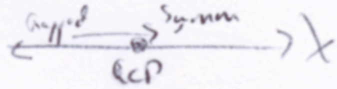
Michael Kolodrubetz, KITP Talk, 9/6/12

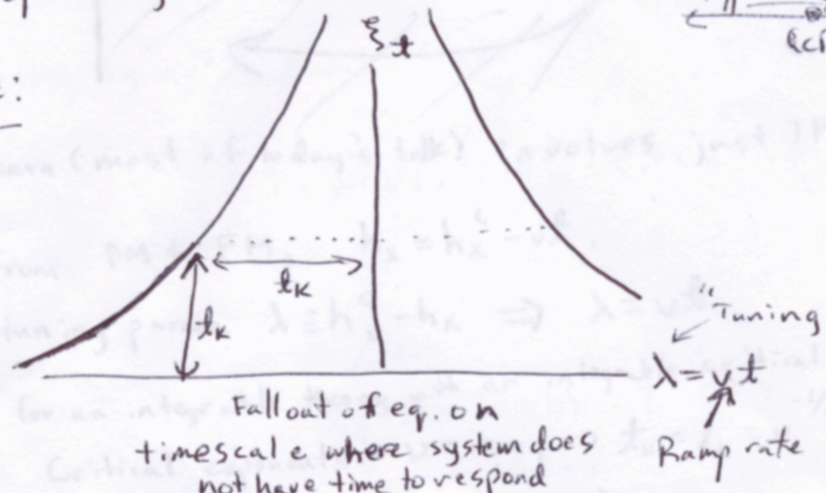
Title: Universality and Dephasing within the Kibble-Zurek Mechanism

Talk has three major goals:

1) Provide numerical evidence that standard scaling arguments (including universality) apply to certain non-equilibrium protocols, such as Kibble-Zurek (KZ)
2) Discuss certain phenomena which appear only within the KZ scaling theory, i.e., the existence/non-existence of dephasing as a fn. of protocol.
3) Describe how KZ and its (natural) extensions may be useful to probe universal aspects of dephasing and thermalization.



Basic ideas of KZ:



$$t_K = \xi_t(t_K) \overset{\text{Algebra}}{\Rightarrow} \boxed{t_K = v^{\frac{-\nu z}{1+\nu z}}} \Rightarrow l_K = t_K^{1/z} = v^{\frac{-\nu}{1+\nu z}}.$$

Original formulation (Kibble 1976, Zurek 1985) – l_K sets density of defects after ramp

Modern perspective (Deng et al. 2008, Chandran et. al. 2012, ...) – t_K & l_K set scales in well-defined neq. scaling limit $\xi_t^{eq} \to t_K$, $\xi^{eq} \to l_K$

$\Downarrow$

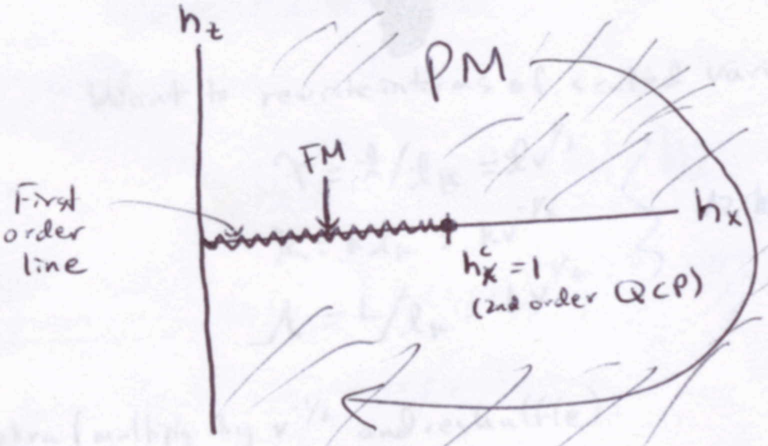
Also exists similar story for non-linear ramps (Barankov & Polk. 2008, Sen et. al. 2008)

$$\text{If } \lambda = v|t|^r \operatorname{sgn}(t), \text{ then } t_K \to v^{\frac{-\nu z}{1+r\nu z}}$$

Special case $r=0 \Leftrightarrow$ symmetric quench across CP (v plays the role of quench depth)

The model: (Extended) transverse-field Ising ~~model~~ chain

$$H = -\sum_j \left[\sigma^z_j \sigma^z_{j+1} + h_x \sigma^x_j + h_z \sigma^z_j\right]$$



Simplest case (most of today's talk) involves just TFI chain, $h_z = 0$.

Ramp from PM to FM, $h_x = h_x^c - vt$.

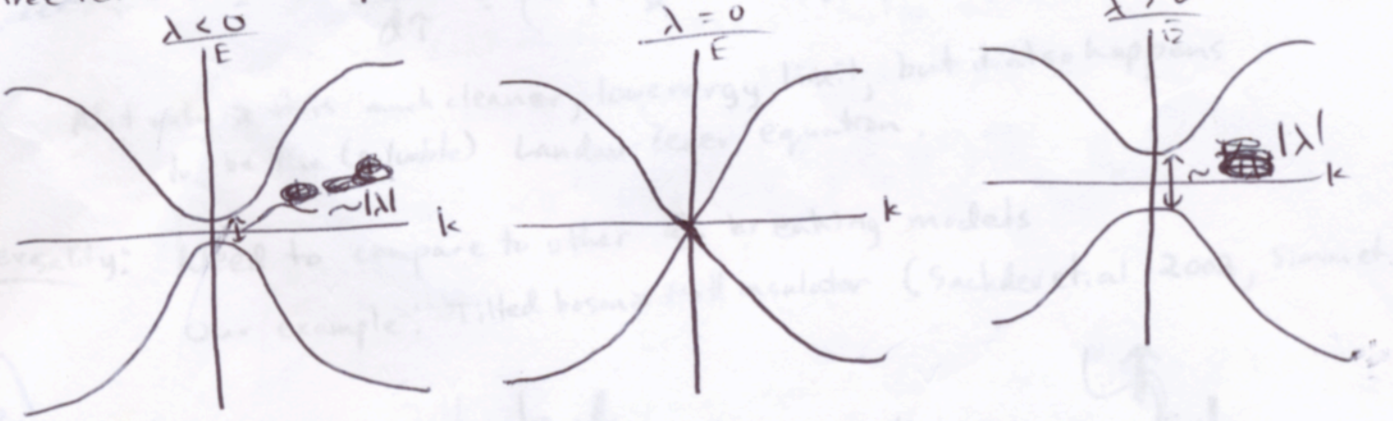
Define tuning param. $\lambda \equiv h_x^c - h_x \Rightarrow \lambda = vt$

This is a ramp for an integrable theory with an integrable critical point

Critical exponents: $\nu = z = 1 \Rightarrow t_k = l_k = v^{-1/2}$ (note that $t_k, l_k \to \infty$ as $v \to 0$)

Solution is via Jordan-Wigner (cf. Sachdev 1999)

Gives free fermions with disp. rel:



$$H_k = \left[-\lambda + \mathcal{O}(k^2)\right]\sigma^z_k + \left[k + \mathcal{O}(k^3)\right]\sigma^x_k$$

$$E_k = \pm\sqrt{\lambda^2 + k^2}$$

Dynamic scaling can be motivated from Schrödinger eqn:

$$i \frac{d|\psi_k\rangle}{dt} = \left[(-vt + O(k^2))\sigma^z_k + (k + O(k^3))\sigma^x_k\right]|\psi_k\rangle$$

Want to rewrite in terms of scaled variables:

$$\tau = t/\ell_K = t v^{1/2}$$
$$\kappa = k\ell_K = k v^{-1/2}$$
$$\Lambda = L/\ell_K = L v^{1/2}$$

Visible (LEFT)

Via some algebra (multiply by $v^{-1/2}$ and reshuffle):

$$i \frac{d|\psi_\kappa\rangle}{d\tau} = \left[(-\tau + v^{1/2} O(\kappa^2))\sigma^z_\kappa + (\kappa + v\, O(\kappa^3))\sigma^x_\kappa\right]|\psi_\kappa\rangle$$

Scaling limit: $\lim_{\substack{\ell_K \to \infty\ (v \to 0) \\ t/\ell_K,\ k\ell_K,\ L/\ell_K \text{ fixed}}} [\text{Schr. eqn}]$

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$$\to\ i \frac{d|\psi_\kappa\rangle}{d\tau} = (-\tau\sigma^z_\kappa + \kappa\sigma^x_\kappa)|\psi_\kappa\rangle$$

Not only is this much cleaner, low energy limit, but it also happens to be the (solvable) Landau-Zener equation.

Universality: Need to compare to other $\mathbb{Z}_2$ breaking models

Tilted bosonic Mott insulator (Sachdev et. al. 2002, Simon et. al. 20...

Our example: Tilted bosonic Mott insulator

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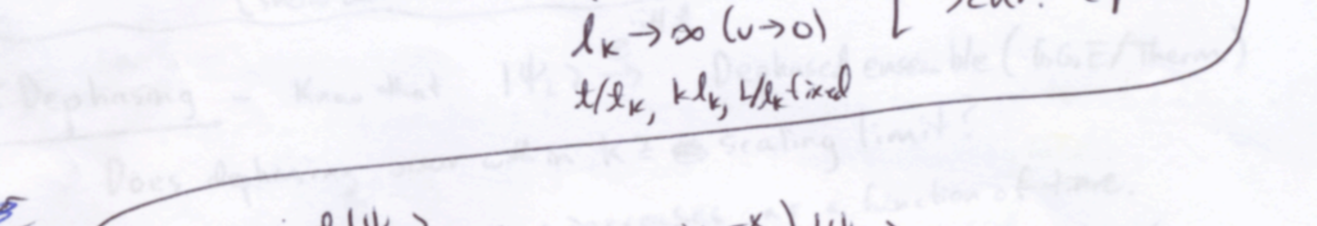


$\mathbb{Z}_2$ symmetry $\longleftrightarrow$ Dipoles on even/odd bonds

In practice map to spin model (dipole = ↑).

Now QPT from ↓↓↓↓ PM to ↑↓↑↓ AFM.

Simulate this model with time-dep DMRG.

Visible (LEFT)

Observables: Need to compare models via observables. Will use two:

Excess heat: $Q(t) \equiv \langle \psi(t)|H(t)|\psi(t)\rangle - \langle \psi_{gs}(t)|H(t)|\psi_{gs}(t)\rangle$

Equal-time Orderparam. corr.: $G(x,t) \equiv \langle \psi(t)| S^z_j \, S^z_{j+x}|\psi(t)\rangle$

Scaling forms look similar to equilibrium

$$\frac{Q}{L} \rightarrow \frac{Q t_K}{L/l_K} = \frac{Q}{vL} = q(\tau, \Lambda)$$

$$G \rightarrow \quad G = X^{-1/4}\, g(\tau, \Lambda, X = x/l_K)$$

(show data for universality on slides)

Part 2: Dephasing – Know that $|\psi_i\rangle \xrightarrow{e^{iHt}}$ Dephased ensemble (GGE/Therm.)

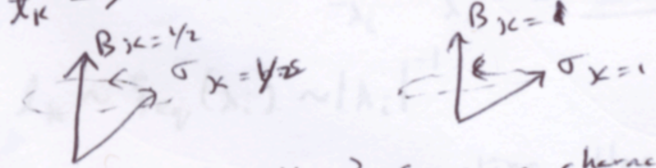
Does dephasing occur within KZ scaling limit?

Naive guess is Yes: because gap increases as a function of time.

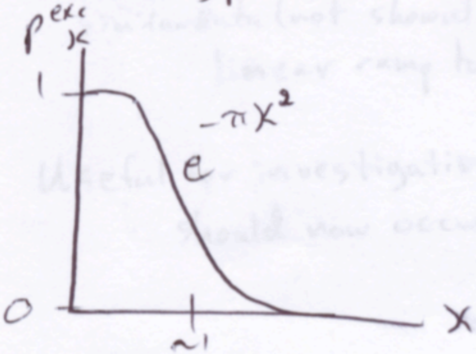
It *is* true that individual modes "precess" faster at later times

But what matters for observables is the phase *difference* between modes

$$H l_K \rightarrow -\tau\sigma^z + X\sigma^x = \tau\left(-\sigma^z + \frac{X}{\tau}\sigma^x\right) = \vec{B}_X \cdot \vec{\sigma}$$



Do they precess together? Compare characteristic modes ($X=0$ & $X=1$)



$$\varphi(X=1) - \varphi(X=0)$$
$$= \int^{\tau_f} d\tau \left[\mathcal{E}(X=1) - \mathcal{E}(X=0)\right]$$
$$= \int^{\tau_f} d\tau \; \tau\left(\sqrt{1 - 1/\tau^2} - 1\right)$$
$$\approx \int^{\tau_f} d\tau \frac{1}{2\tau} = \frac{1}{2}\ln \tau_f$$

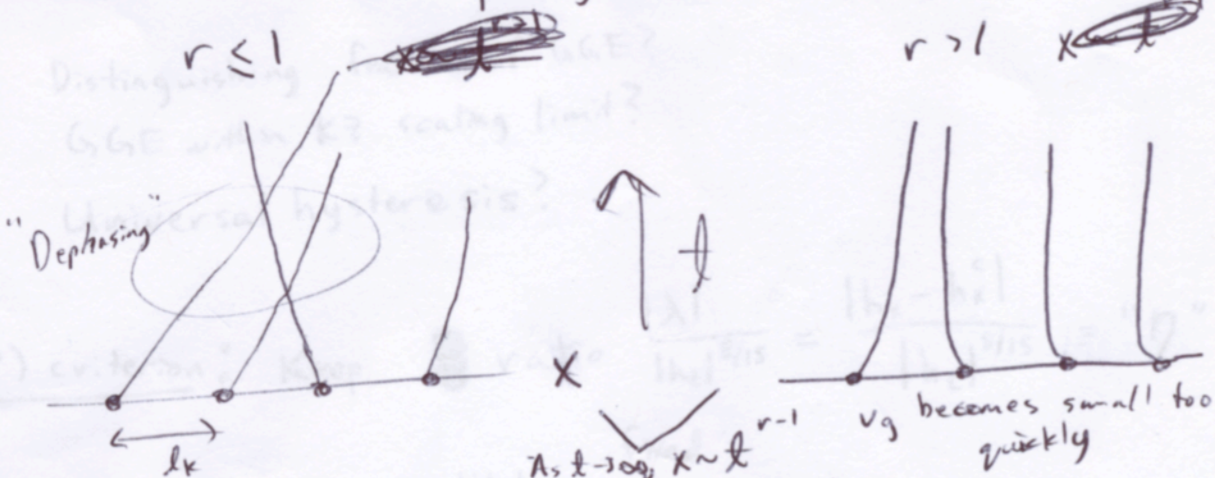
For linear ramp, $\Delta\varphi \rightarrow \infty$ as $\tau_f \rightarrow \infty \Rightarrow$ Dephases

For ramp with arbitrary r, $\Delta\varphi \approx \lim_{\tau_f\to\infty}\int^{\tau_f} d\tau \, \frac{1}{2\tau^r}$

$\overset{r\le 1}{\sim} \infty \Rightarrow$ GGE

$\overset{r>1}{\sim}$ const. $\Rightarrow$ Modes become phase locked (not GGE)

Protocol-tunable distinction between dephasing to GGE & not dephasing



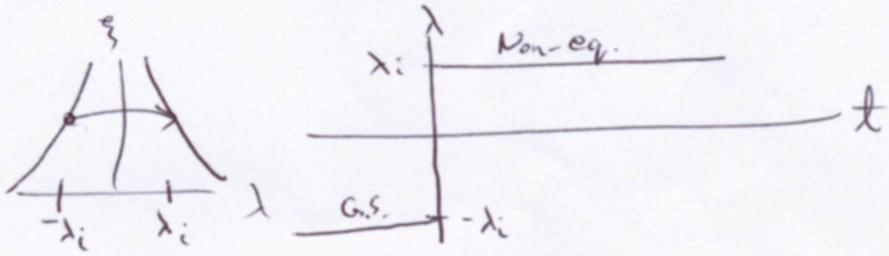
$r = 1$ is marginal case where $x \sim \ln t$ ("$z=\infty$"?)

Part. 3: Extensions

There exist essentially arbitrary protocols to extend these ideas of KZ scaling theory (at least in theorist fantasy land)

One useful scaling direction: Time after quench/ramp

Simplest case is $r=0$:



$$l_K \sim \xi_{eq}(\lambda_i) \sim |\lambda_i|^{-1}$$

Data shows scaling collapse

Similar data (not shown) shows scaling collapse for r linear ramp to $\tau=1$, then waiting.

Useful for investigating dephasing/thermalization, as it should now occur in finite (scaled) time.

Within the Ising model, can investigate other, less trivial scaling directions.

<u>Longitudinal field h_z</u>: $\xi_{eq} \sim |h_z|^{-8/15}$ $(\nu = 8/15)$

Integrable (but non-trivial E_8 field theory, Zamolodchikov 1989, Coldea et. al. 2010)

Distinguishing features of GGE?

GGE within KZ scaling limit?

Universal hysteresis?

<u>Thermalizing (?) criterion</u>: Keep ratio $\frac{|\lambda|}{|h_z|^{8/15}} = \frac{|h_x - h_x^c|}{|h_z|^{8/15}} = $ "η" fixed.

Each value of $\eta \neq 0, \infty$ defines a likely non-integrable field theory.

Does thermalization occur in the KZ scaling limit?

Time scales for GGE/pretherm/therm. for ramp/quenches near the <u>integrable</u> QCP?