Composite object dynamics in interacting 1D lattice systems

PROPAGATION

LOCKING

SCATTERING

"Quantum Bowling"

Masud Haque

Max-Planck Institute for Physics of Complex Systems (MPI-PKS)

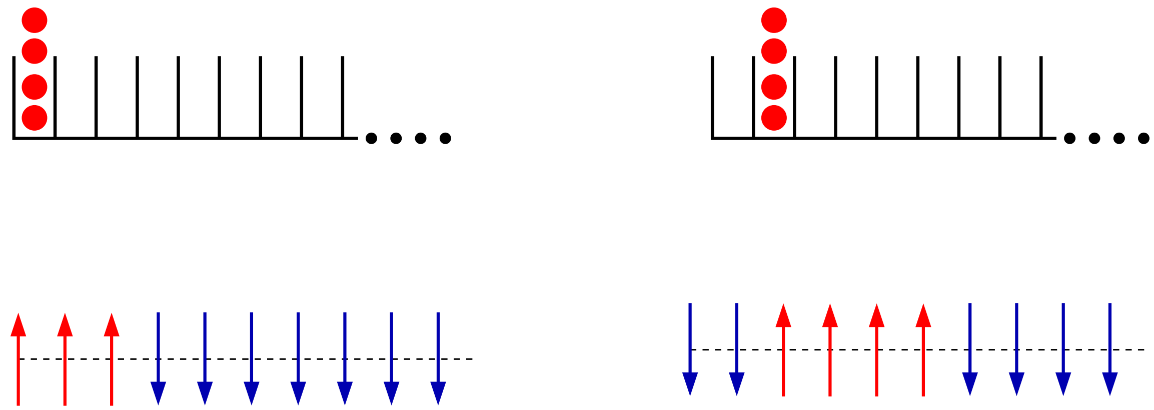
Dresden, Germany

Edge-localization in 1D lattice models

Bose-Hubbard chain

spinless fermion model

XXZ chain



PHYSICS:

Far-from-equilibrium dynamics

Eigenstates far from ground state

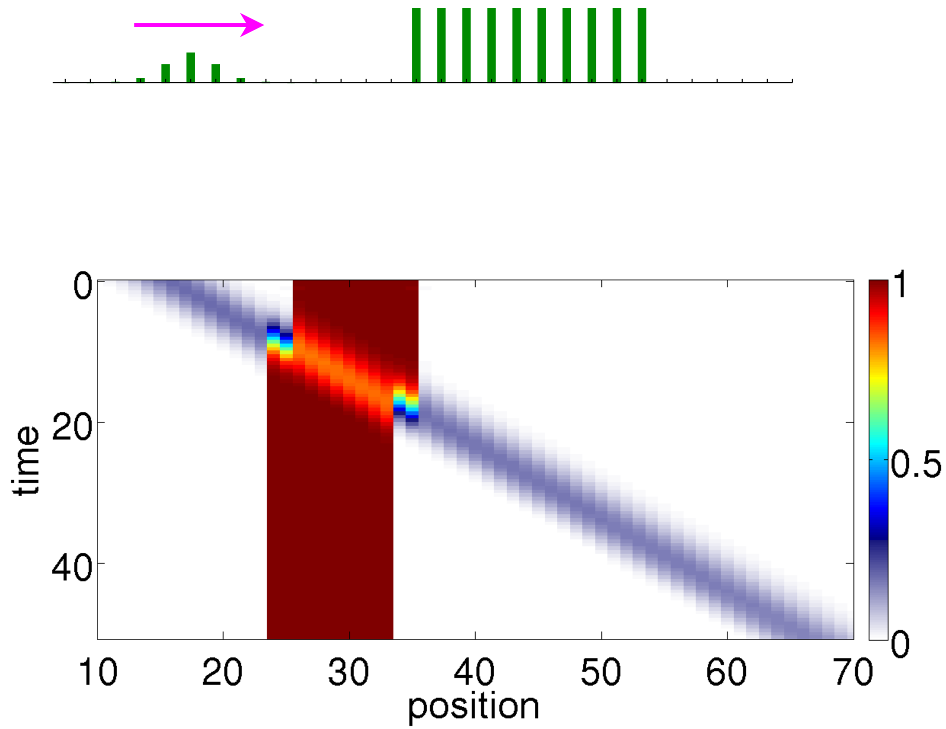
Intricate structures in spectrum (FRACTAL)

QUANTUM CONTROL:

Locking and release of magnetization/state

Designing a quantum switch

Teaser on scattering ('Quantum bowling')



Spinless fermions with nearest-neighbor interactions:

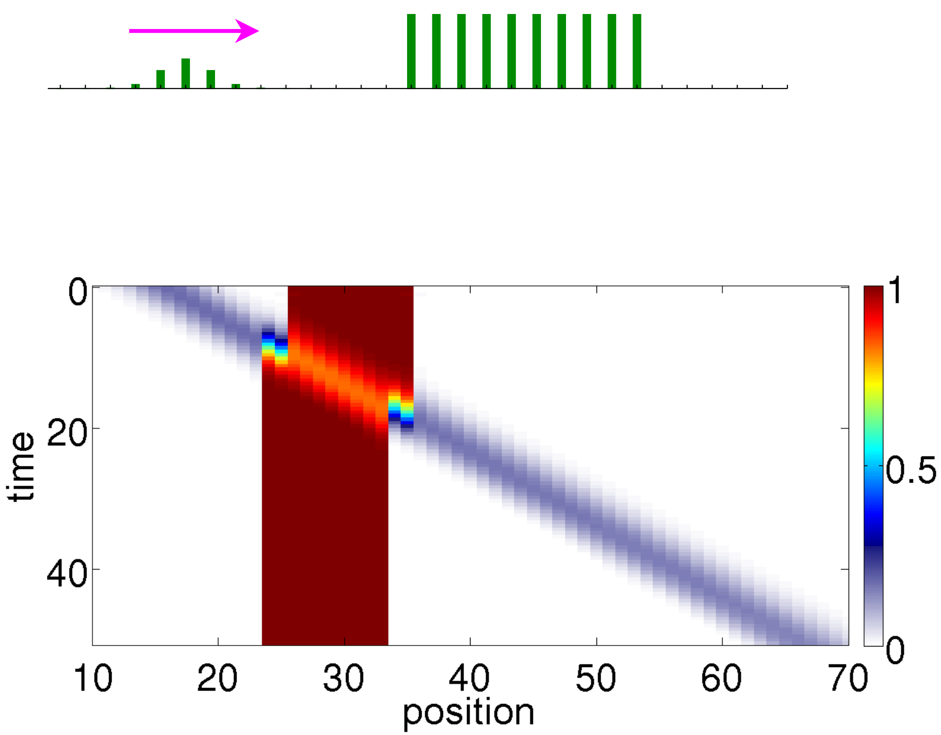
$$H = -t \sum \left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j \right) + V \sum n_j n_{j+1}$$

Strong coupling: $V \gg t$

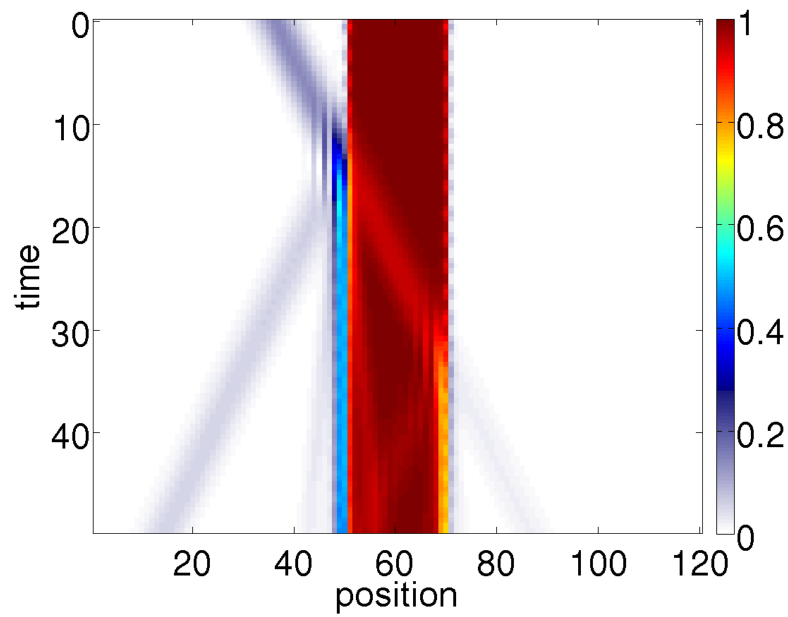
'Same' results for spin chain

(Heisenberg or XXZ chain; large anisotropy)

Teaser on scattering ('Quantum bowling')



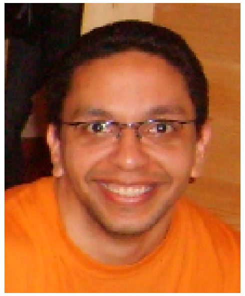
With slightly modified interactions:



Conjecture: Integrability suppresses reflection.

Nice people thank their collaborators

Sergej Flach
MPI-PKS
Dresden



Ricardo Pinto
ex - MPI-PKS

H. G. Evertz
T.U. Graz

Martin Ganahl
T.U. Graz

Motivated by edge phenomena in:

Discrete nonlinear Schrodinger equation (DNLS)

Related issues:

propagation in 1D models

(particles, clusters, magnons, multi-magnons)

For details ....

R. A. Pinto, M. Haque, and S. Flach; Phys. Rev. A **79**, 052118 (2009).

Edge-localized states in quantum one-dimensional lattices.

M. Haque; Phys. Rev. A **82**, 012108 (2010).

Self-similar spectral structures and edge-locking hierarchy in open-boundary spin chains.

M. Ganahl, M. Haque, and H.-G. Evertz; in preparation.

Quantum bowling: particle-hole transmutation in 1D strongly correlated lattice models.

Hamiltonians & small parameters

Hamiltonians:	Small Parameters:
$H_{\text{Bose.Hubbard}} = -t\sum\left(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j\right) + \frac{U}{2}\sum a_j^\dagger a_j^\dagger a_j a_j$	t/U
$H_{\text{sp.ferm.}} = -t\sum\left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j\right) + V\sum c_j^\dagger c_{j+1}^\dagger c_{j+1} c_j$	t/V
$H_{XXZ} = J_x\sum\left[S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z\right]$	$1/\Delta$

I take these Hamiltonians seriously!

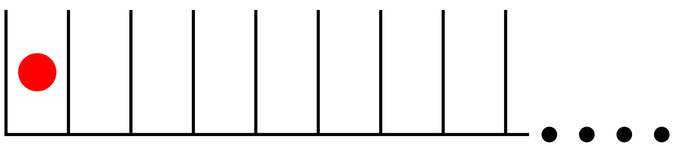
- not only low-energy sector
- no dissipation mechanism

Start with some guessing games

1D Bose-Hubbard model in an OPEN chain (has edges)

$$\hat{H} \;=\; -\,t\sum_{j=1}^{L-1}\left(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j\right) \;+\; \frac{U}{2}\sum_{j=1}^{L} a_j^\dagger a_j^\dagger a_j a_j$$

I'm interested in large U/t.   Data shown for $U = 10$ or $U = -10$

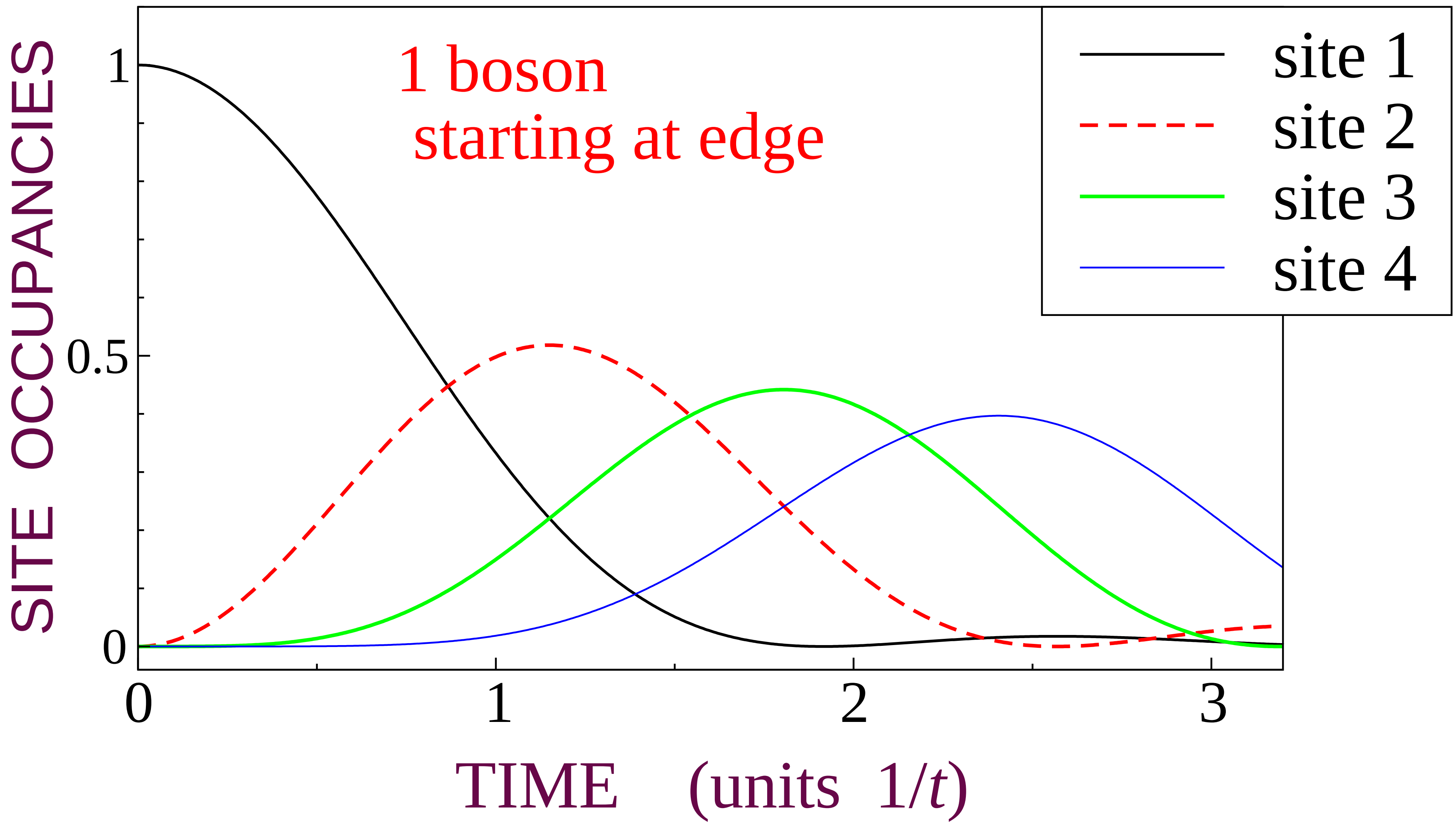


1 0 0 0 0 0 0 0 .....

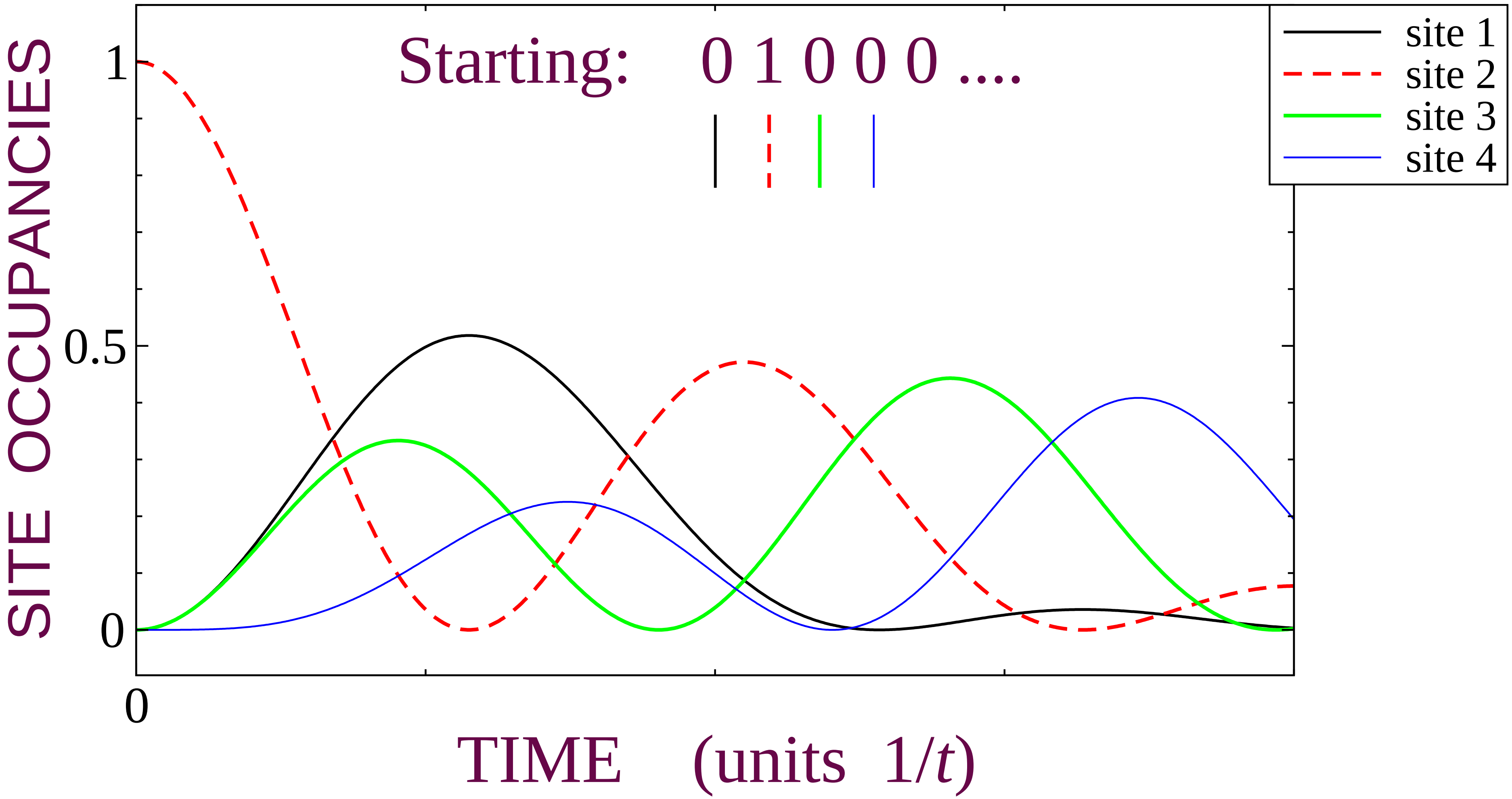
How does this evolve?

At timescales $\sim \hbar/t$

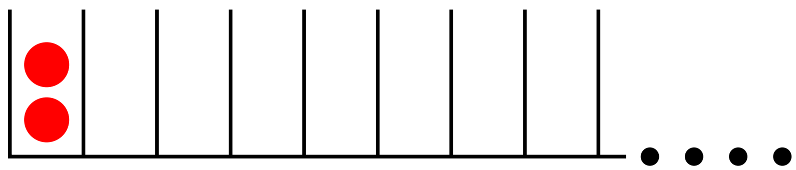
One boson starting at site 1



1 boson starting at site 2 (next-to-edge)



Next: two bosons



2 0 0 0 0 0 0 .....

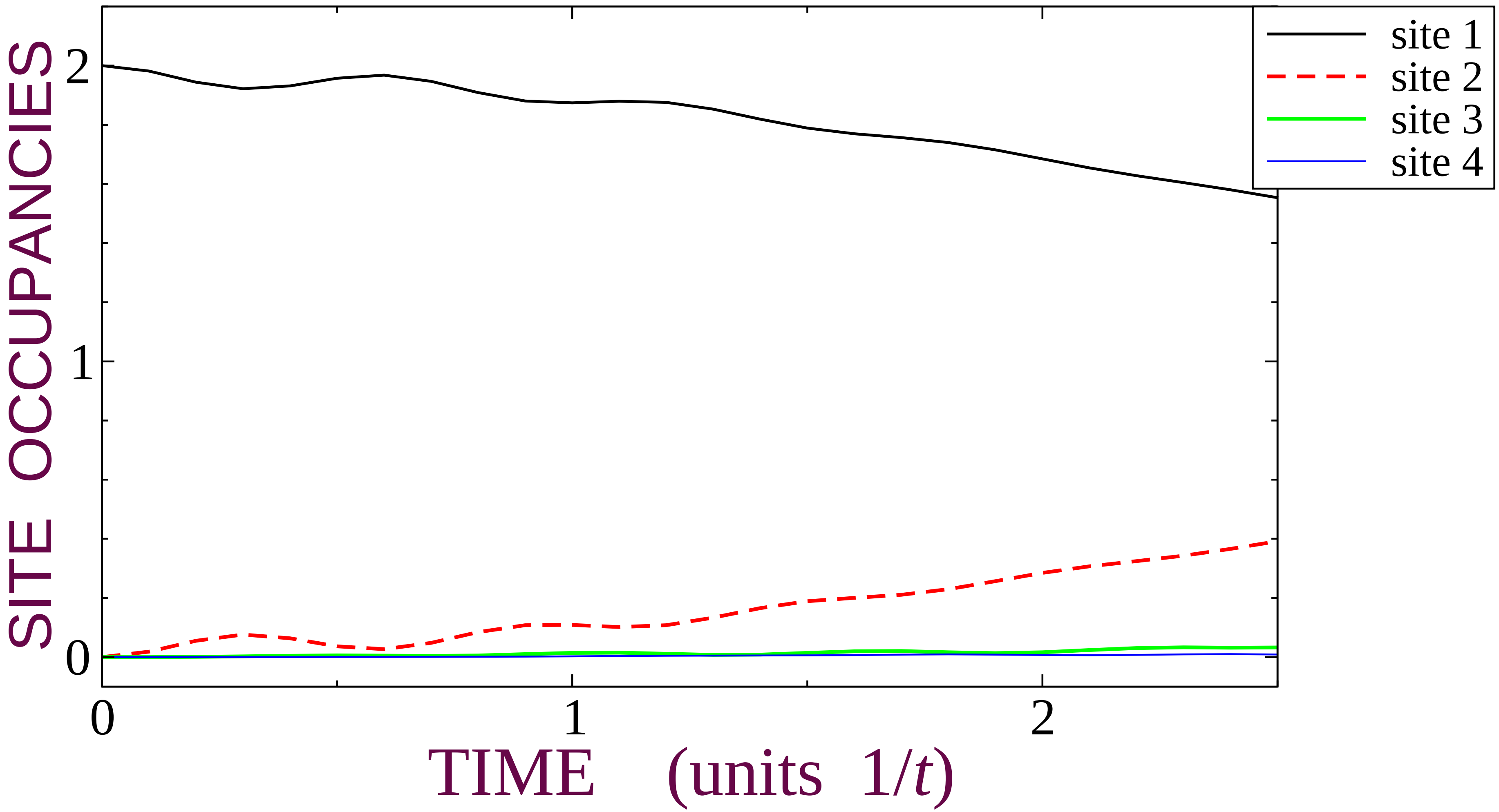
How does this evolve?

At timescales $\sim 1/t \sim 1$

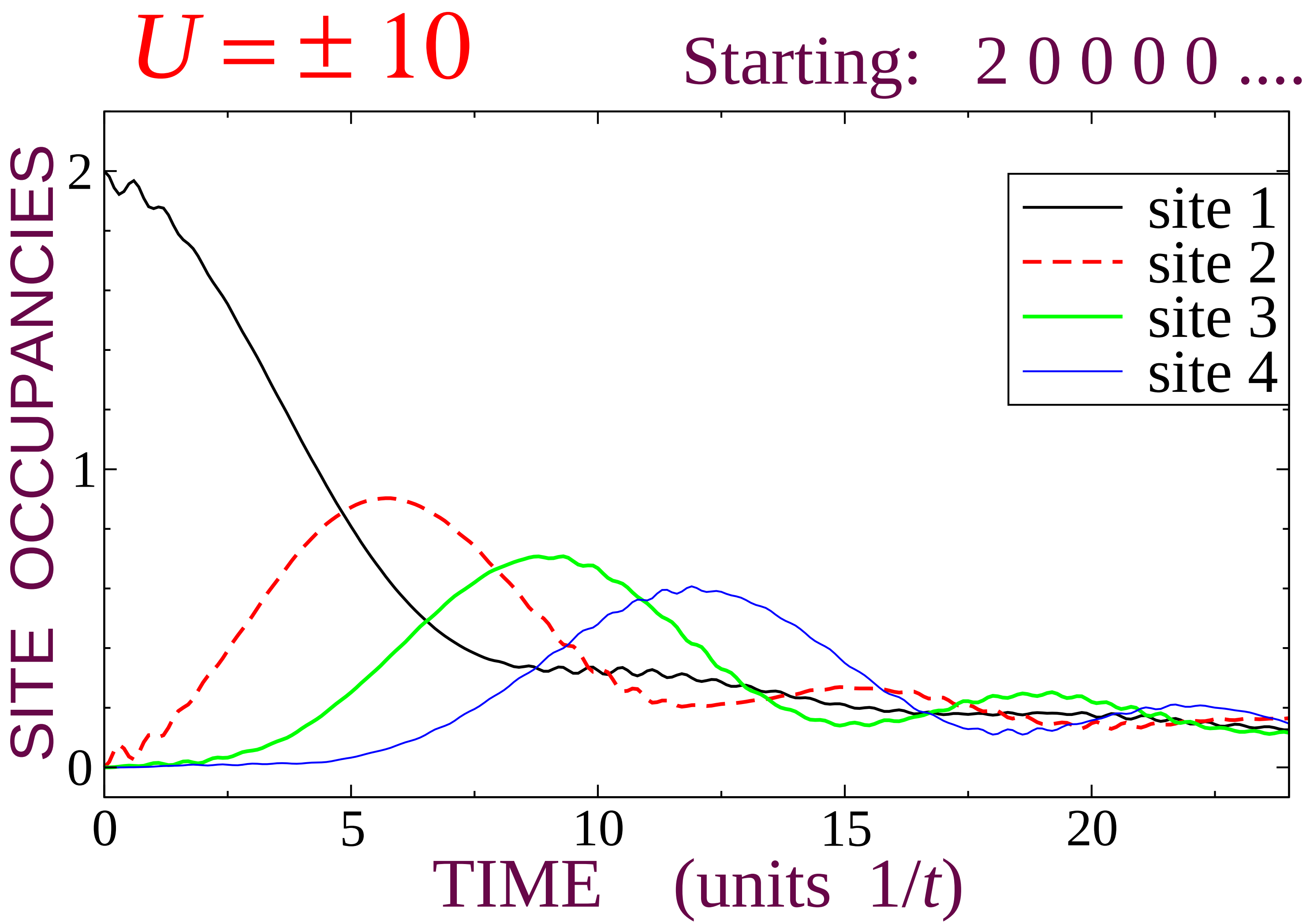
At timescales $\sim 1/(t^2/U) \sim U$

TWO BOSONS AT EDGE: TIMESCALES $\sim 1/t \sim 1$

$U = 10$ Starting: 2 0 0 0 0 ....



2 BOSONS AT EDGE: TIMESCALES $\sim 1/(t^2/U) \sim U$



LARGE U ENCOURAGES CORRELATED PAIR MOTION

Single particle hopping timescale $\sim 1/t \sim 1$

Pair hopping time scale $\sim 1/\left(\frac{t^2}{U}\right) \sim U$

"Repulsively bound pairs"

Triplet hopping time scale $\sim 1/\left(\frac{t^3}{U^2}\right) \sim U^2$

REPULSIVELY BOUND PAIRS

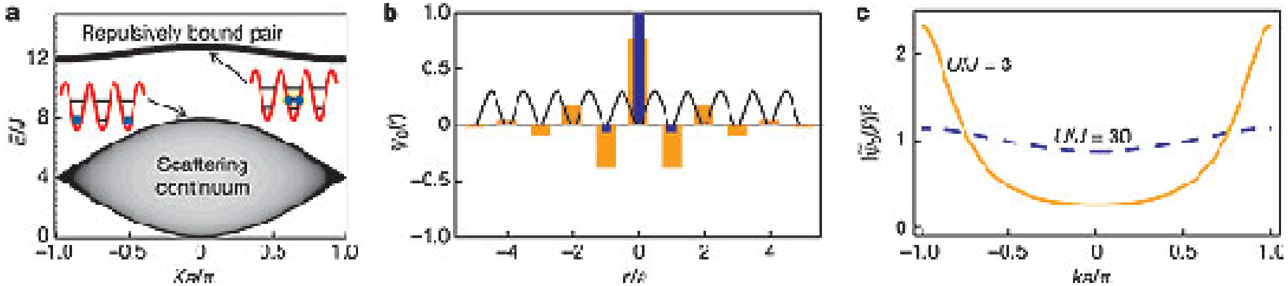
Vol 441|15 June 2006|doi:10.1038/nature04918

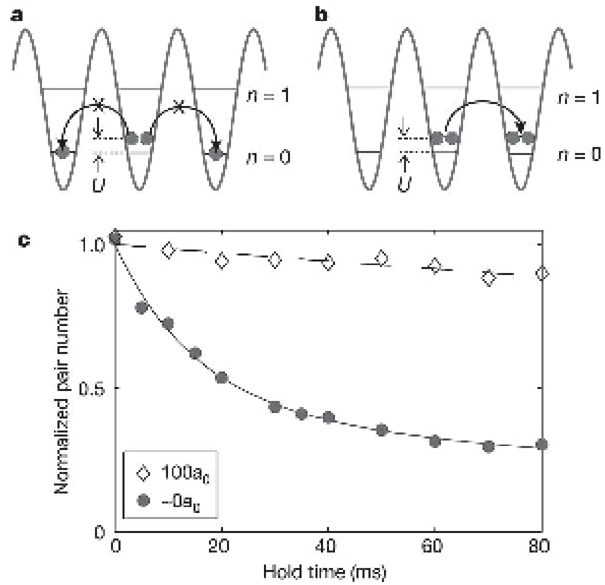
nature

LETTERS

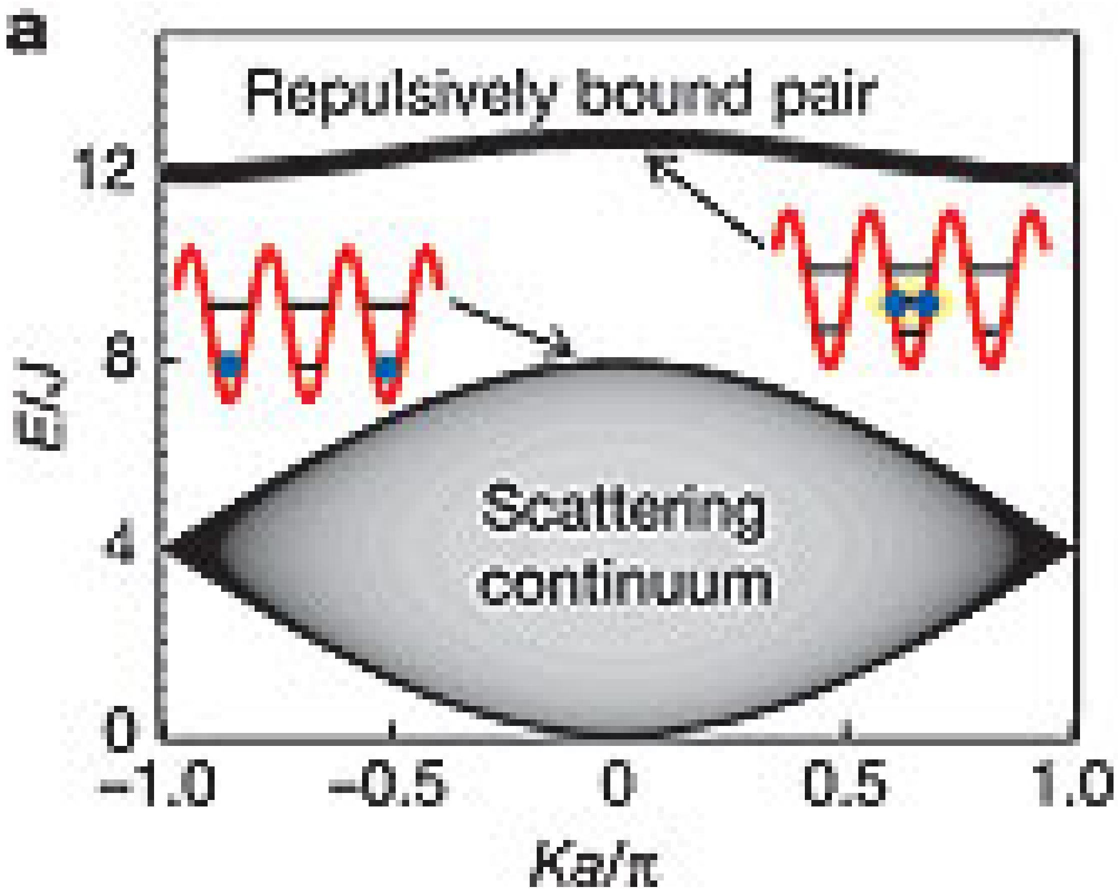
Repulsively bound atom pairs in an optical lattice

K. Winkler[1], G. Thalhammer[1], F. Lang[1], R. Grimm[1,3], J. Hecker Denschlag[1], A. J. Daley[2,3], A. Kantian[2,3], H. P. Büchler[2,3] & P. Zoller[2,3]





“Bands” in energy spectrum, 2 bosons



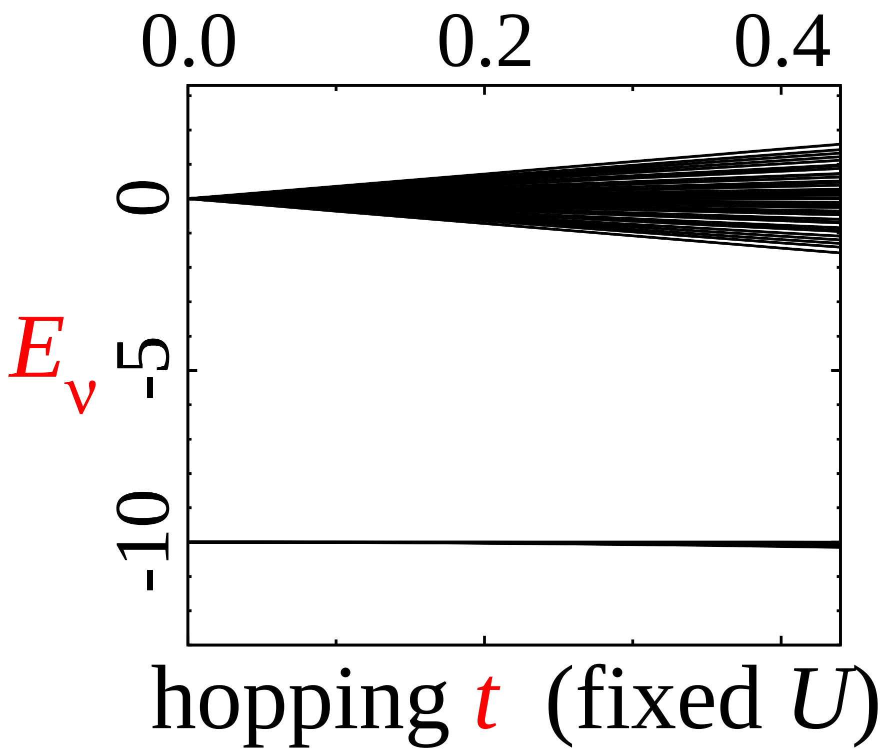
(translation-invariant picture)

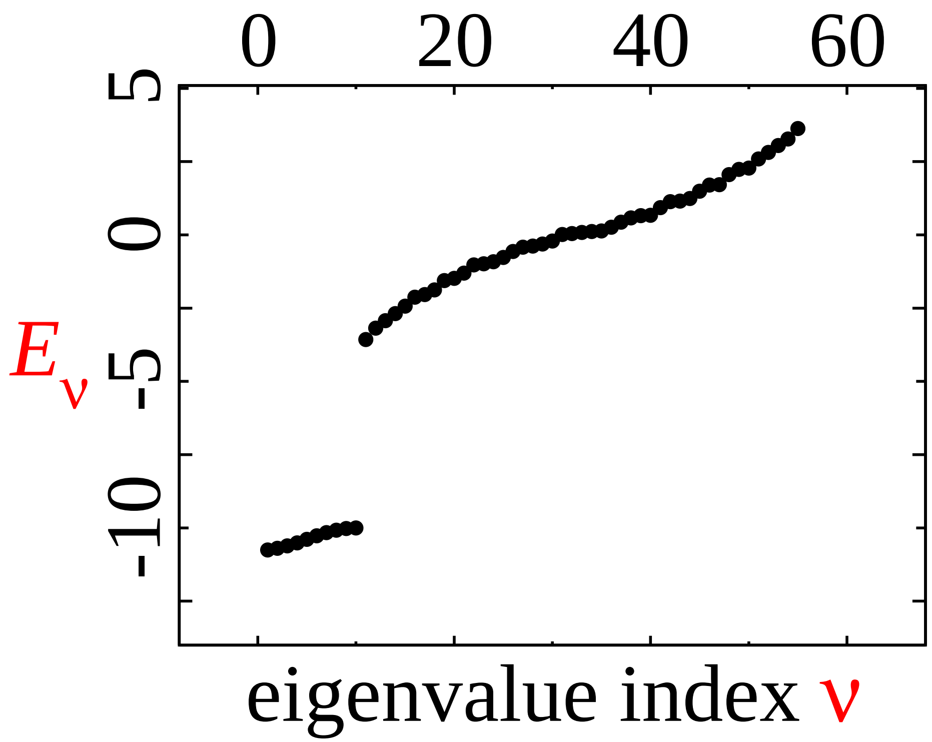
Pairs cannot break without losing energy,

$\Longrightarrow$ without energy relaxation mechanism.

2-BOSON SPECTRUM, BANDS

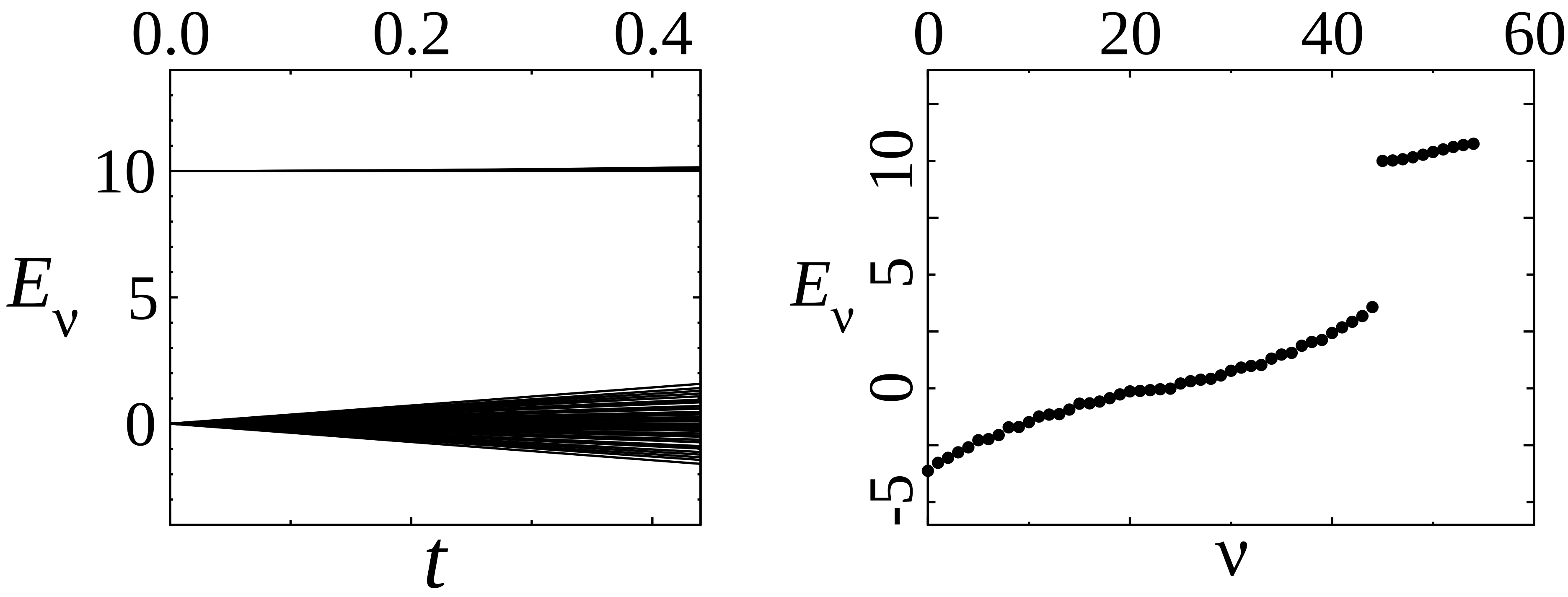
2 Bosons in 10-site open chain. Negative U !! $U = -10$



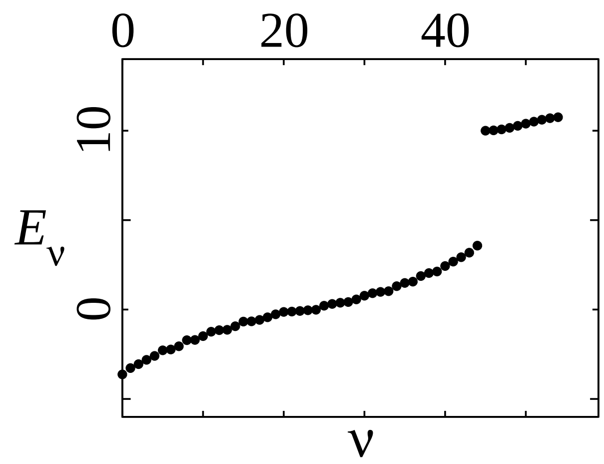


2-BOSON SPECTRUM, BANDS, POSITIVE U

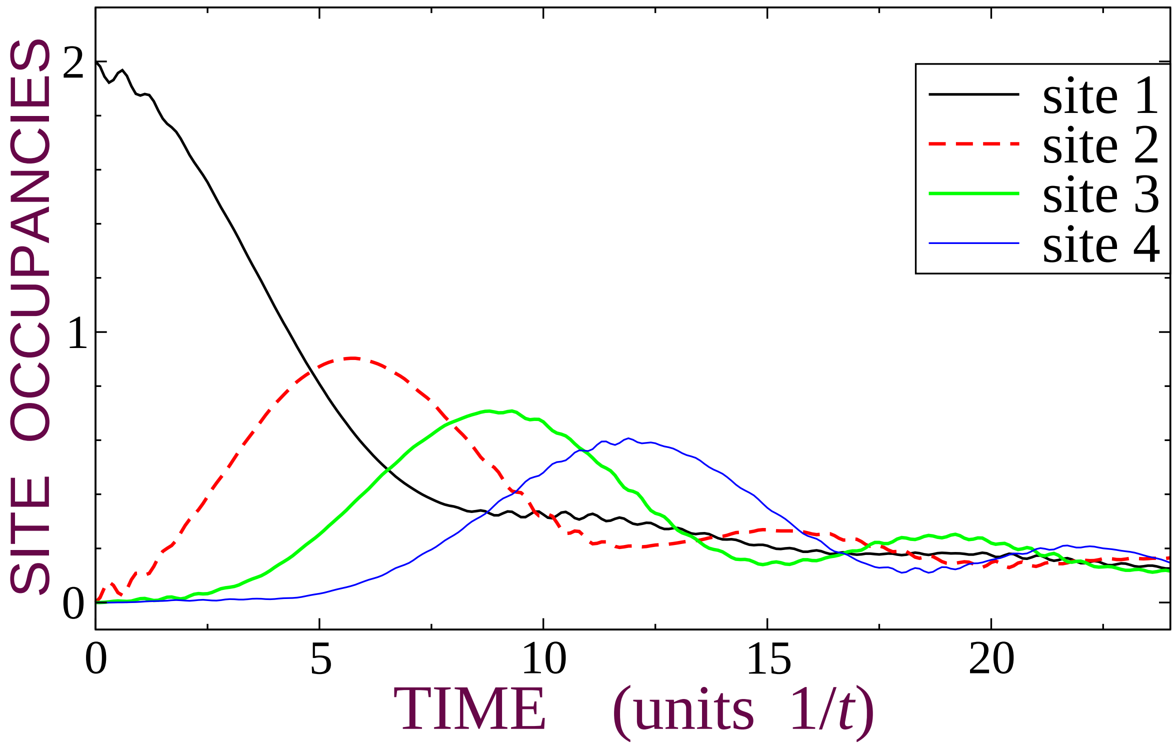
2 Bosons in 10-site open chain. $U = +10$



Two bosons



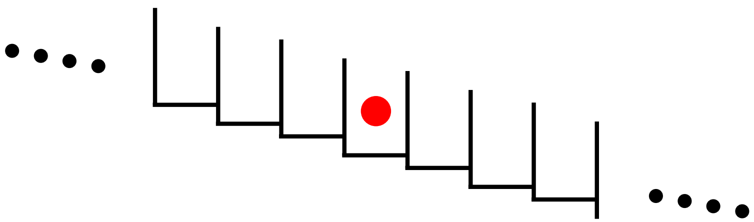
$U = \pm 10$ Starting: 2 0 0 0 0 ....

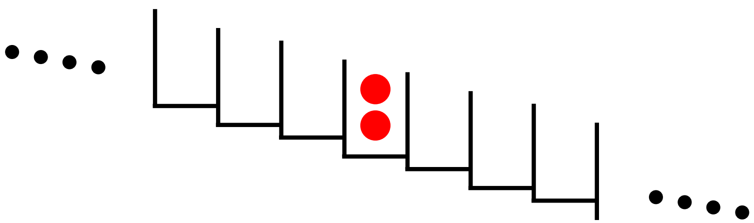


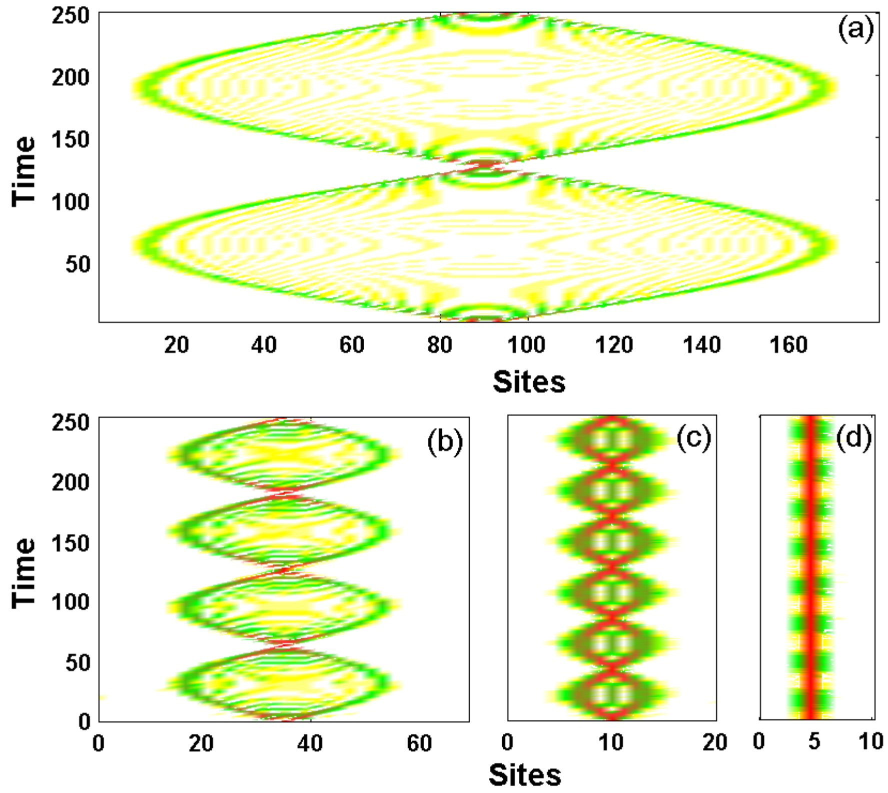
Long time-scale $\rightarrow$ hopping mostly within bound-pair band.

High-frequency oscillations $\rightarrow$ inter-band processes.

Propagation of bound clusters: Bloch oscillations



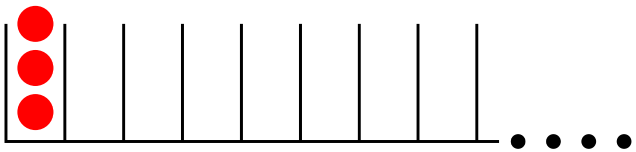




Khomeriki, Krimer, Haque, Flach,

Phys. Rev. A (2010)

Let's move on: THREE bosons

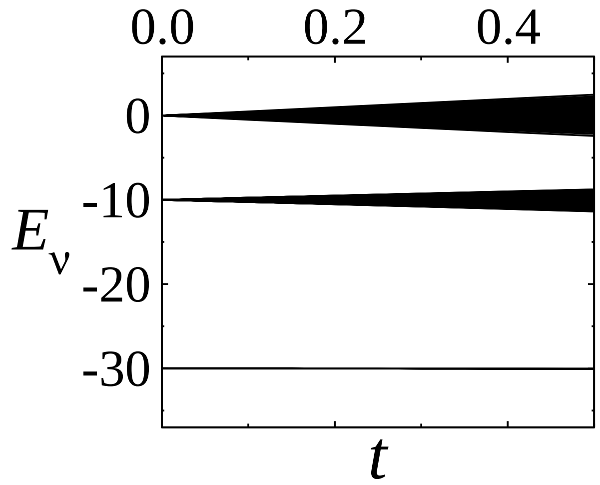


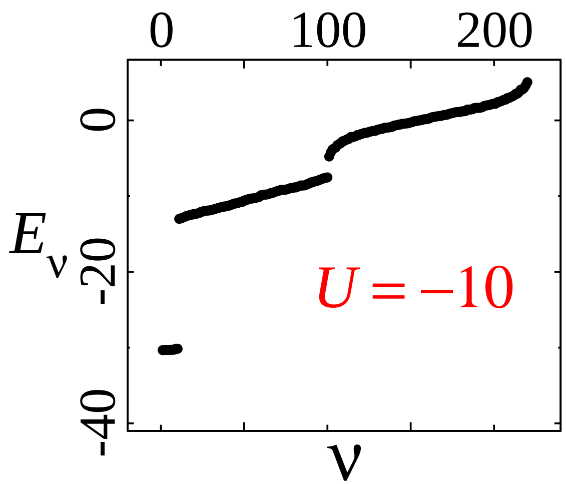
3 0 0 0 0 0 0 ....

How does this evolve?

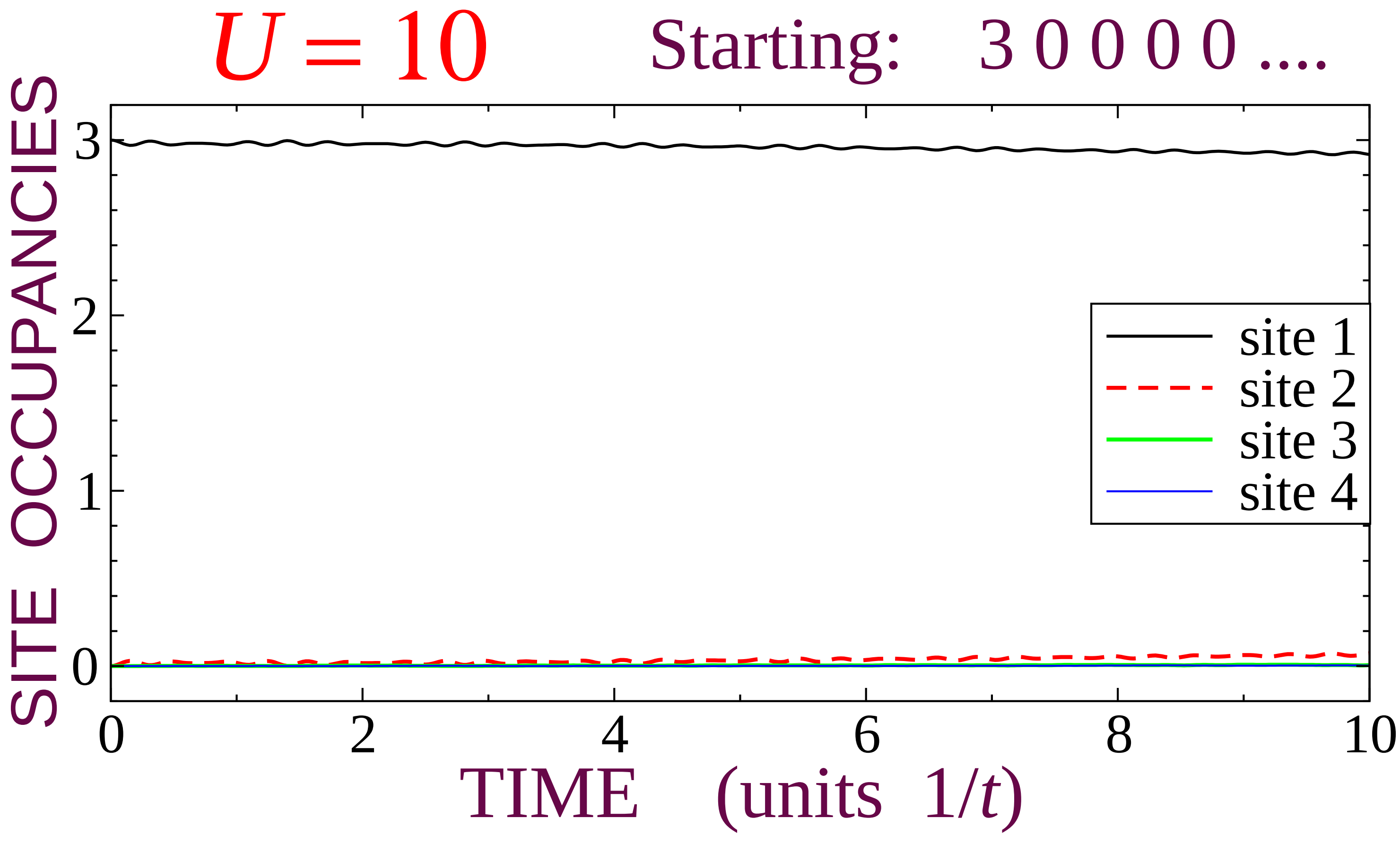
At timescales $\sim 1/t$

At timescales $\sim 1/(t^3/U^2) \sim U^2/t^3$



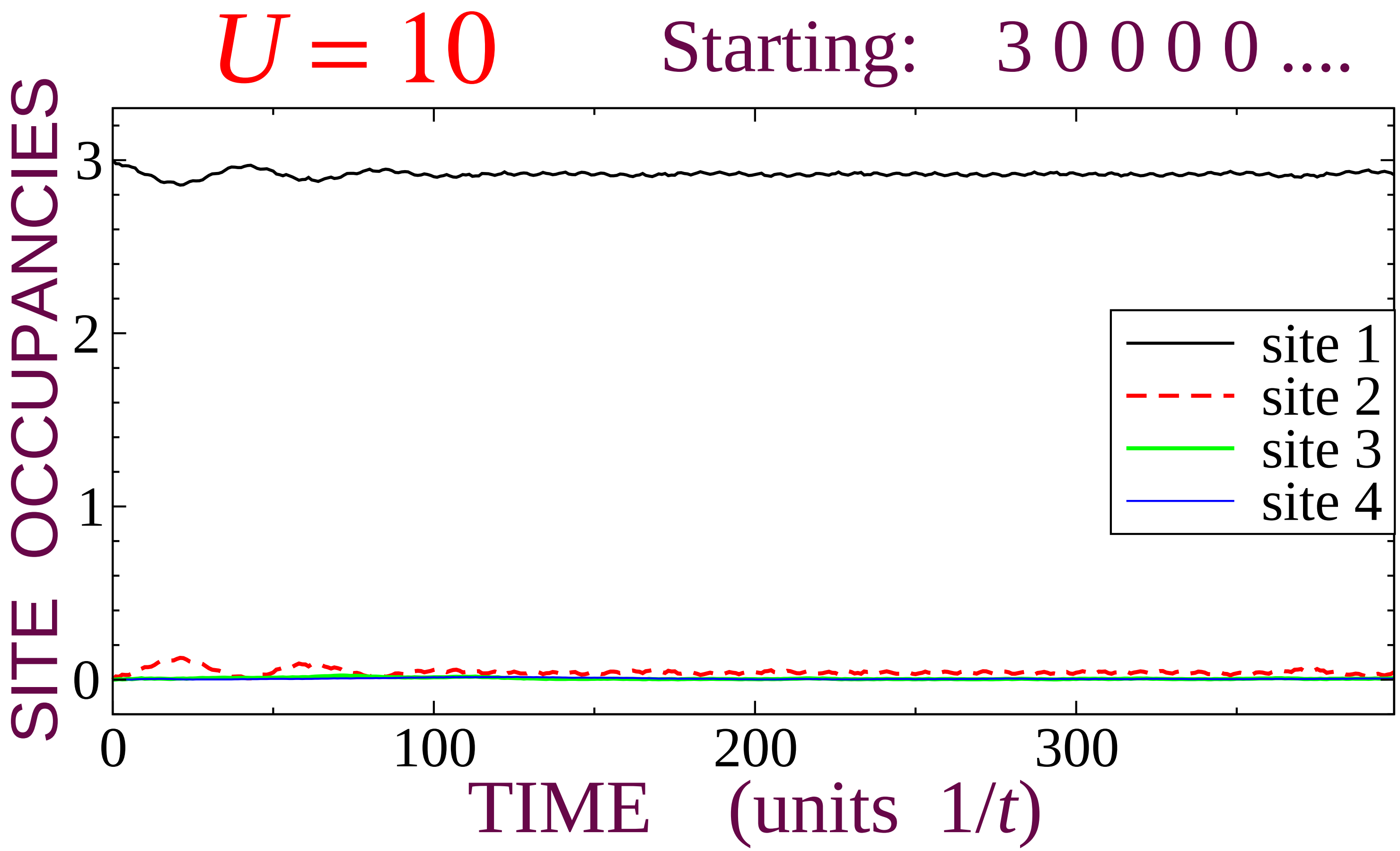


Three bosons at edge: timescales $\sim 1/t$

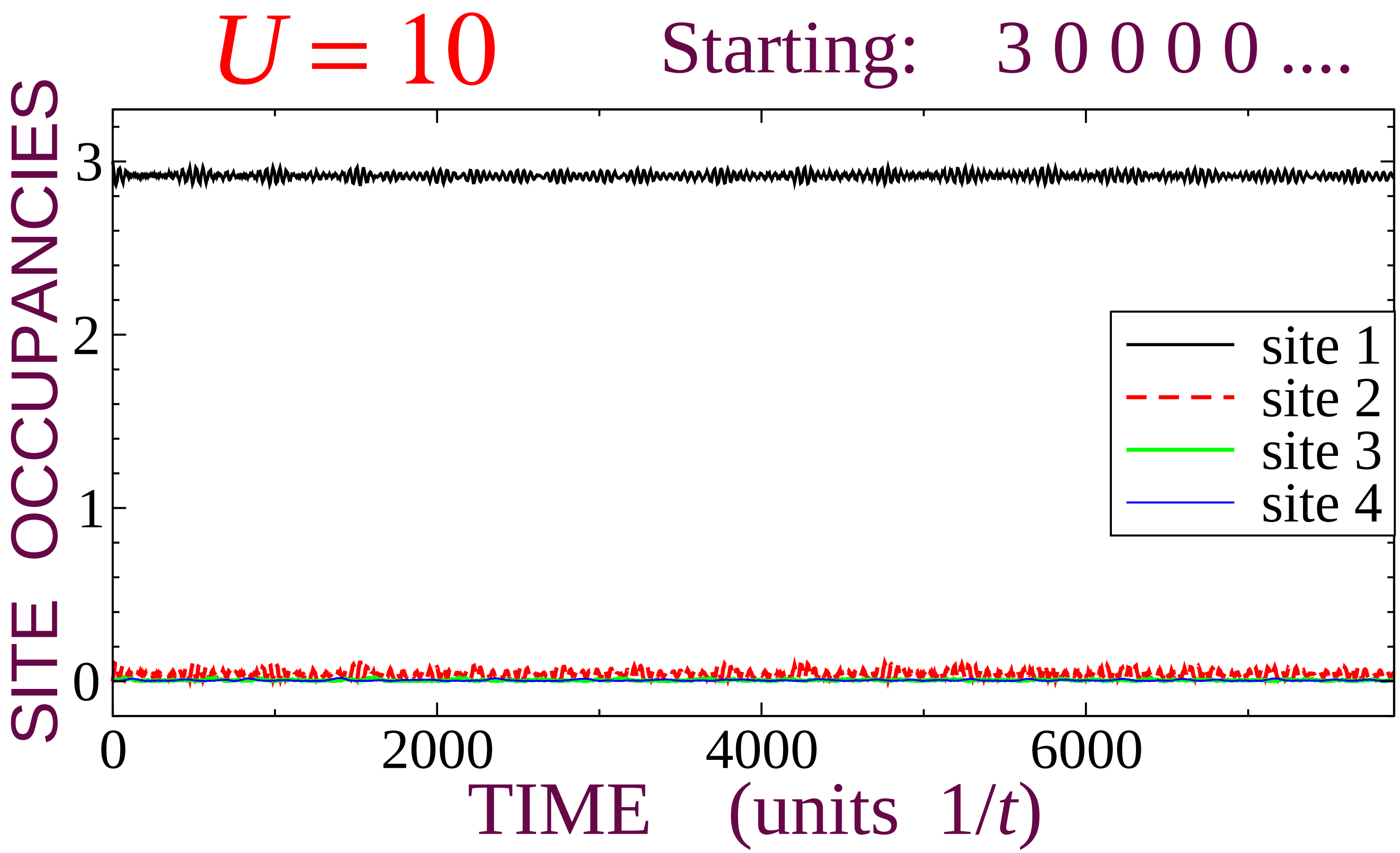


No big surprise.

Three bosons at edge: timescales $\sim U^2$



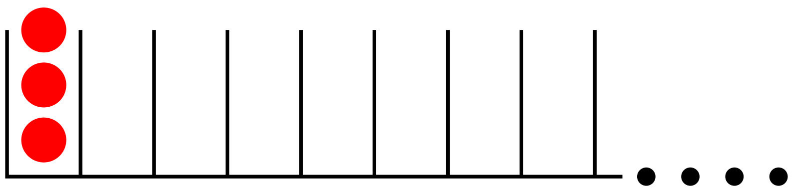
TRYING TIMESCALES $\gg\sim U^2$



? ? ? ? ? ? ? ? ?

You should be surprised

WE'VE FOUND A STABLE STATE

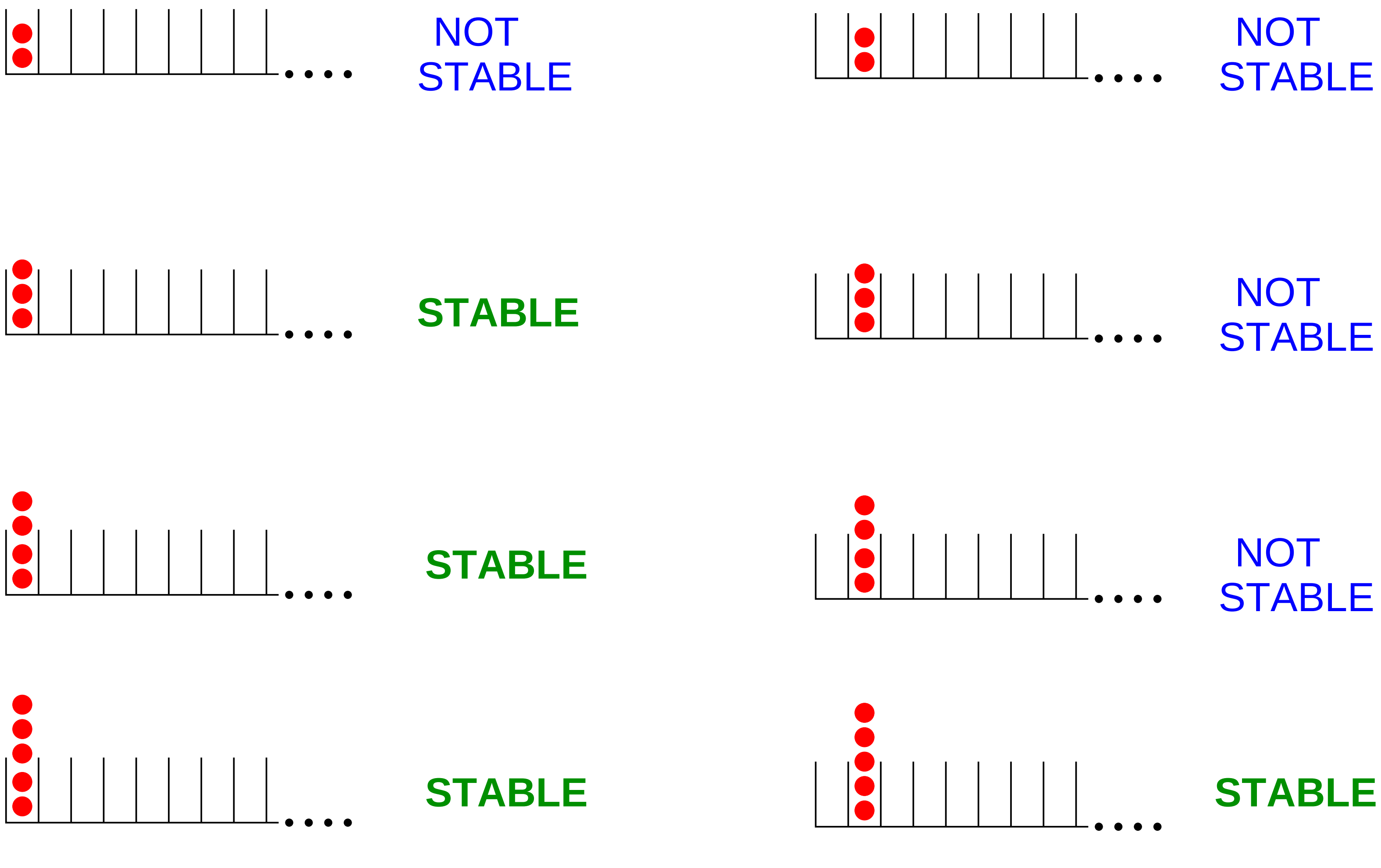


3 0 0 0 0 0 0 ....

For $n \geq 3$ bosons, edge states are stable.

Stable should mean "close" to an eigenstate?

Hierarchy of edge-locked states



Digression

Background Context $\longrightarrow$

Non-equilibrium dynamics in isolated quantum systems

$+$

Advertisement $\longrightarrow$

near-adiabatic ramps in many-body systems

Non-equilibrium dynamics in isolated quantum systems

Isolated Quantum many-body systems

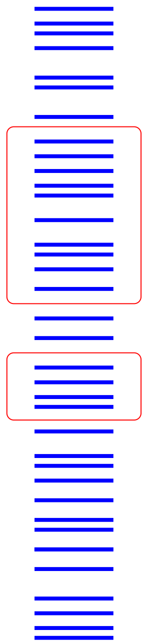
(Cold atoms, some nano-devices):

No external bath

No dissipation!

Unitary quantum dynamics

No tendency toward
ground state



New questions & phenomena

[a] thermalization in isolated systems

Generalized Gibbs Ensemble

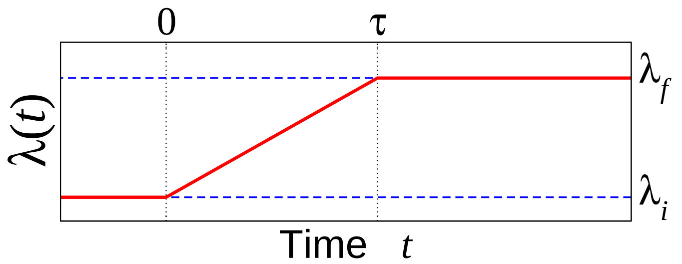
Eigenstate Thermalization Hypothesis

role of Integrability

[b] repulsively bound pairs & clusters

Non-equilibrium dynamics in isolated quantum systems: New questions & phenomena

[c] (Deviation from) adiabaticity in finite-time ramps



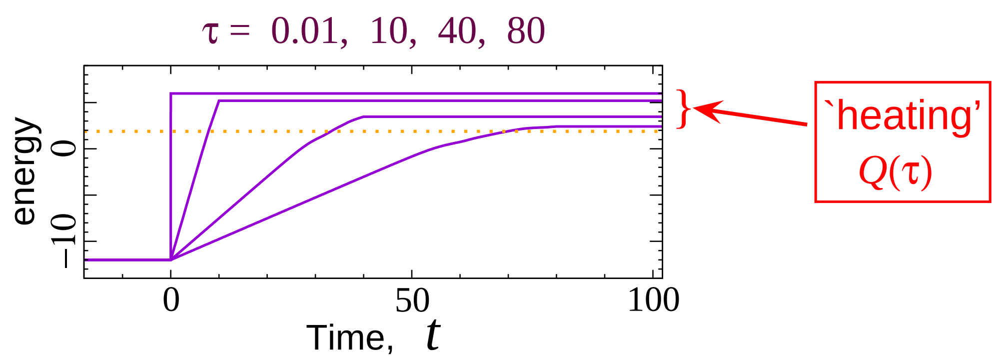
$\tau = 0 \quad \Longleftrightarrow \quad$ sudden quench

$\tau = \infty \quad \Longleftrightarrow \quad$ adiabatic ramp

Finite $\tau \rightarrow$ system doesn't reach final ground state.

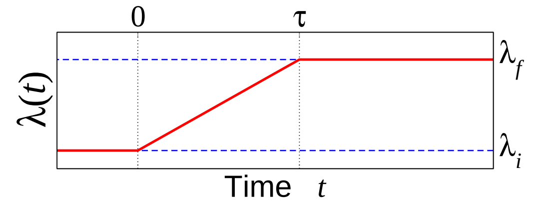
Quantify non-adiabatic through

excess excitation energy over final g.s. energy.



$$Q = E(t \gg \tau) - E_{g.s.}^{(\lambda_f)}$$

DEVIATIONS FROM ADIABATICITY: $Q(\tau)$



Adiabatic theorem: $Q(\infty) = 0$

Asymptotic decay of $Q(\tau)$

$\longrightarrow$ first correction to adiabaticity

$Q(\tau)$ can decay:

Exponentially, as power-law;
With/without oscillations or logarithms

Dóra, Haque, Zaránd; P.R.L. 2011
Pollmann, Haque, Dóra; in prep.

Luttinger Liquid

Venumadhav, Haque, Moessner; P.R.B 2010
Tschischik, Haque, Moessner; arXiv:1209.5534

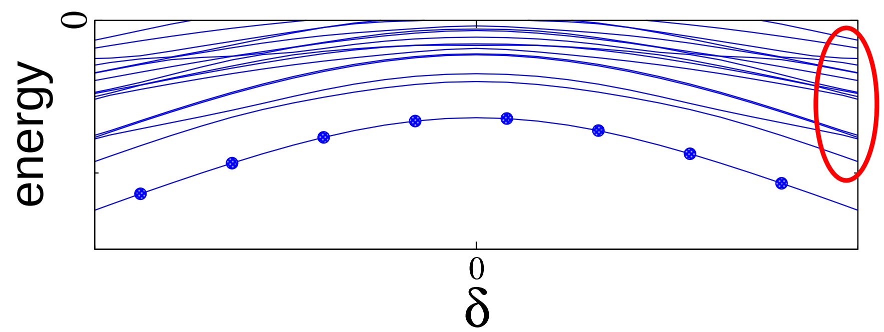
Bose-Hubbard dimers, ladders

Haque & Zimmer, arXiv:1110.0840
Zimmer & Haque, arXiv:1012.4492

Generic interacting trapped systems

Adiabaticity question [$Q(\tau)$]:

meaningful due to lack of dissipation

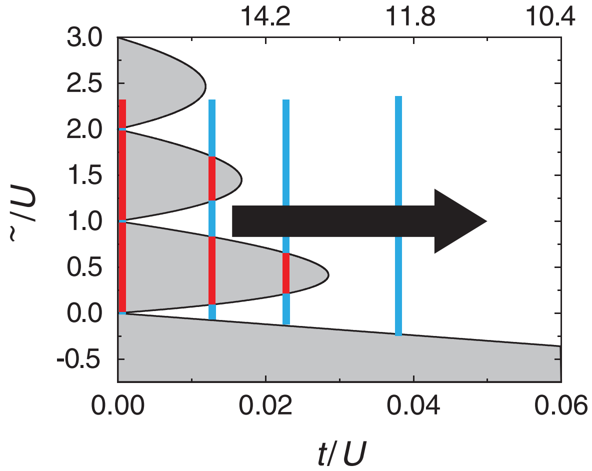


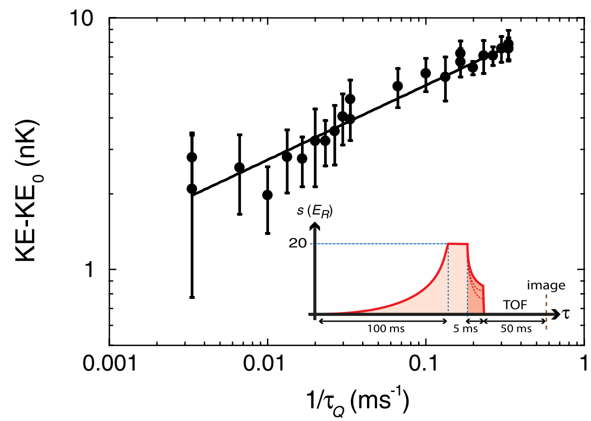
Deviations from adiabaticity: Recent Experiments

UIUC (DeMarco) group

P.R.L. (2011)

Bose-Hubard in trap
ramp of interaction

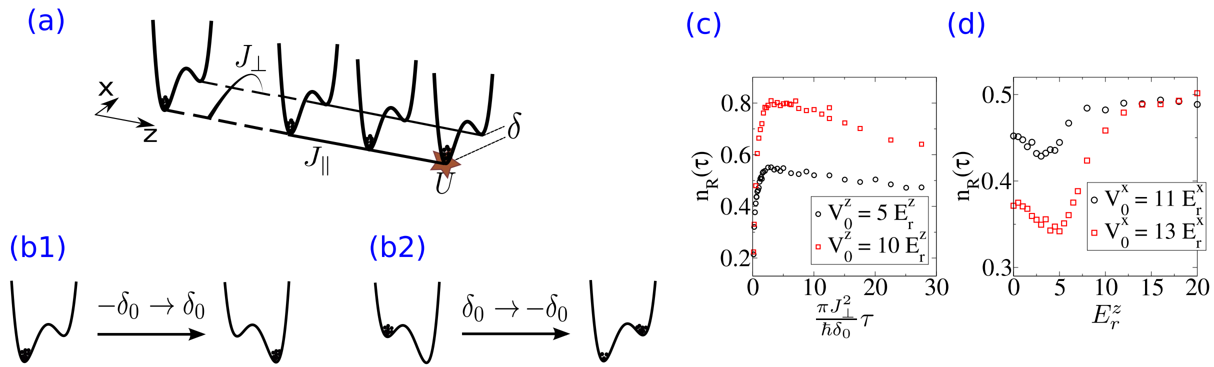




Munich (Bloch) group

Nat. Phys. (2011)

Bose-Hubard in ladder
ramp of bias between legs

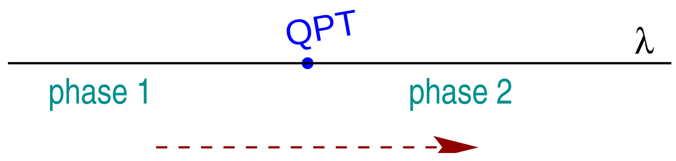


DECAY OF $Q(\tau)$:

UNIVERSALITY IN TRAPPED SYSTEMS

Haque & Zimmer,
arXiv:1110.0840

Zimmer & Haque,
arXiv:1012.4492



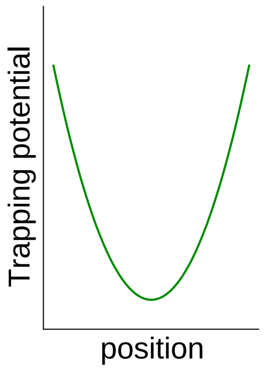
Ramping across quantum phase transitions

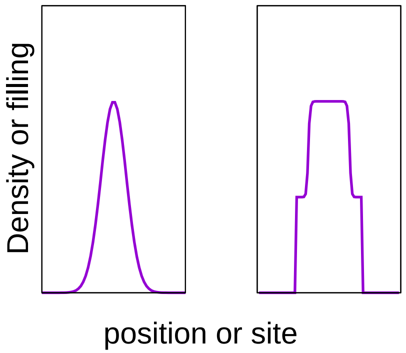
Kibble-Zurek scaling: $Q(\tau) \sim \tau^{-\alpha}$

Uniform systems

Most likely experimental realization:

trapped atoms





HARMONIC CONFINEMENT

$$H = H_{\text{system}} + V_{\text{trap}}$$

INTERACTION RAMPS

Asymptotic decay of $Q(\tau)$ $\longrightarrow$

UNIVERSAL FEATURES

Near-adiabatic ramps [$Q(\tau)$] : Take-home message

Asymptotic decay of $Q(\tau)$ $\longleftrightarrow$ first correction to adiabaticity

Many-particle systems in harmonic trap:

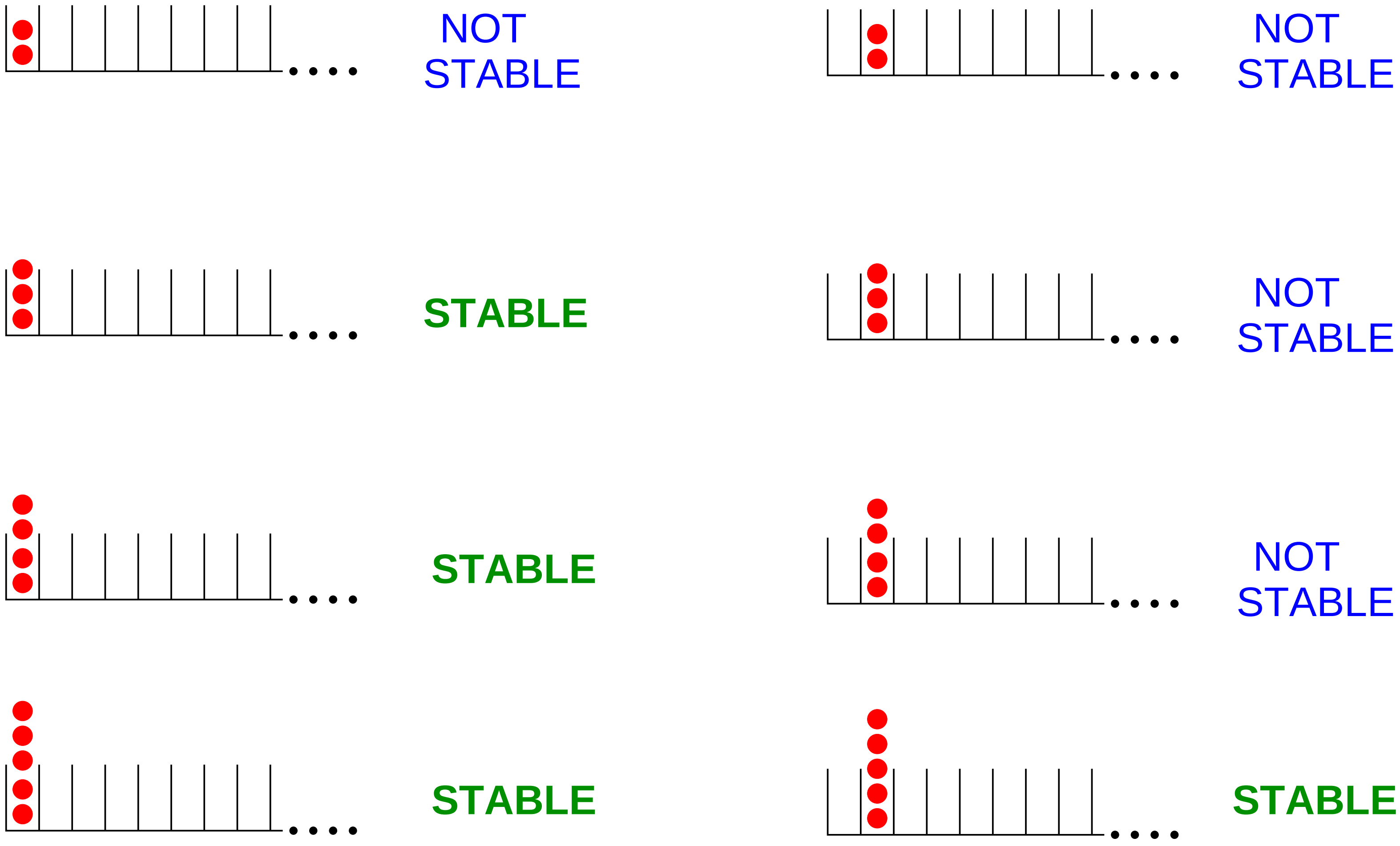
Universal deviations from adiabaticity due to size dynamics

Very different systems, same behavior of $Q(\tau)$

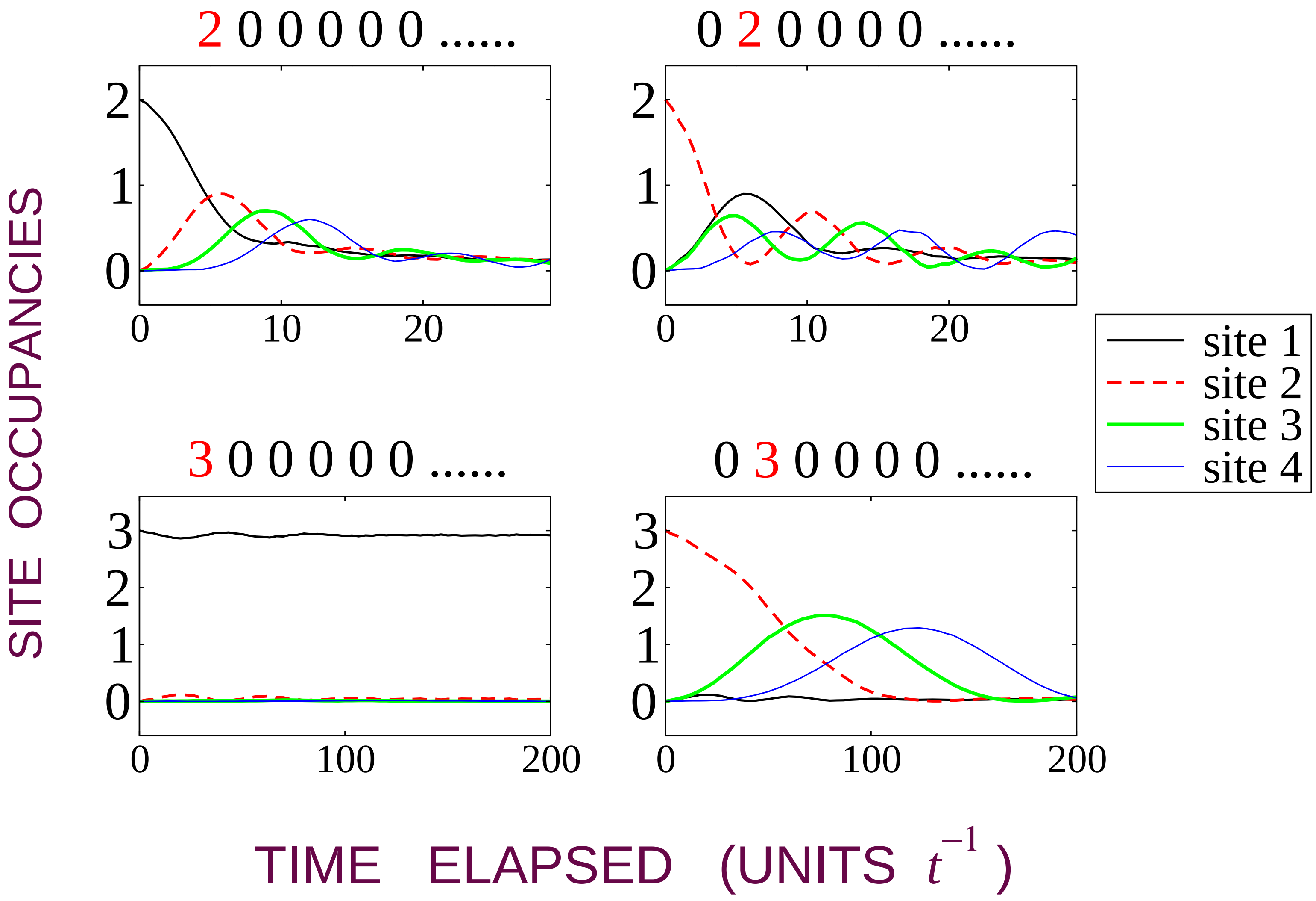
Consequence:

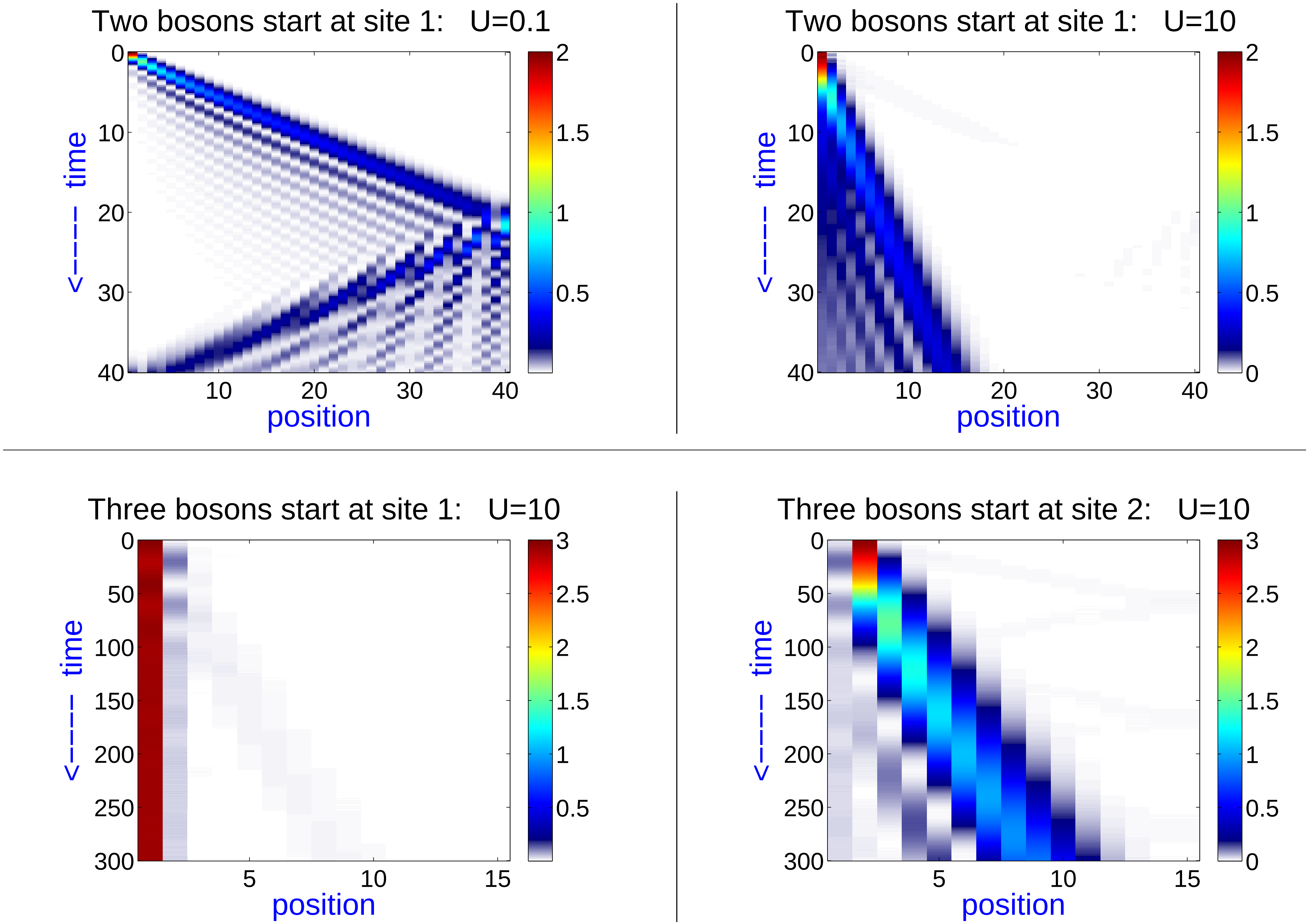
homogeneous-system predictions (Kibble-Zurek scaling etc)
will be hidden or absent in trap experiments

Hierarchy of edge-locked states



Time evolution summary



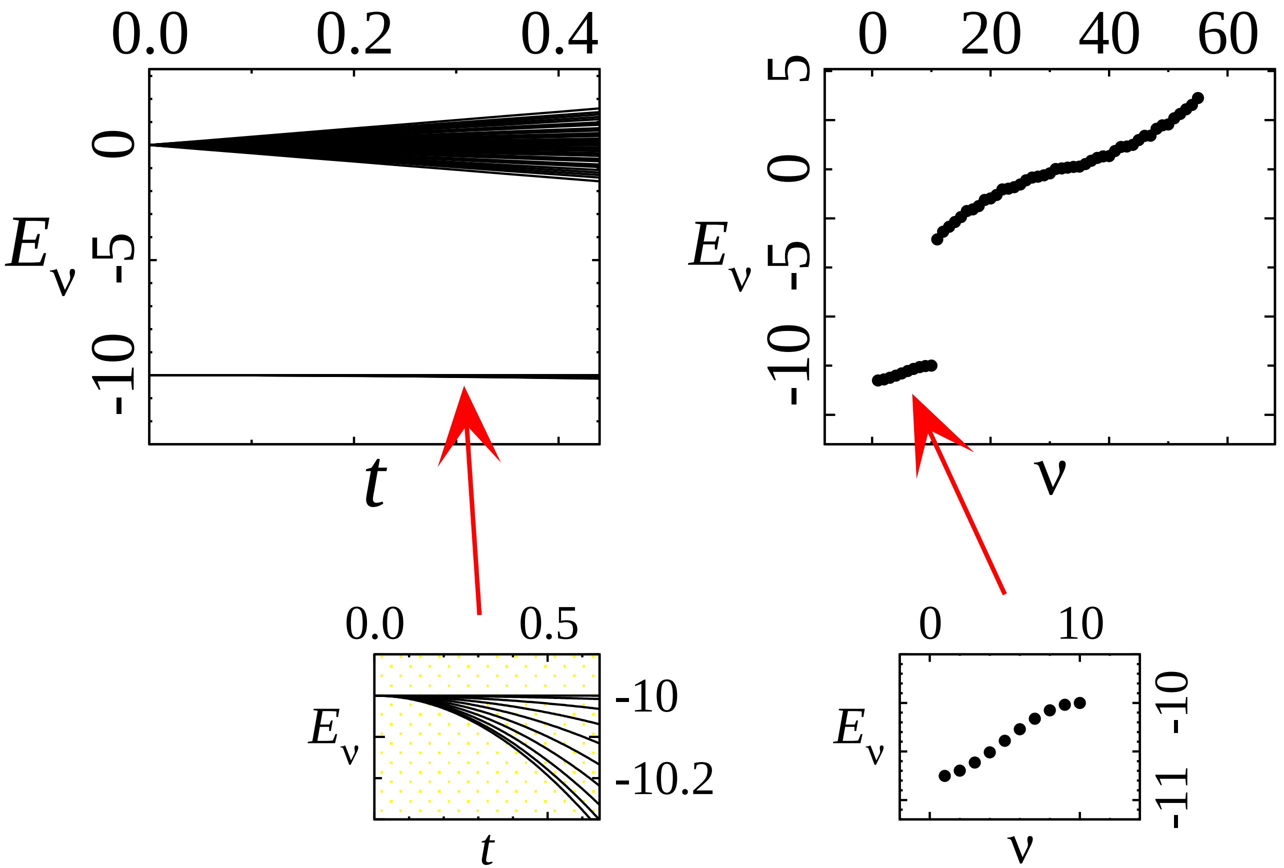

Two bosons start at site 1: U=0.1
Two bosons start at site 1: U=10
Three bosons start at site 1: U=10
Three bosons start at site 2: U=10
time
<-----
position
0
10
20
30
40
0.5
1
1.5
2
50
100
150
200
250
300
5
15
2.5
3

Spectral explanation

To explain geometric locking, examine spectrum

- Focus on “bound” band.
- Compare $n = 2$ and $n = 3$

Structure of 'bound' band: two bosons



Linear combinations of

$|20000.....000\rangle$

$|02000.....000\rangle$

$|00200.....000\rangle$

$|00020.....000\rangle$

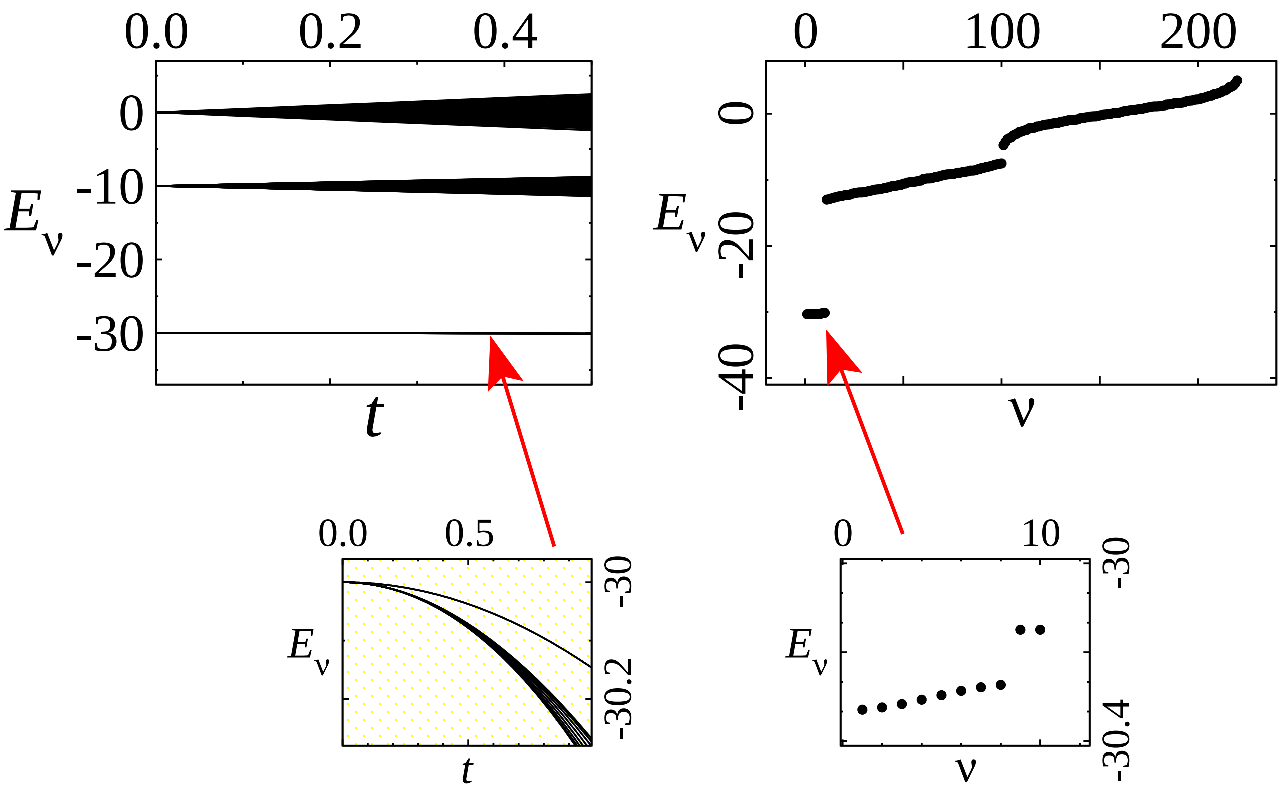
..

..

$|0000.....002\rangle$

... plus tiny non-bound contributions

‘Bound’ band: three bosons



Linear combinations of

$|03000.....000\rangle$

$|00300.....000\rangle$

$|00030.....000\rangle$

..

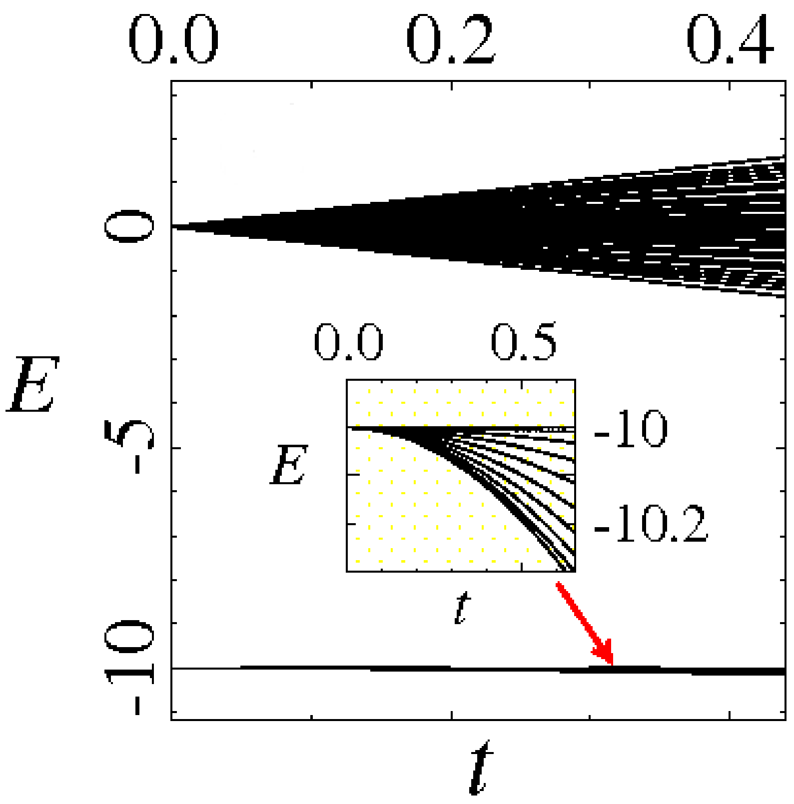
..

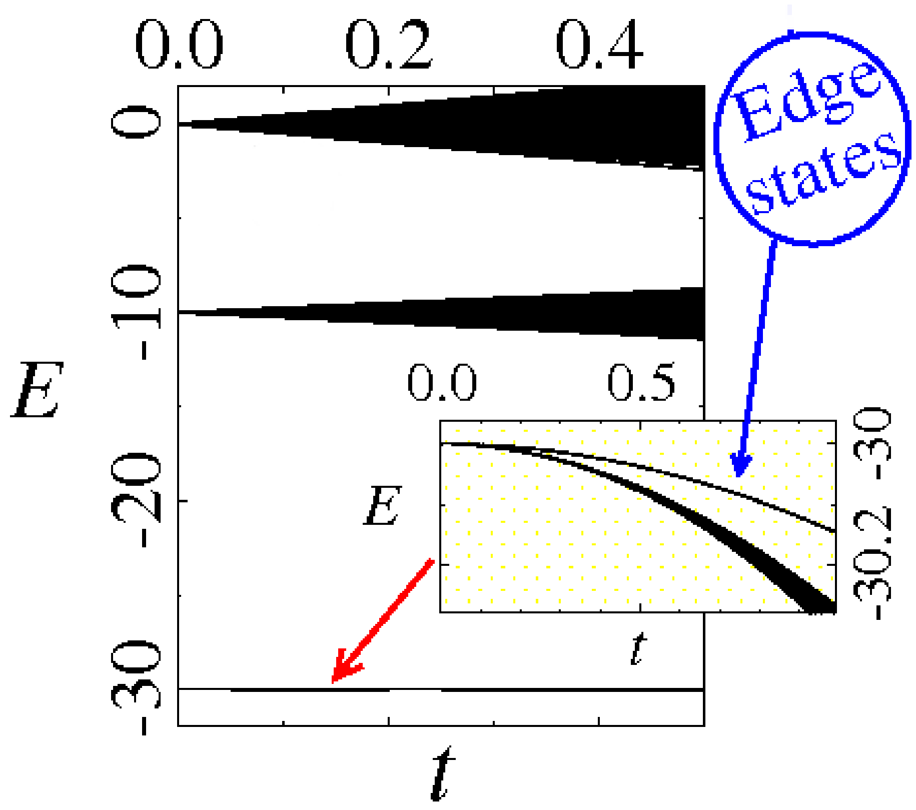
$|0000.....030\rangle$

Separated out from the rest: $|30000.....000\rangle$ and $|0000.....003\rangle$.

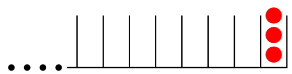
2 bosons

3 bosons





| L > = 

| R > = 

Edge eigenstates: | L > + | R > and | L > − | R >

Tunnel to other edge?

$$|L\rangle = |3000.....00\rangle \qquad \text{and} \qquad |R\rangle = |00.....0003\rangle$$

Question: Why doesn't $|L\rangle$ tunnel to $|R\rangle$?

Answer: It will. After some astronomically long time.

$|L\rangle \leftrightarrow |R\rangle$ tunneling exponentially suppressed.

Splitting between $|L\rangle + |R\rangle$ and $|L\rangle - |R\rangle$ exponentially small.

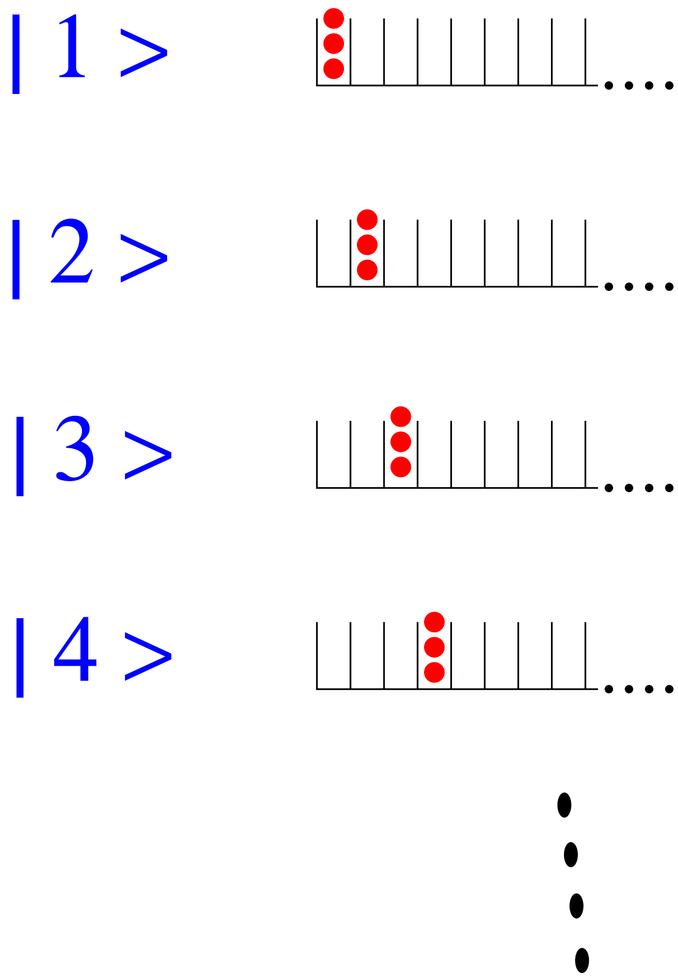
Spectral separation explains stability of edge states

Who ordered the spectral separations?

Degenerate perturbation theory.

Competition between energy shifts at $\mathcal{O}(t^2)$ and manifold mixing at $\mathcal{O}(t^n)$.

Degenerate perturbation theory



Degenerate manifold at $t/U = 0$.

States $|j\rangle$ and $|j+1\rangle$ connect at $\mathcal{O}(t^n)$.

$\Longrightarrow$ mixing / dispersion at $\mathcal{O}(t^n)$.

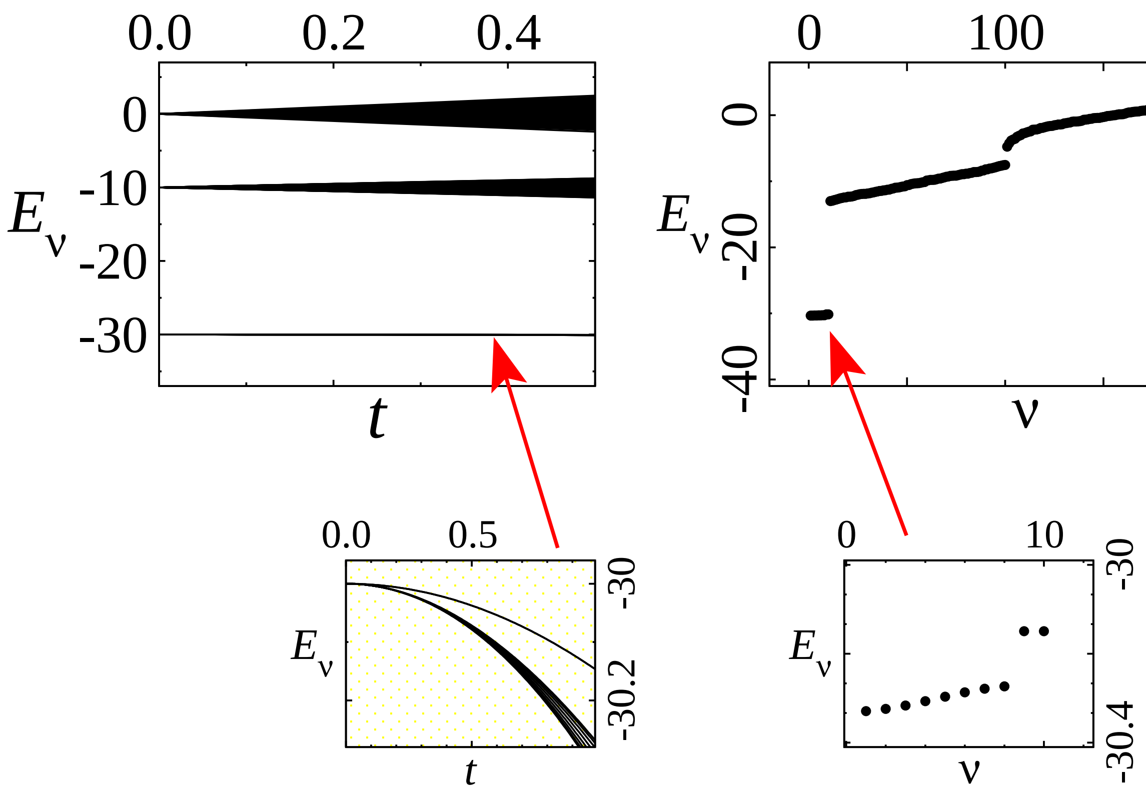
State $|1\rangle$ acquires different shift at $\mathcal{O}(t^2)$.

$\Downarrow$

Spectral separation if $\mathcal{O}(t^2)$ beats $\mathcal{O}(t^n)$.
(1st level of hierarchy)

State $|2\rangle$ acquires different shift at $\mathcal{O}(t^4)$.
(2nd level) ....

THREE BOSONS: $\mathcal{O}(t^2)$ VERSUS $\mathcal{O}(t^3)$



Linear combinations of

$|03000.....000\rangle$

$|00300.....000\rangle$

$|00030.....000\rangle$

..

..

$|0000.....030\rangle$

Separated out from the rest: $|30000.....000\rangle$ and $|0000.....003\rangle$.

What I'm missing....

There should be a sum over histories interpretation	3 0 0 0 0 ...... 0 3 0 0 0 ...... 0 0 3 0 0 ...... 2 1 0 0 0 ...... 2 0 1 0 0 ...... 1 2 0 0 0 ...... 1 0 2 0 0 ...... 1 1 1 0 0 ...... 1 1 0 1 0 ......

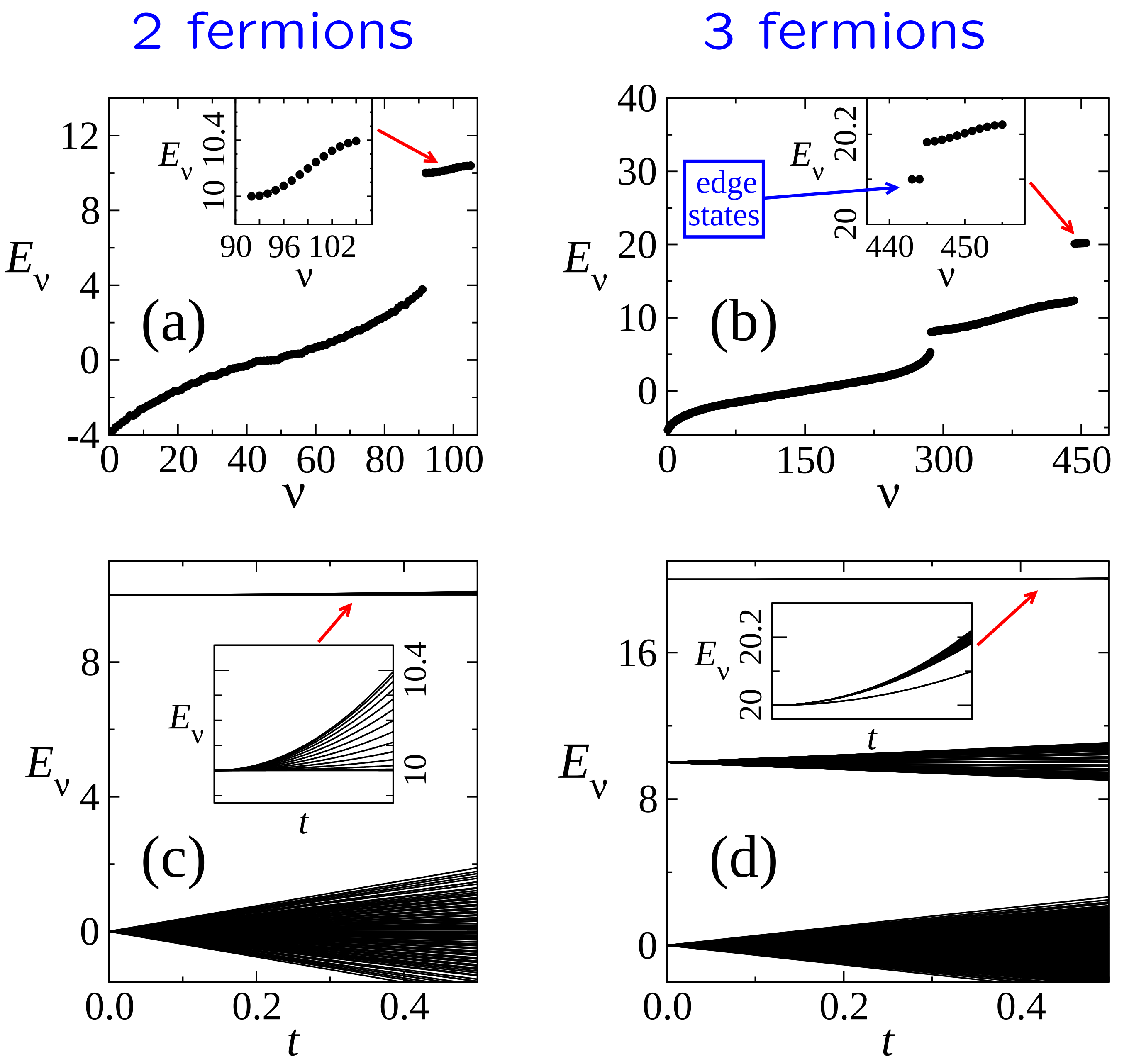
Spinless fermion (t-V) model: similar hierarchy

$$\hat{H} \;=\; -\; t\sum_{j=1}^{L-1}\left(c_j^{\dagger}c_{j+1} \;+\; c_{j+1}^{\dagger}c_j\right) \;+\; V\sum_{j=1}^{L-1} c_j^{\dagger}c_{j+1}^{\dagger}c_{j+1}c_j$$

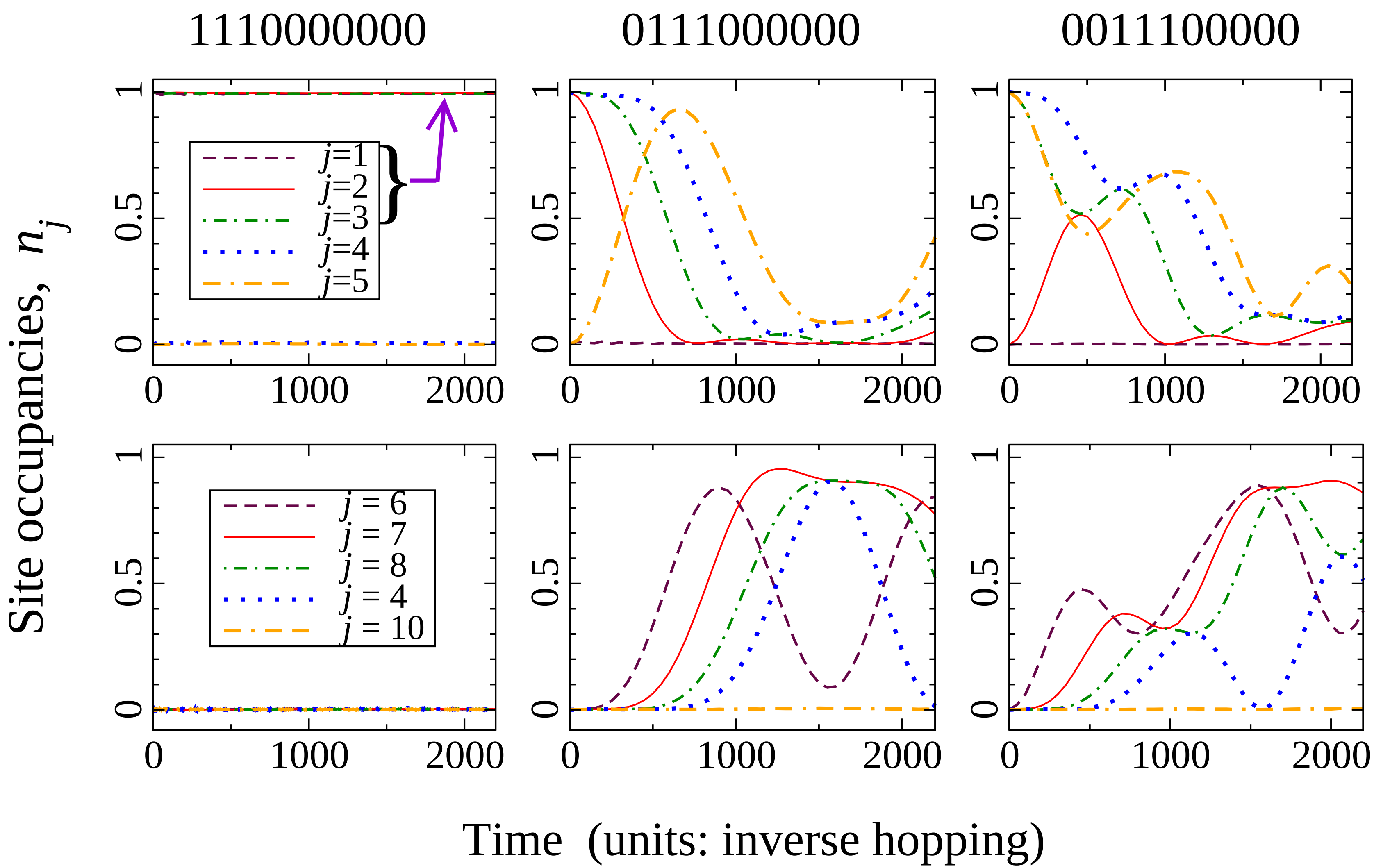
Edge-locked configurations	Not locked
1 1 1 0 0 0 0 0 0 0 0	1 0 0 0 0 0 0 0 0 0 0 ......
1 1 1 1 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0 0 ......
1 1 1 1 1 0 0 0 0 0 0	0 1 0 0 0 0 0 0 0 0 0 ......
1 1 1 1 1 1 0 0 0 0 0	0 1 1 0 0 0 0 0 0 0 0 ......
0 1 1 1 1 1 0 0 0 0 0	0 1 1 1 0 0 0 0 0 0 0 ......
0 1 1 1 1 1 1 0 0 0 0	0 1 1 1 1 0 0 0 0 0 0 ......

Spinless fermions: spectrum

Two, three fermions in 15 sites



SPINLESS FERMIONS: DYNAMICS



DIGRESSION # 2

'Workshop & Seminar' at MPI-PKS Dresden:

Quantum many body systems out of equilibrium

August 12 - 30, 2013

Organized by:

J.S. Caux, Tilman Esslinger, Masud Haque, Corinna Kollath

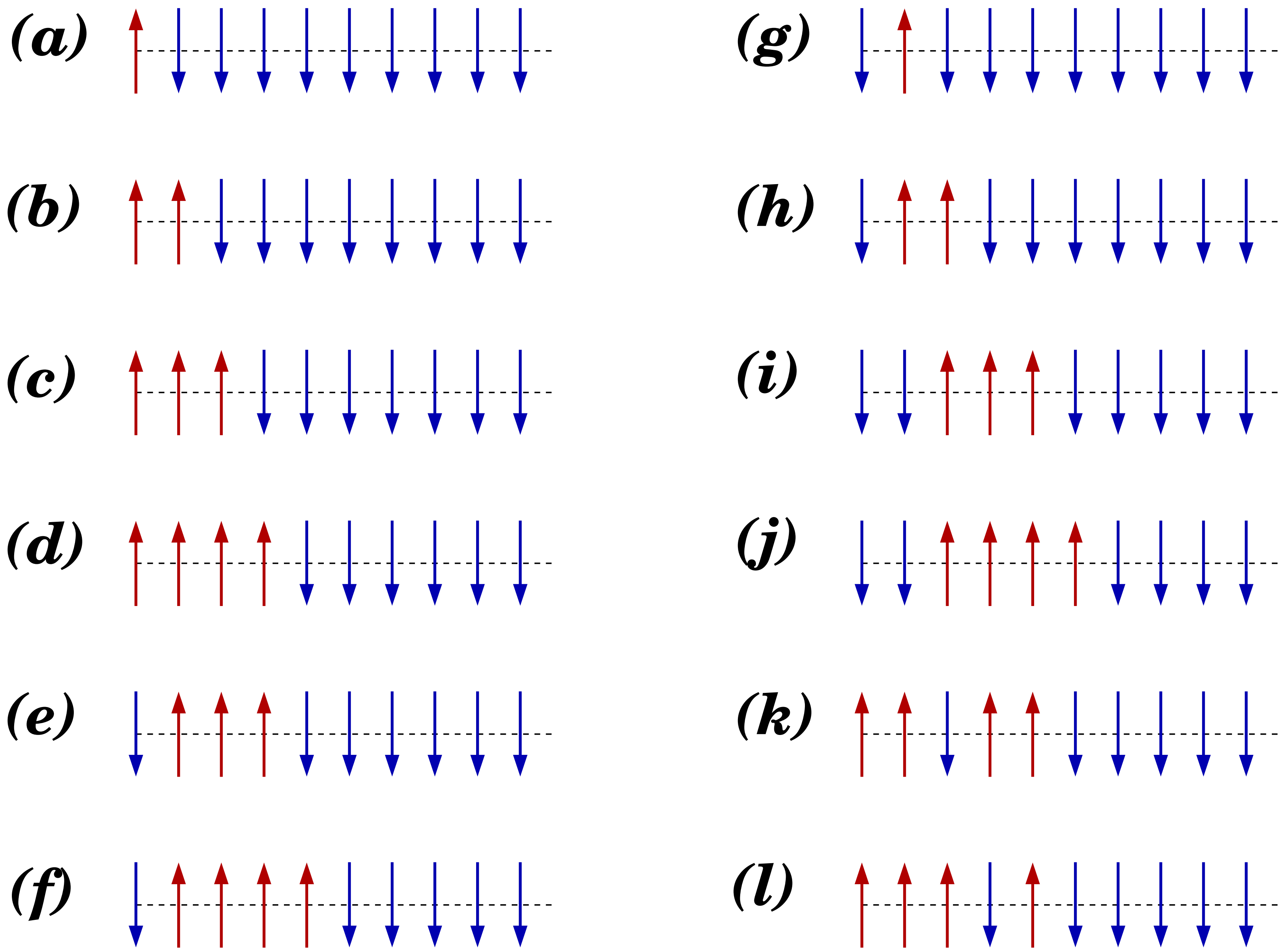
Anisotropic Heisenberg (XXZ) chain

$$H \;=\; J_x \sum_{j=1}^{L-1} \left[S_j^x S_{j+1}^x + S_j^y S_{j+1}^y \;+\; \Delta S_j^z S_{j+1}^z \right]$$

Edge-locking hierarchy $\rightarrow$ surprisingly different from t-V model.

Physical t-V model has $V n_i n_{i+1}$, not $V(n_i - \frac{1}{2})(n_{i+1} - \frac{1}{2})$.

Physical t-V model does not have
empty-empty or empty-occupied energy.
(Only occupied-occupied energy.)

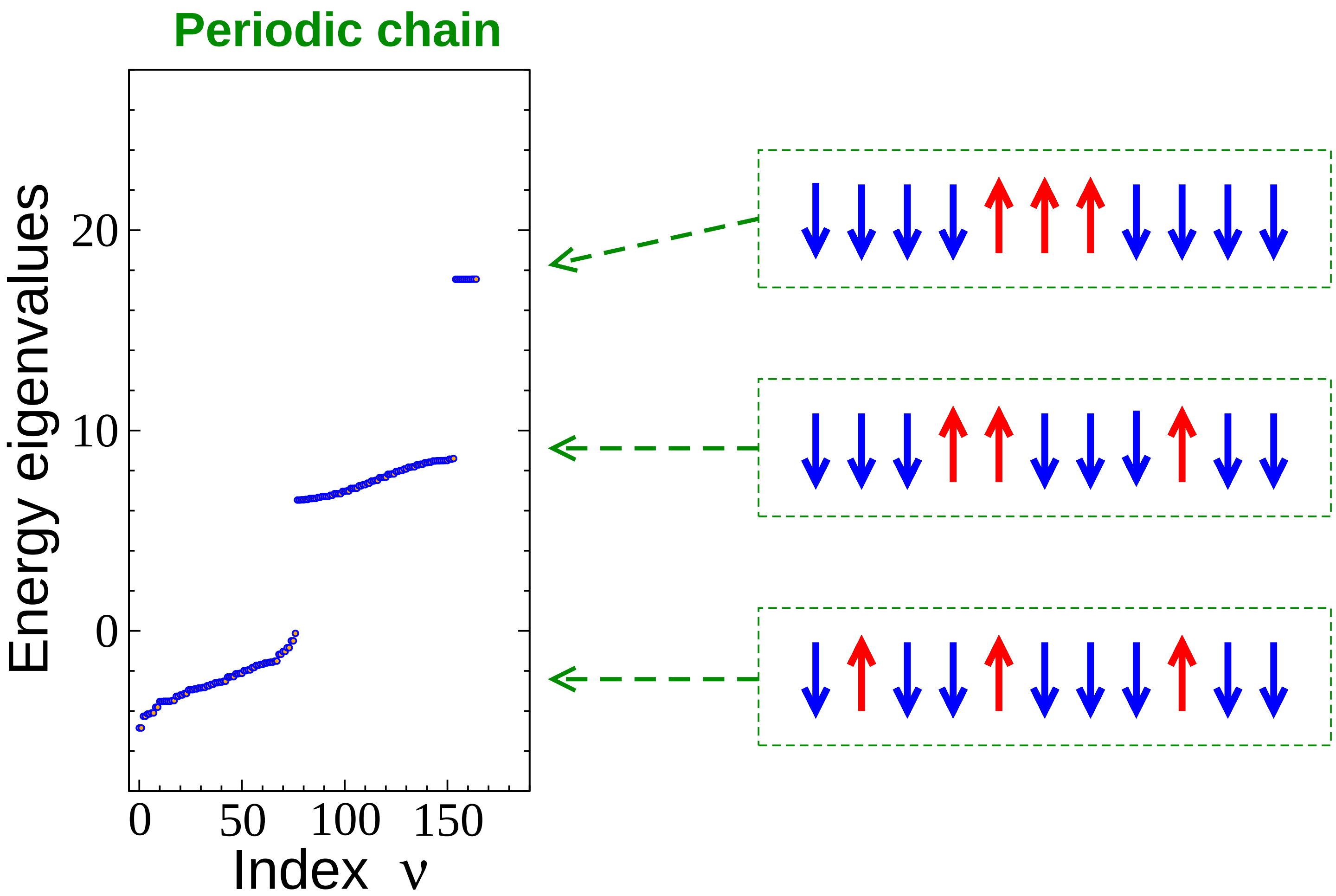

(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)

Spectrum: periodic XXZ chain:

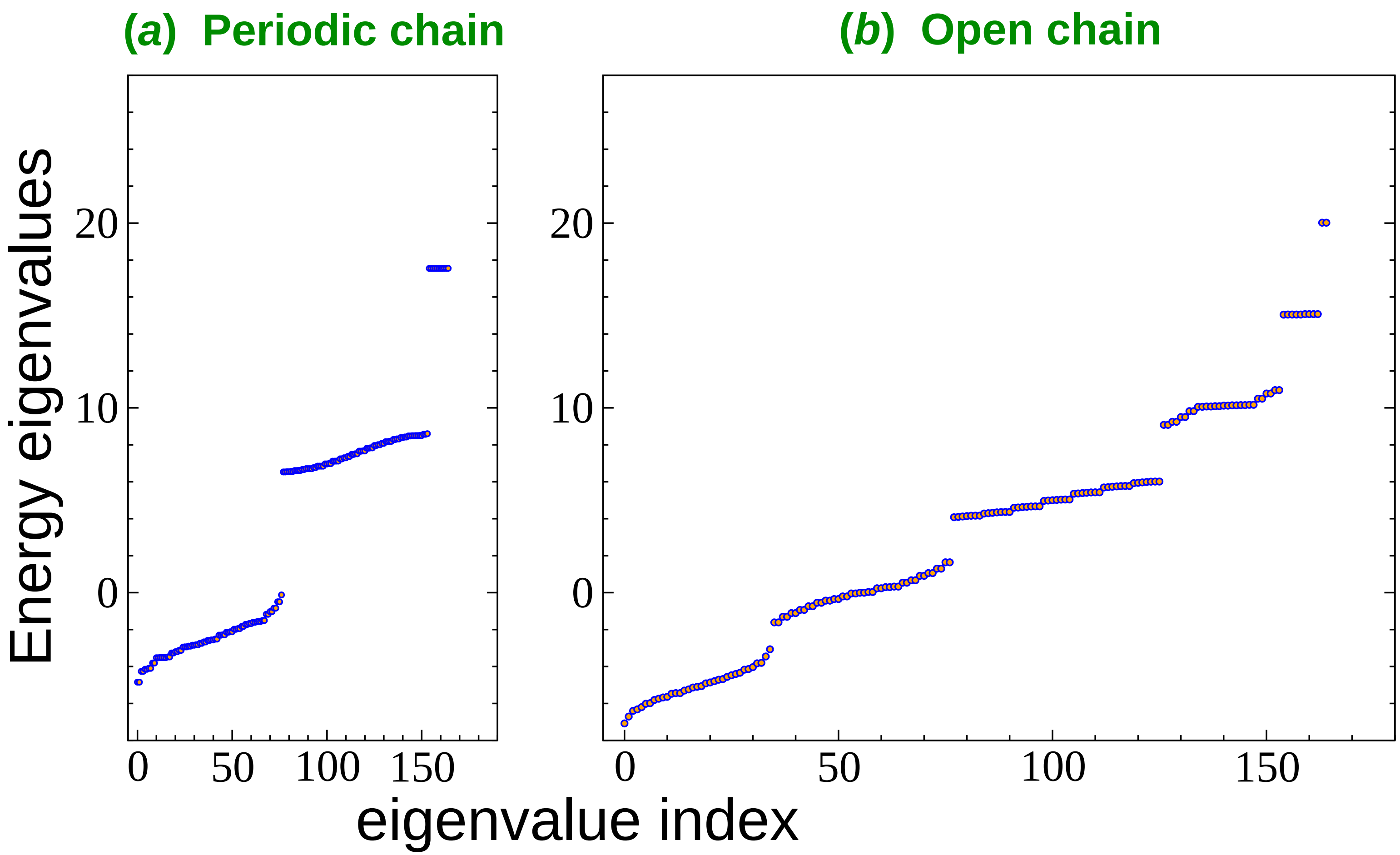
$N_{\uparrow} = 3$

11 sites

$\Delta = 10$



XXZ chain: periodic versus open spectra

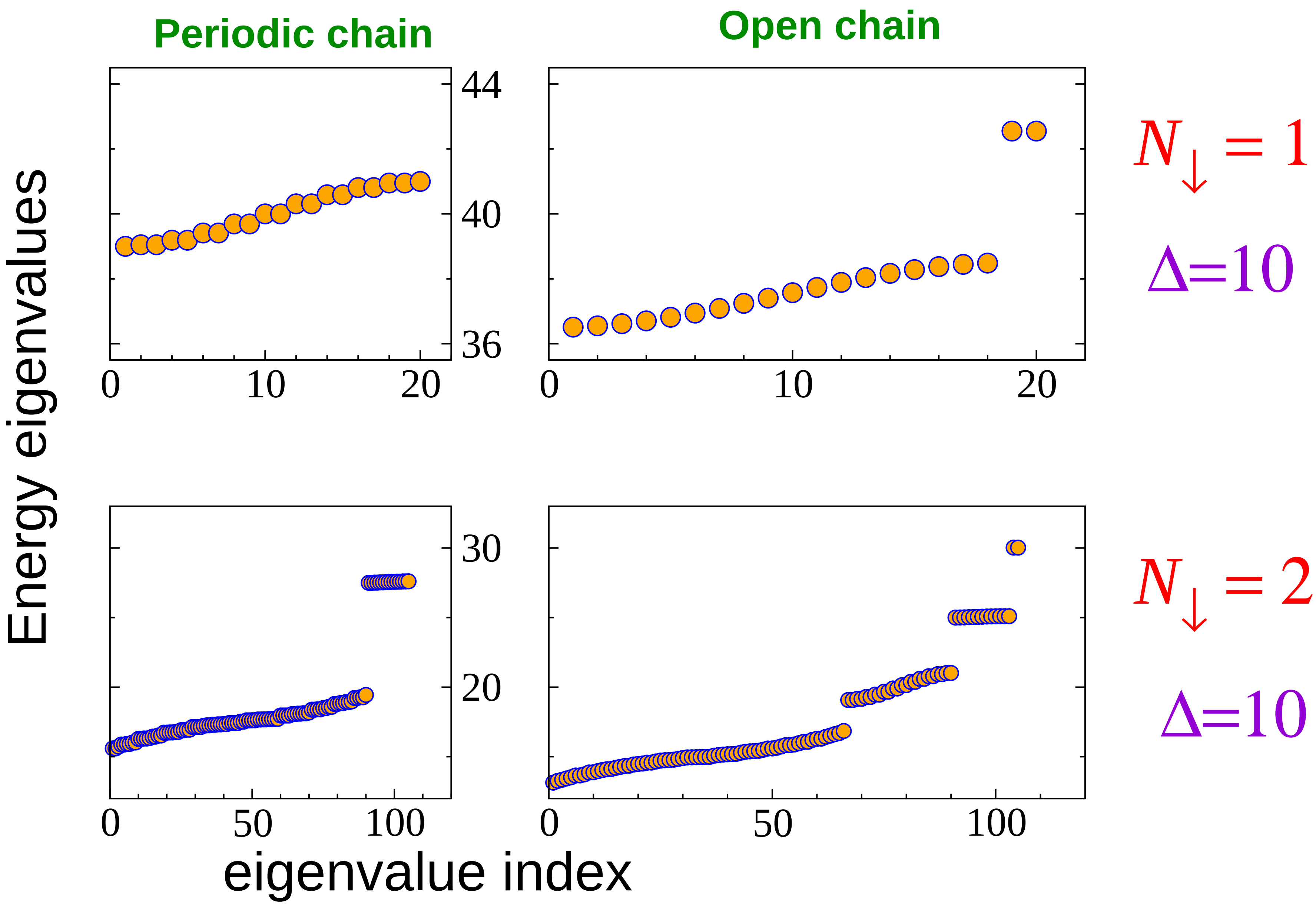


$N_{\uparrow} = 3$

11 sites

$\Delta = 10$

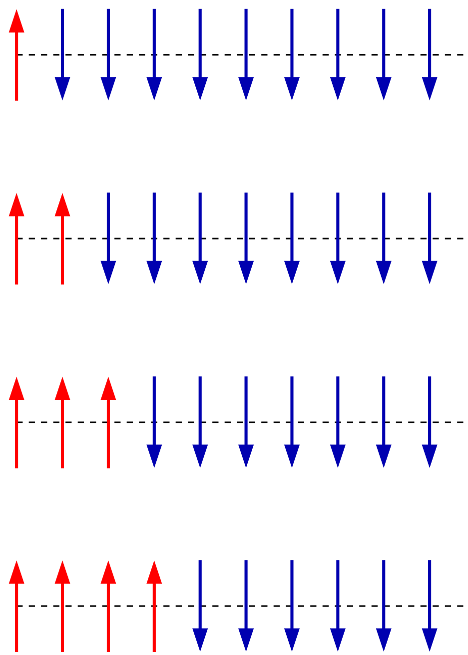
Many extra spectral features in open chain



Periodic chain
Open chain
Energy eigenvalues
44
40
36
0
10
20
0
10
20
$N_{\downarrow} = 1$
Δ=10
30
20
0
50
100
0
50
100
$N_{\downarrow} = 2$
Δ=10
eigenvalue index

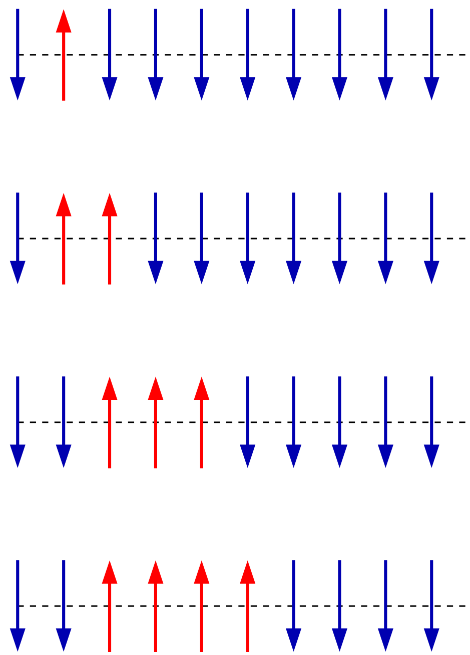
‘Trivially’ locked spin configurations

locked



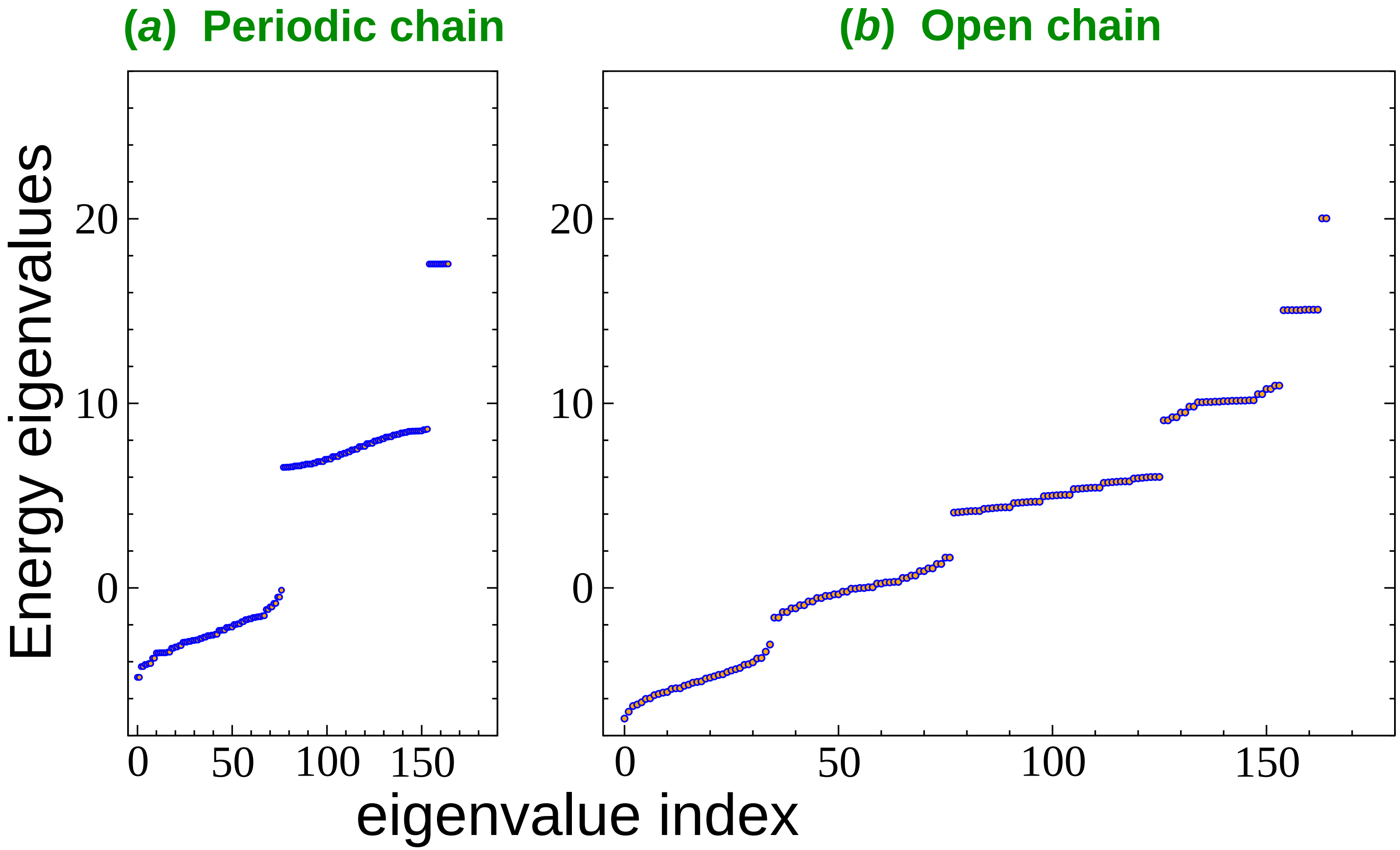
AFM (good) bonds → 1
FM (bad) bonds → (L-2)

not locked



AFM (good) bonds → 2
FM (bad) bonds → (L-3)

XXZ chain: periodic versus open spectra



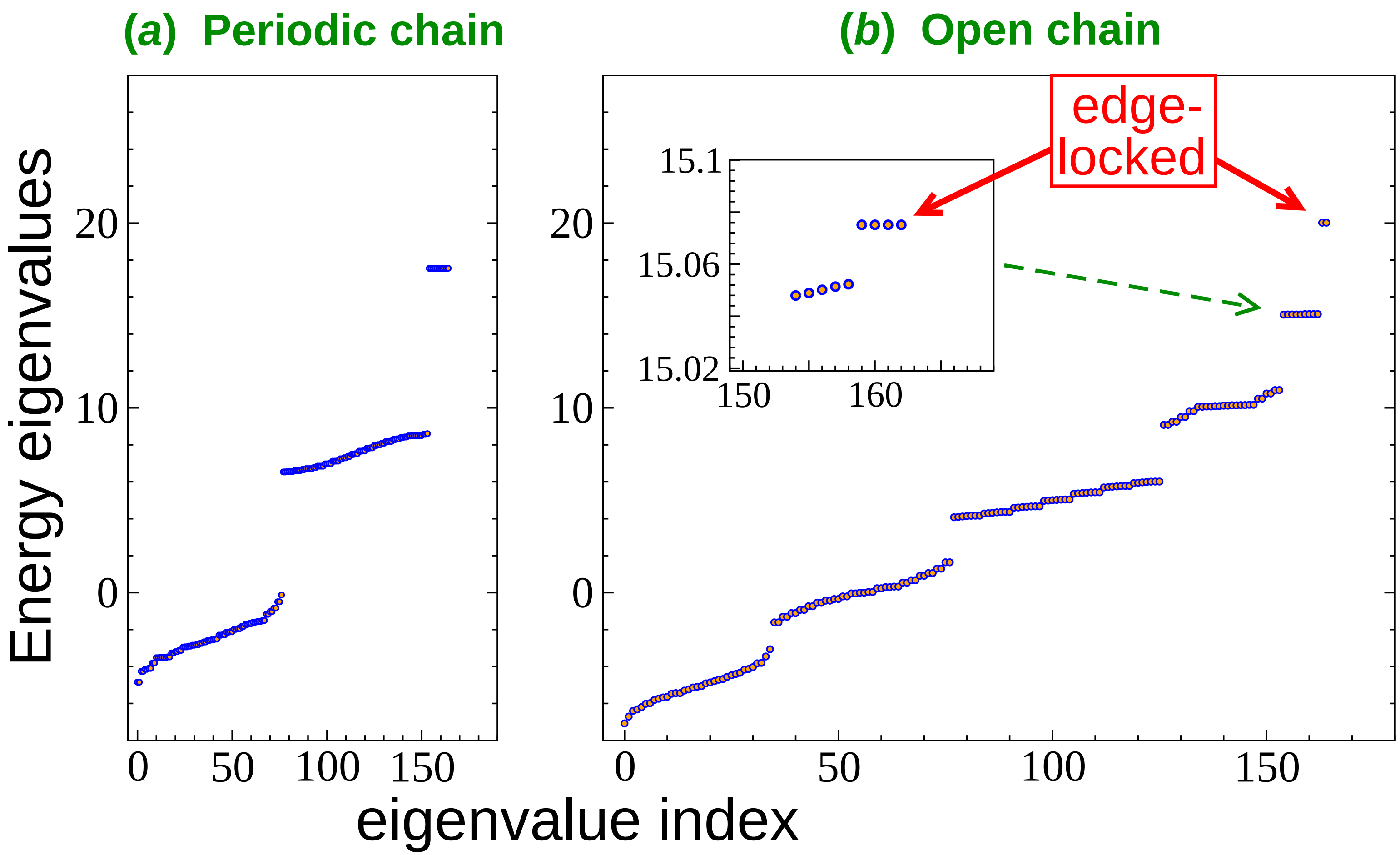
$N_\uparrow = 3$

11 sites

$\Delta = 10$

Many extra spectral features in open chain

XXZ chain: periodic versus open spectra



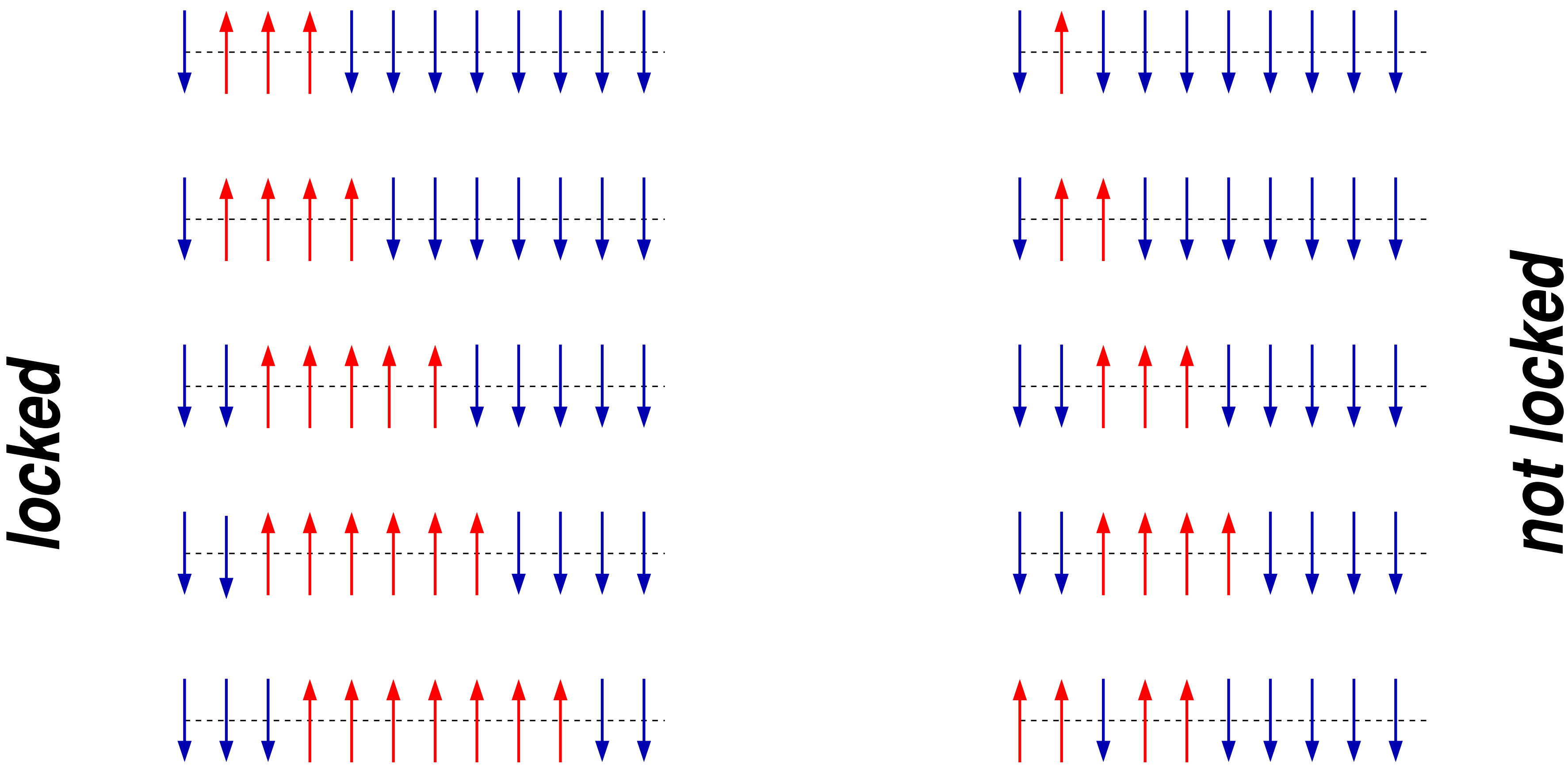
$N_\uparrow = 3$

11 sites

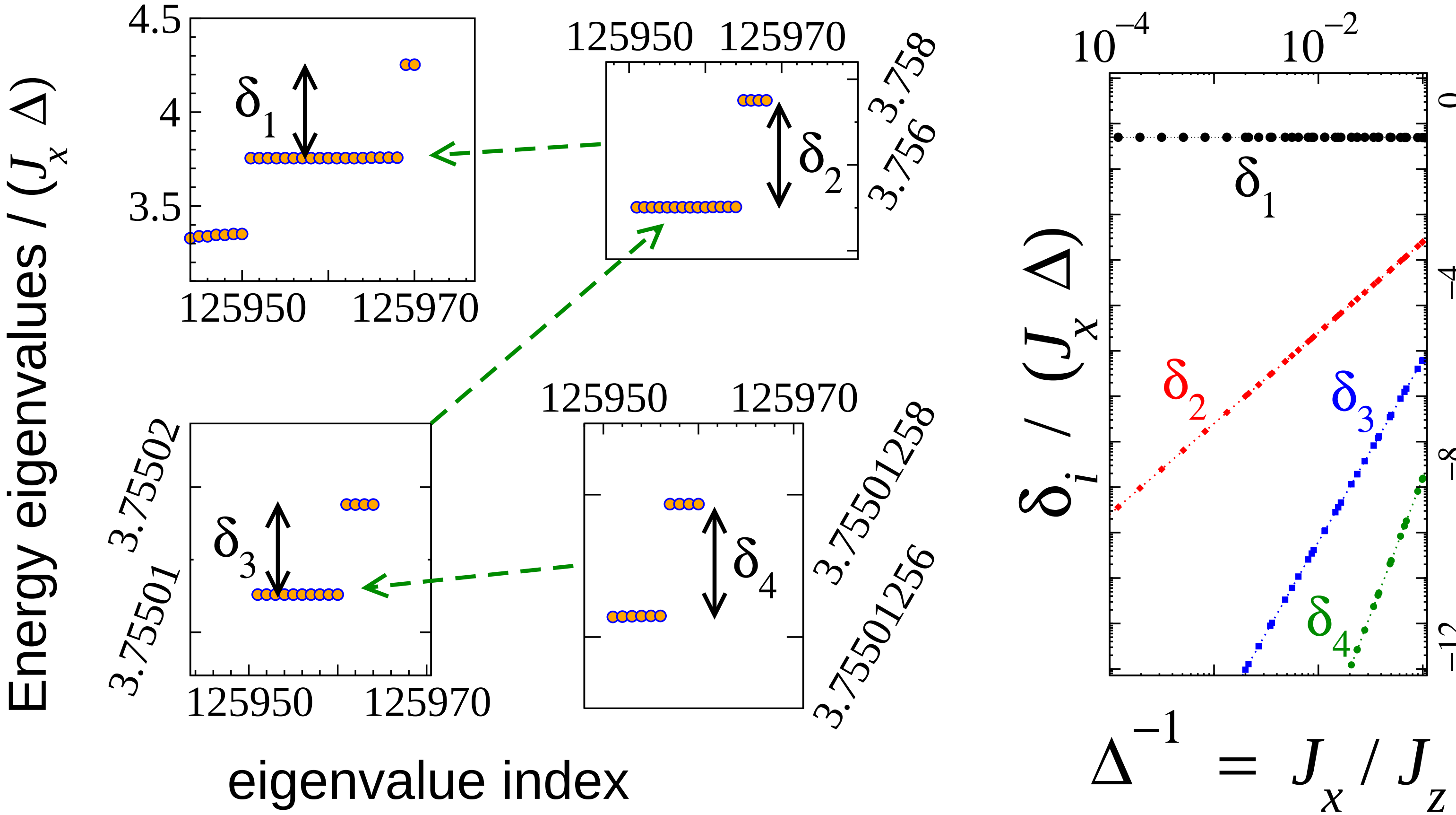
$\Delta = 10$

Many extra spectral features in open chain

XXZ CHAIN: HIERARCHY OF LOCKING EFFECTS



XXZ CHAIN: HIERARCHY



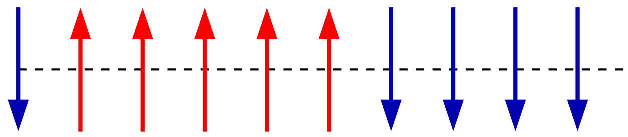
$N_\uparrow = 8$; 20 sites. $\delta_1 \sim \Delta^0$ $\delta_2 \sim \Delta^{-2}$ $\delta_3 \sim \Delta^{-4}$

Hierarchy of edge-localization

Energy spectrum contains structures at
many different scales.

FRACTAL structure in spectrum

"Quantum control" of magnetization transport

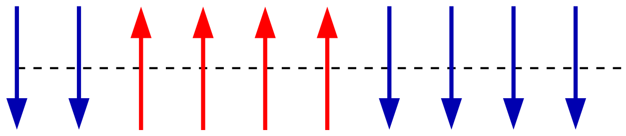


Single-site π-pulse

$\Downarrow$

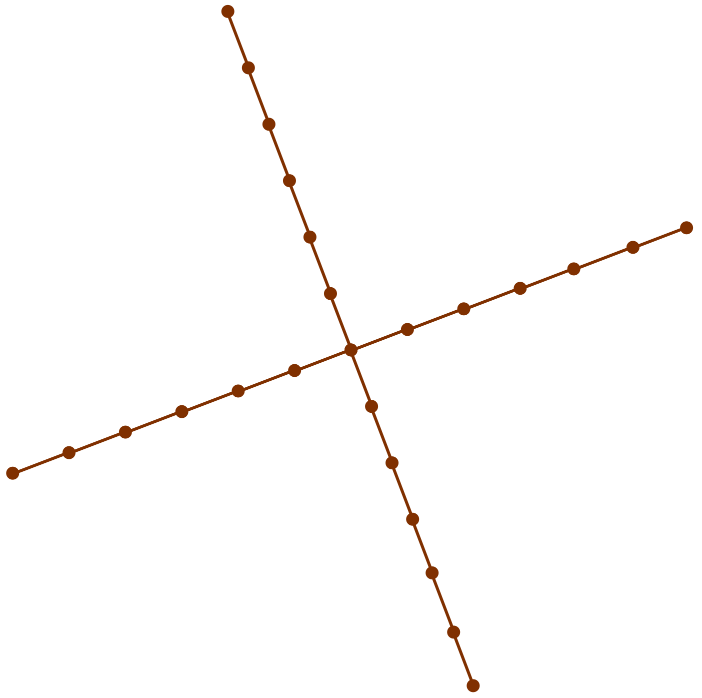
quantum switch

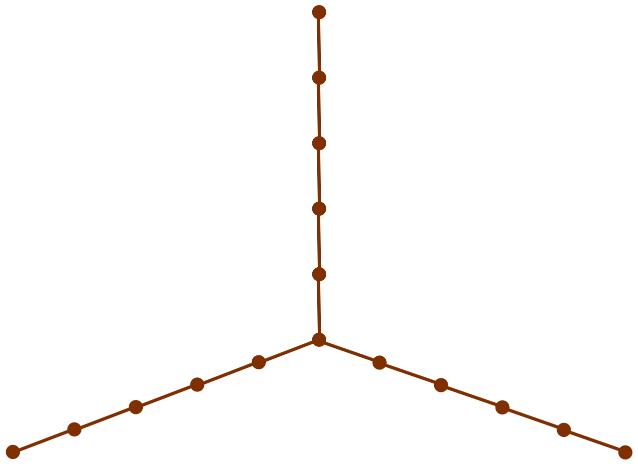
for unlocking magnetization.

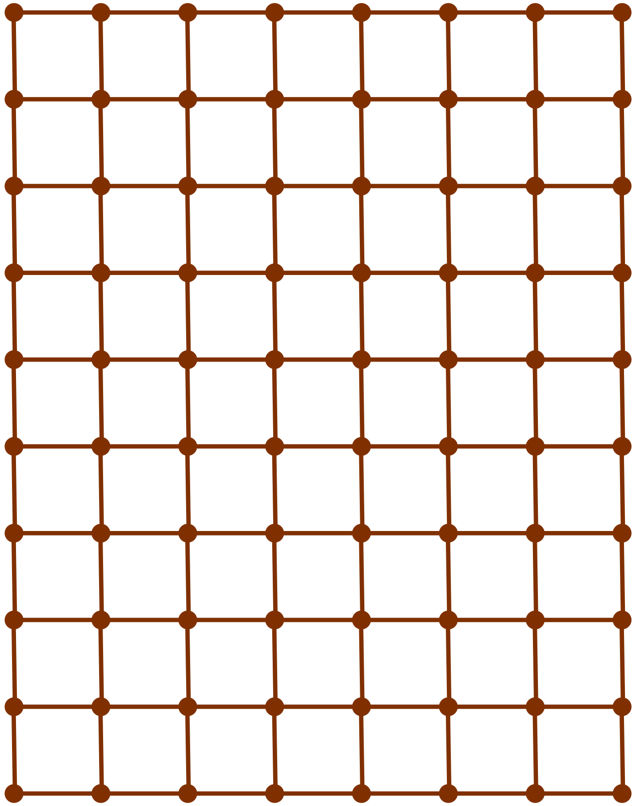


Many other control protocols....

Other geometries: junctions, 2D





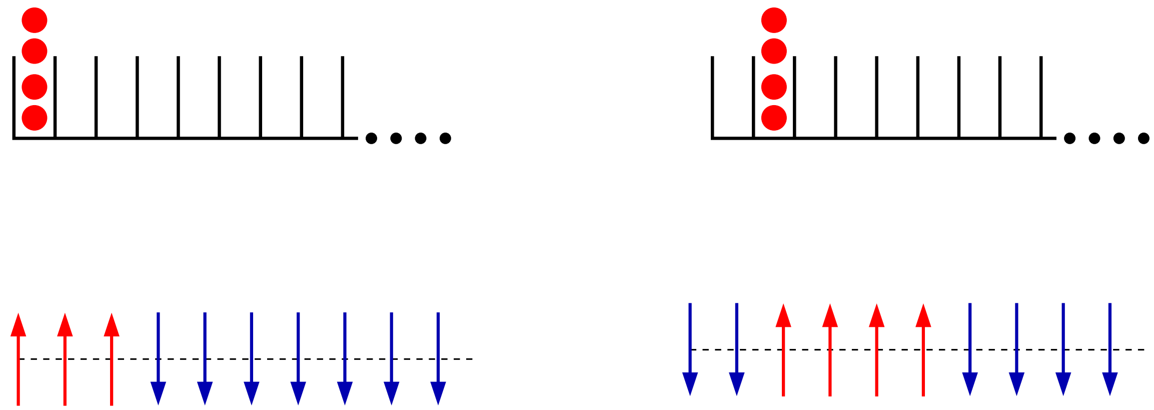


Edge-localization in 1D lattice models

Bose-Hubbard chain

spinless fermion model

XXZ chain



PHYSICS:

Far-from-equilibrium dynamics

Eigenstates far from ground state

Intricate structures in spectrum (FRACTAL)

QUANTUM CONTROL:

Locking and release of magnetization/state

Designing a quantum switch