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VCQ

Vienna Center for Quantum  
Science and Technology

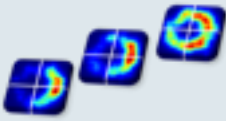


# Pre-thermalisation and Relaxation in a 1d System

David Adu Smith

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[www.atomchip.org](http://www.atomchip.org)

# Non-Equilibrium Systems

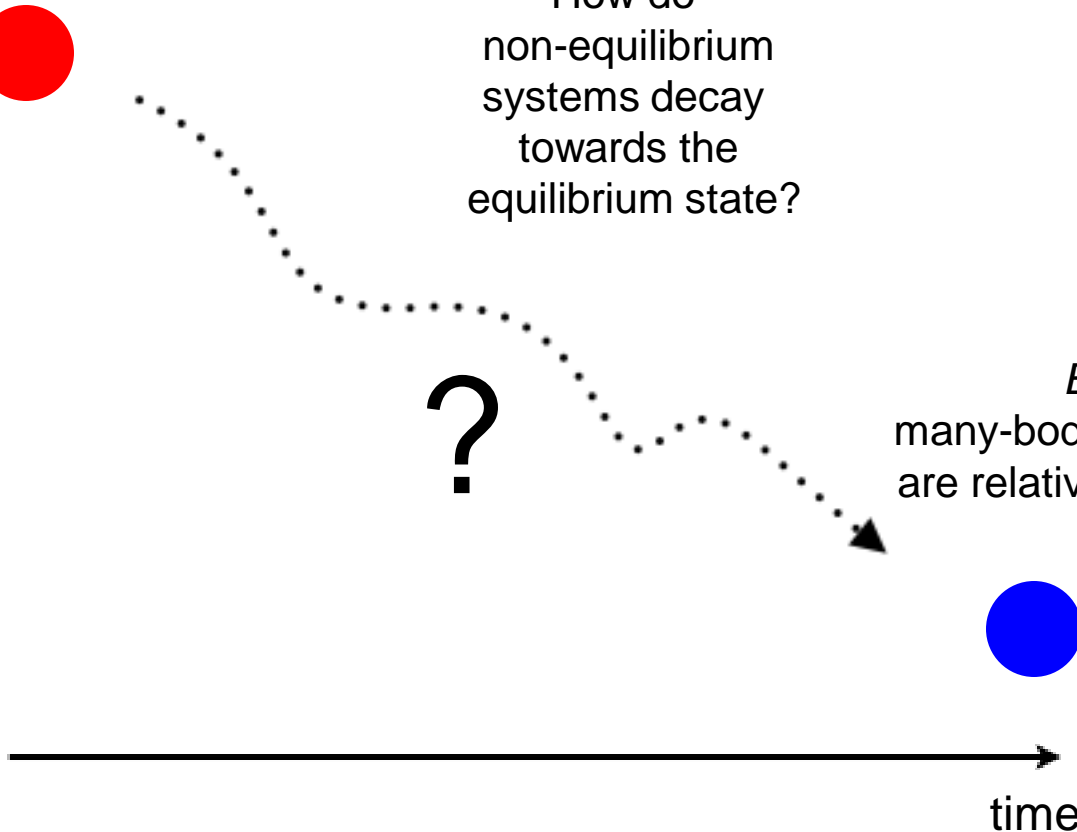


*Non-equilibrium*  
many-body quantum systems  
are *not* well  
understood

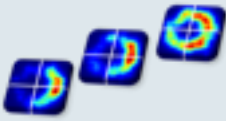
How do  
non-equilibrium  
systems decay  
towards the  
equilibrium state?

?

*Equilibrium*  
many-body quantum systems  
are relatively well understood



# Our Non-Equilibrium System

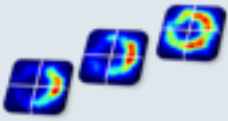


## In an ultracold atom system:

A phase-coherently split 1d Bose gas



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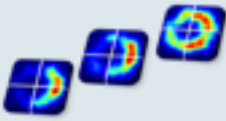
A phase-coherently split 1d Bose gas



1. Prepare an equilibrium 1d Bose gas

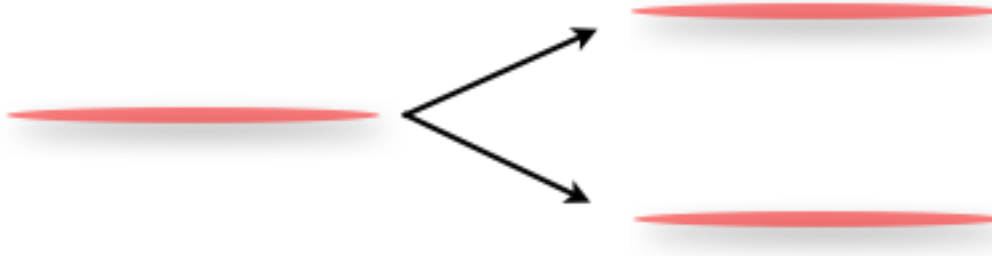


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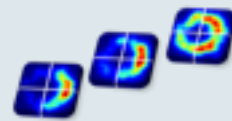


1. Prepare an equilibrium 1d Bose gas

2. Phase-coherently split the gas into two (uncoupled) 1d gases



# Our Non-Equilibrium System



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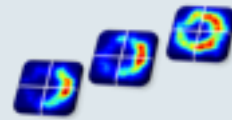


2. Phase-coherently split the gas into two (uncoupled) 1d gases

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# 1d Bose Gases



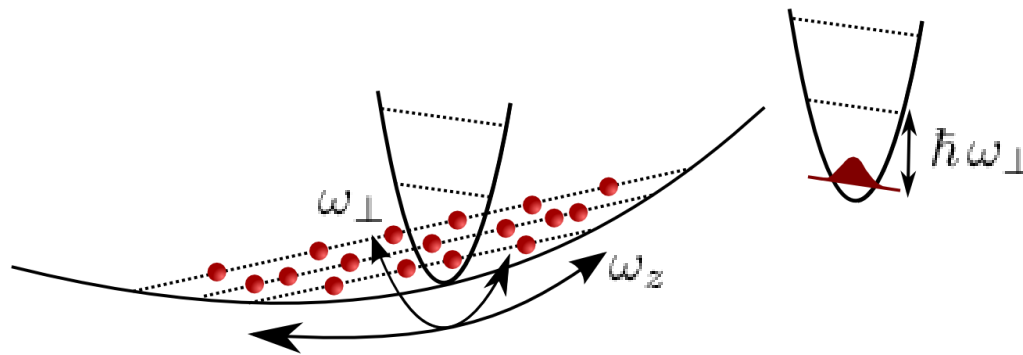
- For an effectively 1d Bose Gas (in a trap)

$$\mu, k_B T \ll \hbar \omega_{\perp}$$

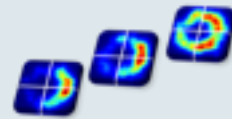
$$\omega_{\perp} \gg \omega_z$$



- Macroscopic occupation of radial ground state
- Longitudinal states populated



# 1d Bose Gases

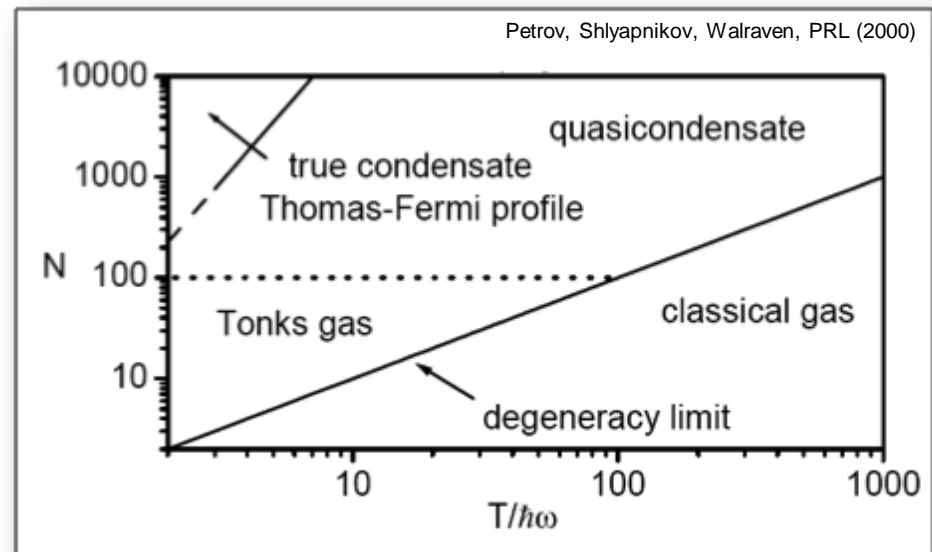
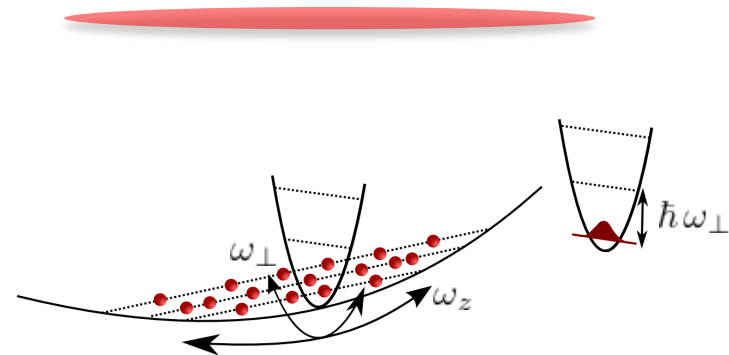


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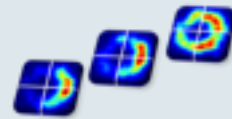
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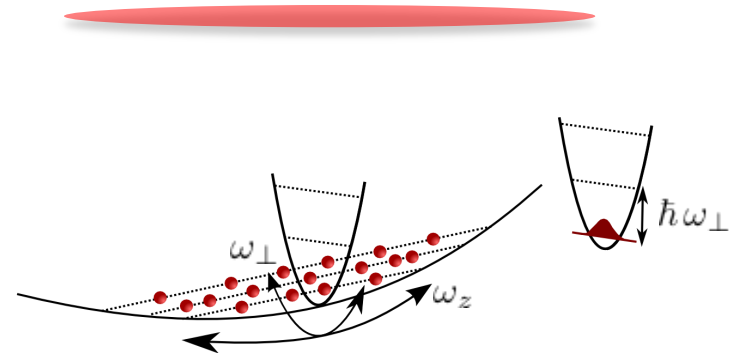


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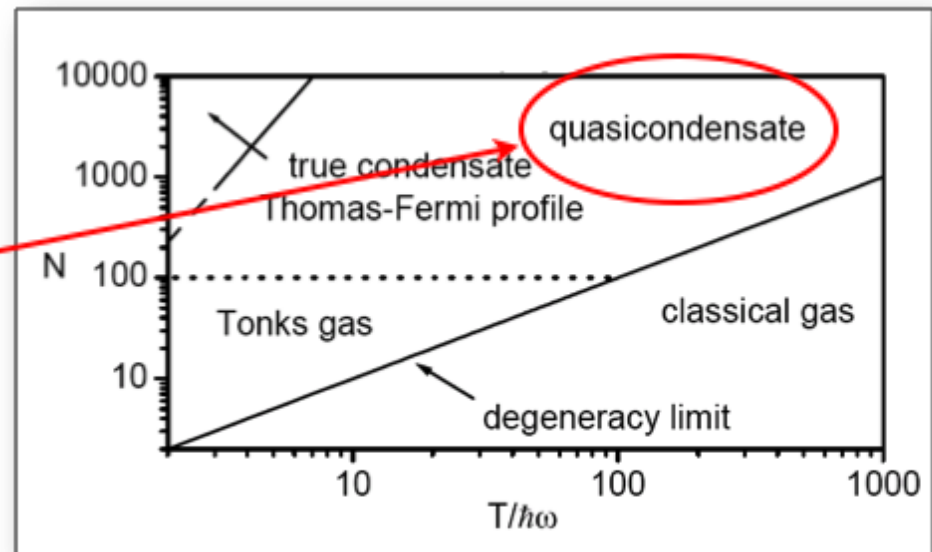
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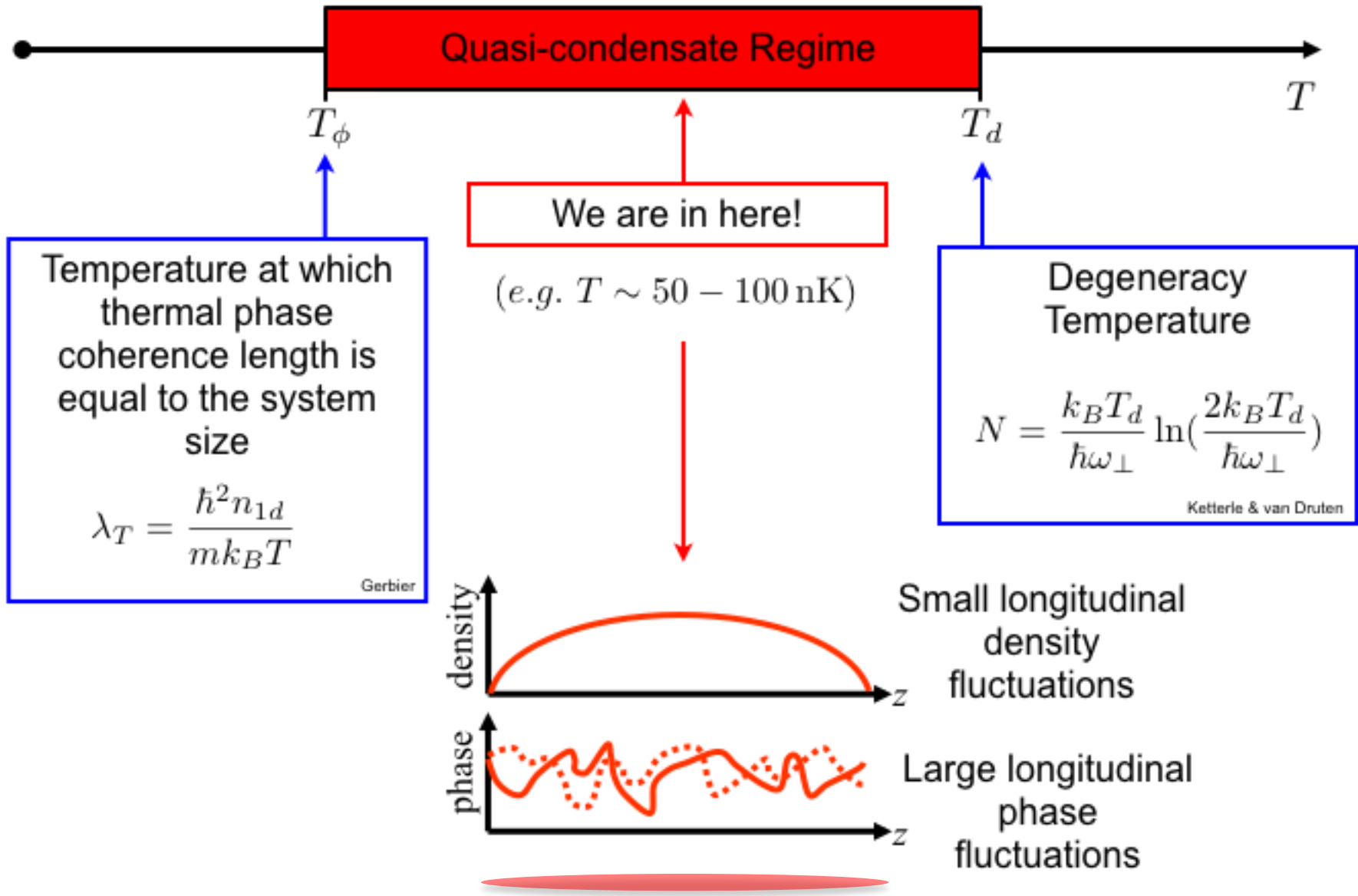
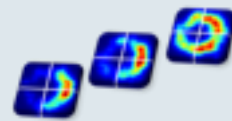


Weakly Interacting Regime

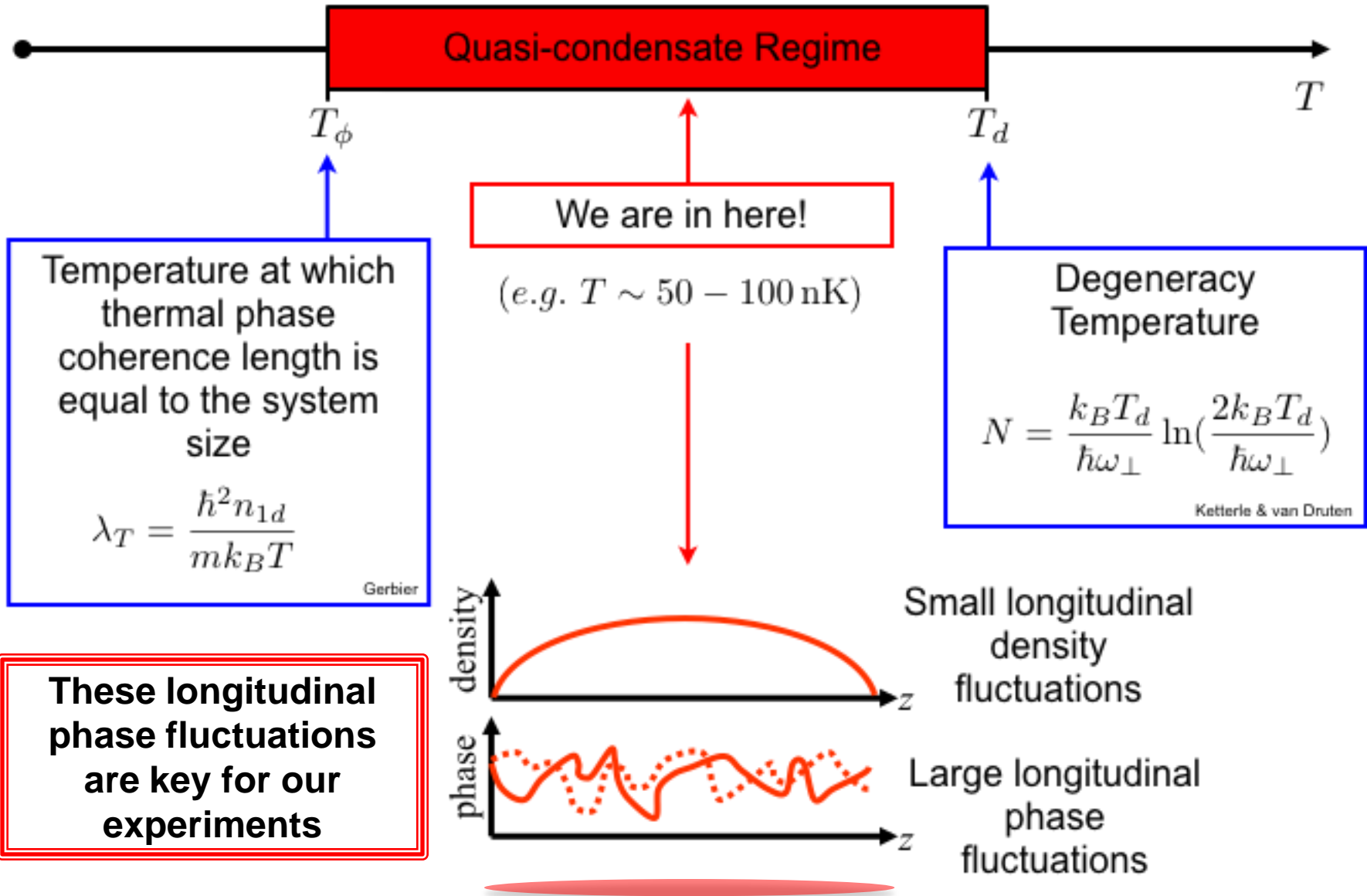
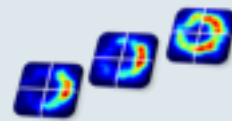
(e.g.  $\gamma = \frac{mg}{\hbar^2 n} \sim 10^{-2}$ )



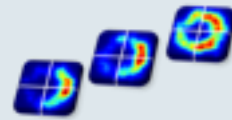
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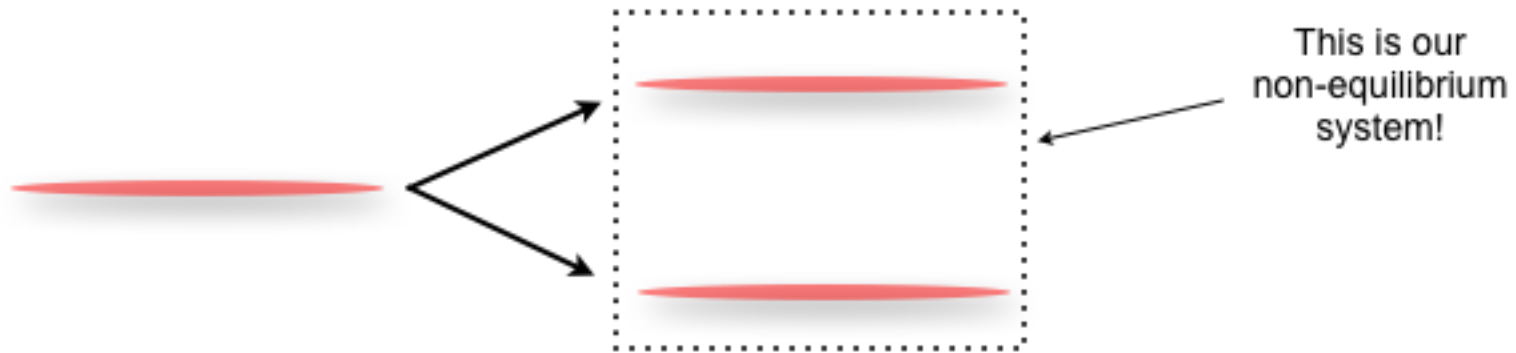


# Non-Equilibrium Systems: Examples



## In an ultracold atom system:

A phase-coherently split 1d Bose gas

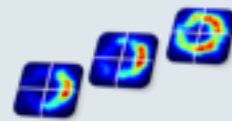


2. Phase-coherently split the gas into two (uncoupled) 1d gases

1. Prepare an equilibrium 1d Bose gas

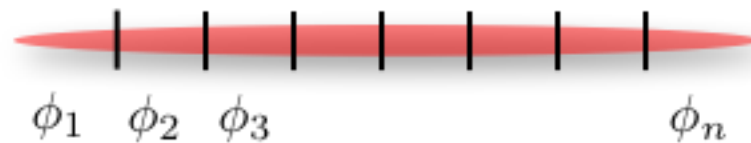


# 1d Bose Gases



Why is this a non-equilibrium system?

1d Bose gas  
in equilibrium

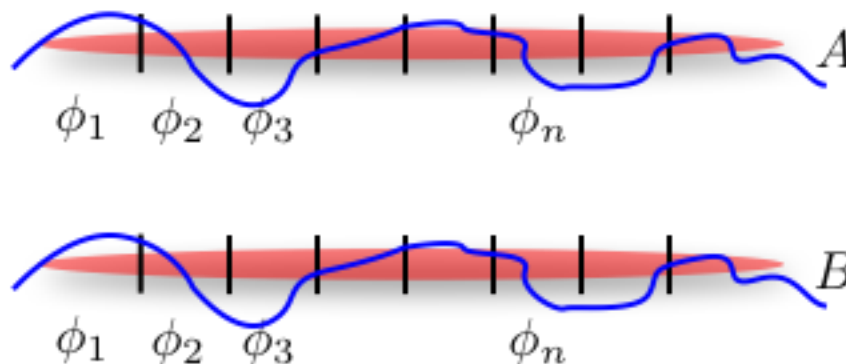


Phase-coherently  
split into  
two 1d gases



phase profiles  
are **correlated**

**difference** is only  
due to **quantum  
shot noise** of  
splitting process

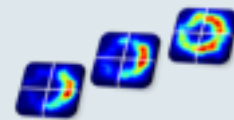


This is our  
non-equilibrium  
system!

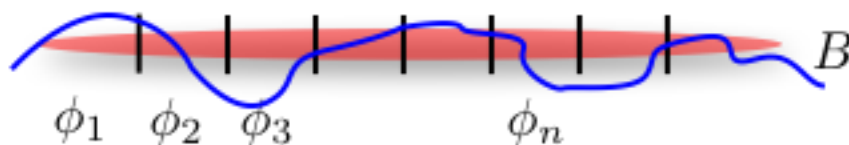
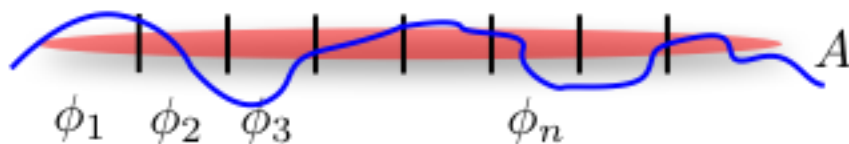
$$\text{At time } t_h = 0, \quad \phi_{nA} \cong \phi_{nB}$$



# 1d Bose Gases

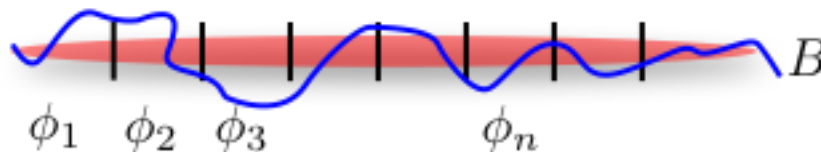
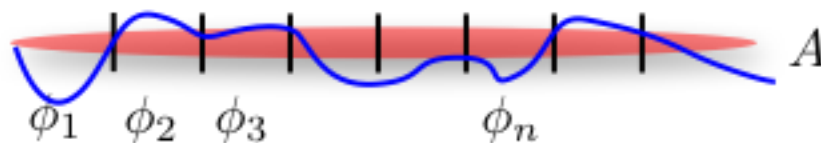


correlated state



At time  $t_h = 0$ ,  $\phi_{nA} = \phi_{nB}$

uncorrelated state



At time  $t_h = t$ ,  $\phi_{nA} \neq \phi_{nB}$



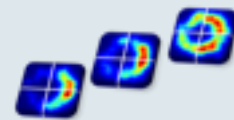
Non-equilibrium

?

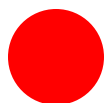
Equilibrium

(two independent gases  
with random relative phase)

# Non-Equilibrium Systems



*Non-equilibrium*  
many-body quantum systems  
are *not* well  
understood



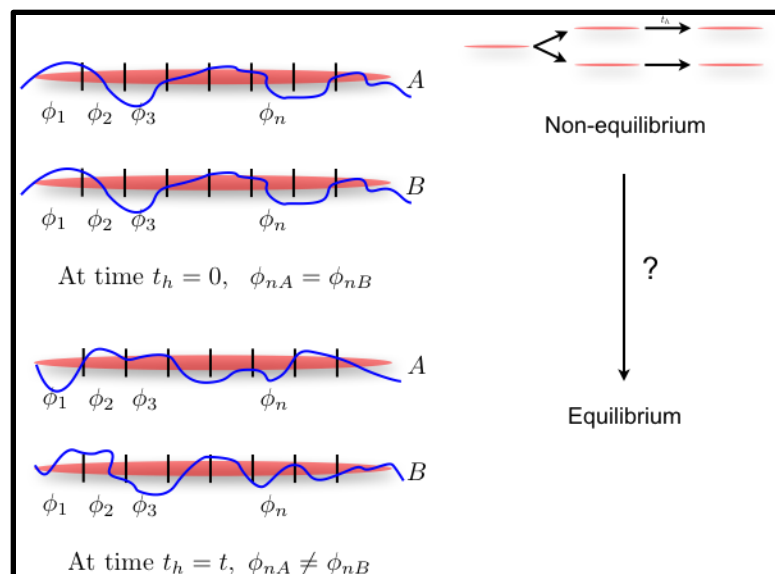
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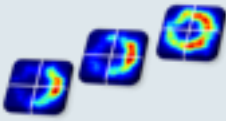
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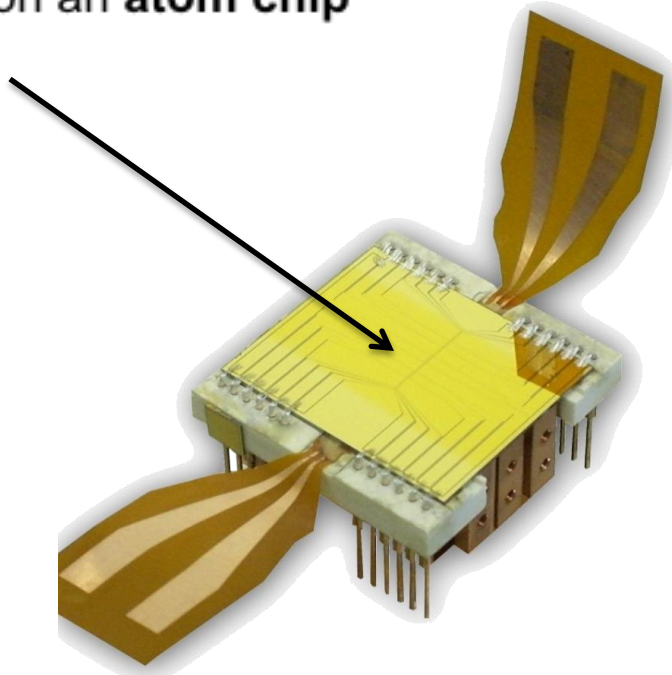
time



# Non-Equilibrium Dynamics



- Load ultracold atoms on an **atom chip**



- Can make **long cigar-shaped traps** (high aspect ratio)
- Can attain **high radial trapping frequencies** (several kHz)
- and **low axial frequencies** ( $< 5$  Hz)

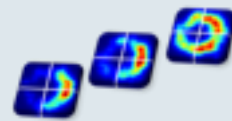


Great for making a **single 1d gas** !

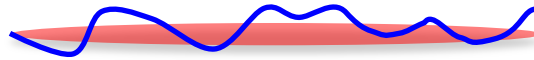




# Non-Equilibrium Dynamics



- Load ultracold atoms on an **atom chip**
- Make an (equilibrium) **1d Bose gas**



$$T = 120 \pm 30 \text{ nK}$$

$$\text{Atom Number} \sim 5000$$

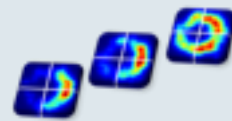
$$\lambda_T \sim 3 \text{ } \mu\text{m} \quad \longleftarrow \lambda_T \propto \frac{\rho}{T}$$

$$\omega_{\perp} = 2\pi \times (2.1 \pm 0.1) \text{ kHz}$$

$$\omega_{\parallel} = 2\pi \times (11 \pm 1) \text{ Hz}$$



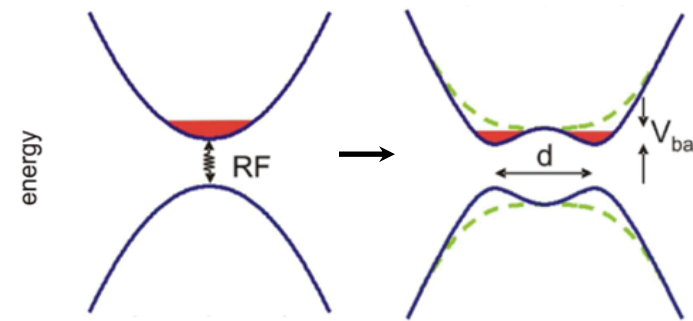
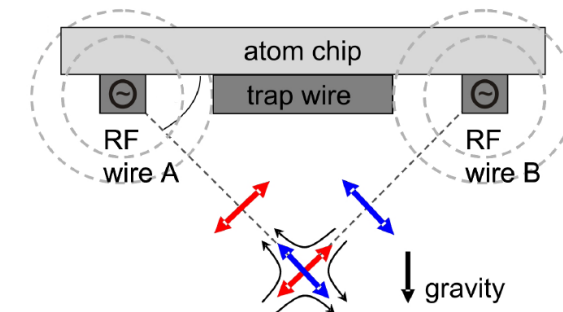
# Non-Equilibrium Dynamics



- Load ultracold atoms on an **atom chip**
- Make an (equilibrium) **1d Bose gas**
- Coherently **split the 1d gas** into two (*uncoupled!*) 1d gases



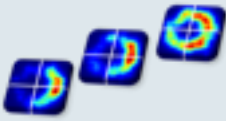
**Radio-frequency (RF) dressing**  
is used to smoothly form a **double-well potential**



Schumm et al., Nature Phys. 1, 57 (2005)



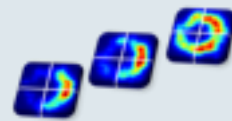
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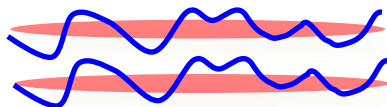
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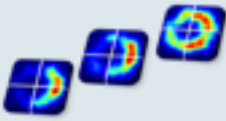
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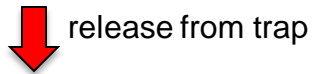
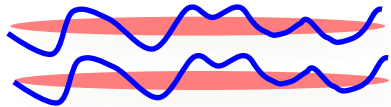
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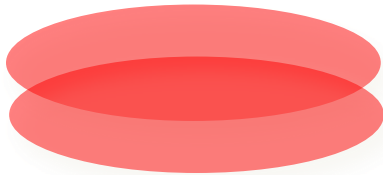
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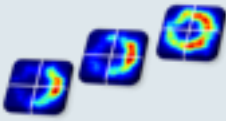
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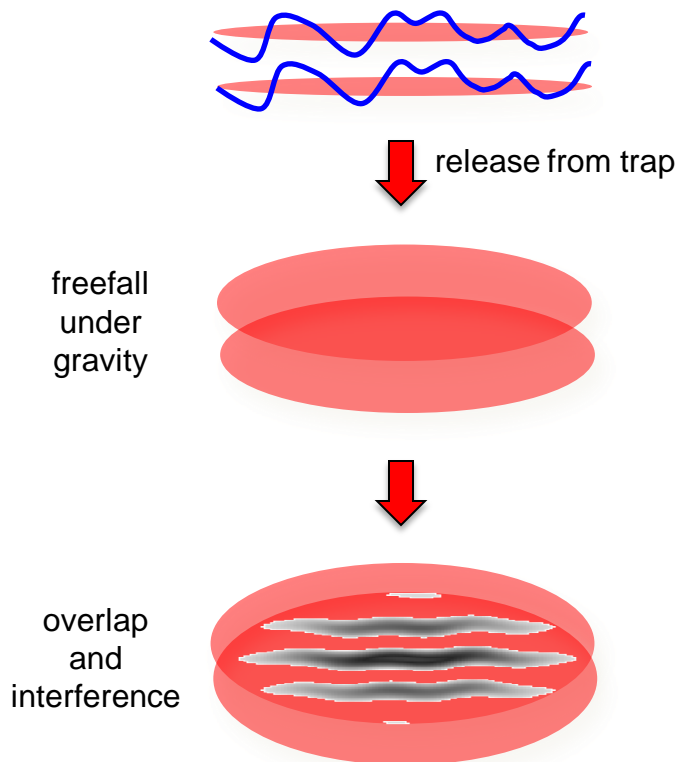
freefall  
under  
gravity



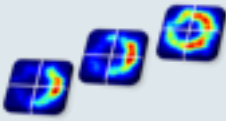
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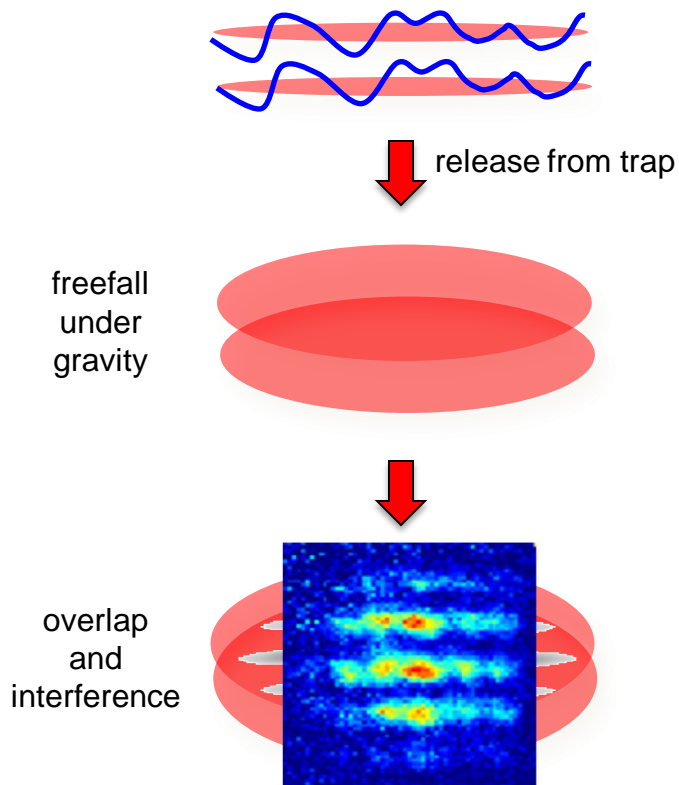
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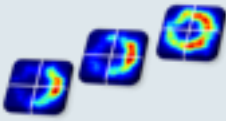
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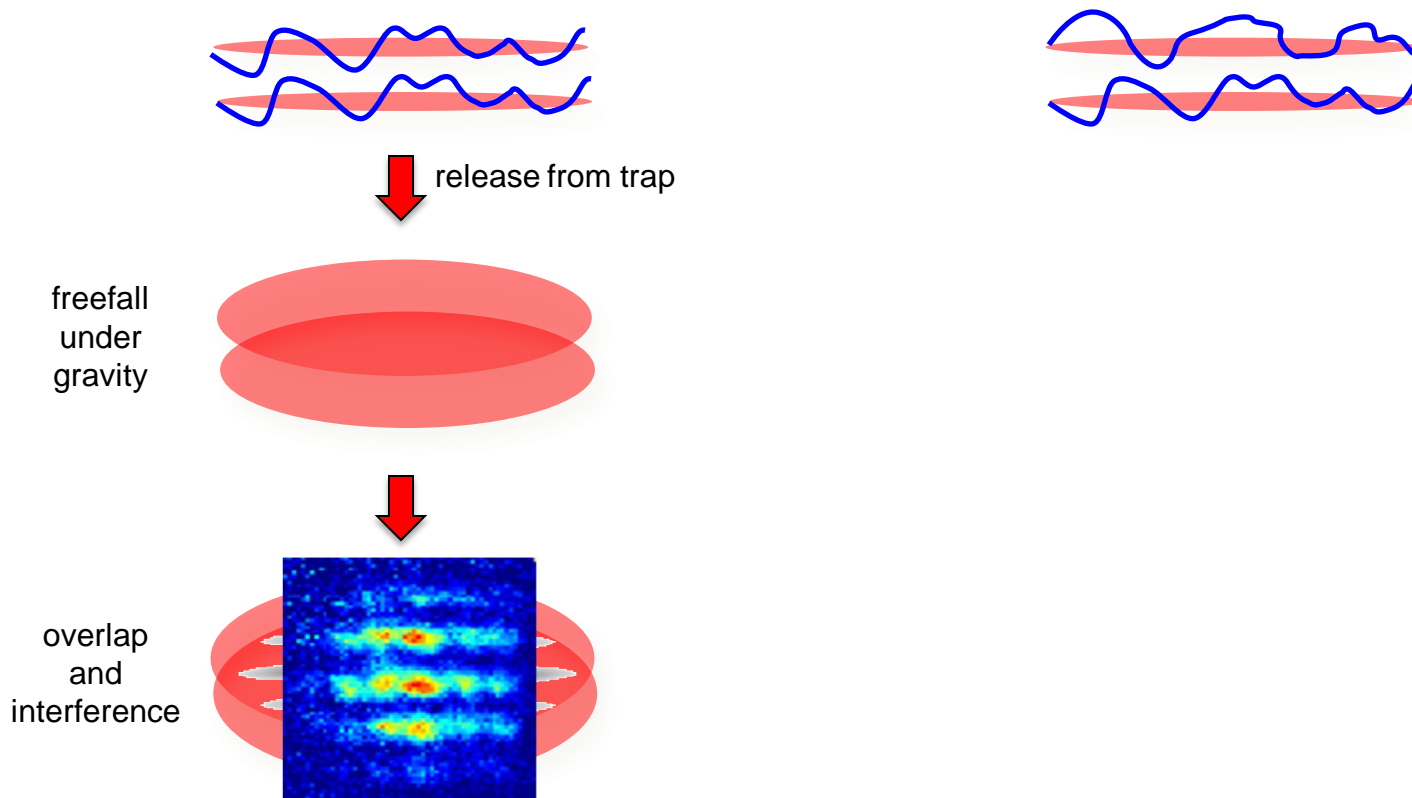
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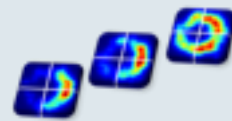


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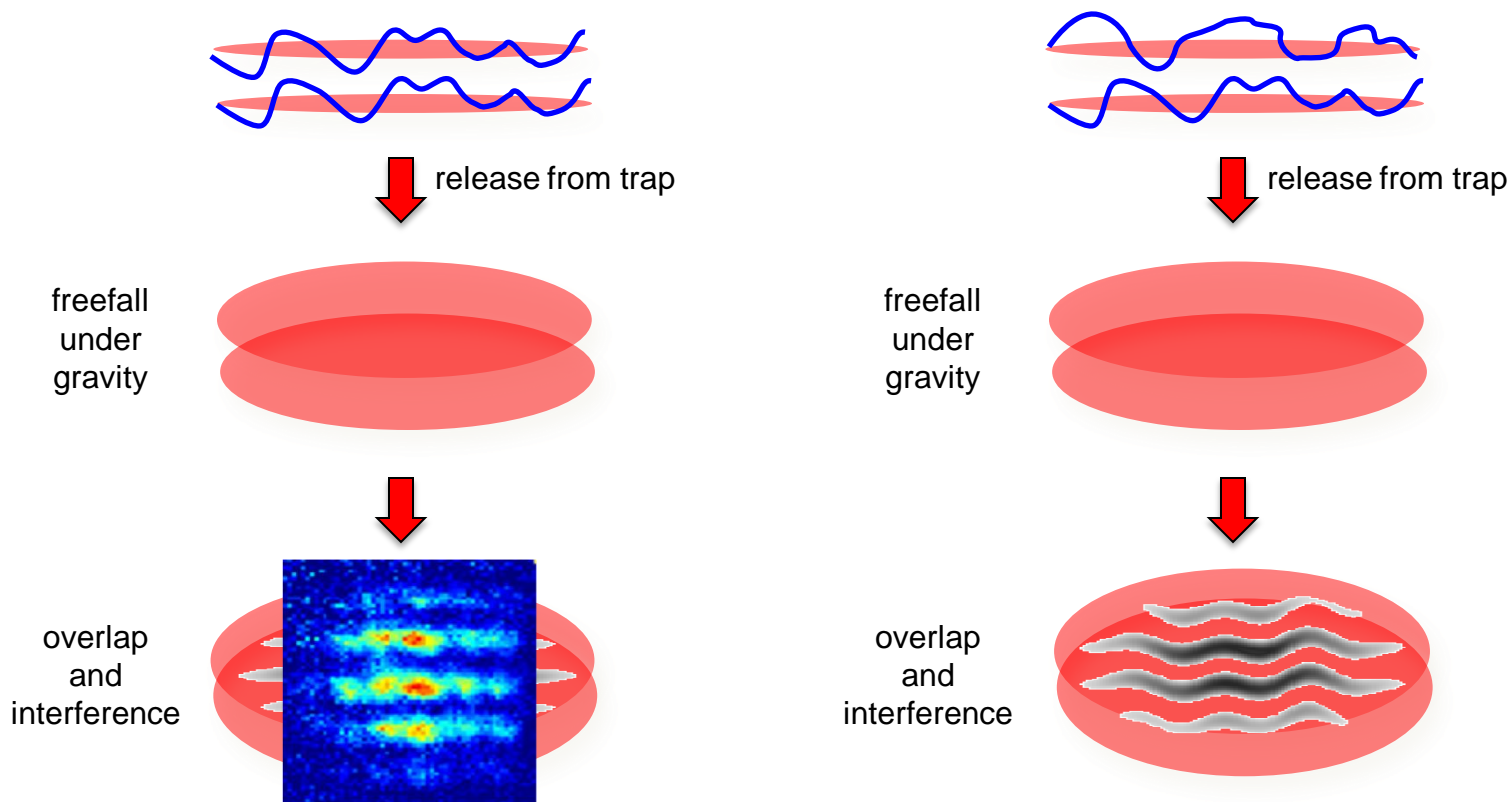




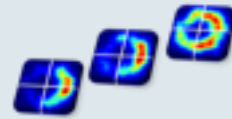
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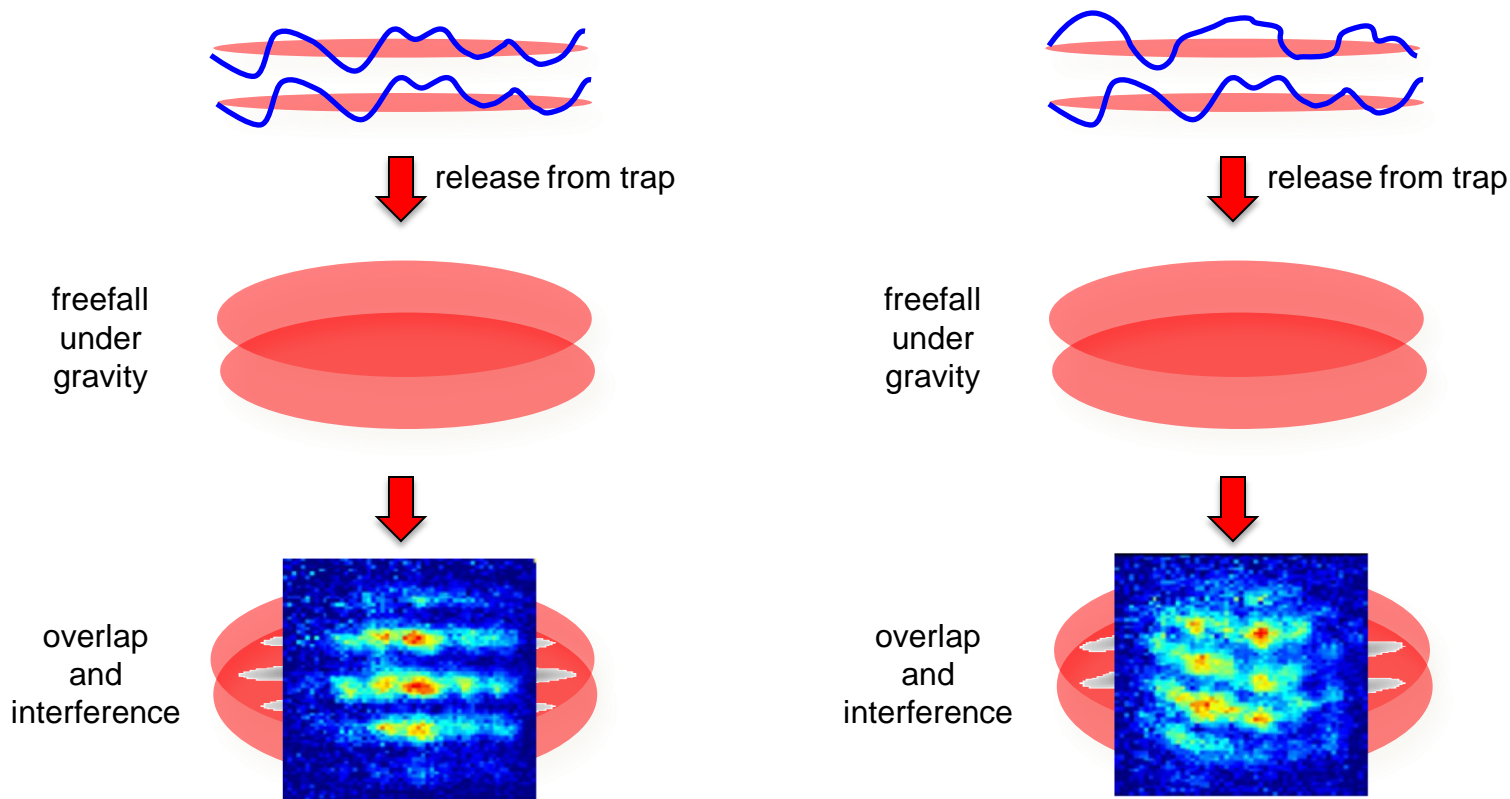
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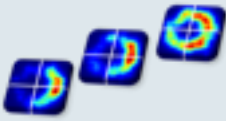
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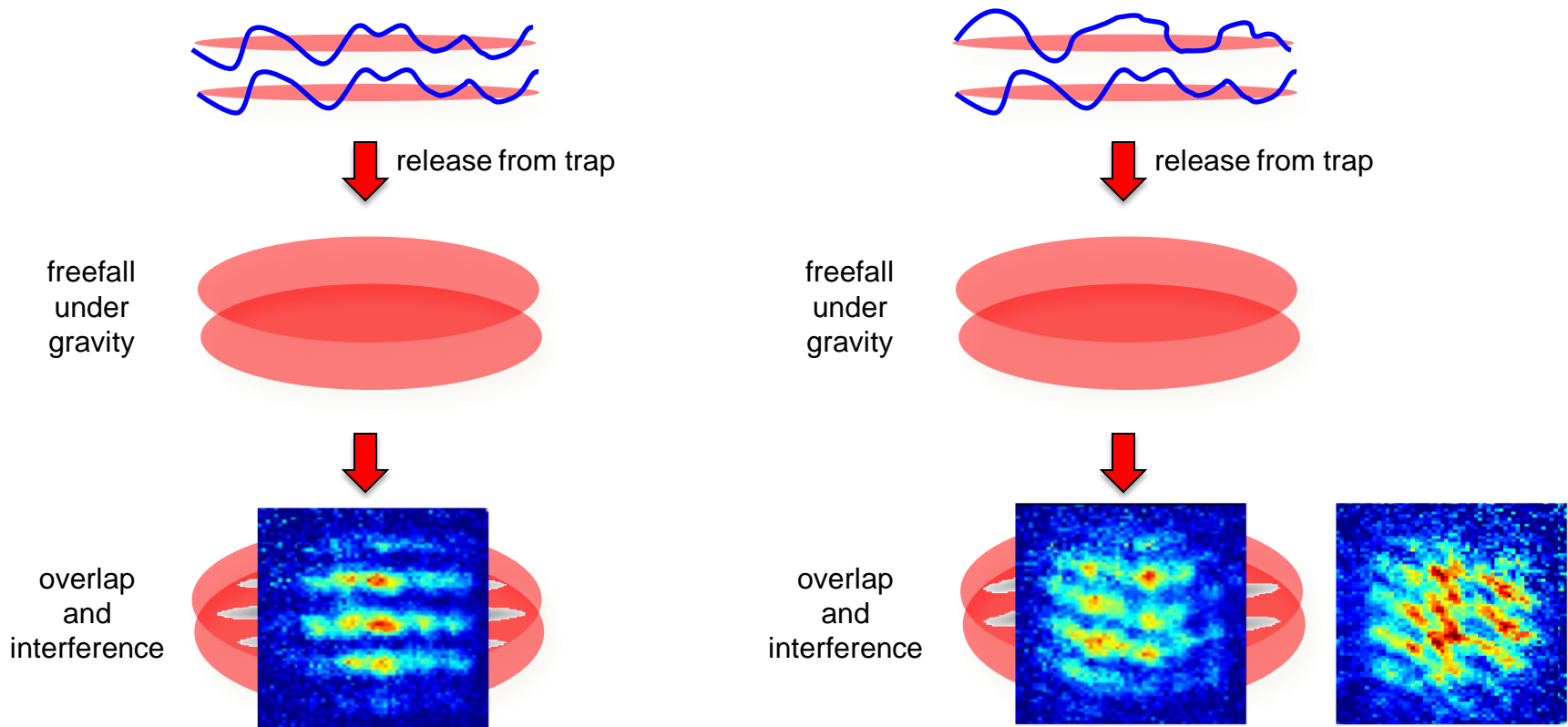
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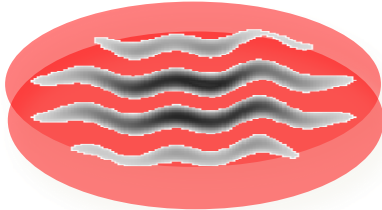
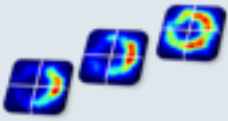
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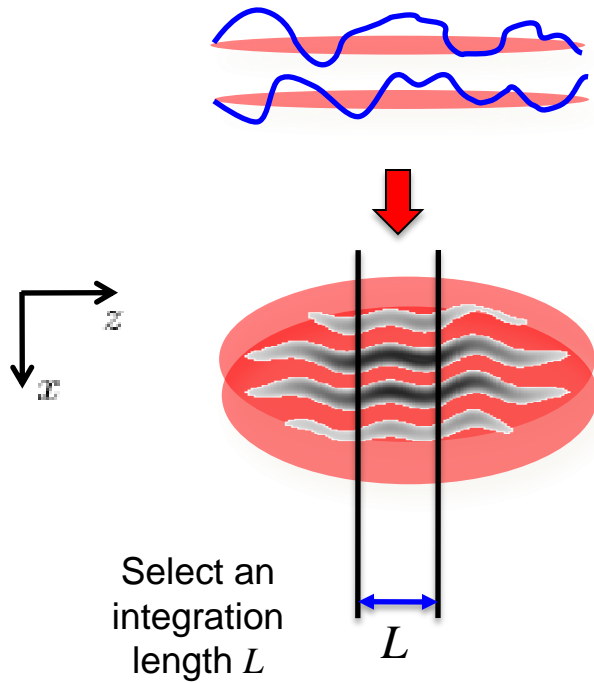
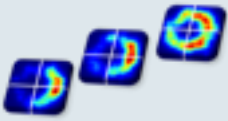
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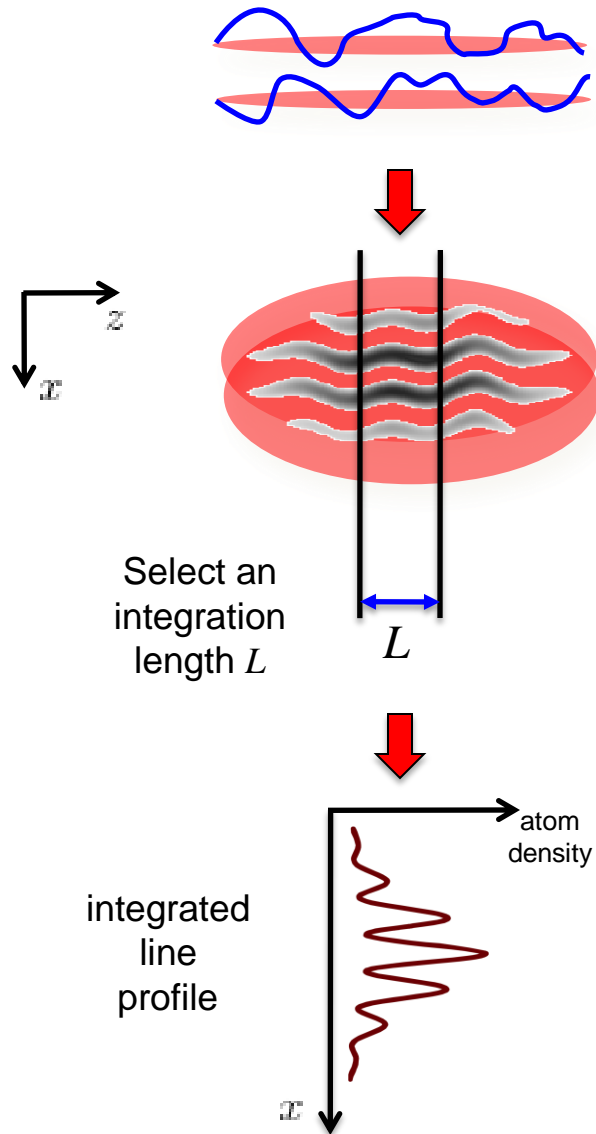
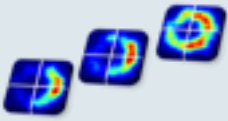
# Extracting Information from Interference Patterns



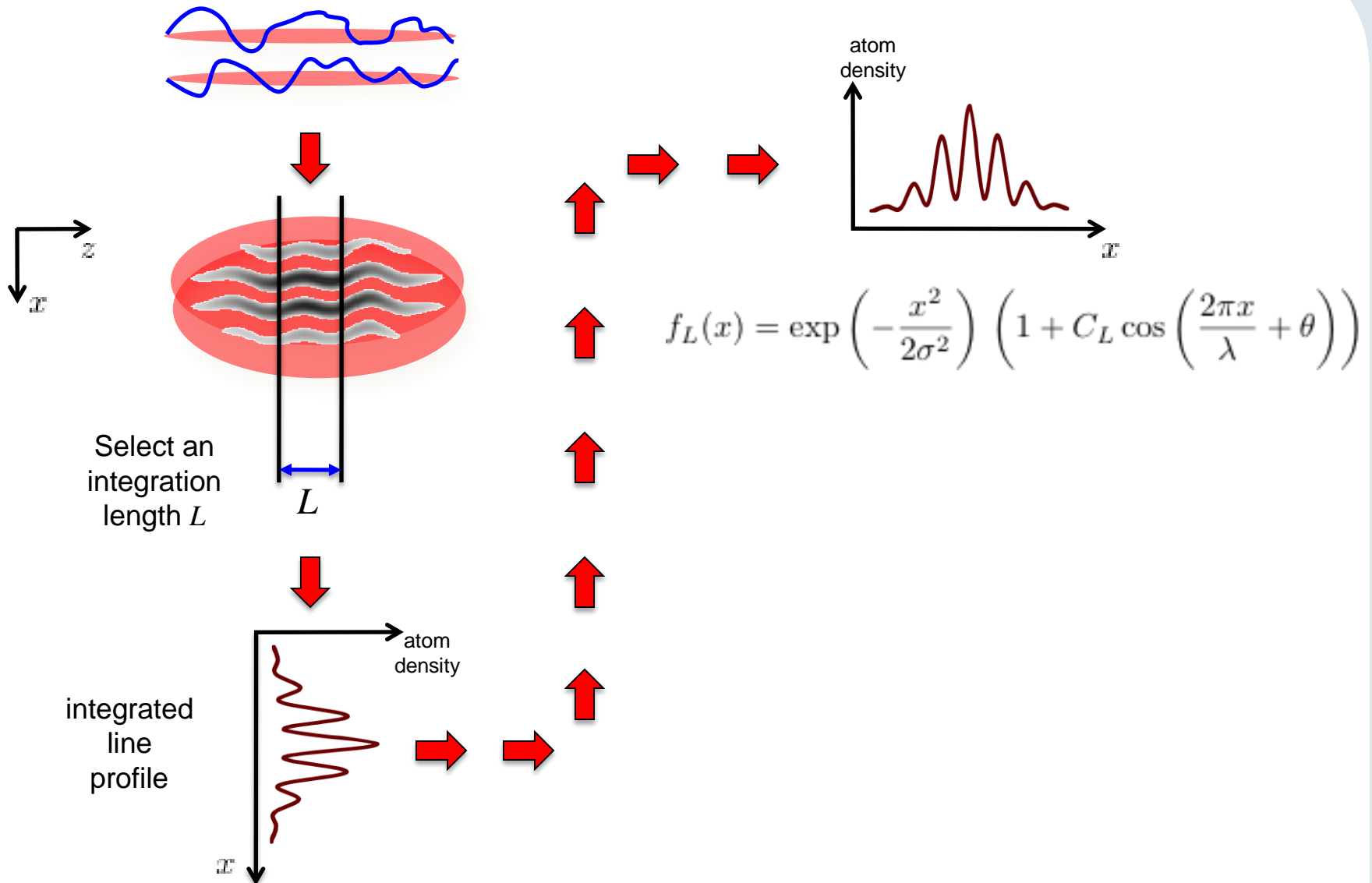
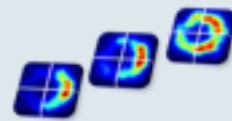
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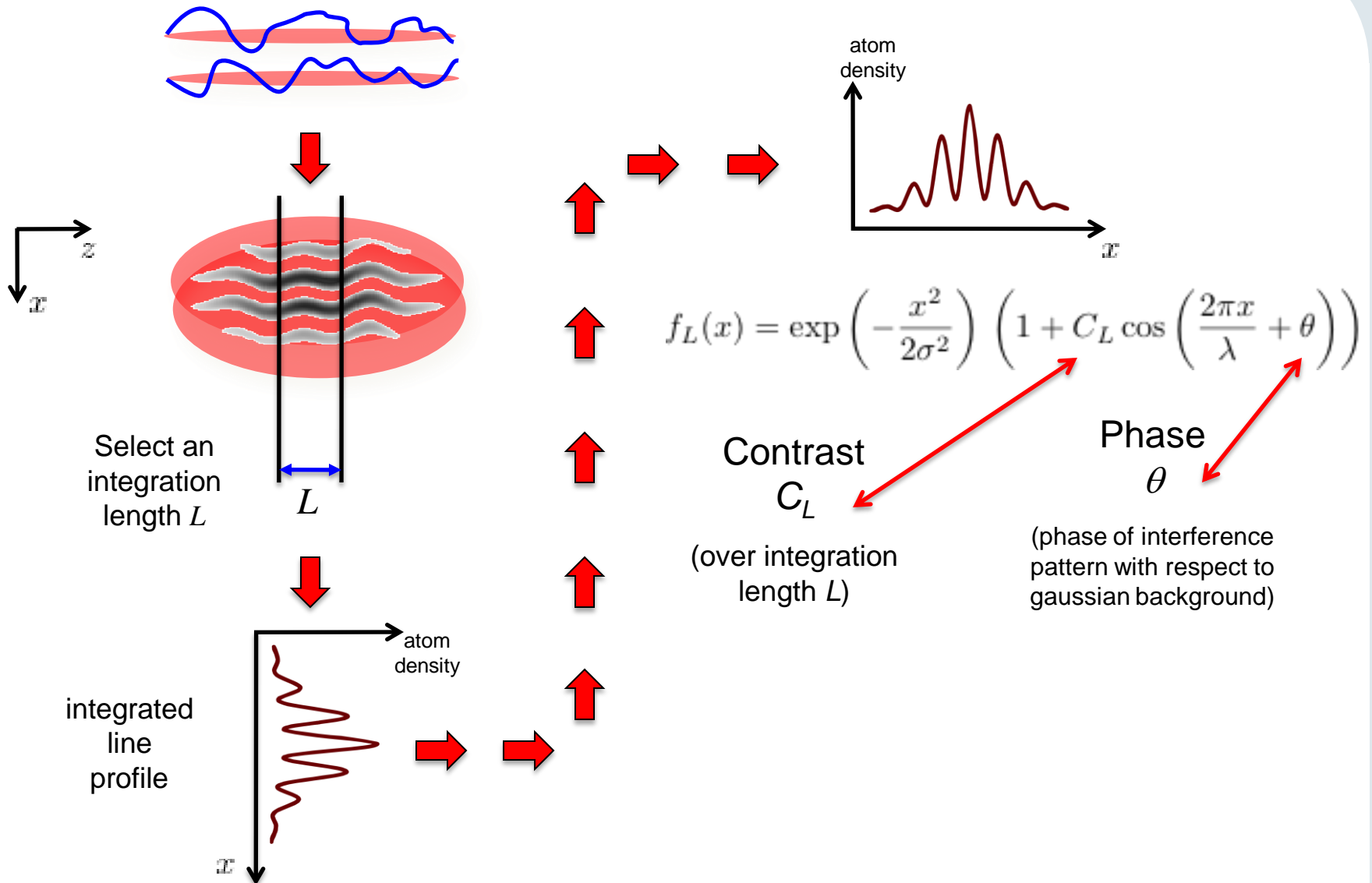
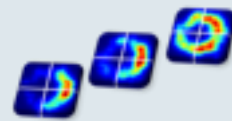
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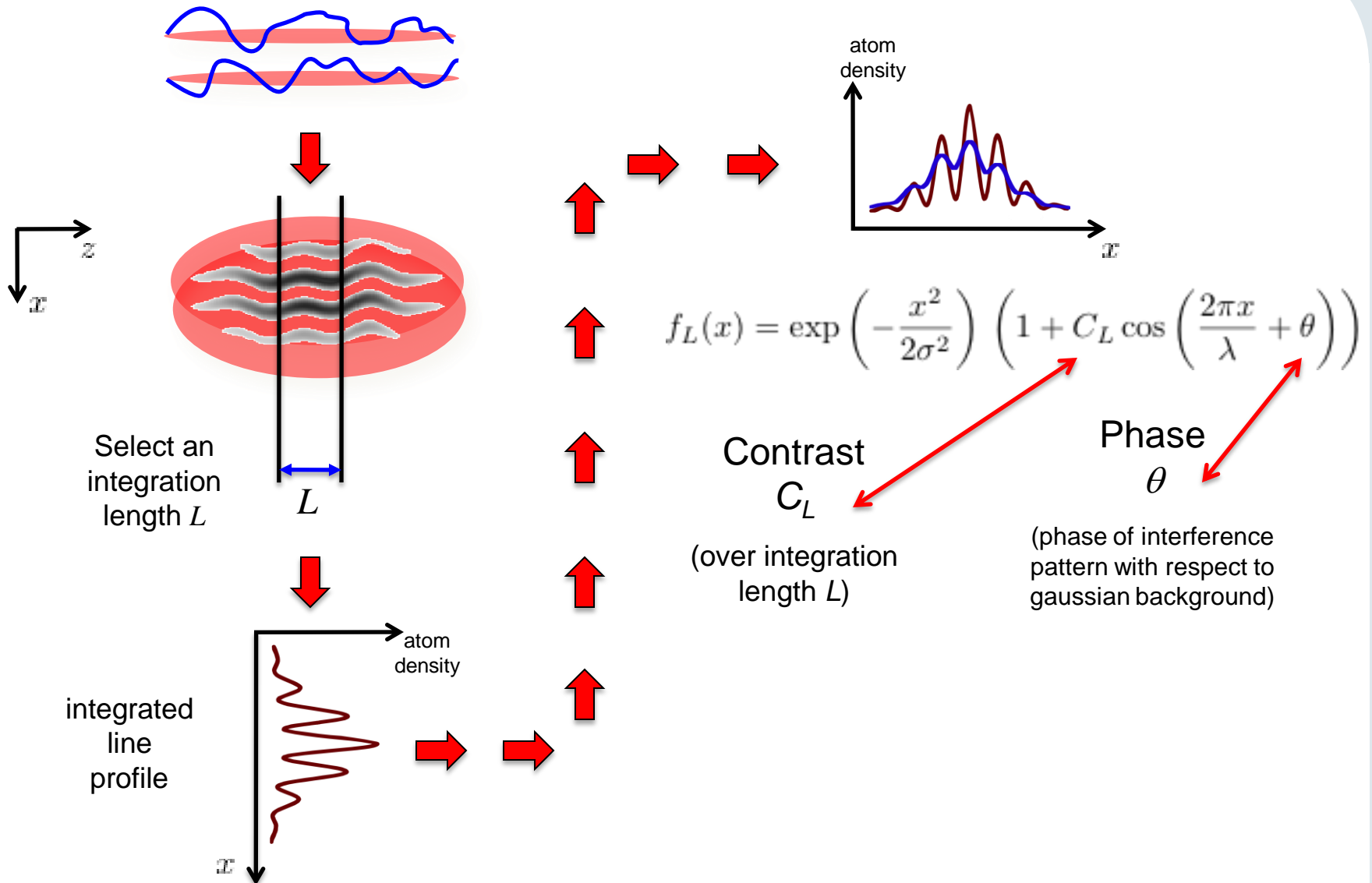
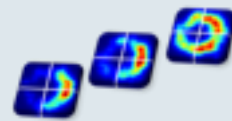


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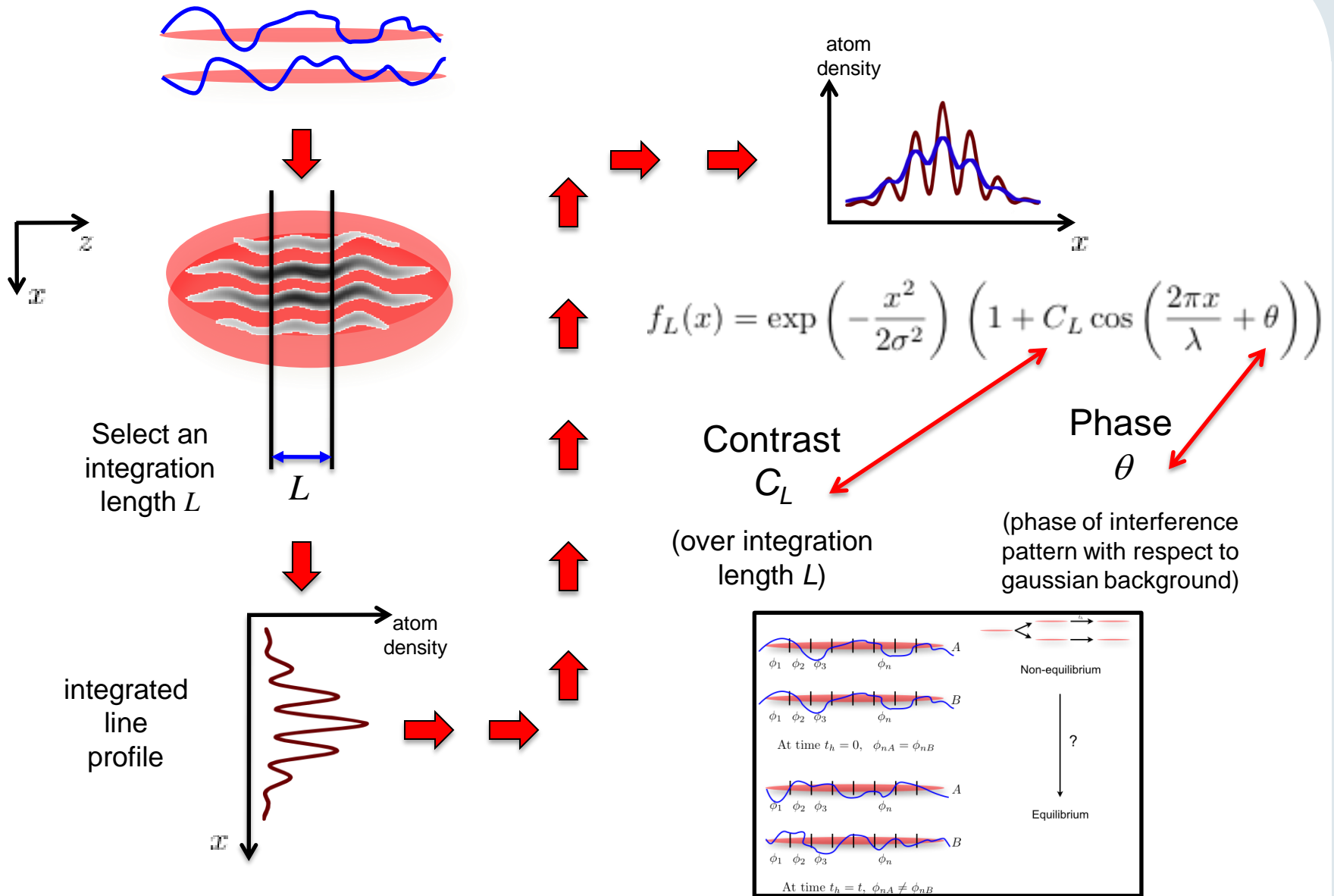
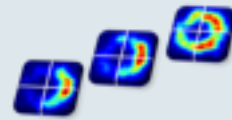




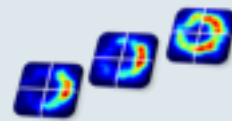
# Extracting Information from Interference Patterns



# Extracting Information from Interference Patterns



# A Previous Experiment

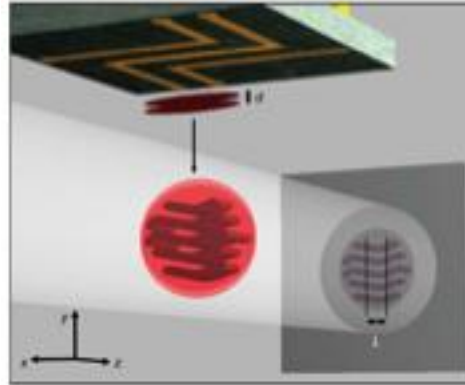


nature

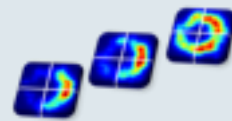
Vol 449 | 20 September 2007 | doi:10.1038/nature06149

## Non-equilibrium coherence dynamics in one-dimensional Bose gases

S. Hofferberth<sup>1,2</sup>, I. Lesanovsky<sup>3</sup>, B. Fischer<sup>1</sup>, T. Schumm<sup>2</sup> & J. Schmiedmayer<sup>1,2</sup>



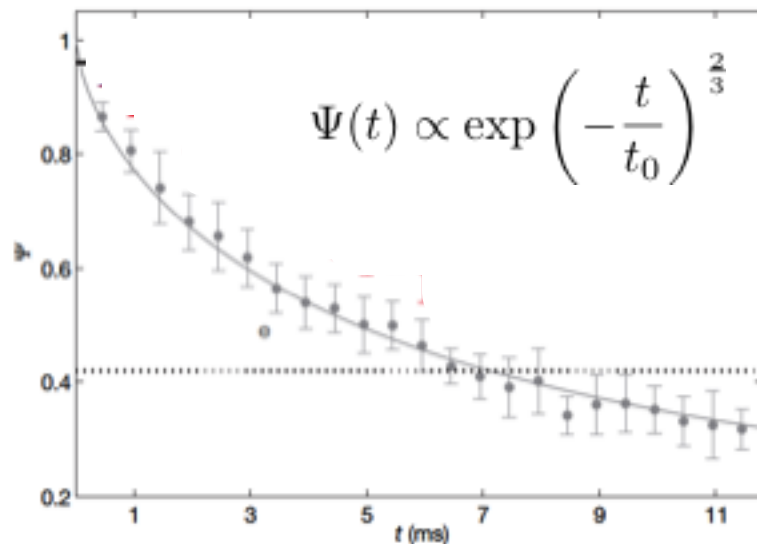
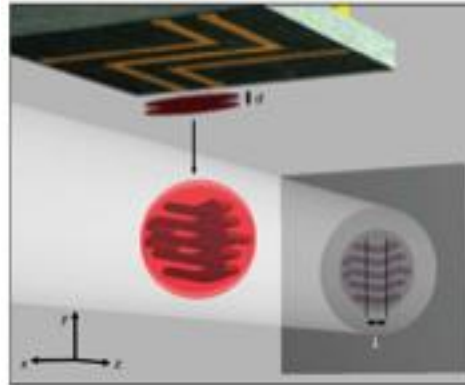
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thermalisation?

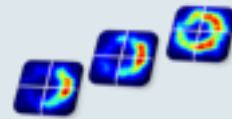
- Measurement of a **mean** coherence factor  $\Psi$
- Application of a theory of **thermalisation**

$$\Psi(t) \propto \exp\left(-\frac{t}{t_0}\right)^{\frac{2}{3}}$$

Burkov et al. Phys. Rev. Lett. **98** 200404 (2007)



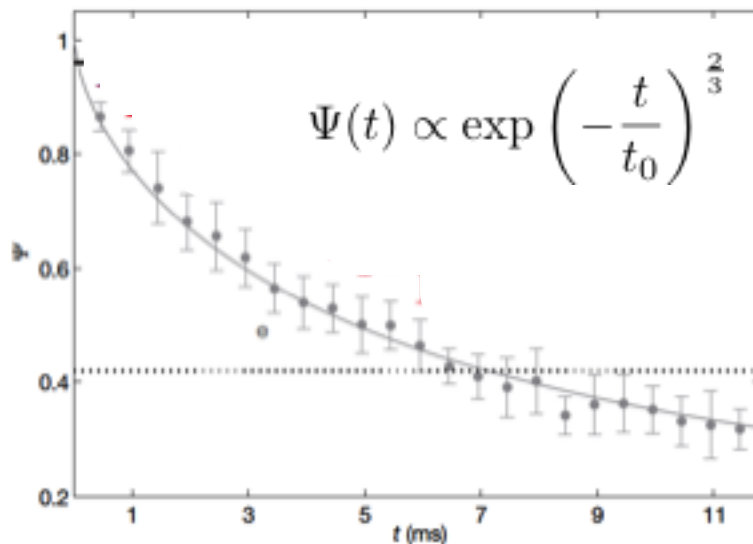
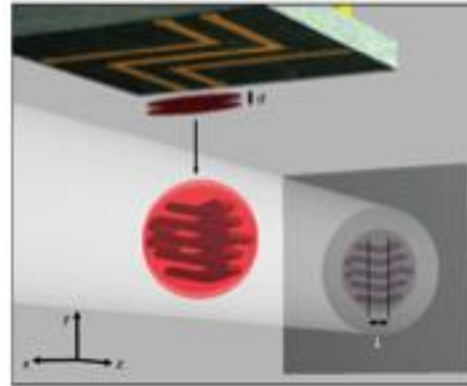
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thermalisation?

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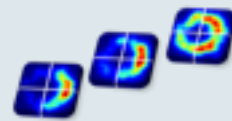
$$\Psi(t) \propto \exp\left(-\frac{t}{t_0}\right)^{\frac{2}{3}}$$

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This attribution of thermalisation to the data was probably incorrect. Why?



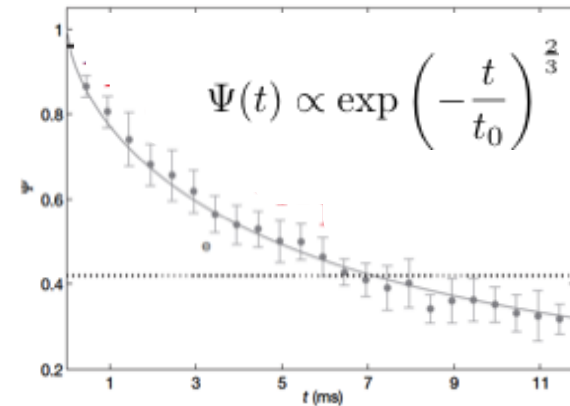
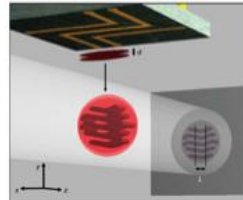
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- Two available theories at the time (2007)

thermalisation

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Burkov et al. Phys. Rev. Lett. **98** 200404 (2007)

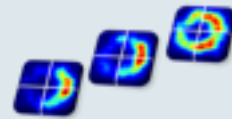
dephasing

$$\Psi(t) \propto \exp\left(-\frac{t}{\tau}\right)$$

Bistritzer & Altman, PNAS **104** 24, 9955 (2007)



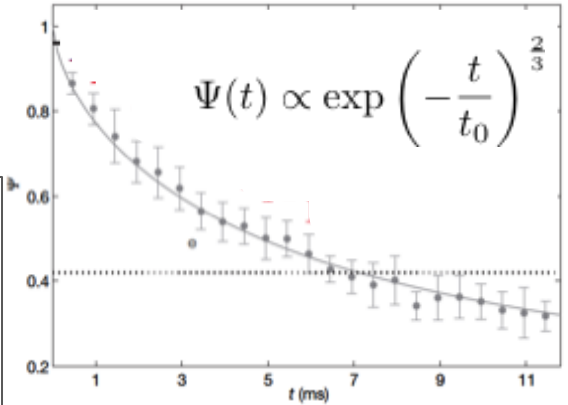
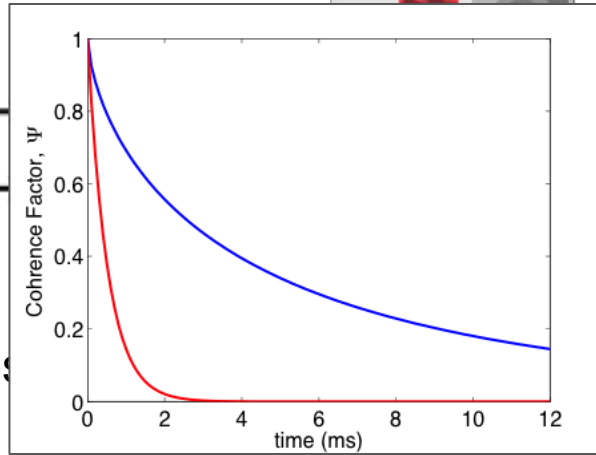
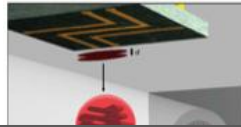
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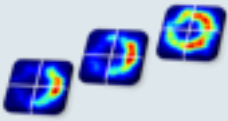
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# A New Experiment in Vienna

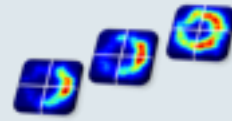


- More controlled splitting process
- Ability to measure for longer evolution times



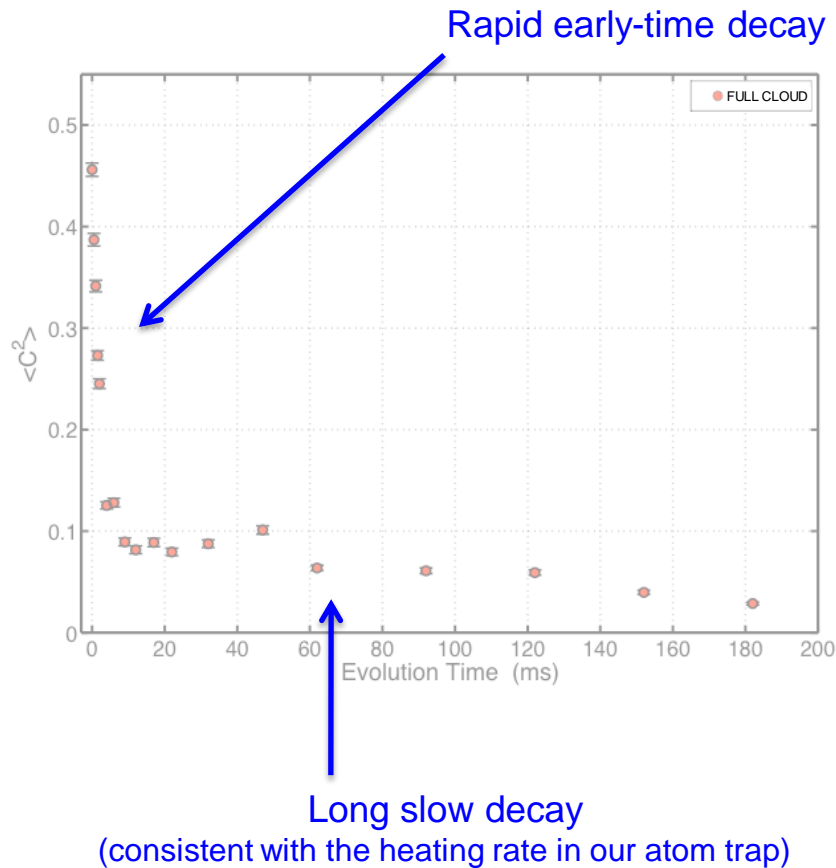


# A New Experiment in Vienna

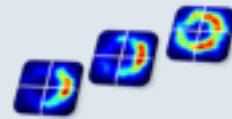


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## Our New Measurements

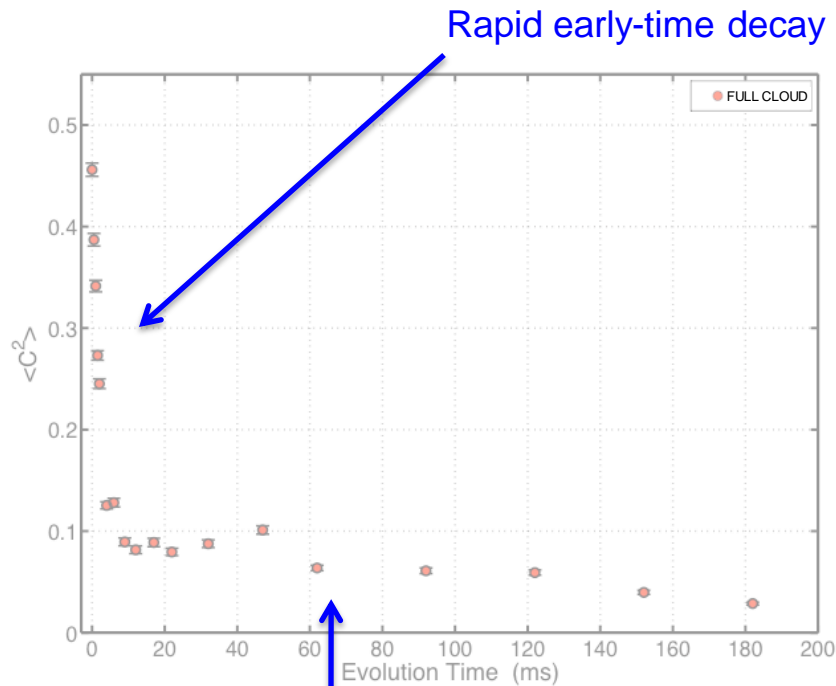


# A New Experiment in Vienna

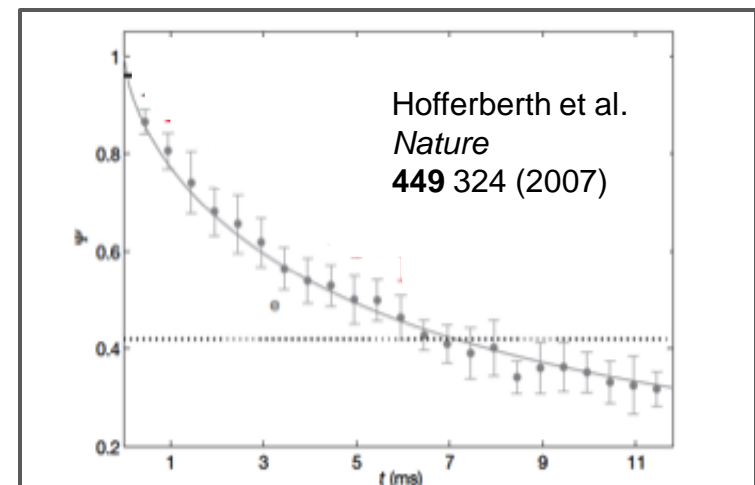
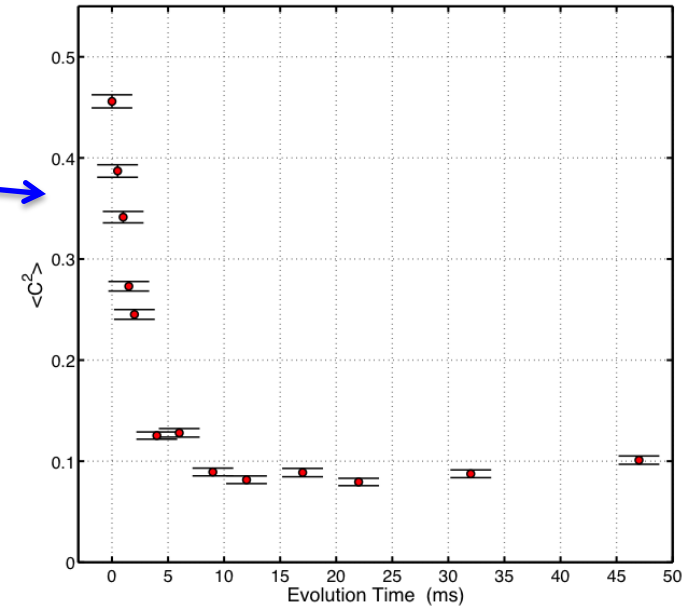


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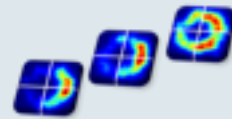
## Our New Measurements



Long slow decay  
(consistent with the heating rate in our atom trap)

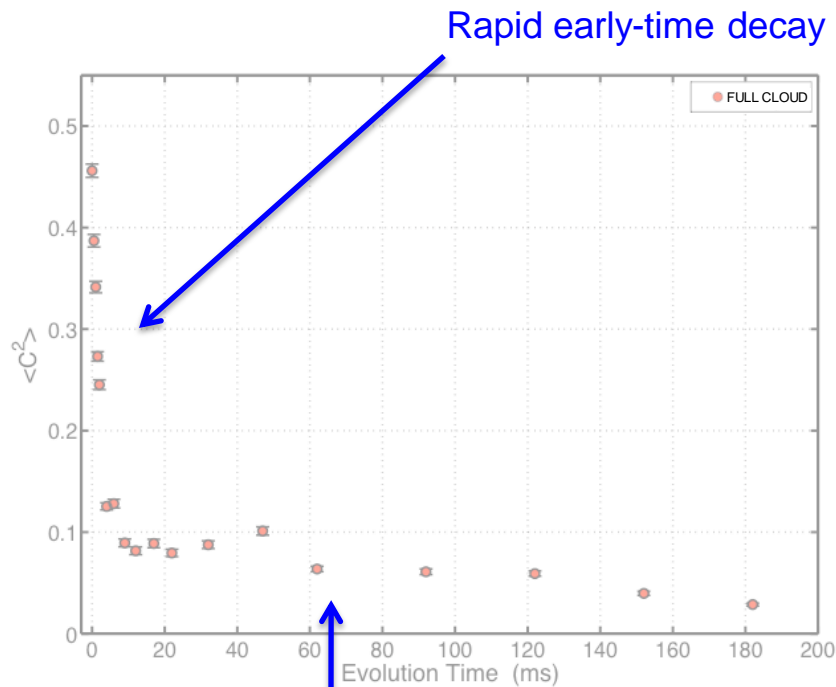


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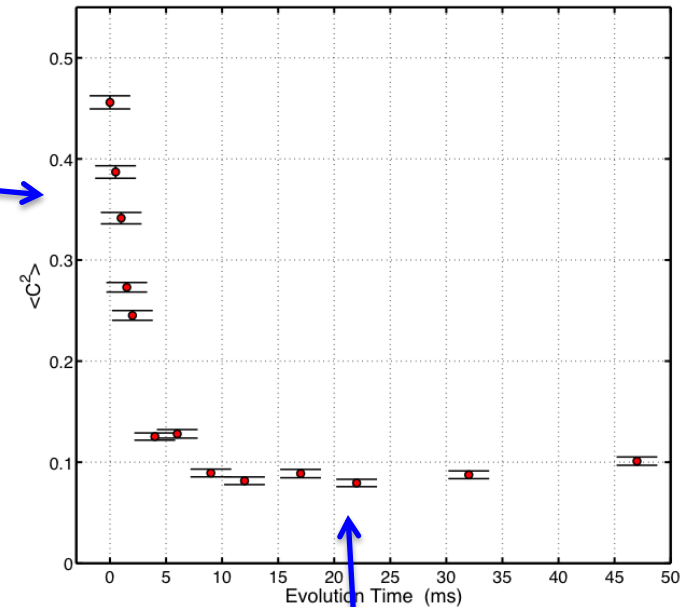


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## Our New Measurements



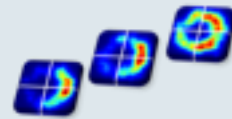
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Thermal  
Equilibrium  
?



# Beyond the Mean: Full Distribution Functions



## For Equilibrium Systems:

nature physics | VOL 2 | OCTOBER 2006 | www.nature.com/naturephysics

ARTICLES

Full quantum distribution of contrast in interference experiments between interacting one-dimensional Bose liquids

VLADIMIR GRITSEV<sup>1\*</sup>, EHUD ALTMAN<sup>2</sup>, EUGENE DEMLER<sup>1</sup> AND ANATOLI POLKOVNIKOV<sup>3</sup>

nature physics | VOL 4 | JUNE 2008 | www.nature.com/naturephysics

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Probing quantum and thermal noise in an interacting many-body system

S. HOFFERBERTH<sup>1,2,3</sup>, I. LESANOVSKY<sup>2,4</sup>, T. SCHUMM<sup>1</sup>, A. IMAMBEKOV<sup>3,5</sup>, V. GRITSEV<sup>3</sup>, E. DEMLER<sup>3</sup> AND J. SCHMIEDMAYER<sup>1,2\*</sup>

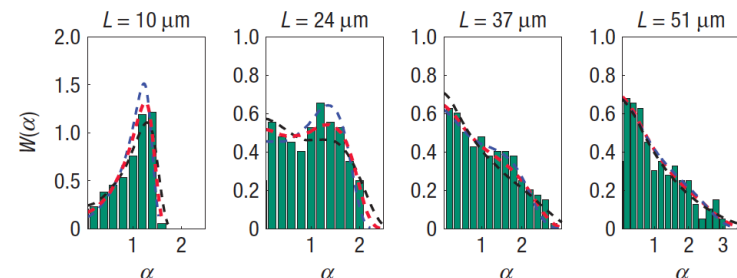
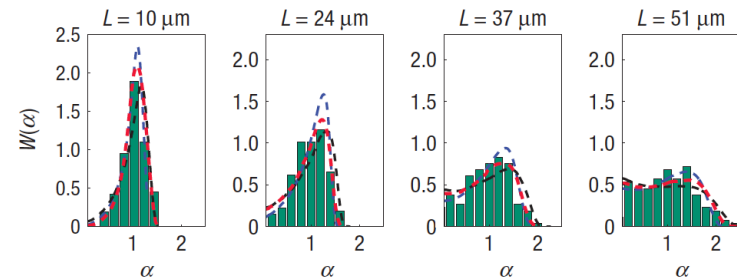
See also:

- Stimming et al. PRL 105, 015301 (2010) [theory]
- Betz et al. PRL 106, 020407 (2011) [experiment]

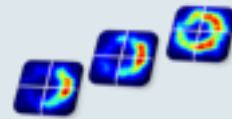
## Theoretical Proposal

## Experimental Realisation

Distributions of Interference Contrast  $C_L$



# Beyond the Mean: Full Distribution Functions



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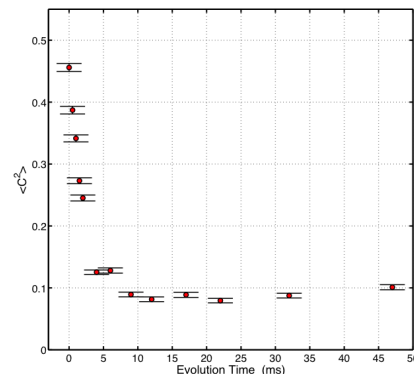
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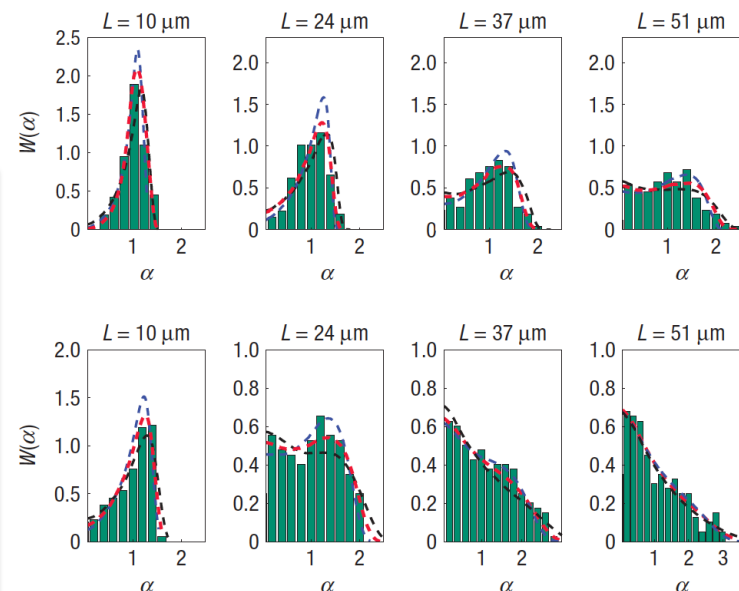
We can use this equilibrium theory to investigate our “*equilibrium?*” state



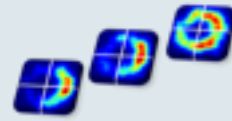
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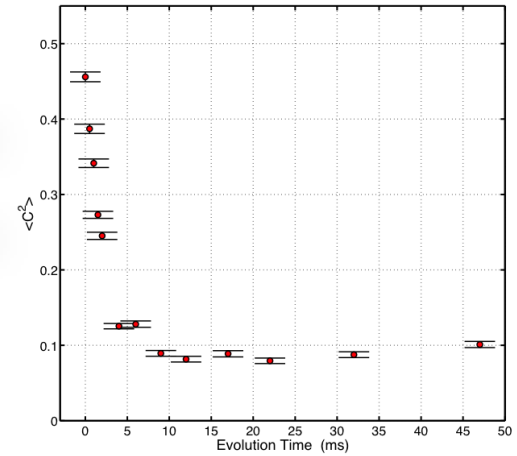
Distributions of Interference Contrast  $C_L$



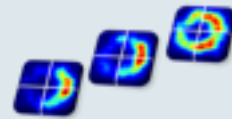
# Applying Equilibrium Theory



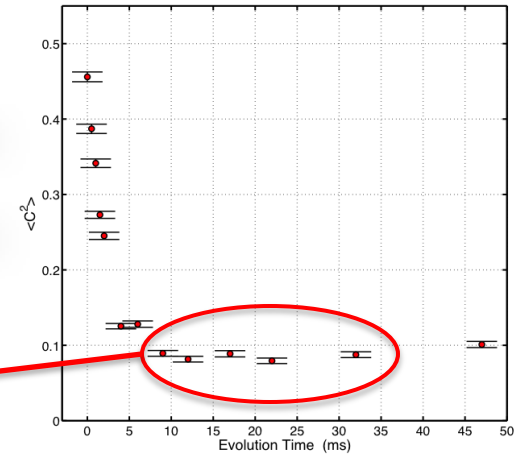
Initial single gas  
 $T = 120 \pm 30$  nK  
 $\lambda_T \sim 3 \mu\text{m}$   
Atom Number  $\sim 5000$



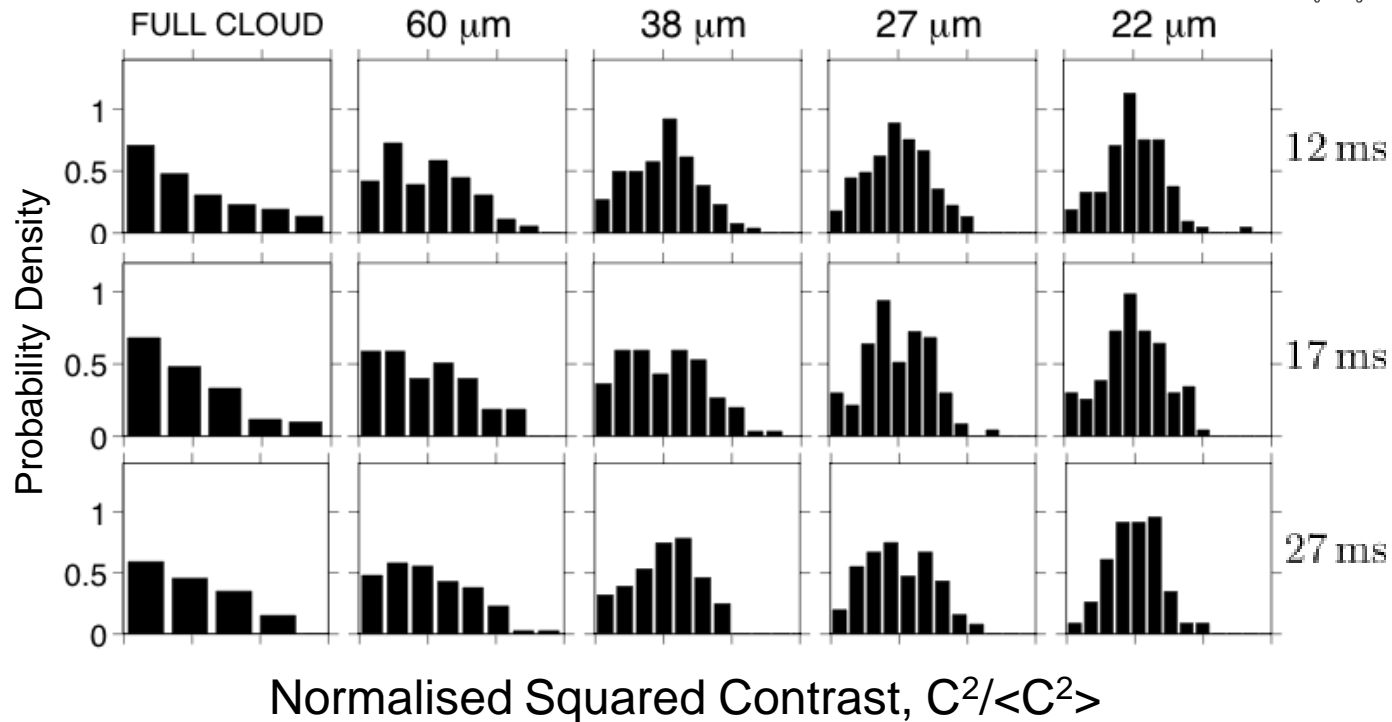
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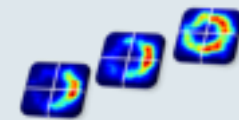
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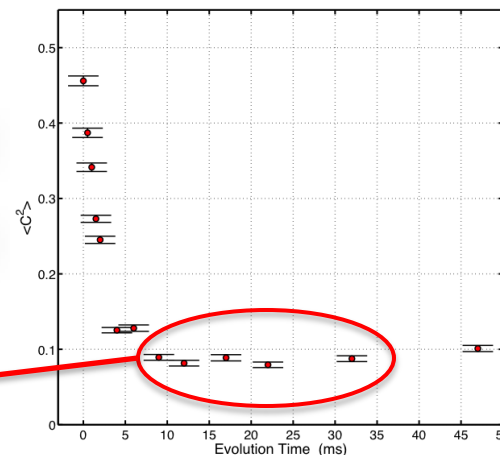
Interference Contrast  $C_L$  Distributions



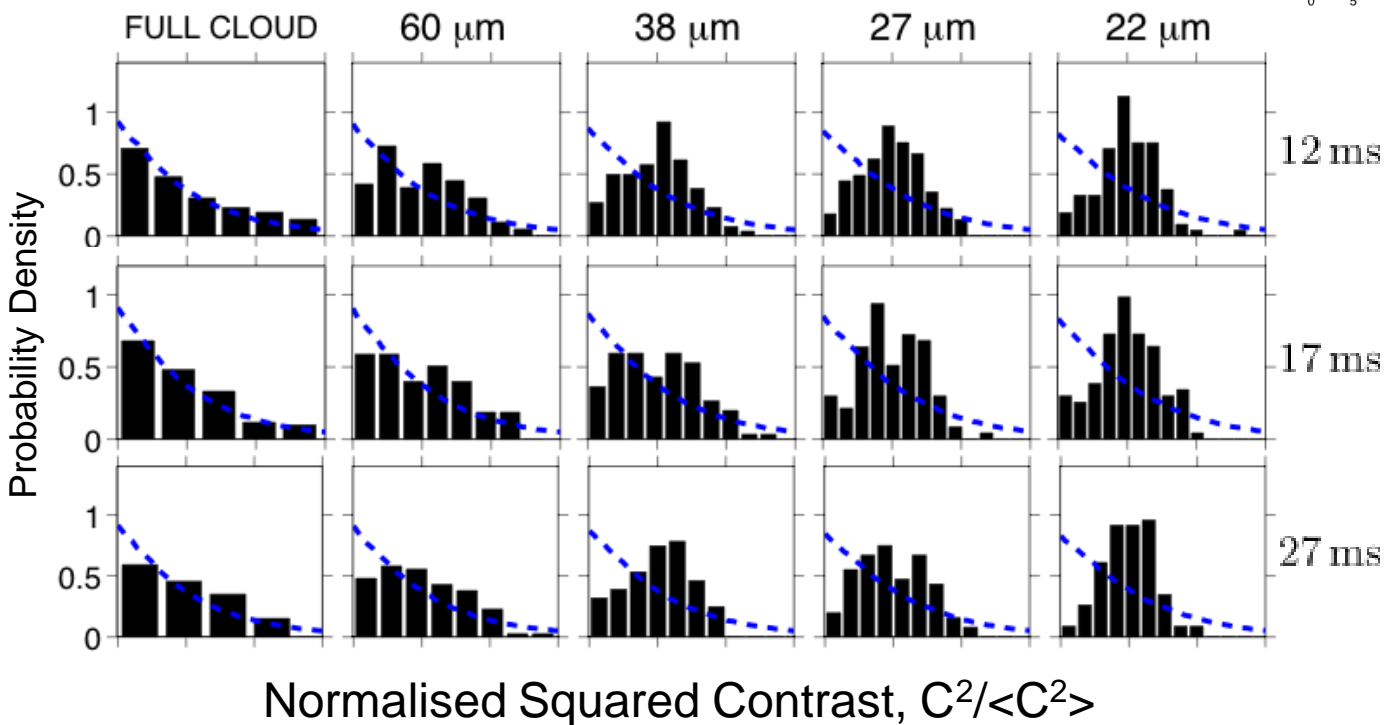
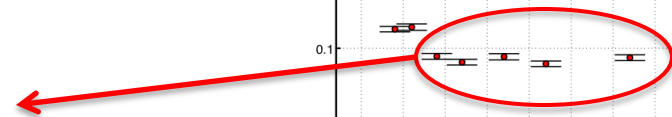
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## Interference Contrast $C_L$ Distributions



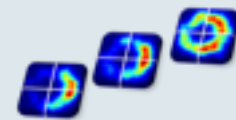
Compare  
 Equilibrium Theory  
 with  
 Non-Equilibrium  
 Experiment

$T = 120$  nK

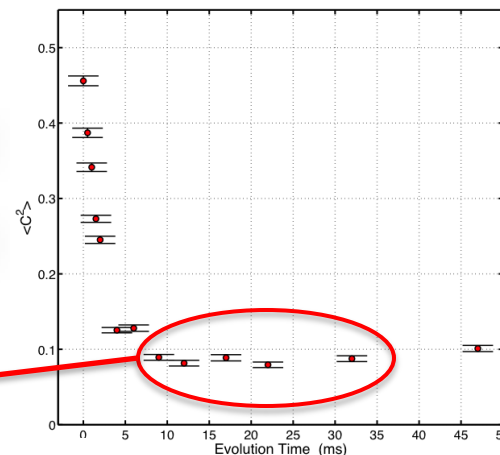




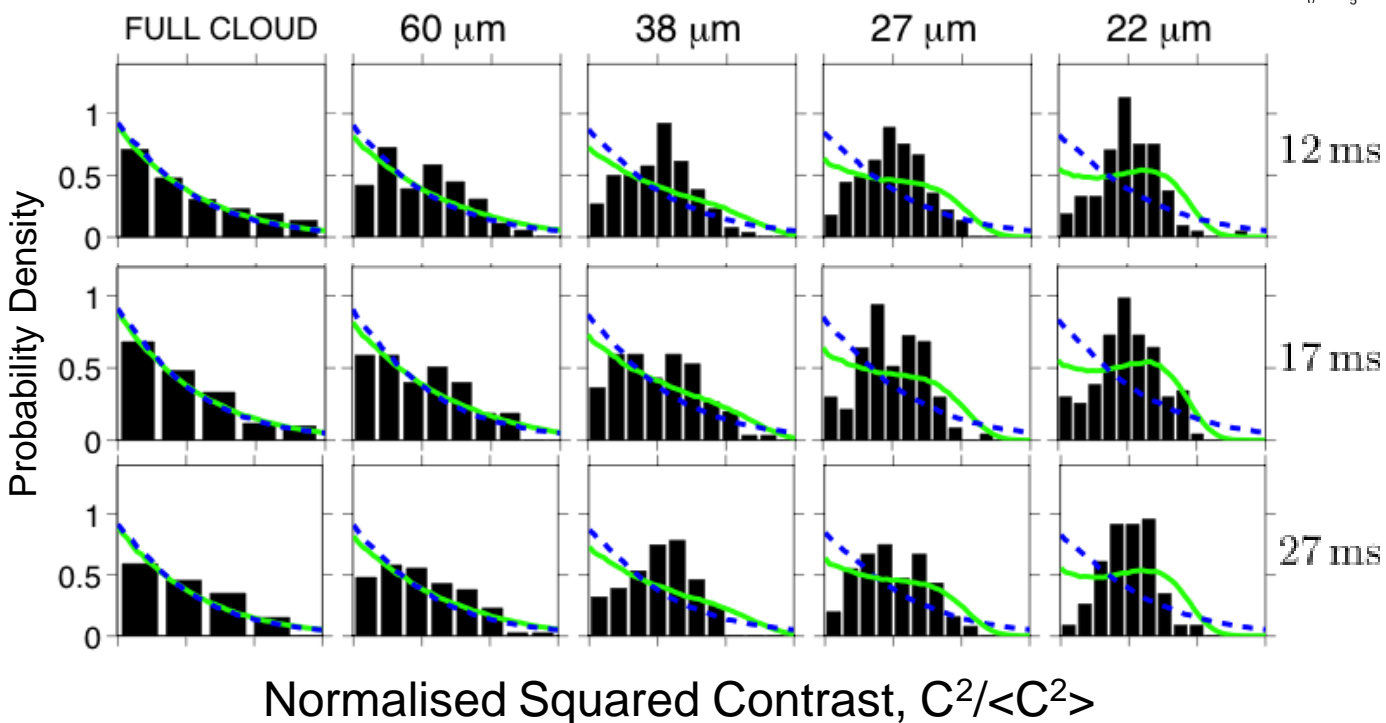
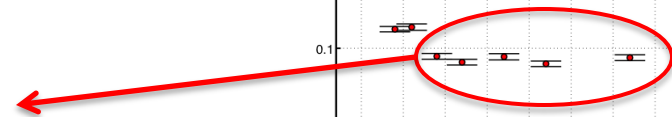
# Applying Equilibrium Theory



Initial single gas  
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 $\lambda_T \sim 3 \mu\text{m}$   
 Atom Number  $\sim 5000$



## Interference Contrast $C_L$ Distributions



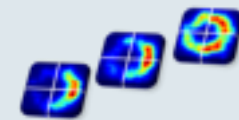
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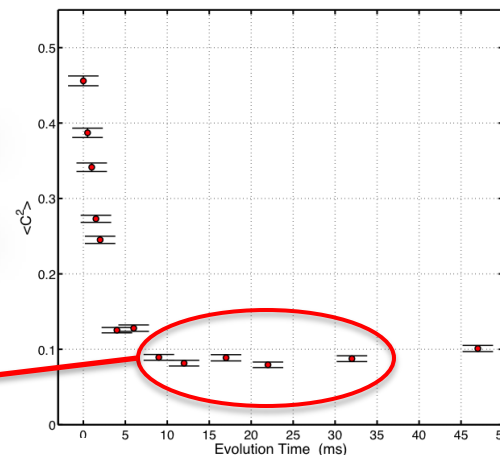
$T = 30 \text{ nK}$



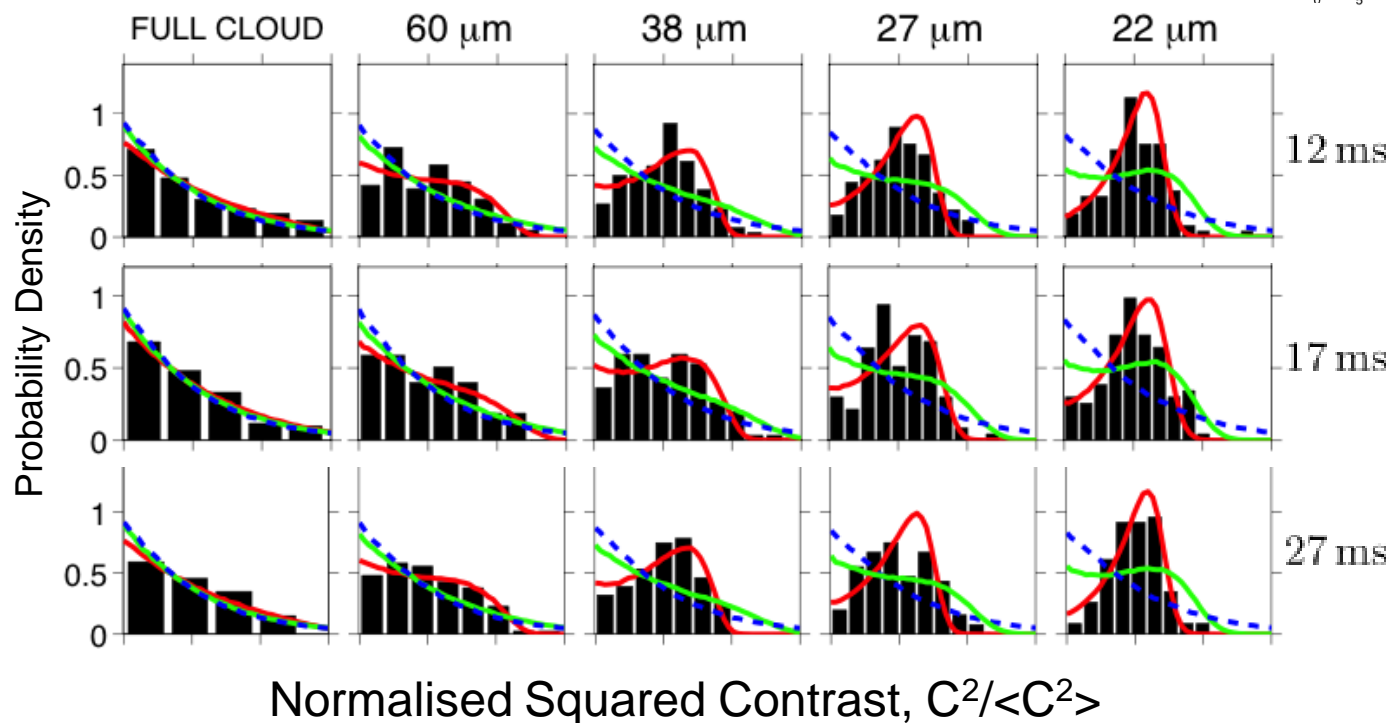
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Initial single gas  
 $T = 120 \pm 30 \text{ nK}$   
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## Interference Contrast $C_L$ Distributions



Compare  
 Equilibrium Theory  
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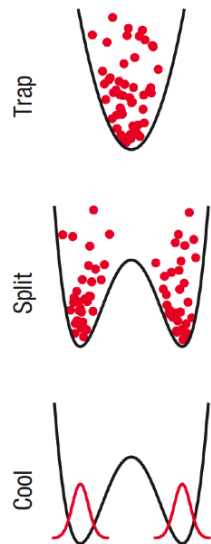
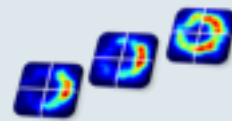
$T = 120 \text{ nK}$

$T = 30 \text{ nK}$

$T_{\text{fit}} = 15 \pm 4 \text{ nK}$



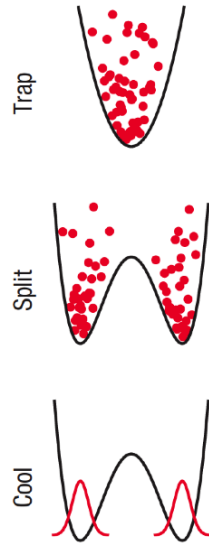
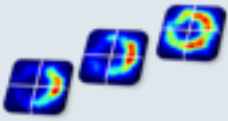
# Measuring Equilibrium Systems



We now want to  
**compare**  
equilibrium theory  
**to**  
equilibrium experiments



# Measuring Equilibrium Systems

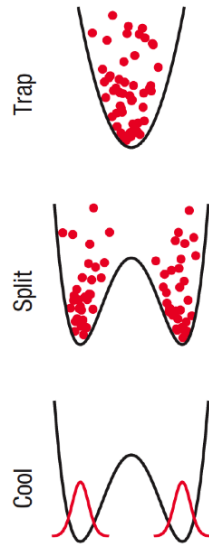
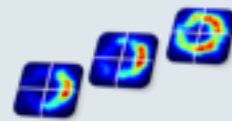


We now want to  
**compare**  
equilibrium theory  
**to**  
equilibrium experiments

create  
two independent  
quasi-condensates



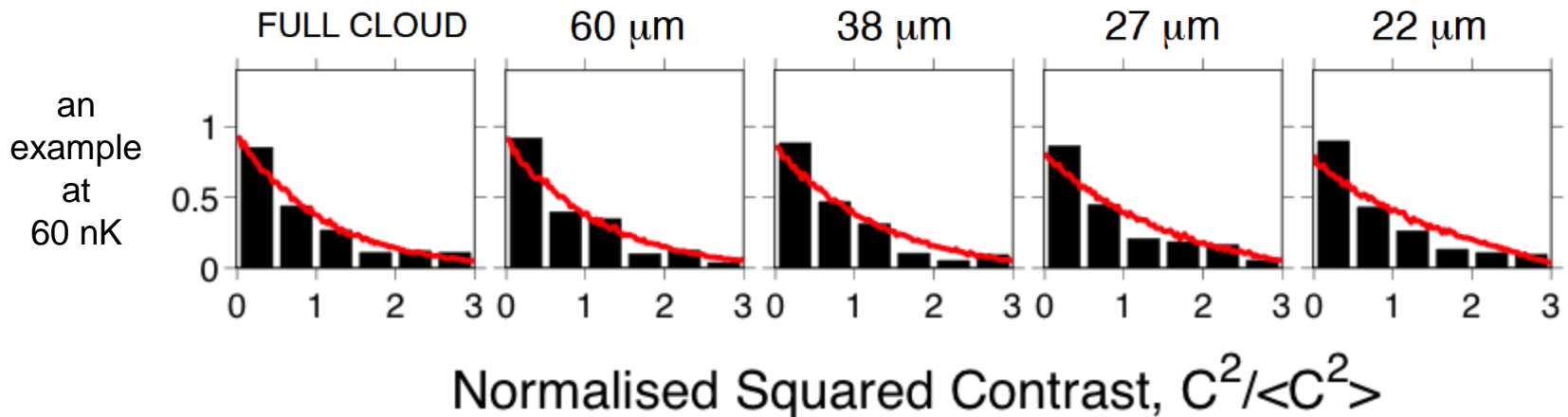
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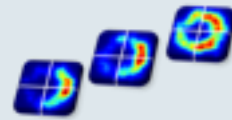
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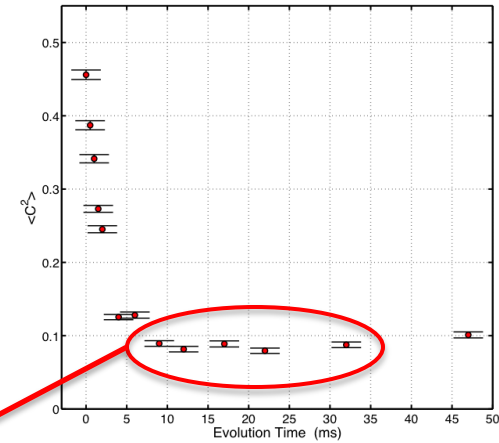
Equilibrium experiments agree with equilibrium theory



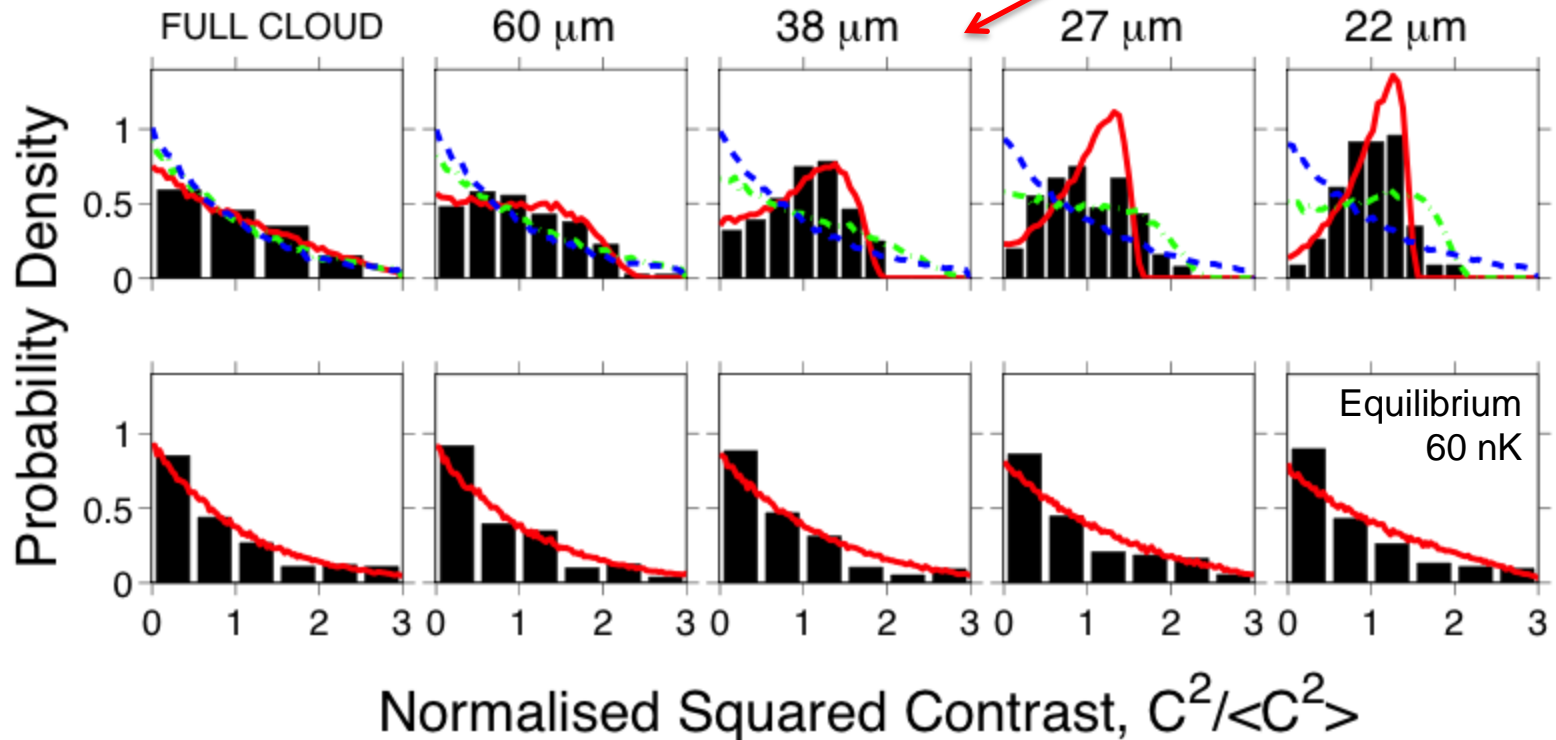
# Equilibrium vs Non-Equilibrium



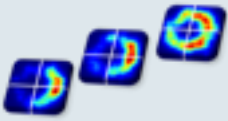
non-equilibrium distributions  
and  
equilibrium distributions



$$T_{\text{fit}} = 15 \pm 4 \text{ nK}$$



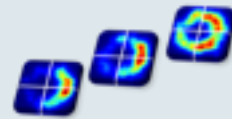
# Non-Equilibrium Evolution



What happens when we change the initial temperature  $T$  of the unsplit system?

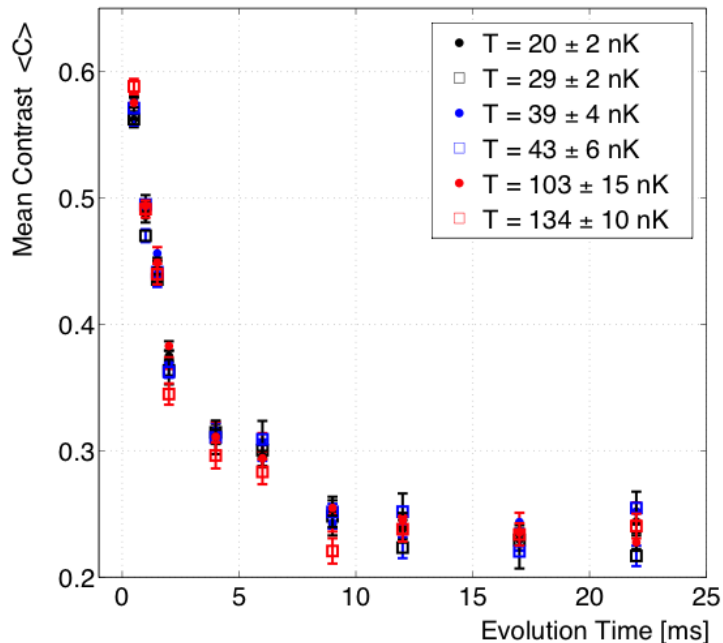


# Non-Equilibrium Evolution



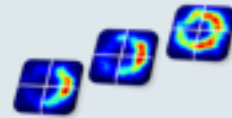
What happens when we change the initial temperature  $T$  of the unsplit system?

EXACTLY the same behaviour



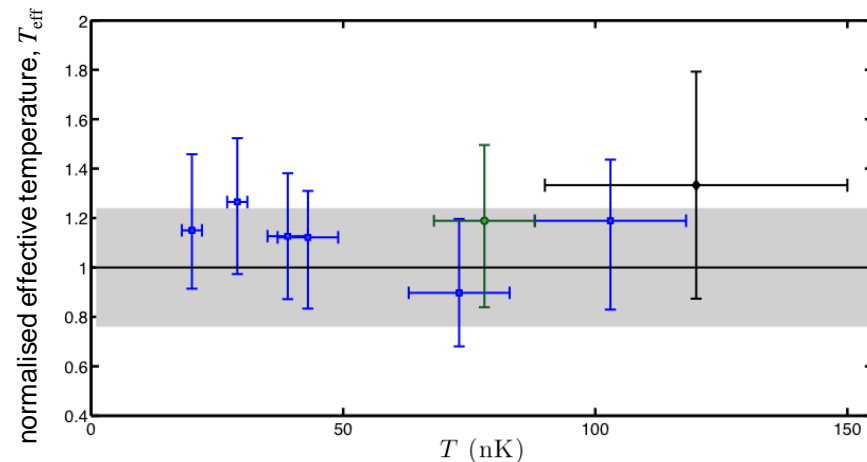
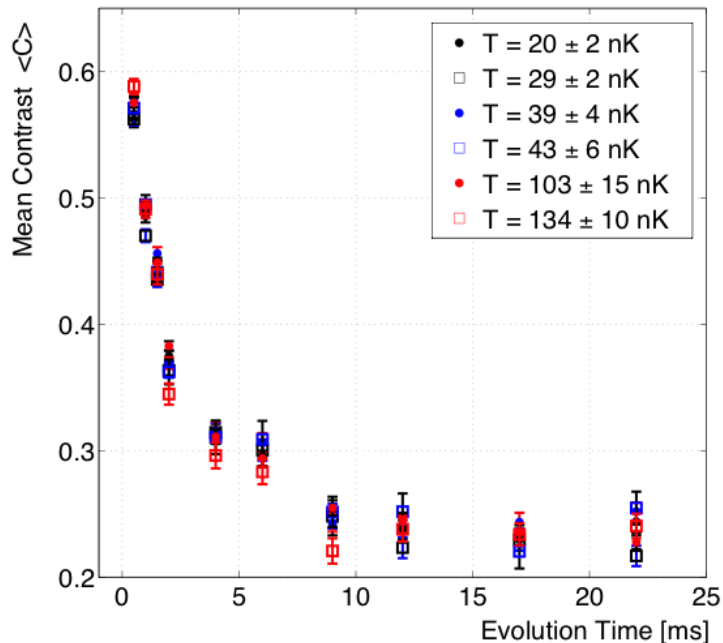


# Non-Equilibrium Evolution

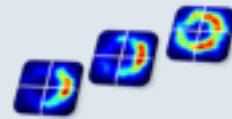


What happens when we change the initial temperature  $T$  of the unsplit system?

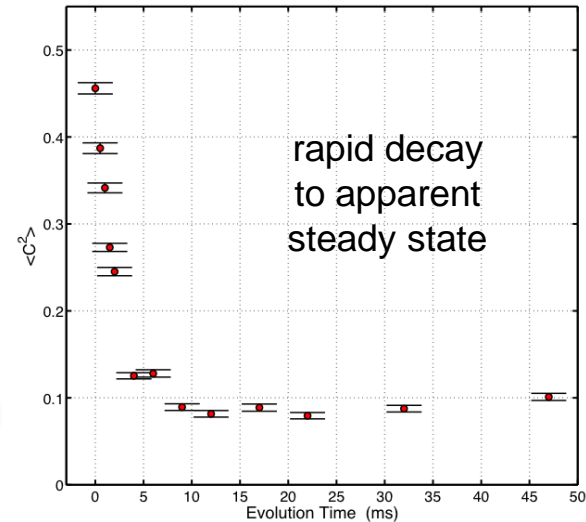
EXACTLY the same behaviour



# Recap



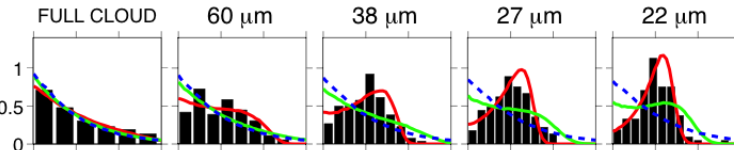
track evolution of non-equilibrium state  
(using matter-wave interference)



BUT  
this is **not** a  
thermal equilibrium state



HOWEVER,  
it is **thermal-like** in form

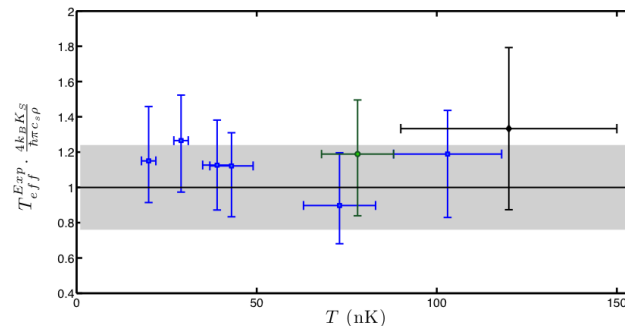
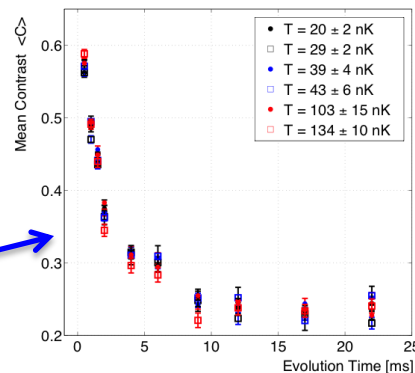


its **effective temperature**  $T_{eff}$   
does not change with  
initial temperature  $T$



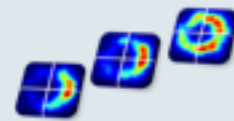
AND

the **initial rapid decay**  
is also **independent** of the  
initial temperature  $T$



This state represents a **fixed point**  
in the **evolution** of the system!





Kitagawa et al. *PRL* **104** 255302 (2010)

PRL **104**, 255302 (2010)

PHYSICAL REVIEW LETTERS

week ending  
25 JUNE 2010

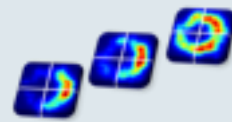
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**Ramsey Interference in One-Dimensional Systems: The Full Distribution Function of Fringe Contrast as a Probe of Many-Body Dynamics**

Takuya Kitagawa,<sup>1</sup> Susanne Pielawa,<sup>1</sup> Adilet Imambekov,<sup>2</sup> Jörg Schmiedmayer,<sup>3</sup> Vladimir Gritsev,<sup>4</sup> and Eugene Demler<sup>1</sup>

+ Kitagawa et al. *New J. Phys.* **13** 073018 (2011)





Kitagawa et al. *PRL* **104** 255302 (2010)  
Kitagawa et al. *New J. Phys.* **13** 073018 (2011)

- Integrable Luttinger-liquid-based theory

$$H_s = \int \left[ \frac{\rho}{8m} (\partial_z \hat{\phi}_s)^2 + g \hat{n}_s^2 \right] dz$$

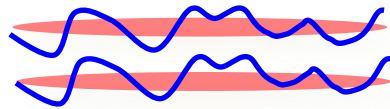
$$H_c = \int \left[ \frac{\rho}{8m} (\partial_z \hat{\phi}_c)^2 + g \hat{n}_c^2 \right] dz$$

$\hat{\phi}_s(z)$  = relative phase

$n_s(z)$  = relative density

$\hat{\phi}_c(z)$  = sum phase

$n_c(z)$  = sum density



Just after splitting

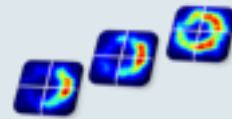
thermally populated

$$\hat{\phi}_c(z) = \hat{\phi}_A(z) + \hat{\phi}_B(z)$$

$$\hat{\phi}_s(z) = \hat{\phi}_A(z) - \hat{\phi}_B(z)$$

populated only by  
quantum shot noise

from the splitting process



Kitagawa et al. *PRL* **104** 255302 (2010)  
 Kitagawa et al. *New J. Phys.* **13** 073018 (2011)

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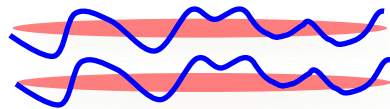
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Equilibrium

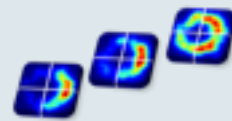
thermally populated

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thermally populated

# Theoretical Description of Non-Equilibrium Dynamics



Kitagawa et al. *PRL* **104** 255302 (2010)

Kitagawa et al. *New J. Phys.* **13** 073018 (2011)

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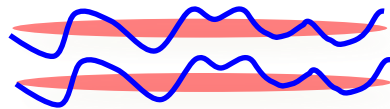
This theoretical model describes the dynamics of the non-equilibrium system through the **dephasing of the momentum modes ( $k$ -modes)** of the system but it **does not describe a thermalisation process**

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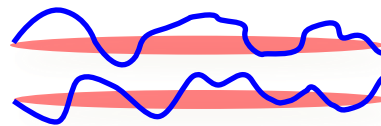
Just after splitting

thermally populated

$$\hat{\phi}_c(z) = \hat{\phi}_A(z) + \hat{\phi}_B(z)$$

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populated only by quantum shot noise from the splitting process



Equilibrium

thermally populated

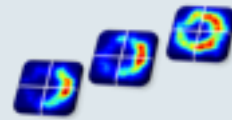
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Kitagawa et al. *PRL* **104** 255302 (2010)

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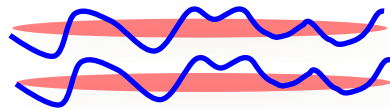
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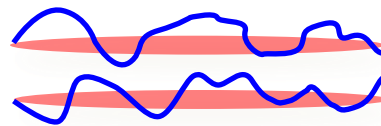
Just after splitting

thermally populated

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Equilibrium

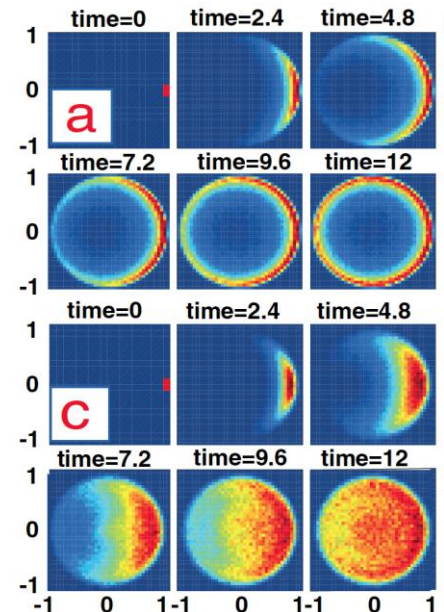
thermally populated

$$\hat{\phi}_c(z) = \hat{\phi}_A(z) + \hat{\phi}_B(z)$$

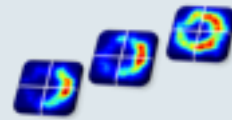
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thermally populated

Two-observable FDFs

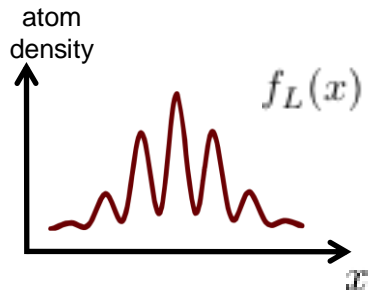


# Theoretical Description of Non-Equilibrium Dynamics



Kitagawa et al. *PRL* **104** 255302 (2010)

- dephasing of the momentum modes (*k*-modes)  
not thermalisation process



$$f_L(x) = \exp\left(-\frac{x^2}{2\sigma^2}\right) \left(1 + C_L \cos\left(\frac{2\pi x}{\lambda} + \theta\right)\right)$$

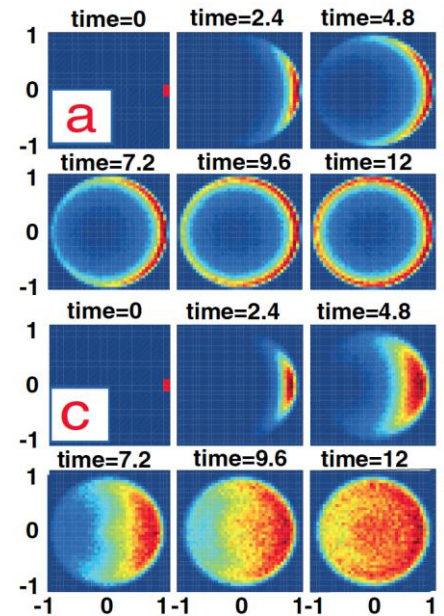
Contrast  
 $C_L$

(over integration  
length  $L$ )

Phase  
 $\theta$

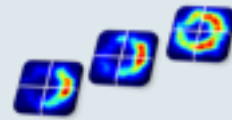
(phase of interference  
pattern with respect to  
gaussian background)

Two-observable FDFs



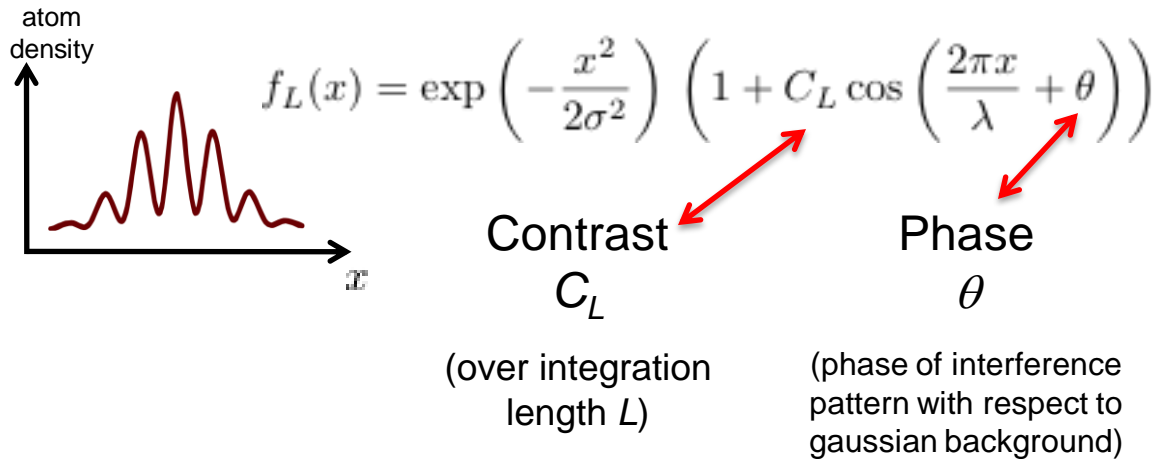


# Theoretical Description of Non-Equilibrium Dynamics

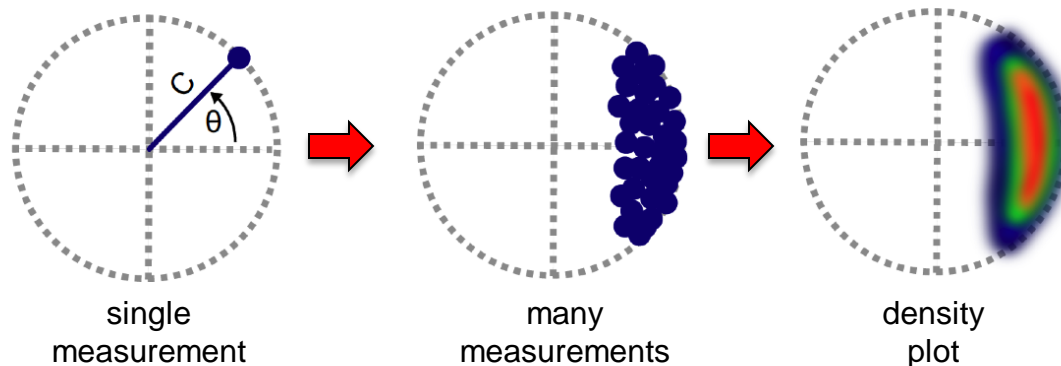
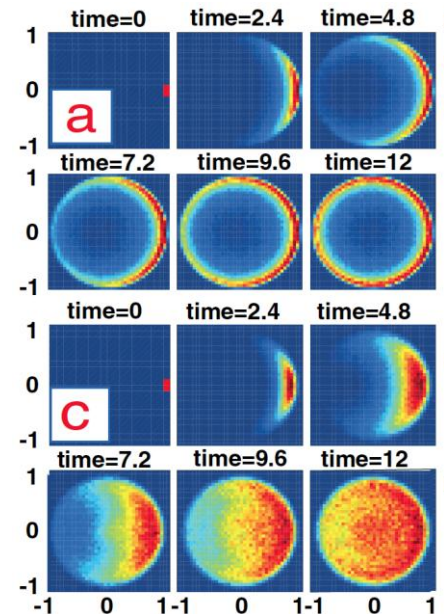


Kitagawa et al. *PRL* **104** 255302 (2010)

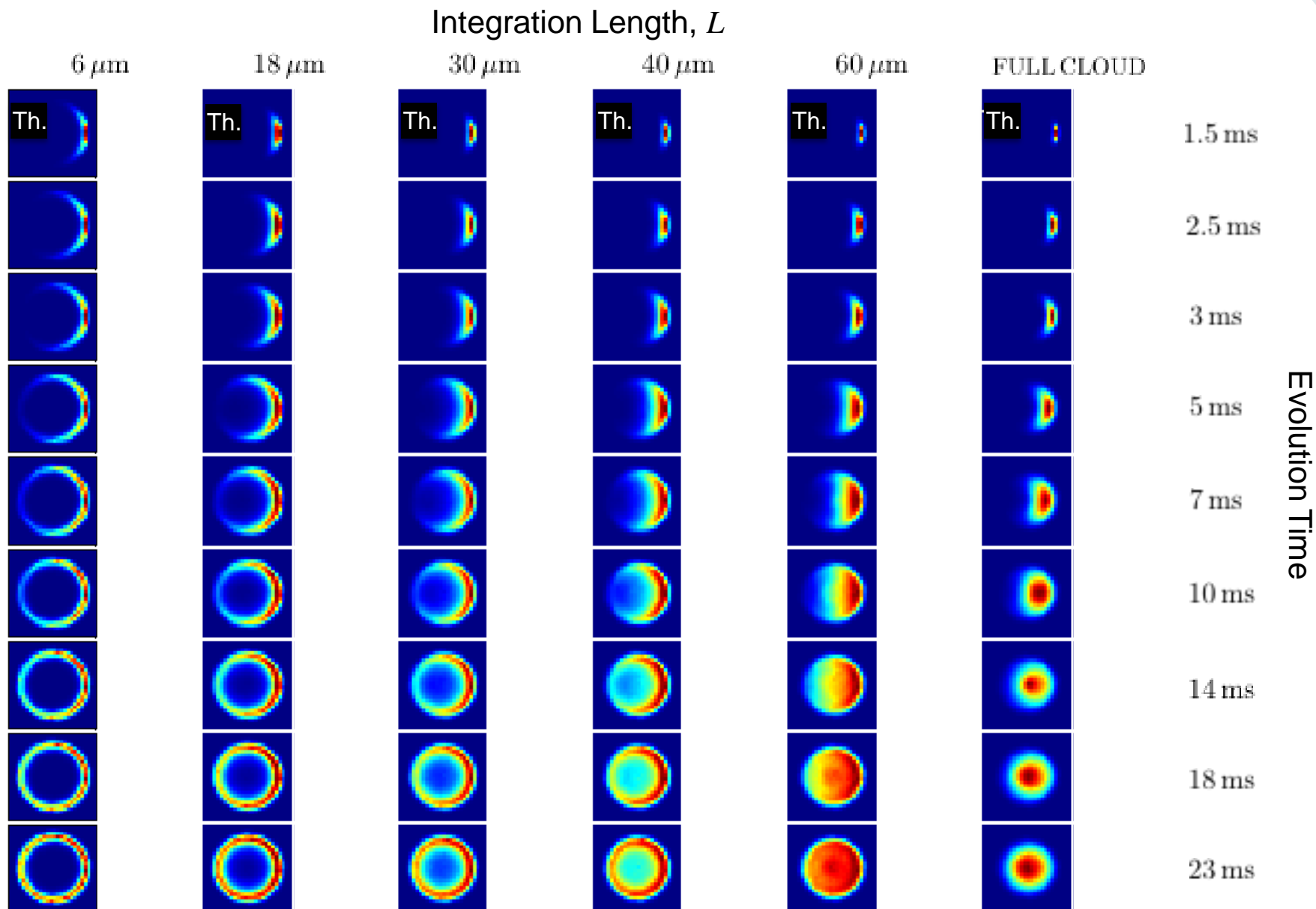
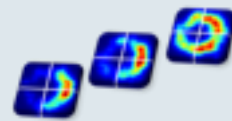
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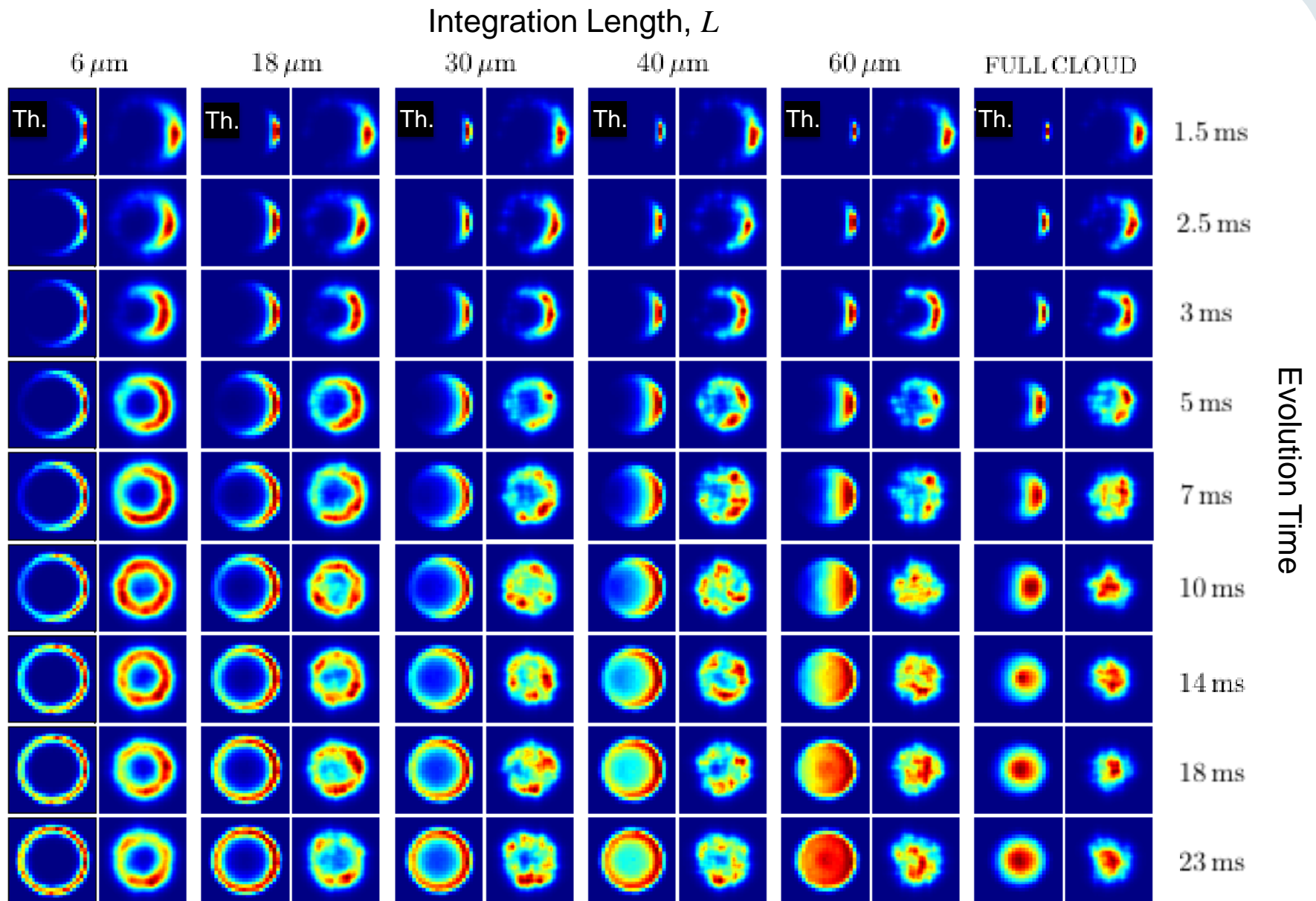
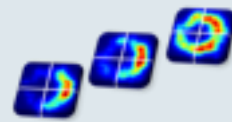
Two-observable FDFs



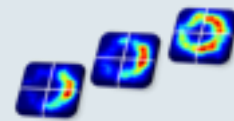
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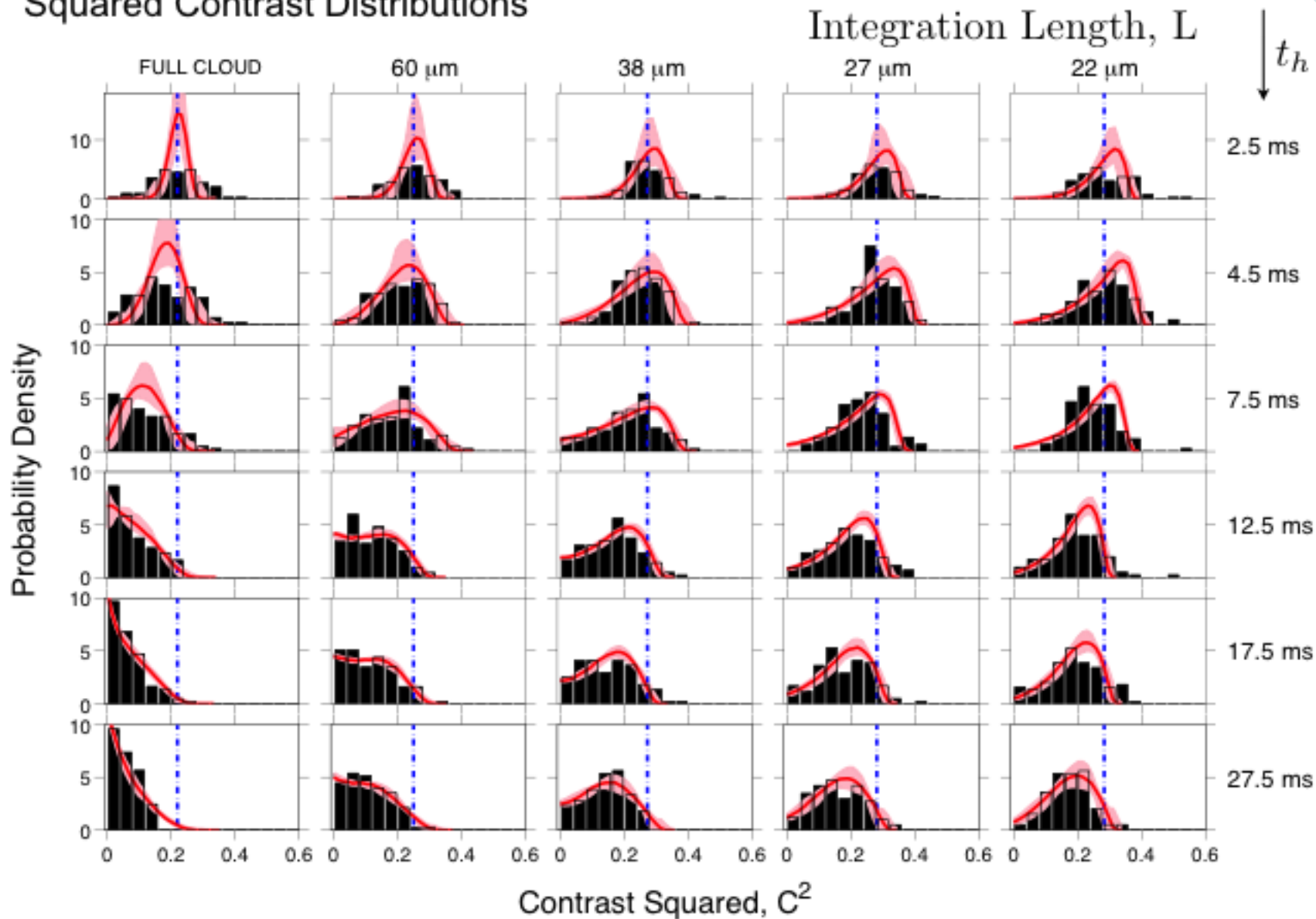
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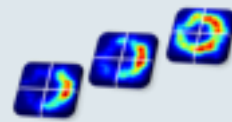
# Theoretical Description of Non-Equilibrium Dynamics



- Squared Contrast Distributions



# Theoretical Description of Non-Equilibrium Dynamics



Prediction for the **effective temperature**  $T_{\text{eff}}$  [Kitagawa et al. *New J. Phys.* **13** 073018 (2011)]

$$T_{\text{eff}} = \frac{g\rho}{2k_B}$$

$$g = 2\hbar\omega_{\perp} a_s$$

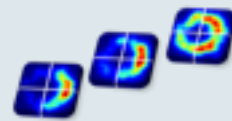
$$\rho = \text{density}$$

$\omega_{\perp}$  – radial trapping frequency

$a_s$  – scattering length



# Theoretical Description of Non-Equilibrium Dynamics



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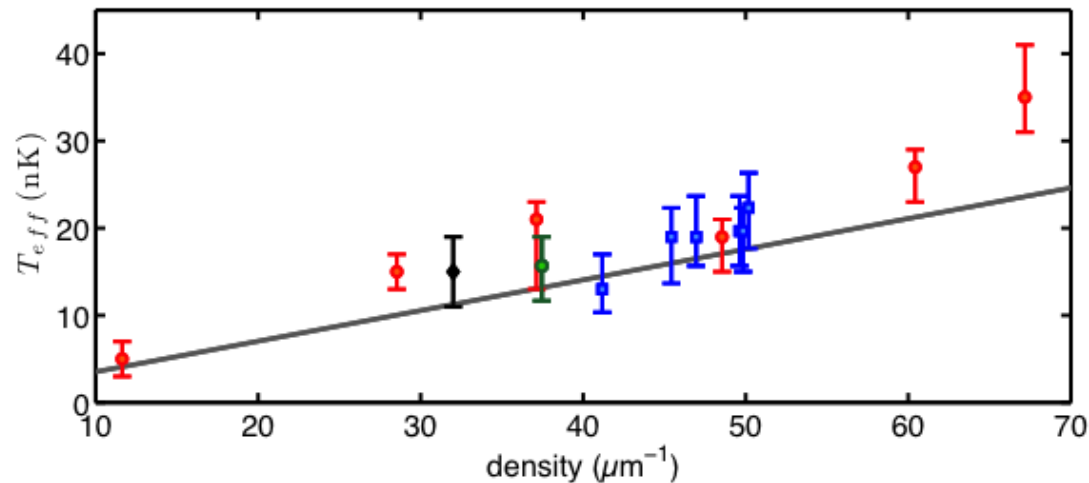
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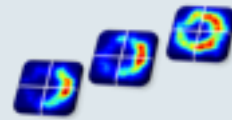
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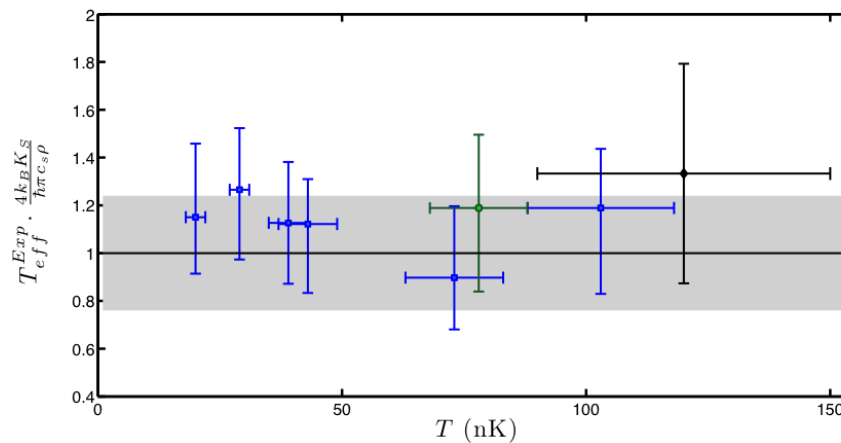
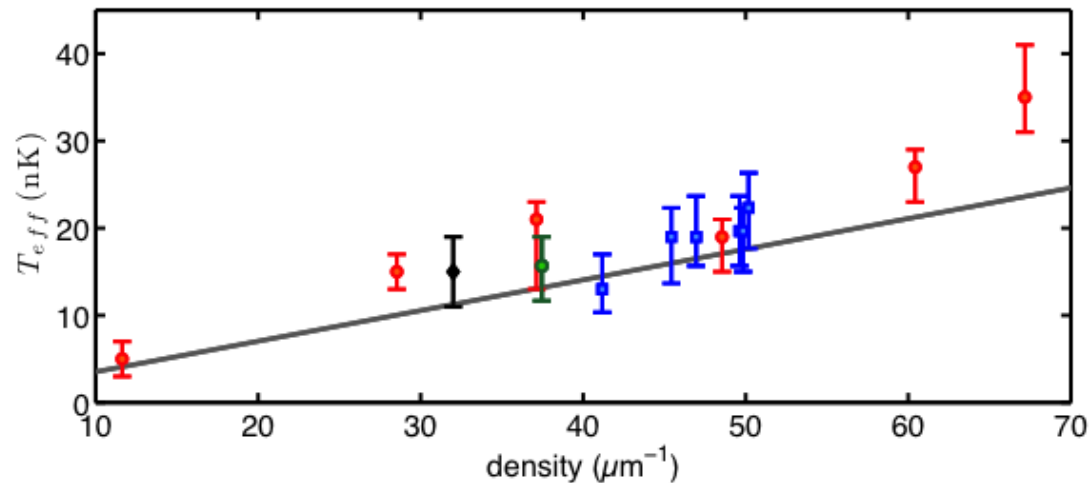
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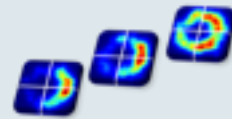
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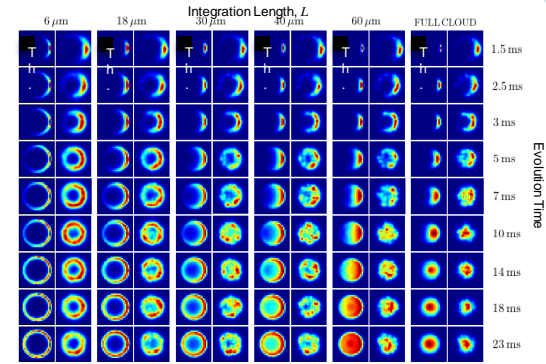
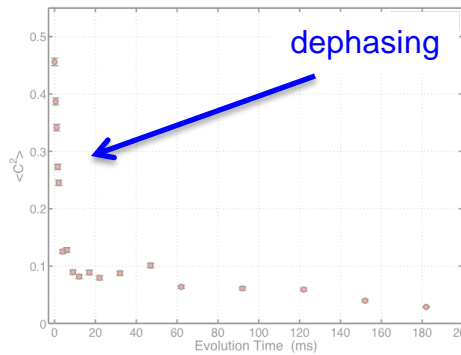
$a_s$  – scattering length



# Pre-thermalisation



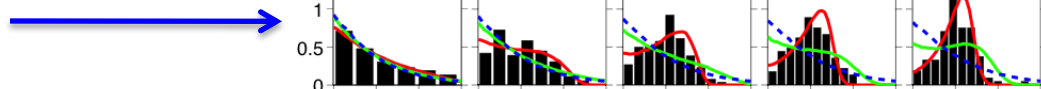
Our **system** decays to a **quasi-stationary state** that is **not a thermal equilibrium state** of the system



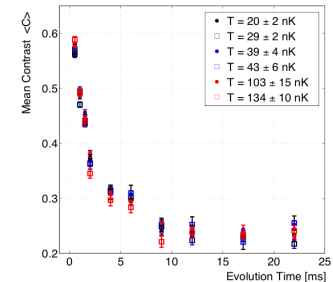
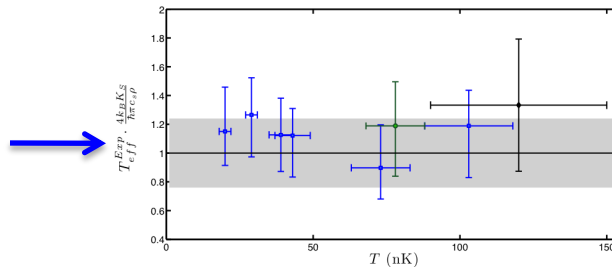
The decay is well described by **dephasing**



to a state that is **thermal-like** in form (observing the interference contrast  $C_L$ )



that displays an **effective temperature  $T_{\text{eff}}$**  and **rapid initial decay** that do not change with initial temperature  $T$  and hence is a **fixed point** in the evolution



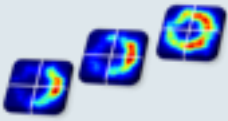
Pre-thermalisation: [Berges, Borsányi, Wetterich, Prethermalization, PRL 93 142002 \(2004\)](#)

- e.g. Moeckel and Kehrein, New J. Phys. 12, 055016 (2010).
- Mathey and Polkovnikov, Phys. Rev. A 81, 033605 (2010).
- Barnett, Polkovnikov, and Vengalattore, Phys. Rev. A 84, 023606 (2011).
- Kitagawa et al. New J. Phys. 13 073018 (2011)
- Kollar, Wolf, Eckstein, Phys. Rev. B 84, 054304 (2011).





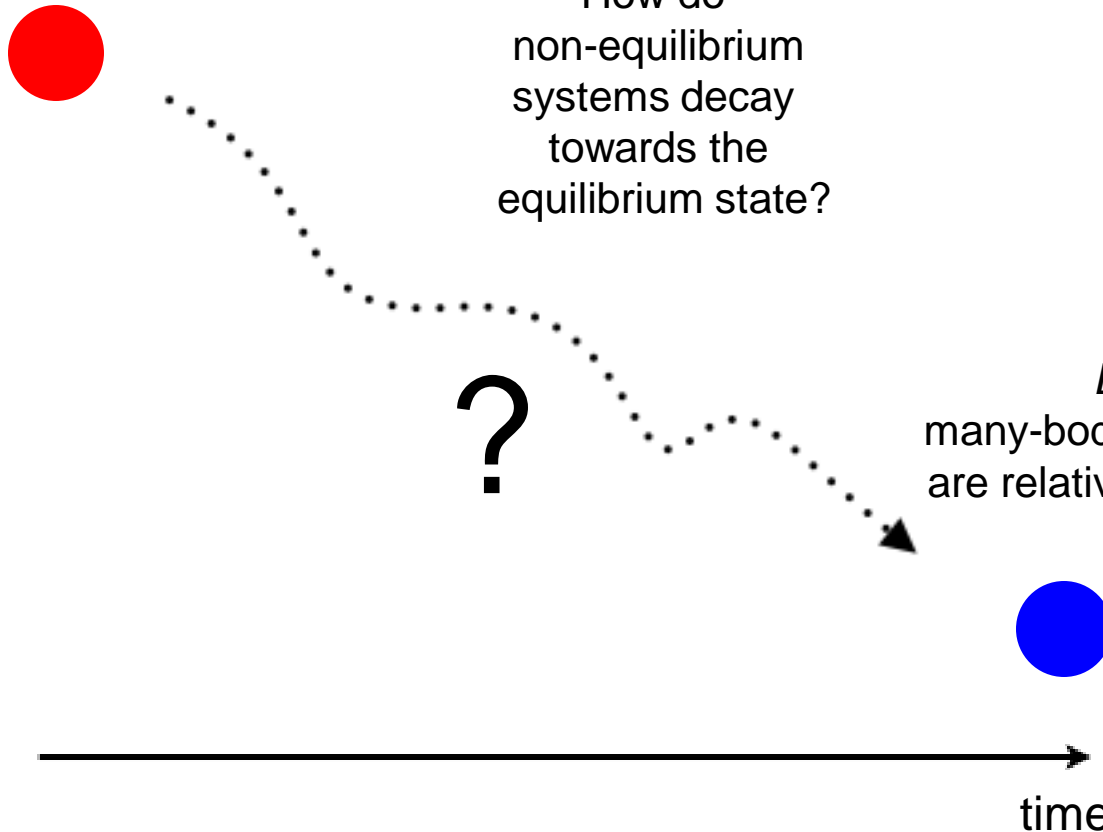
# Non-Equilibrium Systems



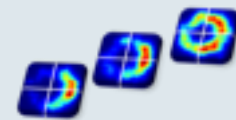
*Non-equilibrium*  
many-body quantum systems  
are *not* well  
understood

How do  
non-equilibrium  
systems decay  
towards the  
equilibrium state?

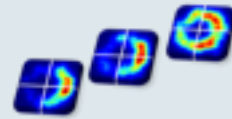
*Equilibrium*  
many-body quantum systems  
are relatively well understood



# Thank you.



# Open Questions

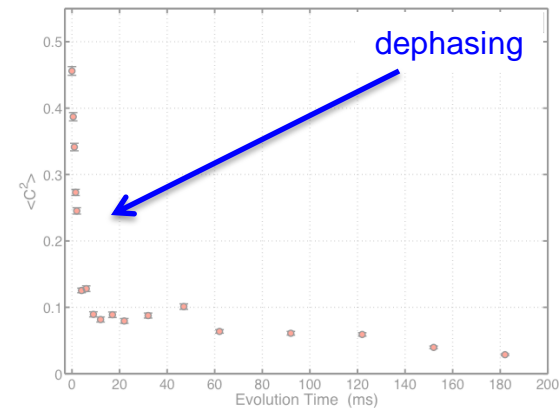


## Thermalisation

What is thermalisation?

How does it occur in a 1d system?

It is a subtle process in our case, if it exists at all.



## Generalized Gibbs Ensemble (GGE)

It is predicted that the prethermalised state can be described by a GGE.  
e.g. Kollar, Wolf, Eckstein, Phys. Rev. B 84, 054304 (2011).

How can we prove this with an experiment? Can we prove this?

## Prethermalization

What is the most general definition of prethermalization?

Berges, Borsányi, Wetterich, *Prethermalization*, PRL **93** 142002 (2004)

Moeckel and Kehrein, New J. Phys. 12, 055016 (2010).

Mathey and Polkovnikov, Phys. Rev. A 81, 033605 (2010).

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