PUZZLE OF THE INTERMEDIATE (BAD METAL) PHASE IN JOSEPHSON ARRAYS

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Experiment: M. Gershenson group at Rutgers.

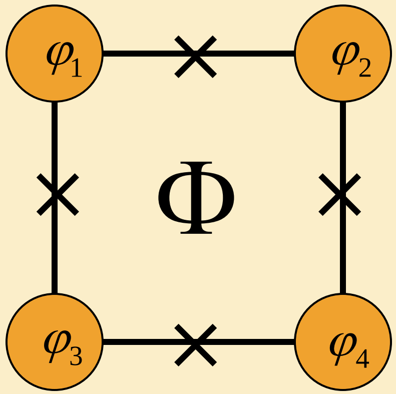
PLAN

1. Introduction: simplistic models of SIT transition in Josephson junction arrays
2. Experimental data
3. Missing ingredients: charge disorder.
4. Numerical solution of finite small systems.
5. Unconventional glass.
6. Conclusions.

SIMPLISTIC MODEL OF THE SIT TRANSITION IN JOSEPHSON JUNCTION ARRAYS

JOSEPHSON ARRAYS

Elementary building block



Efetov
Altshuler, Efetov & Syzranov

Ideal Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_{ij}^{-1} q_i q_j + E_J \cos(\varphi_i - \varphi_j - 2\pi \frac{\Phi_{ij}}{\Phi_0}) \quad q_i = 2e\, i \frac{d}{d\varphi_i}$$

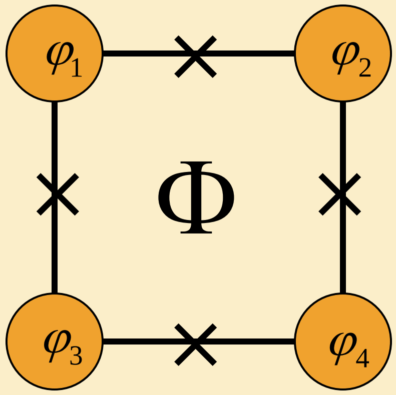
C_{ij} - capacitance matrix E_J - Josephson energy

Simplest case: large ground capacitance of individual islands ($C_0 \gg C$):

$C_{ij} = C_0 \delta_{ij} + C D_{ij}$ where D_{ij} is discrete Laplace operator

JOSEPHSON ARRAYS

Elementary building block



Toy Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_0^{-1} q_i q_i + E_J \cos(\varphi_i - \varphi_j - 2\pi \frac{\Phi_{ij}}{\Phi_0}) \quad q_i = 2e\, i \frac{d}{d\varphi_i}$$

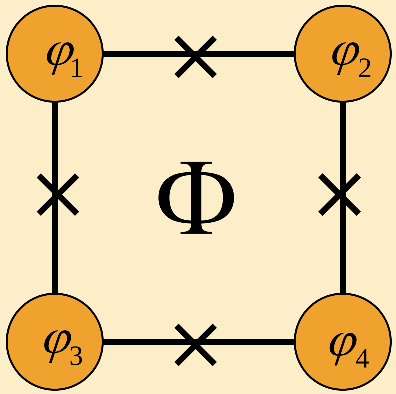
C_0 - ground capacitance E_J - Josephson energy

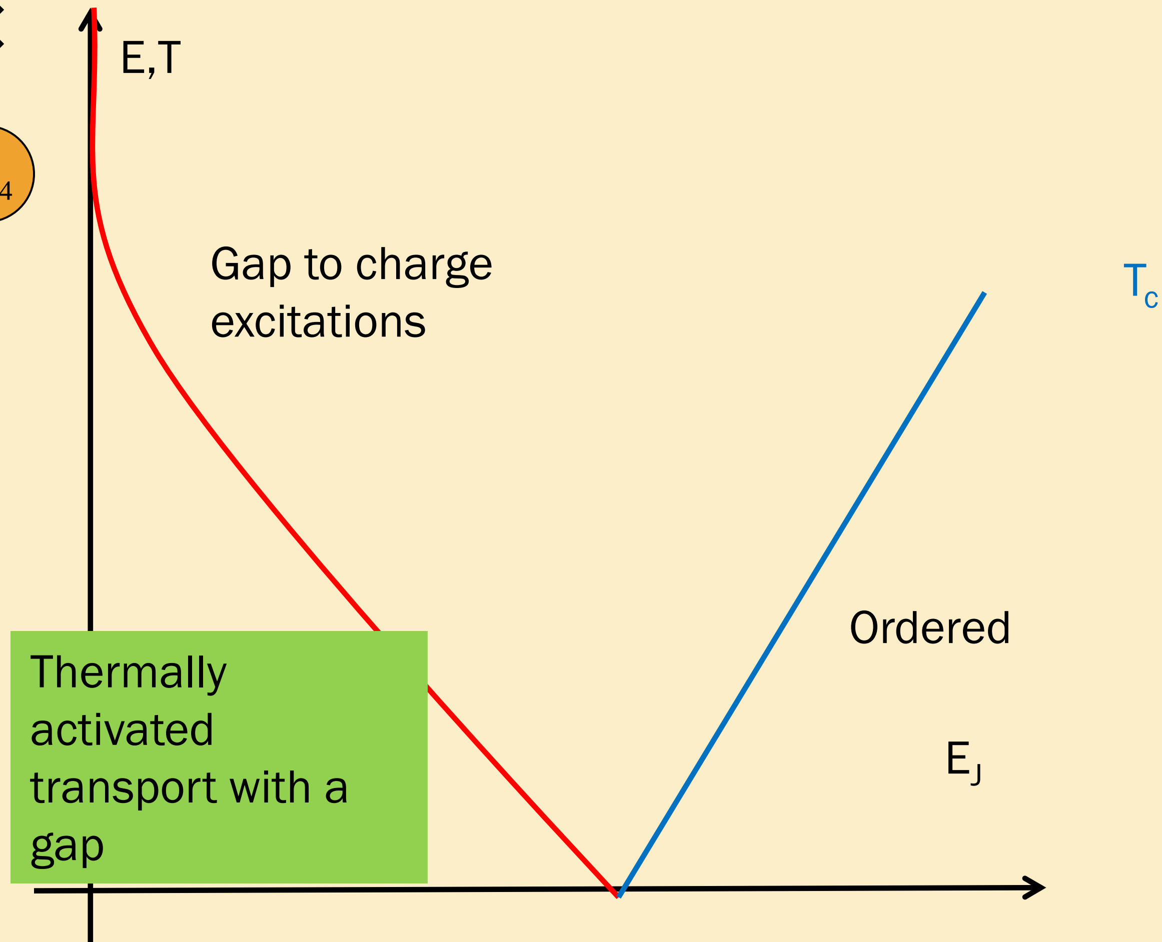
At $E_J > E_C = e^2/2C$ expect long range order in phase (superconductor)

At $E_J < E_C = e^2/2C$ ground state $q = 0$ excitations are separated by a gap that closes exactly at transition.

JOSEPHSON ARRAYS

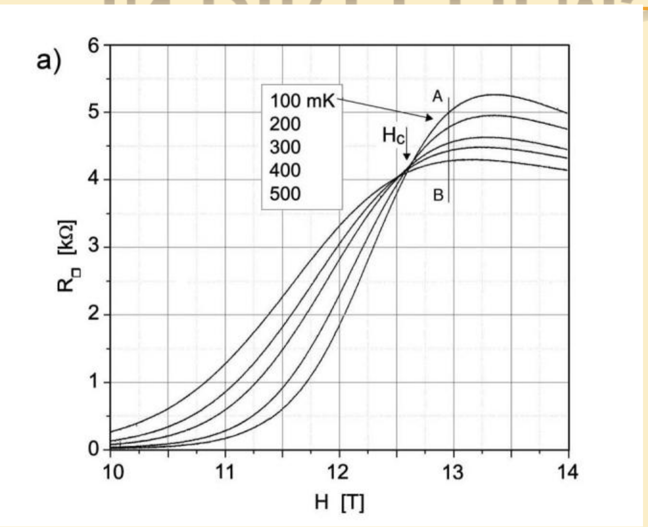
Elementary building block



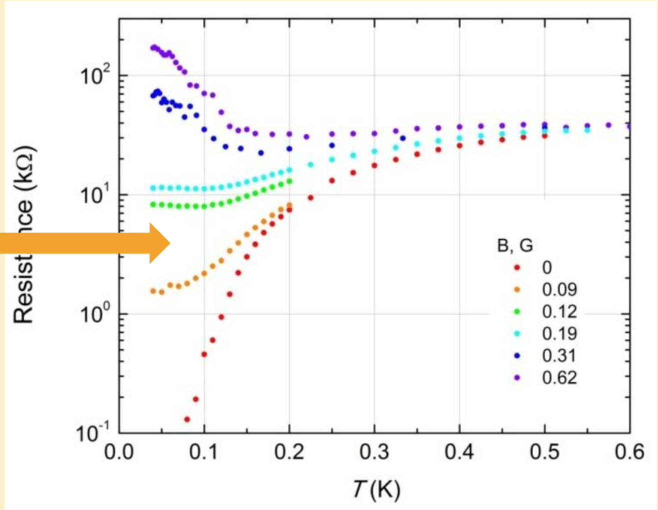


EXPERIMENTAL DATA

SUPERCONDUCTOR-INSULATOR TRANSITION IN DIRTY FILMS AND ARRAYS.

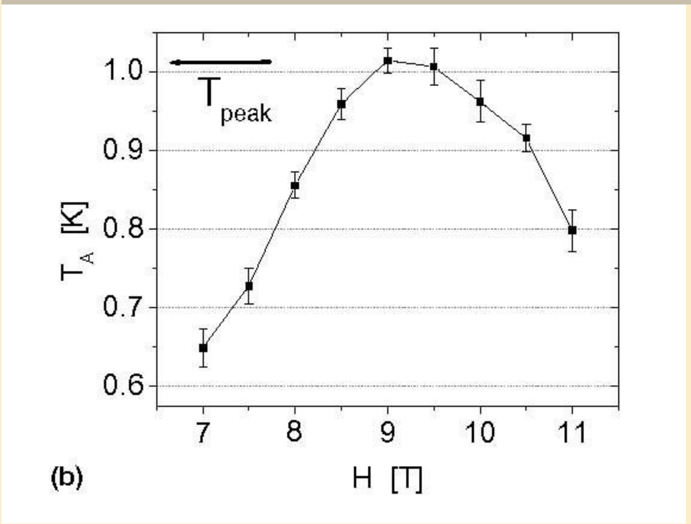


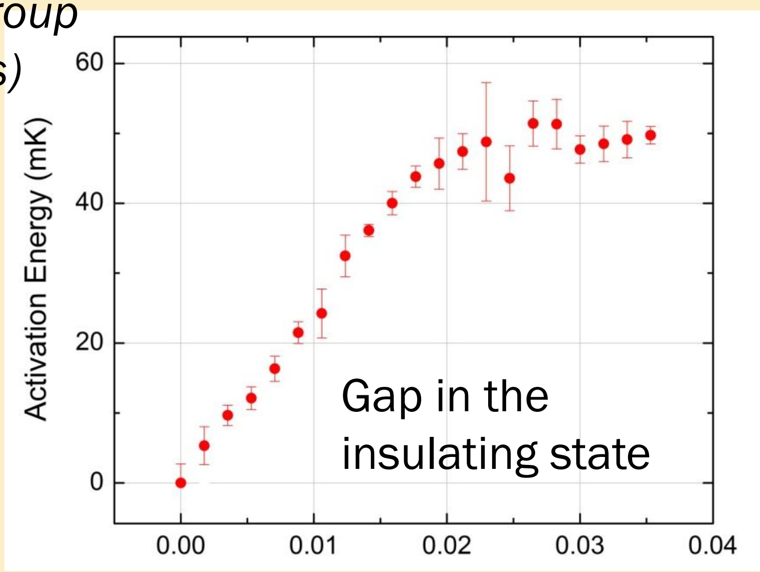
Large regime of T-independent resistance



Quantum critical behavior as a function of magnetic field (*Kapitulnik 2008*). Self dual quantun critical point (M. Fisher)

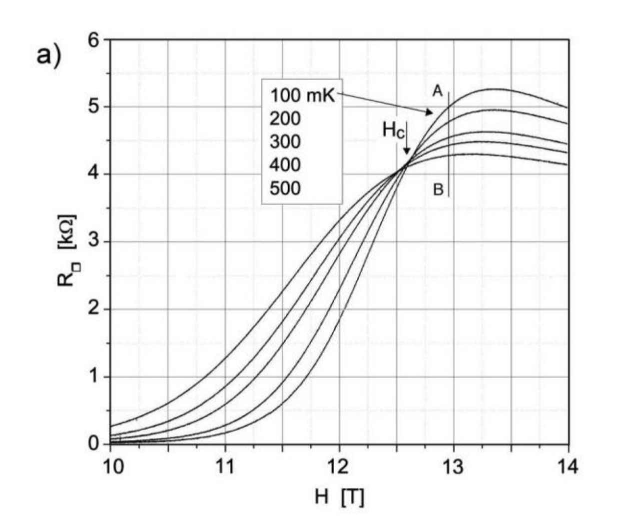
Data from Gershenson group (2010, Rutgers)



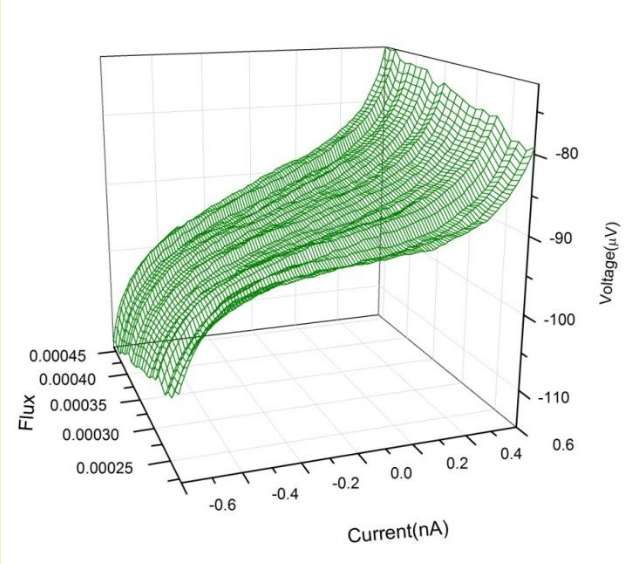


$$\frac{\Phi - \Phi^*}{\Phi_0}$$

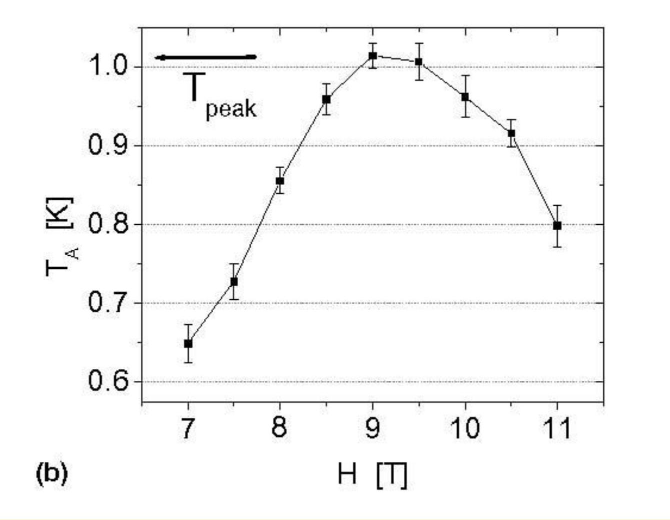
SUPERCONDUCTOR-INSULATOR TRANSITION IN DIRTY FILMS AND ARRAYS.

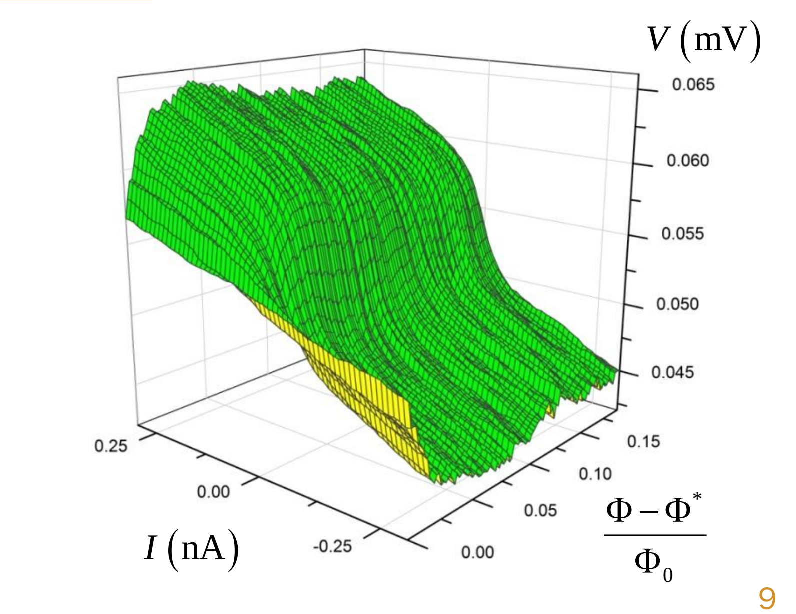


Featureless V(I) in pseudo normal state



Quantum critical behavior as a function of magnetic field (*Kapitulnik 2008*). Self dual quantun critical point (M. Fisher)

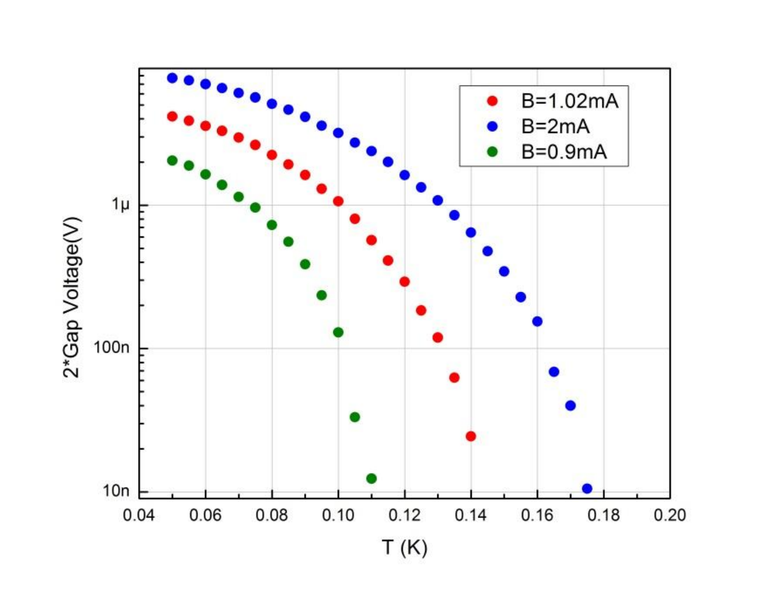


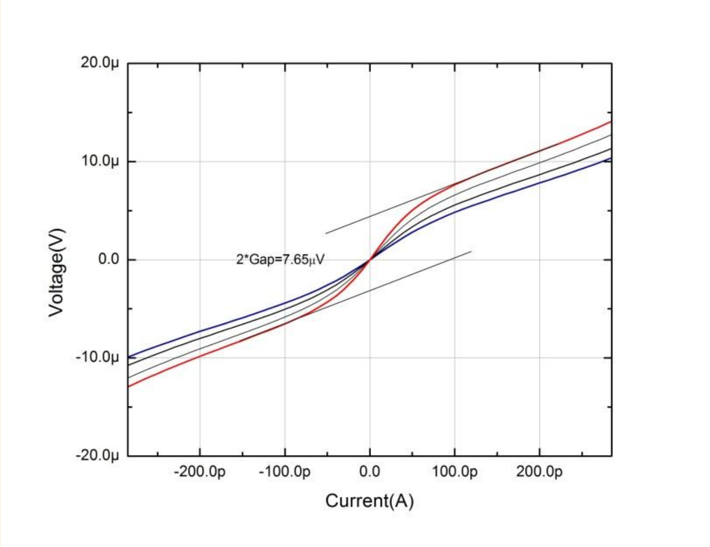


JOSEPHSON ARRAYS VS. FILMS

- Films show direct supercondutor-insulator transition while some arrays show broad intermediate “normal” phase for large E_C/E_J.
- Both arrays and films show insulating gap decreasing continuously away from the critical point.
- Both arrays and films show large inhomogeneity of superconducting properties despite nominally homogeneous normal state.

Quantum critical behavior of the insulating gap:

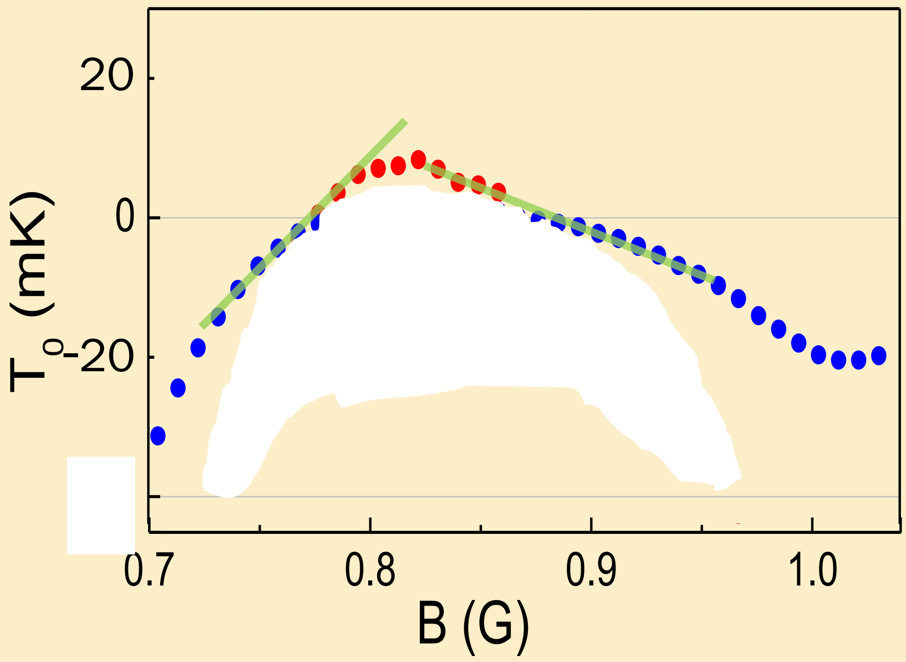


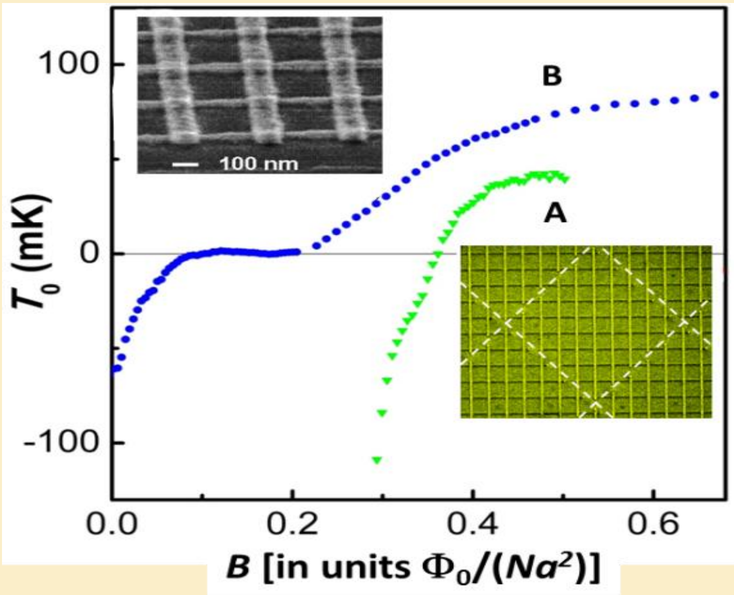


Gap in the array is of the order of temperature and coincides with Arrenius when determined from the current-voltage characteristics of the *whole* sample!

JOSEPHSON ARRAYS VS. FILMS

- Multiple phase transitions in films as a function of field for small E_C/E_J?

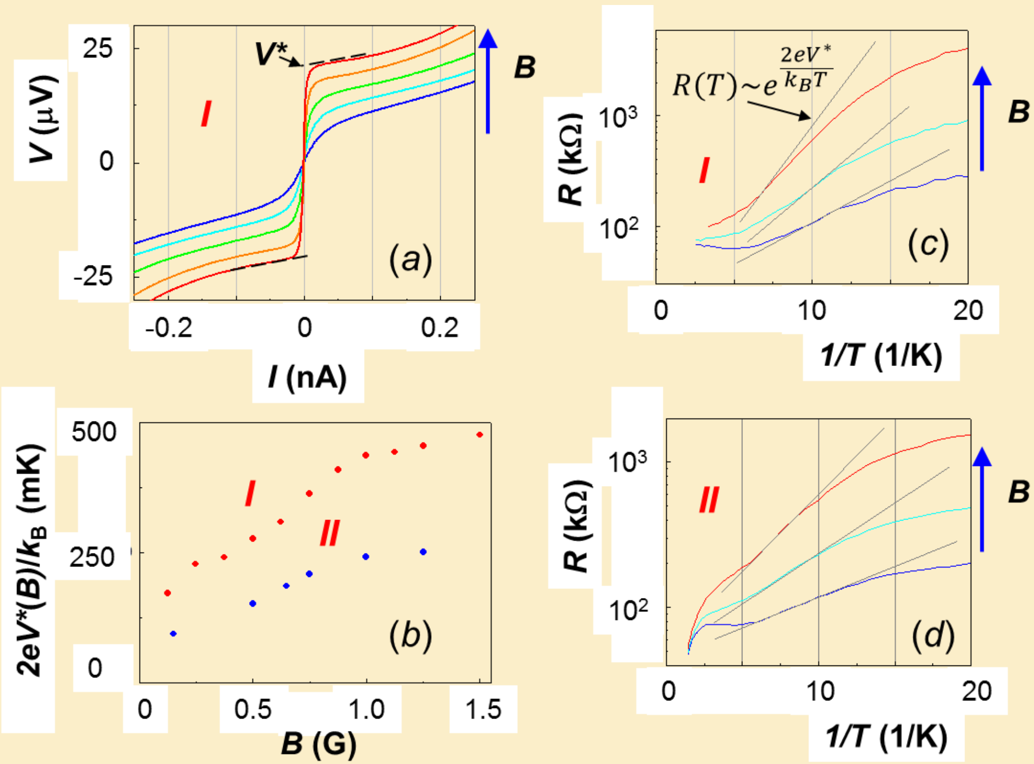




Direct SI transition in low R arrays

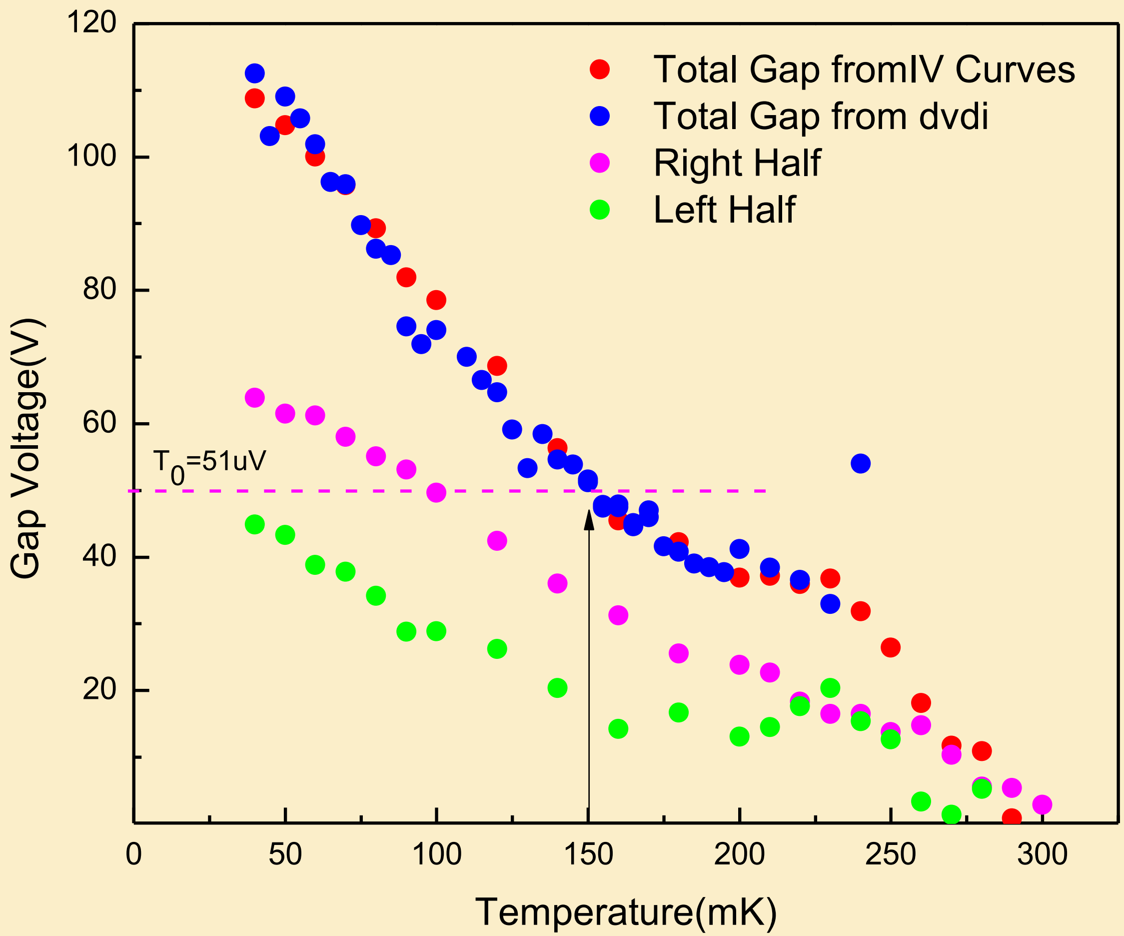
Insulator – “normal” in high R arrays

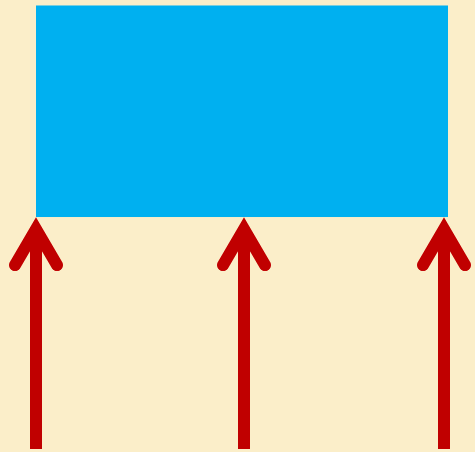
EMERGENCE OF LARGE SPATIAL SCALES



Gap extracted from the current-voltage characteristics and activation energies coincide for the whole arrays!
Indication that transport is dominated by a single weak link.

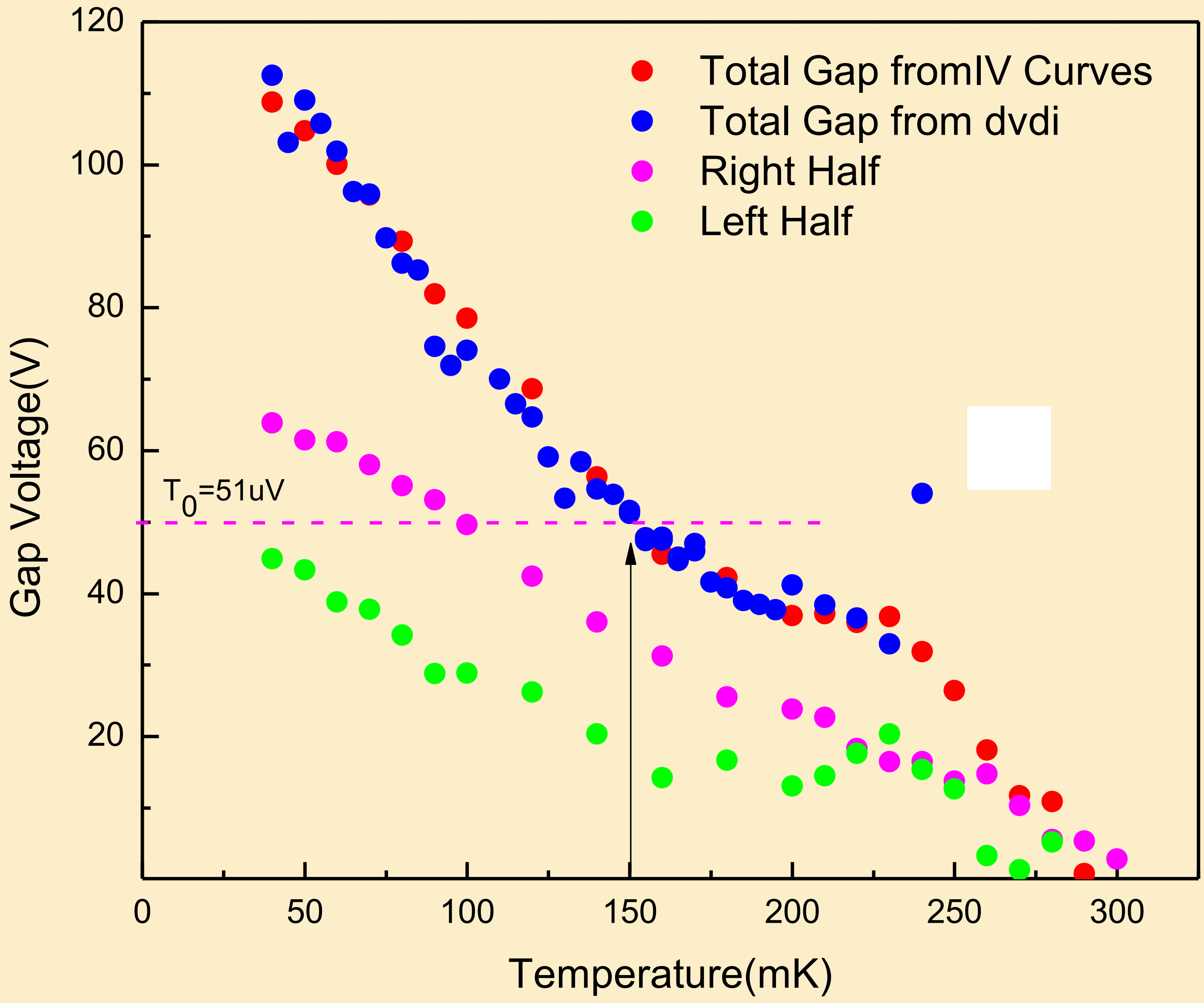
NO EVIDENCE FOR WEAK LINK





No difference between right and left half of the array!

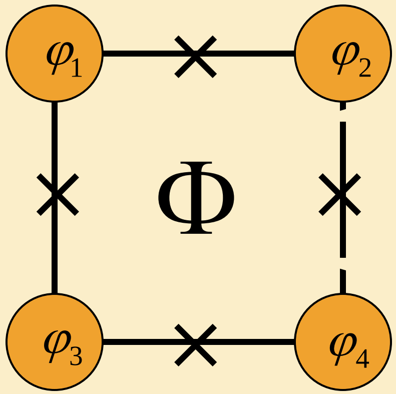
OFFSET VOLTAGE IVCS AT FULL FRUSTRATION

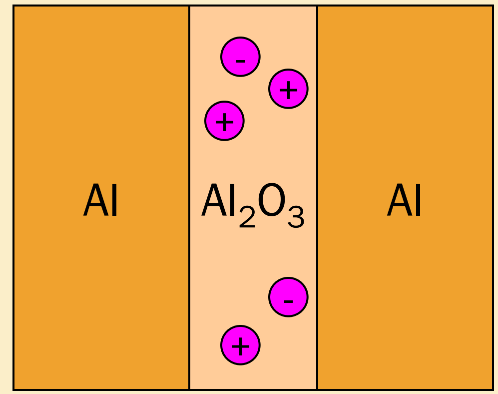


MISSING INGREDIENT: DYNAMICAL CHARGE DISORDER

TYPE 3 TRANSITION - JOSEPHSON ARRAYS

Elementary building block





Realistic Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_{ij}^{-1}(q_i - Q_i)(q_j - Q_j) + (E_J + \delta E_J)\cos(\varphi_i - \varphi_j - 2\pi\frac{\Phi_{ij} + \delta\Phi}{\Phi_0}) \quad q_i = 2e\, i\frac{d}{d\varphi_i}$$

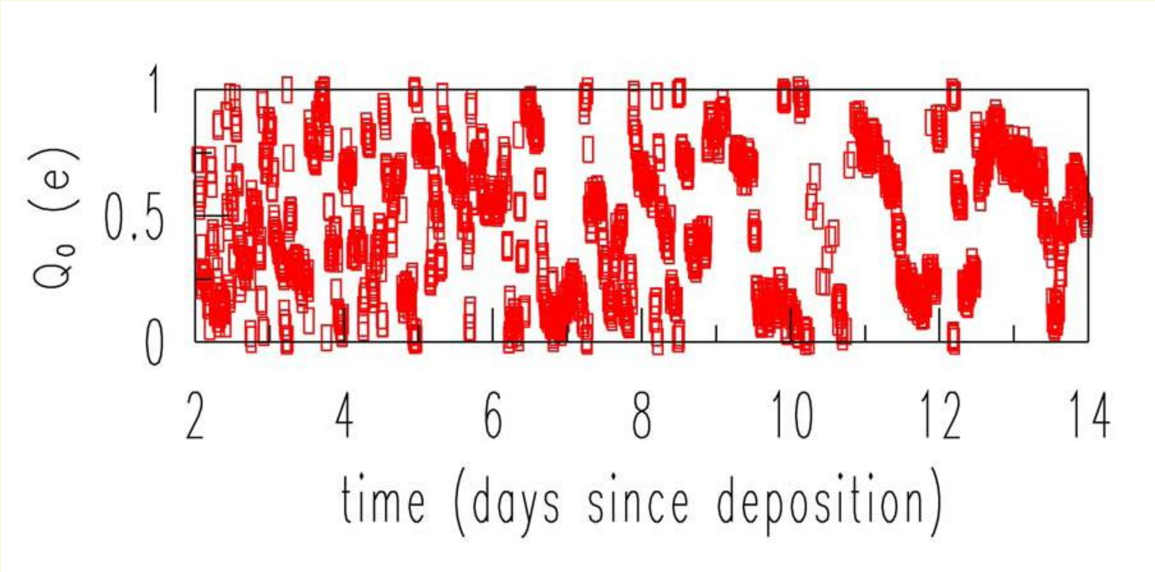
C_{ij} - capacitance matrix E_J - Josephson energy

$Q_i = Q_i^0 + Q_i(t)$ - induced charge (static and fluctuating) - large effect

$\delta\Phi = \delta\Phi^0 + \delta\Phi(t)$ - static flux due to area scatter and flux noise $\delta\Phi(t) \sim 2-5\times 10^{-6}\Phi_0$

$\delta E_J = \delta E_J^0 + \delta E_J(t)$ - static scatter of Josephson energies and their time dependent fluctuations. $\delta E_J(t)/E_J \lesssim 10^{-6}$

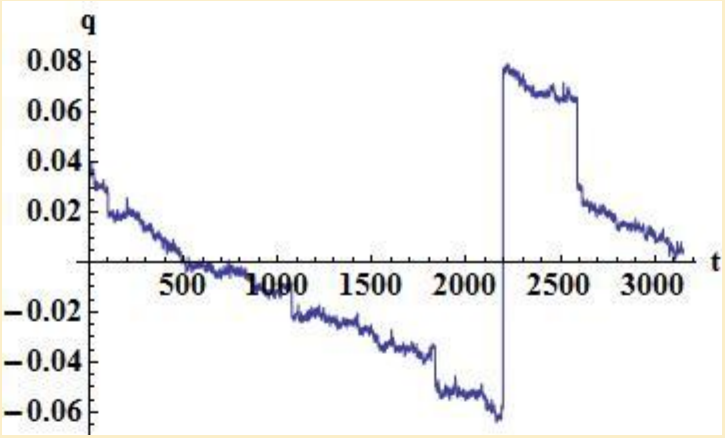
OFFSET CHARGE DYNAMICS



Direct studies of
Ultra-small junctions

1. Conventional SET
Zimmerman 2008

2. Suspended SET
Pashkin et al (unpublished)



Conclusion – charge is completely random at a given moment of time

In larger arrays time scales are about 1ms
Manucharyan et al

CONCLUSIONS FOR THE MODEL

The relevant terms in the Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_{ij}^{-1}(q_i - Q_i)(q_j - Q_j) + E_J \cos(\varphi_i - \varphi_j - 2\pi \frac{\Phi_{ij}}{\Phi_0}) \quad q_i = 2e\, i \frac{d}{d\varphi_i}$$

C_{ij} - capacitance matrix E_J - Josephson energy

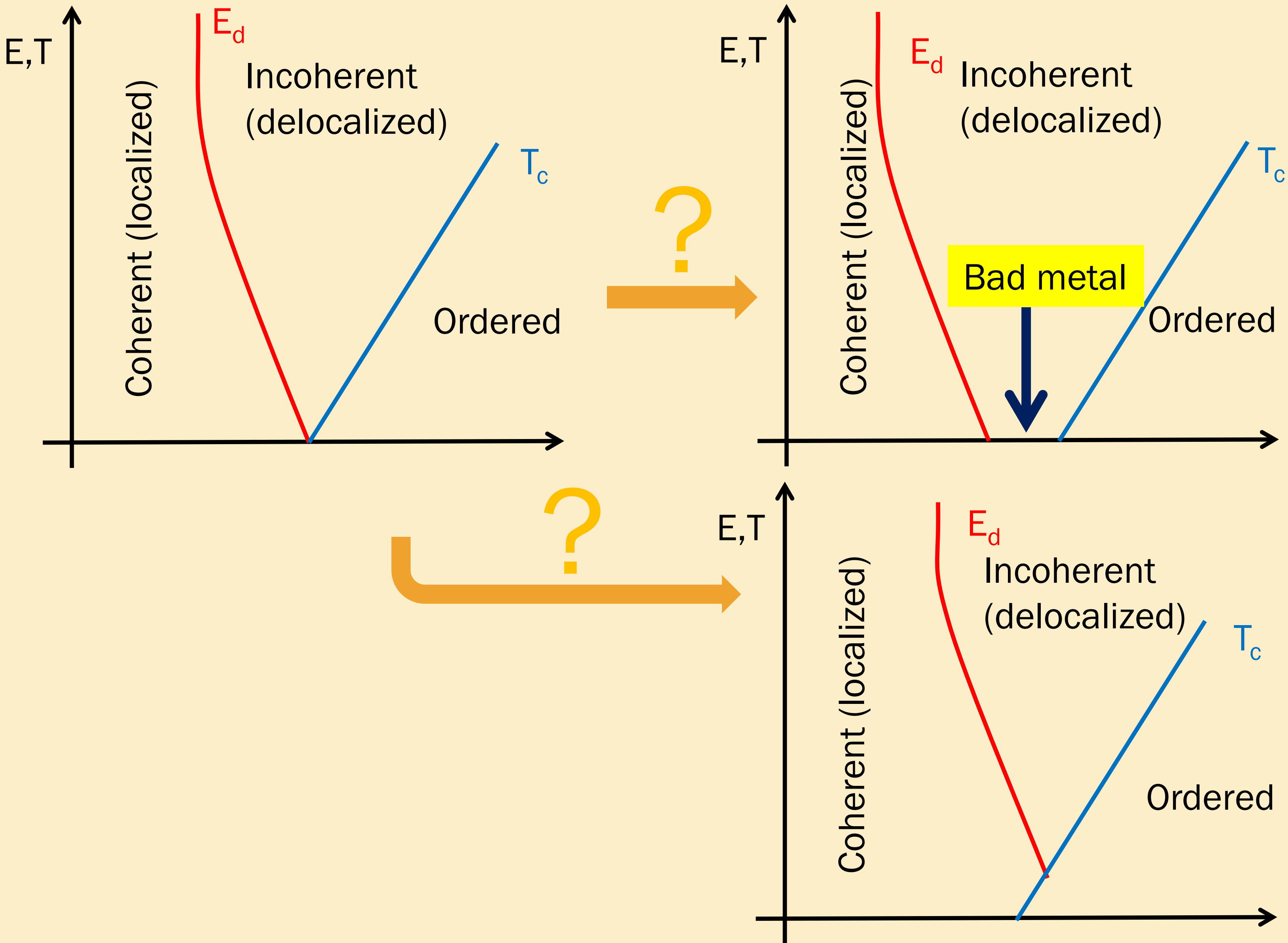
Q_i - completely random pseudostatic offset charge

To compare with the experiments results should be averaged over Q_i

because typical experiment cannot resolve the times scales at which Q_i varies

NUMERICAL SOLUTION OF FINITE (SMALL) SYSTEMS.

Q: WHICH TYPE OF BEHAVIOR TO EXPECT?



NUMERICAL TOOL: LEVEL STATISTICS

Spins located on random graph

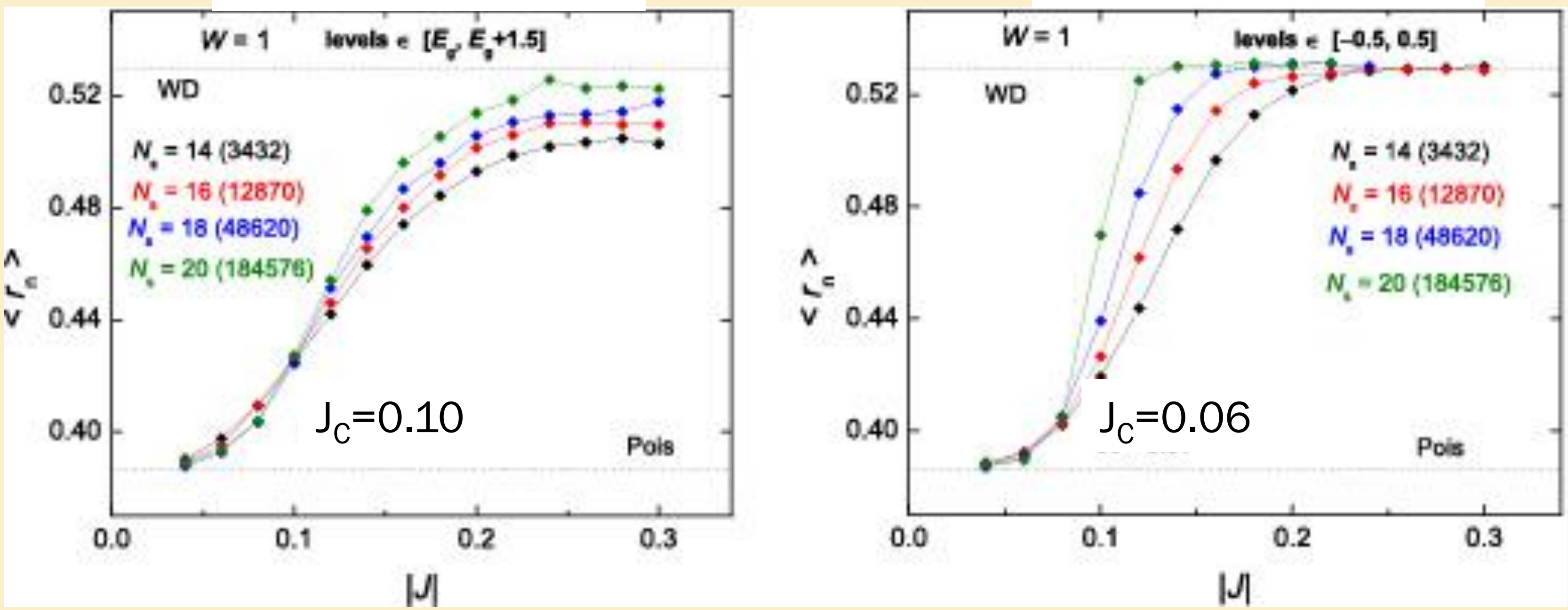
$$H = -\sum_j \xi_j \sigma_j^{\,z} - \sum_{(ij)} \left[g(\sigma_i^{\,+}\sigma_j^{\,-} + \sigma_i^{\,-}\sigma_j^{\,+}) \right]$$

$$r_n = \min\{\delta_n, \delta_{n+1}\} / \max\{\delta_n, \delta_{n+1}\}$$

K=2

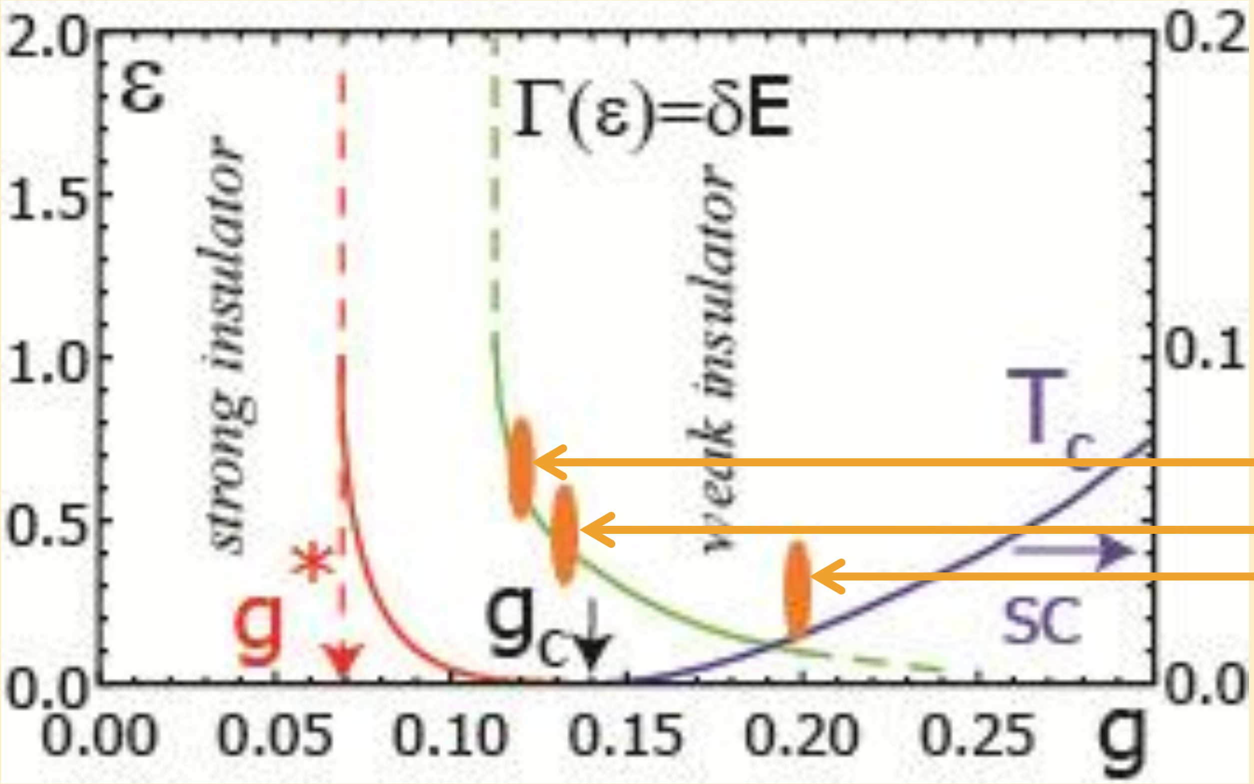
Low energy levels

High energy levels



Level correlations as a function of interaction evidence the transition between localized regime at small J and delocalized regime at large J

DETAILED COMPARISON OF NUMERICAL RESULTS AND ANALYTICAL EXPECTATIONS



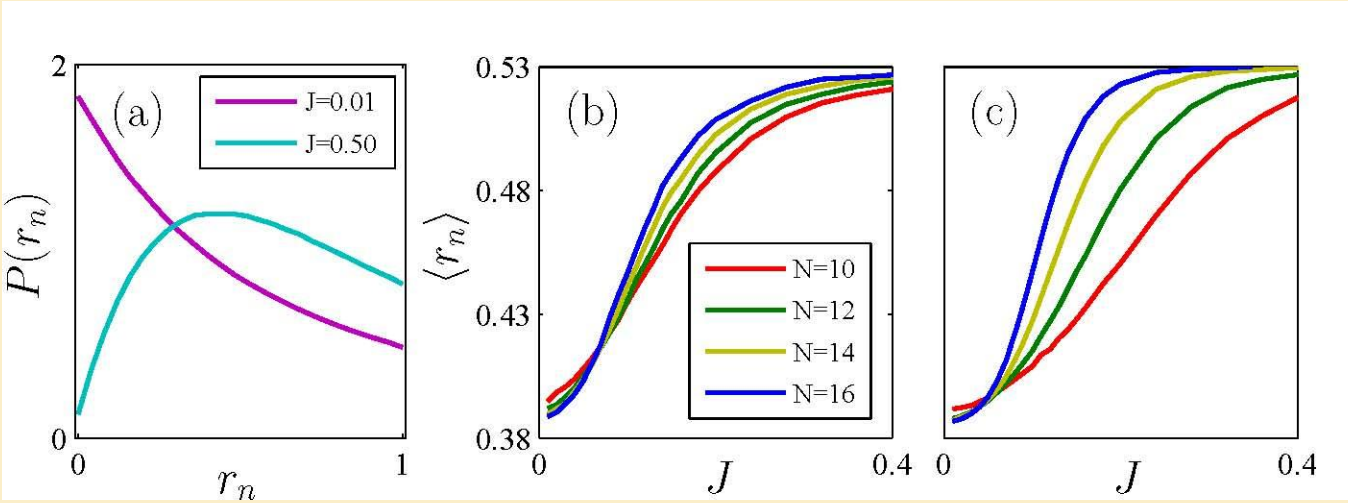
K=2

Transition on a finite random graph as evidences by the change in the spectral statistics

In a finite system one expects the level repulsion when $\Gamma(\varepsilon) \sim \textit{level spacing}$ with probability O(1). This happens at $g > g_{cr}(\varepsilon)$ (true transition in infinite system).

Conclusion: perfect agreement with the leading order recursive equations for $\Gamma(\varepsilon)$
No evidence for $\varepsilon_{cr} \sim N$

NUMERICAL EVIDENCE FOR BAD METAL

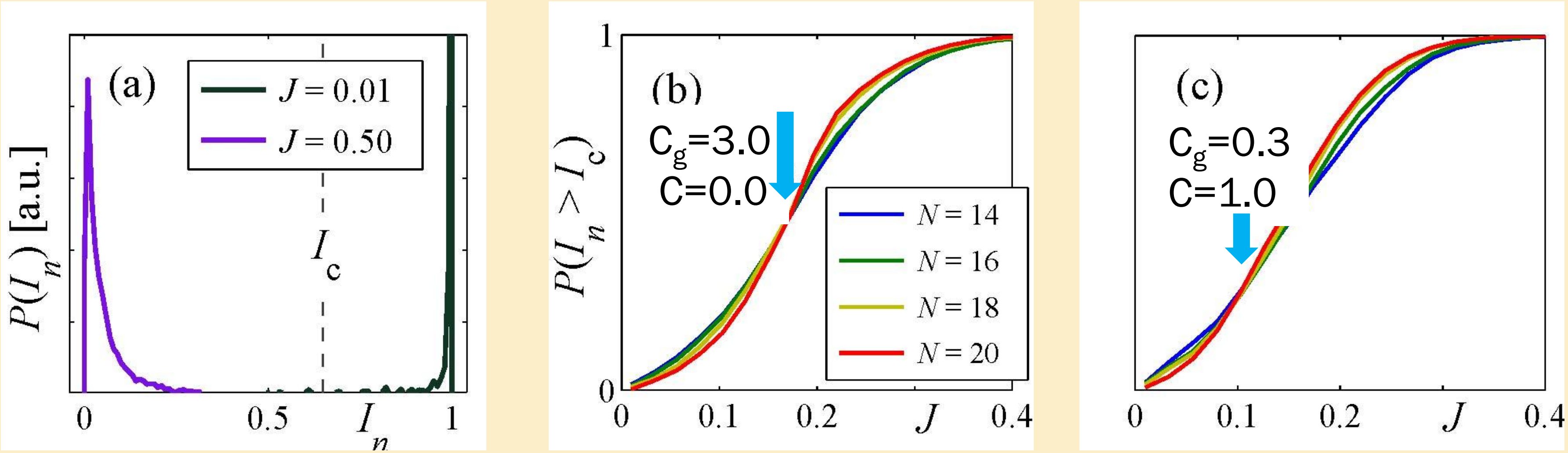


Level statistics in the presence of Coulomb interaction shows transition from coherent to incoherent phases

$J_d = 0.065$ for $E \in [1 \ldots 2]$ and $J_d = 0.037$ for $E \in [3 \ldots 4]$

TRANSITION FOR LOW ENERGY STATES

Practical problem: low energy states have low density of states + their properties vary with energy ⟶ poor statistics.
Resolution: check the low energy results by computing participation ratio.



Probability to find large participation ratio shows a well defined crossing point which coincides with the one found from level statistics at high energies (when the latter is accurate.

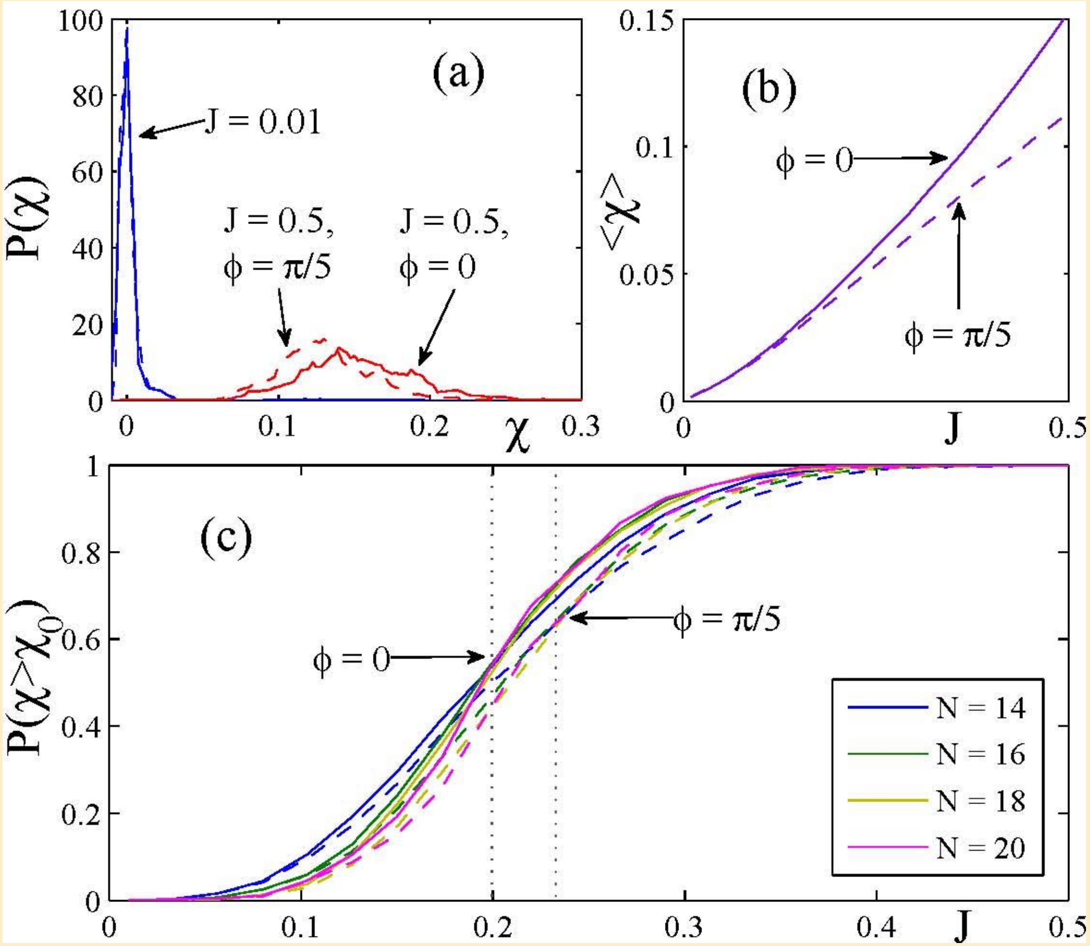
Conclusion: transition at very low energies happens at

$J_{\mathrm{d}} = 0.10$ for $E \to 0$ while $J_{\mathrm{d}} = 0.065$ for $E \in [1 \ldots 2]$ and $J_{\mathrm{d}} = 0.037$ for $E \in [3 \ldots 4]$

SUPERCONDUCTIVITY

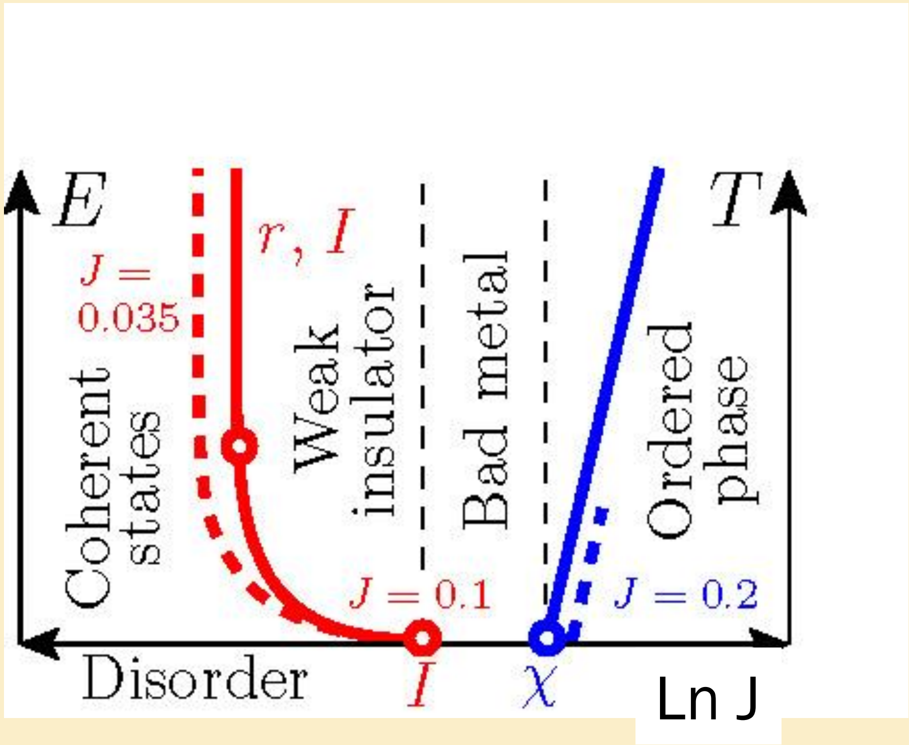
Compute susceptibility in the ground state

$$\chi = \frac{1}{N^2} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} \langle 0 | \hat{S}_i^- \hat{S}_j^+ | 0 \rangle$$



Susceptibility (normalized to spin) shows well defined transition at much larger J.

PHASE DIAGRAM (NUMERICAL SIMULATIONS)



Conclusion from numerical simulations:

The presence of Coulomb repulsion leads to the appearance of the intermediate phase between superconductor and coherent insulator!

The regime of 'bad metal' shrinks to zero as $C/C_g \rightarrow 0$

EVIDENCE FOR UNCONVENTIONAL GLASS.

WHAT IS THE STATE AT J=0?

Classical Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_{ij}^{-1}(q_i - Q_i)(q_j - Q_j)$$

$Q_i \in (0,1)$, $q_i = 0,1$ (one can show that larger variations of q_i are irrelevant)

$C_{ij} = C_0\delta_{ij} + CD_{ij}$ where D_{ij} is discrete Laplace operator

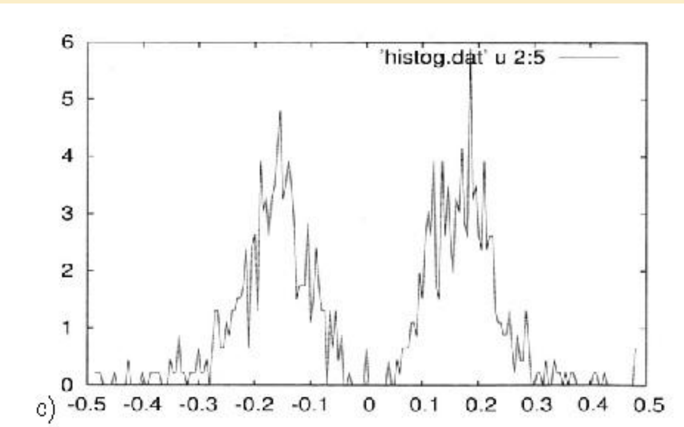
$C_0 \ll C \rightarrow$ long range interaction in finite d>1

Main question: is long range interaction and disorder sufficient to form a glass?
The interaction (inverse Laplace) is largest in d=1.
Q: What is the ground state in d=1 for C_0 =0?
A: Trivial because interaction becomes local in variables

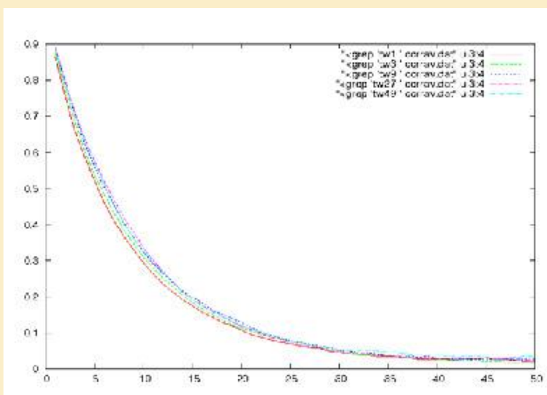
$$N_i = \sum_{j=0}^{i} Q_j \quad n_i = \sum_{j=0}^{i} q_j$$

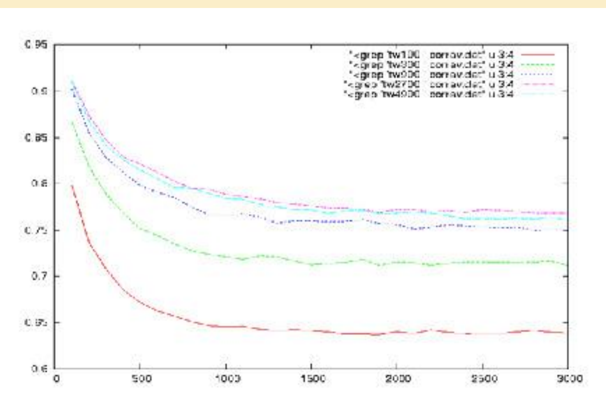
$$H = \frac{1}{2C}\sum_i (N_i - n_i)^2$$

DATA FOR D>1



Density of states shows a Coulomb gap
- as expected -
Compare to Sherrington-Kirkpatrick model where
$$\nu(E) \propto E$$





Fast relaxation of magnetization in d=1

Slow relaxation of magnetization in d>1
Does not obey any sort of known time dependence such as ageing

SUMMARY

1. Experimental data indicate that an intermediate phase might be formed between the superfluid and insulating Josephson arrays as a function of frustration or E_J/E_C ratio.
2. In the intermediate phase the resistance is weakly (if it all) temperature dependent, certainly the activation gap becomes zero. Current voltage characteristics in this regime are featureless at low current or voltages.
3. Numerical and analytical solutions for the models on random graph show that such phase does not appear in the absence of Coulomb interaction.
4. Numerical data show that a wide regime in which the activation gap is zero but the superconductivity is absent might appear in the presence of Coulomb interaction.
5. It is likely that the existence of the intermediate phase is due to the glassy nature of the classical state formed by Coulomb interaction.
6. There is no real theory…