Measurements of spin diffusion in a homogeneous solid via NMR

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.. it is those laws, which we call the laws of nature...these therefore we must not seek from uncertain conjectures, but learn them from observations and experiments... (Sir Isaac Newton).

Outline

- Problem statement
- The spin system
- Controlling a many body quantum spin system in NMR
 - Average Hamiltonian Theory
 - Examples from NMR: Hahn echo & 'magic echo'
- Spin diffusion on a lattice
- Experimental protocol
- Discussion of experimental results
- Conclusions

Problem statement

Wish to explore how spin diffusion evolves a well-defined microscopic property (a spin state) in the long-time, many-body limit.

The spin system used in this study: single crystal of calcium fluoride

Image taken from http://www.gemrockauctions.com

Image taken from http://www.indiamart.com

Image taken from http://www.crystalinks.com

About the spin system

•¹⁹F nuclear spins are coupled by a dipolar interaction which in a strong magnetic field ($H_Z >> H_D$) is given by

$$H_{D} = \frac{1}{4} \sum_{i < j}^{N} \gamma^{2} \hbar^{2} \frac{(1 - 3\cos^{2} \vartheta_{ij})}{r_{ij}^{3}} \left[2I_{z}^{i}I_{z}^{j} - \frac{1}{2} \{I_{+}^{i}I_{-}^{j} + I_{-}^{i}I_{+}^{j}\} \right]$$

•Sparse paramagnetic impurities: irreversible processes

•Wish to explore how the highlighted term evolves a well-defined microscopic property (a spin state) in the macroscopic limit.

The dynamics of the flip-flop interaction: spin diffusion

$$H_{ff} = \frac{1}{2} \sum_{i < j}^{N} b_{ij} [I_{+}^{i} I_{-}^{j} + I_{-}^{i} I_{+}^{j}]$$
$$b_{ij} = \frac{1}{4} \gamma^{2} \hbar^{2} \frac{(1 - 3\cos^{2} \vartheta_{ij})}{r_{ij}^{3}}$$

An example: Can spatially propagate the state of one spin, to a dipolar coupled neighbor

$$E_{+}^{i} \otimes 1^{j} \xrightarrow{H_{ff}} 1^{i} \otimes E_{+}^{j}$$

where

$$E_{+}^{i} = (1 + 2I_{z}^{i})/2$$

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Average Hamiltonian Theory



A celebrated example (WAHUHA sequence) demonstrating the ability to effectively turn off the strong nuclear dipolar interaction allowing for the measurement of the (much weaker) chemical shift interaction

Average Hamiltonian Theory

- A well known and established method for controlling how a system evolves under a desired effective Hamiltonian by manipulating the dynamics with an experimentally controllable time dependent Hamiltonian.
- Not tied to Nuclear Magnetic Resonance(NMR), though NMR is a very suitable system for its application and has demonstrated the applicability and great success of the theory through numerous investigations such as in spectroscopy and imaging.

Average Hamiltonian Theory in NMR

- To replace a sequence of Radio Frequency (RF) pulses (indicated by in the figure) and evolutions under an internal Hamiltonian by a **time independent** effective Hamiltonian, H_{eff}, and experimentally implement a desired evolution of the spin system.
- H_{eff} evolves the system from the state $|\Psi(t=0)\rangle$ to $|\Psi(t=T)\rangle$.



Overview of Average Hamiltonian Theory One aspect of the theory involves analyzing the transformation of an interaction (H_{int}) in a frame rotating with the applied RF pulses.

The time evolution of the density matrix is given by

$$\rho(t_c) = U_{RF} U_{int} \rho(0) U_{int}^{-1} U_{RF}^{-1}$$

 U_{RF} represents the interaction associated with the sequence of RF pulses applied over a time $t_{c.}$

H_{int} refers to the systems' internal Hamiltonians (chemical shifts, field inhomogeneity, dipolar coupling)

An appealing aspect of the formalism is that a variety of Hamiltonians can be accounted for such as : pulse errors, pulse transients & finite pulse width effects

$$U_{RF} = \dots U_{2}U_{1}U_{0} \qquad Series$$

$$U_{int} = (t_{c}) = \exp[-it_{c}(\overline{H}_{int}^{0} + \overline{H}_{int}^{1} + \dots)]$$

$$\overline{H}_{int}^{0} = \frac{1}{t_{c}} \int_{0}^{t_{c}} \widetilde{H}_{int} Magnus$$

$$\overline{H}_{int}^{1} = \frac{-i}{2t_{c}} \int_{0}^{t_{c}} \widetilde{H}_{int}(\tau), \int_{0}^{t} H_{int}(\phi) d\phi] d\tau$$

$$\overline{H}_{int}^{2} = \dots$$

 $\widetilde{H}_{int}(t) = U_{RF}^{-1} H_{int} U_{RF}$

The Hahn-echo sequence



If the system is sampled at interval 2τ it will appear to have evolved under an effective Hamiltonian which is twice the dipolar interaction. The chemical shift interaction is 'turned-off'.

A sample calcium fluoride spectrum

• Without multiple pulse one experimentally observes a broad line-width due to strong dipolar interactions.



Using a magic echo to 'turn off' the dipolar coupling in our spin system

• Under **perfect experimental conditions**, the cycle time-reverses the evolution of a system of nuclear spins that are coupled by a dipolar Hamiltonian (H_D) by a change of sign of the effective Hamiltonian of the sequence.

 $H_{eff} = -1/2H_D$

The Magic Echo Cycle



Under this cycle the system will appear to not evolve under the dipolar interaction as approximated to zeroth order of the Magnus Expansion.

Accounting for errors in average Hamiltonian calculation

• H_{eff} affected by experimental errors

(RF pulse transients, finite pulse widths, pulse errors ...).

$$H_{eff} = -1/2H_D + errors$$

- AHT allows one to account for a variety of pulse errors that may plague a multiple pulse cycle by considering their effect on H_{eff}, and then design cycles which remove their contribution to perform the desired transformation.
- Considered the effects of finite pulse width in the $\pi/2$ pulse, pulse flip errors, and symmetric phase transients.

Removing H_D and correcting for errors(1)

- Can remove the effect of H_D using a pulse sequence with $H_{eff} = -H_D$
- The $\pi/2$ pulse widths are not negligible but easily accounted for in the calculation of H_{eff} .

Contribution of
Finite pulse widths

$$H_{eff} = -1/2H_D + \alpha (I_x^i I_x^j - I_y^i I_y^j)$$

 2α is the $\pi/2$ pulse width

Resulting line narrowing: 361 Hz

Removing H_D and correcting for errors (2)

• Can remove the effect of finite pulse widths by alternating the phases of pulses in a well defined way, and then making a measurement over a larger cycle which removes their effect on the spin evolution.



Simplified pulse sequence diagram

$$H_{eff} = -1/2H_{D} + \alpha (I_{x}^{i}I_{x}^{j} - I_{y}^{i}I_{y}^{j})$$

$$\int I_{eff} = -1/2H_{D} - \alpha (I_{x}^{i}I_{x}^{j} - I_{y}^{i}I_{y}^{j})$$

$$H_{eff} (total) = -H_{D}$$

G.S. Boutis, P. Cappellaro, H. Cho, C. Ramanathan, D. G. Cory. "Pulse error compensating symmetric magic echo trains" (2003) J. Magn. Reson. 161, 132-137.

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A single crystal of calcium fluoride

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Zeeman order

Nuclear spins are aligned along a strong external magnetic field, at room temperature

$$\rho = 1 - \varepsilon \sum_{i} I_{z}^{i} \qquad \begin{array}{c} \varepsilon = \gamma B/kT \\ \rho = density \\ operator \end{array}$$

Dipolar order

Nuclear spins are aligned along local dipolar fields of other nuclear spins

$$\rho = 1 - cH_D$$
 c is constant

In a strong external magnetic field the dipolar ordered state scales as H_D .

A toy model of spin diffusion

Purpose of the model

- a. Highlight the method of measurement
- b. Highlight the dynamics of spin-diffusion
- c. Do <u>NOT</u> want to calculate a diffusion rate

•In general

$$\rho(\tau) = 1 - \varepsilon \sum_{k} c_{k} I_{z}^{k} e^{i(\varphi_{j} - \varphi_{k})} = 1 - \varepsilon \sum_{k} c_{k} I_{z}^{k} e^{i\Delta\varphi_{jk}}$$

•The detected signal $\langle I_z \rangle = Tr[\rho I_z]$

$$\langle I_{z} \rangle = \varepsilon \left\langle \sum_{k} c_{k} e^{i \Delta \varphi_{jk}} \right\rangle = \varepsilon \left\langle 1 - \frac{(\Delta \varphi)^{2}}{2} + \dots \right\rangle$$

•Long time many spin limit

 $-\Delta \varphi$ small and random

-spread of the phase behaves diffusively

$$\left\langle \left(\Delta \varphi \right)^2 \right\rangle / 2 = k^2 \left\langle \left(\Delta z \right)^2 \right\rangle / 2 = k^2 D u$$

Spin diffusion of dipolar order

$$\rho(o) = 1 - \{ e^{i\varphi_j} \} E_+^j \otimes 1^k \otimes 1^l \otimes 1^m \dots$$

Create dipolar order

$$\rho(o) = 1 - \varepsilon c e^{i\varphi_j} (3I_z^j I_z^k - I^j \cdot I^k) \otimes 1^l \otimes 1^m \otimes 1^n \dots$$

Evolution under H_D for a variable time ' τ '

$$\rho(\tau) = 1 - \varepsilon c(e^{i\varphi_j}) 1^j \otimes 1^k \otimes (3I_z^l I_z^m - I^l \cdot I^m) \otimes 1^n \dots$$

Spin diffusion of dipolar order $\rho(\tau) = 1 - \varepsilon c(e^{i\varphi_j}) 1^j \otimes 1^k \otimes (3I_z^l I_z^m - I^l \cdot I^m) \otimes 1^n \dots$ Convert back to Zeeman order for detection $\rho(\tau) = 1 - \varepsilon \begin{bmatrix} w_i (e^{i\varphi_j}) 1^j \otimes 1^k \otimes I_z^l \otimes 1^m \dots \\ + w_m (e^{i\varphi_j}) 1^j \otimes 1^k \otimes 1^k \otimes 1^l \otimes I_z^m \dots \end{bmatrix}$ <u>Readout</u>: -gradients, $H_D = 0$ $\rho_{readout}(\tau) = 1 - \varepsilon \begin{bmatrix} w_{l}(e^{i(\varphi_{j} - \varphi_{l})})1^{j} \otimes 1^{k} \otimes I_{z}^{l} \otimes 1^{m} \dots \\ + w_{m}(e^{i(\varphi_{j} - \varphi_{m})})1^{j} \otimes 1^{k} \otimes 1^{l} \otimes I_{z}^{m} \dots \end{bmatrix}$ •The detected signal $\langle I_z \rangle = Tr[\rho I_z]$

•In general

$$\left\langle I_{z}\right\rangle = \varepsilon \left\langle \sum_{l} w_{l} e^{i\Delta\varphi_{jl}} + \sum_{m} w_{m} e^{i\Delta\varphi_{jm}} \right\rangle$$

•Long time many spin limit

 $-\Delta \varphi$ small and random

-spread of the initial phase behaves diffusively

$$\left\langle I_{z} \right\rangle = \varepsilon \left\langle 1 - \left(\varDelta \varphi \right)^{2} + \dots \right\rangle$$
$$\left\langle \left(\varDelta \varphi \right)^{2} \right\rangle = k^{2} \left\langle \left(\Delta z \right)^{2} \right\rangle = 2k^{2} D\tau$$

$$\left\langle I_{z}\right\rangle_{dipolar} = \varepsilon \left\langle 1 - \left(\varDelta \varphi\right)^{2} + \dots \right\rangle$$

$$\left\langle I_{z}\right\rangle_{Zeeman} = \varepsilon \left\langle 1 - \frac{\left(\Delta \varphi\right)^{2}}{2} + \dots \right\rangle$$

- •Model indicates a difference due to spin correlations in case of dipolar order.
- •Spin system is actually more complex.

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Why this experiment is challenging

Spin diffusion rate has been predicted to be very slow:

D~1 x 10⁻¹² cm $^{2}/s$

The displacement of spin coherence is therefore approximately 1 µm in about 1 hour.



The spatially encoded profile needs to have a wavelength on this length scale to probe the dynamics.

Attenuation due to diffusion needs to be larger than that due to relaxation.

Sample sizes are ~ 1mm^3 . The precisions required to encode and then refocus the residual phase encoding is therefore on the order of 1 part in 10^5 .

State preparation

• Encode profile with wavelength on the order of 1 micron

-Suppress internal dipolar coupling (including spin diffusion and any offset) using average Hamiltonian methods

-Apply short, strong pulsed gradients in well-defined times of RF pulse sequence.

Experimental techniques

- Improved a radio-frequency pulse sequence for controlling nuclear spin degrees of freedom by correcting for finite pulse width errors, RF transients, and other pulse errors, to better decouple the nuclear spins.
- Pulsed gradient fields to the sample (Solid state NMR probe with an efficient gradient & RF coil)
- High pulsed currents (25-50 A) for the gradients, in short times (20-30 μs) & reproducibility must be less than 1 in10⁵ (electronic circuitry).

Design of the gradient coil (schematic does not include RF coil for manipulating spin dynamics)

Conditions for uniform gradient

$$z_1 = 0.44R$$
 $\frac{S_2}{S_1} = 7.47$ $z_2 = 1.19R$

Advantages of this coil configuration

A very strong gradient field can be created

Conventional gradient coils produce ~50 to 100 G/cm. 1m

This gradient coil design ~ 20,000 G/cm @ 55 Amp





B.H. Suits and D.E. Wilken "Improving magnetic field gradient coils for NMR imaging" (1989) *J. Phys. E: Sci. Instrum.*, 22, 565-573. W. Zhang and D.G. Cory "Pulsed gradient NMR probes for solid state studies" (1989) *J. Magn. Reson.* 132, 144-149.

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Experimental results

- $\mathbf{D}_{\mathbf{z}}$ Diffusion constant of Zeeman order
- $\mathbf{D}_{\mathbf{E}}$ Diffusion constant of dipolar order
- **T**₁ Relaxation time of Zeeman order
- T_{1D} Relaxation time of dipolar order

Experimental Measurements

Zeeman Diffusion

Dipolar Diffusion



FIG. 2. Experimental results of the spin diffusion rate of Zeeman order with the crystal aligned along (a) the [111] direction—the pulsed gradient fields generated are \bigcirc , 2886 G/cm; *, 5661 G/cm; \triangleleft , 8510 G/cm; \square , 8510 G/cm; and (b) the [001] direction—the pulsed gradient fields generated are \square , 2886 G/cm; *, 5661 G/cm; \triangleright , 8695 G/cm; \bigcirc , 7474 G/cm. The error in *k* is approximately 3%.



FIG. 3. Experimental results of the spin diffusion rate of dipolar order with the crystal aligned along (a) the [001] direction—the pulsed gradient fields generated are \triangleright , 5698 G/cm; \Box , 5698 G/cm; \bigcirc , 8473 G/cm; *, 8399 G/cm; and (b) the [111] direction—the pulsed gradient fields generated are *, 2886 G/cm; \bigcirc , 8436 G/cm; \triangleleft , 5661 G/cm; \Box , 8362 G/cm. The error in *k* is approximately 3%.

Summary of the experimental results for the diffusion rates of Zeeman and dipolar order

Measurements performed in this study	[001]	[111]
$D_{\rm E} [x \ 10^{-12} \ {\rm cm^{2/s}}]$	29 ± 3	33 ± 4
$D_{Z} [x \ 10^{-12} \ cm^{2}/s]$	6.4 ± 0.9	4.4 ± 0.5
T ₁ [s]	256.2 ± 3.4	287.5 ± 7.0
T _{1D} [s]	9.1 ± 0.1	9.0 ± 0.2

G.S. Boutis, D. Greenbaum, H. Cho, D. G. Cory, C. Ramanathan "Spin diffusion of correlated two-spin states in a dielectric crystal" (2004) *Phys. Rev. Lett.* 92: 13 137201-137205

Zeeman order: Comparison with theory & simulation

Theoretical studies of D _Z	D_z [001] x 10 ⁻¹² [cm ² /s]	D_Z[111] x 10 ⁻¹² [cm ² /s]	$\frac{D_{z}[001]}{D_{z}[111]}$
Moment Calculation (Redfield and Yu)	8.22	6.71	1.22
Irreversible Statistical Mechanics (Borckmans and Walgraef)	6.98	4.98	1.40
Simulation of classical gyro- magnets (Tang and Waugh)	7.43	No data	No data
Hydrodynamic Aproach (D. Greenbaum et. al.)	8.1	7.4	1.1

Experimental Measurement:

 $\frac{D_z[001]}{D_z[111]} = 1.4 \pm 0.2$

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A.G. Redfield, W. N. Yu "Moment-Method Calculation of Magnetization and Interspin-Energy Diffusion" (1968) Phys. Rev. 169, 443–450.

P. Borckmans, D. Walgraef "Irreversibility in Paramagnetic Spin Systems: Free Induction Decay and Spin Diffusion" (1968) Phys. Rev. 167, 282–288.

C.Tang, J. S. Waugh "Dynamics of classical spins on a lattice: Spin diffusion" (1992) Phys. Rev. B 45, 748–754

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Ratio of the diffusion coefficients and relaxation times reflect local anisotropy

$$\frac{D_z[001]}{D_z[111]} = 1.4 \pm 0.2$$

$$\frac{T_1[111]}{T_1[001]} = 1.1 \pm 0.1$$

•Angular variation due to anisotropy in the dipolar coupling; dipolar coupling stronger along the [001] direction than [111] direction

$$b_{ij} = \frac{1}{4} \gamma^2 \hbar^2 \frac{(1 - 3\cos^2 \vartheta_{ij})}{r_{ij}^3}$$

Dipolar order: Comparison with theory & simulation

Theoretical studies of D_E	D _E [001] x 10 ⁻¹² [cm ² /s]	D _E [111] x 10 ⁻¹² [cm ² /s]	$\frac{D_E[001]}{D_E[111]}$	$\frac{D_E[001]}{D_Z[001]}$	$\frac{D_E[111]}{D_Z[111]}$
Irreversible Statistical Mechanics (Borckmans and Walgraef)	8.53	7.43	1.16	1.22	1.49
Simulation of classical gyro-magnets (Tang and Waugh)	13.3	No data	-	1.79	-
Hydrodynamic Aproach (D. Greenbaum et. al.)	19.1	11.4	1.7	2.35	1.54

Experimental results

$\frac{D_E[001]}{2} = 4.5 \pm 0.8$	$\frac{D_E[111]}{2} = 7.5 \pm 1.2$	$\frac{D_E[001]}{0} = 0.9 \pm 0.1$
$D_{Z}[001]$	$D_{Z}[111]$	$D_{E}[111]$ 0.5 ± 0.1

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Experimental results

$$\frac{D_E[001]}{D_Z[001]} = 4.5 \pm 0.8 \quad \frac{D_E[111]}{D_Z[111]} = 7.5 \pm 1.2 \quad \frac{D_E[001]}{D_E[111]} = 0.9 \pm 0.1$$
$$\frac{T_1[001]}{T_{1D}[001]} = 28 \pm 0.5 \quad \frac{T_1[111]}{T_{1D}[111]} = 30 \pm 1.0 \quad \frac{T_{1D}[001]}{T_{1D}[111]} = 0.9 \pm 0.1$$

•Spin diffusion rate of dipolar order is isotropic with respect to crystal orientation.

•Differences in the spin diffusion rate of dipolar order to Zeeman order do not account for differences in corresponding relaxation times (perhaps other processes are involved?).

Simple schematic highlights why diffusion of dipolar order is less quenched



If spins 1 and 2 are initially in the same state (both \uparrow or both \downarrow), no evolution takes place



For the diffusion of dipolar order along the z direction, there are the two possibilities: both spins 1 and 3, and spins 2 and 4, can flip, or spins 1 and 2, and spins 3 and 4, can flip

Even if states 3 and 4 are initially \uparrow and \downarrow , two different evolution paths are present. Spins 1 and 2 can flip and spins 3 and 4 can flip or spins 1 and 4 and spins 2 and 3 can flip.

Conclusions

- The spin diffusion rate of a two spin correlated spin state in single crystal calcium fluoride was measured using pulsed gradient schemes.
- Coherent averaging techniques were applied to correct for a variety of pulse errors in the 'magic-echo' cycle resulting in more accurate control over the nuclear spin degrees of freedom. These methods are also applicable for investigations relating to spectroscopy and imaging.
- The experimental methodology allows for further tests of many-body physics ex: studies of spin diffusion as a function of polarization or dimensionality.

Acknowledgements

David Cory (Thesis Advisor) J. S. Waugh (Thesis Advisor) S.H. Chen

Timothy Havel Pabitra Sen Uli Scheven

Hyong-Joon Cho
Sekhar Ramanathan
Joseph Emerson
Marco Pravia
Evan Fortunato
Grum Teklemariam
Nicolas Boulant
Paola Capellaro

Gabriella Leu Phillip Sun Amro Farid Wurong Zhang Shyamal Somaroo

Research supported by NSF, DARPA & Cambridge/MIT Institute

A postdoctoral position is expected to be available starting May 2013

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