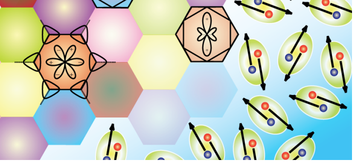

Dynamic Kosterlitz-Thouless transition in 2D Bose mixtures

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KITP, September 2012

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LEXI
LANDESEXZELLENZINITIATIVE HAMBURG

- Dynamic Kosterlitz-Thouless transition

  with K. Günter, J. Dalibard, A. Polkovnikov
  - Renormalization group dynamics, vortex unbinding, reverse Kibble-Zurek
  - Experimental proposal for mixtures of hyperfine states

cond-mat/1112.1204

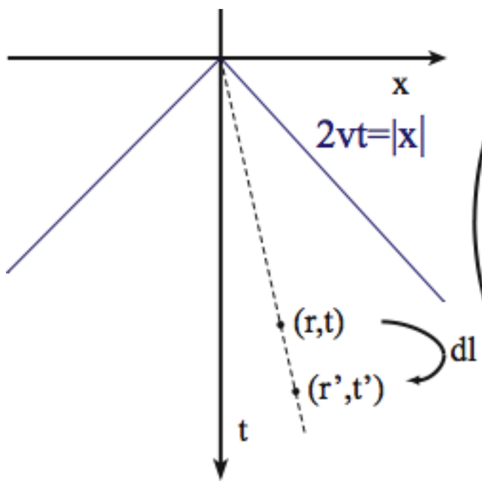
LM and A. Polkovnikov, Phys. Rev. A 81, 033605 (2010)

LM and A. Polkovnikov, Phys. Rev. A 80, 041601(R) (2009)

Renormalization group framework for critical dynamics

LM and A. Polkovnikov, Phys. Rev. A 81, 033605 (2010)

We rescale real-time and real space, and correct for it up to 1-loop order



$$\begin{cases} \dot{\mathbf{p}}(\mathbf{r},\mathbf{t}) = -\partial_{\mathbf{q}}\mathbf{H}(\mathbf{r},\mathbf{t},\lambda,\mathbf{g}) \\ \dot{\theta}(\mathbf{r},\mathbf{t}) = \partial_{\mathbf{p}}\mathbf{H}(\mathbf{r},\mathbf{t},\lambda,\mathbf{g}) \end{cases}$$

$$dl = dt/t = dr/r$$

$$\begin{cases} \dot{\mathbf{p}}(\mathbf{r}',\mathbf{t}') = -\partial_{\mathbf{q}}\mathbf{H}(\mathbf{r}',\mathbf{t}',\lambda',\mathbf{g}') \\ \dot{\theta}(\mathbf{r}',\mathbf{t}') = \partial_{\mathbf{p}}\mathbf{H}(\mathbf{r}',\mathbf{t}',\lambda',\mathbf{g}') \end{cases}$$

$$\mathrm{I}: \quad \frac{d\tau}{dl} = \alpha g^2$$

$$\mathrm{II}: \quad \frac{dg}{dl} = (2 - 2/\tau) g$$

$t = t_0 e^l$

This generates RG flow equations in real-time. To predict the dynamic behavior, we fix non-universal constants and time scales, and integrate the flow equations in time.

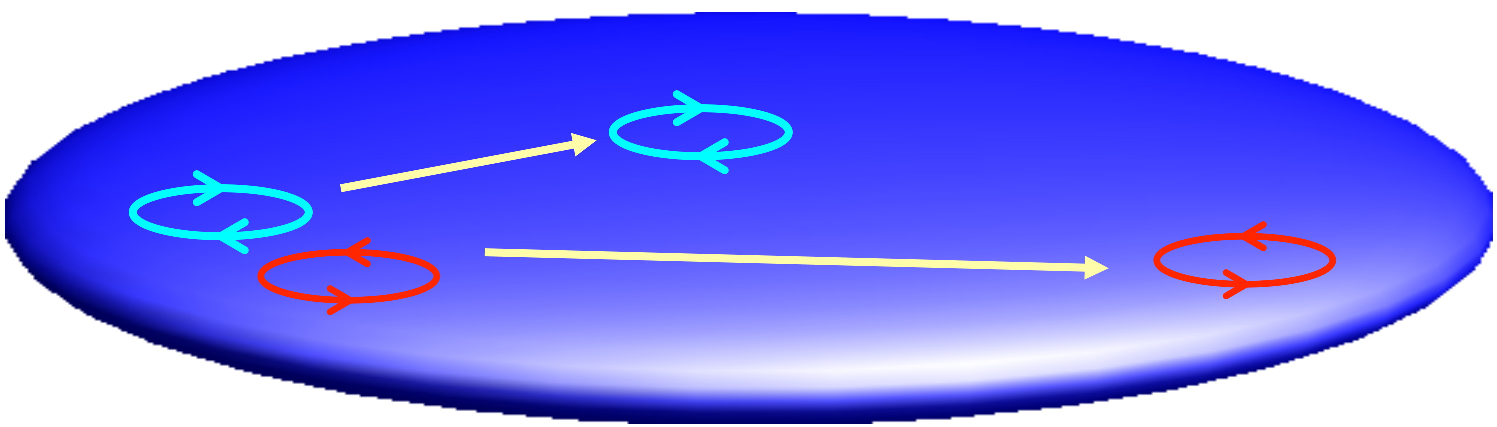
Bose gas in 2D: Kosterlitz-Thouless transition

Quasi-Order $G(x) = \langle \psi^{+}(x)\psi(0) \rangle \sim |x|^{-\frac{\tau}{4}}$ $\tau^{-1} \approx \frac{\pi\rho}{2mT} + C$

Prokof'ev, Ruebenacker, Svistunov, '00, '01, '02

Kosterlitz-Thouless transition driven by vortex unbinding

Disordered phase $G(x) = \langle \psi^{+}(x)\psi(0) \rangle \sim \exp(-|x| / x_0)$

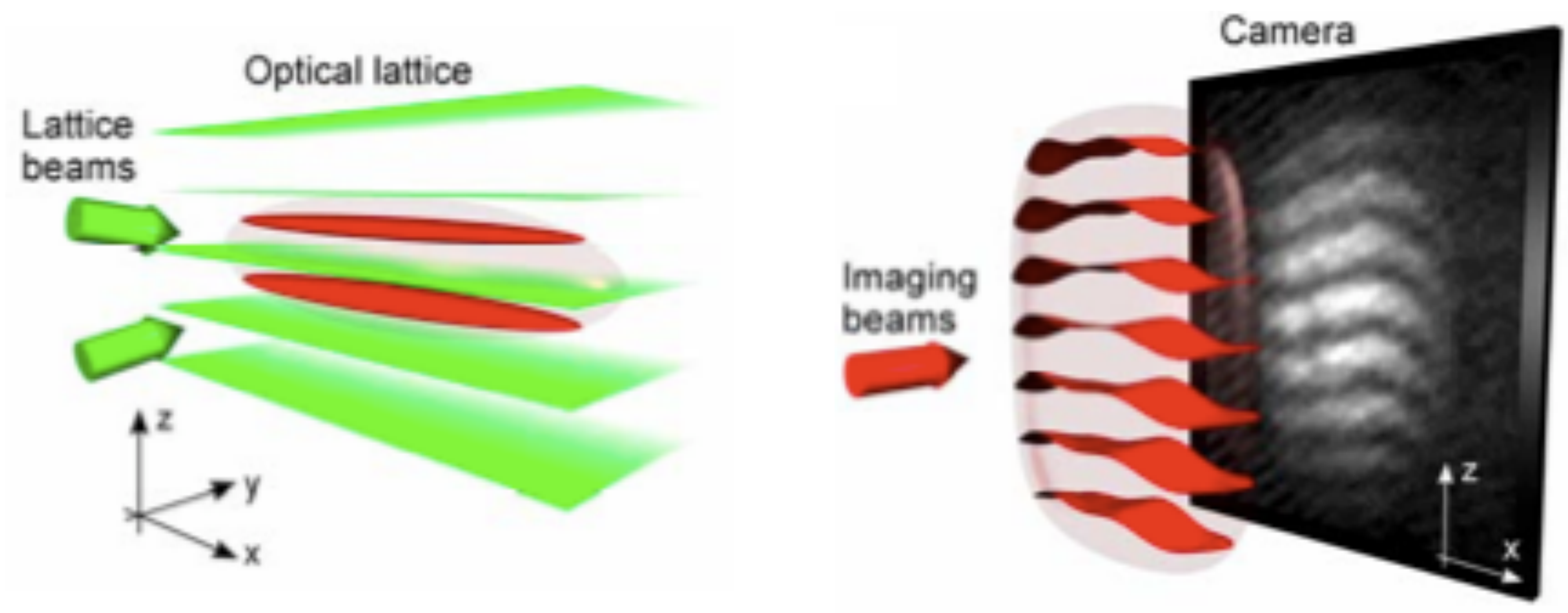


Berezinskii 1972, Kosterlitz and Thouless 1973

Interfering 2D Bose gases

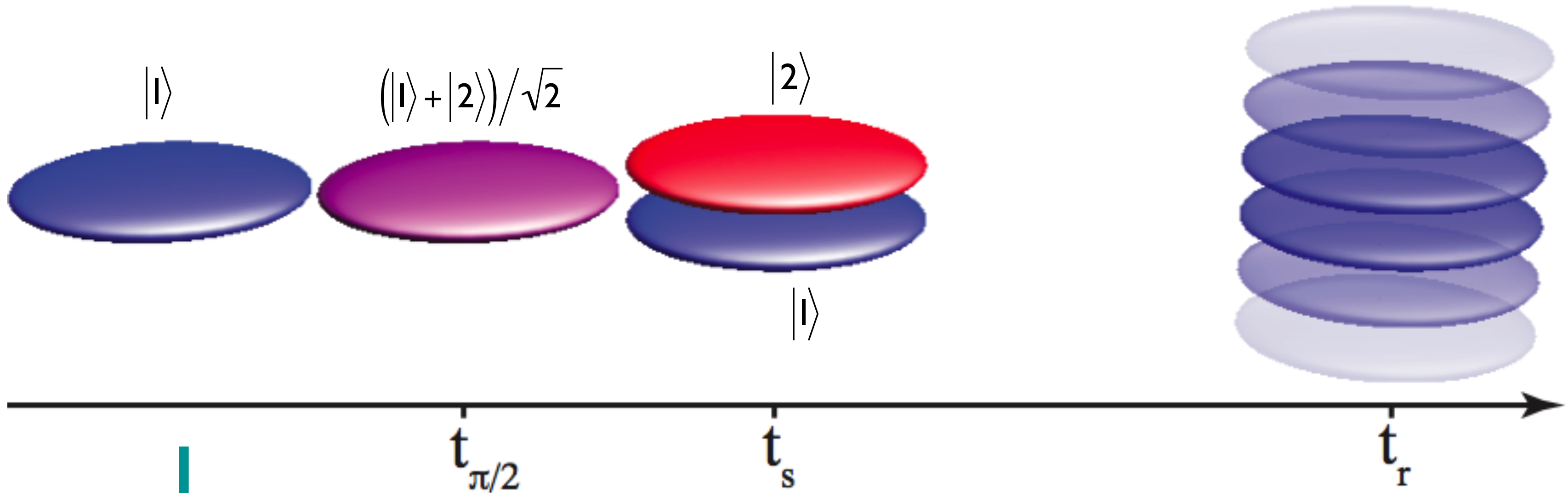
- Realization of quasi-condensates and KT transition
- algebraic → exponential scaling. Vortices

Dalibard group, Phillips/Helmerson group, Chin group, Jin group, …



Review: Z. Hadzibabic, J. Dalibard, Rivisto del Nuovo Cimento, 34, 389 (2011)

Preparation and measurement sequence



Prepare a 2D superfluid in internal state $|1\rangle$

Apply π/2 pulse to create a superfluid in $(|1\rangle + |2\rangle)/\sqrt{2}$

Apply field gradient, turn off the inter-species interaction

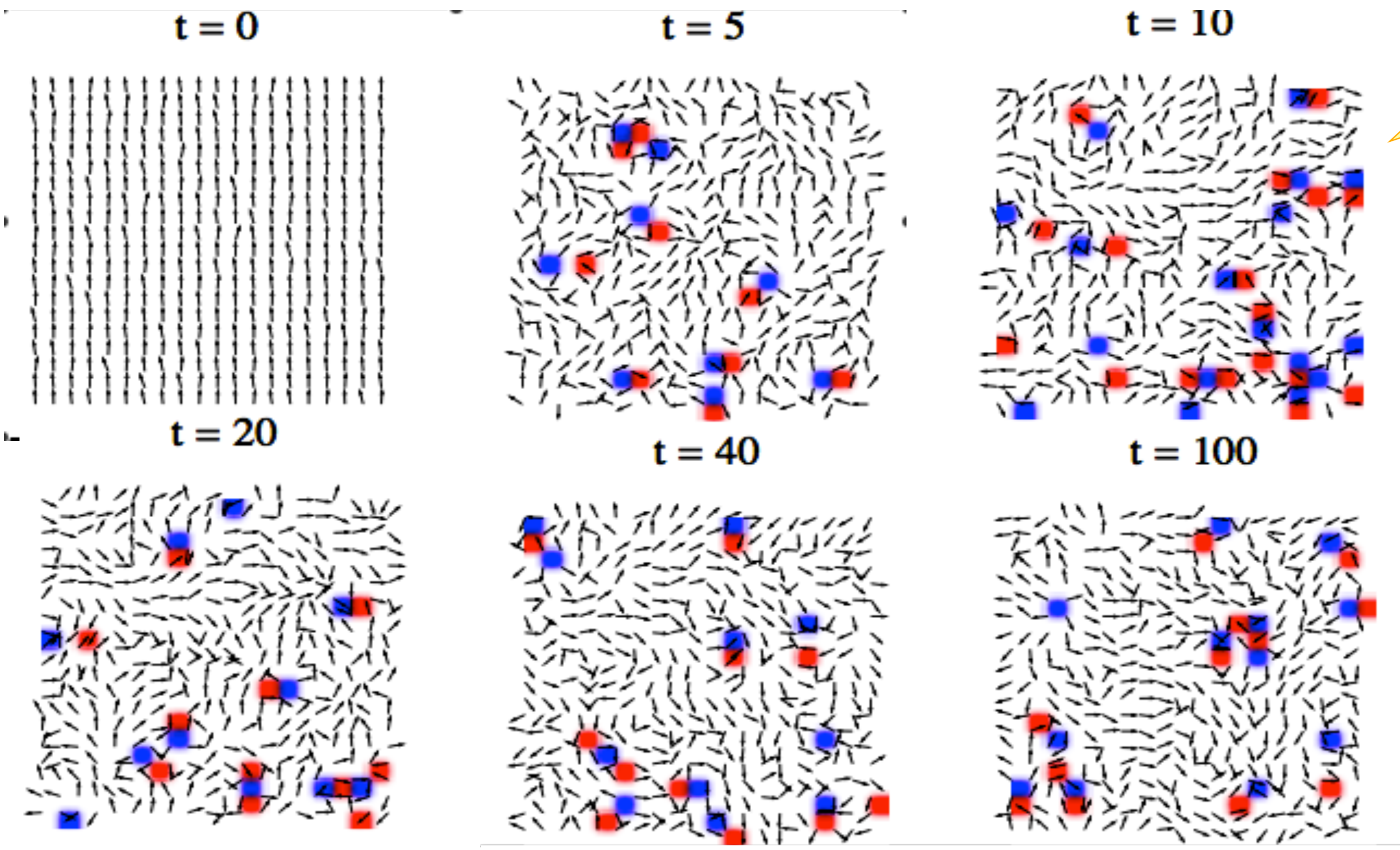
Measure interference to detect critical dynamics

1D experiment: Schmiedmayer group

Numerical Approach: Truncated Wigner approximation

Sample over many Gross-Pitaevskii solutions according to the Wigner distribution of the initial state. Includes quantum and thermal fluctuations.

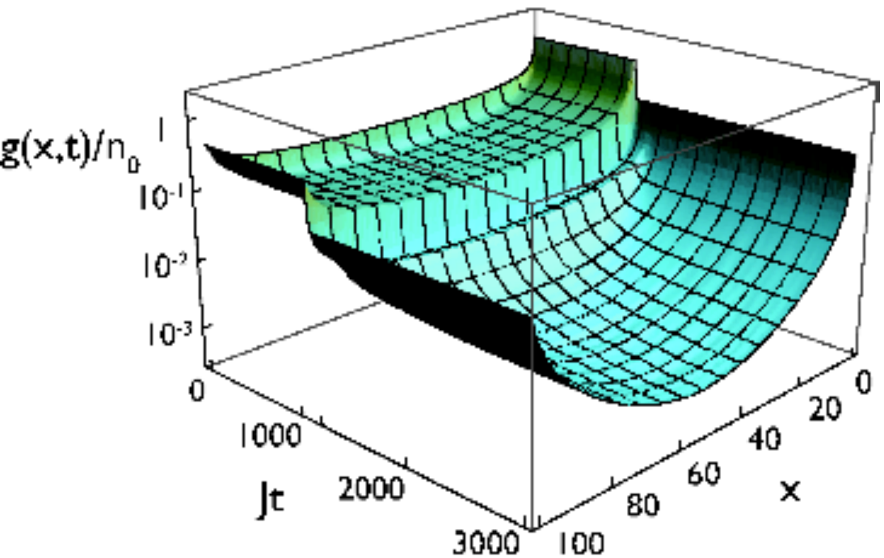
● Vortex ● Anti-vortex

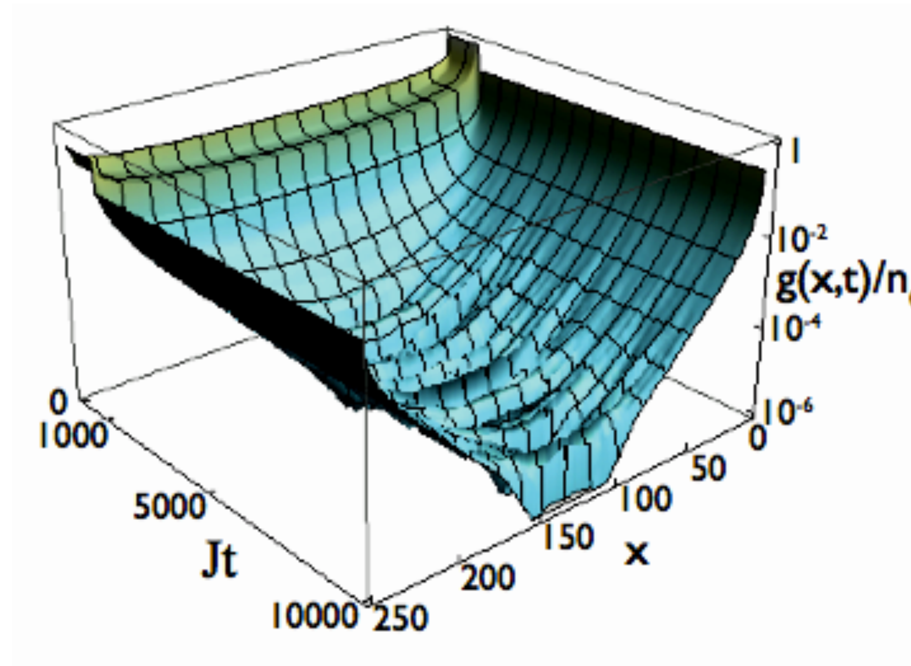


20*20 lattice

Phys. Rev. A 80, 041601(R) (2009); 81, 033605 (2010)
A. Polkovnikov, Annals of Phys. 325, 1790 (2010)

Dynamics of G(x,t) on different time scales





On intermediate scales, a metastable supercritical state with algebraic scaling emerges.

On a very long time scale, the scaling changes from algebraic to exponential

→ dynamic KT transition!

$$\hbar/J = 0.3ms \quad \rho_0 = 50/\mu m^2 \quad l = 0.3\mu m \quad {}^{87}Rb$$

Quantifying the change of functional form

We fit g(x,t) with an algebraic and an exponential fitting function:

Algebraic: $\sim |x|^{-\tau/4}$

$$f_a(x) = C\left(|\sin(\pi x/L)|\,L/\pi\right)^{-\tau/4}$$

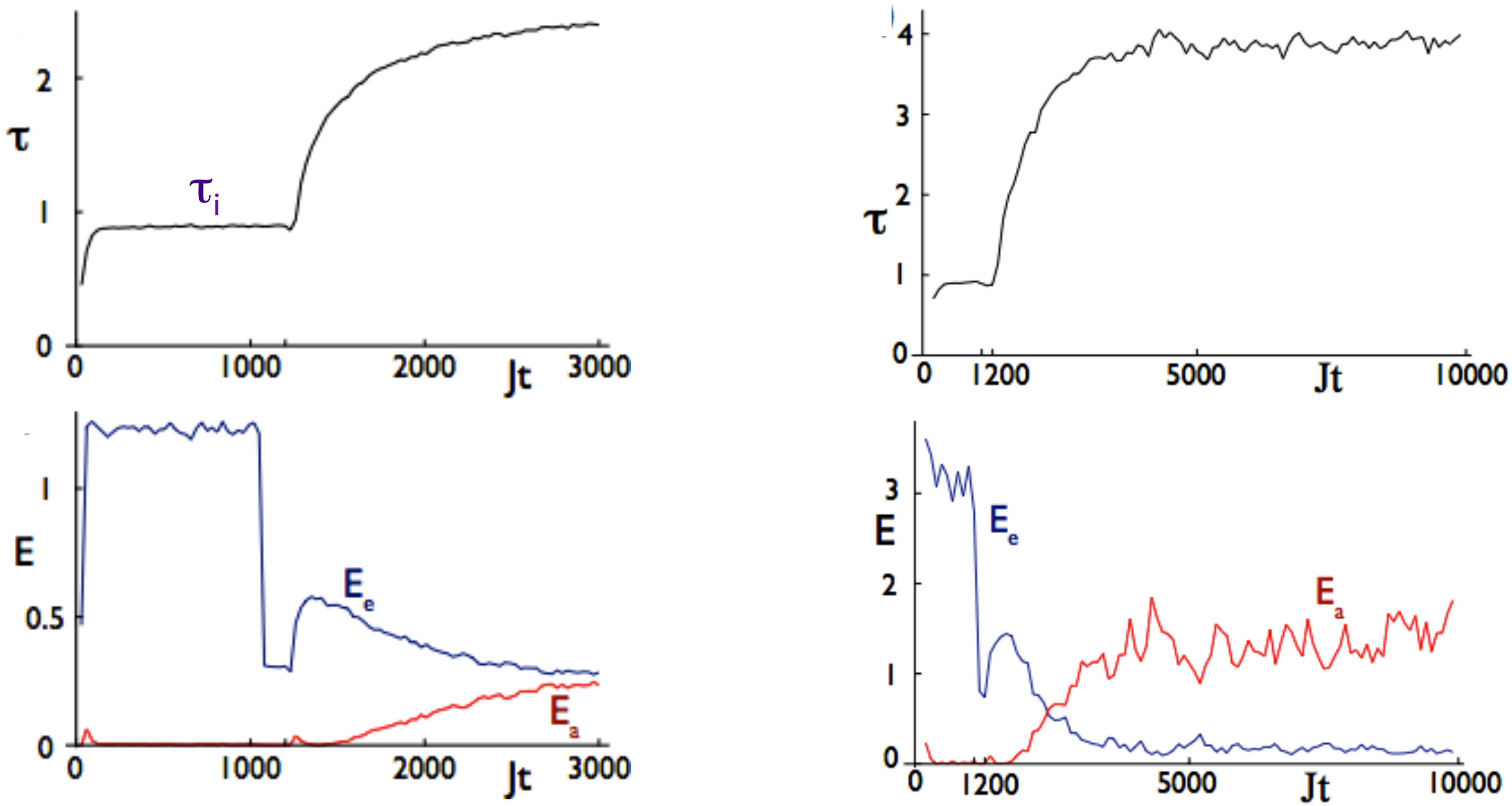
Exponential: $\sim \exp\left(-|x|/x_o\right)$

$$f_e(x) = C\exp\left(-|\sin(\pi x/L)|/x_o\right)$$

We define the two fitting errors:

$$E_{e,a}(t) = \sum\left(g(x,t) - f_{e,a}(x)\right)^2$$

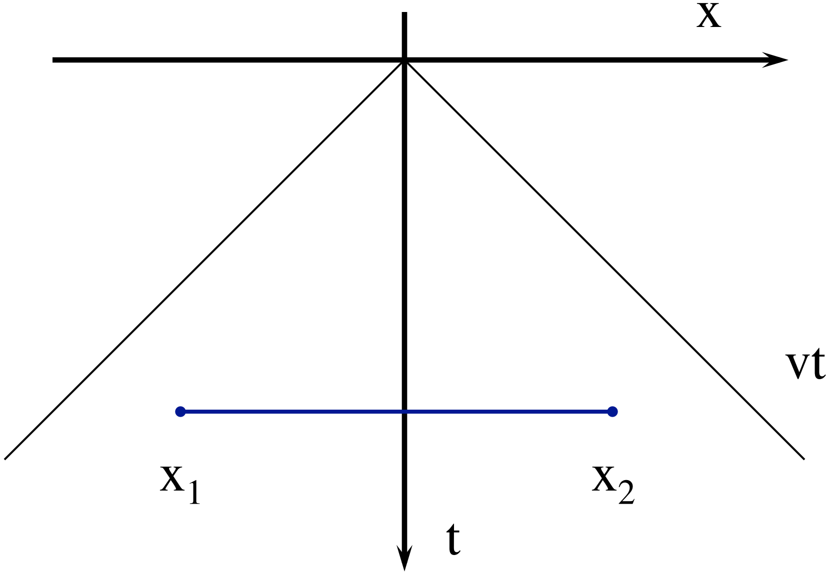
Fitting results

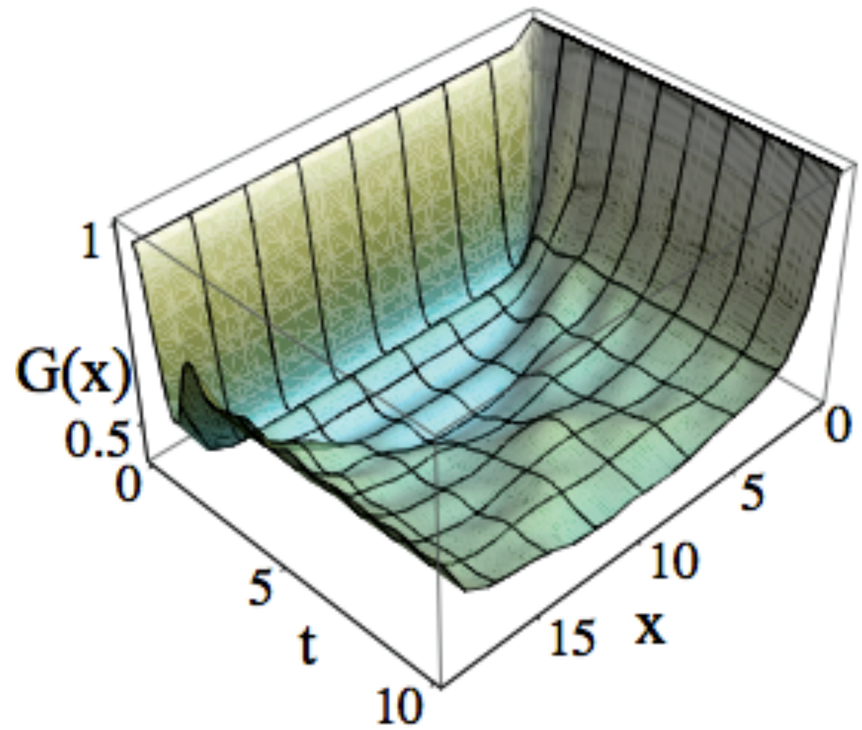


- The system relaxes to a steady state with exponent τ_i
- A supercritical superfluid state is observed
- The relaxation to the disordered groundstate is slowed down critically

Light cone dynamics

Dynamics separates G(x,t) into connected and disconnected part.



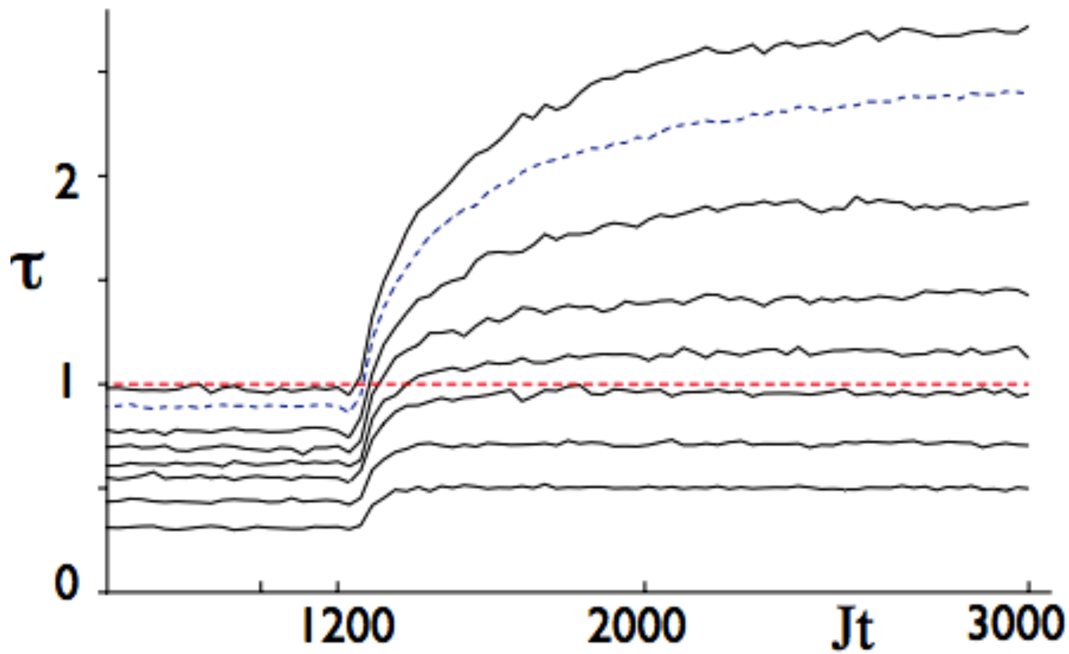


After the quench, G(x,t) is only piece-wise algebraic, thus the fitting error spikes up.

Phys. Rev. A 80, 041601(R) (2009); 81, 033605 (2010)

Mapping out the dynamical phase transition

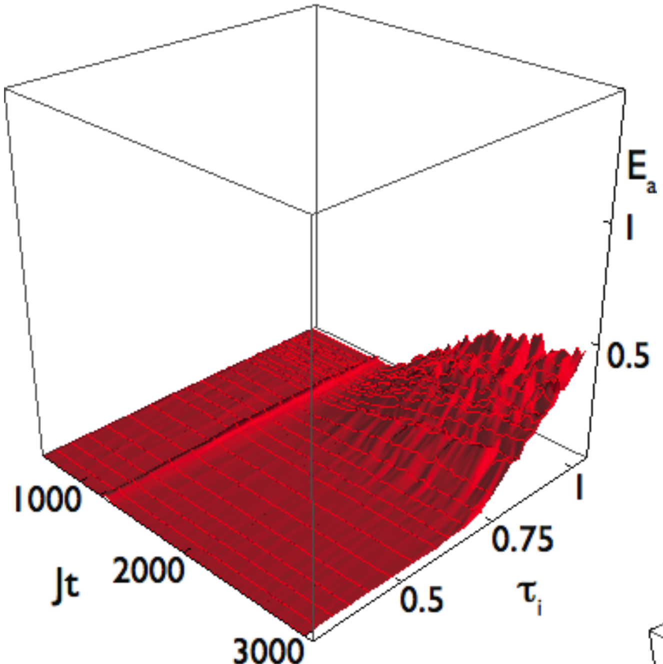
- $\tau(t)$, with τ_I as initial value



- For small τ_i, the system equilibrates at some final $\tau_f > \tau_i$

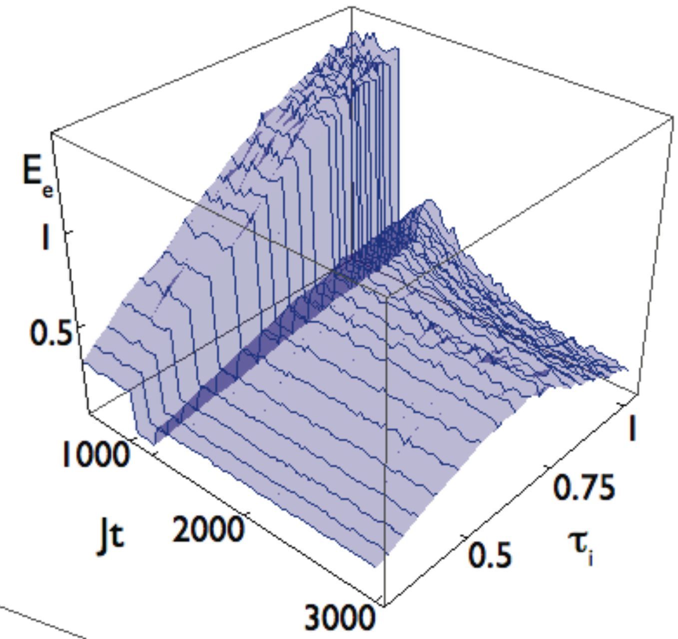
- For larger τ_i, the system continues to increase, and eventually relaxes to thermal equilibrium via vortex unbinding

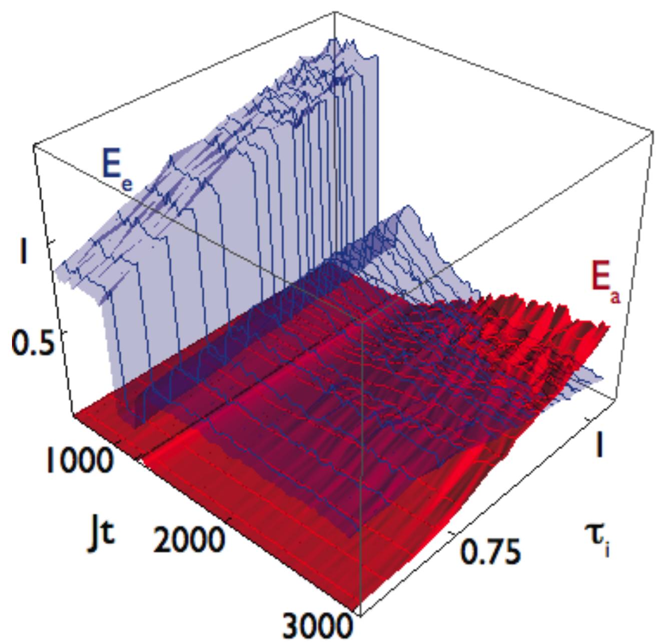
Errors indicate change of functional form



Algebraic error

Dynamic
KT transition!





Exponential
error

Scaling exponent of the metastable state

- The spectrum separates into total and relative density+phase fluctuations

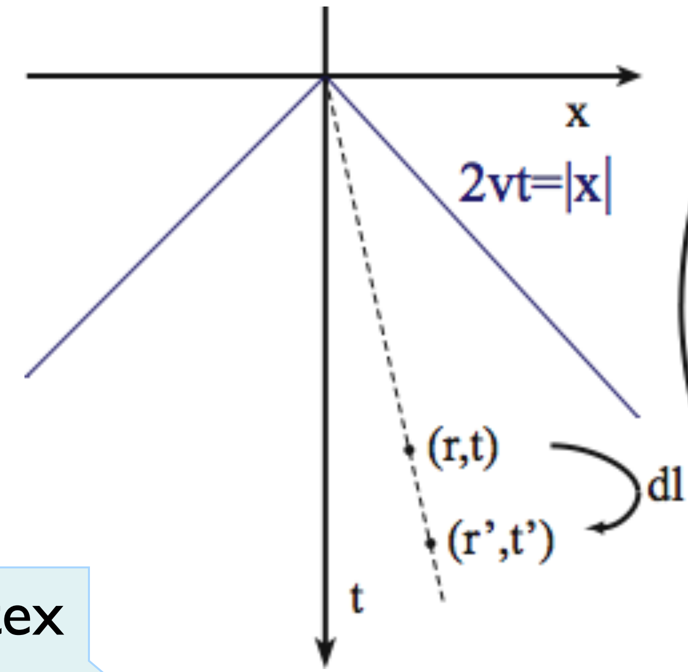
$$\psi_{1,2} \sim (n_0/2 + \delta n_{1,2})^{1/2} \exp(i\phi_{1,2})$$

- Symmetric and anti-symmetric sector equilibrate on a time scale shorter than the vortex unbinding scale.
- For T >> U $n_0/2$, the total energy scales as T^2
- So $T_f = T_i / \sqrt{2}$
- But $\tau^{-1} \approx \frac{\pi\rho}{2mT} + C$, so

$$\tau_f = \left(\frac{1}{\sqrt{2}\tau_i} + D \right)^{-1}$$

For 1D gases, sym and anti-sym sector stay out-of-equilibrium for a much longer time, see T. Kitagawa, et al., NJP 2011

Dynamics as a renormalization group process



$$\begin{cases} \dot{\mathbf{p}}(\mathbf{r},\mathbf{t}) = -\partial_{\mathbf{q}} \mathbf{H}(\mathbf{r},\mathbf{t},\lambda,\mathbf{g}) \\ \dot{\theta}(\mathbf{r},\mathbf{t}) = \partial_{\mathbf{p}} \mathbf{H}(\mathbf{r},\mathbf{t},\lambda,\mathbf{g}) \end{cases}$$

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g: vortex fugacity

$$\mathrm{I}: \quad \frac{d\tau}{dl} = \alpha g^2$$

$$\mathrm{II}: \quad \frac{dg}{dl} = (2 - 2/\tau) g$$

$t = t_0 e^l$

→ **Dynamic KT transition**

A few steps of the derivation

LM and A. Polkovnikov, Phys. Rev. A 81, 033605 (2010)

Sine-Gordon model in 2D, i.e. dual theory of the XY model (see e.g. X-G Wen)

$$\frac{1}{\mu}\partial_{tt}\theta = \lambda\triangle\theta + \frac{g}{a^2}\sin\theta$$

We split the field θ into high- and low-energy modes, $\theta = \theta^{<} + \theta^{>}$, i.e. into momentum modes between $\Lambda - d\Lambda$ and Λ for $\theta^{>}$ and below $\Lambda - d\Lambda$ for $\theta^{<}$.

We solve the EoMs for $\theta^{>}$ up to first order in g:

$$\theta^{>}(t) = \theta_0^{>}(t) + \theta_1^{>}(t)$$

We introduce this into the EoM of $\theta^{<}$

$$\frac{1}{\mu}\partial_{tt}\theta \approx \lambda\triangle\theta + \frac{g}{a^2}(\cos\theta^{<})\theta^{>} + \frac{g}{a^2}\sin\theta^{<}\Big(1 - \frac{(\theta^{>})^2}{2}\Big)$$

We now integrate out $\theta^>$ and get

$$g \to g\Big(1 - \frac{(\theta^>)^2}{2}\Big) = g\Big(1 - \frac{1}{4\pi\lambda}\Big) = g(1 - 2/\tau)$$

The $\theta_1^>$ correction in the second term gives rescaled λ and μ. So the low-energy harmonic oscillators are renormalized

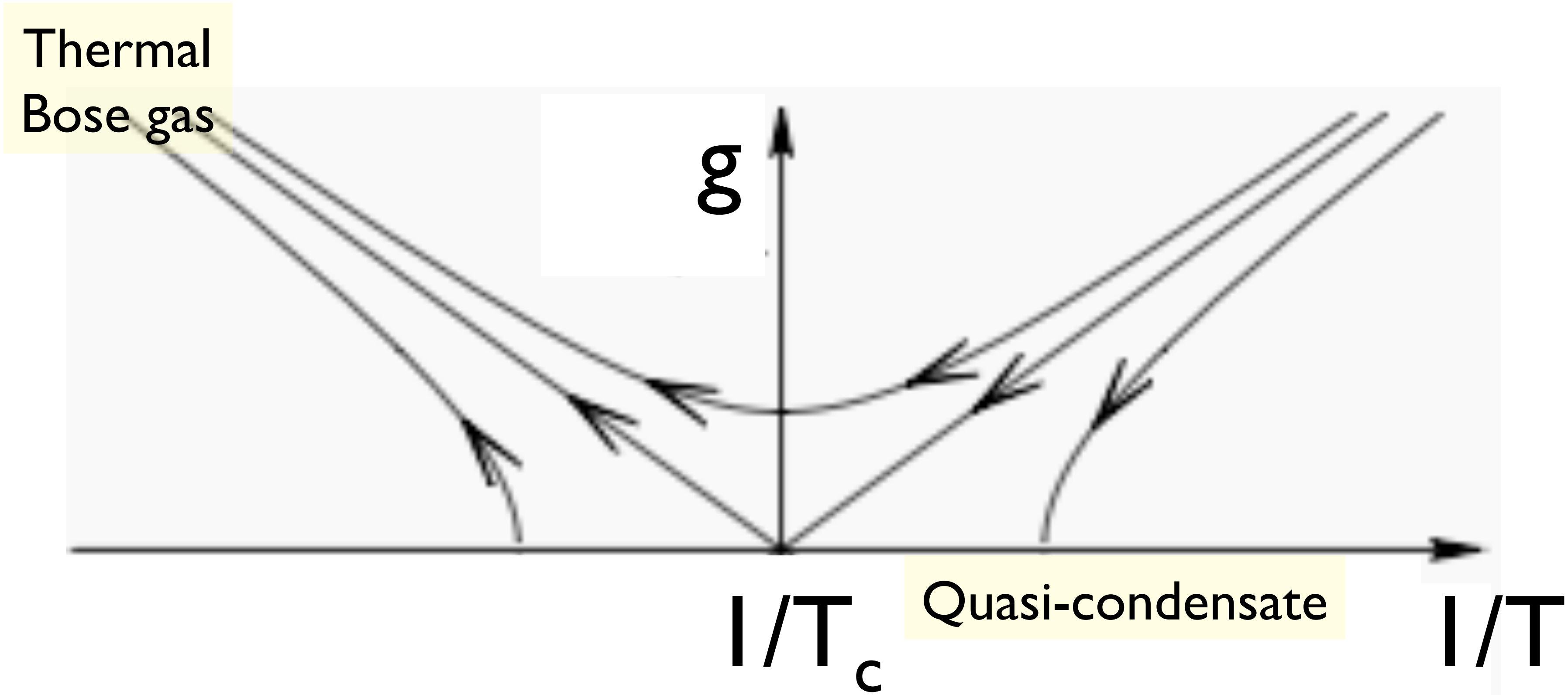
$$\omega_k \to \omega_k^{'}$$

We now use the separation of time scales between the cut-off energy and the $k \to 0$ regime. According to the theorem on adiabatic invariants (LL, Theor. Mech.), the ratio

$$I_k = \frac{E_k}{\omega_k}$$

is invariant under slower deformations of ω_k. We thus find a slow drift of the energy stored in mode k, corresponding to slow heating. This is consistent with the slow increase of τ. Combining all these arguments, and writing them in differential form, we arrive at the KT flow equations.

Renormalization group flow vs. dynamics

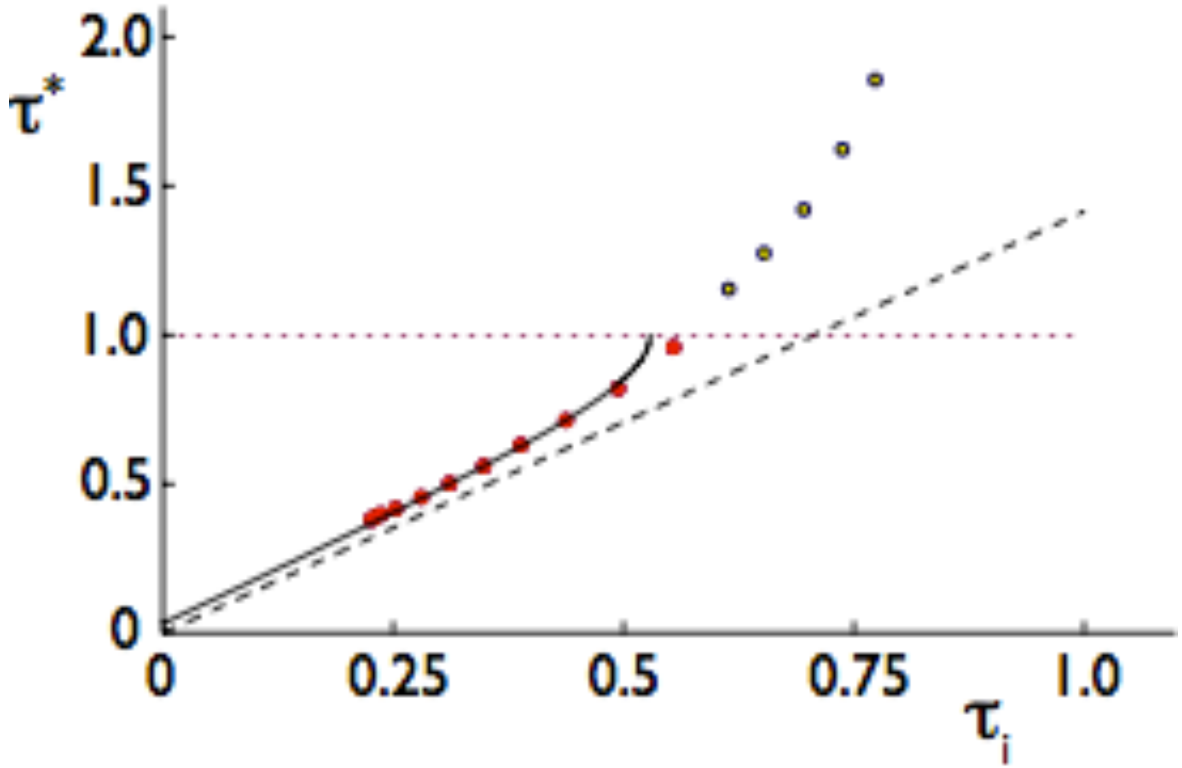


Dynamics resemble the RG flow of the equilibrium system

Phys. Rev. A 80, 041601(R) (2009); 81, 033605 (2010)

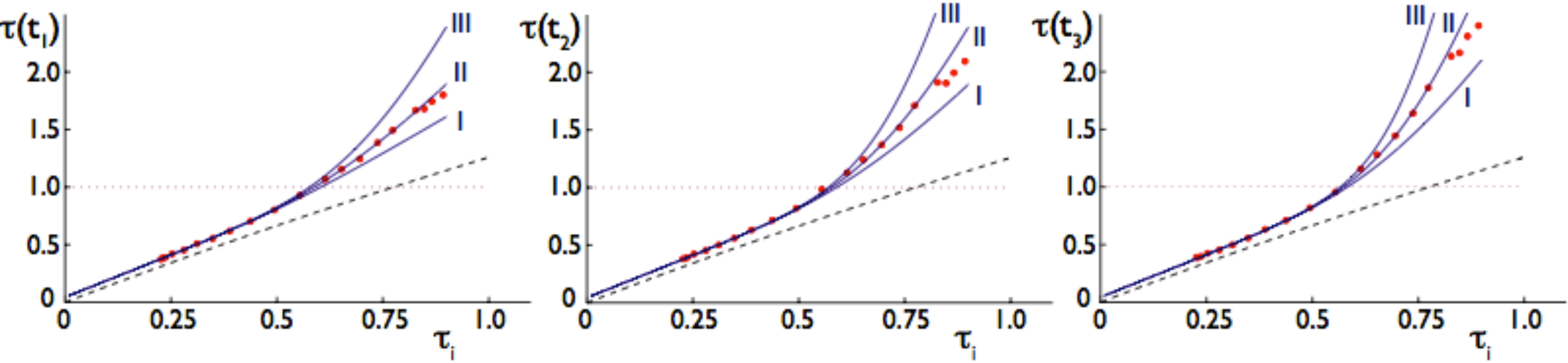
Subcritical regime, fixing $\alpha g^2(0)$

- We write $$\frac{d(\alpha g^2)}{dl} = (4 - 4/\tau)(\alpha g^2)$$
- By using the asymptotic form of τ, and fitting it to the subcritical data, one can extract $\alpha g^2(0)$ and C.



Predicting critical dynamics via real-time RG

- We integrate the flow equations and find agreement with the numerics.



Red: τ data at times $t_1 = 300$, $t_2 = 1200$, $t_3 = 1800$.

Blue: RG prediction; ‘II’ is the correct prediction, ‘I’ and ‘III’ are two near-by solutions for visible comparison.

Analytical description of critical dynamics!

Time scales of vortex unbinding

From RG, we derive the time scale

Near criticality:

$$t^{*} \sim \exp\left(\frac{\exp(-E_c/2T)}{\sqrt{1-T/T_c}}\right)$$

Away from criticality:

$$t^{*} \sim \exp\left(\frac{E_c}{T-T_c}\right)$$

→ Exponential increase at criticality

The energy range of suppression is given by the vortex core energy E_c

Conclusions

- Critical dynamics can be described using a novel RG approach
- Realistic, experimental proposal to create a dynamic Kosterlitz-Thouless transition of 2D superfluids
- Metastable, supercritical state state emerges
- Dynamical vortex unbinding (Reverse Kibble-Zurek effect)