MINIMAL DEFECT PRODUCTION PAST A QUANTUM CRITICAL POINT

Rosario Fazio

Scuola Normale Superiore, Pisa - ITALY
&
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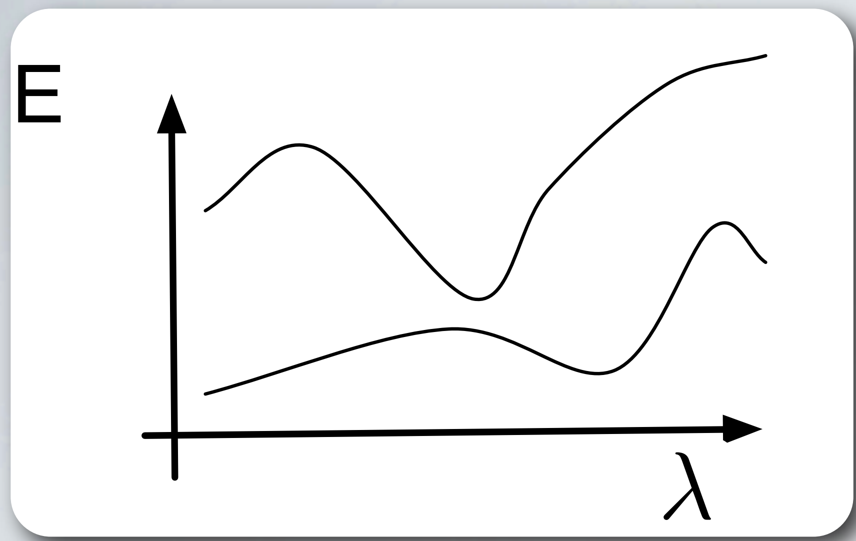
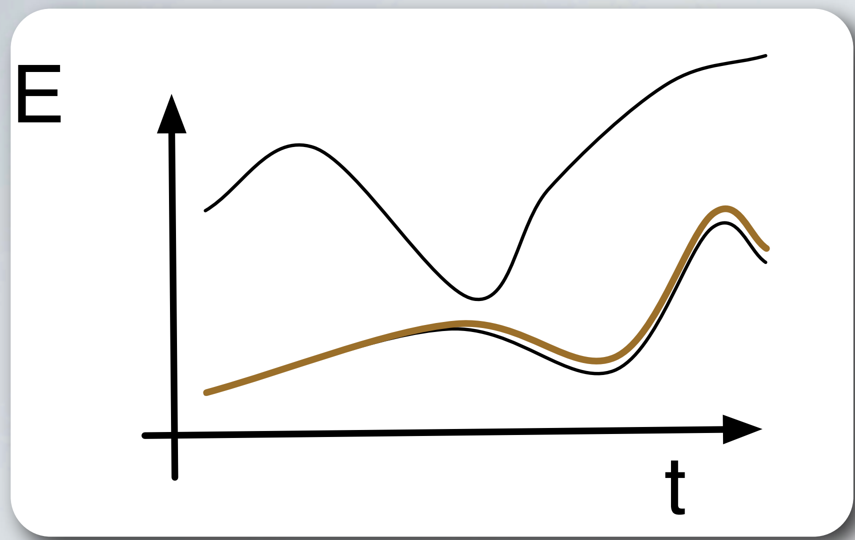


ADIABATIC DYNAMICS FOR A CONTROLLED EVOLUTION OF MANY-BODY SYSTEMS



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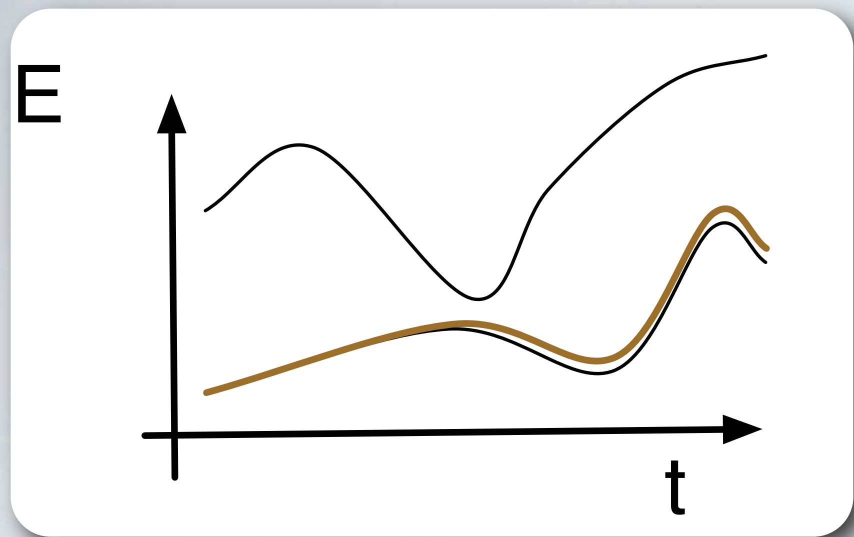
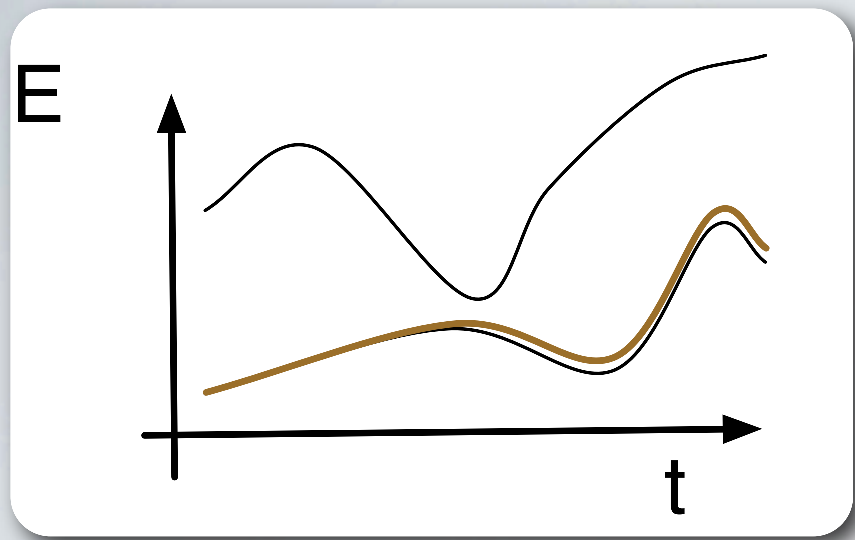


ADIABATIC DYNAMICS FOR A CONTROLLED EVOLUTION OF MANY-BODY SYSTEMS

- State Preparation (loading atoms in optical lattices)



ADIABATIC DYNAMICS FOR A CONTROLLED EVOLUTION OF MANY-BODY SYSTEMS

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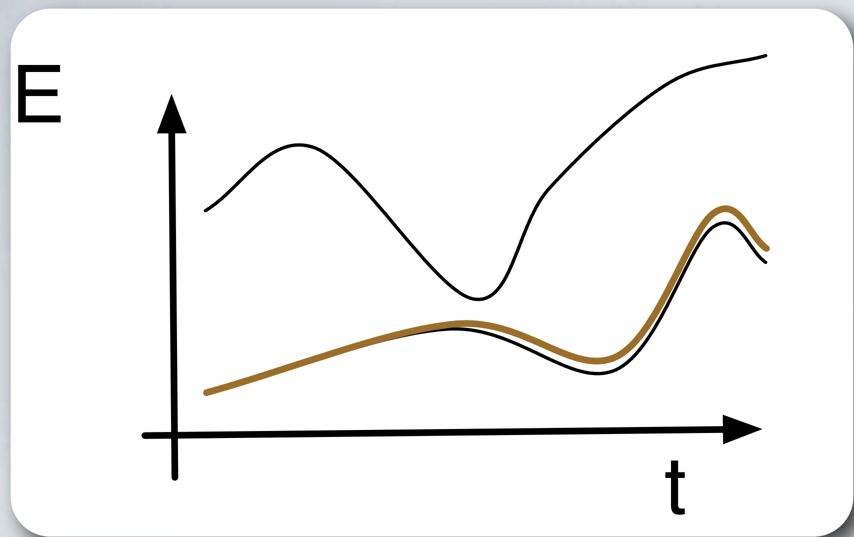
- Adiabatic Quantum Computation



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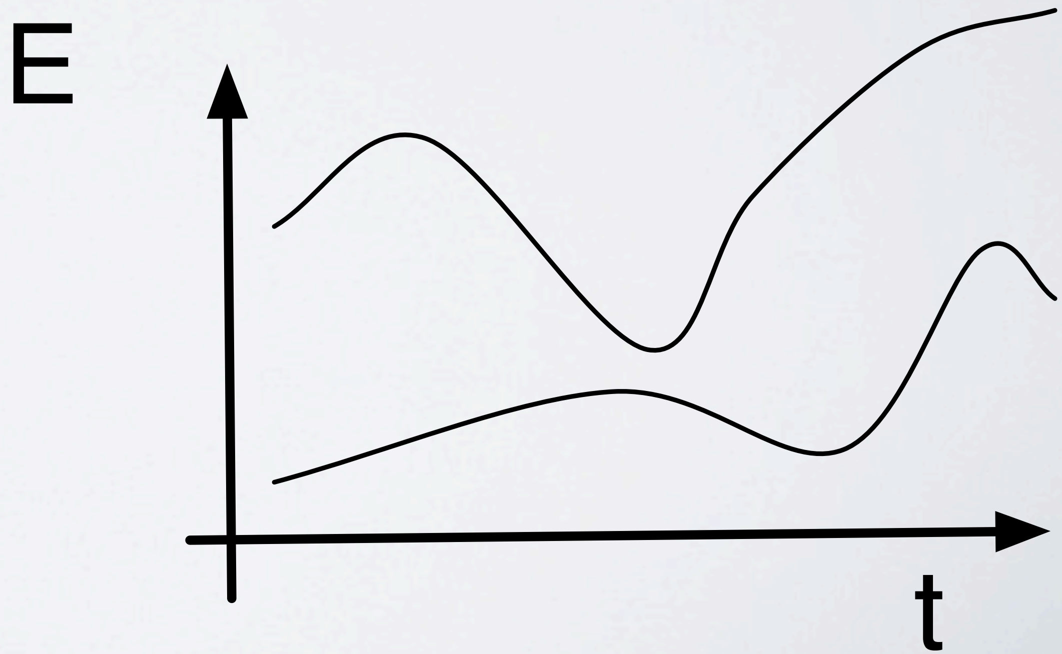

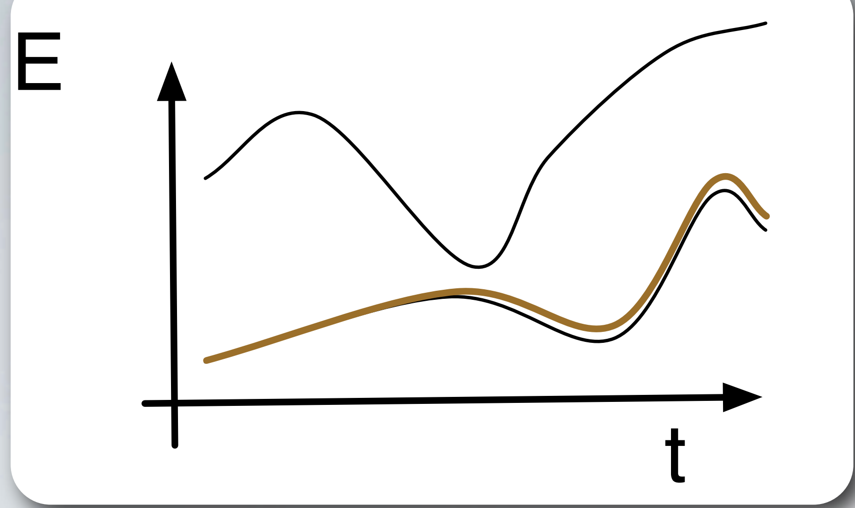

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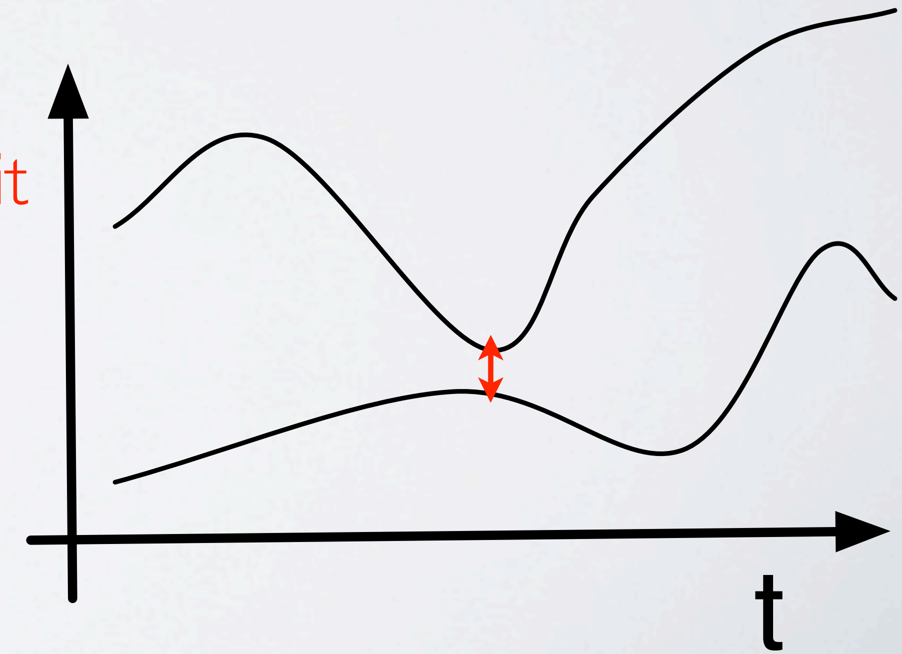
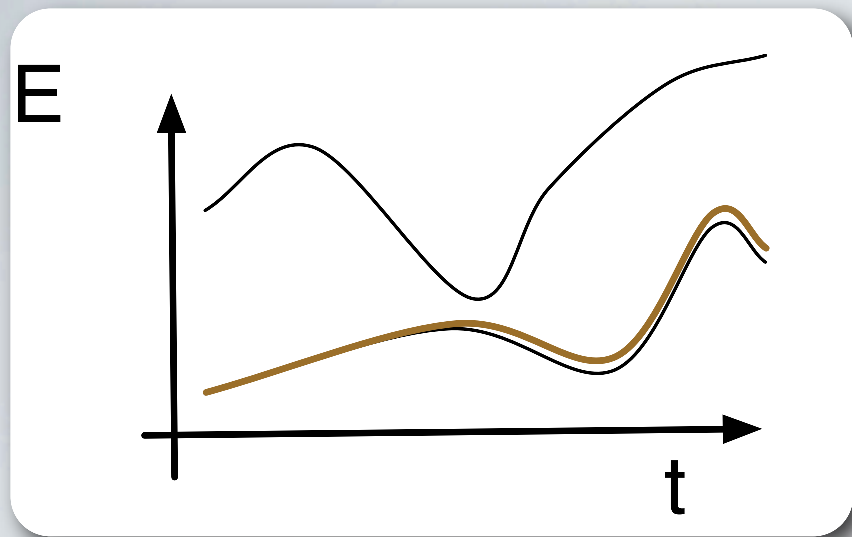
Gaps close in the thermodynamic limit





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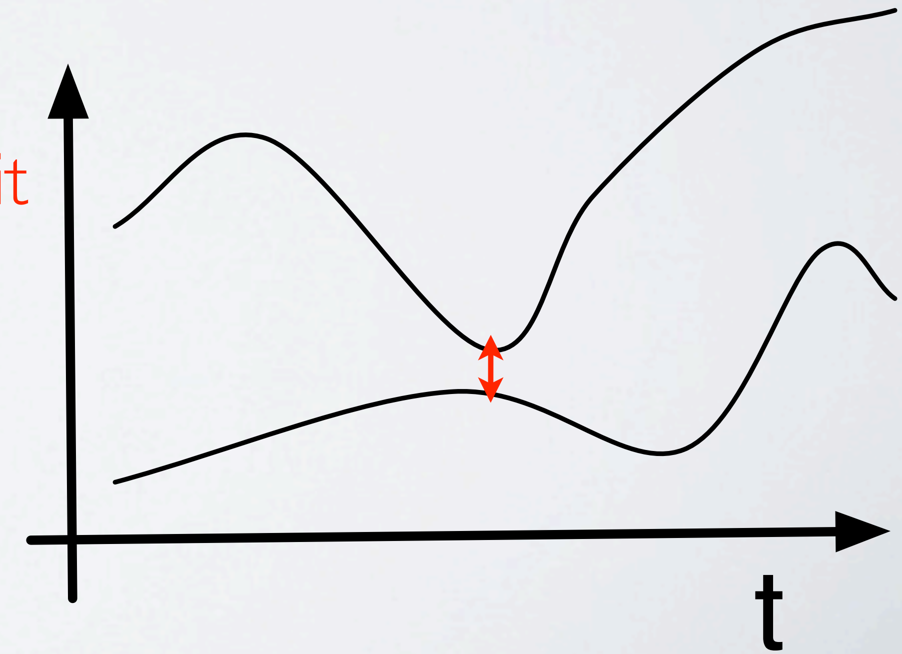

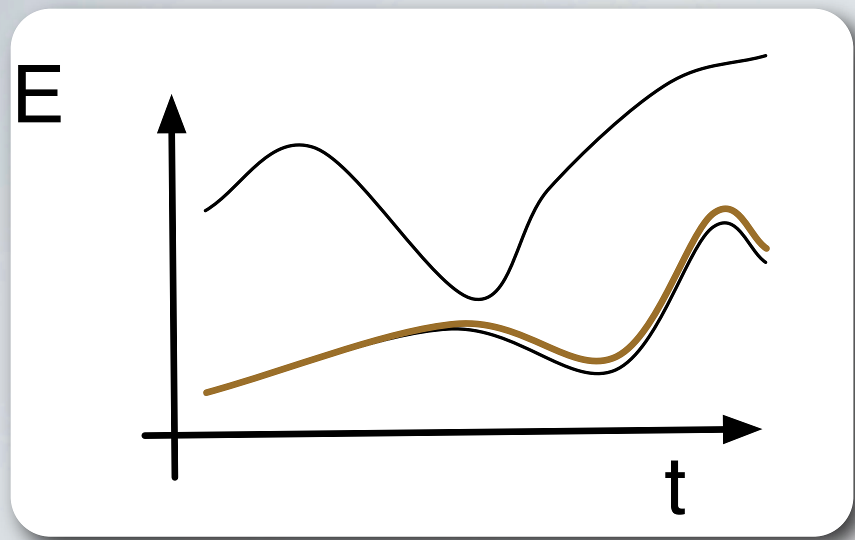

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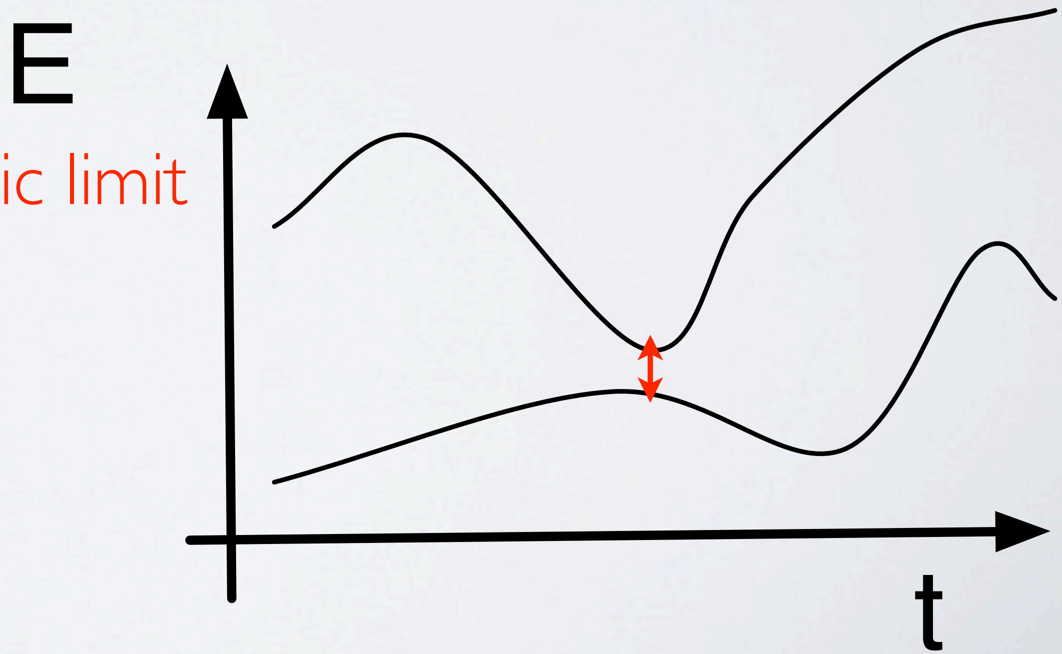
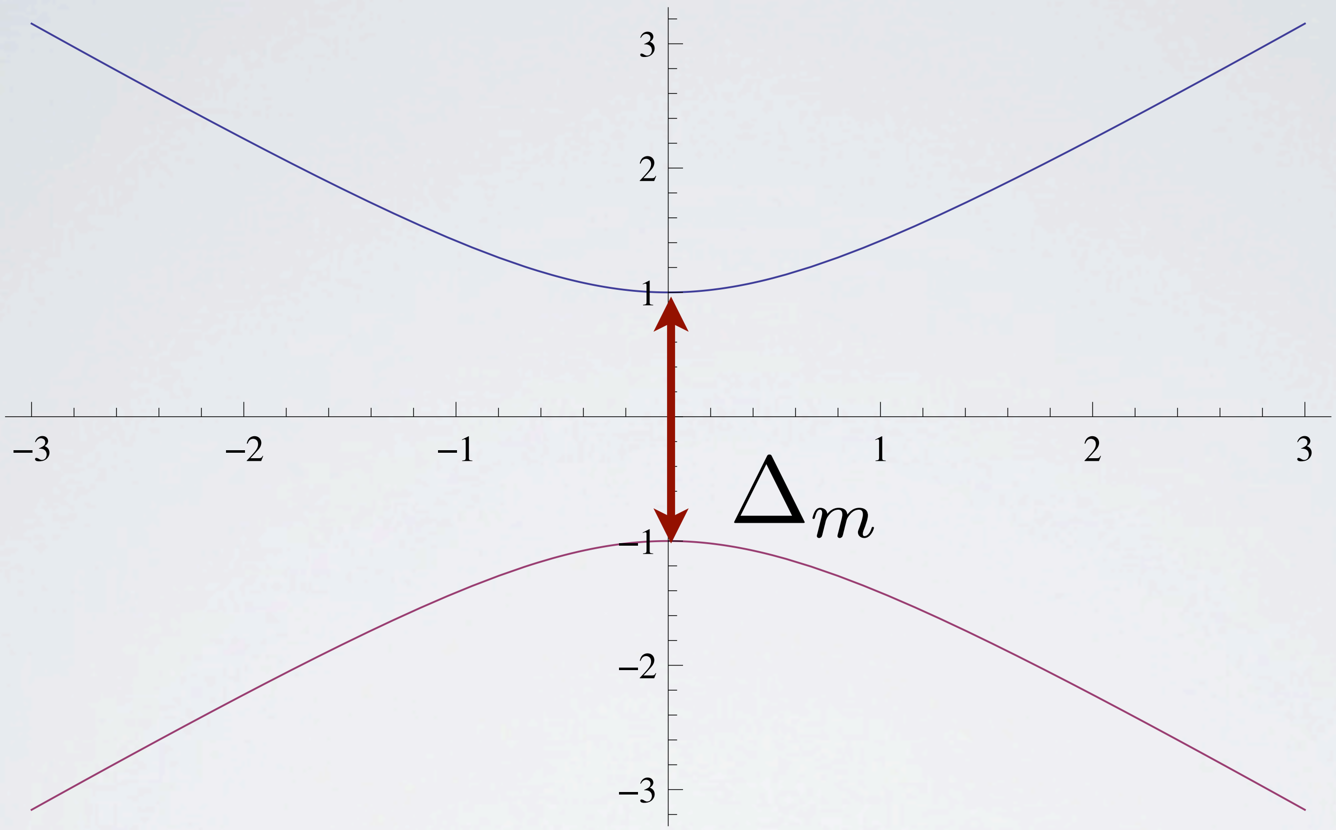
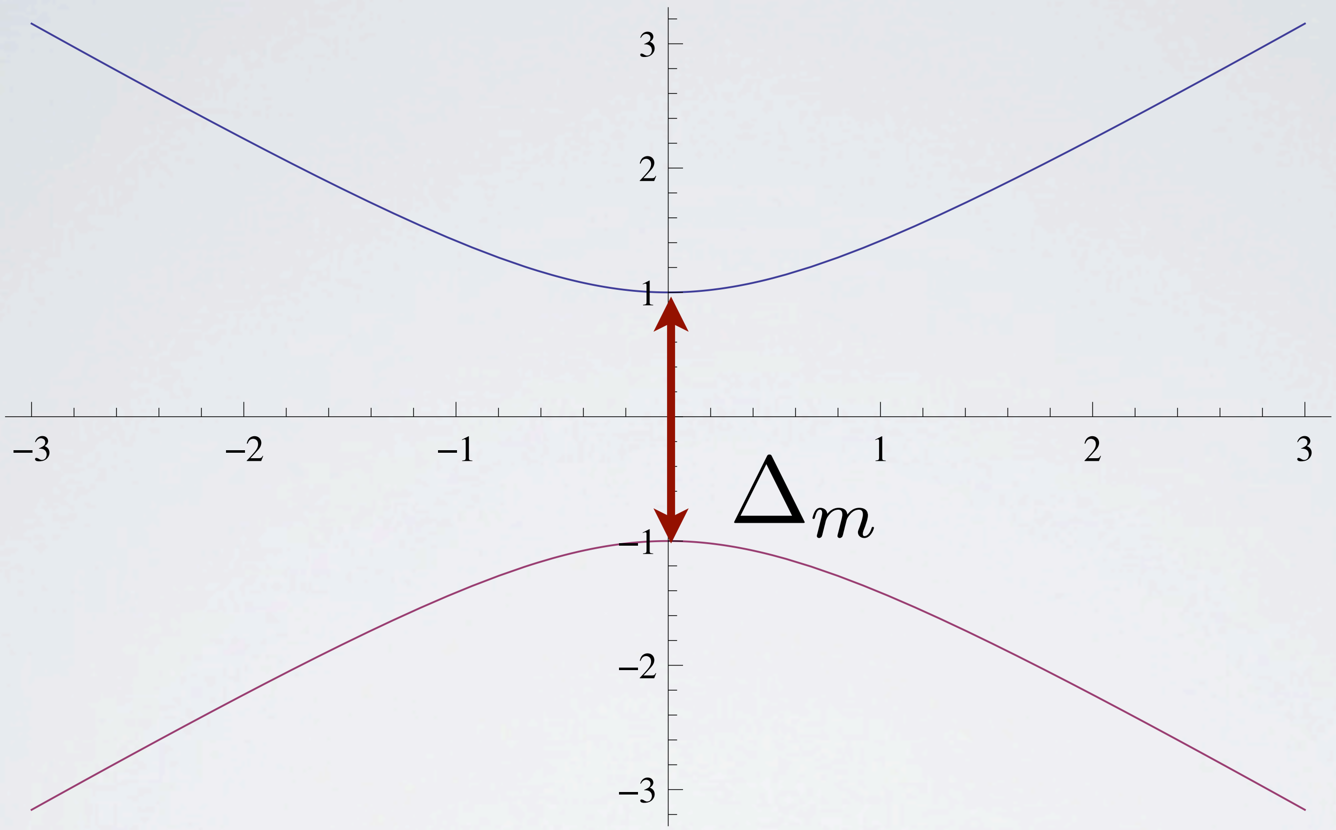
↓

Defect production

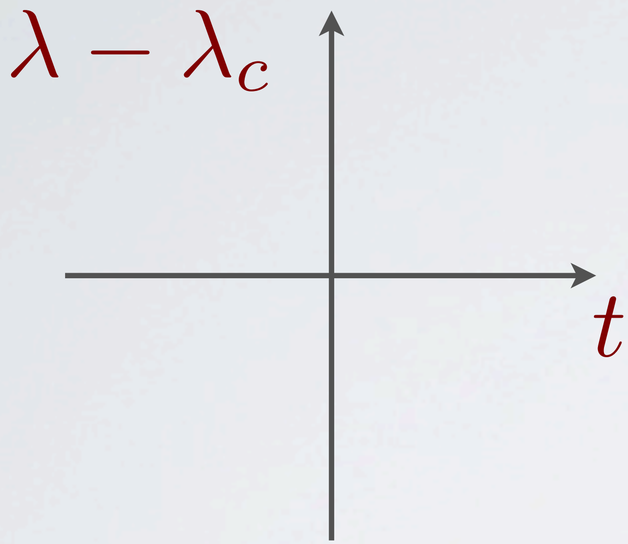



3
2
1
−1
−2
−3
−3
−2
−1
1
2
3
Δ_m



$$\Delta_m \sim \frac{1}{N^\eta}$$

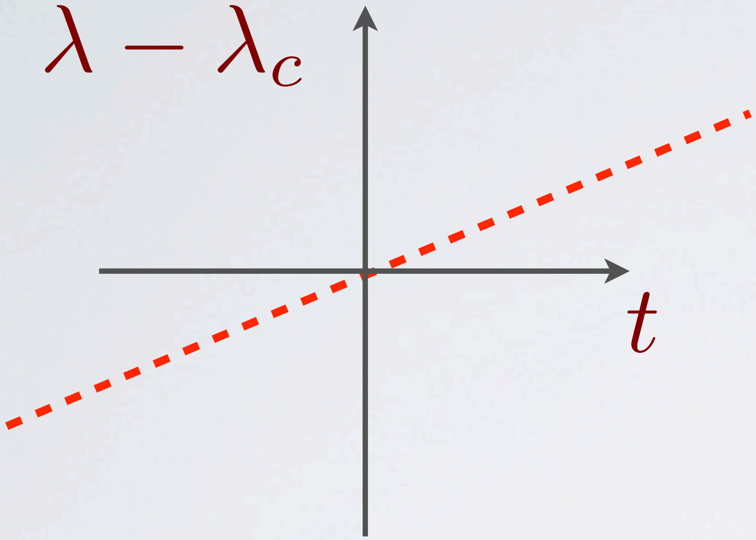
ADIABATIC DYNAMICS AND QPTs



$$\lambda - \lambda_c = vt$$

– W. Zurek, U. Dorner and P. Zoller (2005)
– A. Polkovnikov (2005)

ADIABATIC DYNAMICS AND QPTs



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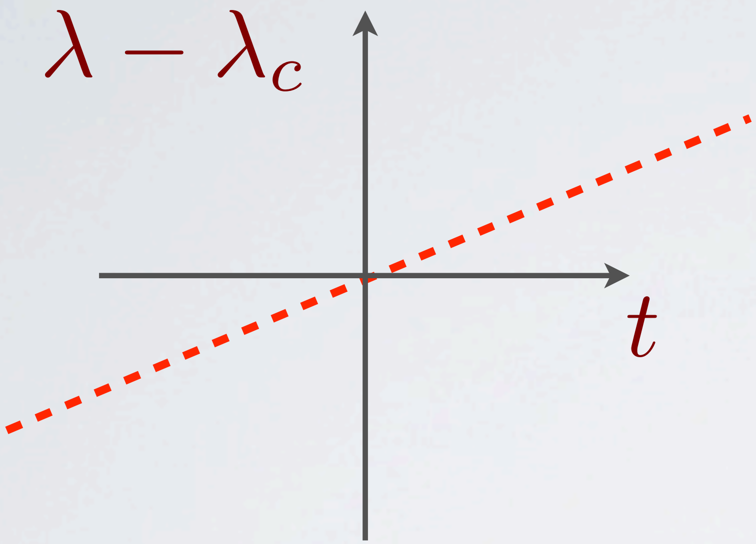
ADIABATIC DYNAMICS AND QPTs



$$\lambda - \lambda_c = vt$$

$$\Delta E \sim v^{\frac{d\nu}{z\nu+1}}$$

– W. Zurek, U. Dorner and P. Zoller (2005)
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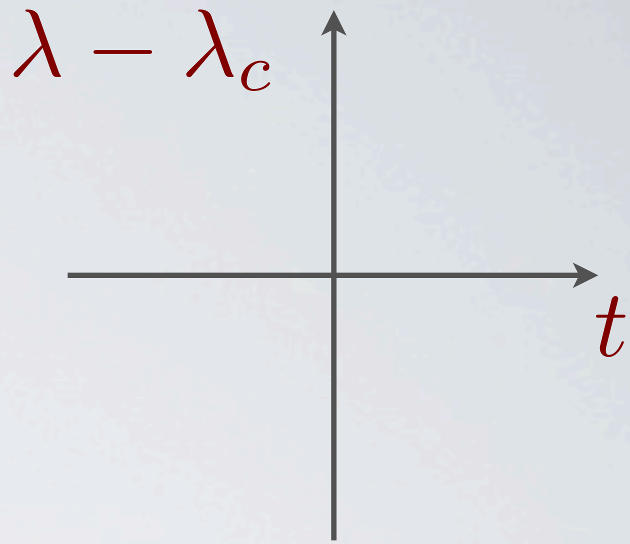
Is it possible to minimize the defect production on crossing a phase transition?

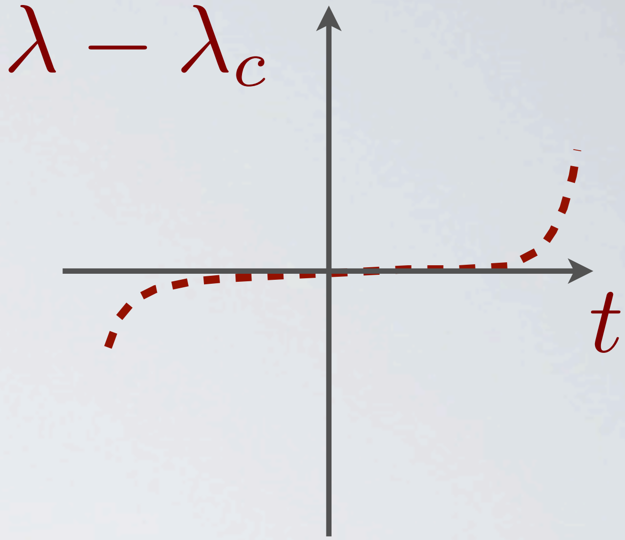
NON-LINEAR SWEEPS



$$\lambda - \lambda_c \sim |\, vt \,|^\alpha \, \mathrm{sign}(t)$$

– R. Barankov and A. Polkovnikov (2008)
– D. Sen, K. Sengupta, and S. Mondal (2008)

NON-LINEAR SWEEPS



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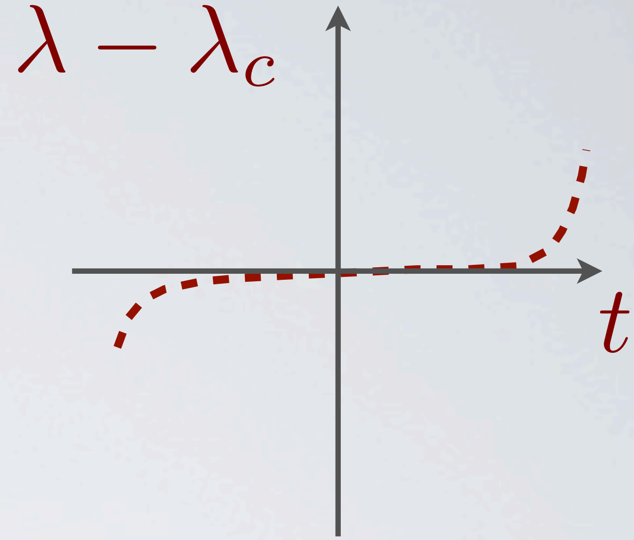

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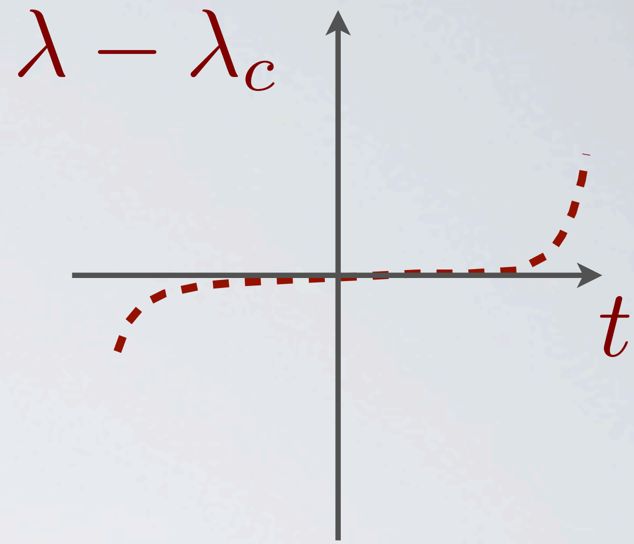


$$\lambda - \lambda_c \sim |\, vt \,|^{\alpha} \, \mathrm{sign}(t)$$

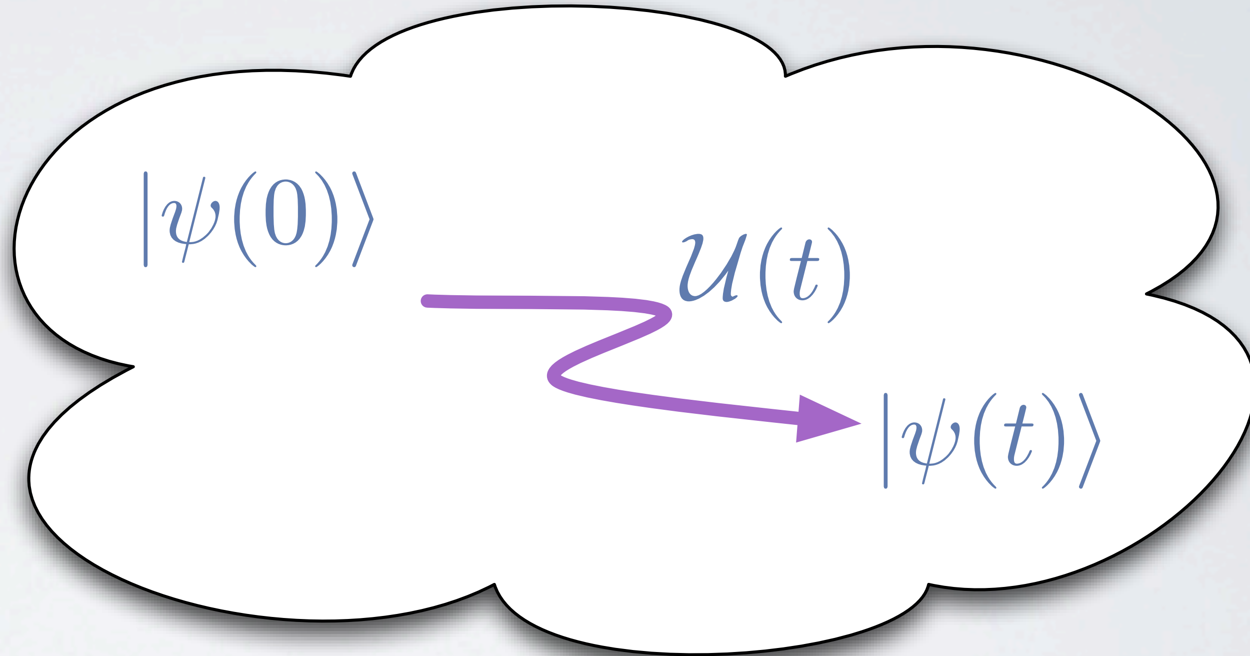
- Excess energy: $\Delta E \sim v^{\frac{\alpha d \nu}{\alpha z \nu + 1}}$

- Given a total passage time there is an optimal value of the non-linearity which depends on T and on the critical exponents of the transitions

– R. Barankov and A. Polkovnikov (2008)
– D. Sen, K. Sengupta, and S. Mondal (2008)

Thursday, August 23, 2012

Optimal control applied to defect formation across a QPT



– T. Caneva, T. Calarco, R. F., G.E. Santoro, and S. Montangero (2011)

OPTIMAL CONTROL

$|\psi_0\rangle$

$$\mathcal{H} = \mathcal{H}(\{d_j(t)\}, t)$$

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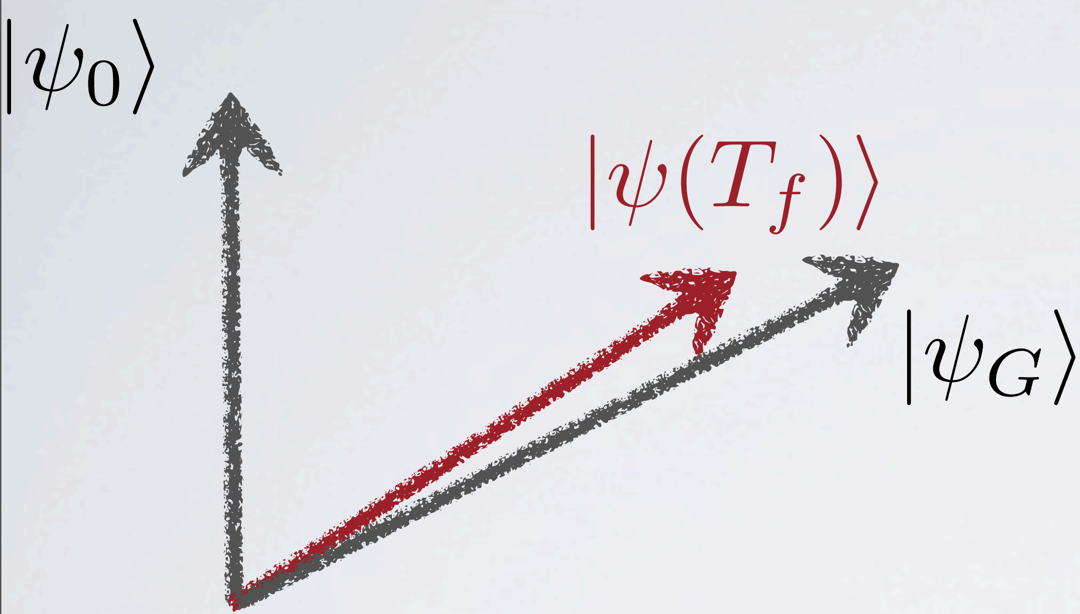
OPTIMAL CONTROL



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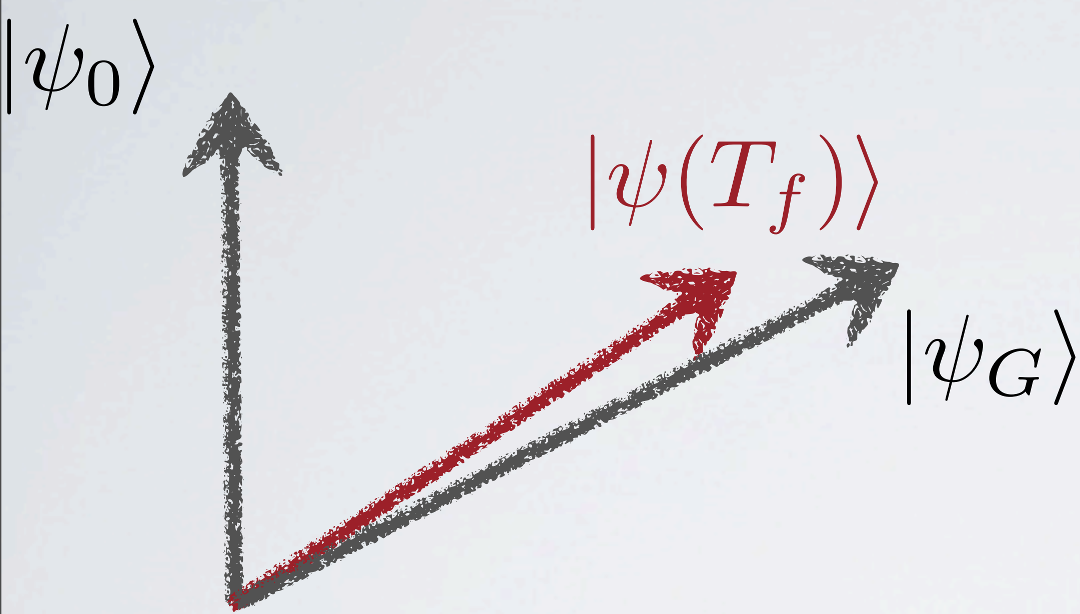
OPTIMAL CONTROL



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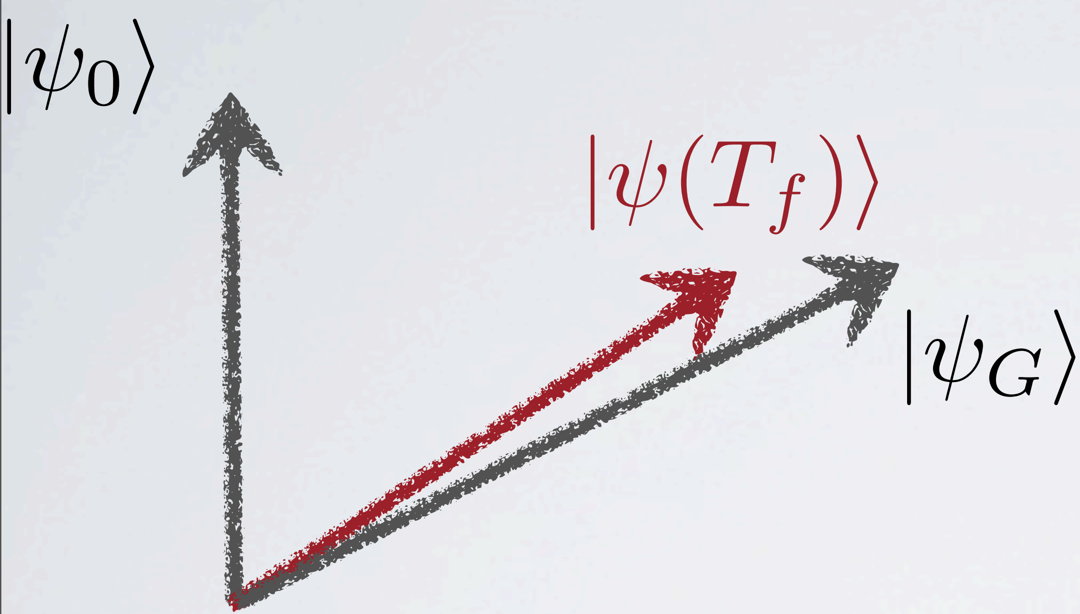
Pulse sequence?

OPTIMAL CONTROL



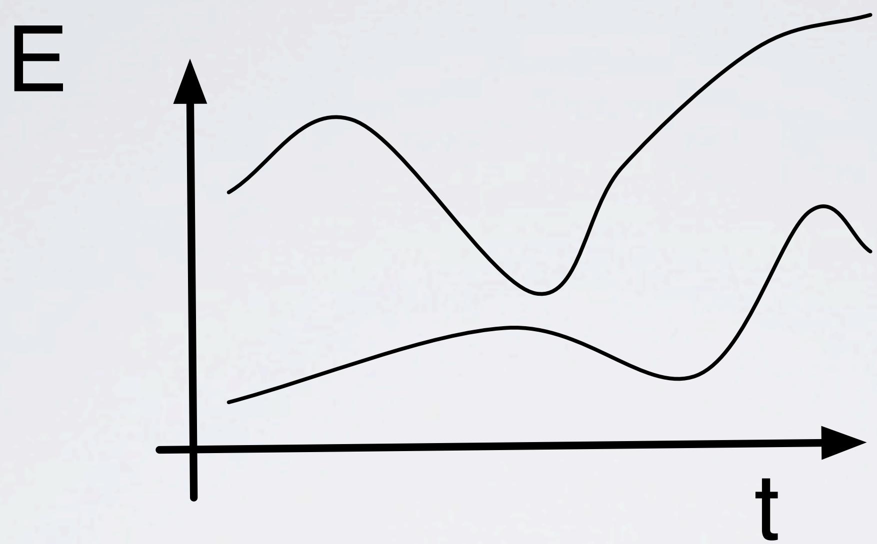
Maximal overlap:

$$\mathcal{F} = |\langle\psi_G|\psi(T_f)\rangle|^2$$

$$\mathcal{H} = \mathcal{H}(\{d_j(t)\}, t)$$

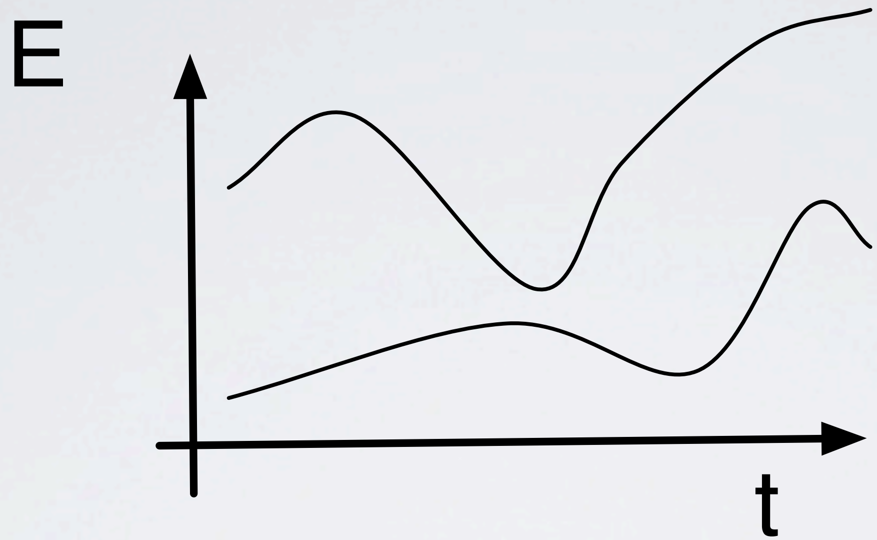
Pulse sequence?

OPTIMAL DYNAMICS: A CARTOON



OPTIMAL DYNAMICS: A CARTOON

Slow



Adiabatic strategy

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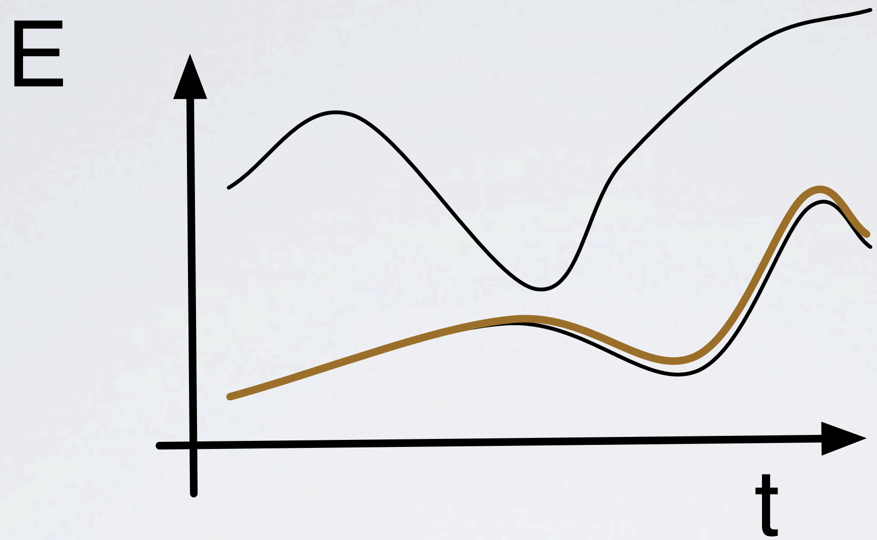

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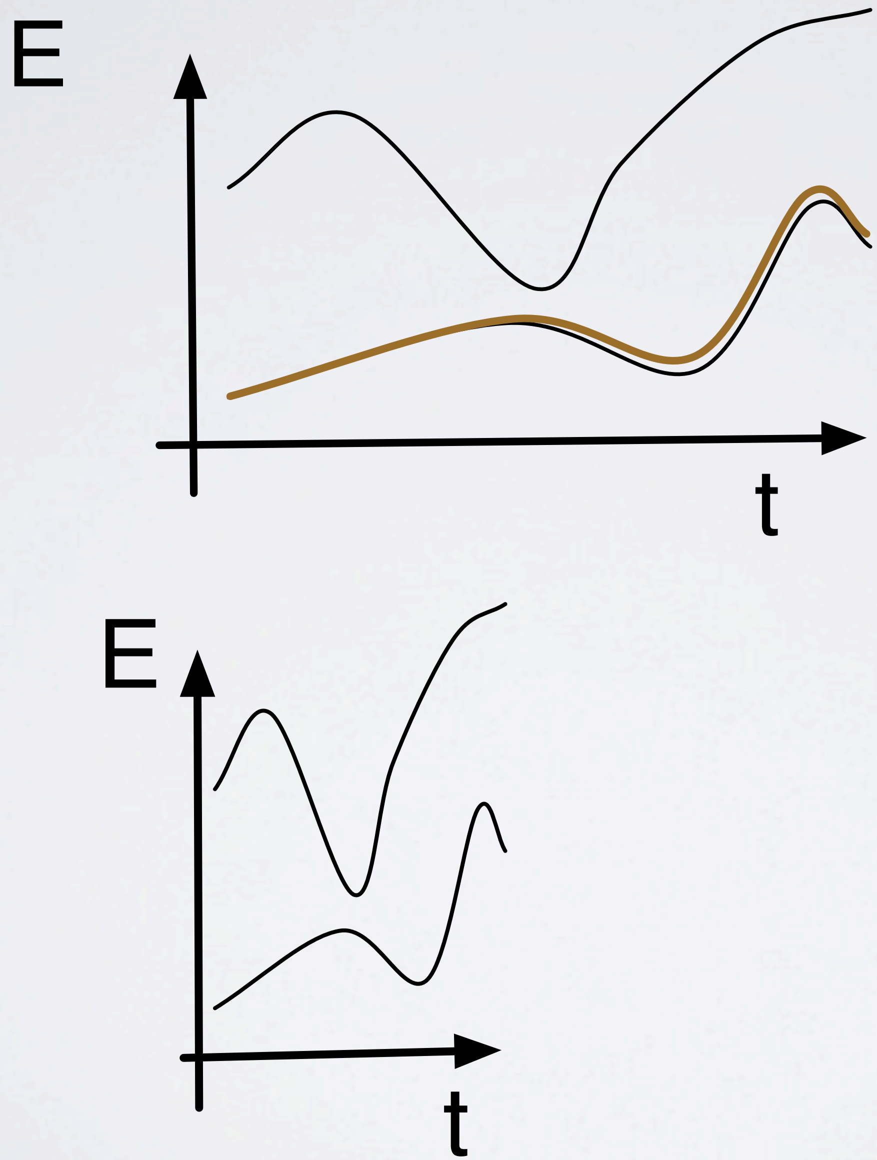

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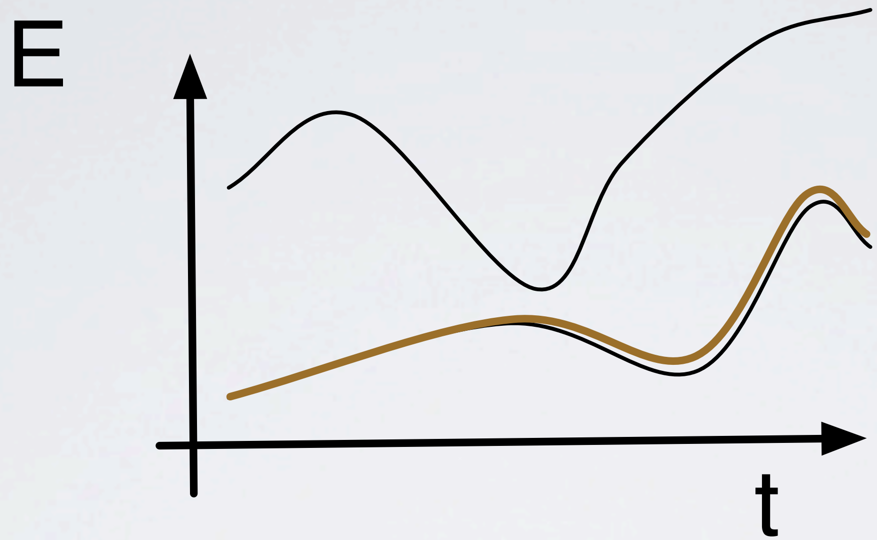
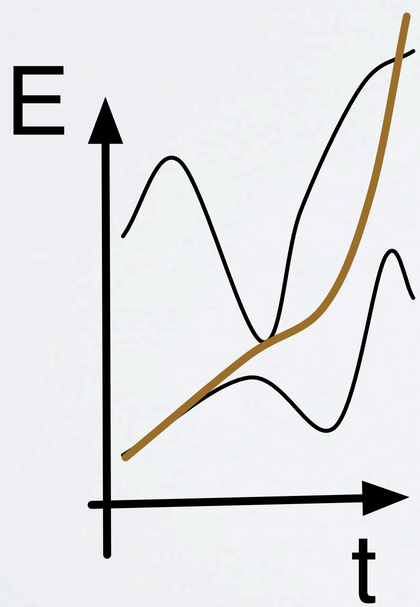


Adiabatic strategy

Fast



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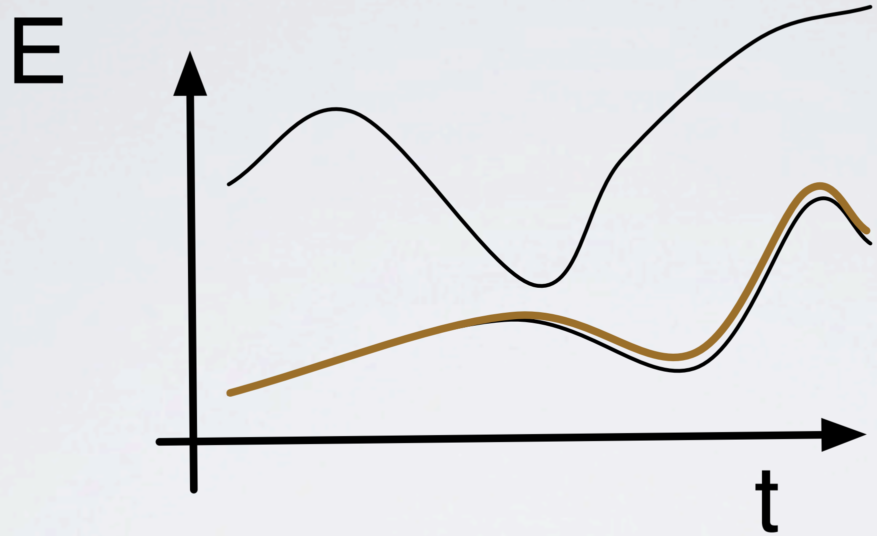

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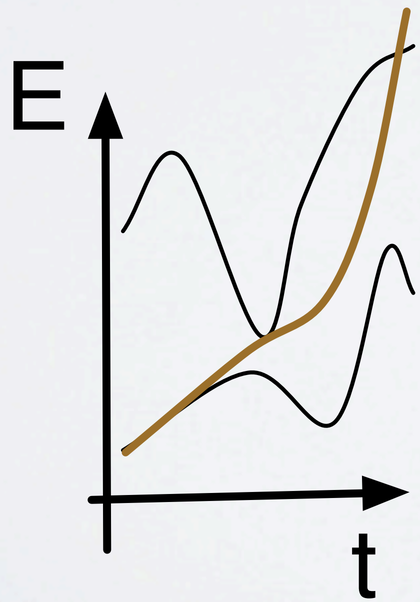

Optimal control

OPTIMAL DYNAMICS: A CARTOON

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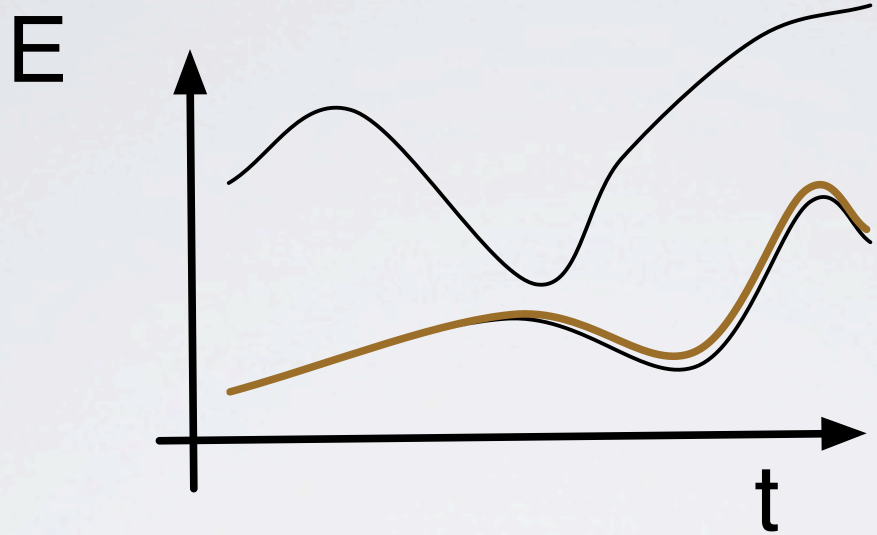
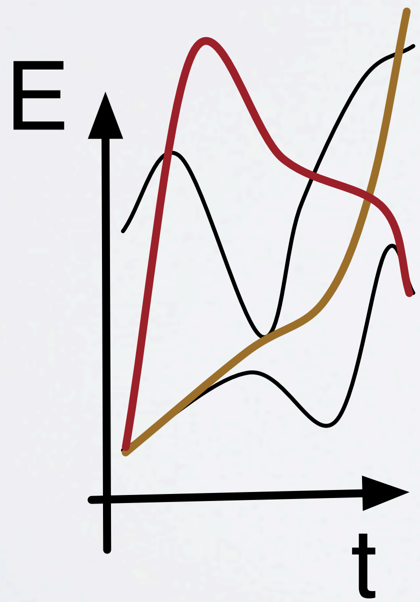


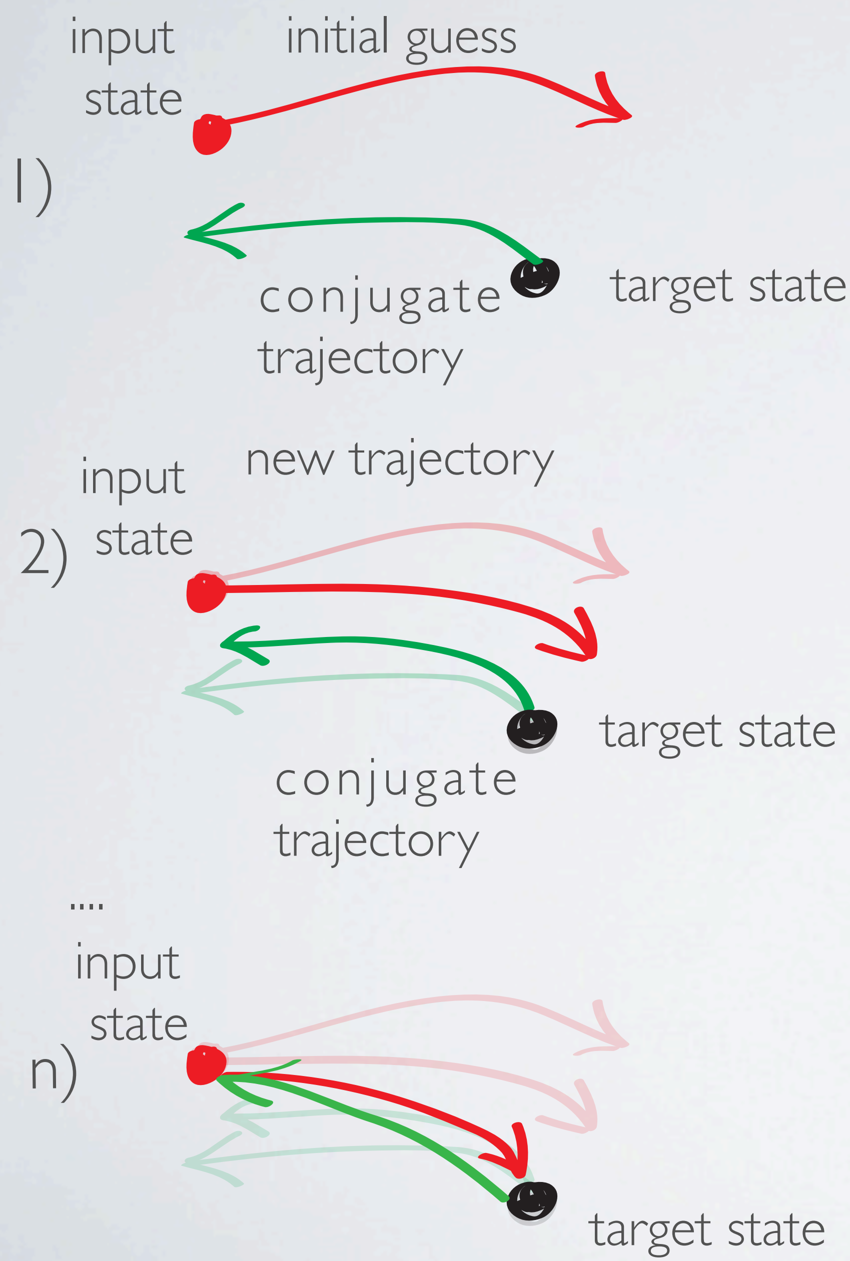
Adiabatic strategy

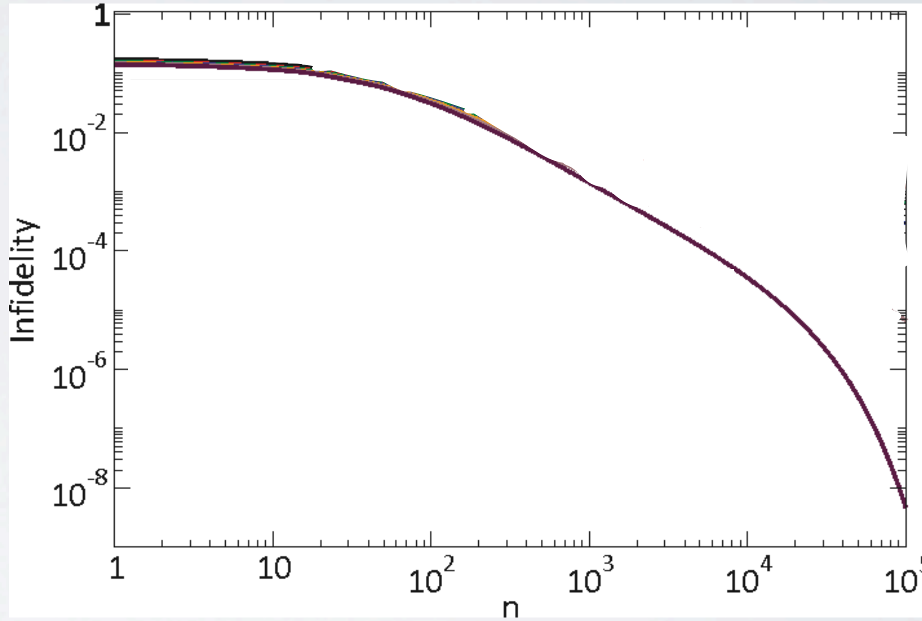
Fast



Optimal control

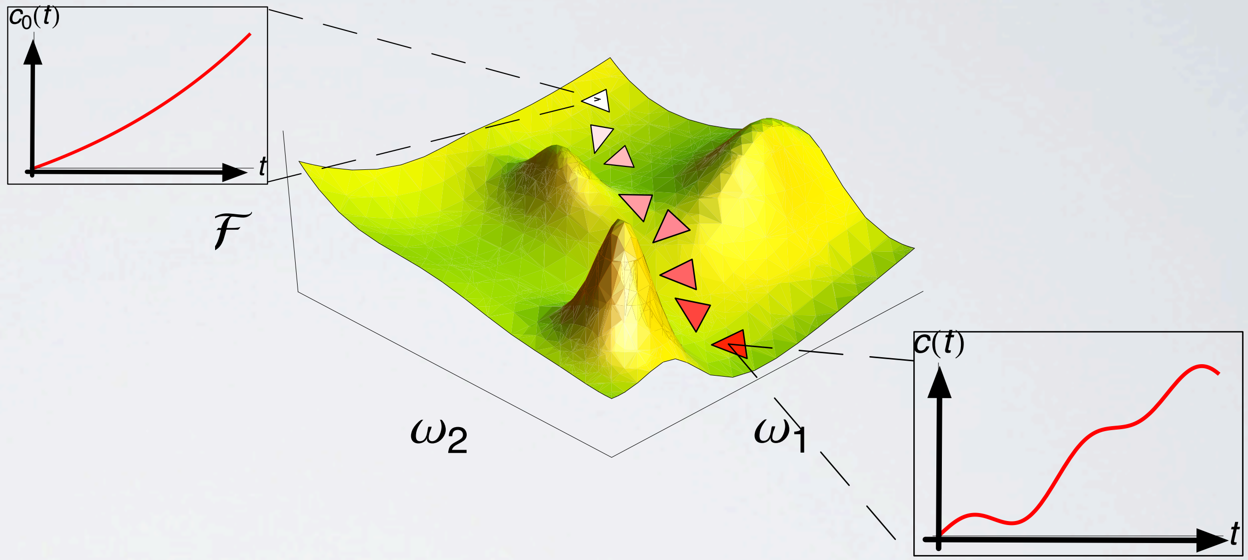
KROTOV ALGORITHM

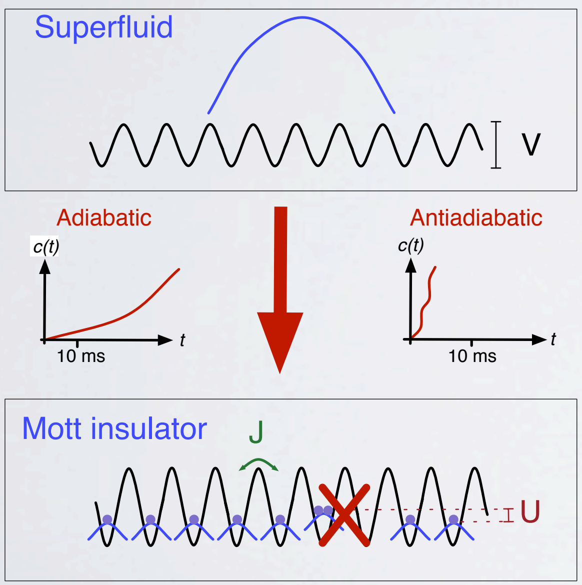




number of iterations
of the algorithm

CHOPPED RANDOM BASIS (CRAB)





J/U >> 0.1

J/U << 0.1

– P. Doria, T. Calarco, and S. Montangero (2011)

THE MODELS

THE MODELS

GROVER SEARCH (GSA)

$$\mathcal{H} = [1 - \Gamma(t)](1 - |s\rangle\langle s|) + \Gamma(t)(1 - |\bar{\psi}\rangle\langle\bar{\psi}|)$$

$|s\rangle$ completely symmetric superposition

THE MODELS

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LIPKIN- MESHKOV-GLICK (LMG)

$$\mathcal{H} = -\frac{1}{N}\sum_{i<j}\sigma_i^x\sigma_j^x - \Gamma(t)\sigma_i^z$$

THE MODELS

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$$\mathcal{H} = -\frac{1}{N}\sum_{i<j}\sigma_i^x\sigma_j^x - \Gamma(t)\sigma_i^z$$

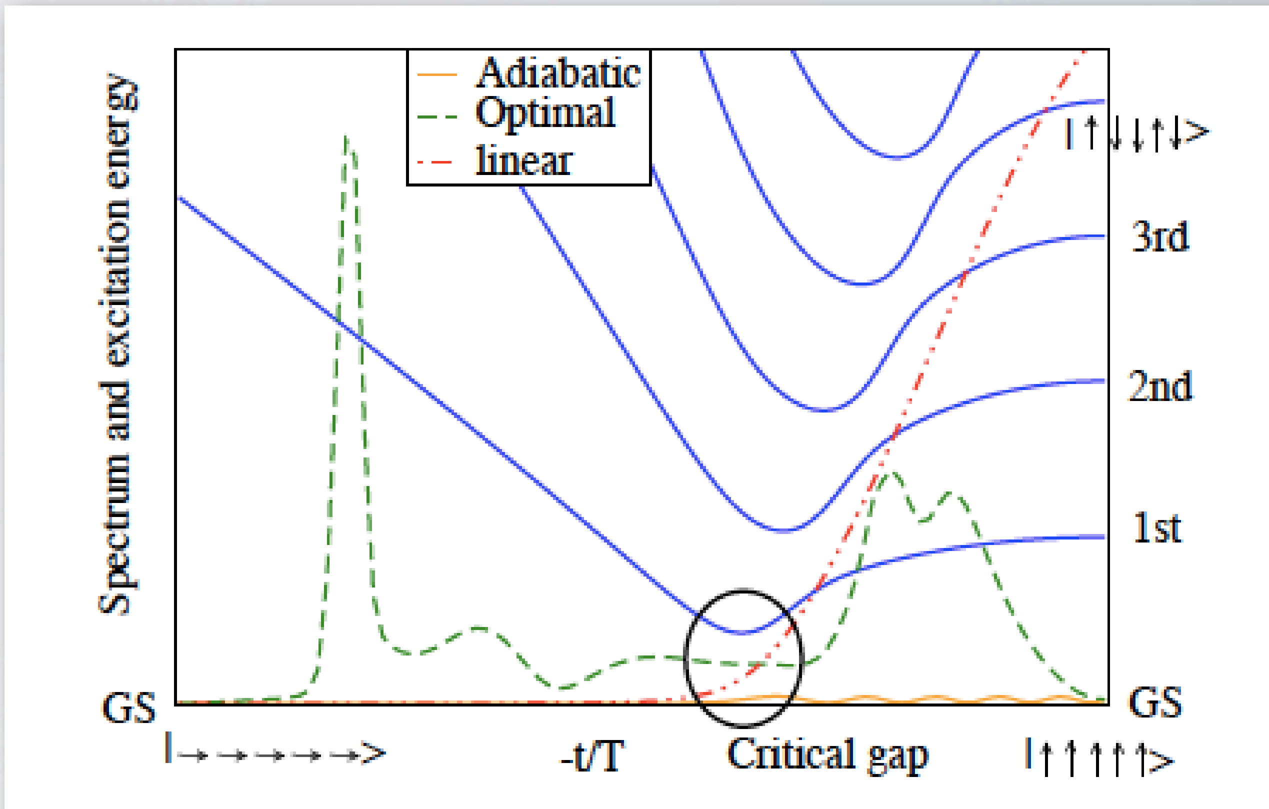
LANDAU-ZENER (LZ)

$$\mathcal{H} = \Gamma(t)\sigma^z + \omega\sigma^x$$

INFIDELITY

$$I(T) = 1 - |\langle \psi_G | \psi(T) \rangle|^2$$

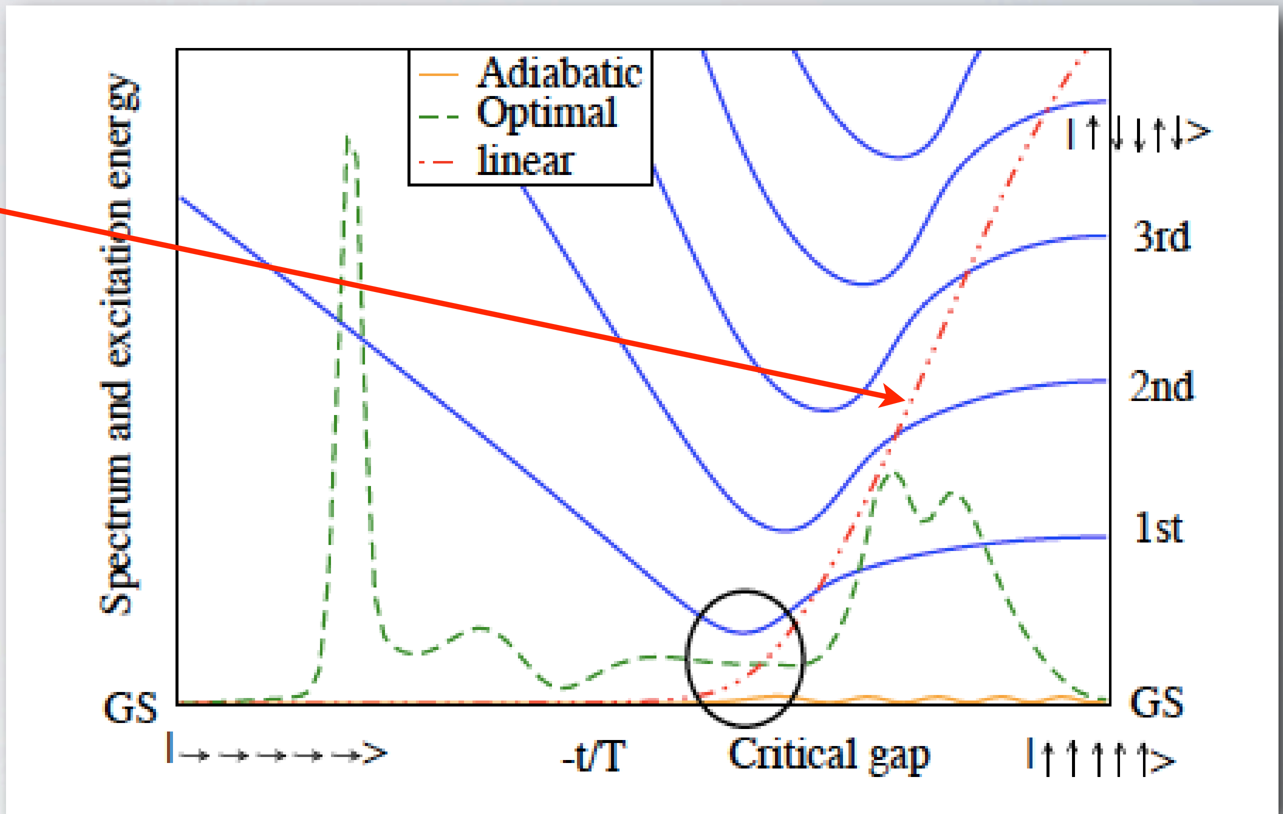
LMG MODEL



Adiabatic
Optimal
linear
Spectrum and excitation energy
GS
|→→→→→>
-t/T
Critical gap
|↑↑↑↑↑>
GS
1st
2nd
3rd
|↑↓↓↑↓>

LMG MODEL

Linear quench for a "short" time T



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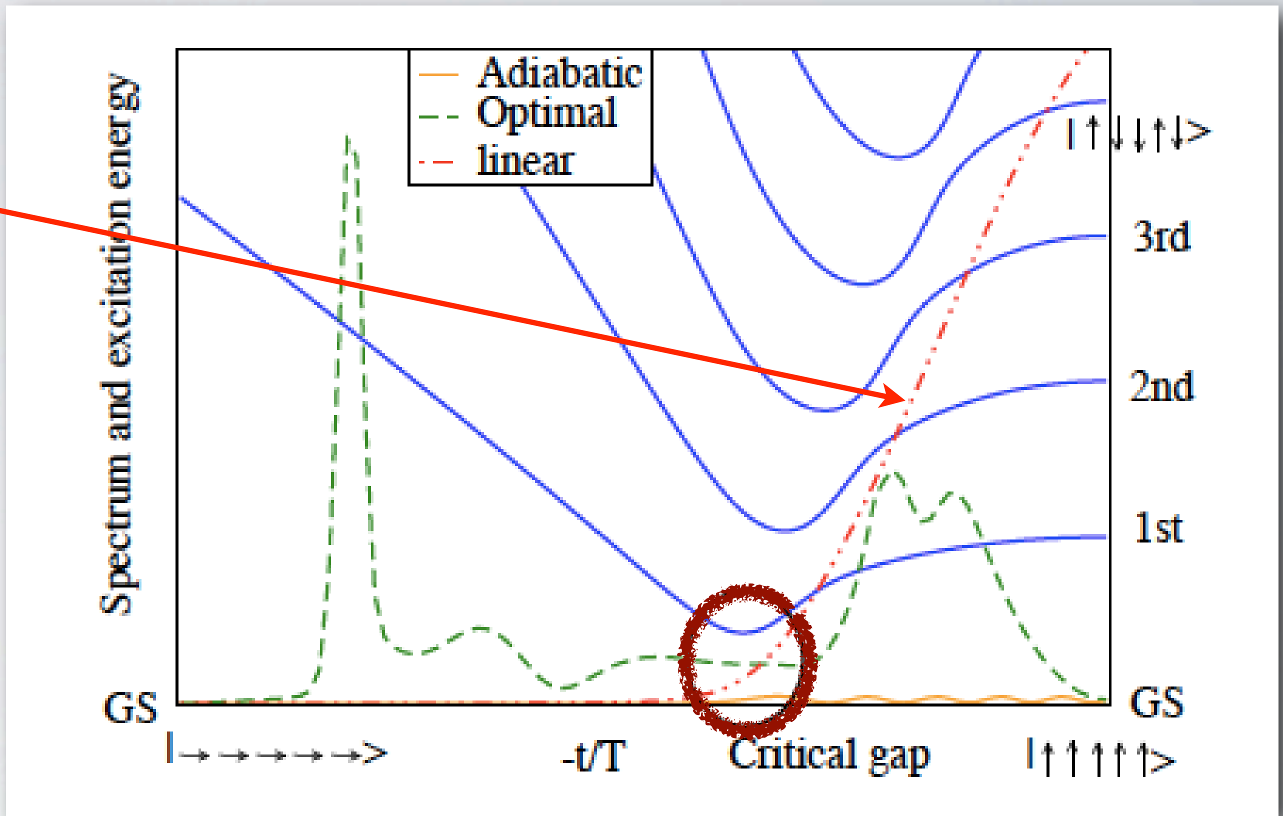
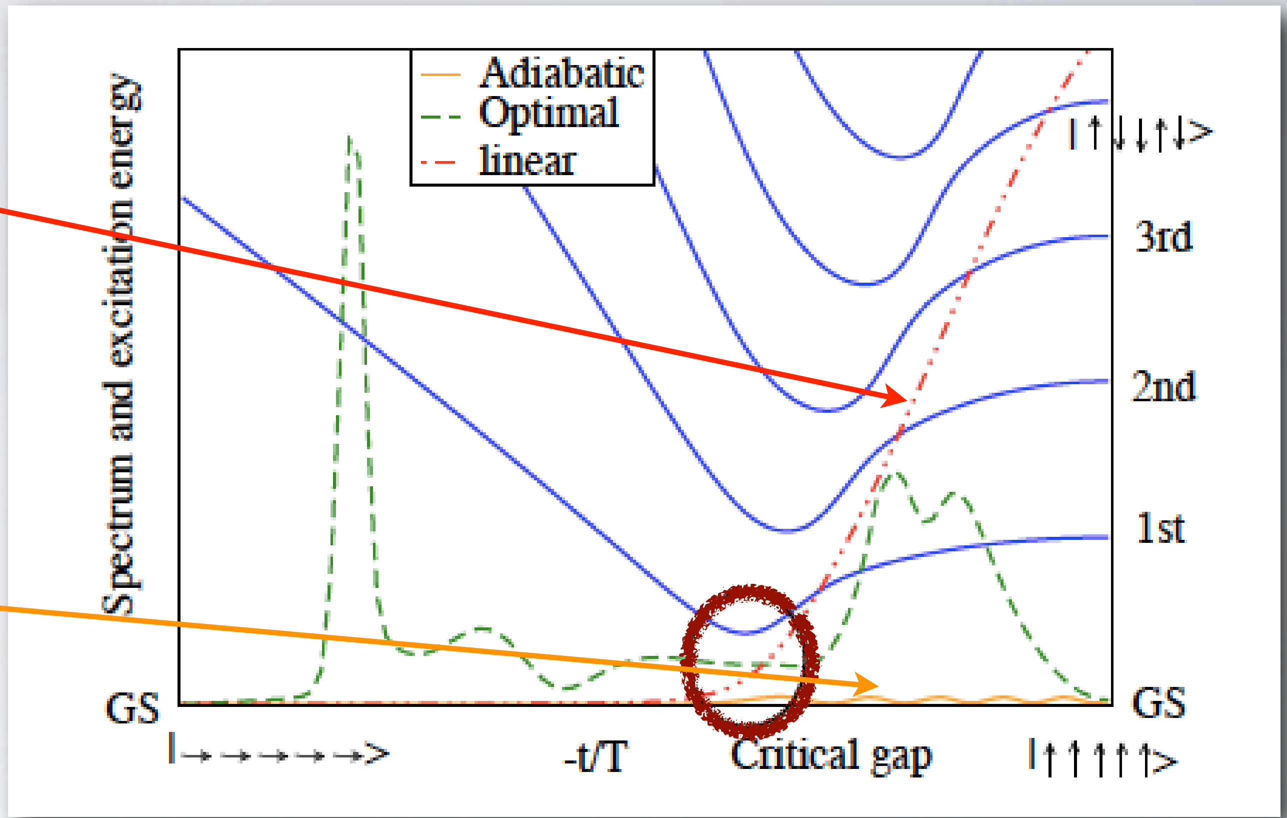


LMG MODEL

Linear quench for a "short" time T

Linear quench in the adiabatic limit $T \sim T_{ad}$

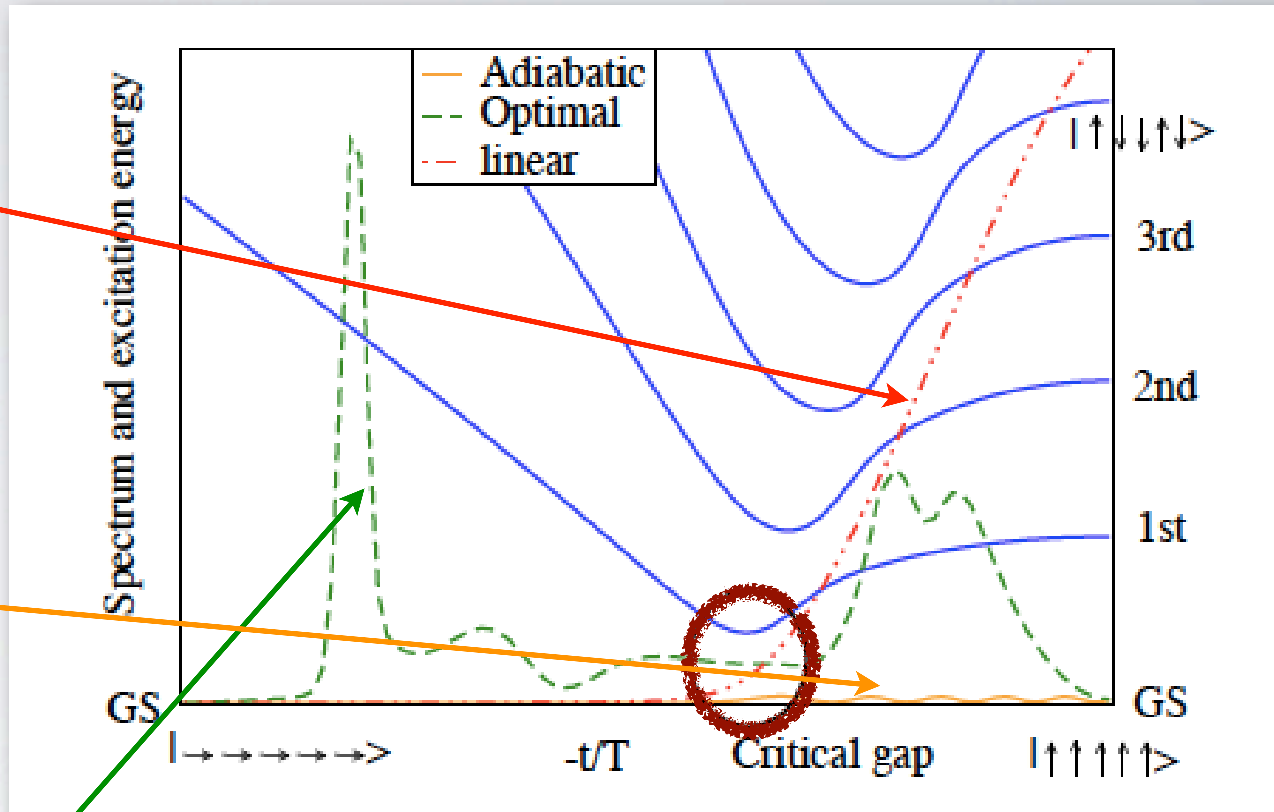


LMG MODEL

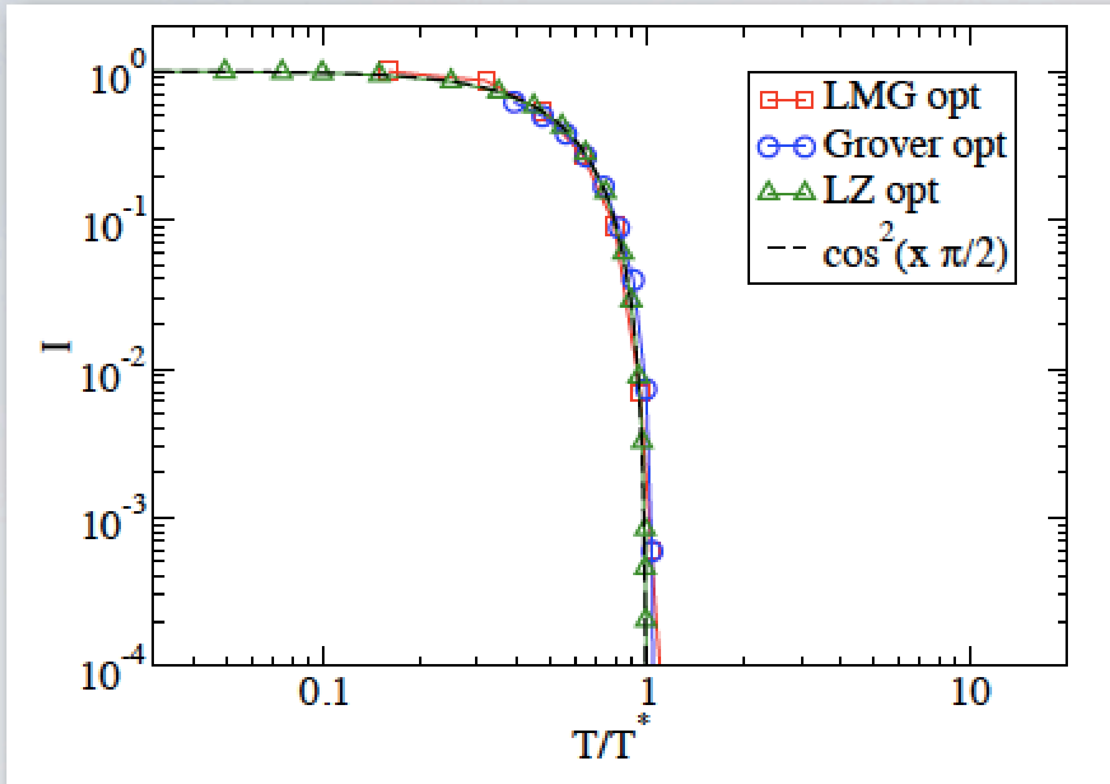
Linear quench for a "short" time T

Linear quench in the adiabatic limit $T \sim T_{ad}$

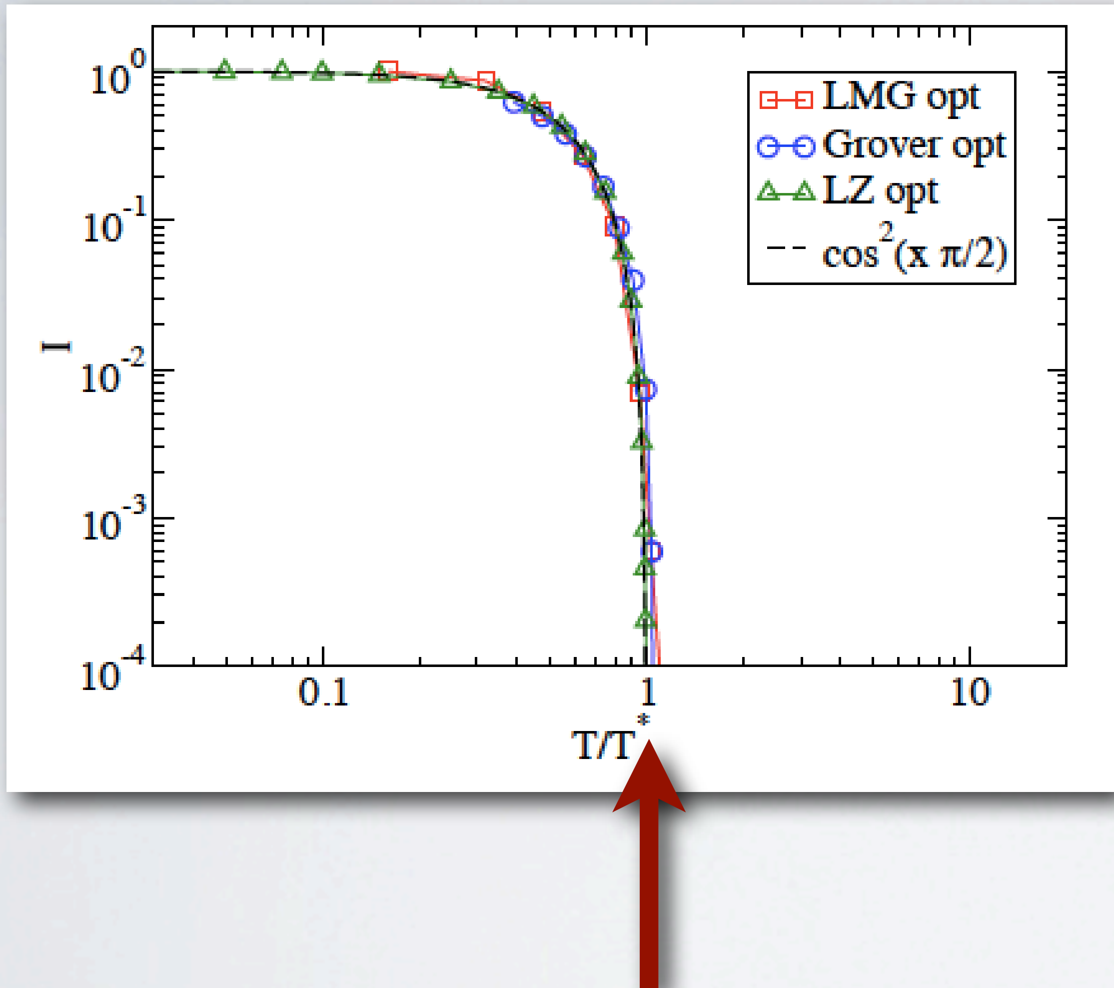
Optimal pulse at time $T \ll T_{ad}$



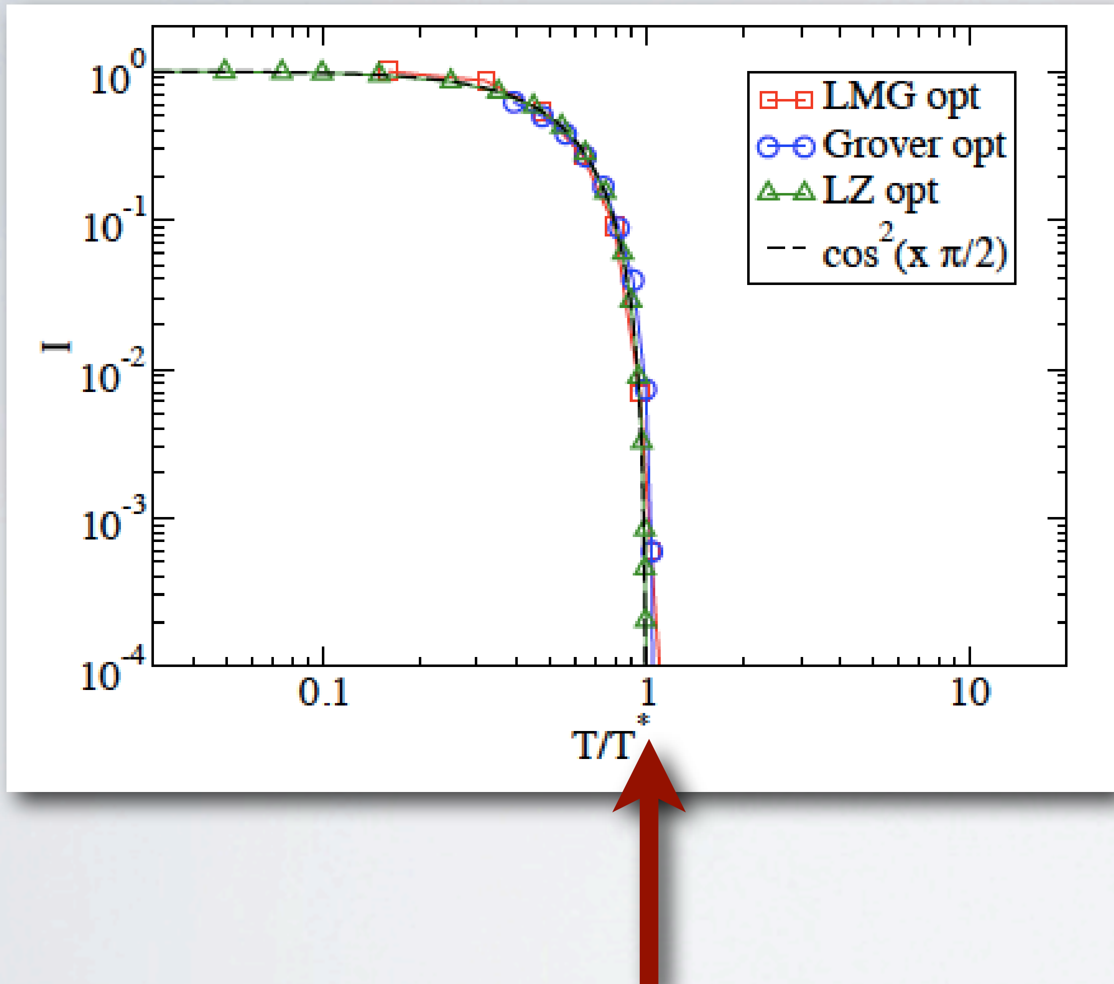
SPEED LIMIT AND QPTs



SPEED LIMIT AND QPTs



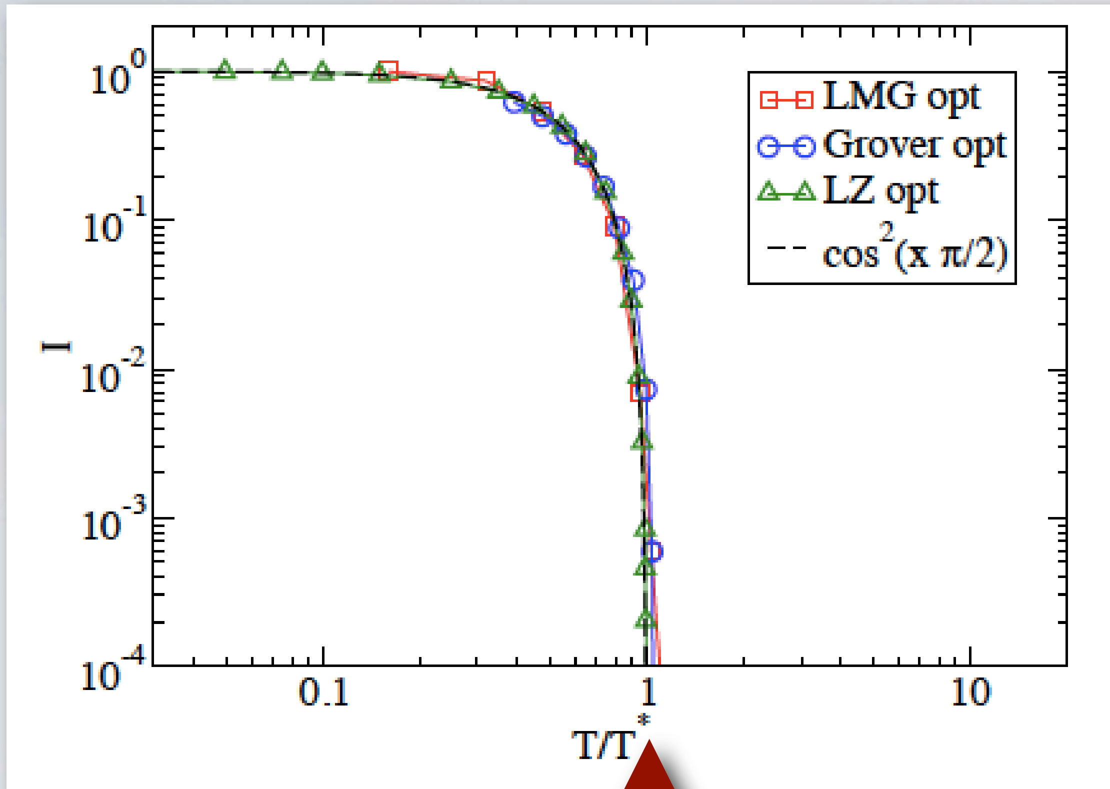
SPEED LIMIT AND QPTs

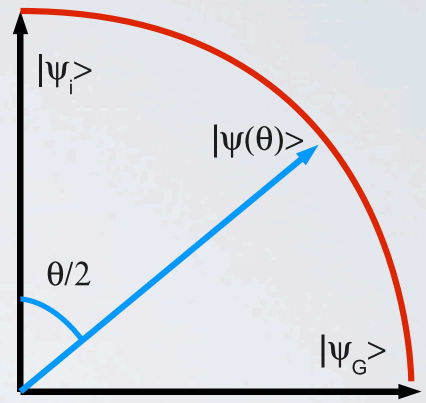


There is a minimum time, associated to the quantum speed limit below which the optimization is ineffective

– T. Caneva, M. Murphy, T. Calarco, R. F., S. Montangero, V. Giovannetti, and G.E. Santoro (2009)

SPEED LIMIT AND QPTs





An optimized evolution then can be interpreted as a uniform motion along a geodesic with speed π/T^*

There is a minimum time, associated to the quantum speed limit below which the optimization is ineffective

– T. Caneva, M. Murphy, T. Calarco, R. F., S. Montangero, V. Giovannetti, and G.E. Santoro (2009)

QUANTUM SPEED LIMIT

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Time-independent case

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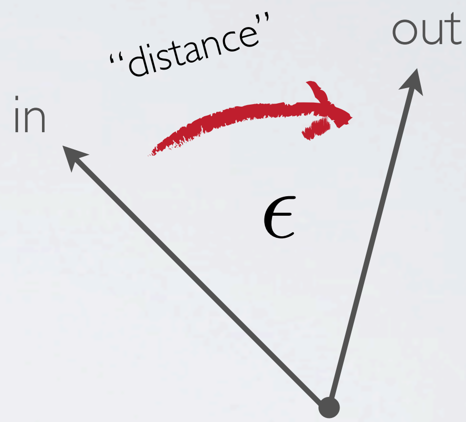
Time-independent case

Determine the Minimum time required for a quantum state to evolve to a different one placed at a certain distance from it.



$$E = \langle\Psi|H|\Psi\rangle \qquad \Delta E = \sqrt{\langle\Psi|(H-E)^2|\Psi\rangle}$$

Initial energy

Initial state

Energy variance

- T. K. Bhattacharyya (1983)
- P. Pfeifer (1993)
- N. Margolus and L.B. Levitin (1998)
- V Giovannetti, S Lloyd, and L Maccone (2003)
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QUANTUM SPEED LIMIT

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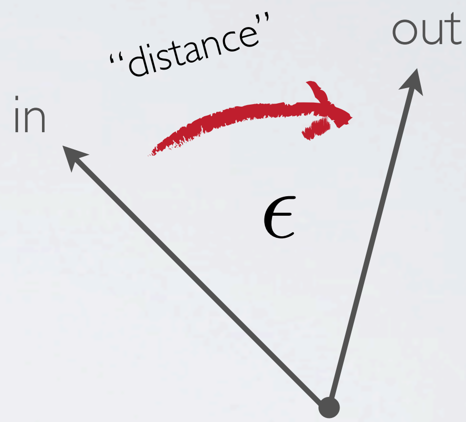

$$E = \langle \Psi | H | \Psi \rangle \qquad \Delta E = \sqrt{\langle \Psi | (H - E)^2 | \Psi \rangle}$$

Initial energy

Initial state

Energy variance

$$T^* = \max\left(\alpha(\epsilon)\frac{\pi}{2E}, \beta(\epsilon)\frac{\pi}{2\Delta E}\right)$$

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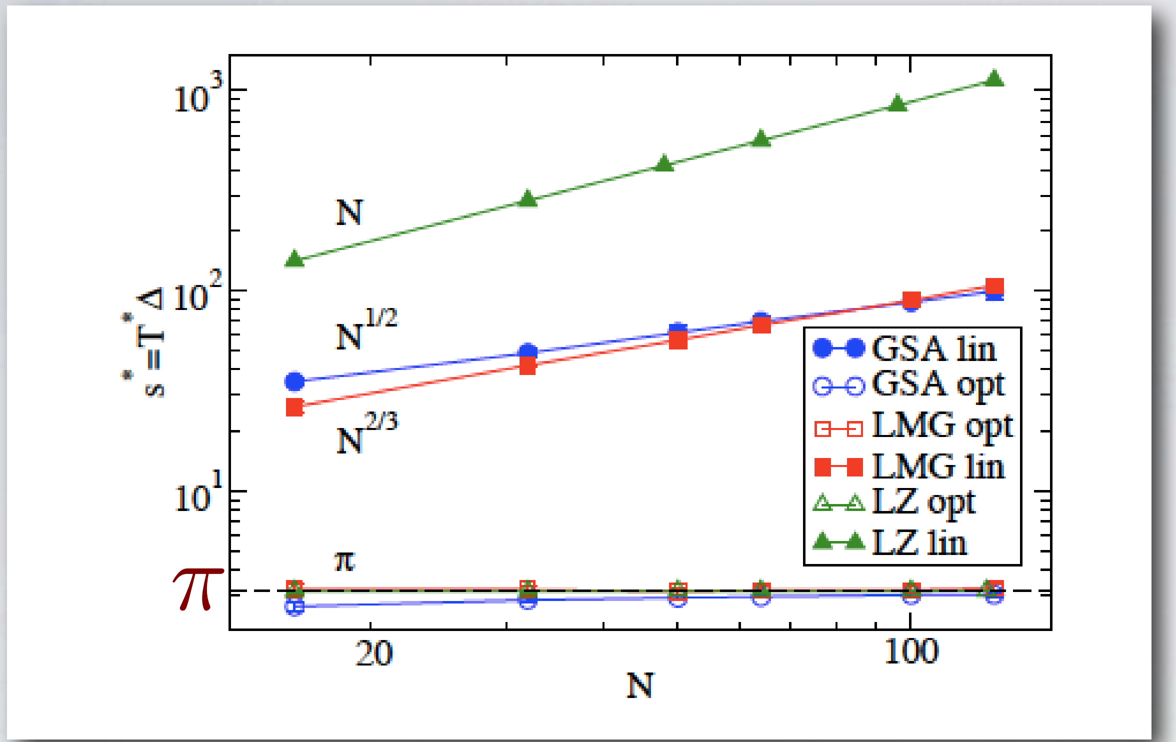
- A. Carlini et al (2006)

SPEED LIMIT AND QPTs

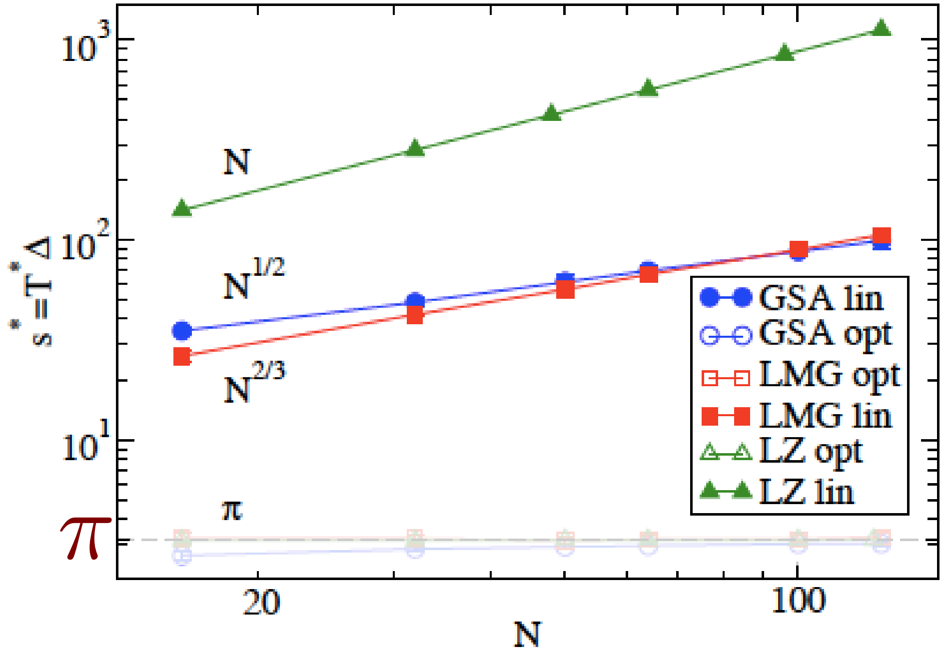


Comparison of optimized and non-optimized evolutions

T^*

minimum time to achieve infidelity $I \sim 10^{-3}$

SPEED LIMIT AND QPTs



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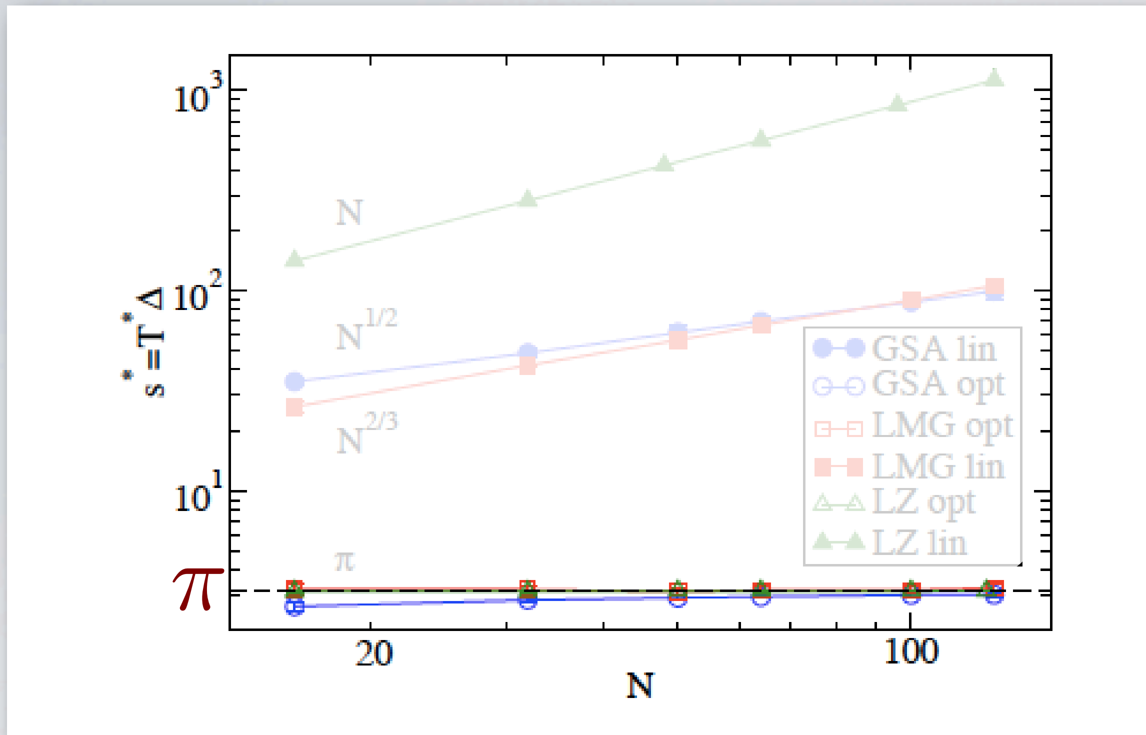

Comparison of optimized and non-optimized evolutions

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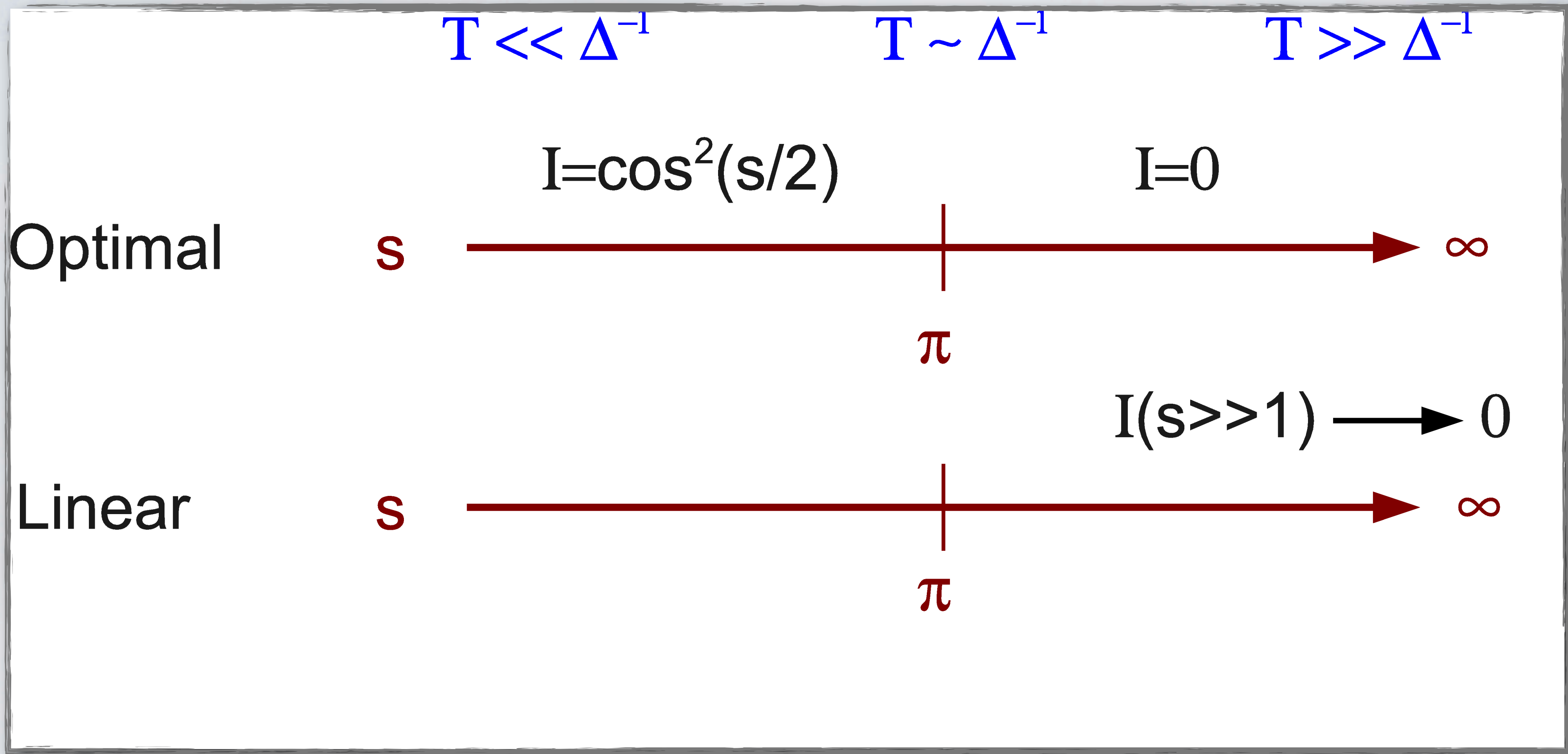
- Linear: scaling with N

- Optimal: motion along the geodesic at constant speed

SPEED LIMIT AND QPTs



SUPERADIABATIC EVOLUTION

M.V. Berry

Requirement $$|\psi_0[\Gamma(t)]\rangle$$

$$\tilde{H} = H + \delta H$$

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Transitionless quantum driving through a critical point

– A. del Campo, M.M. Rams, and W.H. Zurek (2012)

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– M.G. Bason, M. Viteau, N. Malossi, P. Huillery, E. Arimondo, D. Ciampini, R.F., V. Giovannetti, R. Mannella, and O. Morsch (2012)

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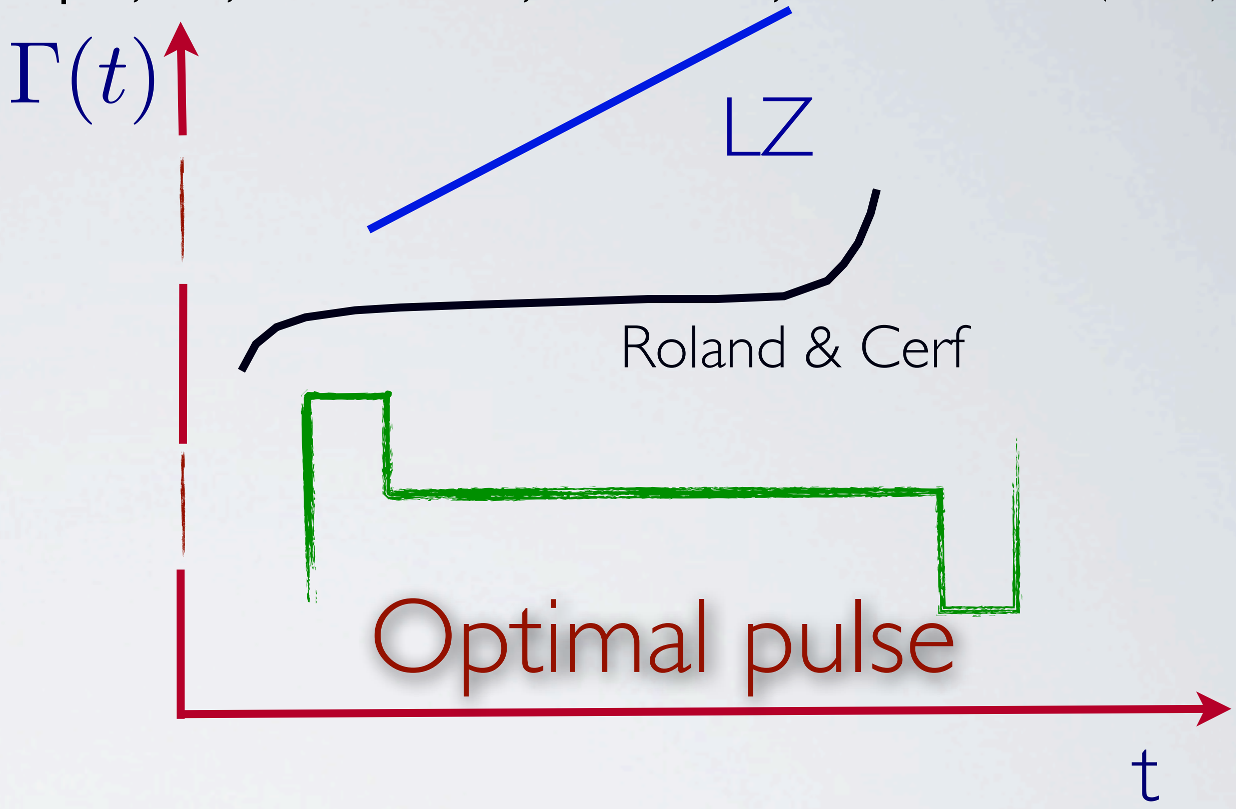

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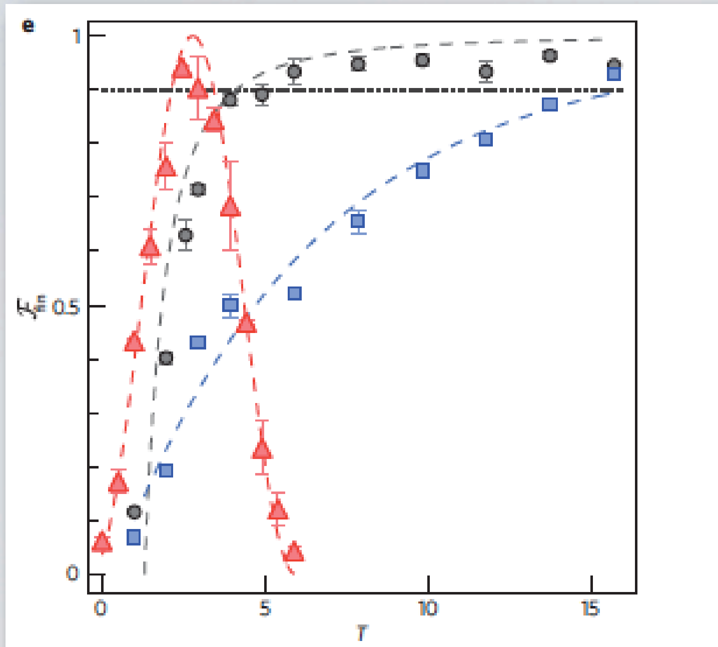

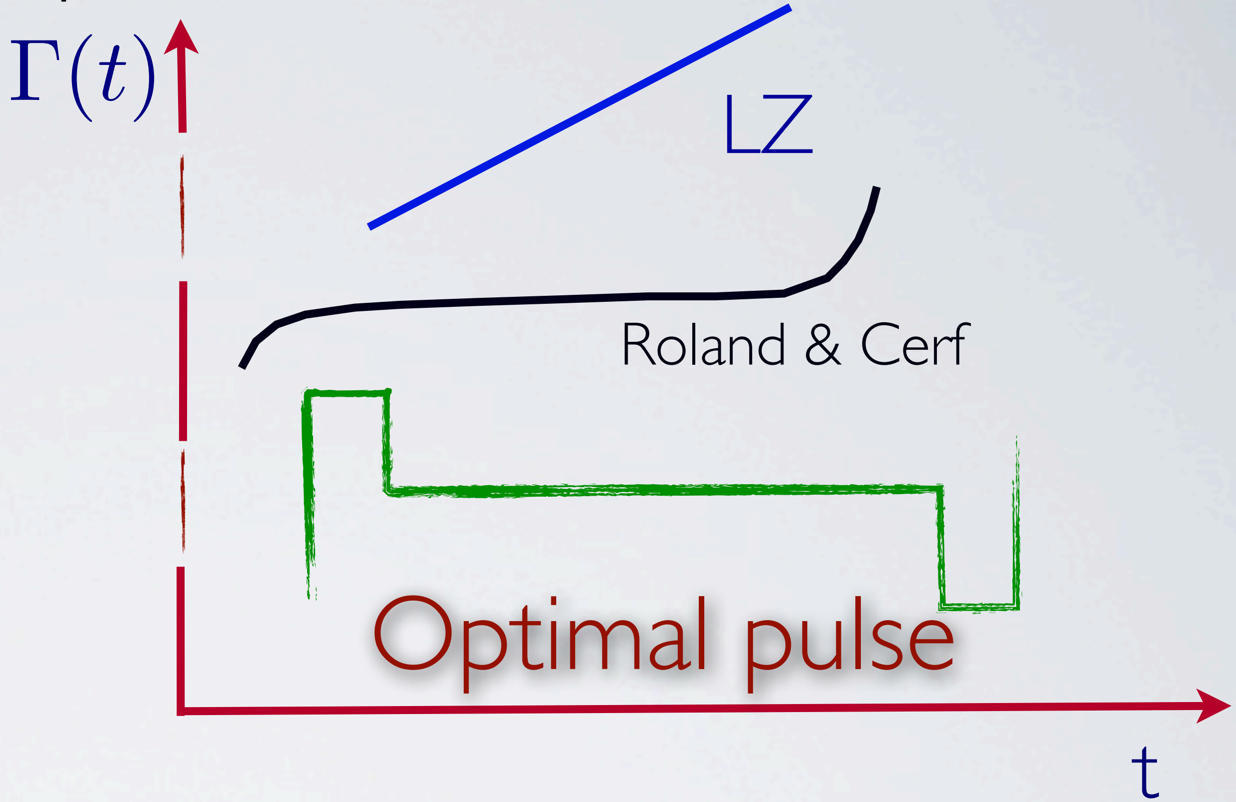

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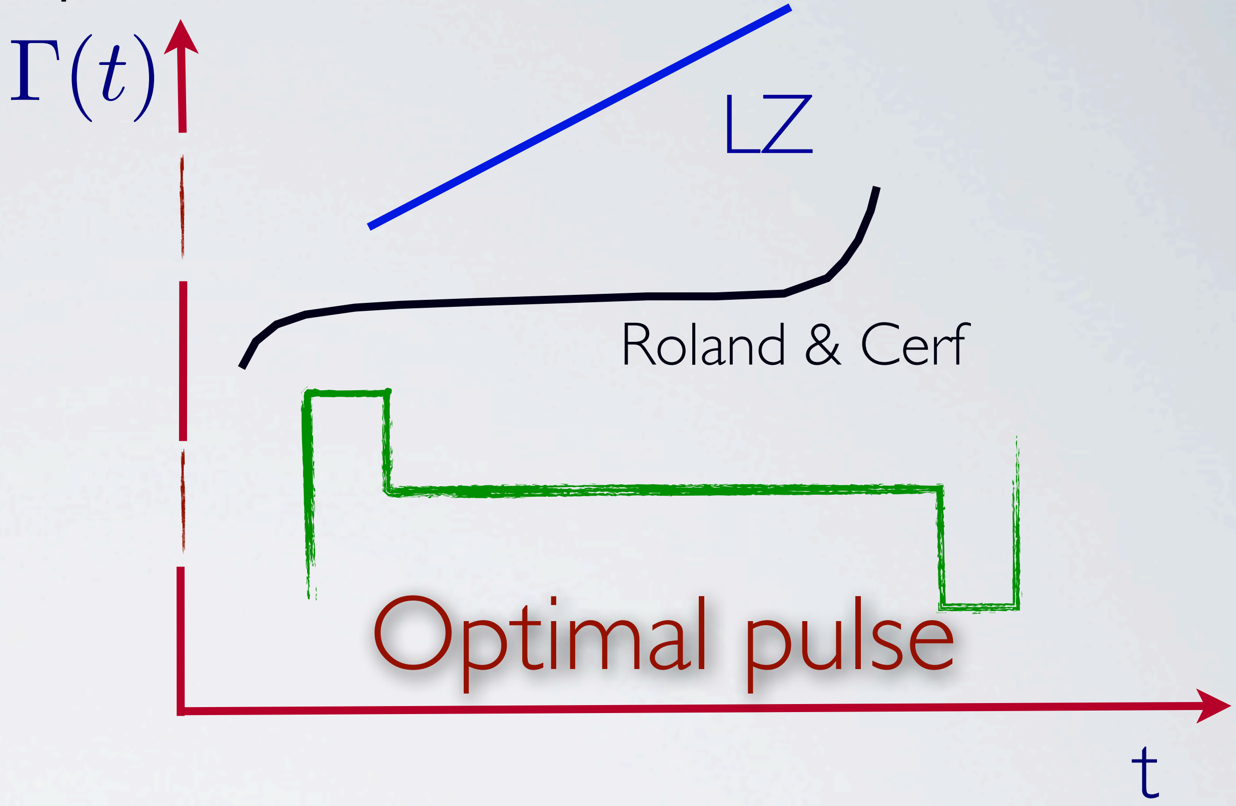

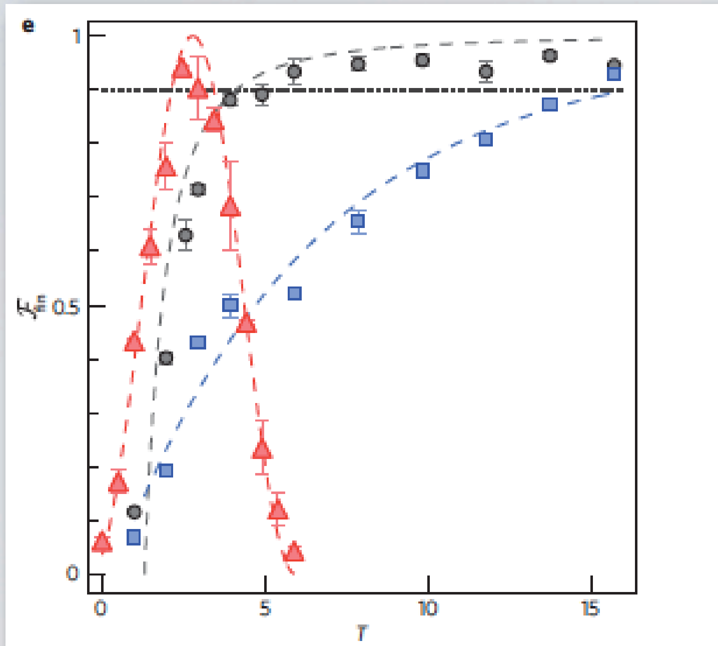

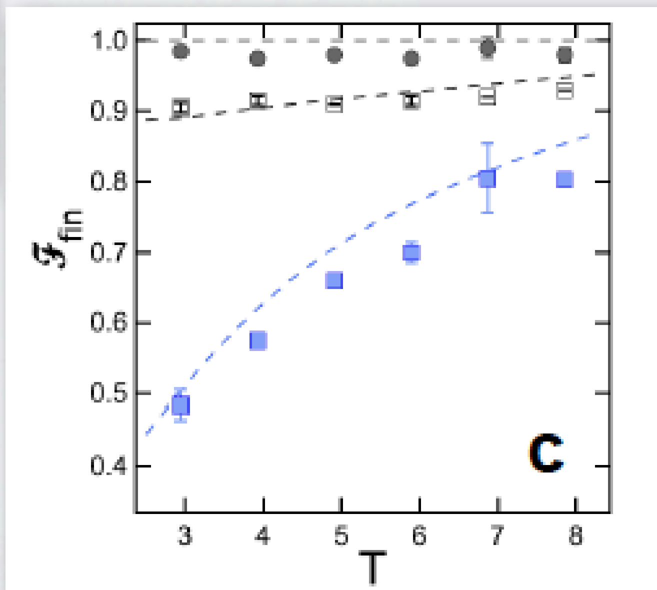

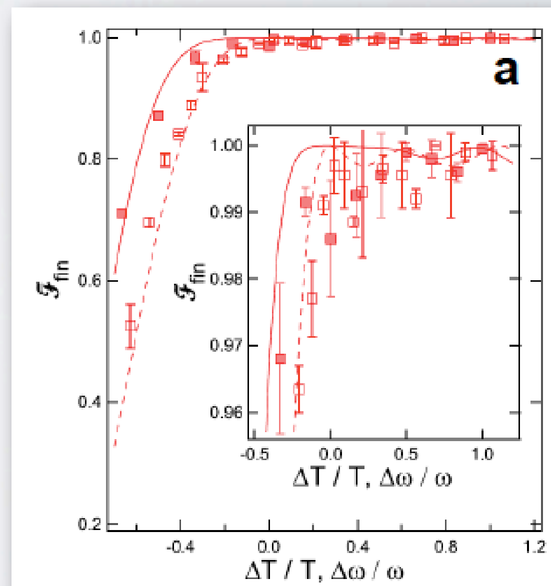

SUMMARY

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- Minimal defect formation by optimal quantum control
- Quantum speed limit related to the minimum gap
- Simple description in terms of two-level dynamics

IN COLLABORATION WITH

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- Vittorio Giovannetti
- Alessandro Silva, Giuseppe Santoro
- Ennio Arimondo, Mark Bason, Donatella Ciampini, Riccardo Mannella, Nicola Malossi, Oliver Morsh, Matthieu Viteau
- Luigi Amico

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