# Work, large deviations, universality and Casimir effect in quantum quenches


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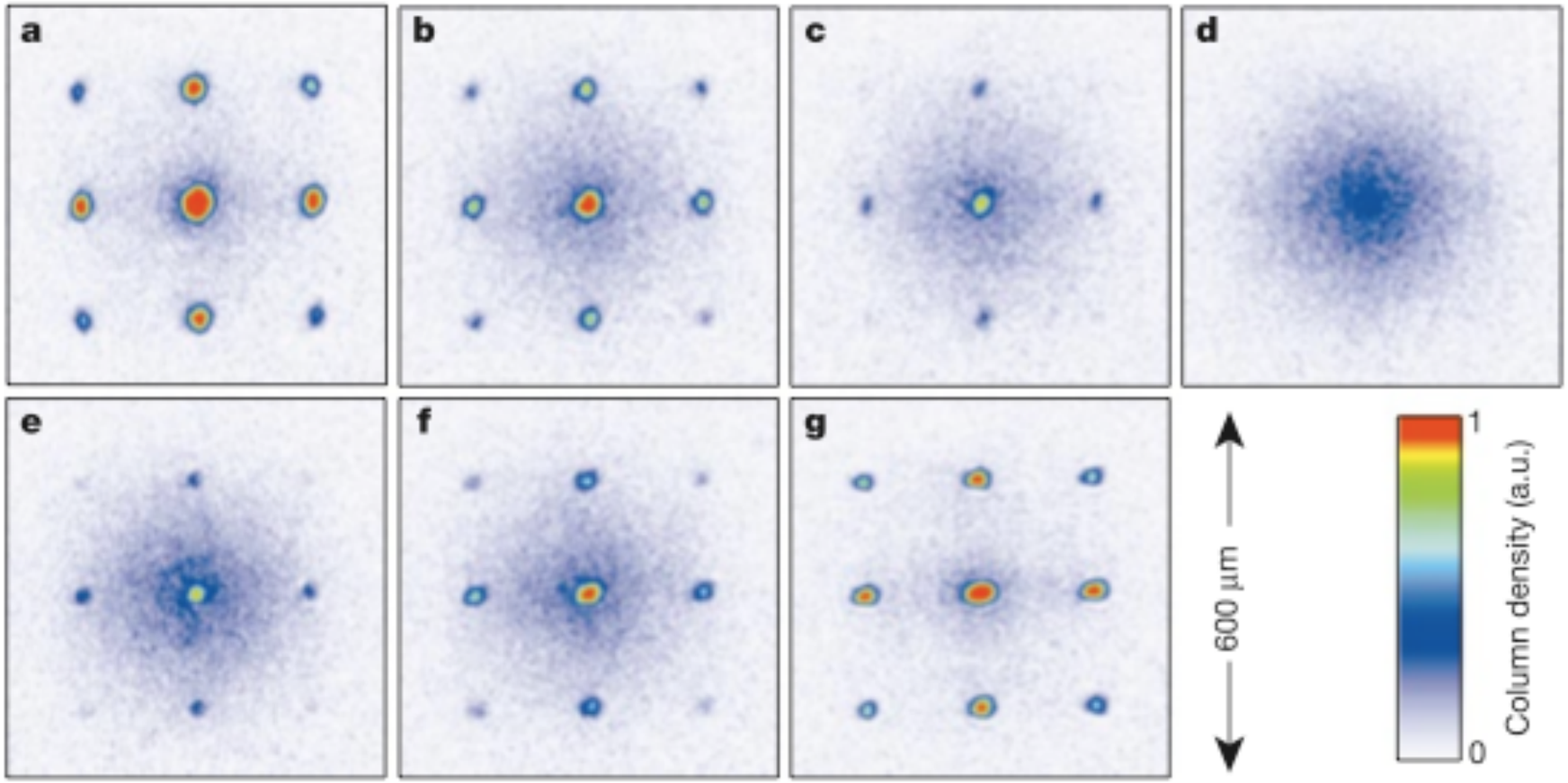
# Quantum isolated systems (many-body)

Revived interest → ultra cold atoms

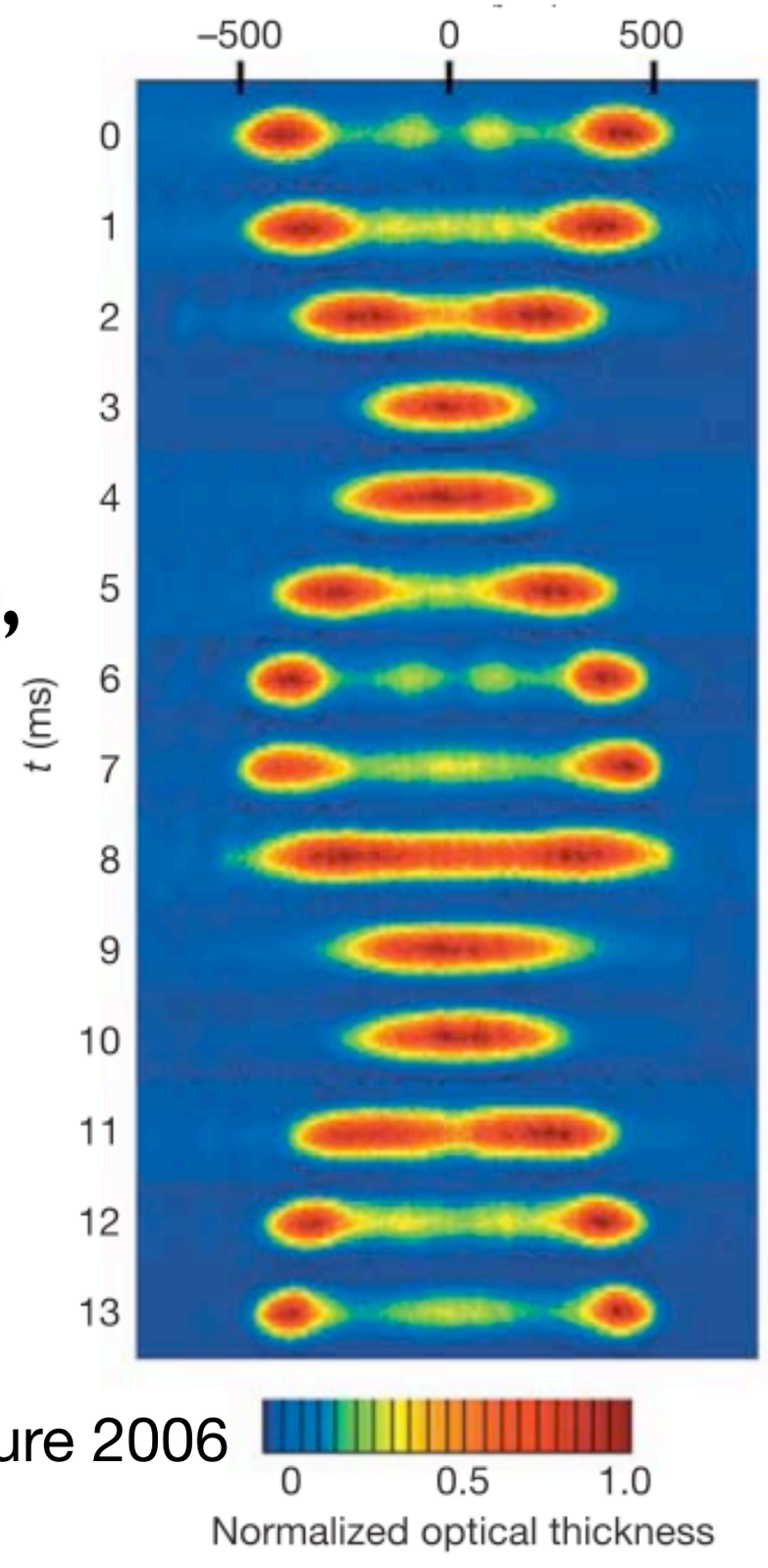
- Control of H
- Coherent dynamics




Greiner et al., Nature 2002

- Approach to equilibrium
  → Foundation, cosmology
- Lack of thermal behavior
  → localization, integrability ...
- Quantum annealing: Quant Comp, Kibble-Zurek mechanism
- Exploration of highly excited states

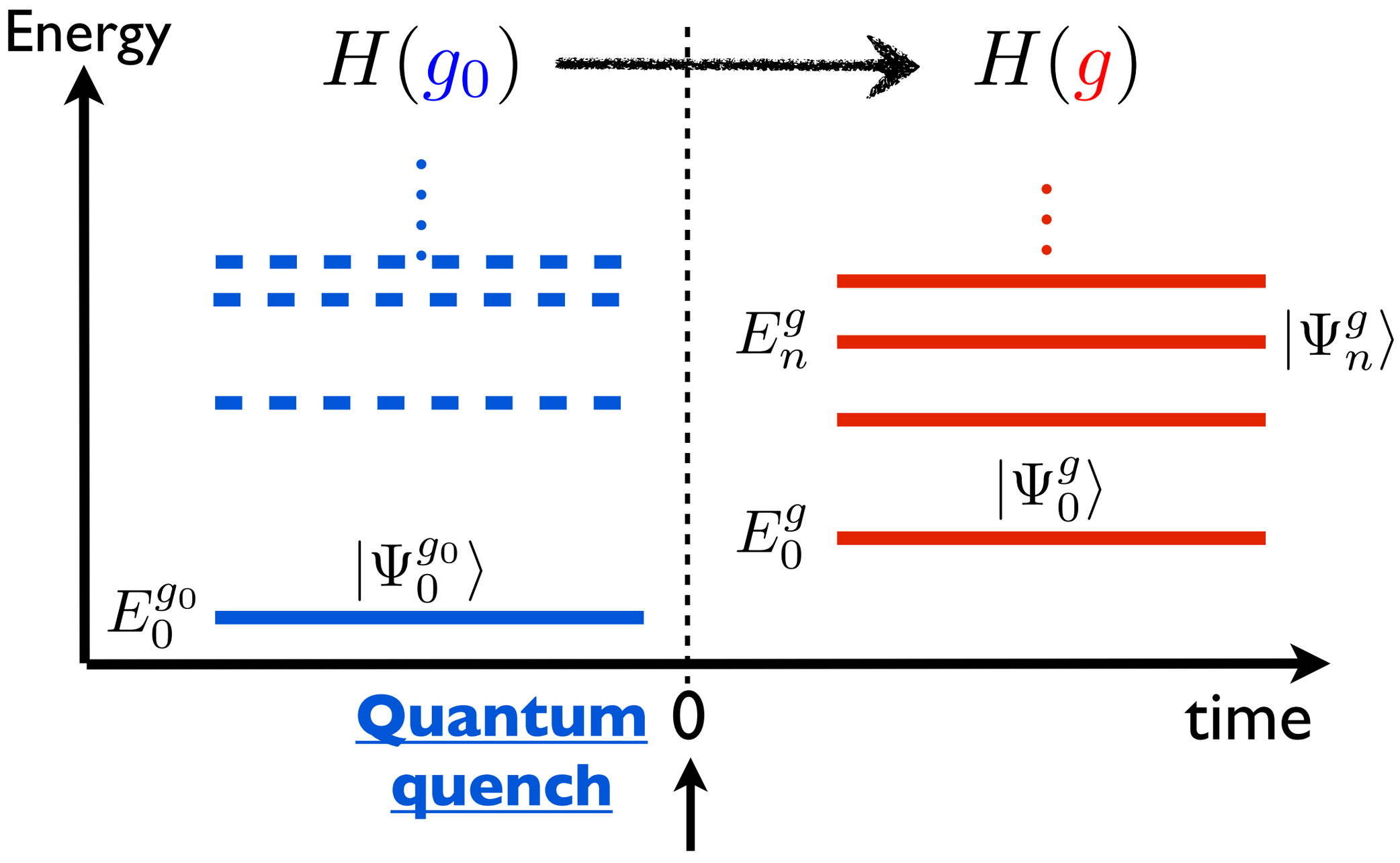
[Polkovnikov,Sengupta,Silva,Vengalattore,RMP'11]




Kinoshita et al., Nature 2006

# Quantum quench (global)

$$|\psi(t)\rangle = \mathrm{e}^{-iH(g)t}|\Psi_0^{g_0}\rangle$$



# Issues...

- Dynamical transitions

  Biroli, Sciolla - Schirò, Fabrizio - Gambassi, Calabrese -...
  Heyl,Polkovnikov,Kehrein.

stationary state?

- Long-lived non-equilibrium states

  Kinoshita, Wenger, Weiss - Kollath,Läuchli, Altman -...

dimensionality?
cons. laws?

- Relaxation of inhomogeneity, aging

  Shütz, Trimper - Iglói, Rieger - Carleo, Becca, Schirò, Fabrizio - ...

integrability?

- Thermalization

  Srednicki - Biroli,Kollath,Läuchli - Rigol,Dunjko,Olshanii - Berges -
  Calabrese,Cardy - Rossini,Silva,Mussardo,Santoro - Calabrese,Essler,Fagotti -...
  Mitra,Giamarchi -...
  Foini,Cugliandolo, Gambassi -...

effective temp.s?

# ...look at...

- Expectation values & correlators of observ.s

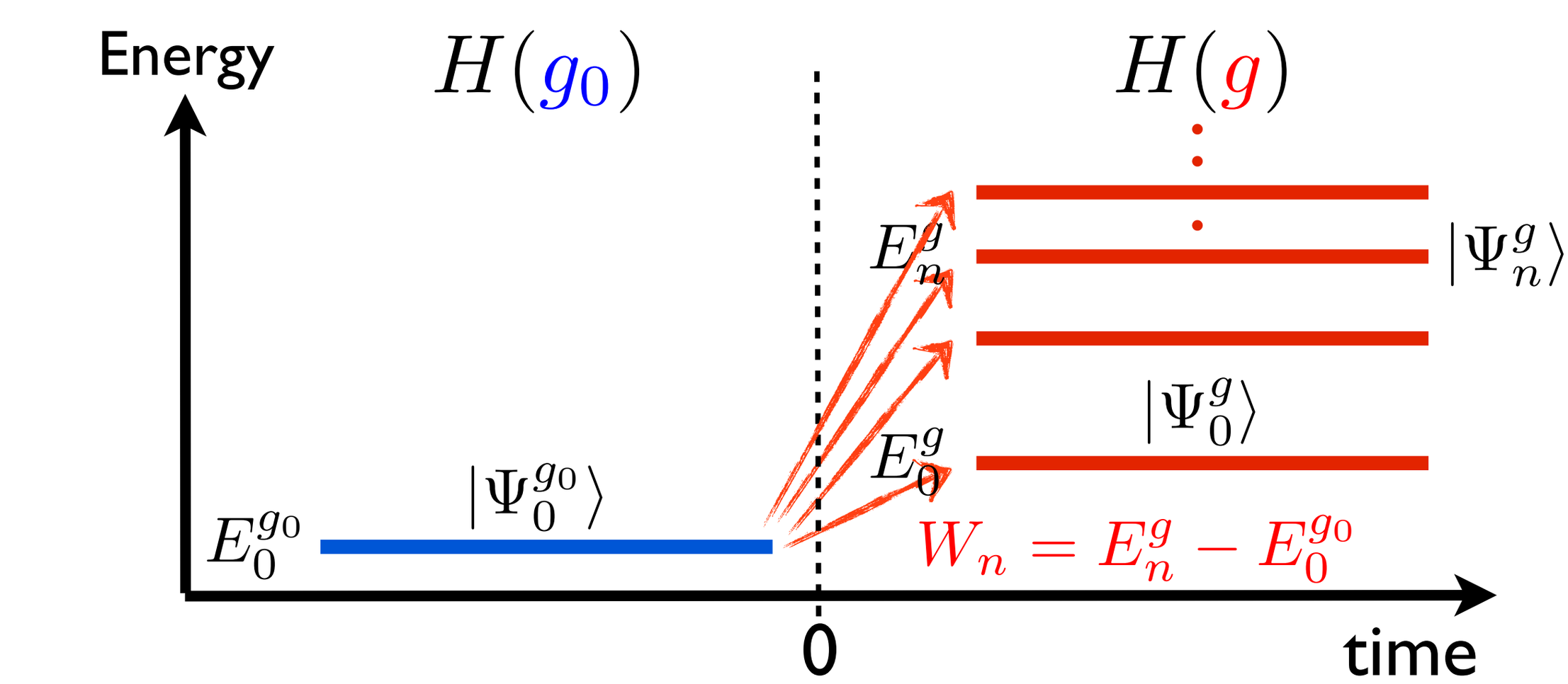
Igloi, Riegel '01 - Altman, Auebarch '02
Sengupta, Powell, Sachdev '04 - Polkovnikov '05
Zurek, Dorner, Zoller '05
Calabrese, Cardy '06 - Gritsev, Polkovnikov '07
Rossini, Silva, Mussardo, Santoro '08-'09
Fagotti, Essler, Calabrese '11-'12
Foini, Cugliandolo, Gambassi '11-'12

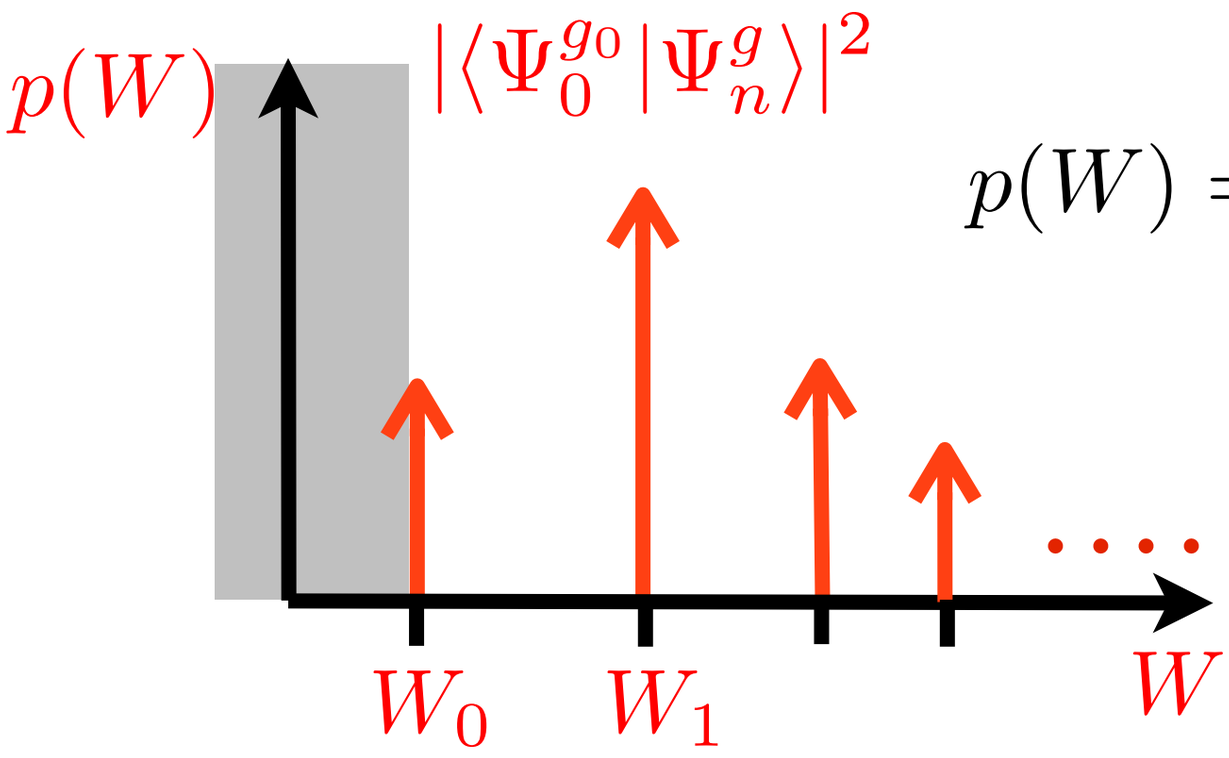
- Statistics of excitations & observ.s

Polkovnikov '08 - Silva '08 - Barankov, Polkovnikov '09
Kehrein '09-'10 - Kitagawa '11 - Gring et al. '11
Canovi et al. '12

here: **WORK** $\Delta E$

$W$



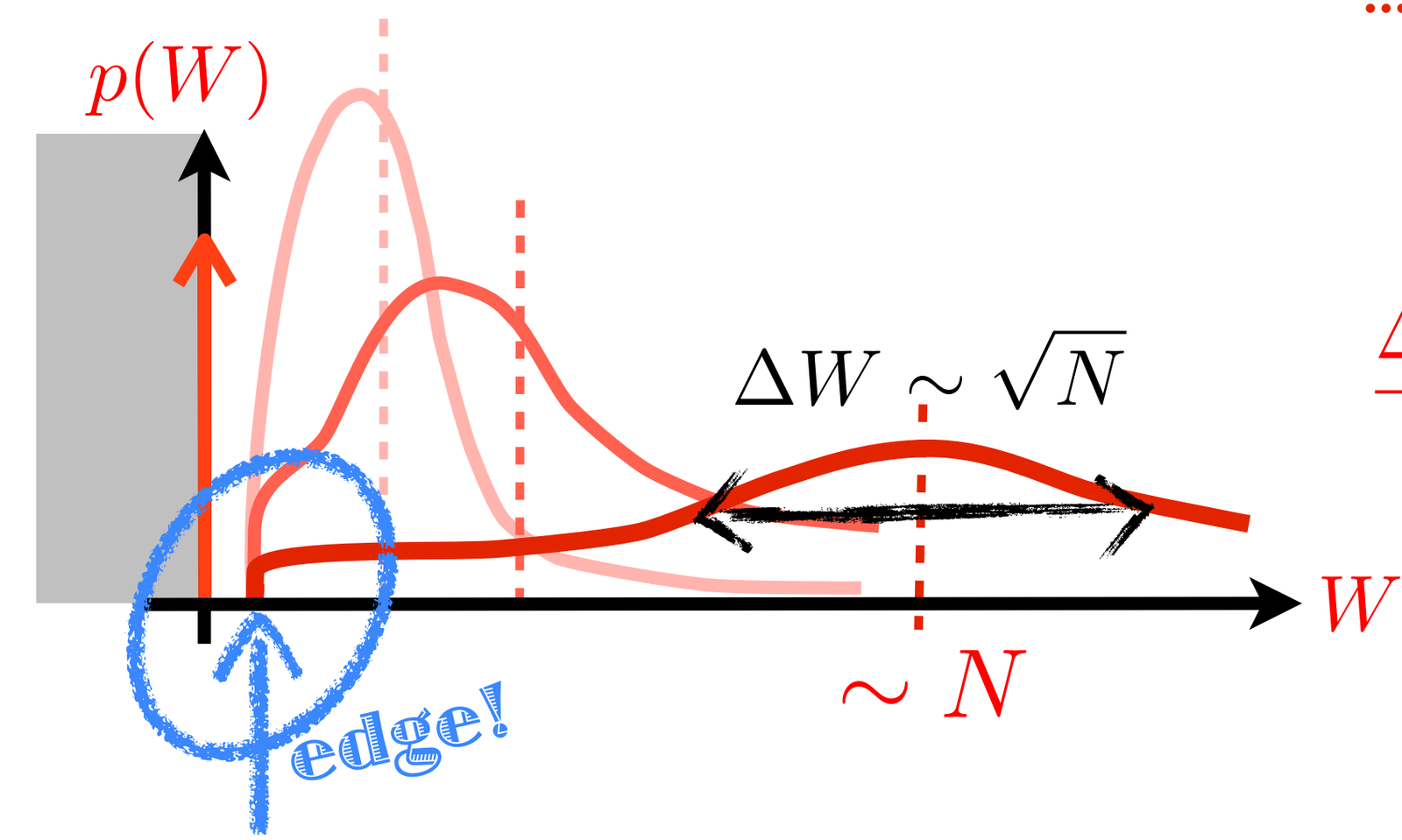

$$p(W) = \sum_{n\geq 0} |\langle\Psi_0^{g_0}|\Psi_n^g\rangle|^2 \delta(W - W_n)$$

$\langle W\rangle \propto N$

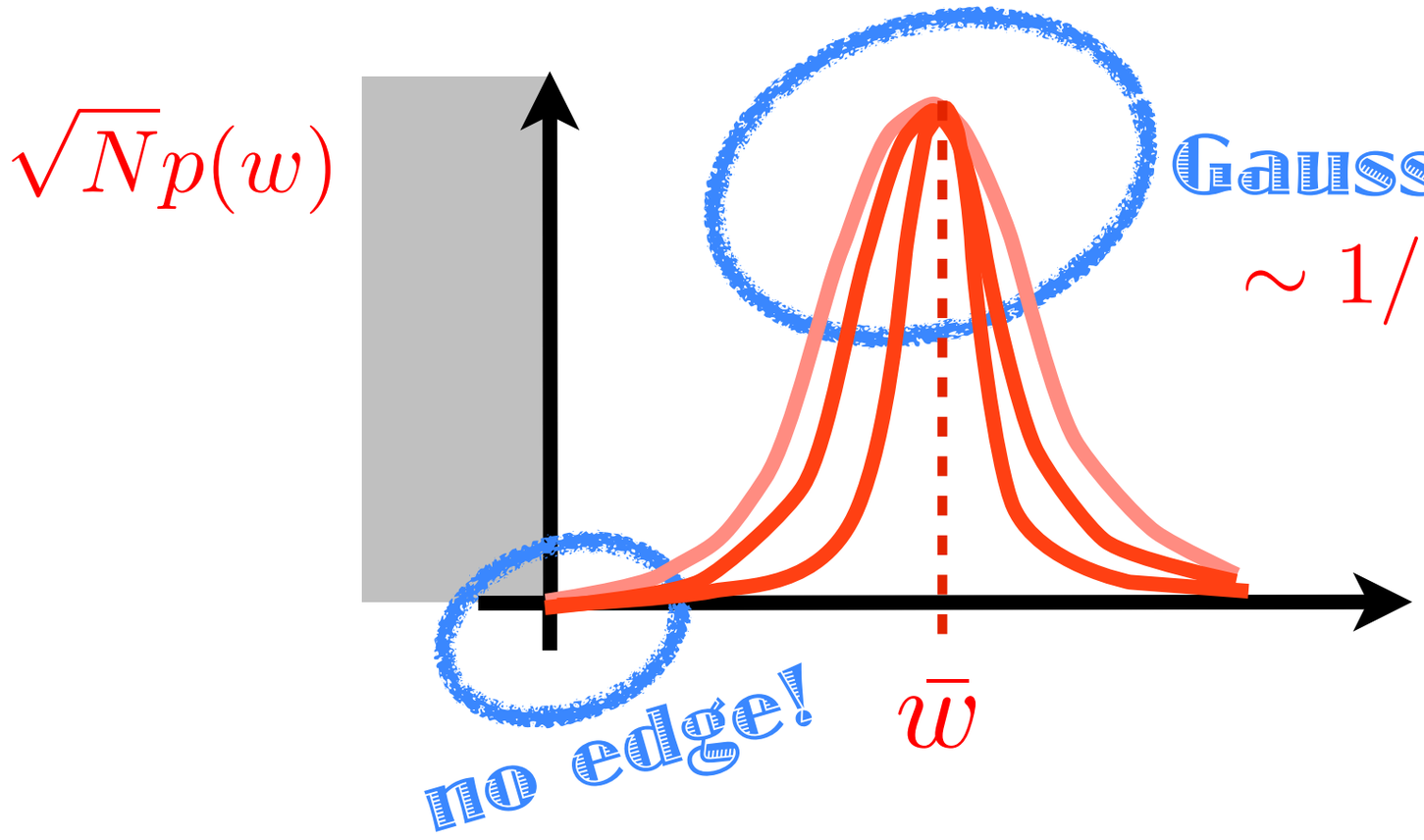
Extensive!

$W \to W - W_0$

.....*generic* features




$$\frac{\Delta W}{W} \sim \frac{1}{\sqrt{N}}$$




Gaussian >>triv. univ!

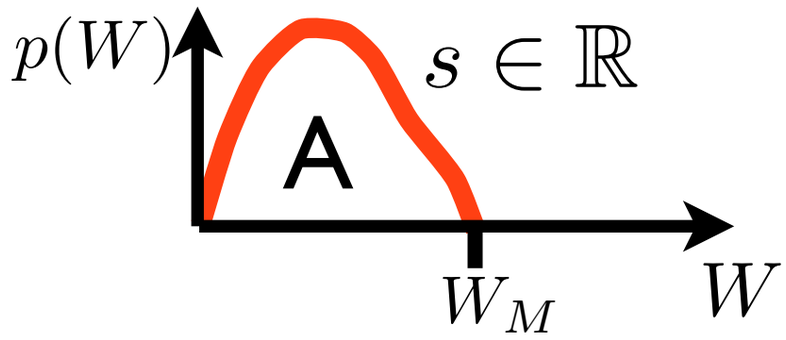
$\sim 1/\sqrt{N}$ >>large dev.!

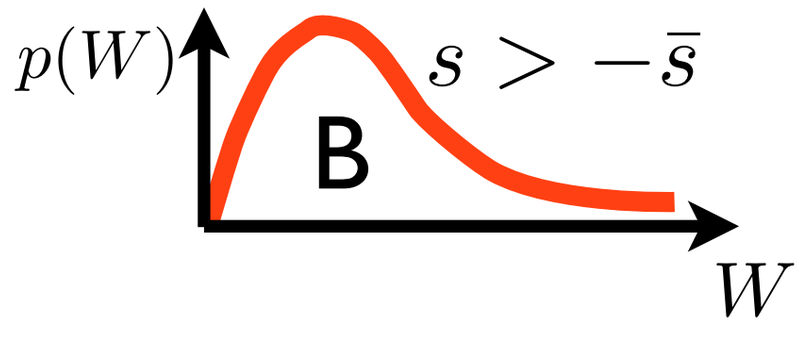
$w \equiv W/N$

Intensive!

$W > 0$

$$G(s) \equiv \langle \mathrm{e}^{-sW} \rangle \quad \text{mgf}$$




$$= \langle \Psi_0^{g_0} | \mathrm{e}^{-s[H(g)-E_0^g]} | \Psi_0^{g_0} \rangle$$

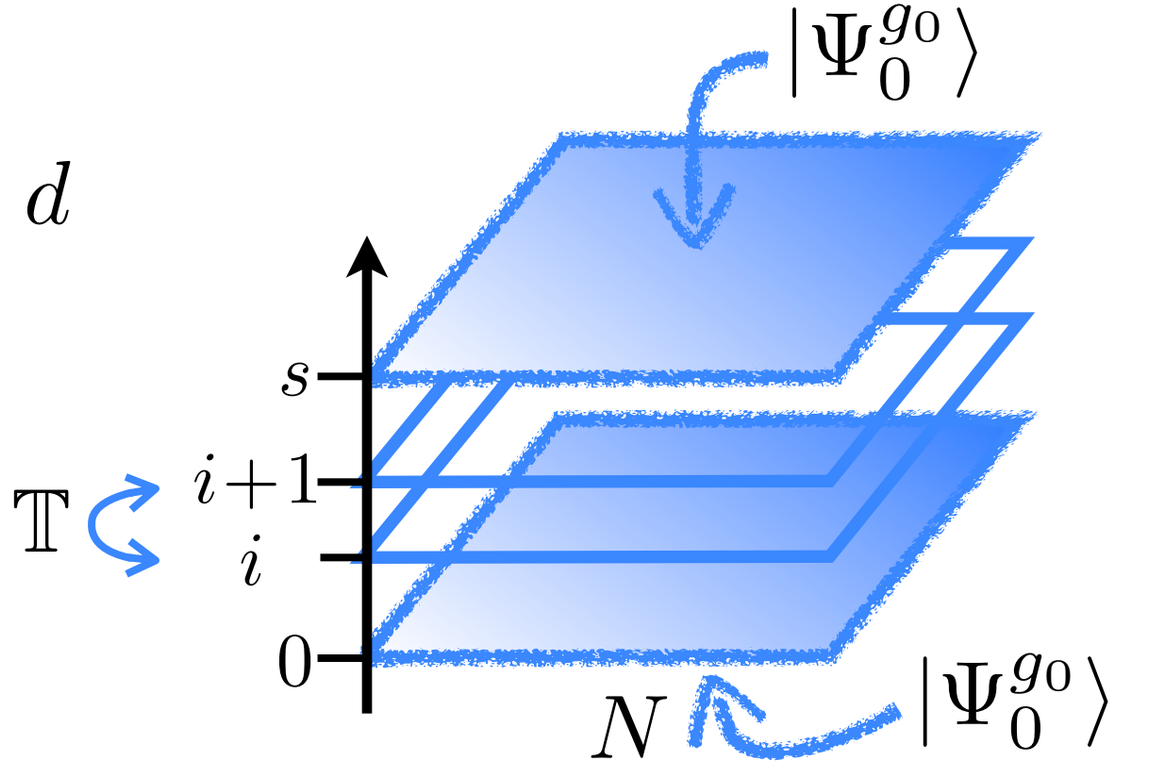
$$\langle \Psi_0^{g_0} | \mathrm{e}^{-sH(g)} | \Psi_0^{g_0} \rangle \qquad \text{quantum } d$$

$$\mathbb{T} \equiv \mathrm{e}^{-H(g)}$$

$$\langle \Psi_0^{g_0} | \mathbb{T}^s | \Psi_0^{g_0} \rangle \qquad \text{classical } d+1$$

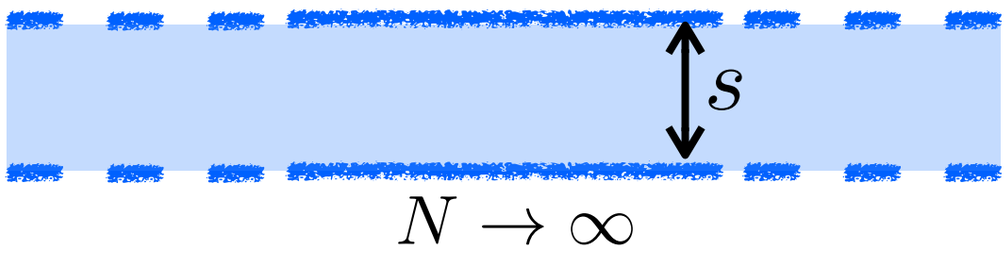
$$\text{film } N \times s$$

$$Z_{N \times s} \equiv \mathrm{e}^{-\mathcal{F}}$$




[Silva'08 - Gambassi, Silva'11]

confined systems: Barber - Fisher - Cardy - .. >'83



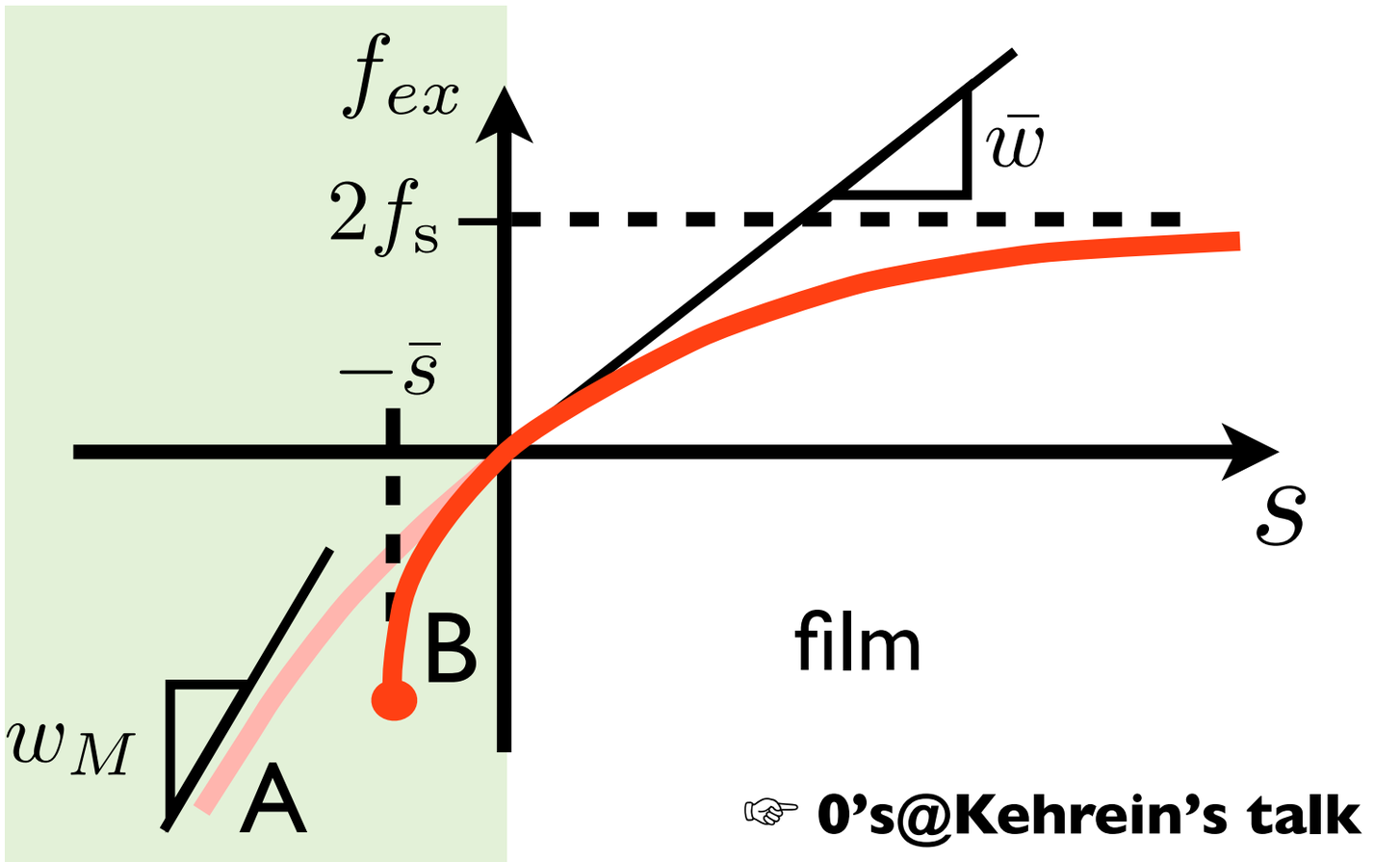

$$\mathcal{F} \overset{s\to\infty}{=} \underbrace{\text{volume}\times f_{\rm b}}_{sN} + \underbrace{\text{surface}\times f_{\rm s}}_{2N} + \underbrace{\text{finite-size corr. } (s)}_{\propto N}$$

$$E_0^g \qquad -\ln \text{fidelity} = N\, f_{ex}(s) \qquad f_{ex}(s) \overset{s\to\infty}{\to} 2f_{\rm s}$$

[Venuti,Zanardi - You, Li, Gu - DeGrandi, Gritsev,Polkovnikov]

$$G(s) = \mathrm{e}^{-N f_{ex}(s)}$$

- $f_{ex}$ concave
- $f_{ex}(0) = 0$




☞ **0's@Kehrein's talk**

# Edge singularities & Casimir eff.

[Gambassi, Silva'11]

Lapl. trans.

$$p(W) \leftrightarrow G(s) \leftrightarrow f_{ex}(s)$$

$W \to 0$ thres.

$s \to \infty$

$$\left.\begin{array}{l} H(g \to g_c) \\ \text{gap} \to 0 \\ \sim |g-g_c|^{\nu z} \end{array}\right\} \quad \begin{array}{l} \xi \to \infty \\ \sim |g-g_c|^{-\nu} \end{array}$$

bulk **critical** point

finite-size corr.

$$s \gg a \quad \xi \gg a$$

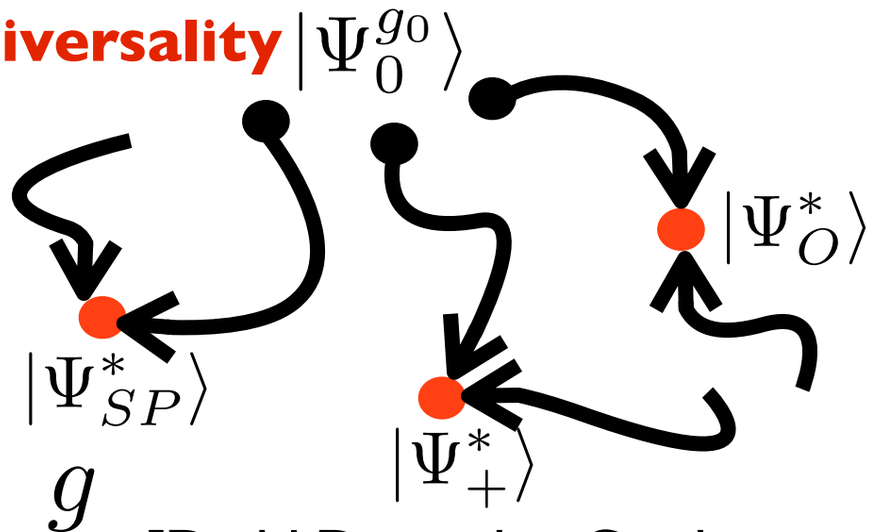
$$f_{ex}(s) = 2f_s + s^{-d}\Theta(s/\xi) + \dots$$

Universal!

$d+1$ bulk UC

critical Casimir eff.

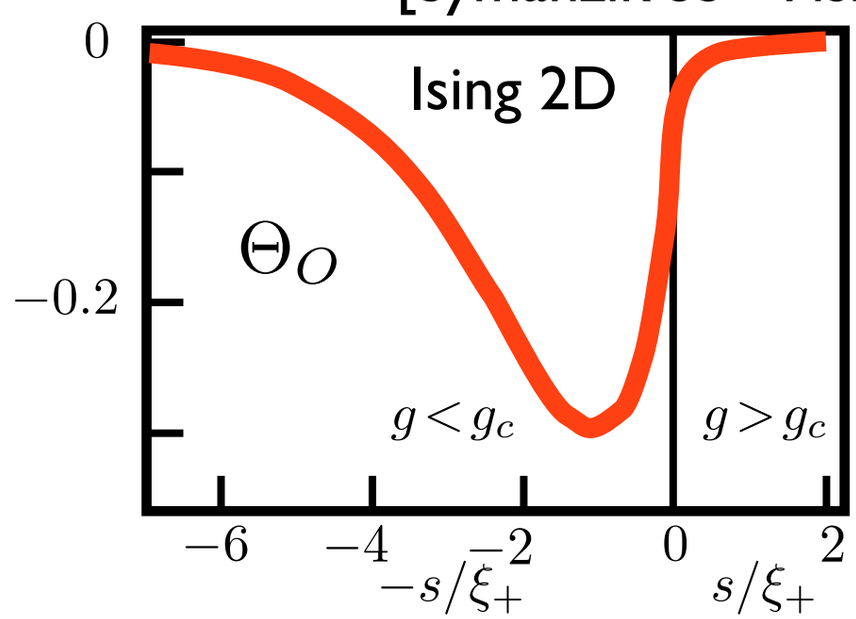
**surface universality**




[Diehl,Dietrich - Cardy - ....>'83]

[Symanzik'83 - Fisher,deGennes'78]

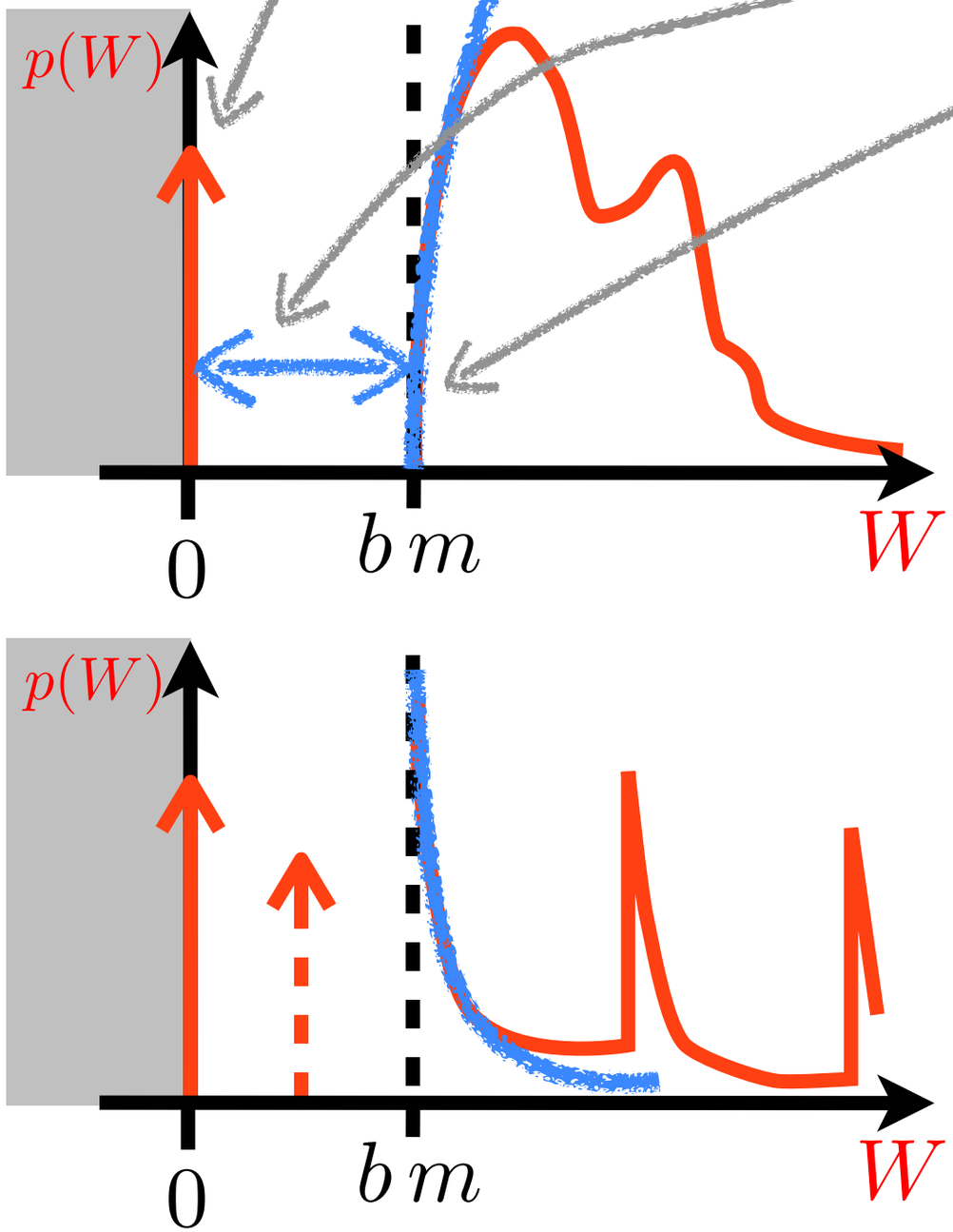
[Evans,Stecki'94]

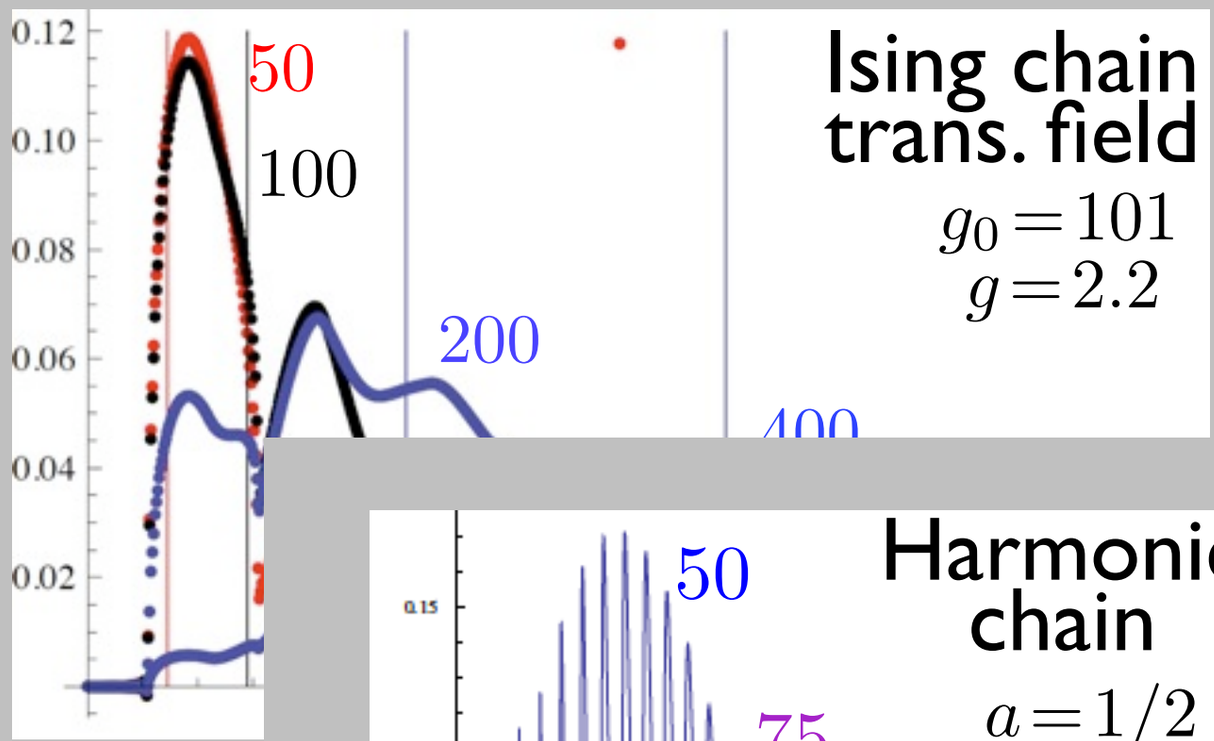


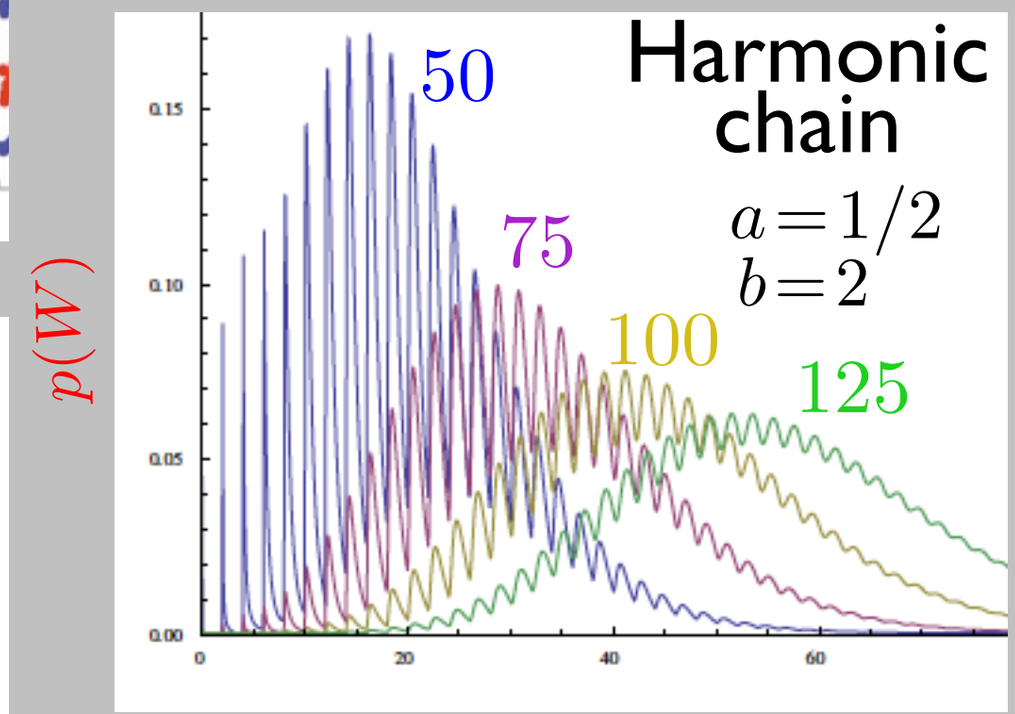

Ising 3D:
[Vasilyev,etal'08]

$$\Theta(x \gg 1) = \mathcal{C}\, x^a\, \mathrm{e}^{-b\,x} \qquad m = m(g) = \xi^{-1}$$

$$p(W) = \mathrm{e}^{-N 2 f_\mathrm{s}} \left[\delta(W) + C\,\theta(W - b\,m)(W - b\,m)^{d-a-1} + \ldots\right]$$



# Large deviations

[Touchette,Phys.Rep.'09]

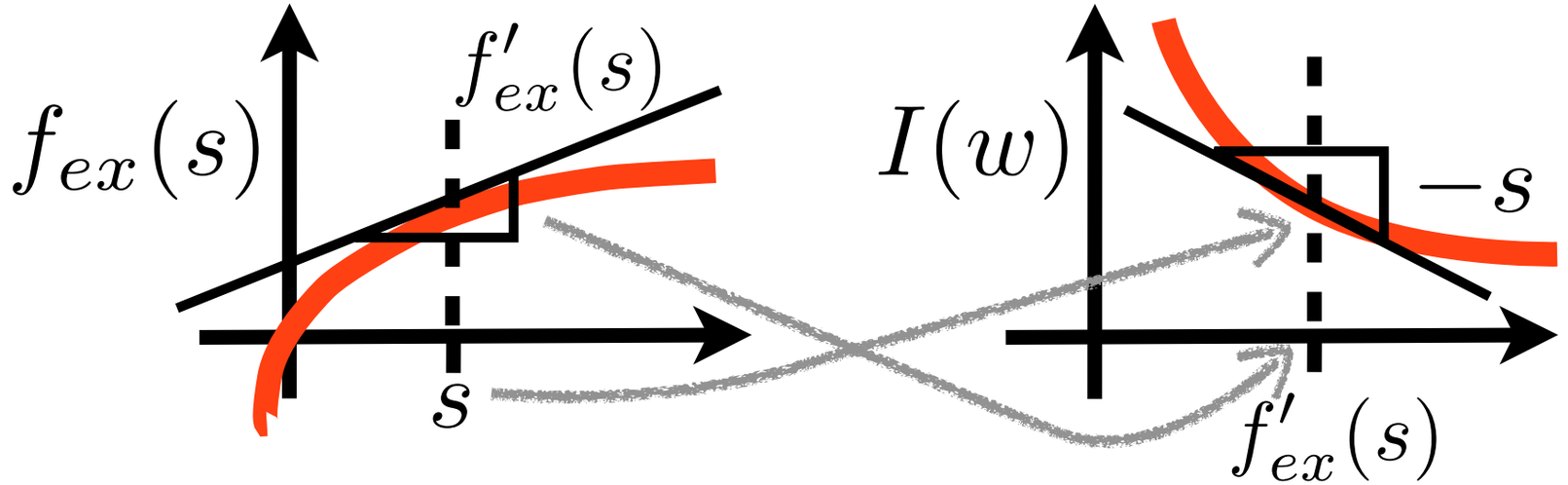
$$G(s) \equiv \langle \mathrm{e}^{-sW} \rangle = \mathrm{e}^{-N f_{ex}(s)}$$

$$p(w) \propto \int \mathrm{d}s \; \mathrm{e}^{sNw} G(s)$$

$$\Rightarrow \quad p(w) \propto \mathrm{e}^{-N I(w)}$$

Legendre, Fenchel:

$$I(w) = -\inf_{s \in \mathbb{R}} \{ s\, w - f_{ex}(s) \}$$

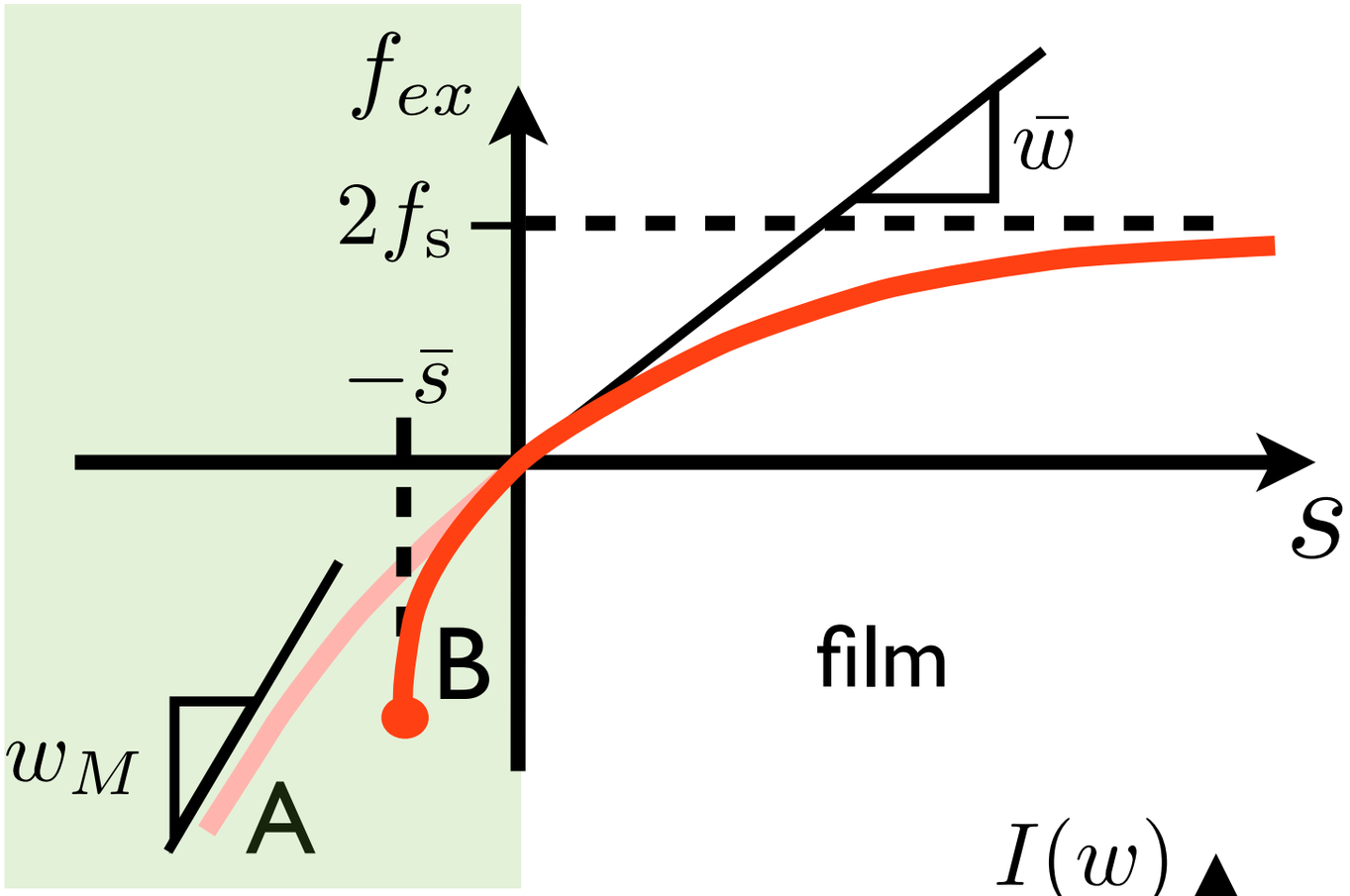


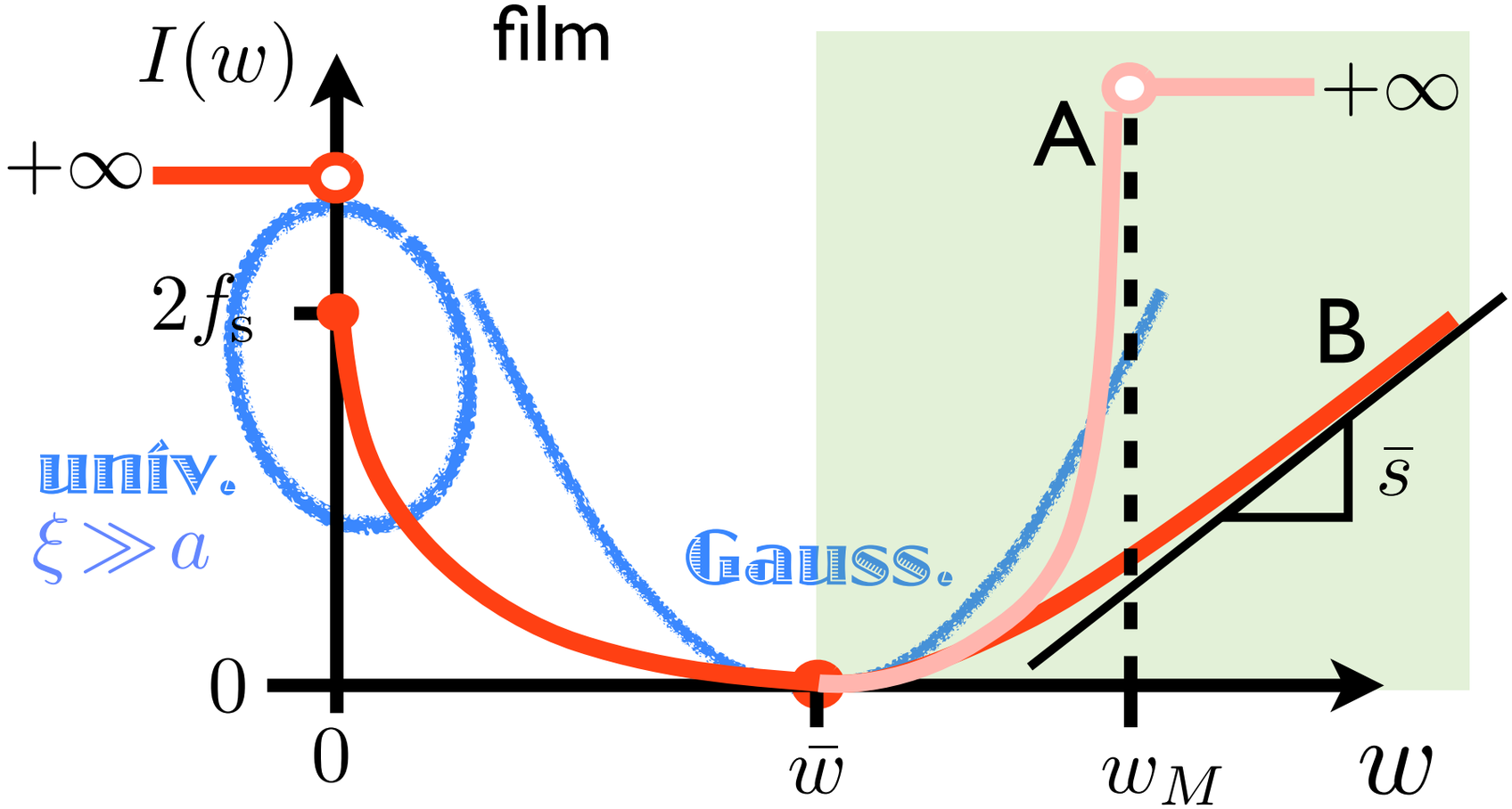

$$I(w<0) = +\infty$$

$$I(w \to 0^+) \Leftarrow f_{ex}(s \to \infty)$$

universal!

[Gambassi,Sotiriadis,Silva'12]


$f_{ex}$
$2f_{\rm s}$
$-\bar{s}$
$\bar{w}$
$s$
B
$w_M$
A
film


film
$I(w)$
$+\infty$
$2f_{\rm s}$
univ.
$\xi \gg a$
Gauss.
0
0
$\bar{w}$
$w_M$
$w$
A
$+\infty$
B
$\bar{s}$

$$\begin{matrix} s \gg a \\ \xi \gg a \end{matrix} \qquad f_{ex}(s) = 2f_{\rm s} + s^{-d}\Theta(s/\xi) + \dots \qquad \xrightarrow{\text{L-F}} \qquad I(w \to 0^+)$$

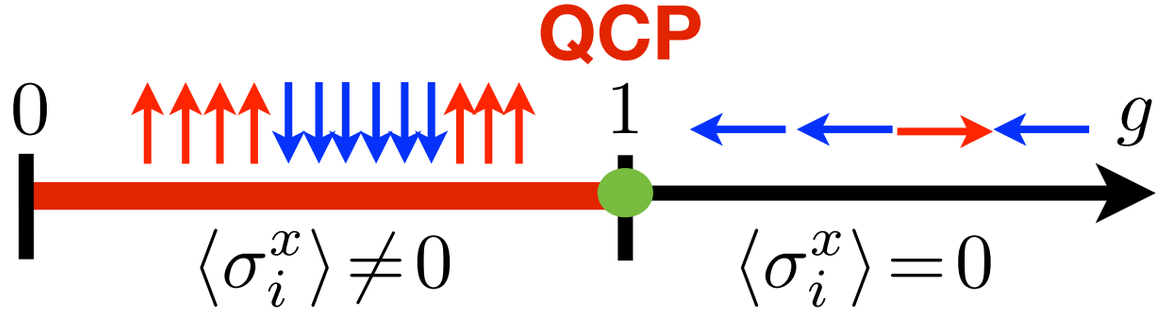
@CP: $$I(w \to 0) = 2f_{\rm s} - \underbrace{\frac{d+1}{d}\Delta\left(\frac{w}{\Delta}\right)^{d/(d+1)}}_{\text{univ.}} + \dots \qquad \Delta = d|\Theta(0)|$$

**critical Casimir amplitude**

> *Beyond? model-dependent....*

## Quantum Ising chain:

$$H(g) = -\sum_i \left[\sigma_i^x \sigma_{i+1}^x + g\sigma_i^z\right]$$




QP: $\epsilon_k(g)$ + quench: $\Delta_k(g, g_0)$

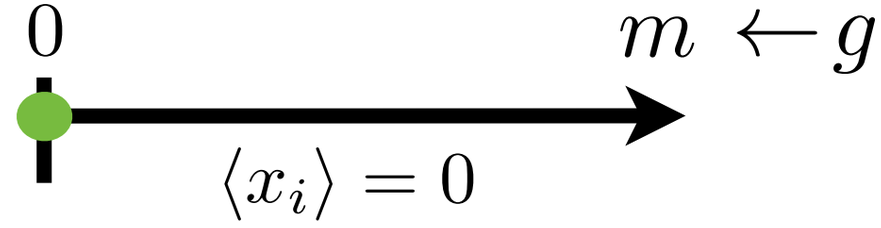
$$f_{ex}(s) = -\int_0^\pi \frac{{\rm d}k}{(2\pi)} \ln\left[\frac{1 + (\tan\Delta_k)^2\, {\rm e}^{-\epsilon_k\, s}}{1 + (\tan\Delta_k)^2}\right] \qquad \Rightarrow \quad \text{case A}$$

2d class. Ising

## "Phonons": (free bosonic field) $d$-dim. latt. $a$

$$H(m) = \sum_i \frac{\pi_i^2}{2} + \sum_{\langle i,j \rangle} \frac{m^2}{2}(x_i - x_j)^2$$

**QCP**




**QP:** $\omega_k(m)$ **+ quench:** $\lambda_k(m, m_0)$ $\quad |\cdot| \leq 1$ [Gambassi,Sotiriadis,Silva'12]

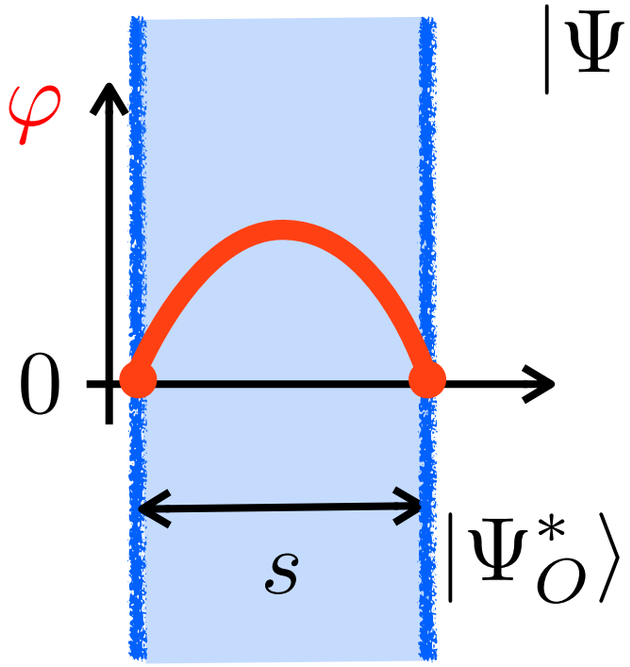
$$f_{ex}(s) = \frac{1}{2}\int \frac{\mathrm{d}^d k}{(2\pi)^d} \ln\left[\frac{1 - \lambda_k^2\, \mathrm{e}^{-2\omega_k\, s}}{1 - \lambda_k^2}\right]$$

→ **case B**
d+1-dim Gauss field $\varphi$

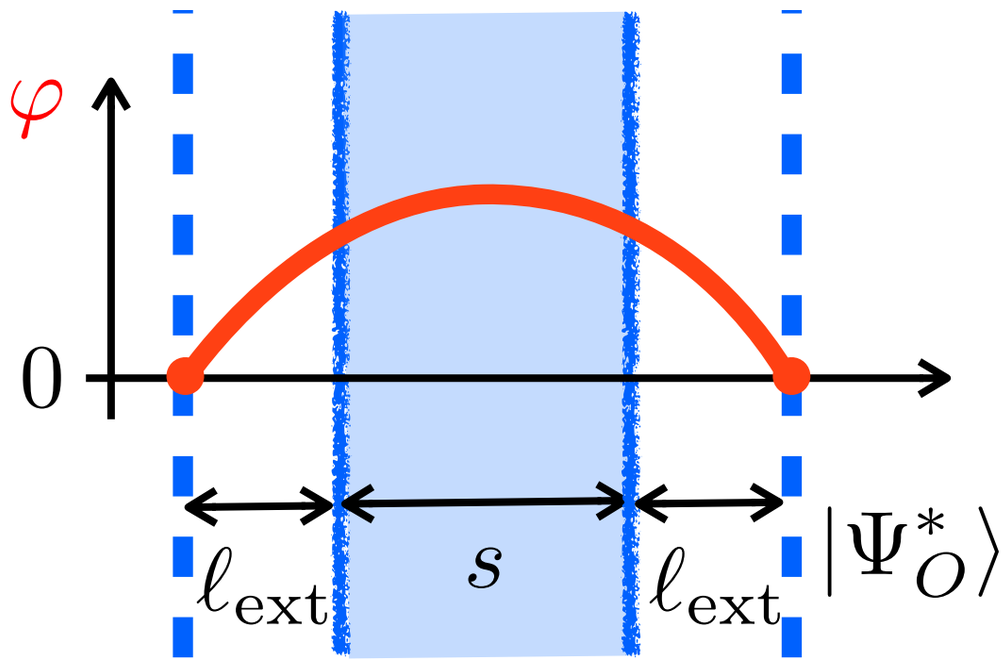
$m_0 a, \;\; m \to 0 \quad (\xi \gg a)$

$|\Psi_0^{m_0}\rangle \rightarrow |\Psi_O^*\rangle$

$\varphi|_{\mathcal{B}} = 0$




$\ell_{\mathrm{ext}} = m_0^{-1}$ extrapol. length [Binder'83]

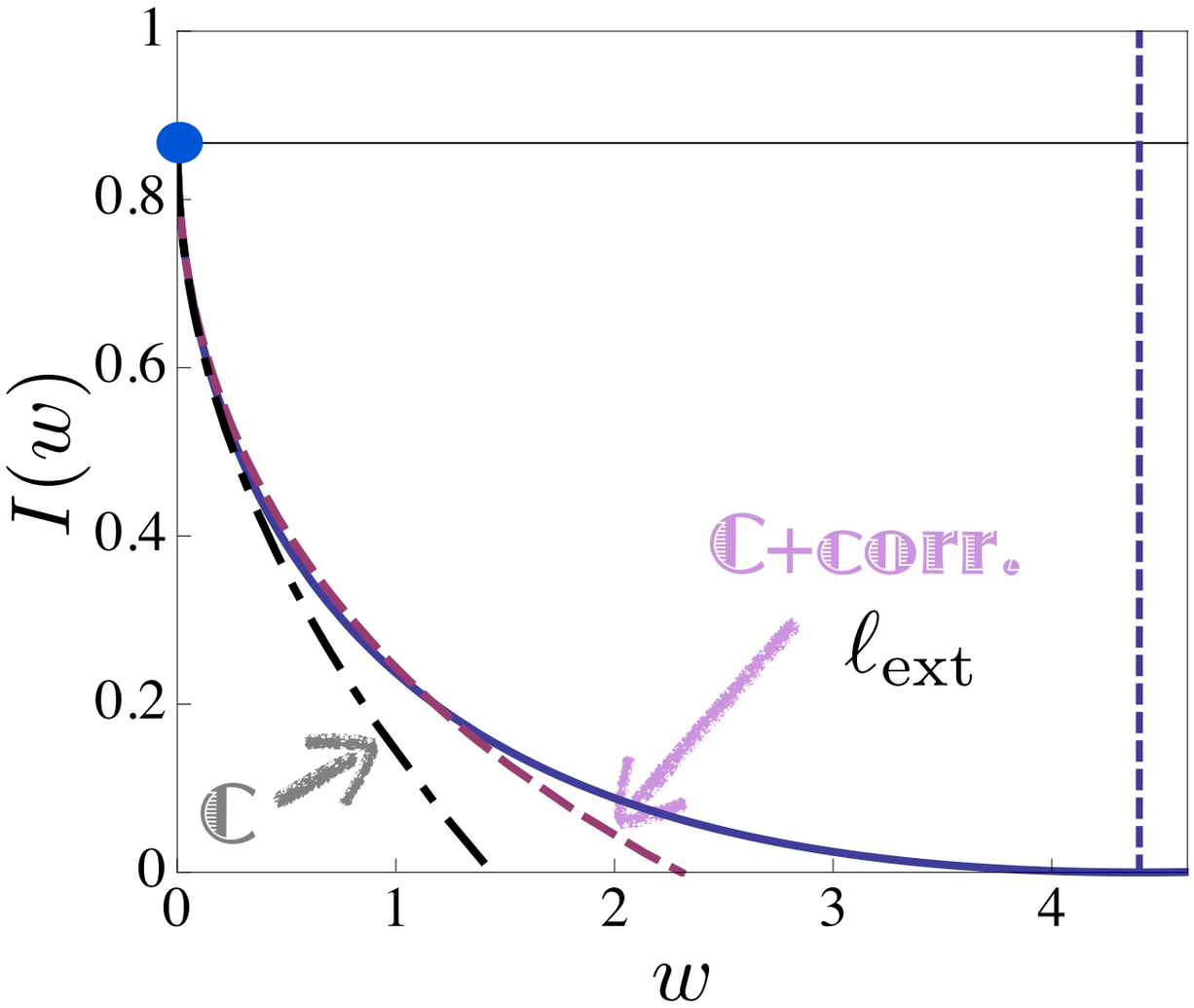


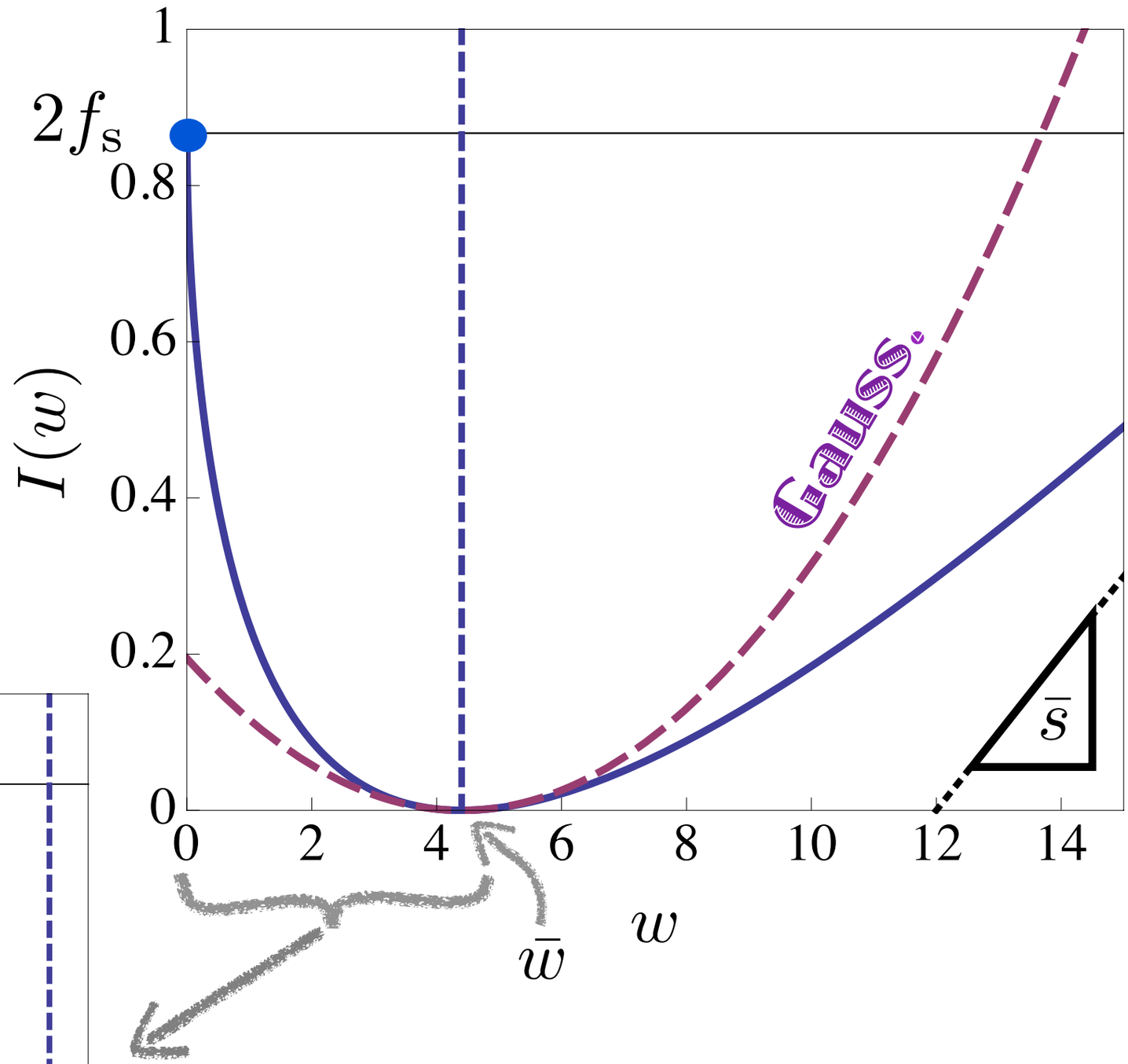
[Gambassi,Sotiriadis,Silva'12]

$d=1$

$m_0 a = 20$

$m=0, \;\; \xi=\infty$




$m\,a \lesssim 1$ **ok**

$m_0 \searrow \Rightarrow \ell_{\rm ext} \nearrow$ **corr!**

$m_0 = m$ **no quench**

*Qu. non-eq.* *Cl. equilib.*

<u>*Obs.:*</u>

$f_{ex}$
$m_0$
???

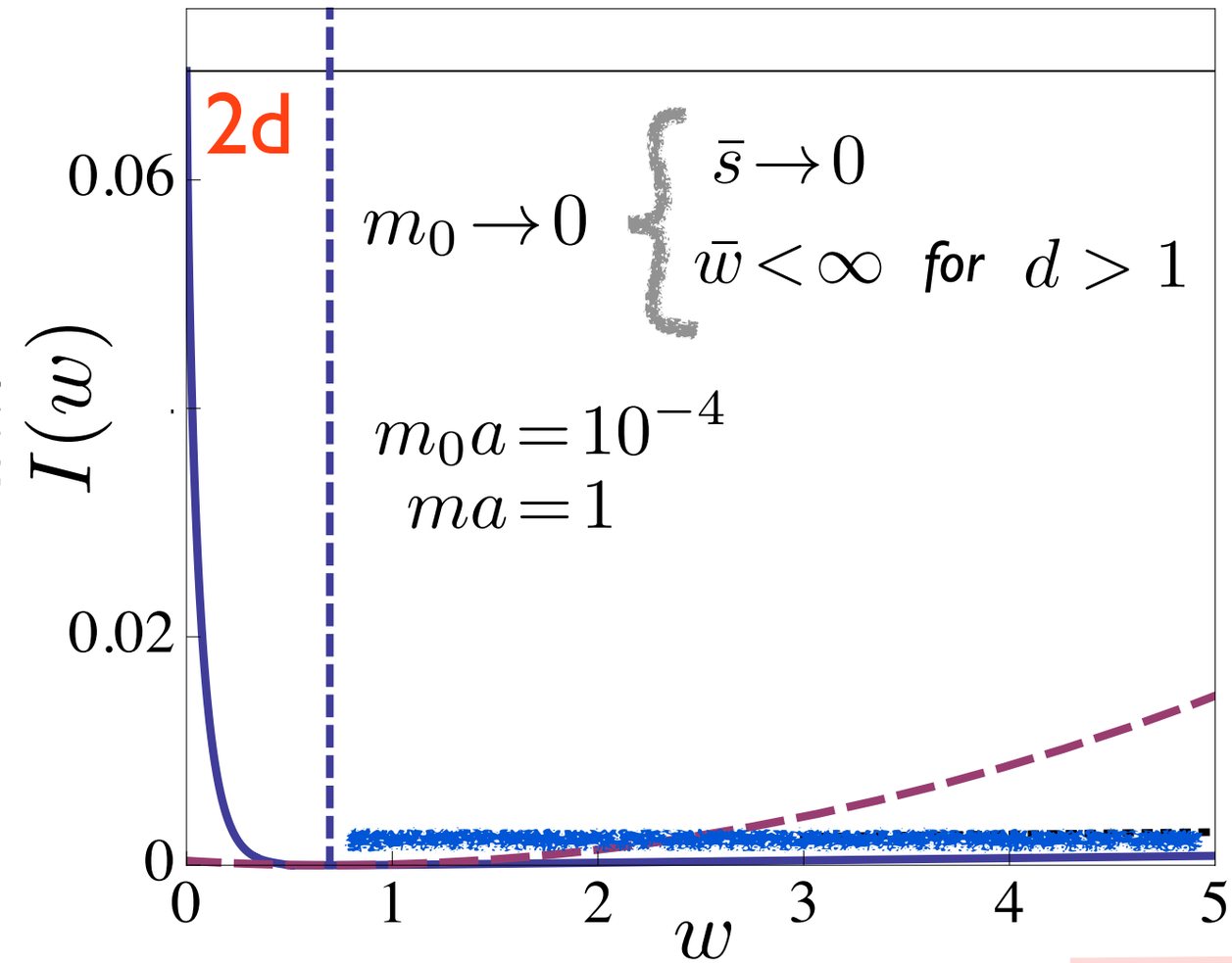
$\Longleftrightarrow$ $\psi$ large dev. of $\rho$
grandcan. $\mu$

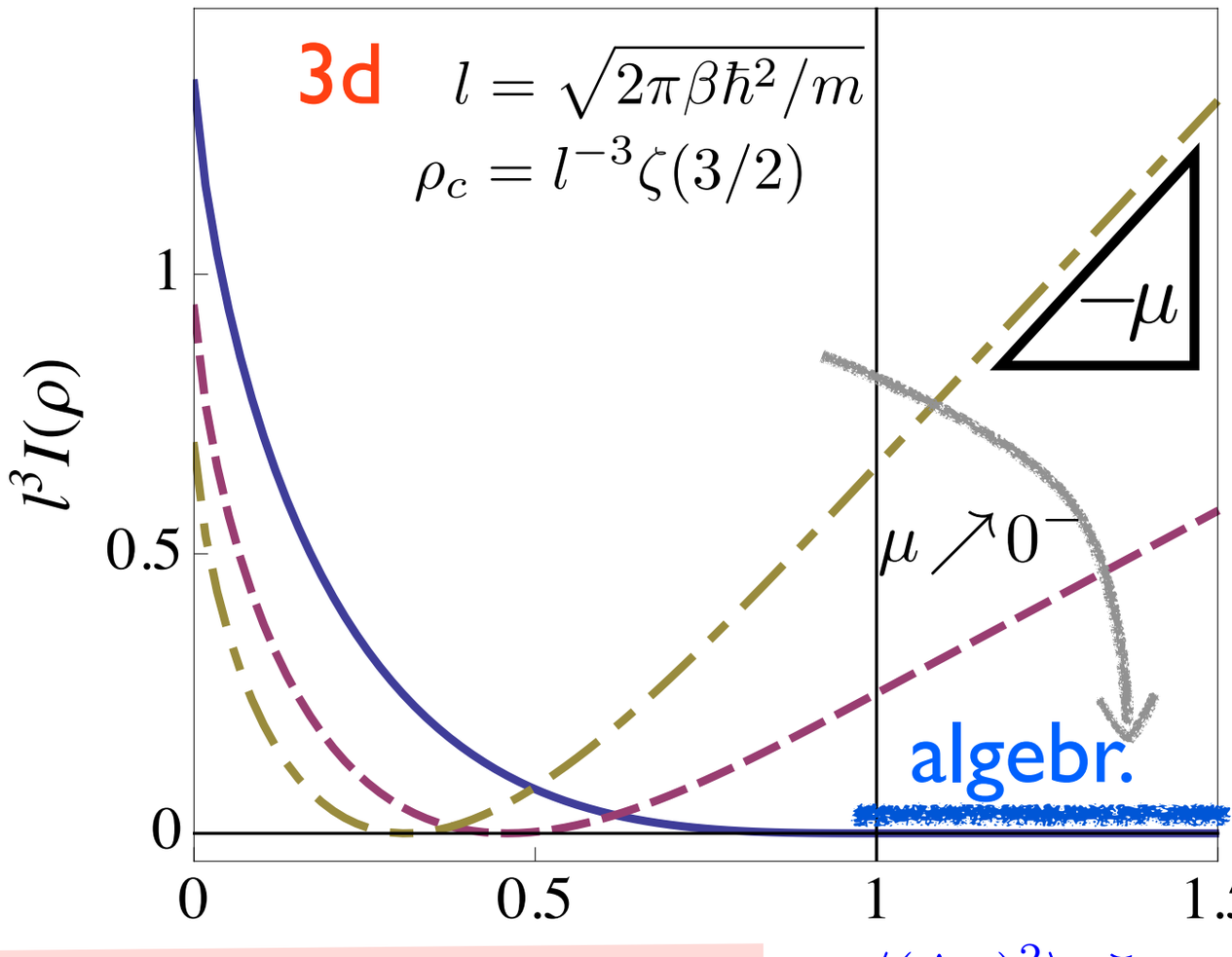
} ideal Bose $\mu \leq 0$

BEC @ $\mu_c = 0$

dim. $> 2$

$\langle n_k \rangle$




algebr. $p(W^{(k\to 0)})$

**Condensation**

$\langle (\Delta\rho)^2 \rangle \nearrow \infty$
condens.

# Conclusions

Take-home message:

- Quantum ⇔ Classical & universality dyn. transitions?
- Edge singularity, large dev. & critical Casimir
- different quantities?
- dimensionality + interaction?
- boundary states? crossover?

Bose:

- condensation?

# Thanks to.....

- **Alessandro Silva**
- **Spyros Sotiriadis**
- **KITP**


*....and to you!*