Composite Fermion Groundstate of Rashba Spin-Orbit Bosons

Alex Kamenev

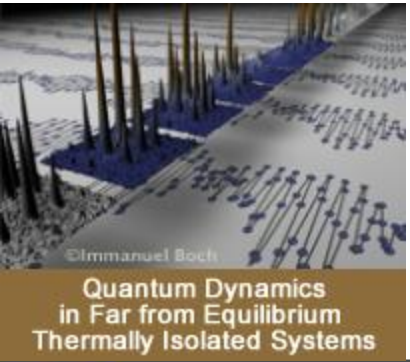
Fine Theoretical Physics Institute,
School of Physics & Astronomy,
University of Minnesota

Tigran Sedrakyan

UNIVERSITY OF MINNESOTA

Leonid Glazman

YALE



ArXiv1208.6266

U.S. DEPARTMENT OF ENERGY

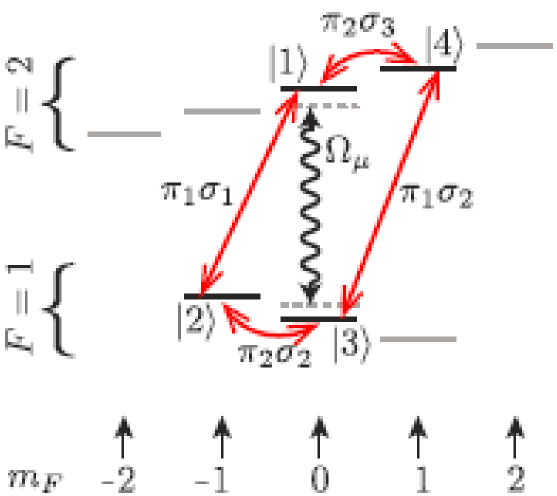
Motivation

PHYSICAL REVIEW A **84**, 025602 (2011)

Realistic Rashba and Dresselhaus spin-orbit coupling for neutral atoms

D. L. Campbell,[1] G. Juzeliūnas,[2] and I. B. Spielman[1]
[1]*Joint Quantum Institute, National Institute of Standards and Technology, and University of Maryland, Gaithersburg, Maryland 20899, USA*
[2]*Institute of Theoretical Physics and Astronomy, Vilnius University, A. Goštauto 12, Vilnius LT-01108, Lithuania*
(Received 19 February 2011; published 30 August 2011)

(a) Coupling diagram



Four-level ring coupling scheme in **87Rb** involving hyperfine states $|F,m_F\rangle$ *Raman-coupled by a total of five* lasers marked *σ1, σ2, σ3, π1, and π2*

Spin-orbit coupling constant

$$\hat{\mathcal{H}}_{SO} = \frac{\mathbf{p}^2}{2m} + v\,(p_x\hat{\sigma}_x + p_y\hat{\sigma}_y)$$

Rashba spin-orbit-coupling

$$H_0 = -\frac{\boldsymbol{\nabla}_{\mathbf{r}}^2}{2m} + iv\hat{\mathbf{z}} \cdot [\boldsymbol{\sigma} \times \boldsymbol{\nabla}_{\mathbf{r}}]$$

$$\varepsilon_{\mathbf{k}} = \frac{k^2}{2m} \pm vk$$

Rotation + two discrete Z_2 symmetries

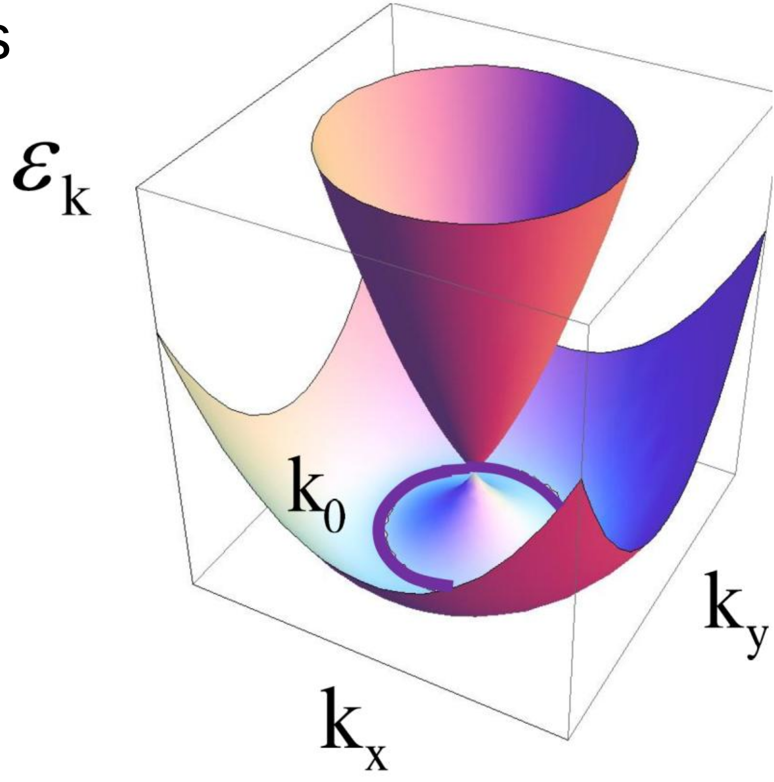
$$\hat{T} \begin{pmatrix} \psi(\mathbf{r}, \uparrow) \\ \psi(\mathbf{r}, \downarrow) \end{pmatrix} = \begin{pmatrix} \bar{\psi}(\mathbf{r}, \downarrow) \\ -\bar{\psi}(\mathbf{r}, \uparrow) \end{pmatrix}$$

time-reversal

$$\hat{P} \begin{pmatrix} \psi(z, \uparrow) \\ \psi(z, \downarrow) \end{pmatrix} = \begin{pmatrix} -i\psi(\bar{z}, \downarrow) \\ i\psi(\bar{z}, \uparrow) \end{pmatrix}$$

parity

$$z = x + iy$$



Bose-Einstein condensates of Rashba bosons

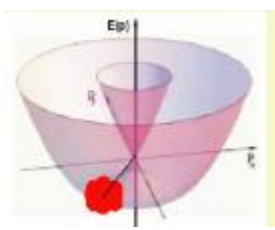
Interaction Hamiltonian:

density operators

$$H_{\text{int}} = \frac{1}{2m}\int d^2r\left(g_0(n_\uparrow + n_\downarrow)^2 + g_2(n_\uparrow - n_\downarrow)^2\right)$$

TRSB

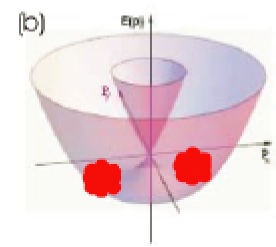
$$g_2 > 0$$



$$\Psi_k \sim \frac{1}{\sqrt{2}} e^{ikr}\begin{pmatrix} 1 \\ -e^{i\arg(k)} \end{pmatrix}$$

Spin-density wave

$$g_2 \leq 0$$



$$\Psi_k \sim \frac{1}{\sqrt{2}}\begin{pmatrix} \cos(kr) \\ i\sin(kr) \end{pmatrix}$$

Chunji Wang, Chao Gao, Chao-Ming Jian, and Hui Zhai, PRL 105, 160403 (2010)

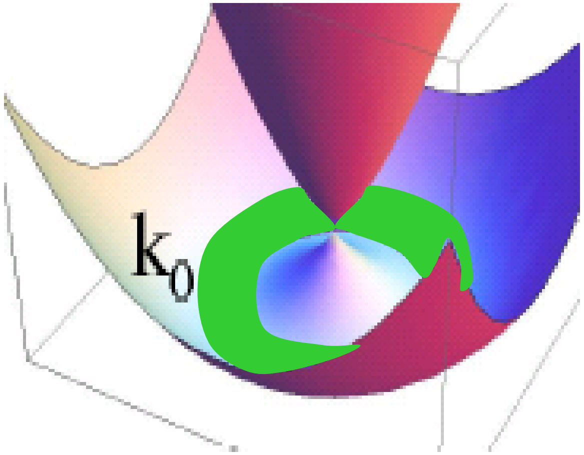
Chemical Potential:

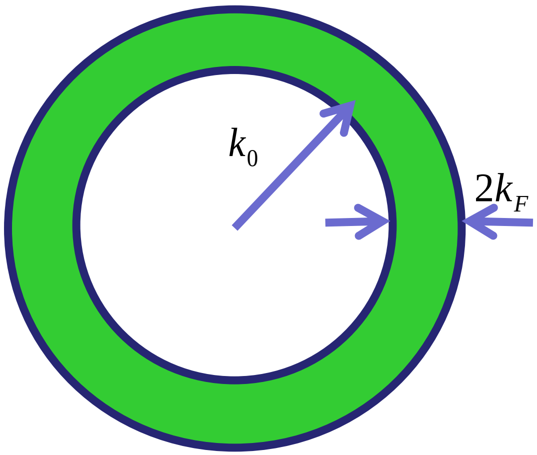
$$E_{\mathrm{int}}^{(0)} = \frac{N^2}{2mV} g_0$$

$$\mu_B = \partial E_{\mathrm{int}} / \partial N \propto n$$

density

However, let us look at Rashba fermions:





Spin-orbit coupling constant

$$k_0 = mv$$

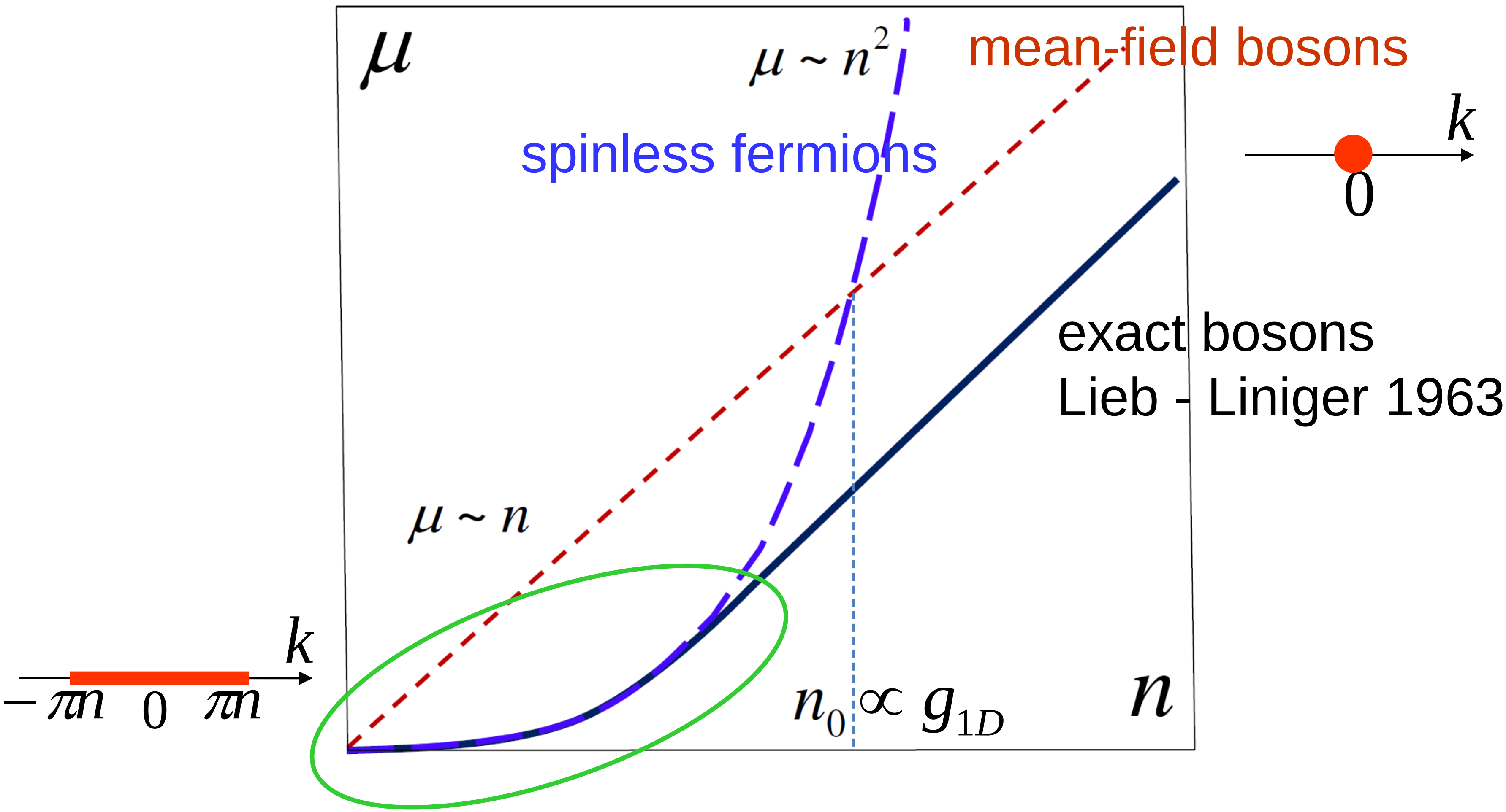
Estimate chemical potential: $(2\pi k_0)(2k_F) \sim 4\pi^2 n$

$$E_F = \frac{k_F^2}{2m} \sim n^2$$

As in 1D!

Near the band bottom, the density of states diverges as $\rho(E) \sim 1/\sqrt{E}$

Reminder: spinless 1D model



Tonks-Girardeau limit

$$\Psi_B(x_1, \ldots, x_N) = \prod_{i,j}^{N} \mathrm{sign}(x_i - x_j) \Psi_F(x_1, \ldots, x_N)$$

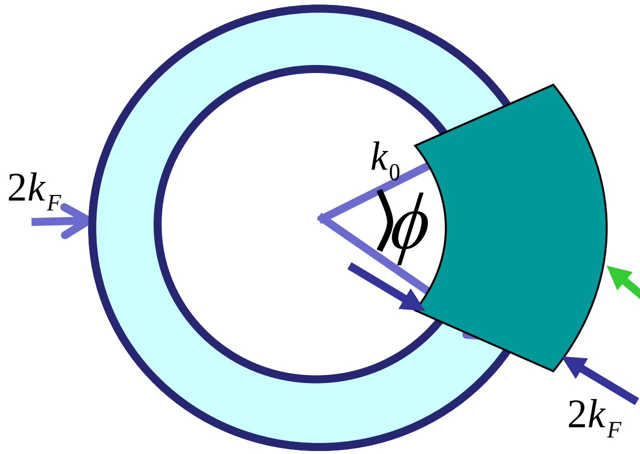
Can one Fermionize Rashba Bosons?

Yes, but…

1. Particles have spin

2. The system is 2D not 1D

Fermions with spin do interact



$$E_{\text{int}} \propto g(1-\cos\phi) \propto g\phi^2$$

Hartree Fock

Variational Fermi surface

$$E_{\text{kin}} \propto k_F^2 \propto \left(\frac{n}{k_0\phi}\right)^2$$

$$E_{\text{int}} \propto gn\phi^2$$

minimizing w.r.t. ϕ

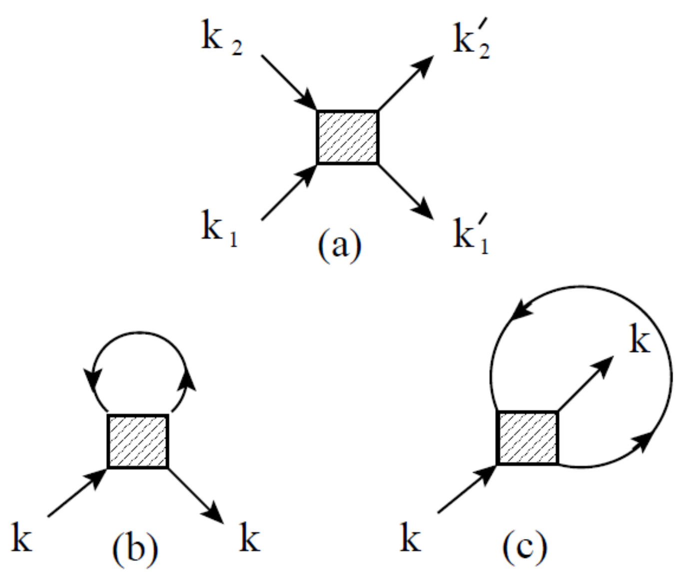
$$\phi = \left(\frac{n}{gk_0^2}\right)^{1/4}$$

$$E_{\text{kin}} \propto E_{\text{int}} \propto \sqrt{\frac{g}{k_0}}n^{3/2} << n$$

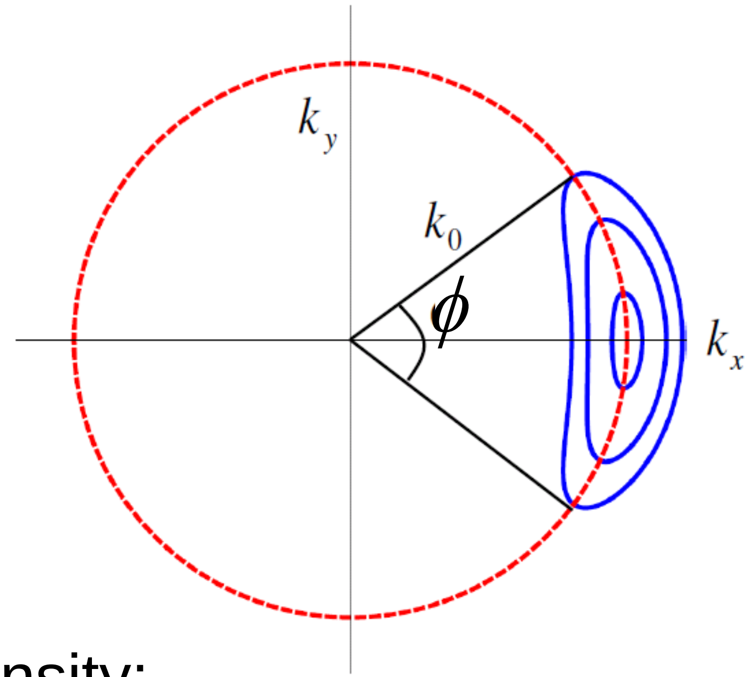
Berg, Rudner, and Kivelson, (2012)

Self-consistent Hartree-Fock for Rashba fermions

Berg, Rudner, and Kivelson, (2012)



nematic state



Elliptic Fermi surface at small density:

$$H_{HF} = \frac{(k_x - k_0)^2}{2m} + \frac{k_y^2}{2m_y}$$

$$m_y = m\frac{k_0^2}{gn} >> m$$

$$\mu \propto \sqrt{\frac{g}{k_0}} n^{3/2} << gn$$

nematic fermions

mean-field bosons

Fermionization in 2D? Chern-Simons!

$$\Psi_{\downarrow\ldots\downarrow}(\mathbf{r}_1,\ldots,\mathbf{r}_N) = \prod_{i<j} e^{\pm i\,\mathrm{arg}(\mathbf{r}_i-\mathbf{r}_j)}\Psi_F(\mathbf{r}_1,\ldots,\mathbf{r}_N)$$

bosonic wave function

Chern-Simons phase

fermionic wave function

- ✓ (plus/minus) One flux quantum per particle
- ✓ Broken parity P
- ✓ Higher spin components are uniquely determined by the ***projection*** on the lower Rashba brunch
- ✓ Fermionic wave function is Slater determinant, minimizing kinetic and interaction energy

Chern-Simons magnetic Field

$$\Psi_{\downarrow\ldots\downarrow}(\mathbf{r}_1,\ldots,\mathbf{r}_N) = \prod_{i<j} e^{\pm i\,\mathrm{arg}(\mathbf{r}_i-\mathbf{r}_j)}\Psi_F(\mathbf{r}_1,\ldots,\mathbf{r}_N)$$

$$\hat{\mathbf{k}} \to \hat{\mathbf{k}} \pm \mathbf{A} \qquad \mathbf{A}_\alpha(\mathbf{r}_j) = \sum_{i\neq j} \epsilon_{\alpha\beta} \frac{(\mathbf{r}_j - \mathbf{r}_i)_\beta}{|\mathbf{r}_j - \mathbf{r}_i|^2}$$

$$B_{CS}(\mathbf{r}_j) = \mathrm{rot}\mathbf{A}(\mathbf{r}_j) = 2\pi\sum_{i\neq j}\delta(\mathbf{r}_j - \mathbf{r}_i) \to 2\pi n$$

mean-field approximation

Particles with the cyclotron mass: $m_c = \sqrt{m_x m_y} = m\sqrt{\frac{k_0^2}{gn}}$

in a uniform magnetic field: $B_{CS} = 2\pi n$

Integer Quantum Hall State

Particles with the cyclotron mass: $m_c = \sqrt{m_x m_y} = m\sqrt{\frac{k_0^2}{gn}}$

in a uniform magnetic field: $B_{\mathrm{CS}} = 2\pi n$

Landau levels: $$\varepsilon_l = \frac{B_{\mathrm{CS}}}{m_c}\left(l + \frac{1}{2}\right)$$

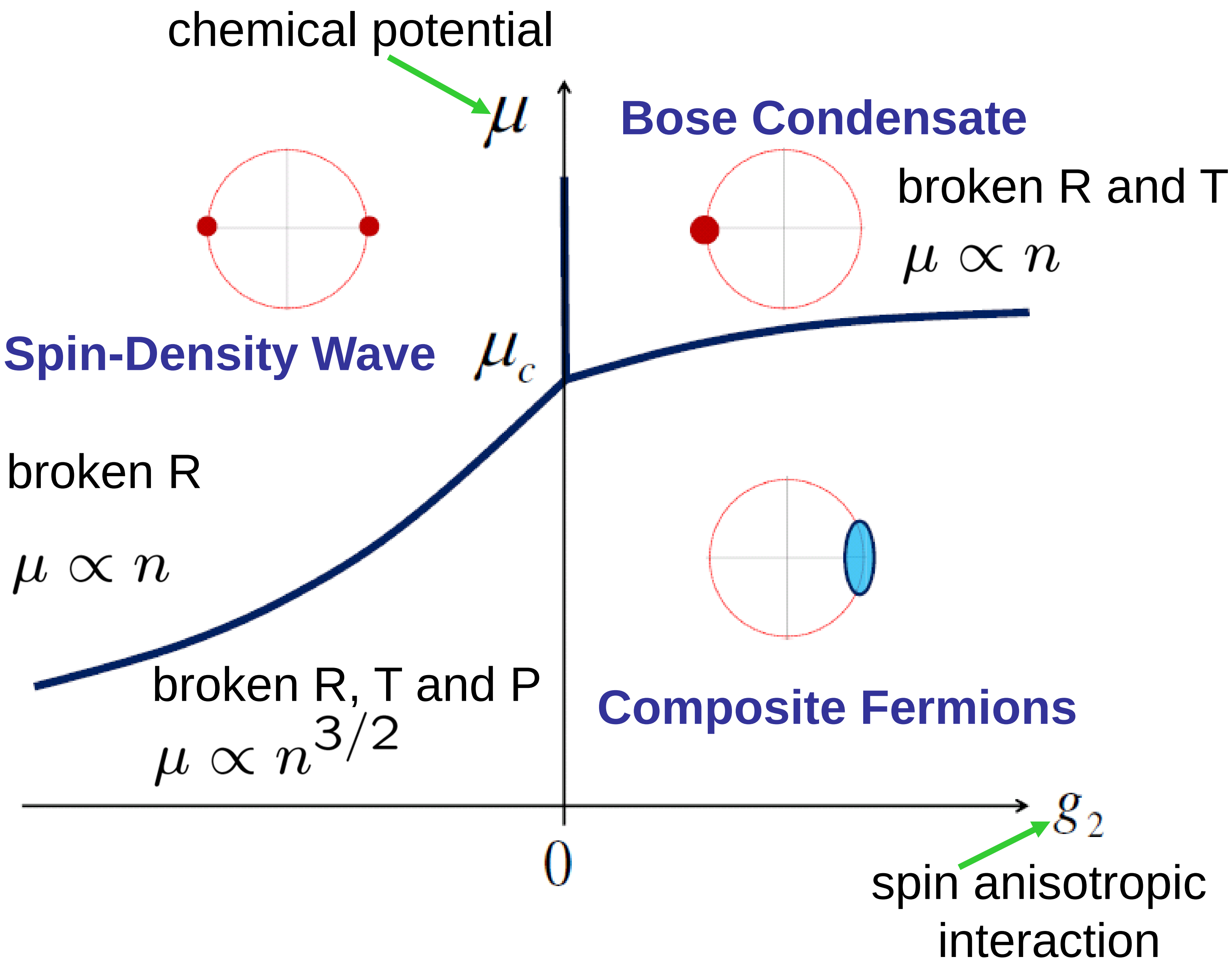
One flux quanta per particle, thus ν=1 filling factor: IQHE

$$\mu = \varepsilon_0 = \frac{\pi\sqrt{g}}{m k_0} n^{3/2}$$

✓ Gapped bulk and chiral edge mode:

interacting topological insulator

Phase Diagram

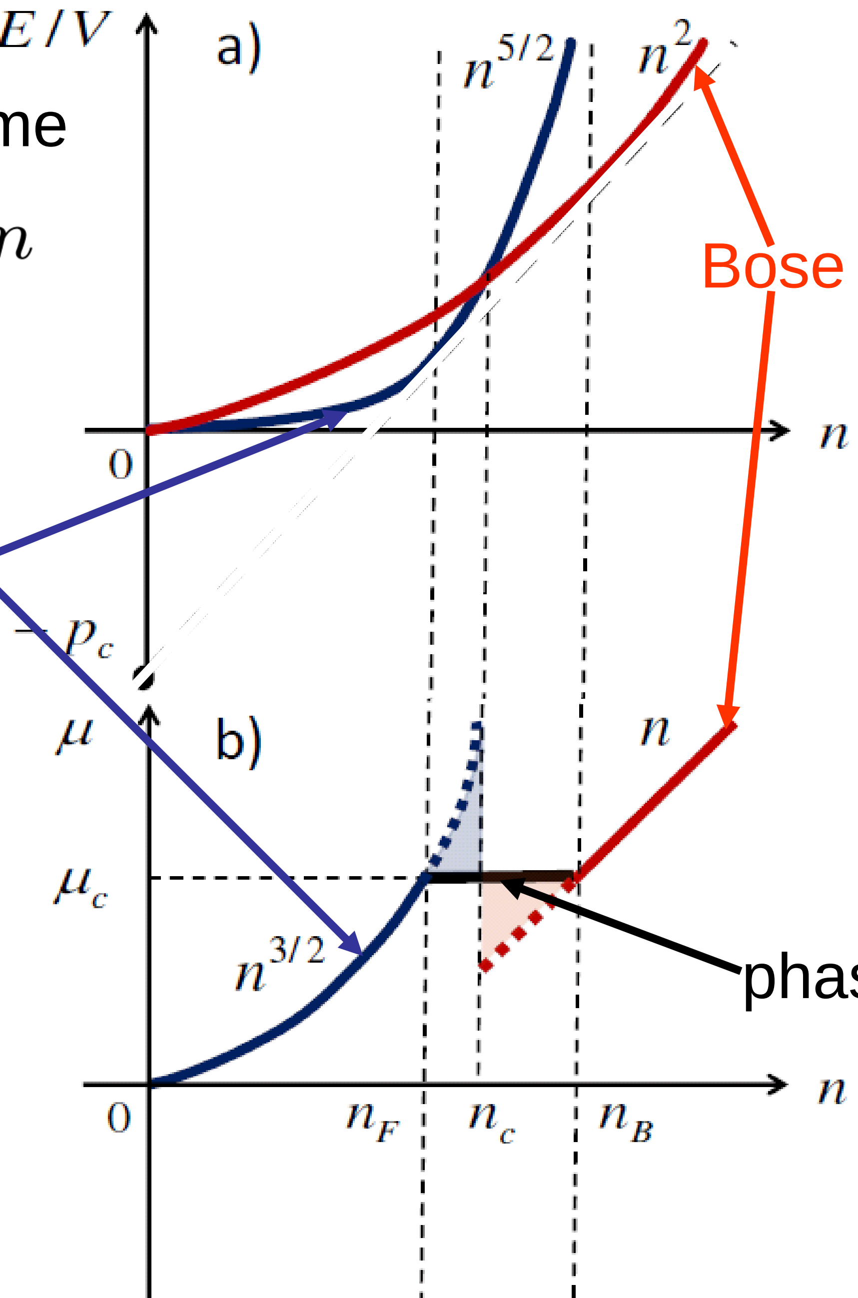


Phase Separation

total energy per volume

$$E(n) = \int^{n} \mu(n) dn$$

composite fermions

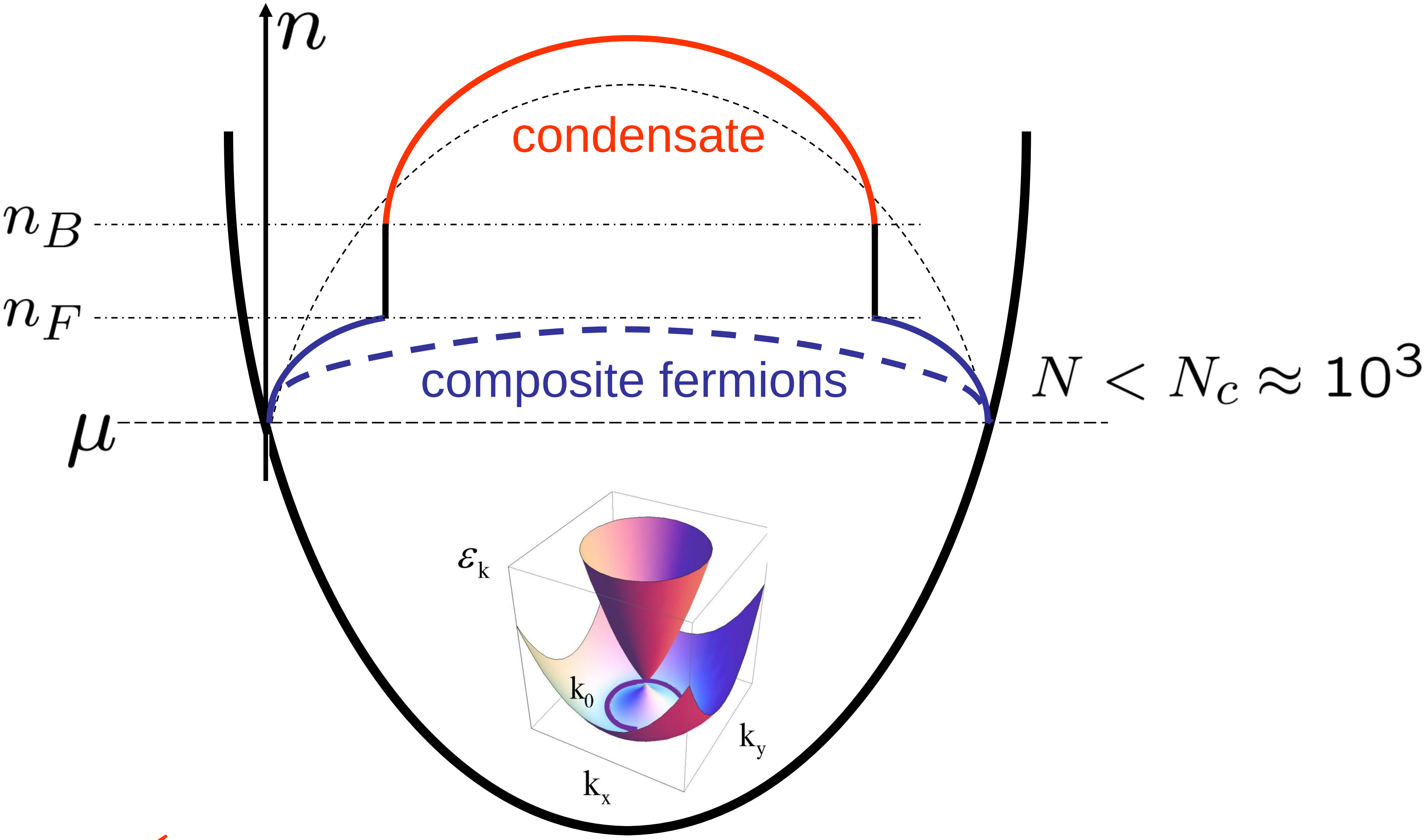


Bose condensate

phase separation

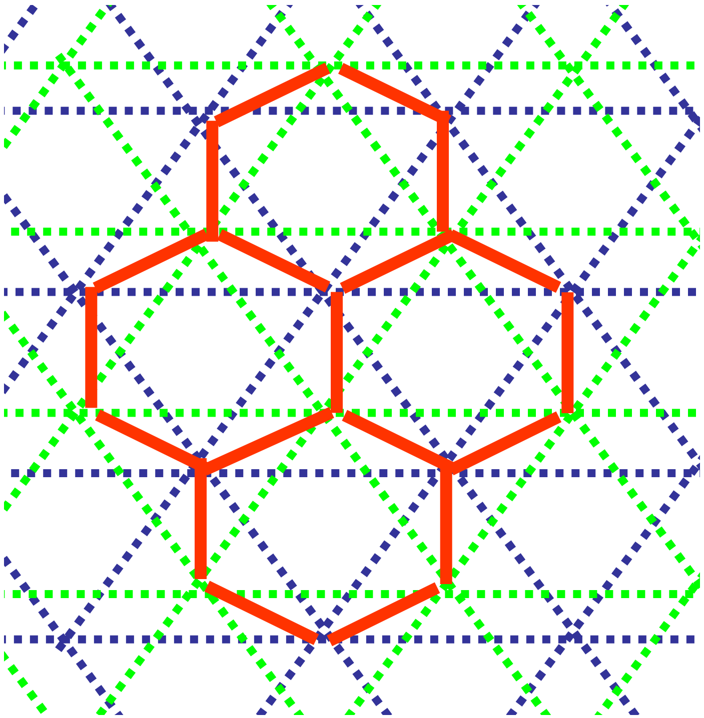
$$n_{B,F} \propto g k_0^2$$

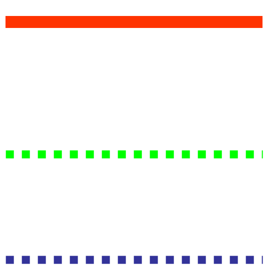
Rashba Bosons in a Harmonic Trap



✓ high density Bose condensate in the middle and low density composite fermions in the periphery

Lattice Model





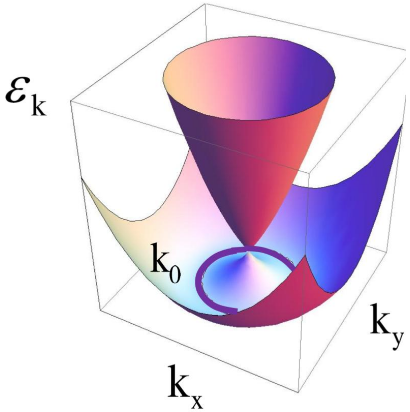
J_1

$\Big\} \; J_2 < 0$

$J_1 \ll |J_2|$

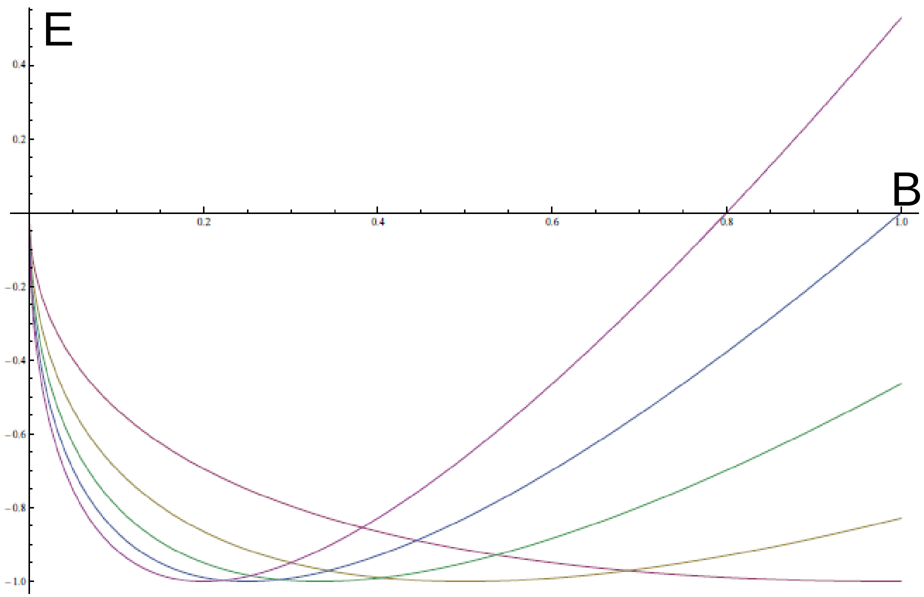
In a vicinity of K and K' points in the Brillouin zone:

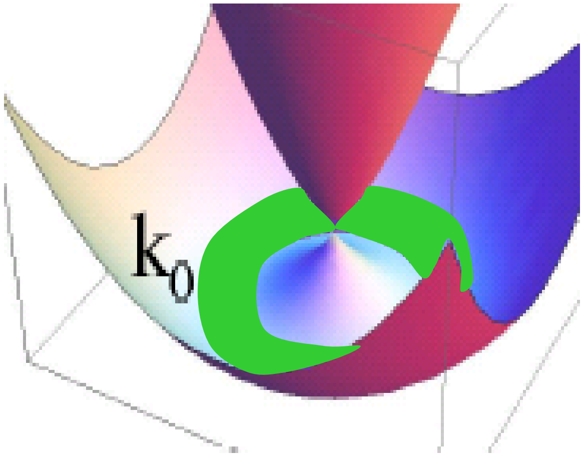
$$\varepsilon_k = \pm J_1 |k| + J_2 k^2$$

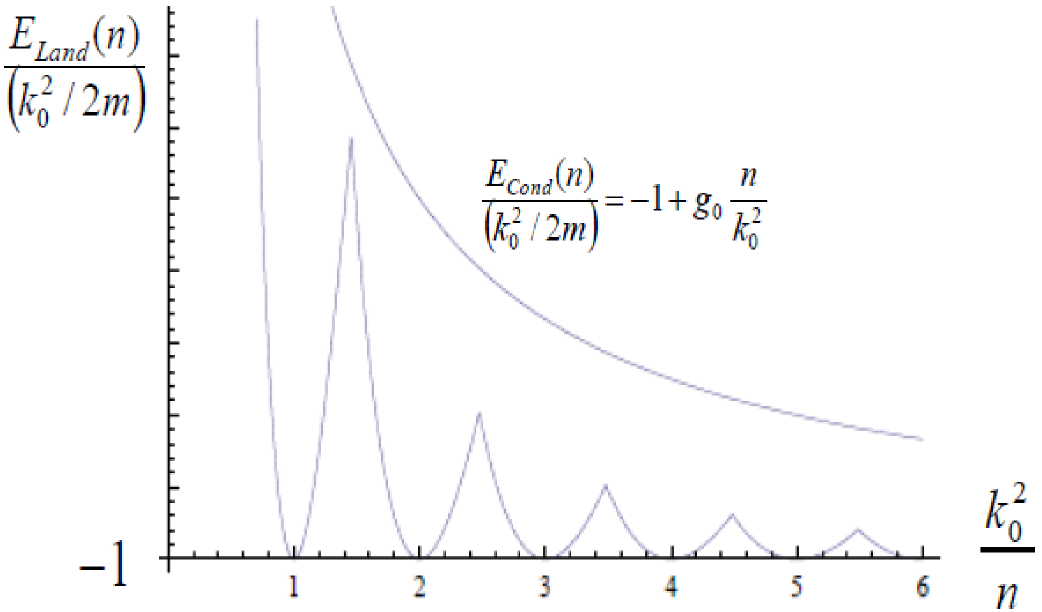


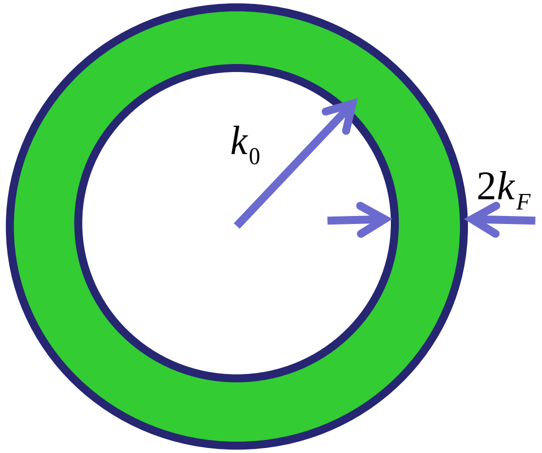
$$k_0 = \frac{J_1}{J_2} \ll 1$$

Landau-Rashba levels









Conclusions

- ✓ At low density Rashba bosons exhibit Composite Fermion groundstate
- ✓ CF state breaks **R, T** and **P** symmetries
- ✓ CF state is gaped in the bulk, but supports gapless edge mode, realizing interacting **topological insulator**
- ✓ CF equation of state: $\mu(n) \propto n^{3/2}$
- ✓ There is an interval of densities where CF **coexists** with the Bose condensate
- ✓ In a trap the low-density CF fraction is pushed to the **edges** of the trap