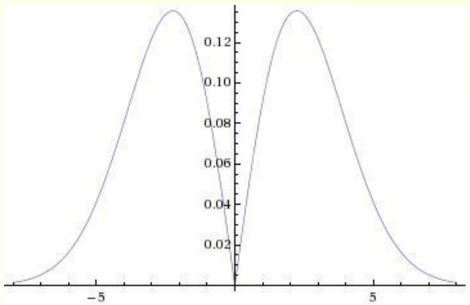
2. Localization of spin flip excitations in the insulator?



Quantum transport at the glassy SI transition

P(y)



y

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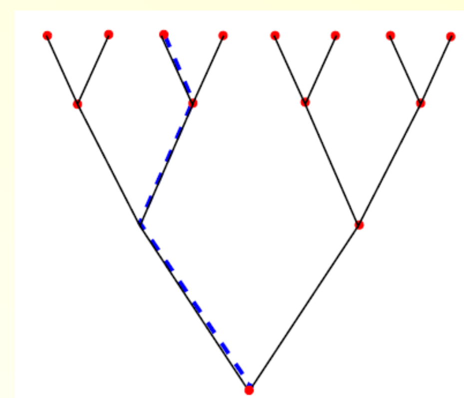
2. Localization of spin flip excitations in the insulator?

(ε):

$$\chi^{(2)}(\omega) = \sum_{\text{paths } P}\prod_{i\in P}\left(\frac{t\,\text{sgn}(\epsilon_i)}{N(\epsilon_i - \omega)}\right)^2$$

$$\chi^{(2)}(\omega_{mob}) \overset{!}{=} 1$$

Mobility edge



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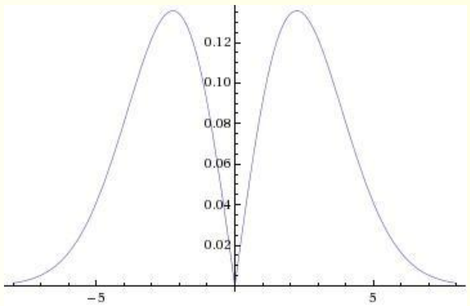

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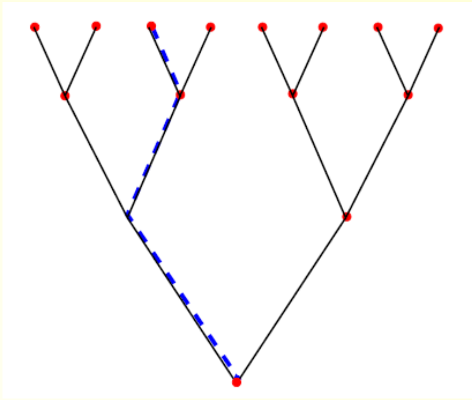
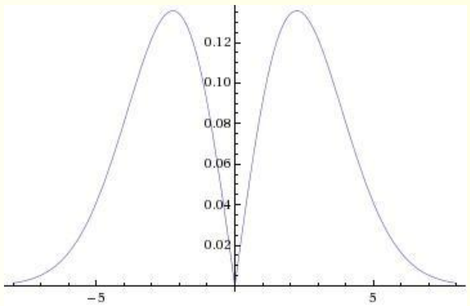


Rare paths dominate.

- **NO** mobility edge in the insulator!
- “Many body localized bosons”!(?)

Quantum transport at the glassy SI transition

P(y)



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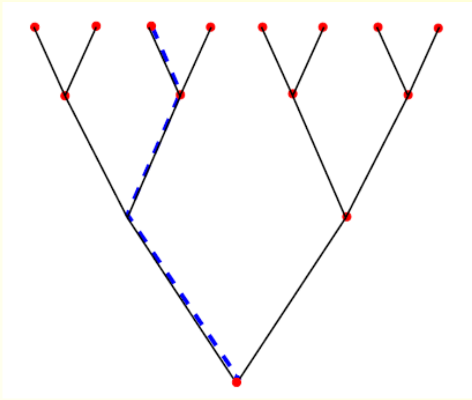

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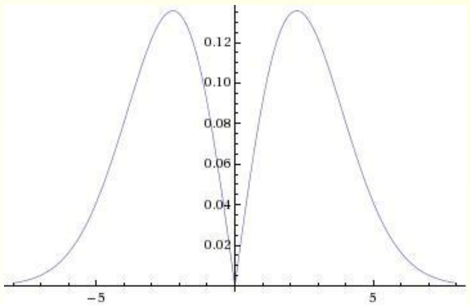

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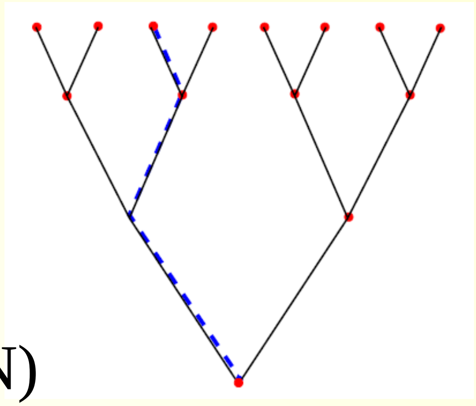


Rare paths dominate. - Due to "Coulomb" gap:

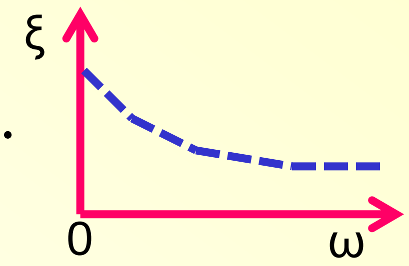
THERE IS a mobility edge! $\omega_{mob} = 0.45/\log(N)$

Conclusions

- Locator expansion for interacting systems in random fields: Interference at low-energy always constructive.

- ξ of bosons shrinks under a B field
  → strong positive magnetoresistance, opposite to fermions.

- ξ(ω) decreases with energy (where controllable).



- Not so if Coulomb gap counteracts this:
  → Finite mobility edge at low E in Bose glass (d>2 but d = 2??)

- But even so: Superfluid emerges without the closing of a mobility gap in general.

- Work in progress: finite T, higher order expansion, resummation