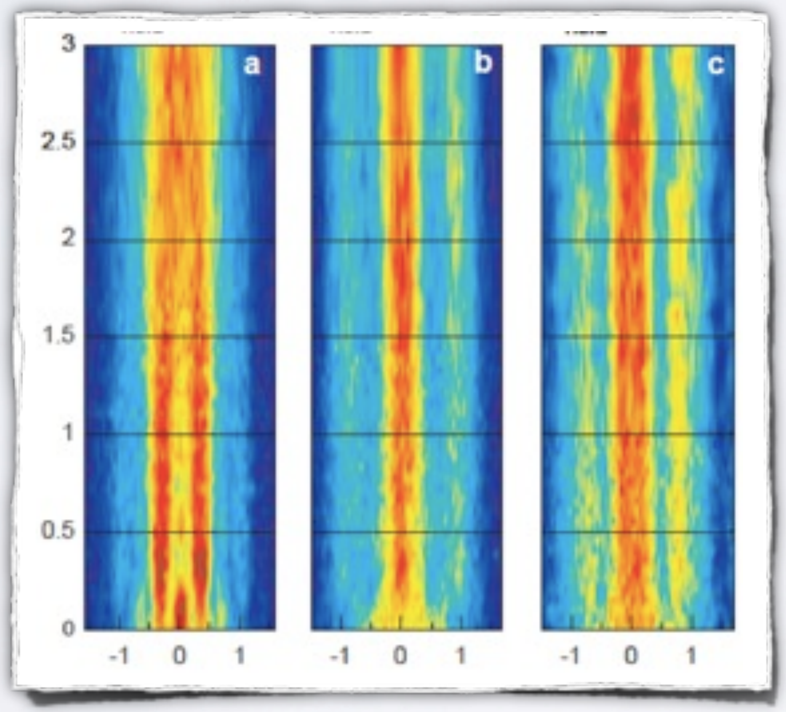
Taming the non-equilibrium:

Equilibration, thermalization and the predictions of quantum simulations



Jens Eisert

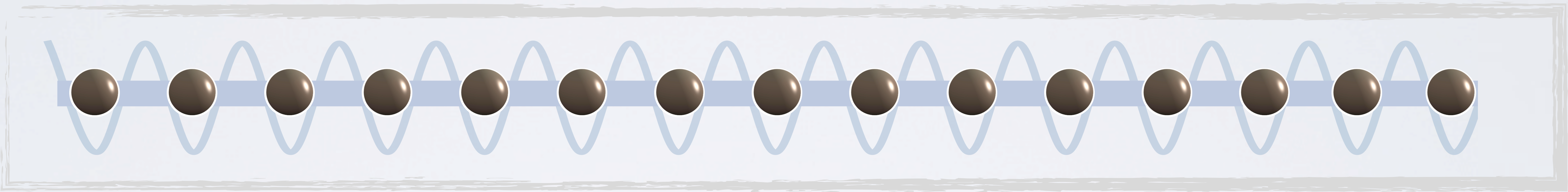
Freie Universität Berlin

KITP, Santa Barbara, August 2012

Dynamics and thermodynamics in isolated quantum systems

Mentions joint work with I. Bloch, S. Trotzky, I. McCulloch, A. Flesch, Y.-U. Chen, C. Gogolin, M. P. Mueller, M. Kliesch, A. Riera

Overview: 1. Equilibration

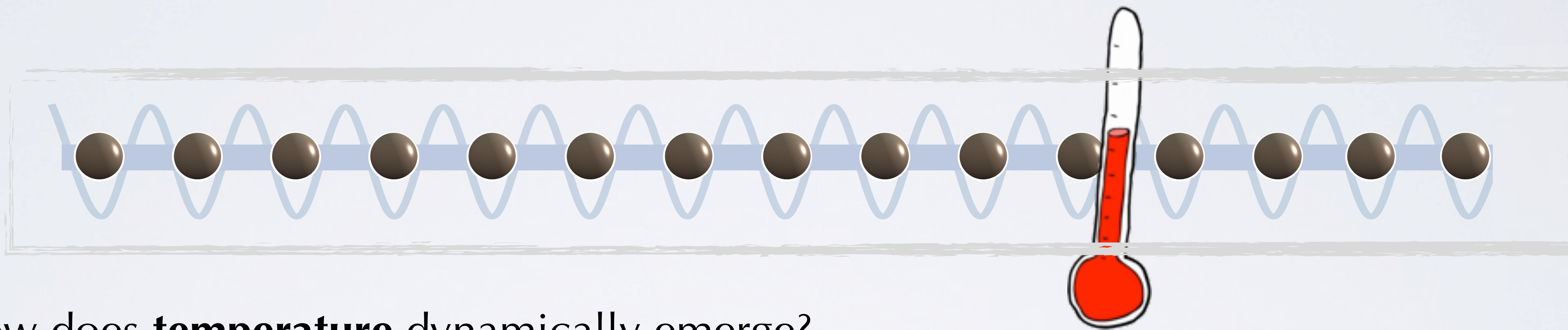


- How do quantum systems **come to equilibrium?**
- Non-equilibrium dynamics after a sudden quench

$$\rho(t) = e^{-iHt}\rho(0)e^{iHt}, \quad H = \sum_i h_i$$

Gogolin, Mueller, Eisert, *Phys Rev Lett* **106**, 040401 (2011)
Cramer, Eisert, *New J Phys* **12**, 055020 (2010)
Cramer, Dawson, Eisert, Osborne, *Phys Rev Lett* **100**, 030602 (2008)

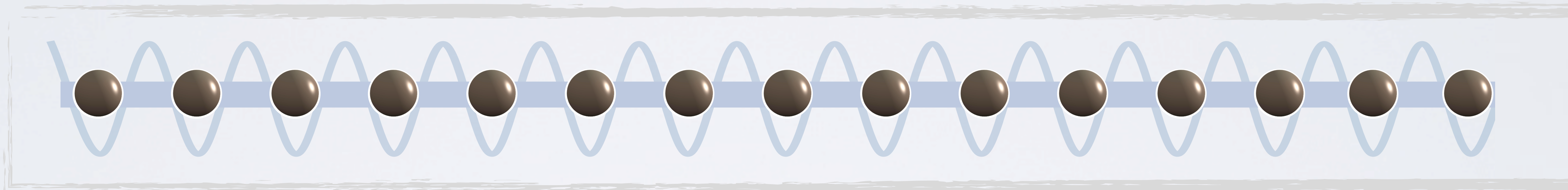
Overview: 2. Thermalization and integrability



- How does **temperature** dynamically emerge?
- Relationship to integrability?

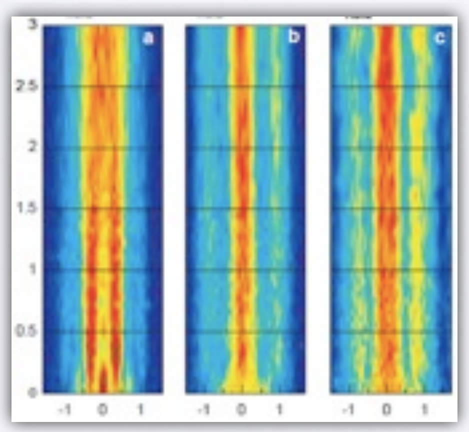
Riera, Gogolin, Eisert, *Phys Rev Lett* **108**, 080402 (2012)
Eisert, Friesdorf, Gogolin, in preparation (2012)
Gogolin, Mueller, Eisert, *Phys Rev Lett* **106**, 040401 (2011)

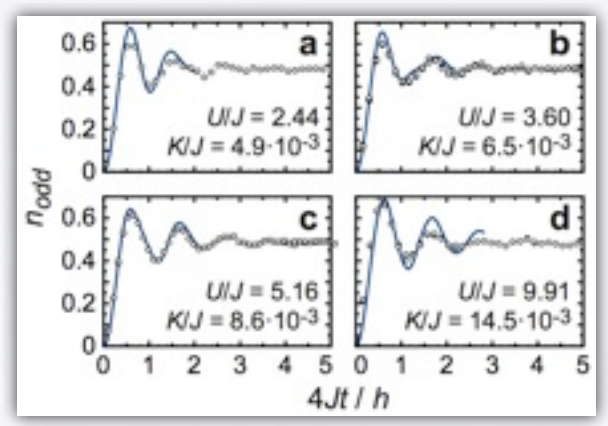
Overview: 3. Quantum simulations



- **Quantum simulation** with **cold atoms**

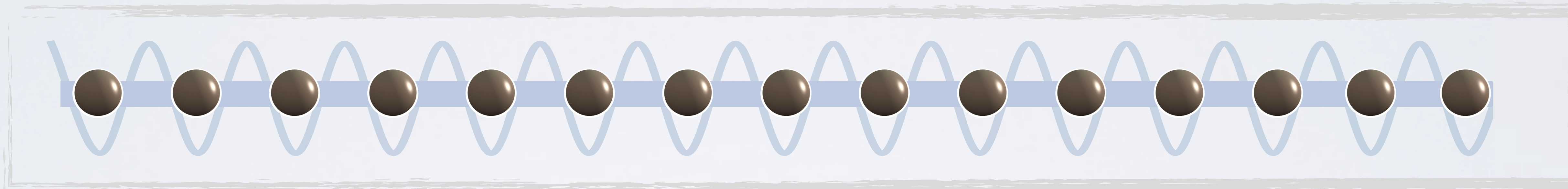
$$H = -J \sum_{\langle j,k \rangle} b_j^\dagger b_k + \frac{U}{2} \sum_k b_k^\dagger b_k (b_k^\dagger b_k - 1) - \mu \sum_k b_k^\dagger b_k$$





Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

Overview: 3. Quantum simulations

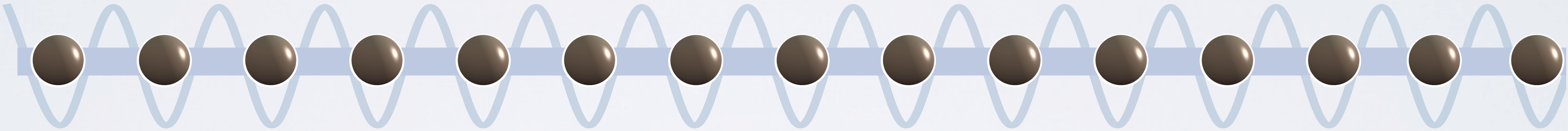


- "A quantum device that outperforms classical computers"

Eisert, Kliesch, in progress (2012)
Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

1. Notions of equilibration

Sudden quenches

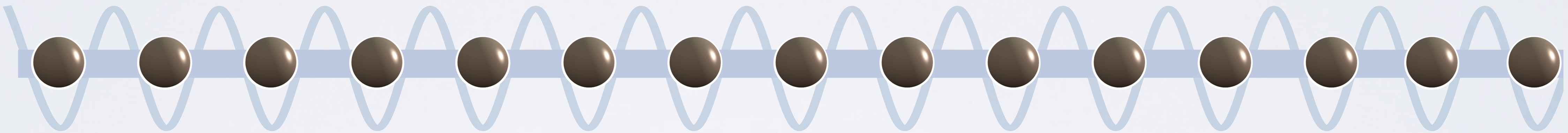


- **Initial state** (clustering correlations, e.g., product state)

- **Then many-body free unitary time evolution**

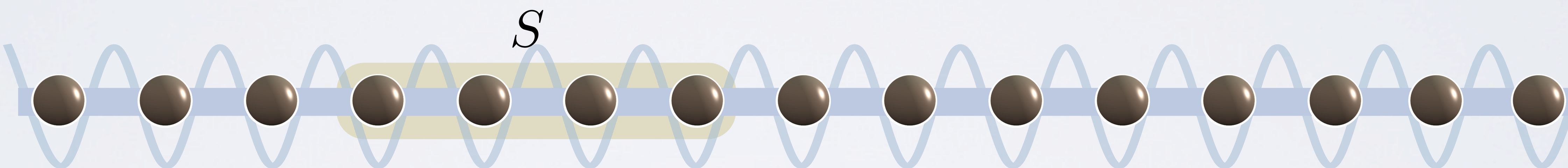
$$\rho(t) = e^{-iHt}\rho(0)e^{iHt}, \quad H = \sum_i h_i$$

Sudden quenches



- **What happens?** Equilibration?

"Strong equilibration"



- **Free bosons** (but **non-Gaussian states):** $H = \sum_{\langle i,j \rangle} (b_i^\dagger b_j + b_j^\dagger b_i)$

- **Observation 1: Strong equilibration**

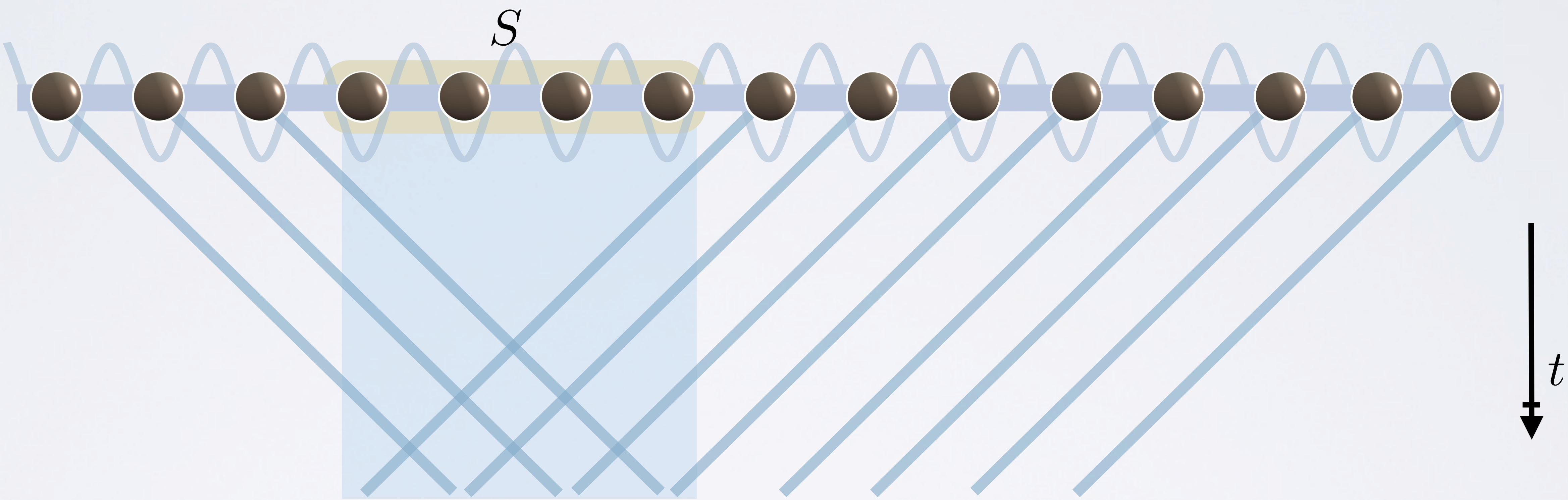
For algebraically clustering correlations (...), for any $\varepsilon > 0$ and any recurrence time t_2 one finds a system size and a relaxation time t_1 such that

$$\|\rho_S(t) - \rho_G\|_1 < \varepsilon, \quad \forall t \in [t_1, t_2]$$

ρ_G is *maximum entropy state* for fixed covariance matrix (linearly many consts of motion, "generalized Gibbs ensemble")

Cramer, Eisert, *New J Phys* **12**, 055020 (2010)
Cramer, Dawson, Eisert, Osborne, *Phys Rev Lett* **100**, 030602 (2008)

Lieb-Robinson bounds and speeds of information propagation

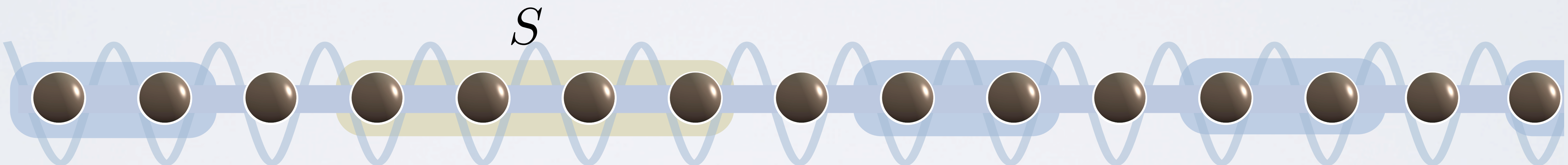


- Finite speed of information propagation (bosonic version of Lieb-Robinson bounds)

(see also Immanuel's talk)

Lieb, Robinson, *Commun Math Phys* **28,** 251 (1972)
Eisert, Osborne, *Phys Rev Lett* **97**, 150404 (2006)
Cramer, Dawson, Eisert, Osborne, *Phys Rev Lett* **100**, 030602 (2008)
Cheneau, Barmettler, Poletti, Endes, Schauss, Fukuhara, Gross, Bloch, Kollath, Kuhr, *Nature* **484**, 481 (2012)

Quantum central limit theorems



Characteristic function of reduced state

$$\chi_{\rho_S(t)}(\beta) = \mathrm{tr}[\rho_S(t)D(\beta)]$$

Formulate non-commutative
Lindeberg **central limit theorem**

Chuck lattice into "rooms" and
"corridors" (Bernstein-Spohn-blocking)

Characteristic function becomes **Gaussian**

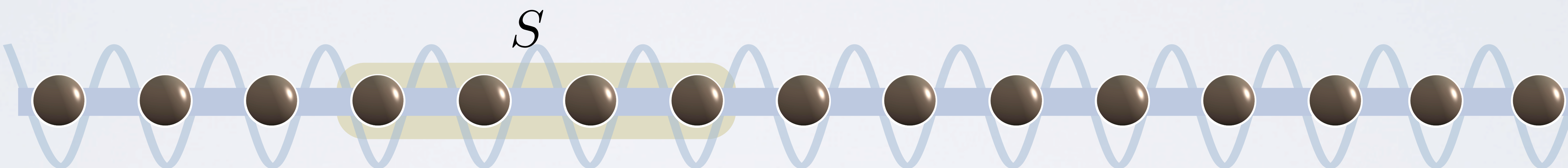
$$\left|\langle D_S\rangle - e^{\mu_S+\sigma_S/2}\right| \leq c_0 \frac{\log(t)}{t^{\frac{\eta_2}{1+\eta}}} + f^{1/2}(t) + e^{f(t)}f(t) + e^{g(t)}g(t)$$

Maximum entropy state

$$\|\rho_S(t) - \rho_G\|_1 < \varepsilon$$

Cramer, Eisert, *New J Phys* **12**, 055020 (2010)
Cramer, Dawson, Eisert, Osborne, *Phys Rev Lett* **100**, 030602 (2008)
Dudnikova, Komech, Spohn, *J Math Phys* **44**, 2596 (2003)

"Weak equilibration"

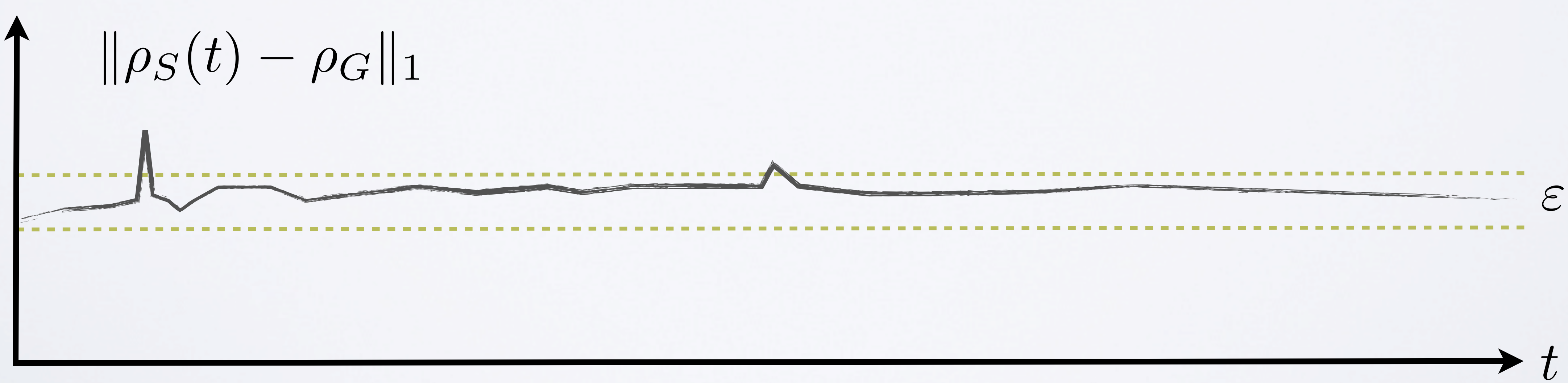


- **Observation 2: Weak equilibration** (true for all Hamiltonians with degenerate energy gaps)

$$\mathbb{E}(\|\rho_S(t) - \rho_G\|_1) \leq \frac{1}{2}\sqrt{\frac{d_S^2}{d^{\text{eff}}}}, \quad d^{\text{eff}} = \frac{1}{\sum_k |\langle E_k|\psi_0\rangle|^4}$$

ρ_G is *maximum entropy state* given *all* constants of motion

Linden, Popescu, Short, Winter, *Phys Rev E* **79**, 061103 (2009)
Gogolin, Mueller, Eisert, *Phys Rev Lett* **106**, 040401 (2011)



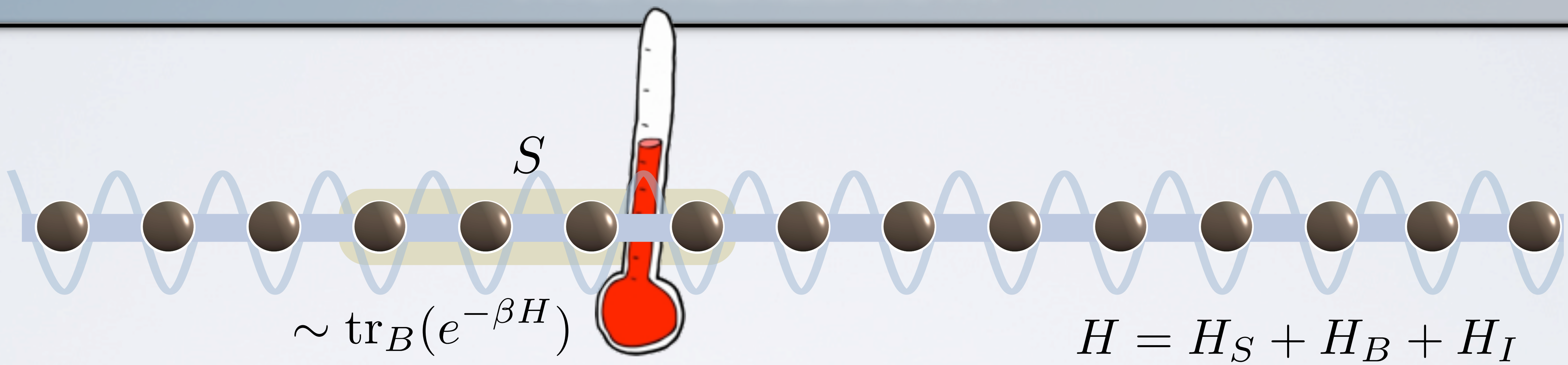
Lessons

- **Lesson**: Systems generically locally "appear relaxed", although the dynamics is entirely unitary

- Proven in *strong sense* for general states in *integrable limit* of Bose-Hubbard model
- True in slightly weaker sense for most times
- Generalized Gibbs ensembles, what conserved quantities?

2. Integrability and thermalization

Thermalization?



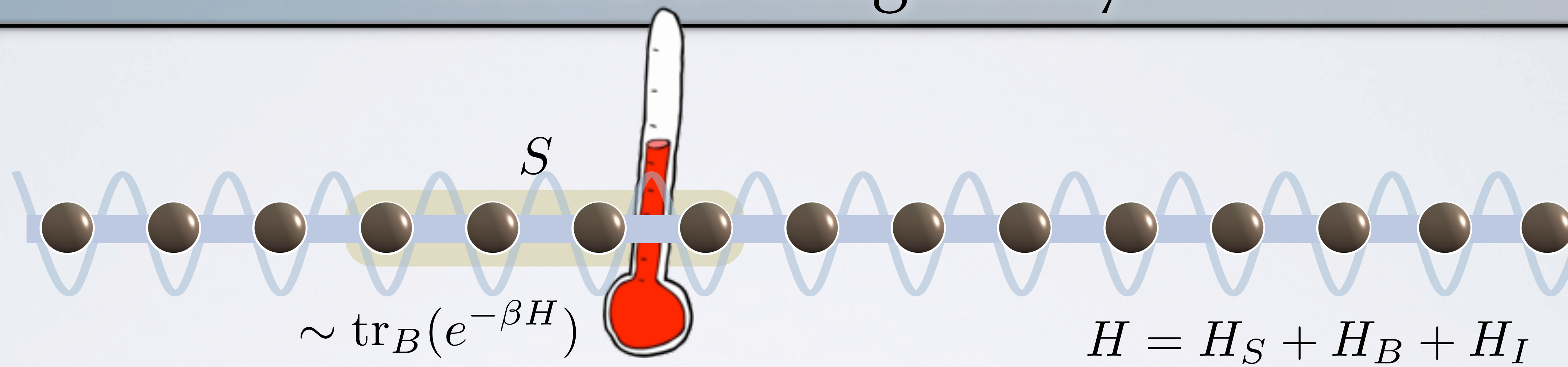
- When do systems **thermalize?**

(See talks by Marcos, Jean-Sebastian, Fabian, ...)

(Progress on thermalization question, ask if interested)

Riera, Gogolin, Eisert, *Phys Rev Lett* **108**, 080402 (2012)

Notions of integrability



- **Notions of integrability**

  (A) Exist *n* independent (local) conserved mutually commuting linearly independent operators (*n* no. of degrees of freedom)

  (B) Like (A) but with linear replaced by algebraic independence

  (C) The system is integrable by the Bethe ansatz

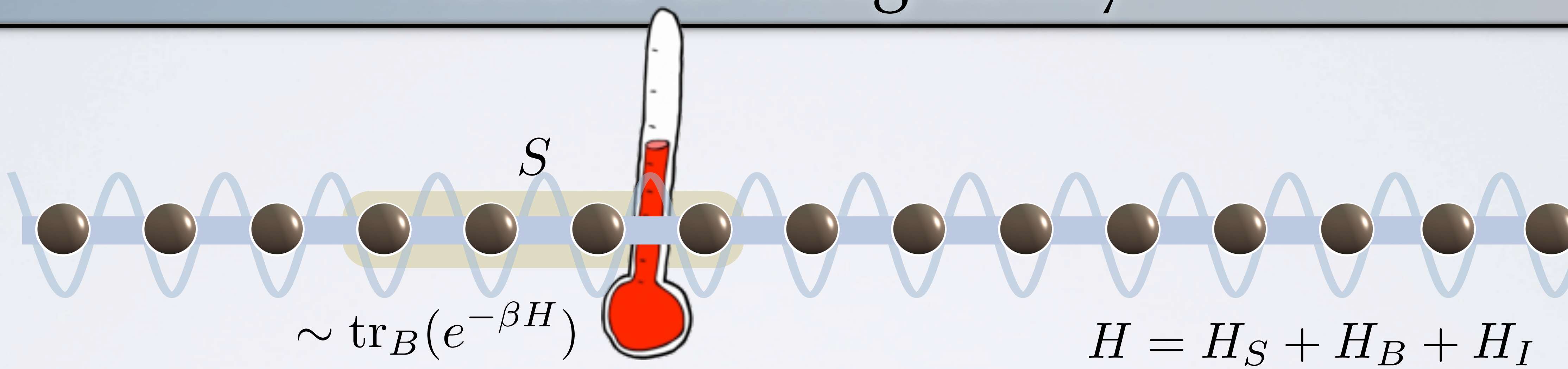
  (D) The system exhibits non-diffractive scattering

  (E) The quantum many-body system is exactly solvable

- **Common intuition:** "Non-integrable models thermalize"

Beautiful models, Sutherland (World Scientific, Singapore, 2004)
Exactly solvable models, Korepin, Essler (World Scientific, Singapore, 1994)

Notions of integrability



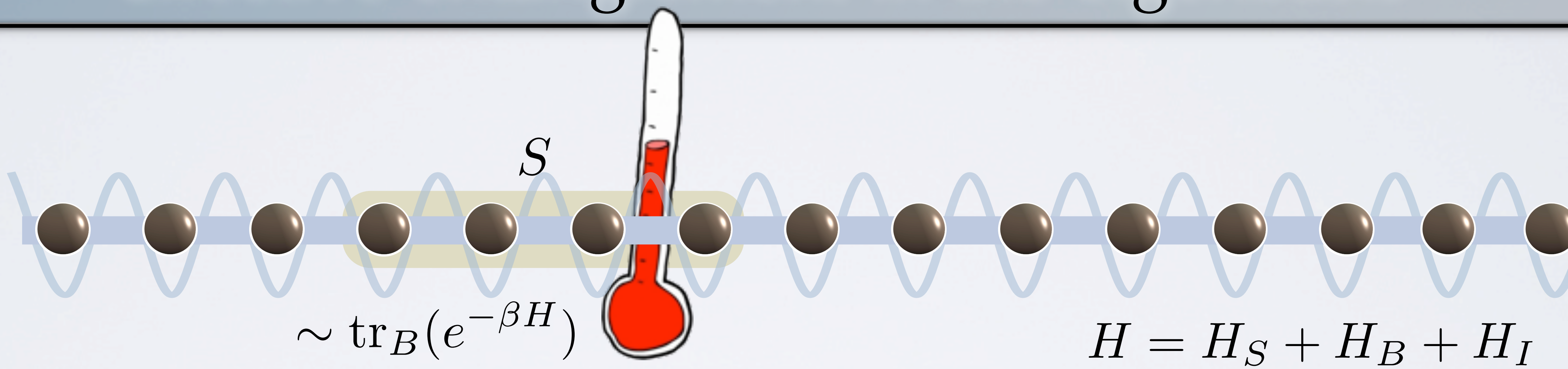
- **Natural candidates?**

- Nearest-neighbor interactions
- Translationally invariant (no disorder)
- No exactly conserved local quantities

Gogolin, Mueller, Eisert, *Phys Rev Lett* **106**, 040401 (2011)

Compare also:
Pal, Huse, arXiv:1103.2613
Canovi, Rossini, Fazio, Santoro, Silva, arXiv:1006.1634
Kollath, Lauchli, Altman, *Phys Rev Lett* **98**, 180601 (2007)
Polkovnikov, Sengupta, Silva, Vengalattore, *Rev Mod Phys* **83**, 863 (2011)
Rigol, Srednicki, *Phys Rev Lett* **108**, 110601 (2012)

Effective entanglement in the eigenbasis



- **Effective entanglement in the eigenbasis**

$$R(\psi_0) = \sum_k |c_k|^2 \| \mathrm{tr}_B |E_k\rangle\langle E_k| - \psi_0^S \|_1, \quad c_k = \langle E_k | \psi_0 \rangle$$

- **Observation 3** (non-thermalization)**:** The physical distinguishability of two local time averaged states $\omega^{S(1)}$ and $\omega^{S(2)}$ of two pure initial product states

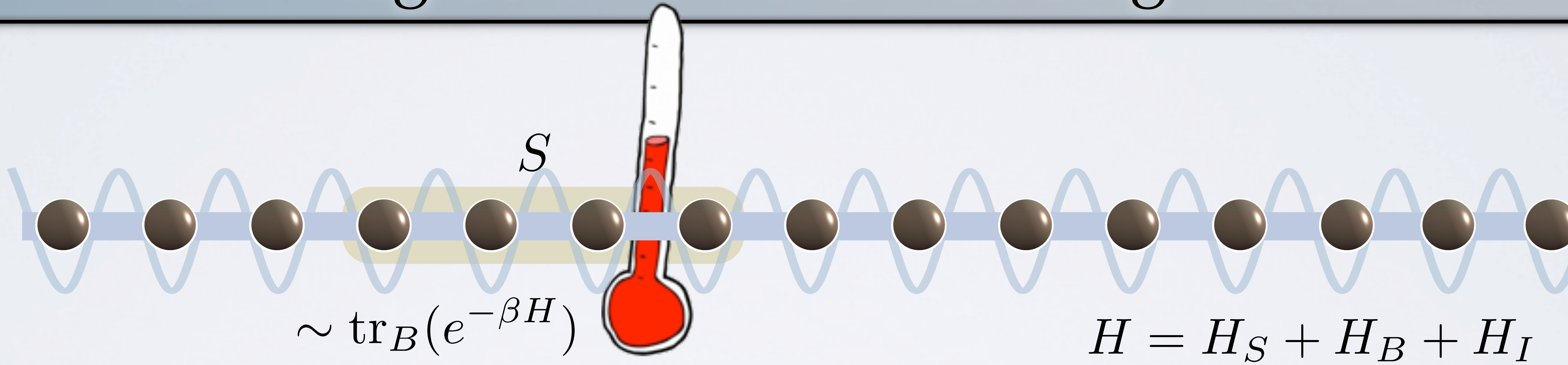
$$\psi_0^{(i)} = \psi_0^{S(i)} \otimes \phi_0^{B(i)}$$

and non-degenerate Hamiltonians is large in that

$$\|\omega^{S(1)} - \omega^{S(2)}\|_1 \geq \|\psi_0^{S(1)} - \psi_0^{S(2)}\|_1) - R(\psi_0^{(1)}) - R(\psi_0^{(2)})$$

Gogolin, Mueller, Eisert, *Phys Rev Lett* **106**, 040401 (2011)

Non-integrable non-thermalizing models



- **Non-thermalization**

> - **Observation 4:** Ex. non-integrable models for which the *memory of the initial condition* remains large for all times

Proof related to Matt Hastings' and Spiros Michalakis' ideas

- So, what is precise relationship? Role of disorder?
- Eigenstate thermalization? Refined concepts of integrability?

Eisert, Friesdorf, Gogolin, in preparation (2012)
Gogolin, Mueller, Eisert, *Phys Rev Lett* **106**, 040401 (2011)

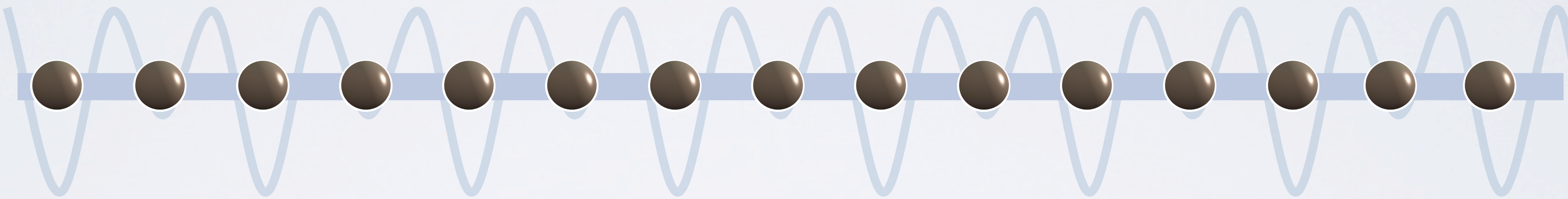
Non-integrable non-thermalizing models

- **Lesson**: Connection between integrability and thermalization may be more intricate than often assumed

Eisert, Friesdorf, Gogolin, in preparation (2012)
Gogolin, Mueller, Eisert, *Phys Rev Lett* **106**, 040401 (2011)

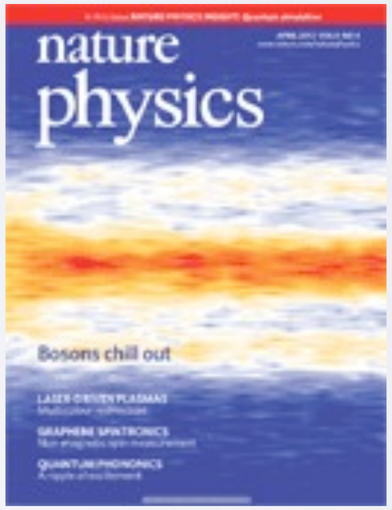
3. Dynamical quantum simulation and "quantum supremacy"

An experiment



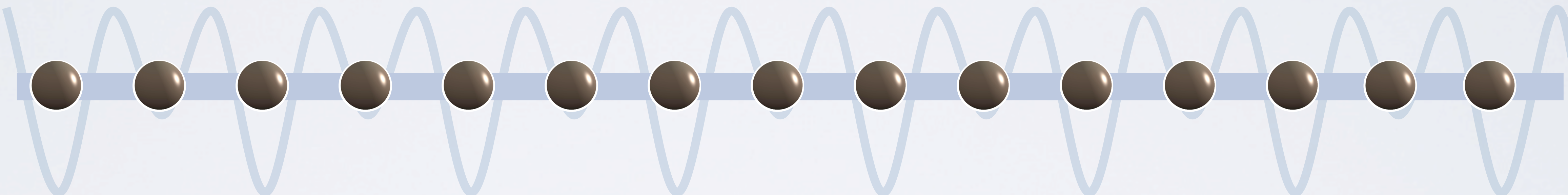
- Quench to full **strongly-correlated Bose-Hubbard Hamiltonian**...

(see also Immanuel's talk)



Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

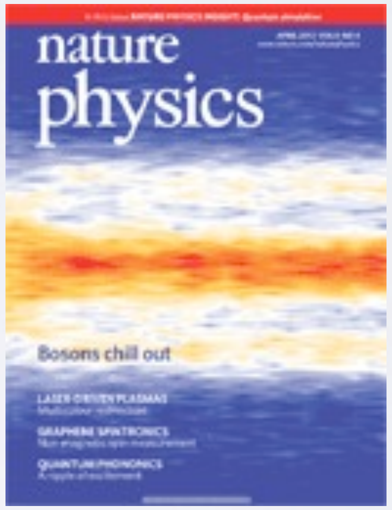
An experiment

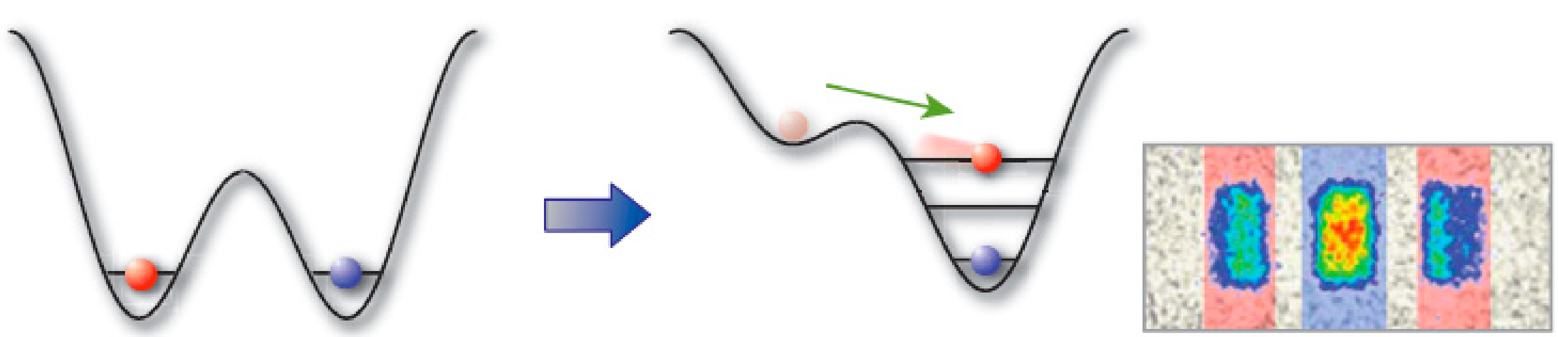


- Quench to full **strongly-correlated Bose-Hubbard Hamiltonian**...
- ... but use **optical superlattices** to circumvent readout problem

$$|\psi(t)\rangle = e^{-iHt}|1,0,1,0,\ldots,1,0\rangle$$

read out with period 2: Densities, correlators, currents...



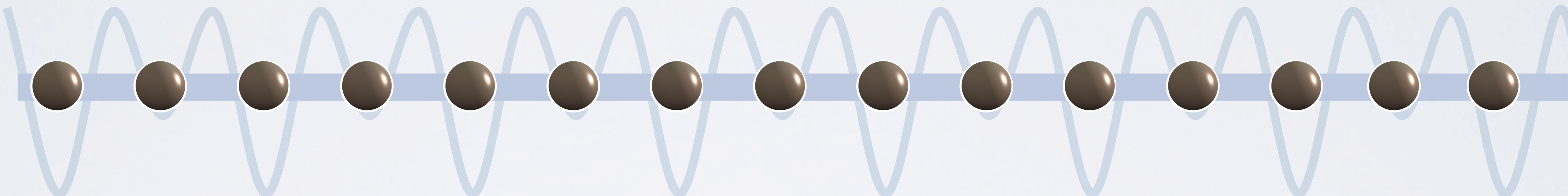


- Bias superlattice
- Unload to higher band
- Time-of-flight measurement: mapping to different Brillouin zones

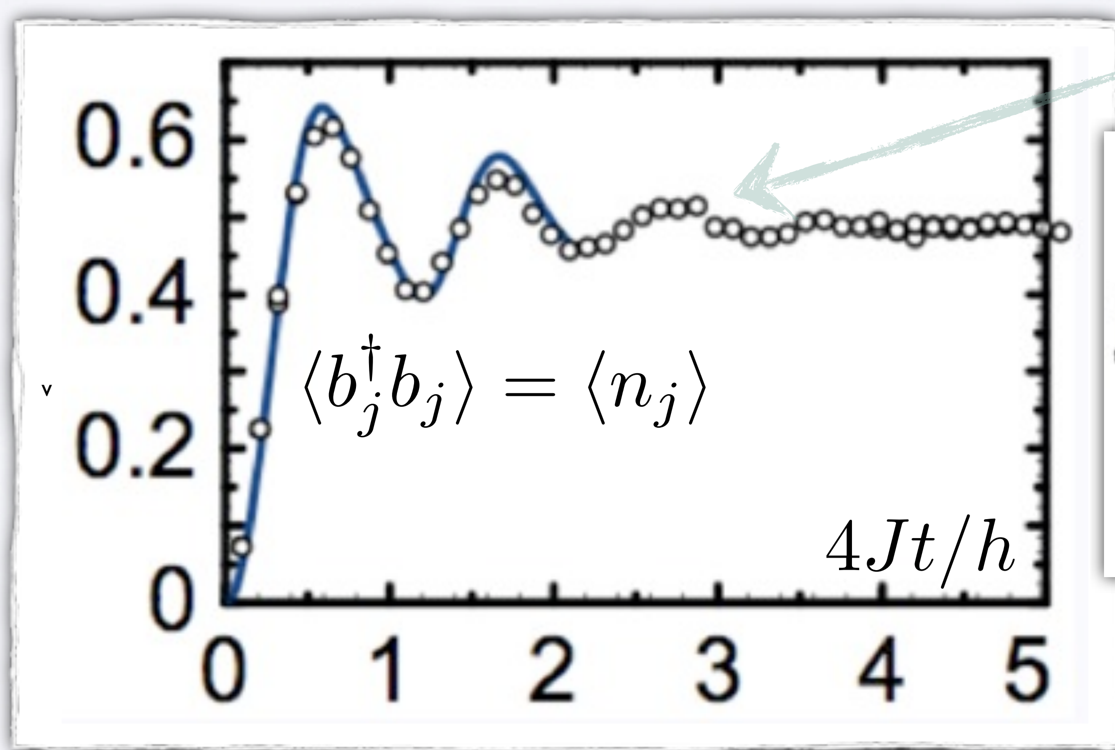
Foelling et al, *Nature Phys* **448**, 1029 (2007)

Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

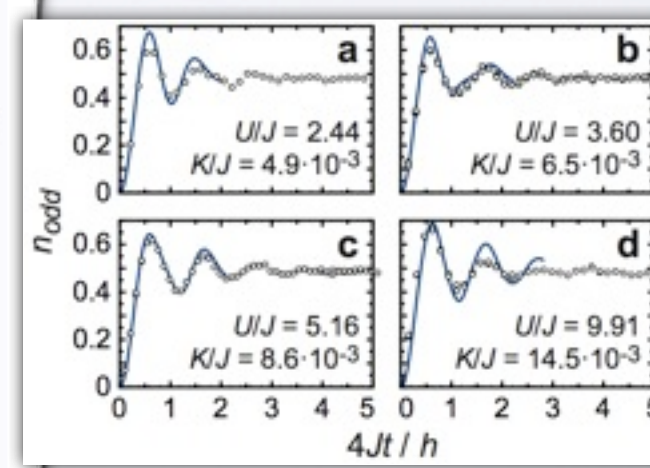
An experiment



- **Densities of odd sites** as function of time

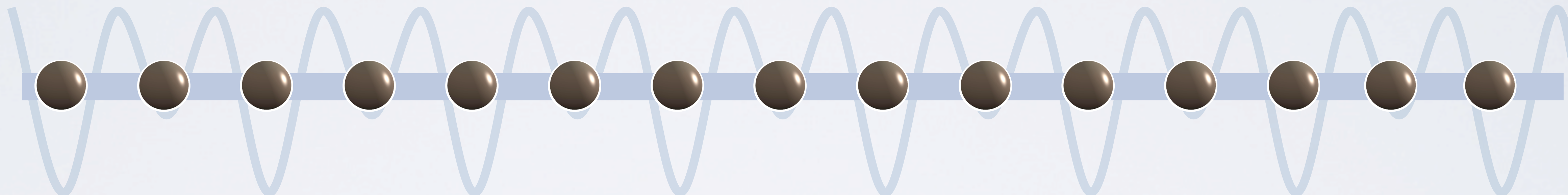


Experimental data

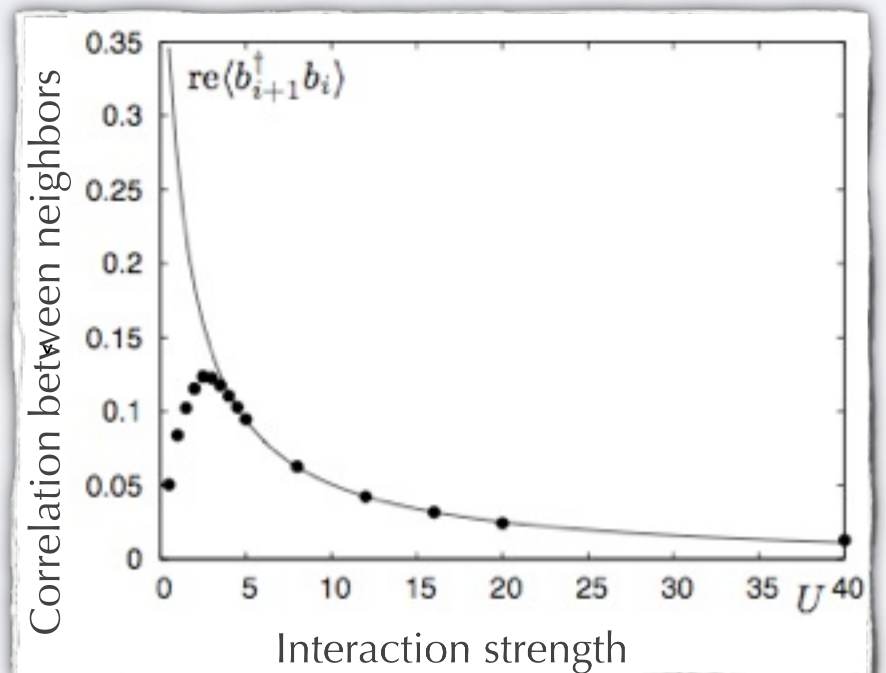


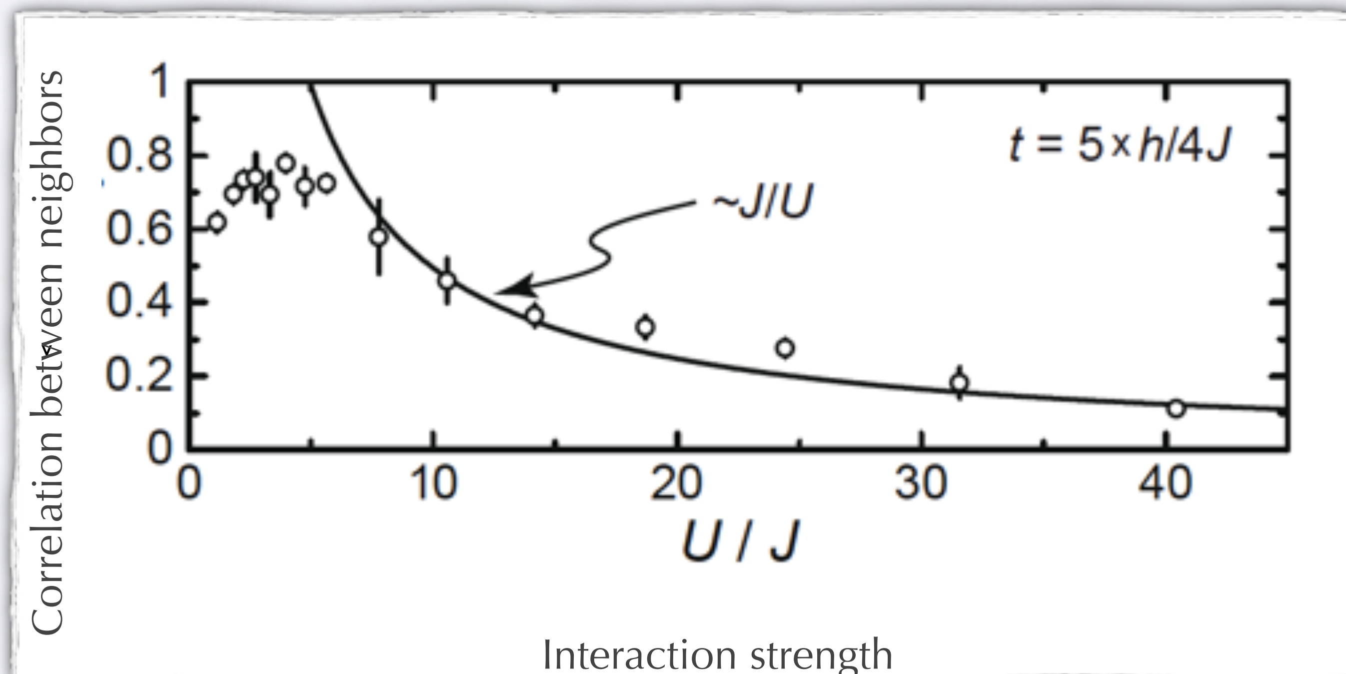
Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

An experiment



- **Visibility** proportional to nearest-neighbor correlations



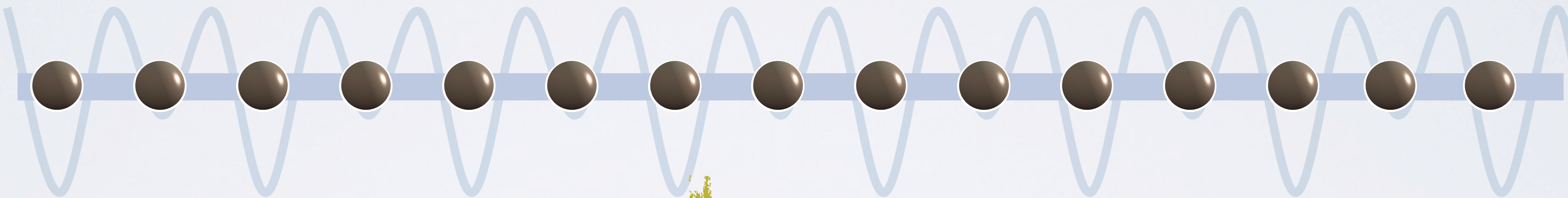


- **Current measurements:** Measure double well oscillations

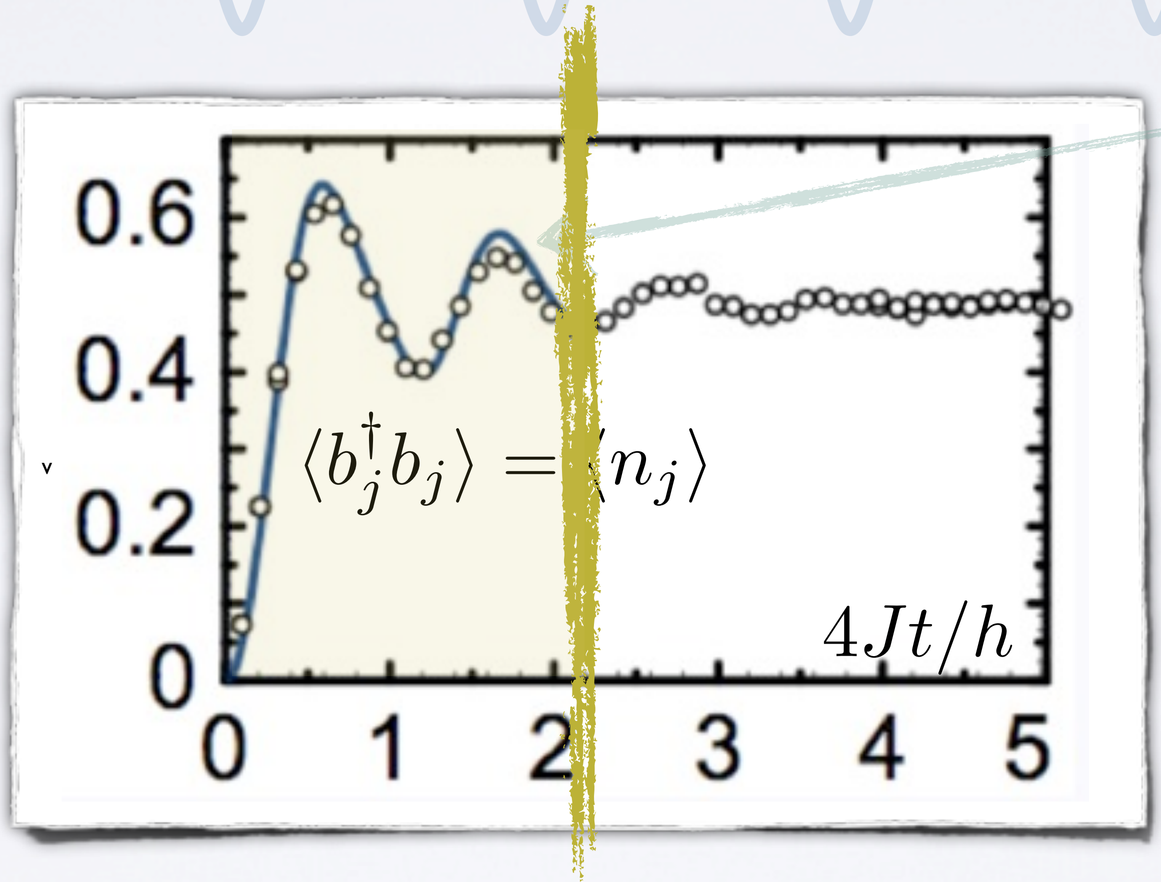
- …

Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

Matrix-product state classical simulation



- **For short times:**



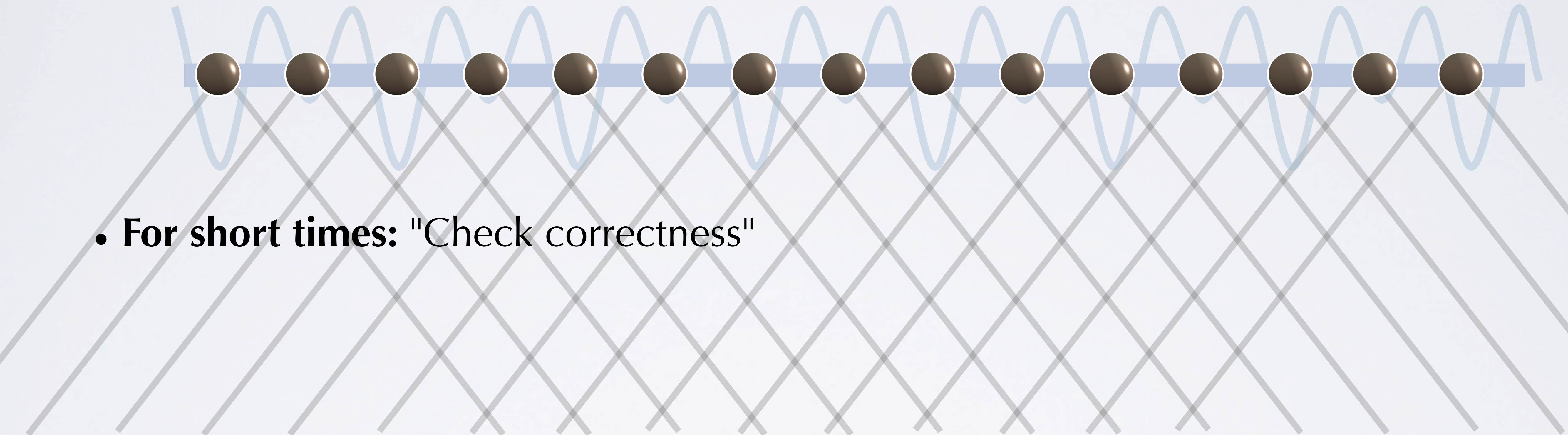
Classical simulation
(up to bond dim. of 5000)

- **Observation 5: Short times matrix-product state (MPS) simulation**

...practically to machine precision with t-DMRG (exponential blow-up of bond dimension in time)

White, *Phys Rev Lett* **69**, 2863 (1992)
Schollwoeck, *Rev Mod Phys* **77**, 259 (2005)
Eisert, Cramer, Plenio, *Rev Mod Phys* **82**, 277 (2010)

Matrix-product state classical simulation



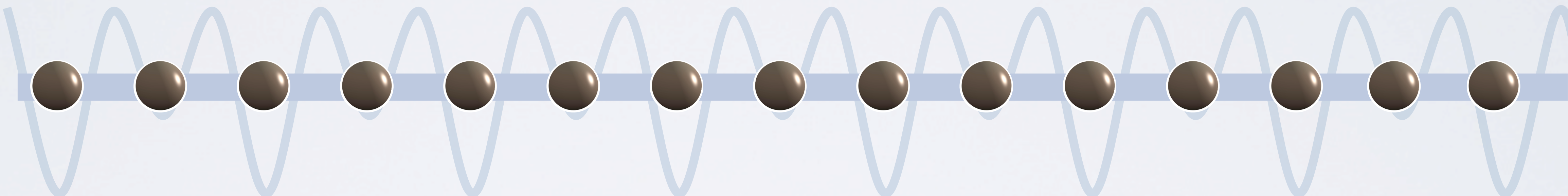
- **For short times:** "Check correctness"

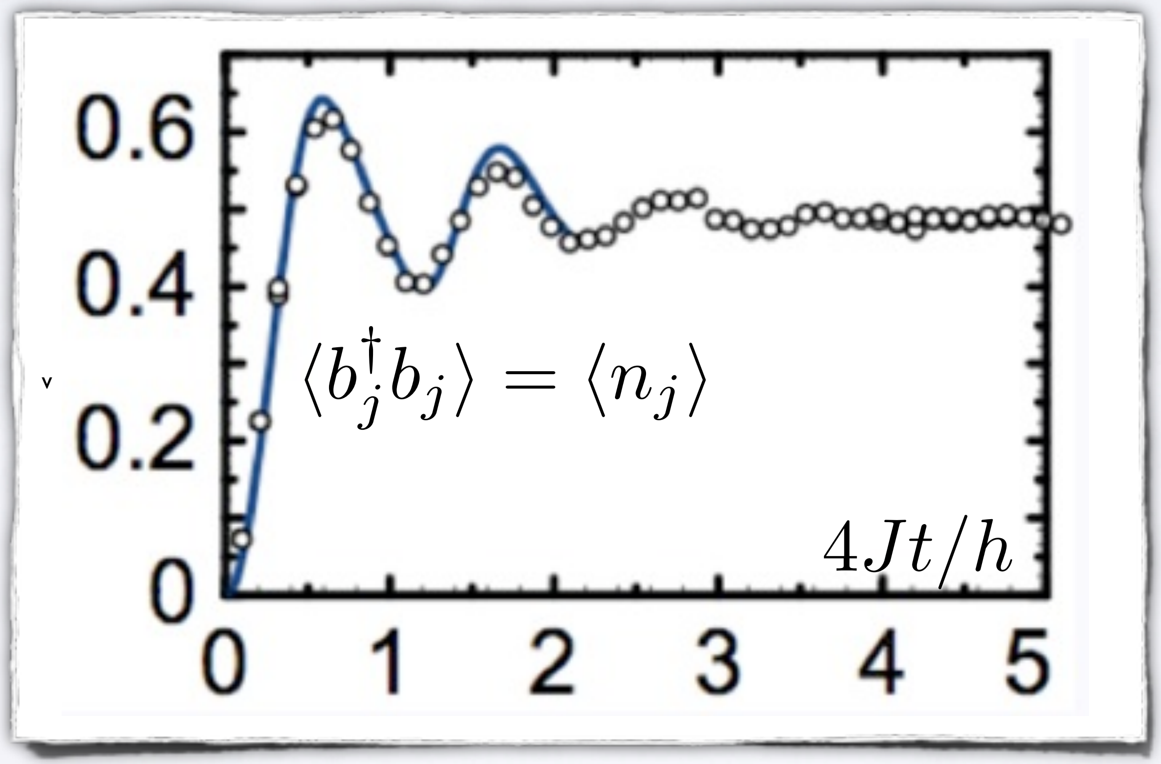
- **Observation 6: Short times matrix-product state (MPS) simulation**

Short time evolution can be efficiently described MPS: Rigorously using quantum cellular automata and Lieb-Robinson bounds

Eisert, Osborne, *Phys Rev Lett* **100**, 030602 (2008)
Osborne, *Phys Rev Lett* **97**, 157202 (2006)

"Quantum simulator"



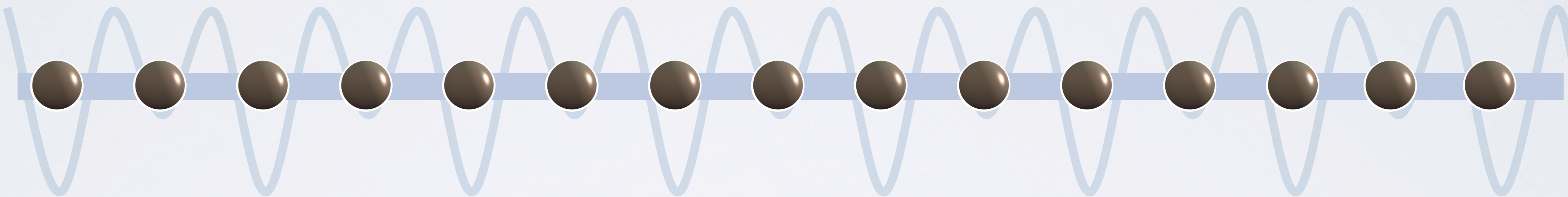


- **Observation 7: Long time dynamics of many-body dynamics in experiment**

Can accurately probe dynamics for longer times (exp vs poly decay, ...)

Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

Devil's advocate



- **Great!** Hmm, easier explanation...?

- ~~Dynamical mean field?~~
- ~~Some Markovian dynamics?~~
- In fact, stronger reduction holds true

Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)
Wolf, Eisert, Cubitt, Cirac, *Phys Rev Lett* **101**, 150402 (2008)

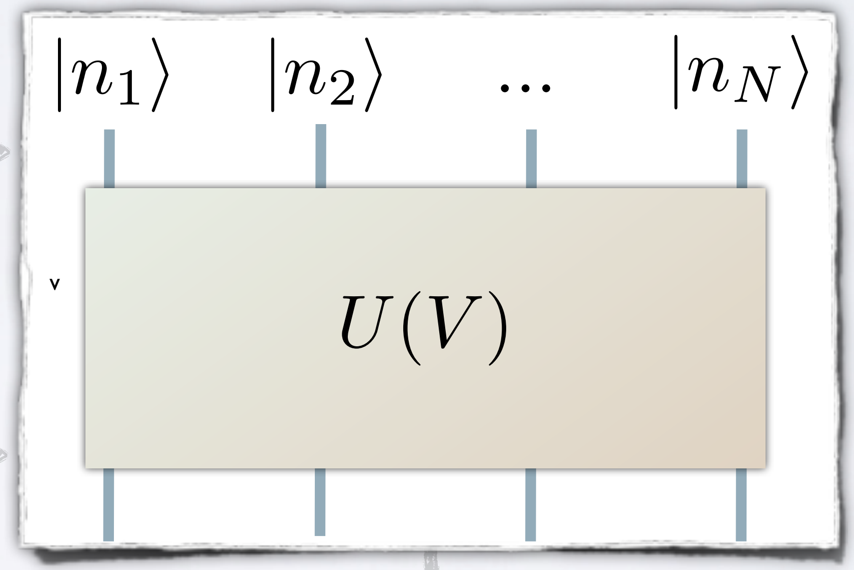
Boson sampling problem

Word of photon numbers, length N

Description of optical network V

$(b_1, \ldots, b_N)^T \mapsto V(b_1, \ldots, b_N)^T$

$\varepsilon > 0$



Sample from output number distribution up to error ε

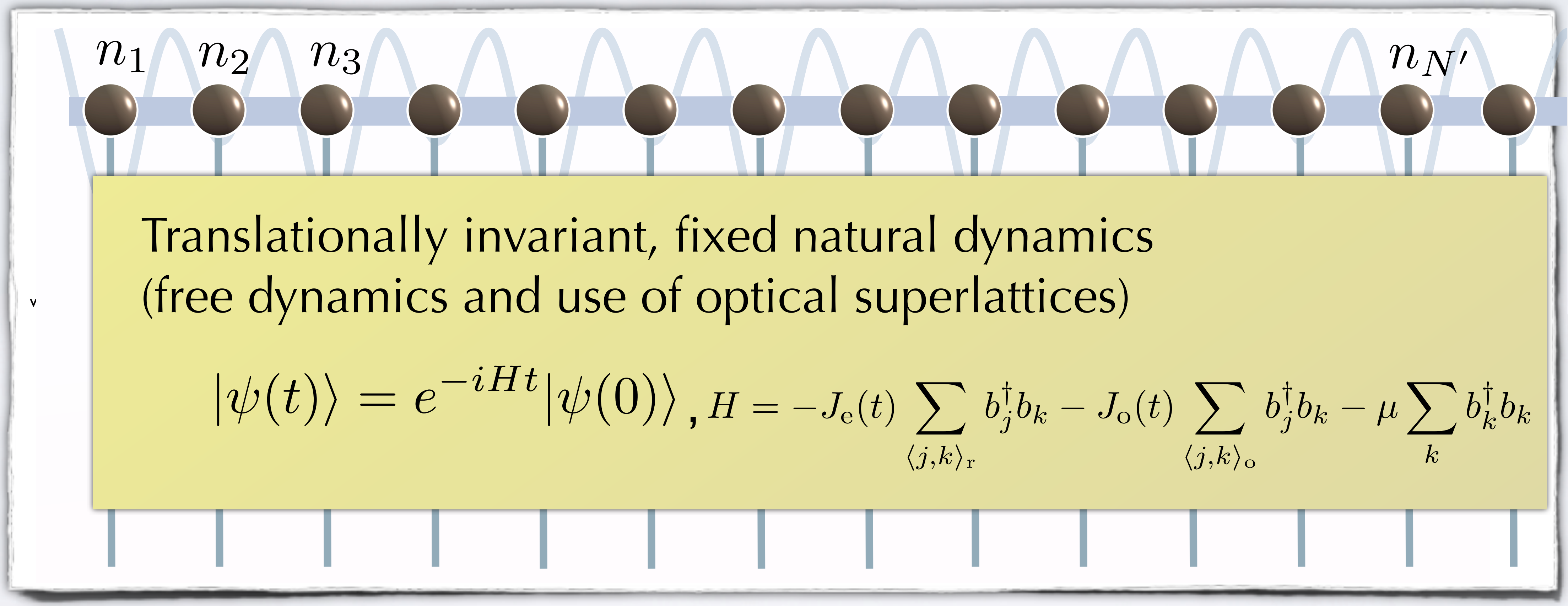
- **Claim:** Not believed to be universal for quantum computing - but, solves sampling problem, **classically intractable*** under plausible assumptions

* Efficient sampling up to exponentially small errors leads to collapse of polynomial hierarchy to third order, with poly accuracy also true, under reasonable conjectures

- **Obvious problems:**
  - Difficult do this optically for large number of modes
  - Arbitrary linear optical networks?

Aaronson, Arkhipov, arXiv:1011.3245
Rohde, Ralph, *Phys Rev A* **85**, 022332 (2012)
Scheel, quant-ph/0406127

Polynomial reduction to boson sampling



Translationally invariant, fixed natural dynamics
(free dynamics and use of optical superlattices)

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle\,,\; H = -J_{\mathrm{e}}(t)\sum_{\langle j,k\rangle_{\mathrm{r}}} b_j^\dagger b_k - J_{\mathrm{o}}(t)\sum_{\langle j,k\rangle_{\mathrm{o}}} b_j^\dagger b_k - \mu\sum_k b_k^\dagger b_k$$

- **Observation 8: Reduction to boson sampling problem using period-2**

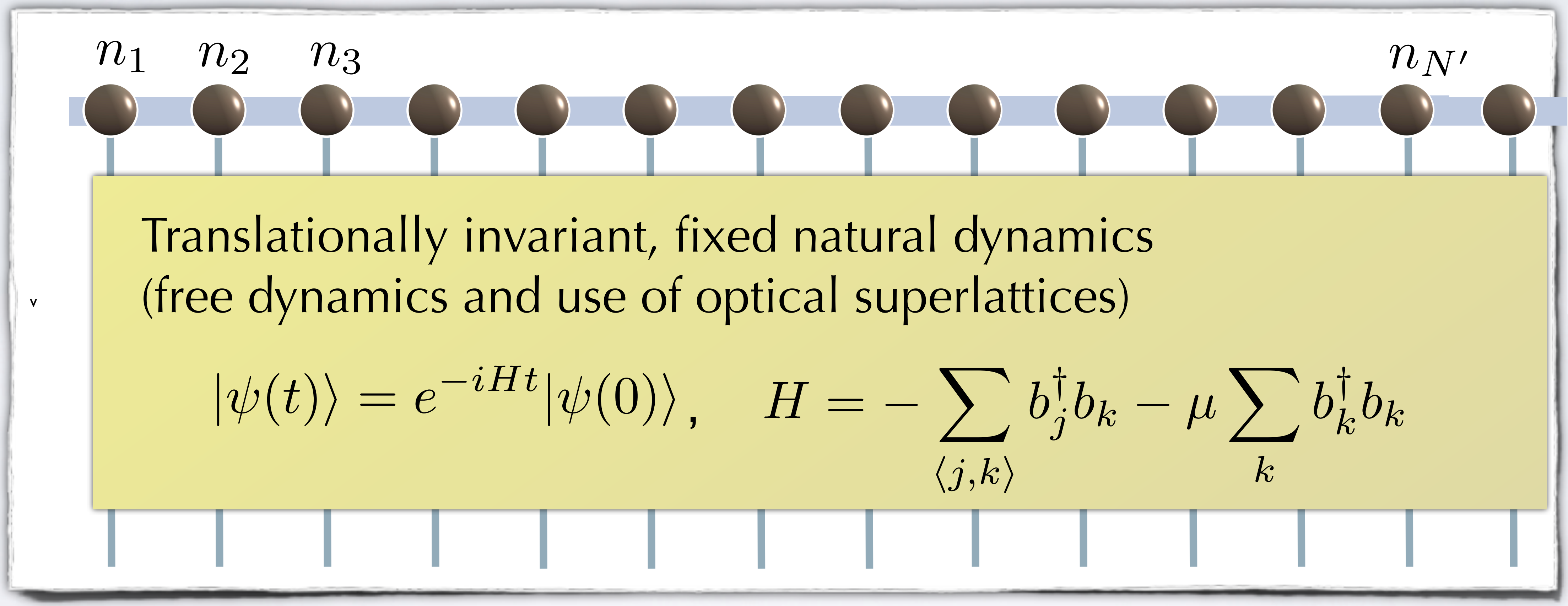
For any instance of the Boson sampling problem there exists an experiment with

- *Initial product state* in optical lattice
- *Natural dynamics under free limit of Bose-Hubbard Hamiltonian* + superlattices
- Measurement of *boson number*

poly overhead, giving rise to same distribution (up to exponentially small) errors

Eisert, Kliesch, in progress (2012)

Polynomial reduction to boson sampling



Translationally invariant, fixed natural dynamics
(free dynamics and use of optical superlattices)

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle, \quad H = -\sum_{\langle j,k\rangle} b_j^\dagger b_k - \mu \sum_k b_k^\dagger b_k$$

- **Observation 9: Reduction to boson sampling problem**

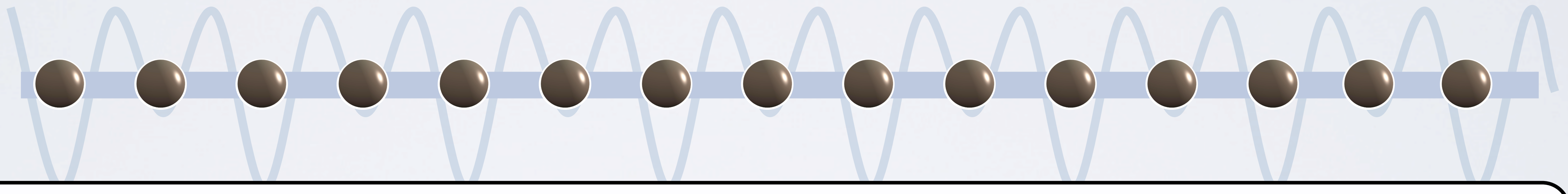
For any instance of the Boson sampling problem there exists an experiment with

- *Initial product state* in optical lattice
- *Natural dynamics under free limit of Bose-Hubbard Hamiltonian*
- Measurement of *boson number*

poly overhead, giving rise to same distribution (up to poly small) errors

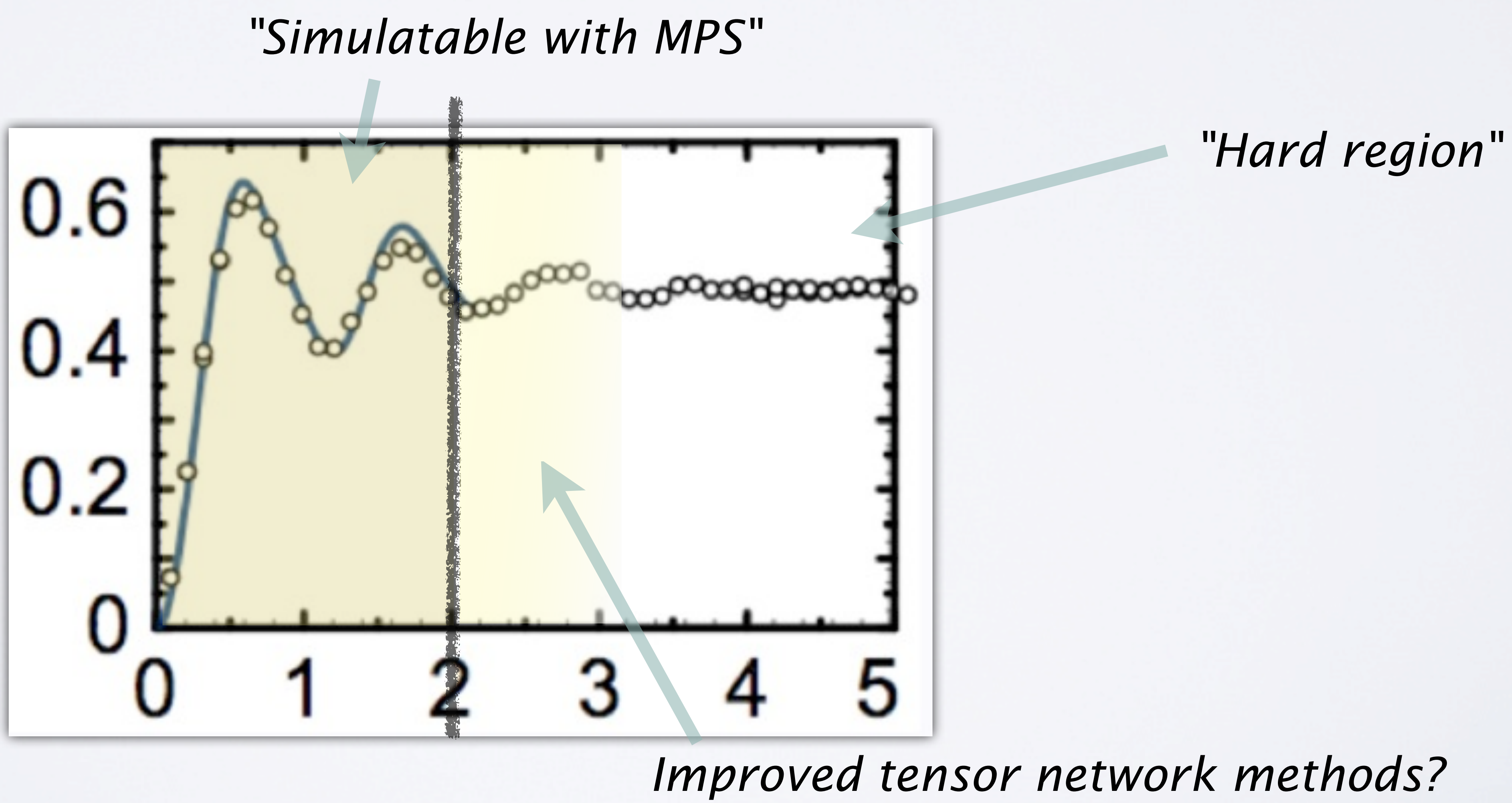
Eisert, Kliesch, in progress (2012)

Quantum dynamical simulator

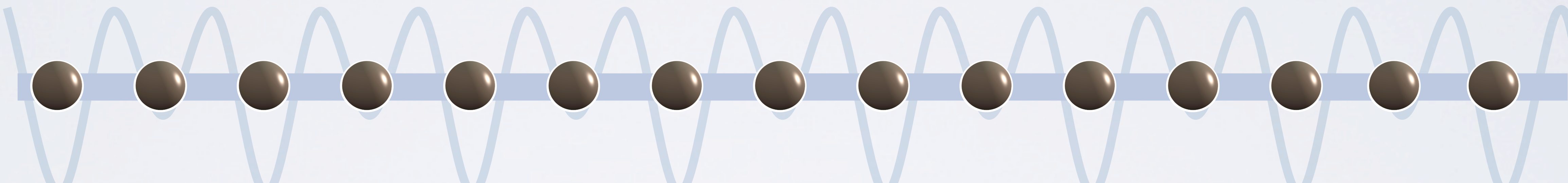


- **Hardness of Bose-Hubbard simulation**

... is (in the above sense) classically a hard problem



Quantum dynamical simulator



- **Hardness of Bose-Hubbard simulation**

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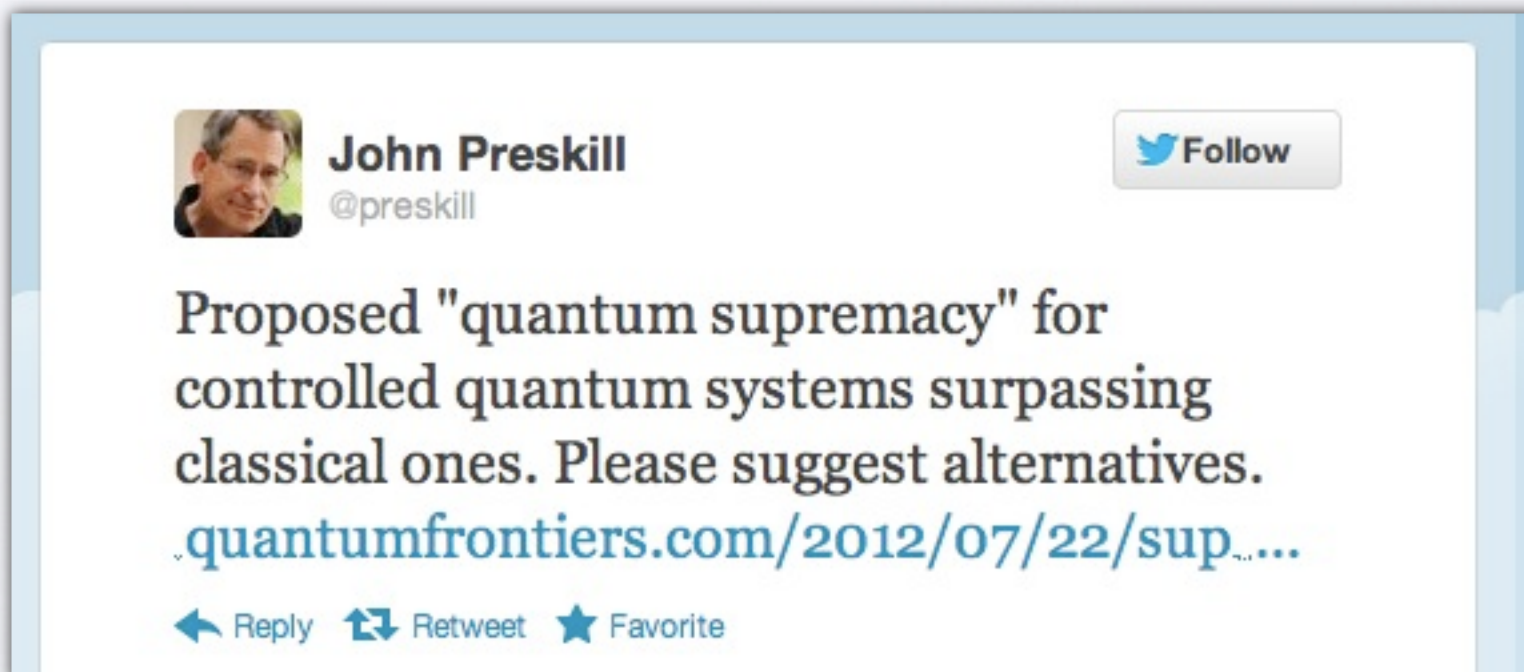
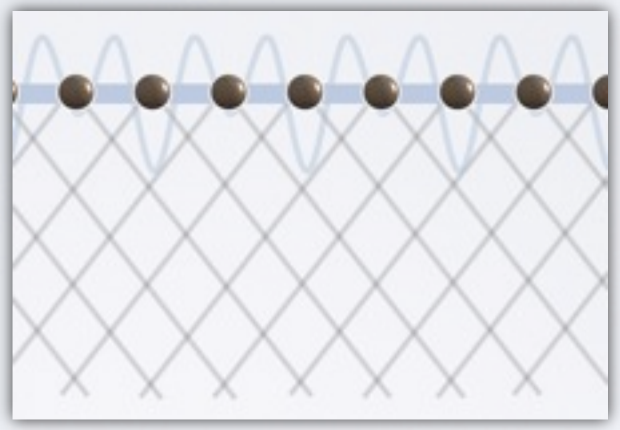
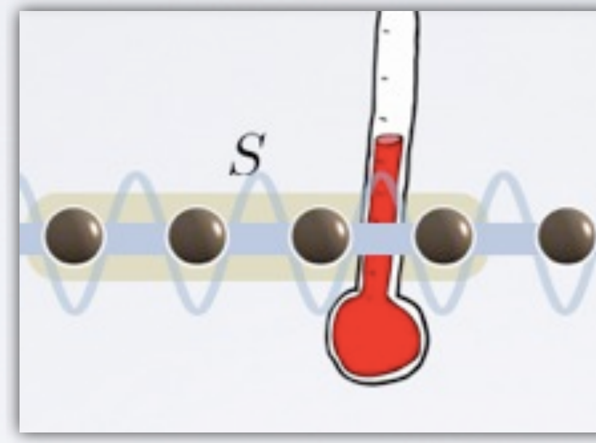


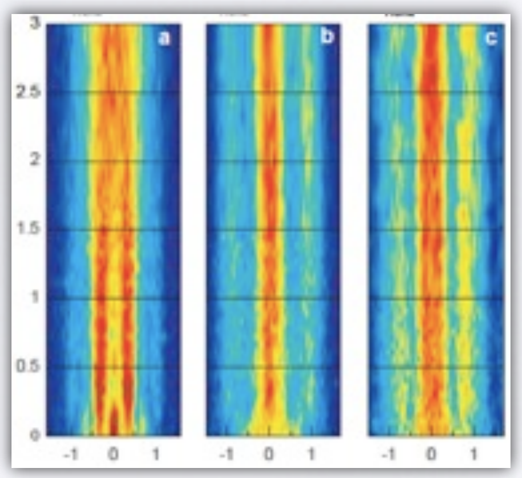
Summary and outlook



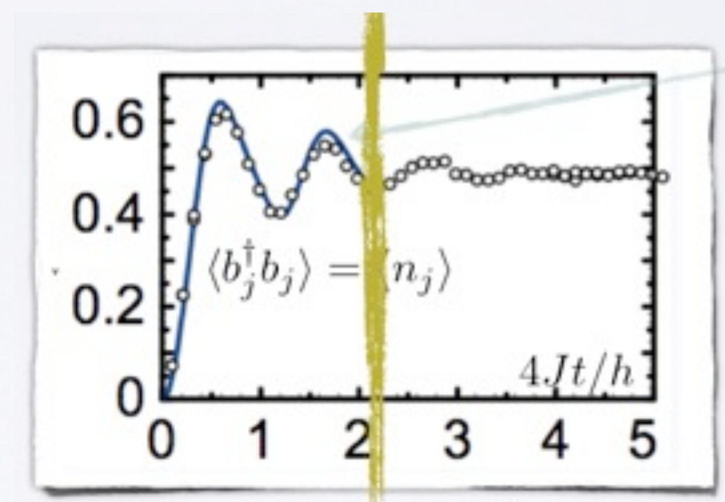
- Equilibration of many-body systems



- Thermalization and integrability



- An experiment



- A "dynamical quantum simulator"

Thanks for your attention!