

Probing Non-Equilibrium Dynamics Using Ultracold Quantum Gases in Optical Lattices

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Outline

Introduction

Single Atom Imaging

Applications

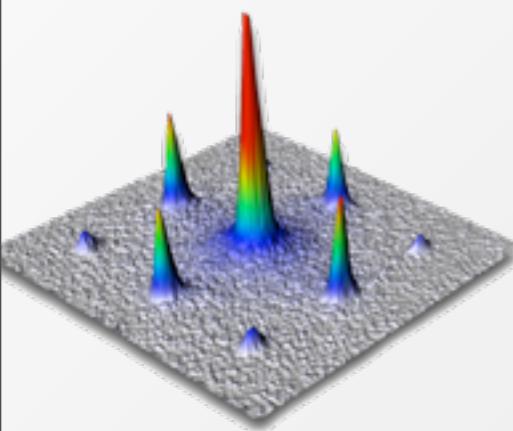
Tracking the Motion of a Single Spin Impurity

Observation of Crystalline States of Rydberg Atoms

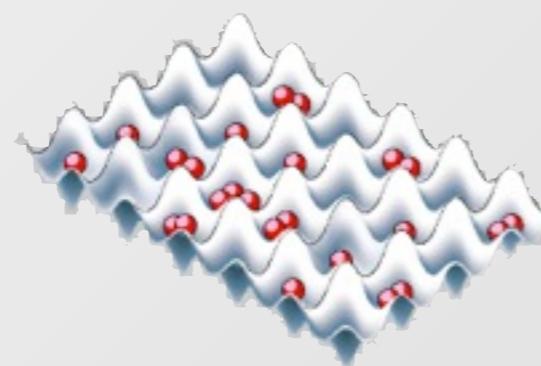
Measuring Zak-Berry Phase of Topological Bloch Bands

From Weak to Strong Interactions

$$\gamma = \frac{\text{Interaction Energy}}{\text{Kinetic Energy}} \gg 1$$



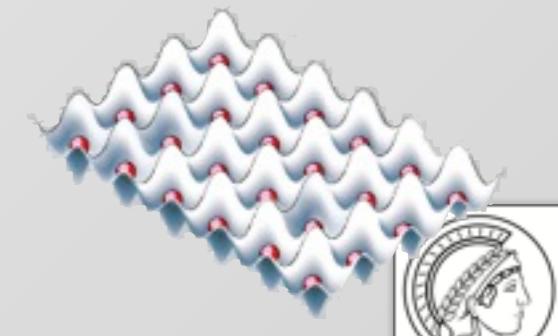
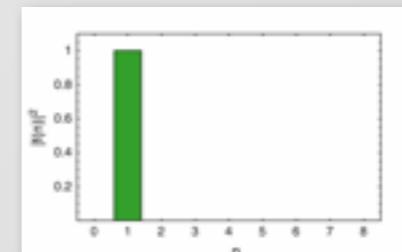
Weak Interactions



Quantum Phase Transition

See S. Sachdev & B. Keimer Phys. Today 2011

Strong Interactions

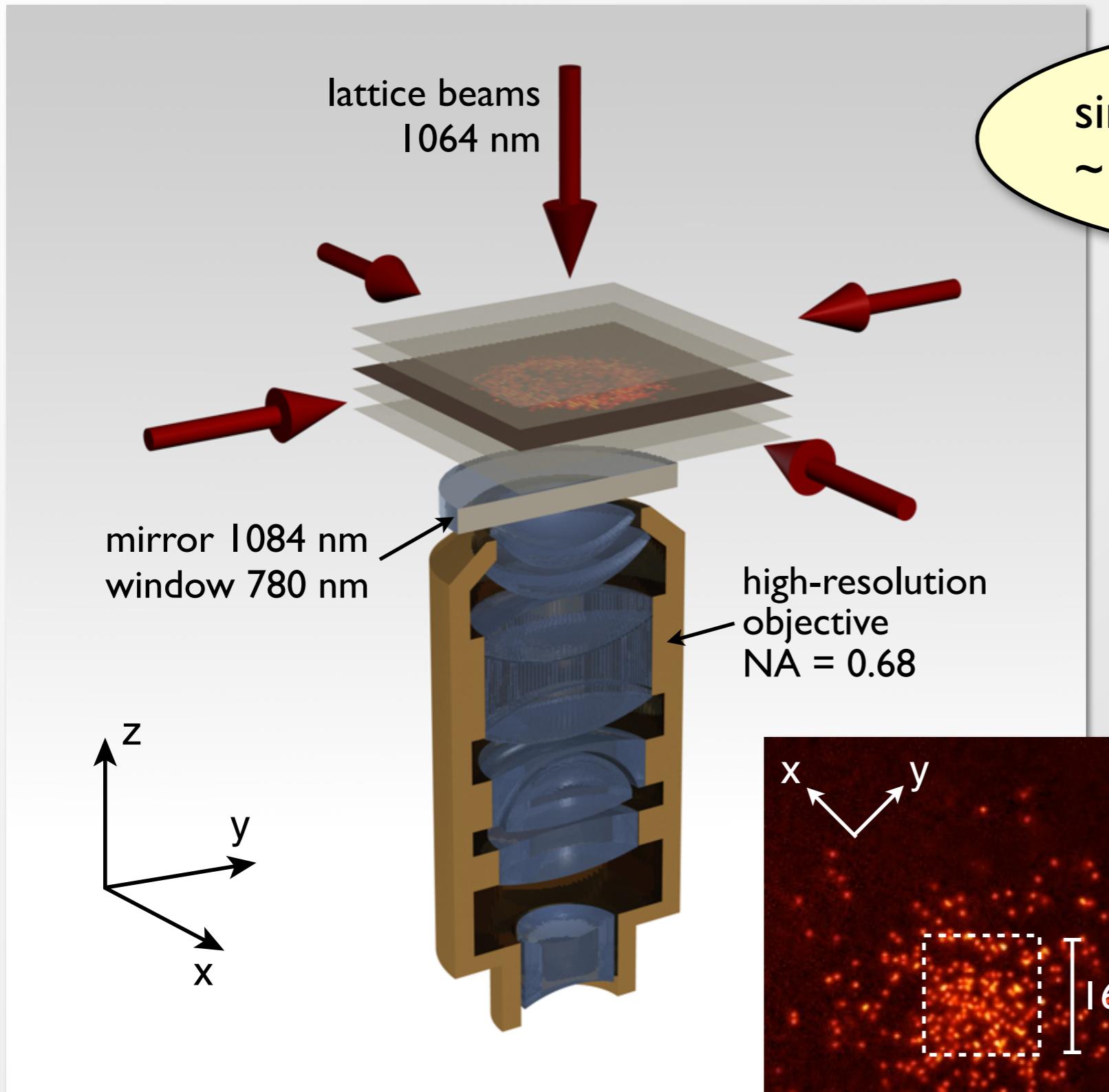


Single Atom Detection in a Lattice

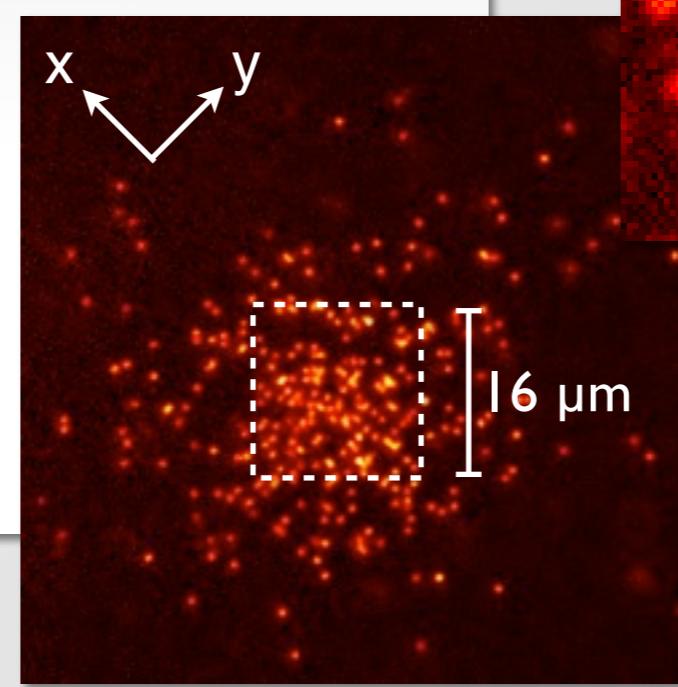
J. Sherson, Ch. Weitenberg, M. Endres, M. Cheneau, T. Fukuhara, P. Schauss, J. Sherson, I.B., S. Kuhr

Sherson et al. Nature 467, 68 (2010),
see also Bakr et al. Nature (2009) & Bakr et al. Science (2010)

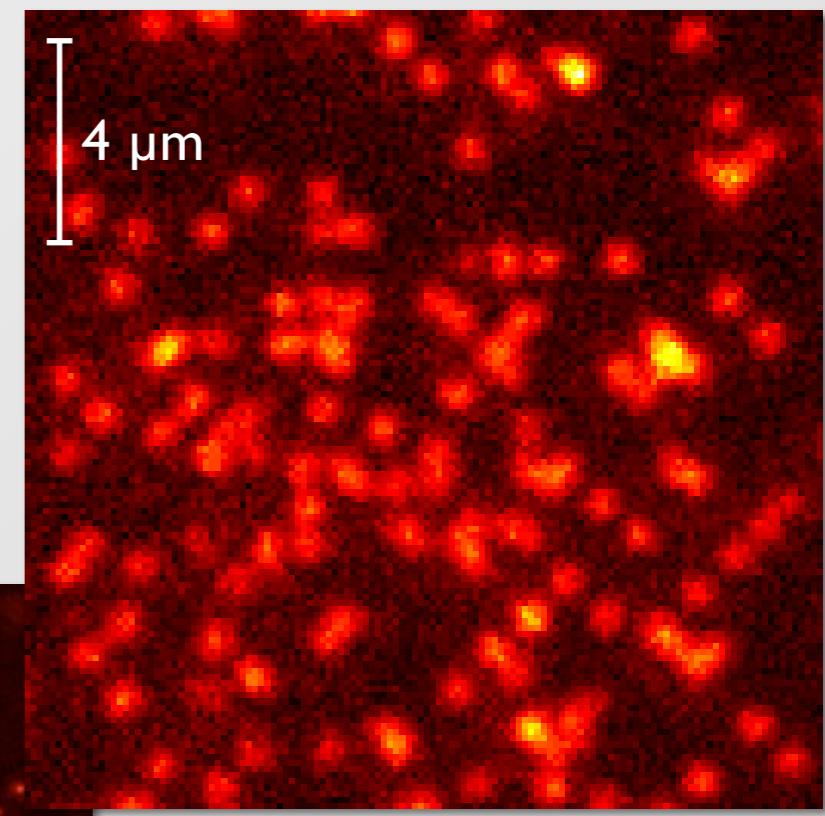
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see also: Harvard (M. Greiner group)

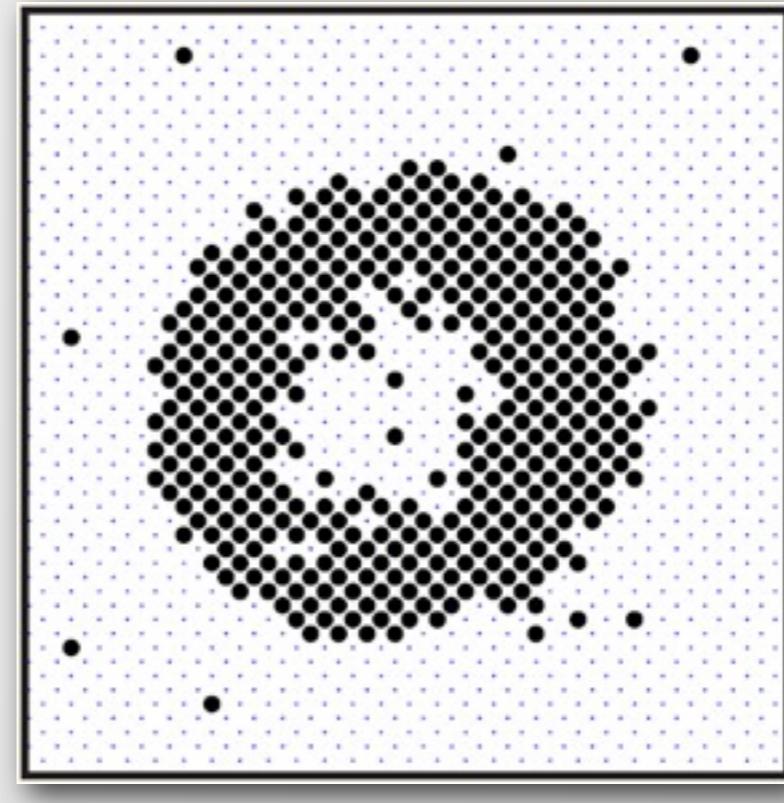
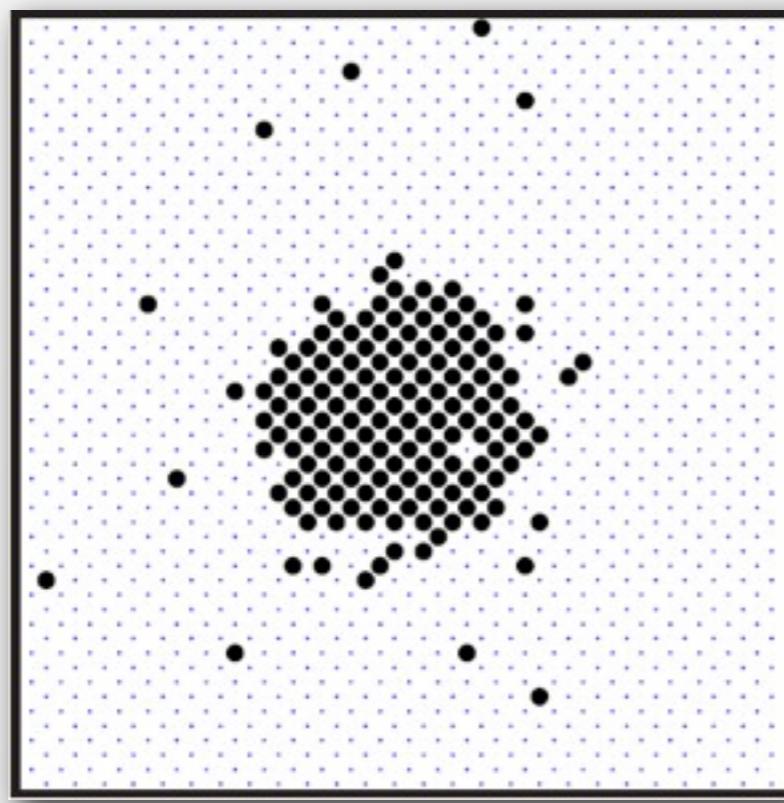
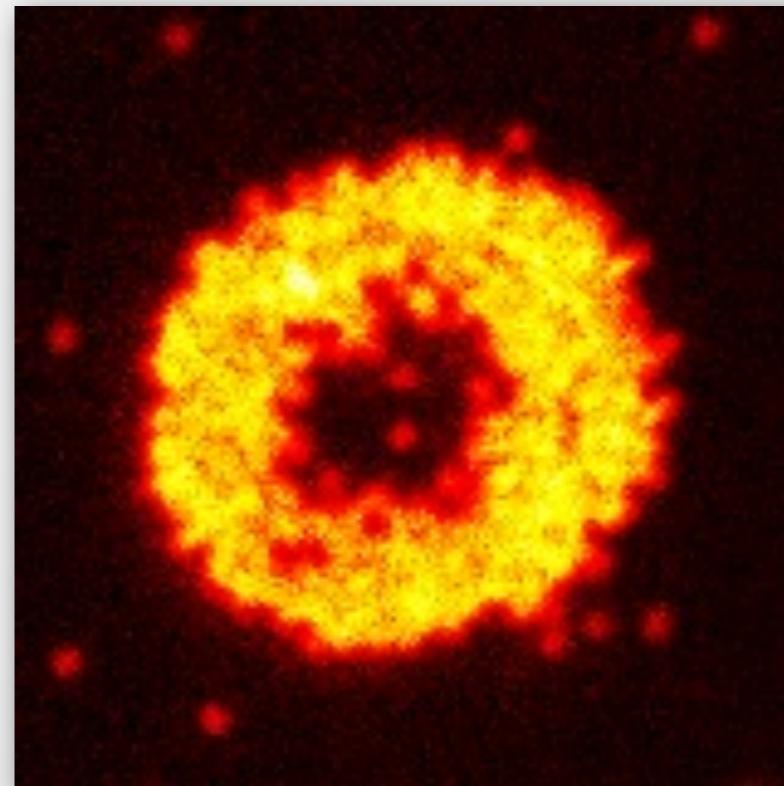
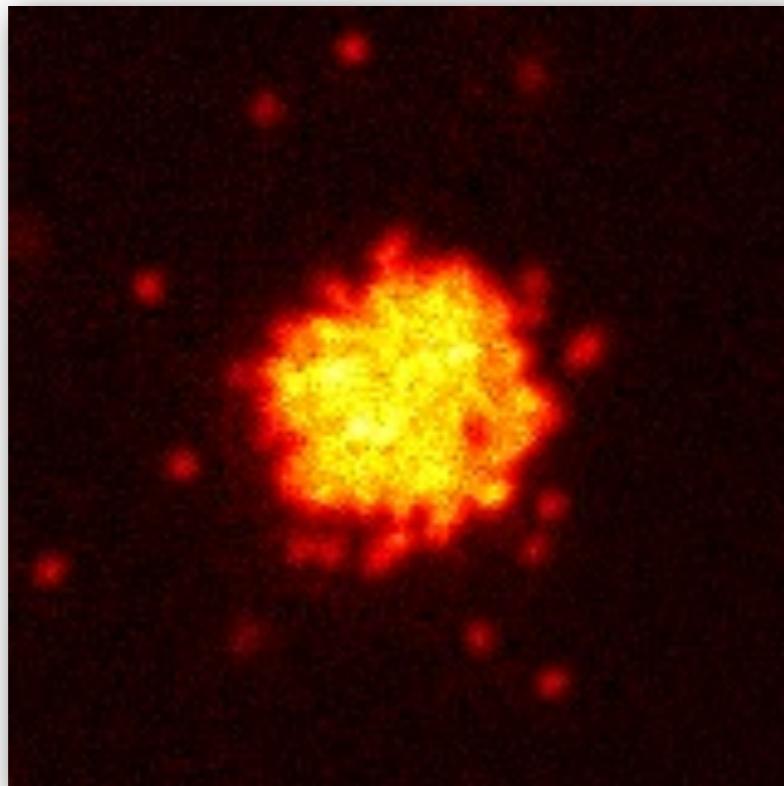


single 2D degenerate gas
~ 1000 ^{87}Rb atoms (bosons)



resolution of the
imaging system:
~700 nm

Images of Mott Insulators

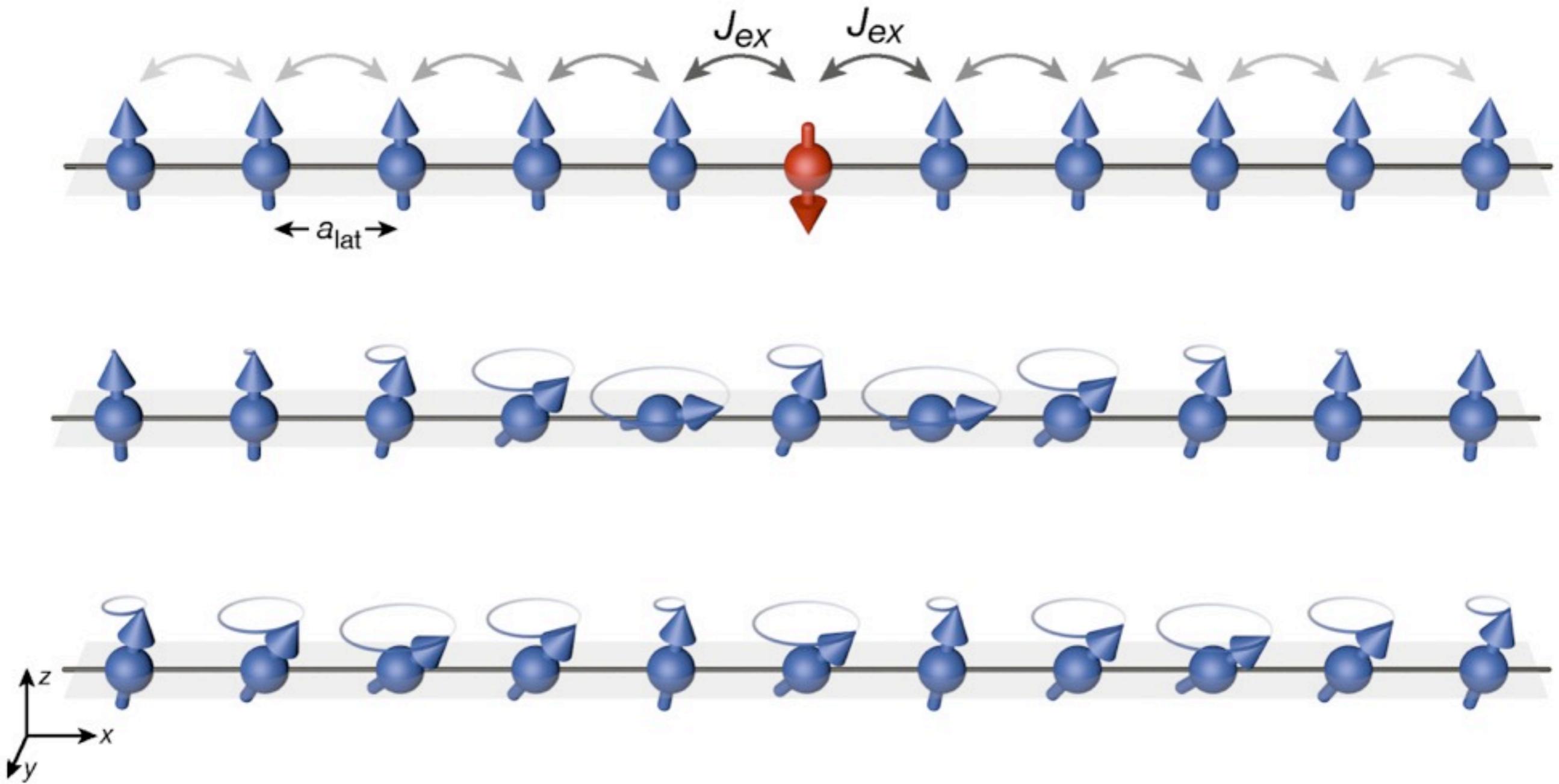


Quantum Dynamics of Mobile Single-Spin Impurity

T. Fukuhara, M. Endres, M. Cheneau P. Schauss, Ch. Gross, I. Bloch, S. Kuhr,
U. Schollwöck, A. Kantian, Th. Giamarchi



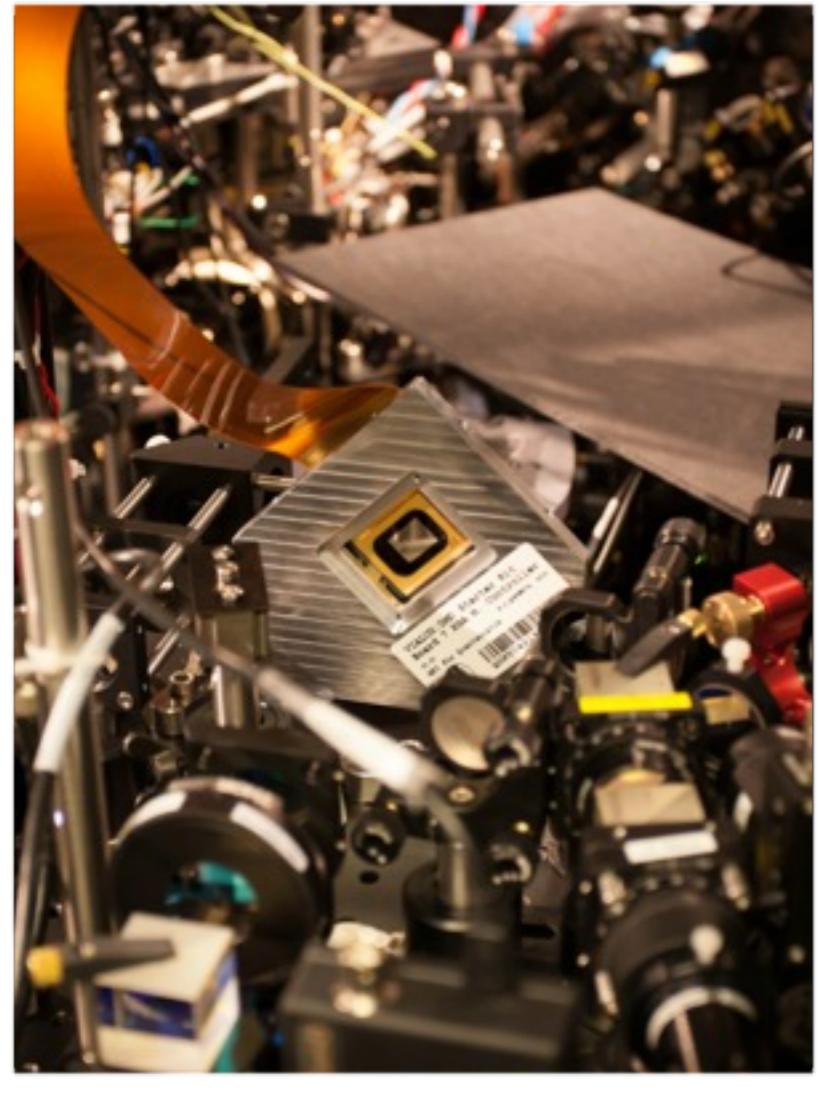
www.quantum-munich.de



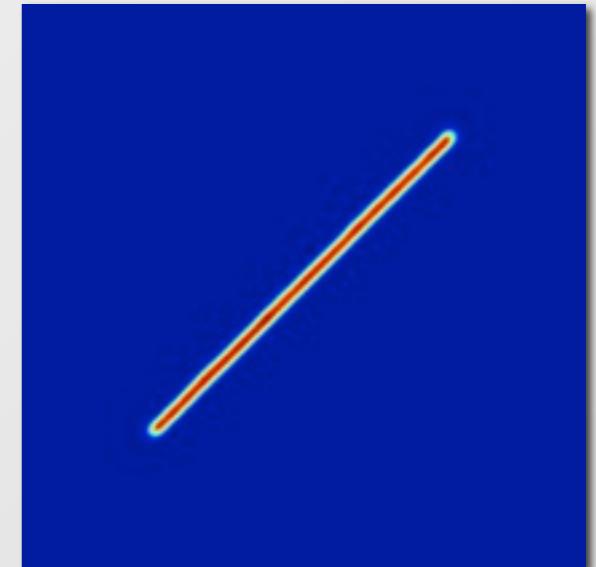
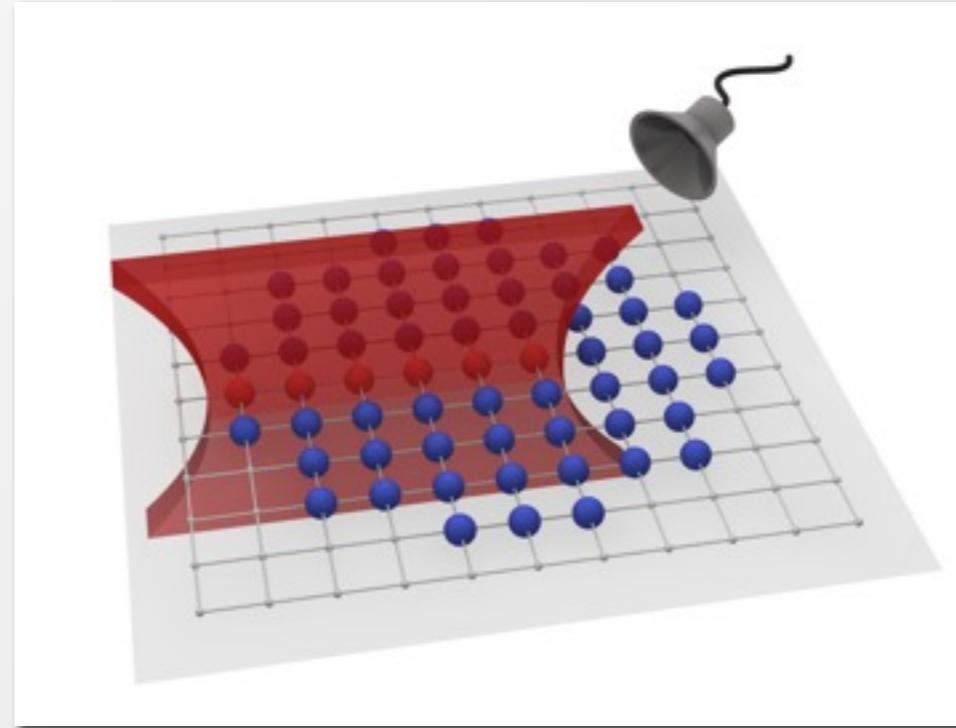
$$-J_{\text{ex}} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Ferromagnetic Heisenberg Interaction

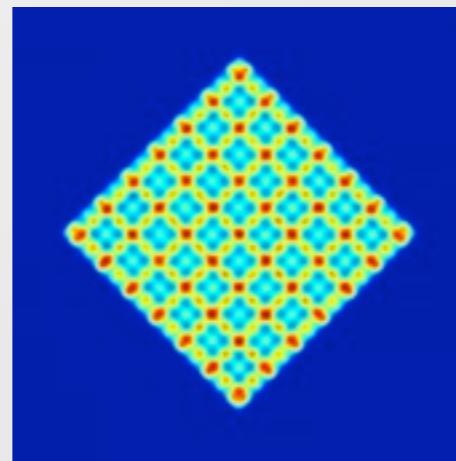
Arbitrary Light Patterns



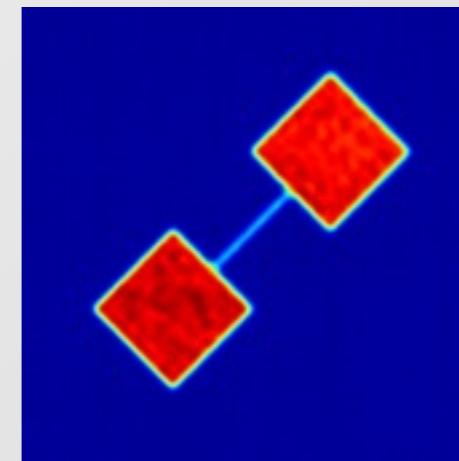
Digital Mirror Device
(DMD)



Measured Light Pattern



Exotic Lattices



Quantum Wires

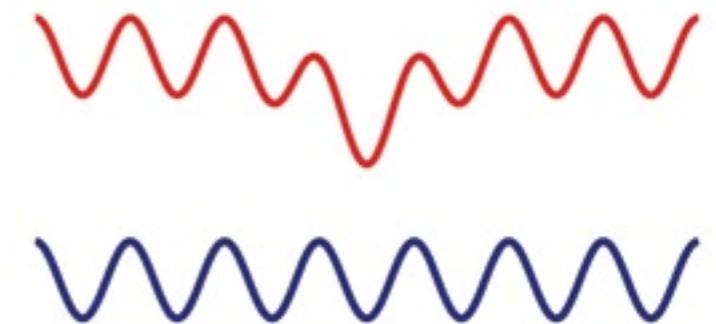
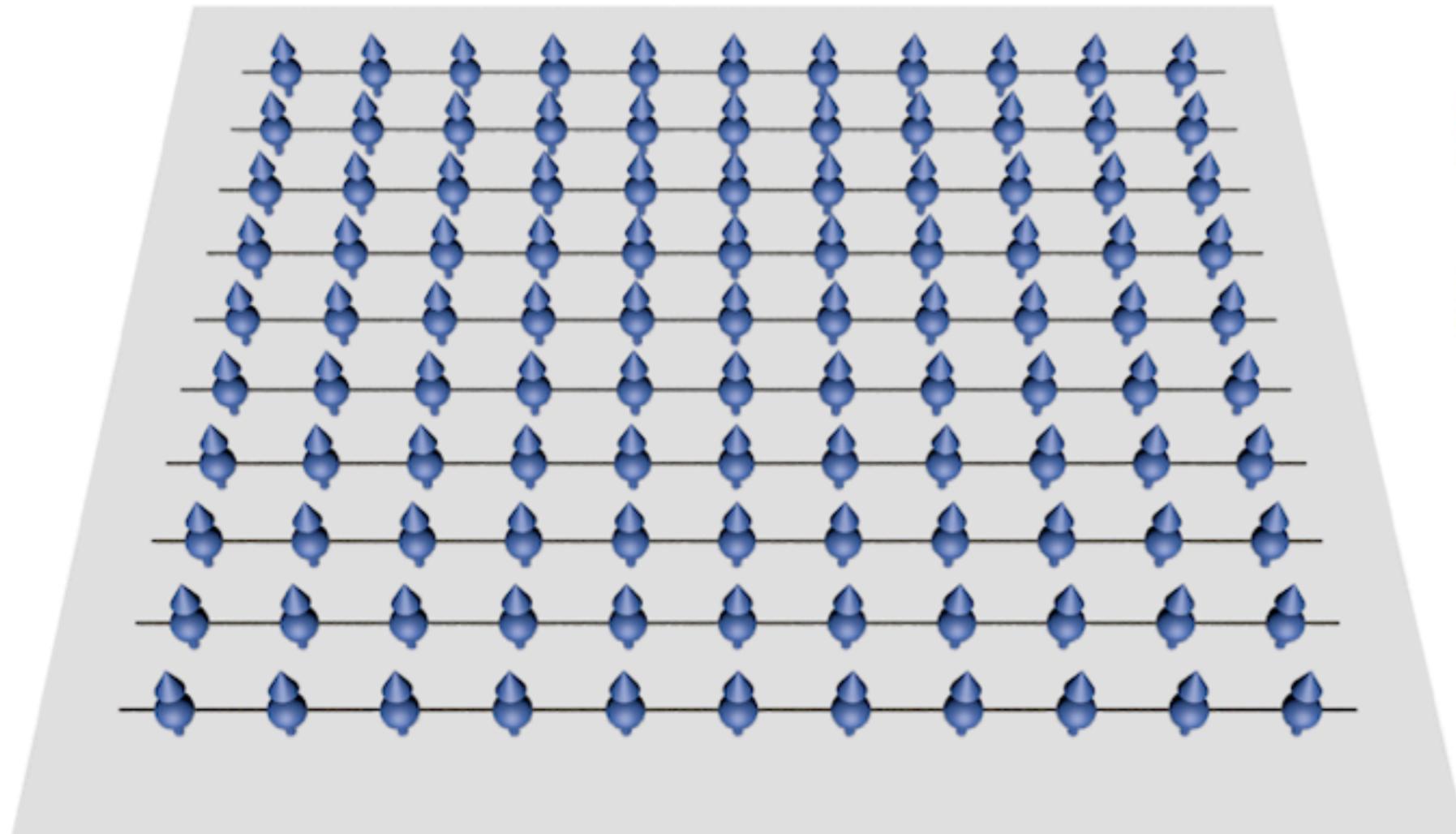


Box Potentials

Almost Arbitrary Light Patterns Possible!

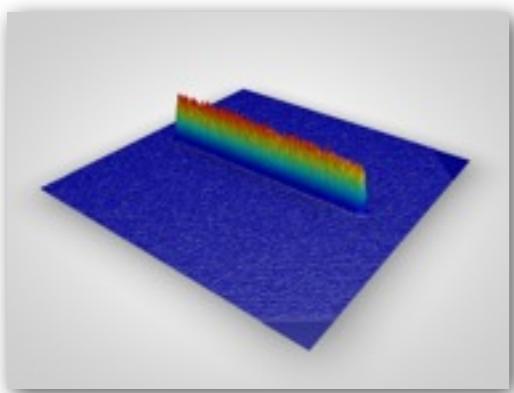
Single Spin Impurity Dynamics, Domain Walls, Quantum Wires, Novel Exotic Lattice Geometries, ...





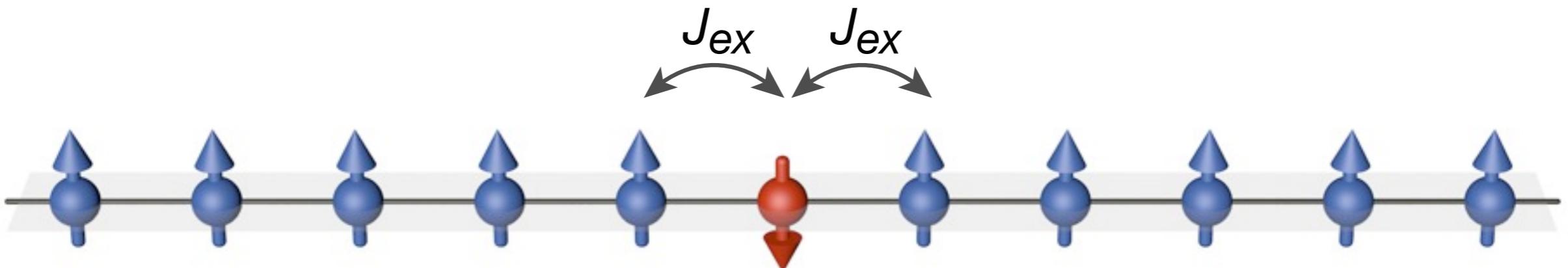
$|2\rangle = |F=2, m_F=-2\rangle$

$|1\rangle = |F=1, m_F=-1\rangle$



Line-shaped light field created with DMD SLM





Heisenberg Hamiltonian

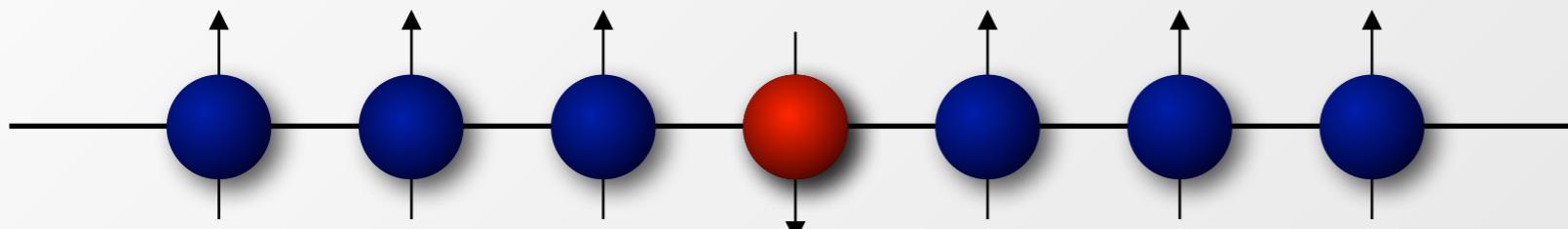
$$\begin{aligned}
 H &= -J_{ex} \sum \mathbf{S}_i \cdot \mathbf{S}_j = -J_{ex} \sum \left(S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z \right) \\
 &= -\frac{J_{ex}}{2} \sum \left(S_i^+ S_j^- + S_i^- S_j^+ \right) \cancel{- J_{ex} \sum S_i^z S_j^z} \quad J_{ex} = 4 \frac{J^2}{U}
 \end{aligned}$$

$$H = -J \sum \left(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_i \hat{a}_j^\dagger \right) \text{ single particle tunneling}$$

Our detection is not spin-resolved.

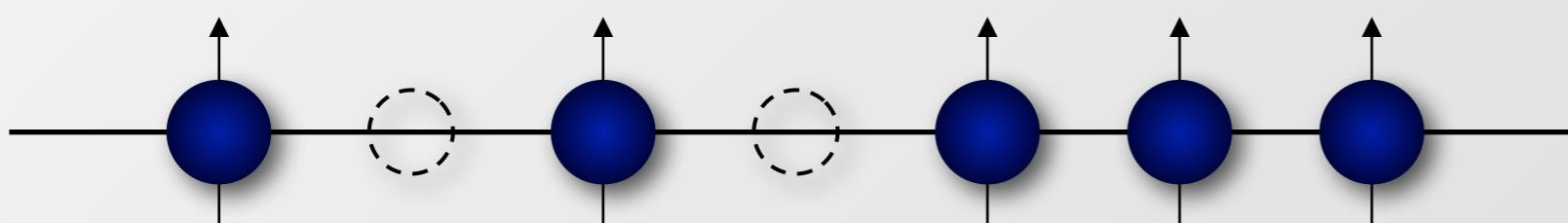
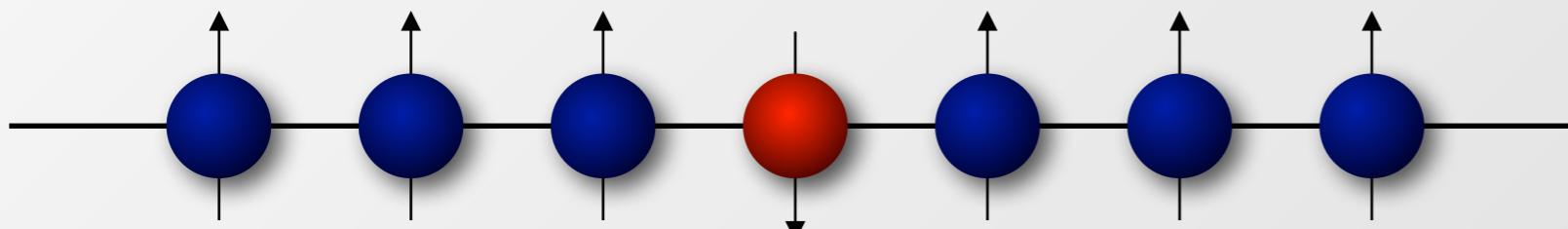
Positive image

Remove the other spin component (bath spin) before detection



Negative image

Measure spin impurities as empty sites by pushing out the impurities



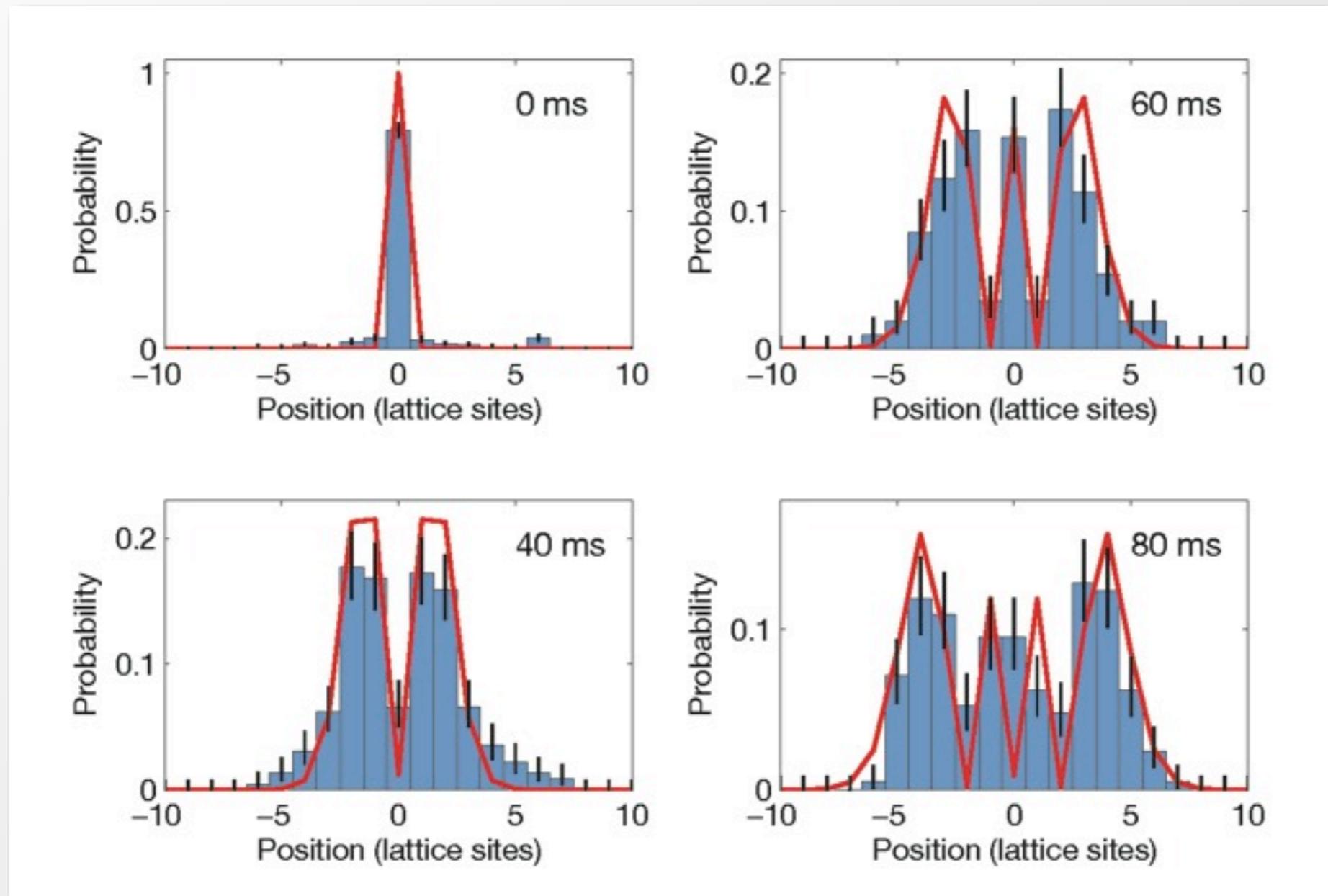
If we only use samples with one empty site ...

- the position of the impurity can be determined
- system has no extra excitation



" zero
temperature"

Coherent quantum dynamics of single spin at zero temperature



$V = 10 \text{ } E_r$
 $U/J = 19$

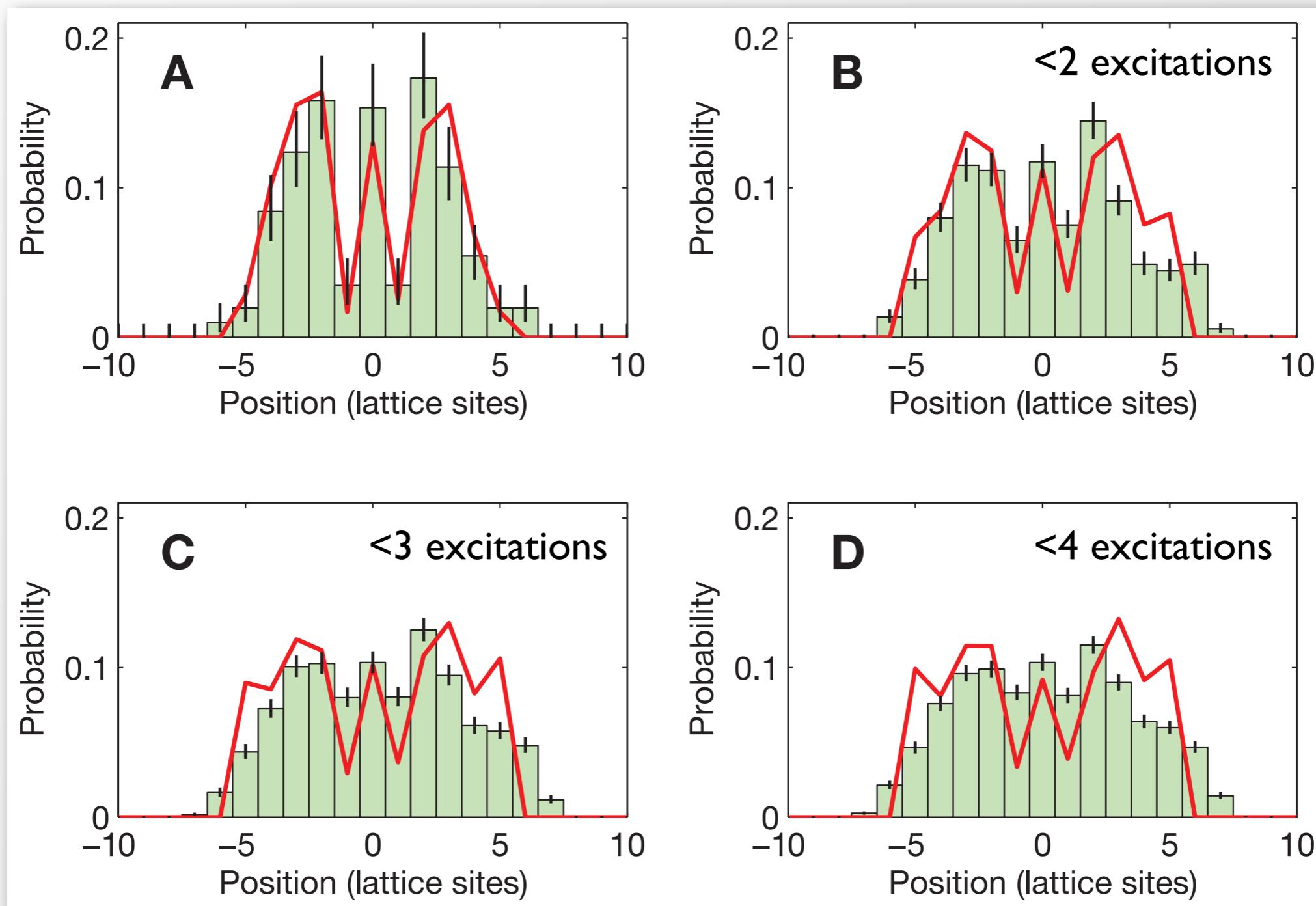
$$H = -\frac{J_{ex}}{2} \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+)$$

$$P_j(t) = \left[J_j \left(\frac{J_{\text{ex}} t}{\hbar} \right) \right]^2$$

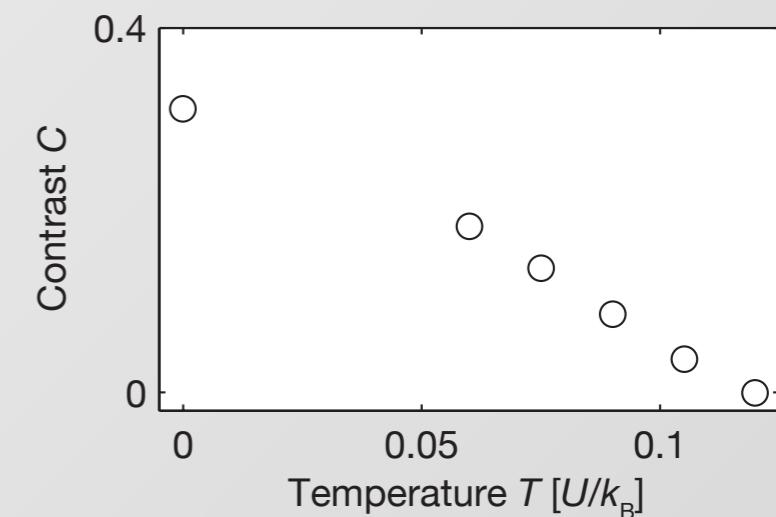
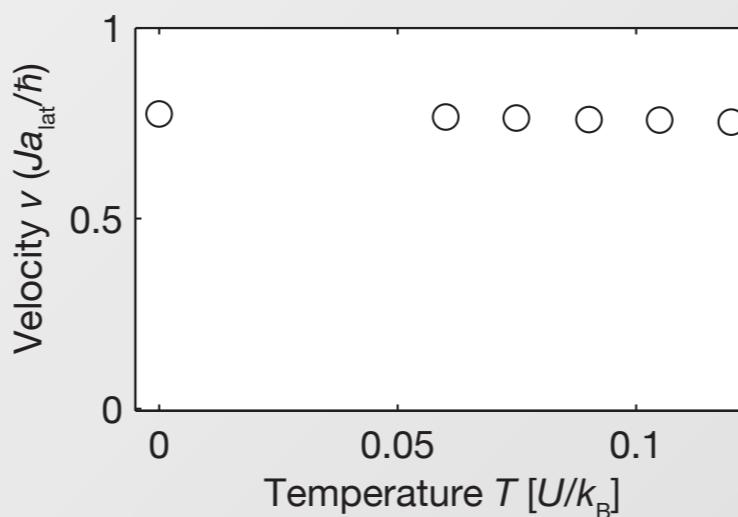
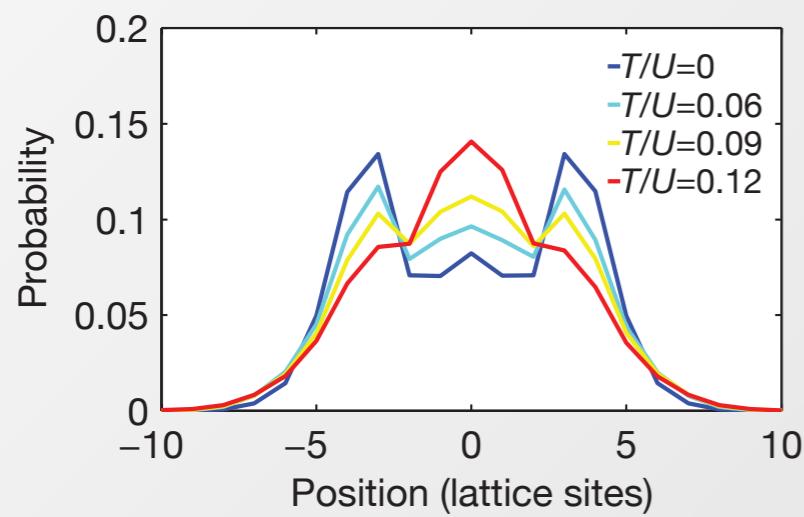
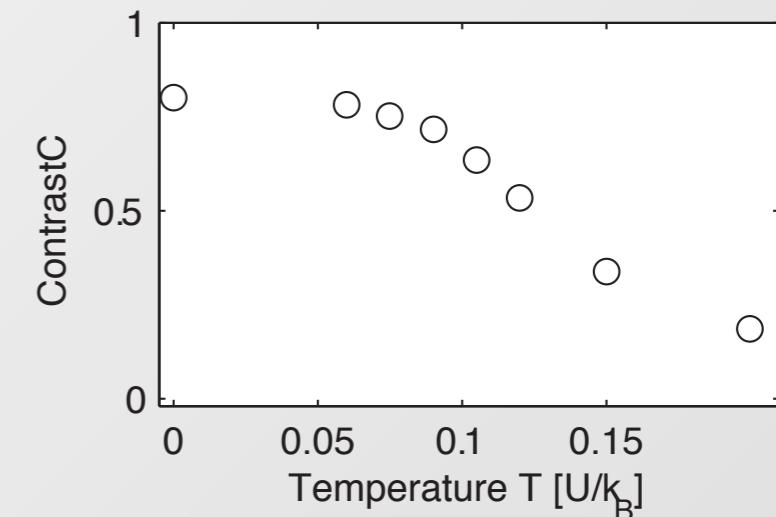
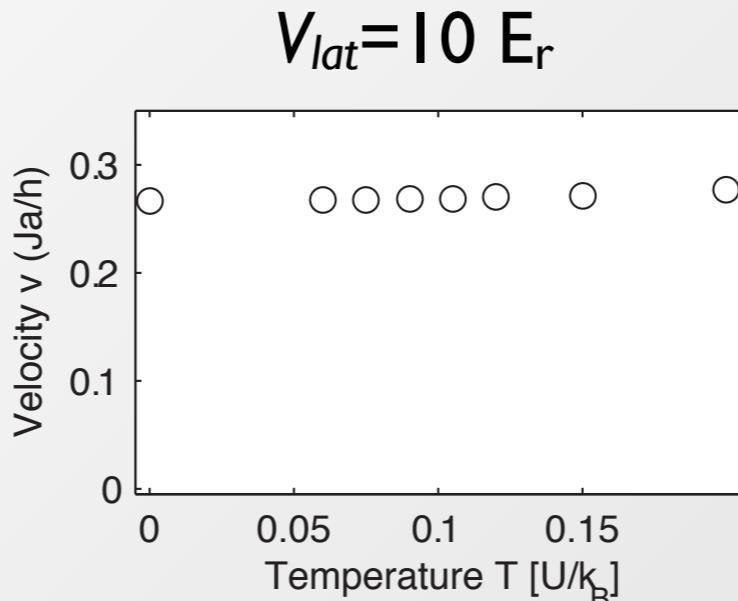
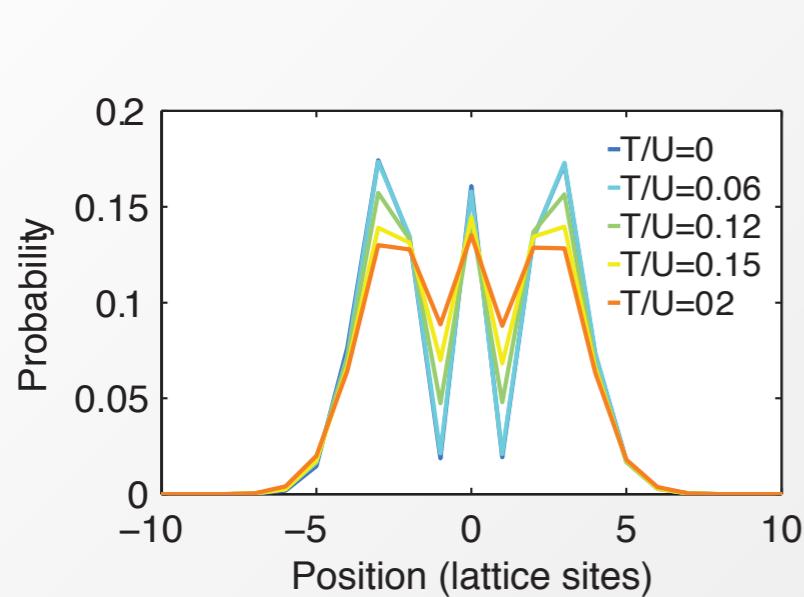
Bessel function of the first kind



Effect from hole excitations



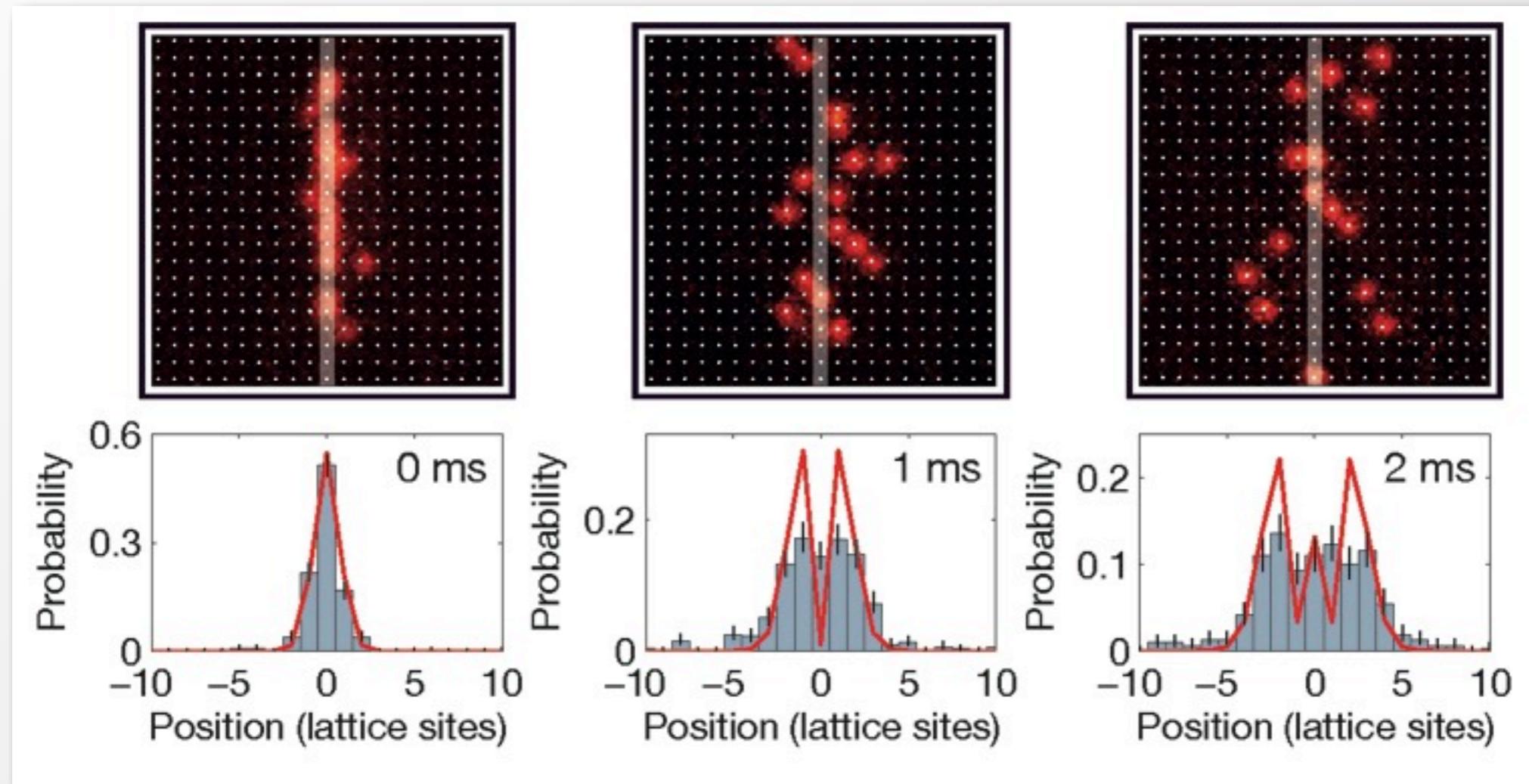
- Only visibility goes down
- Spreading speed almost independent of holes



Propagation speed almost constant, Coherence affected!



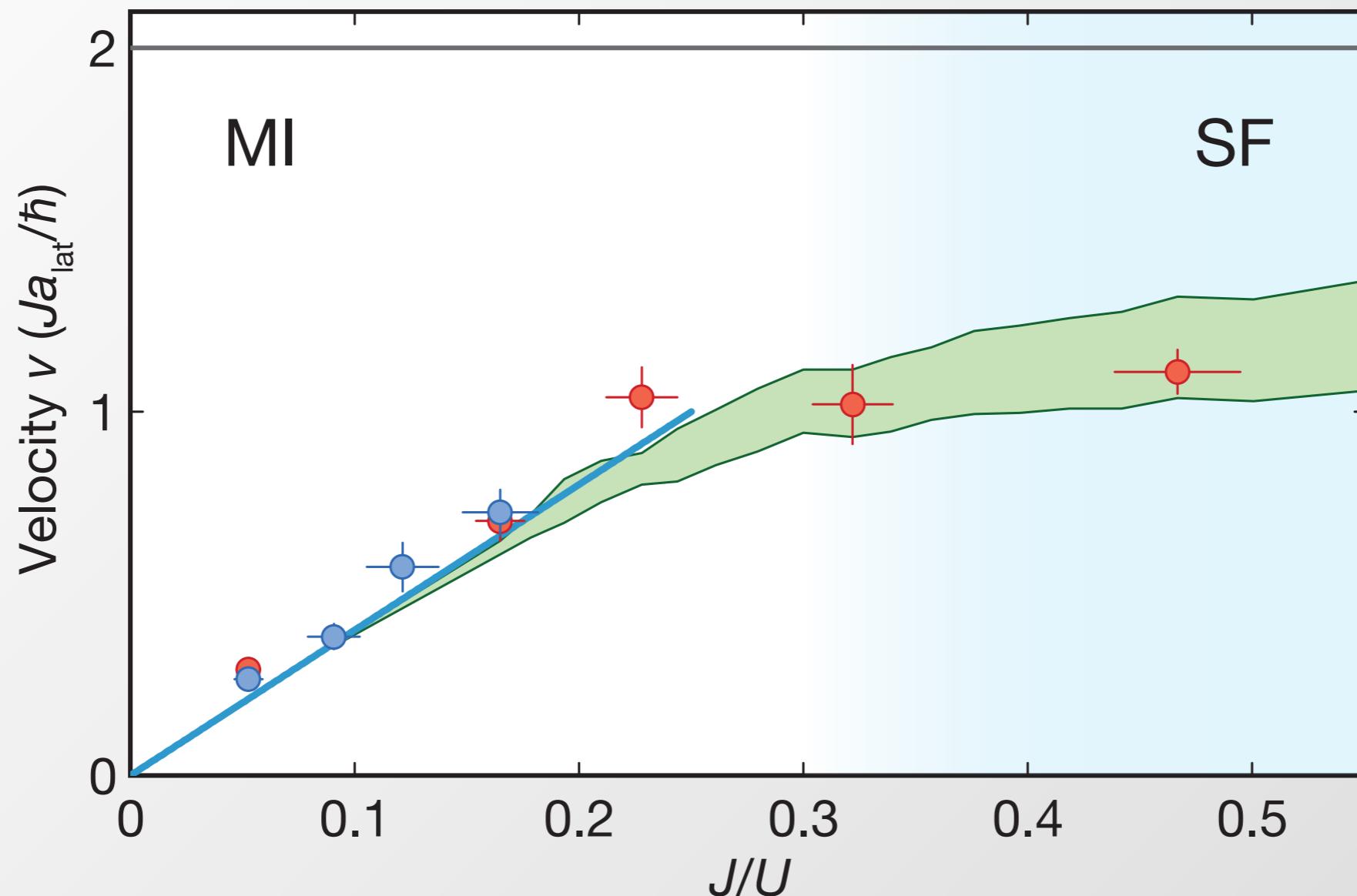
LMU



$$V = 4 \text{ } Er, J/U = 0.32$$
$$(J/U)_c \approx 0.3 \text{ for 1D}$$

Ramp-down time for pinning beam is 1 ms





Reference value:

$$v = \frac{2J a_{\text{lat}}}{\hbar} \quad \text{Free particle tunneling}$$

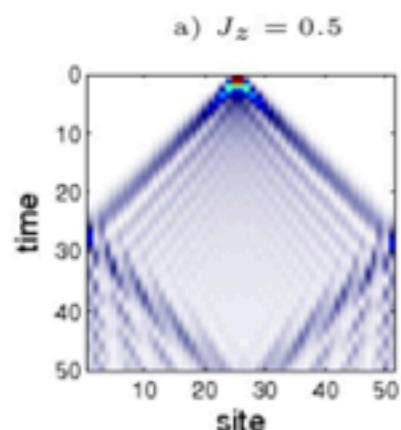


Quantum Dynamics of Interacting Atoms/Spins

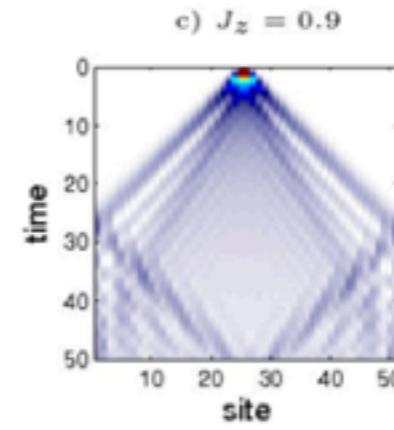
- Effect of Temperature/Holes on Dynamics
- Dynamics of Magnon bound states
- Domain Walls
- Higher Dimensions (1D, 2D, 3D)
- Entropy Transport
- Probe for Quantum Critical Transport
- Direct measurement of Green's function

$$G(x_i, x_j, t) \propto \langle \uparrow | \hat{S}^\dagger(x_j, t) \hat{S}^-(x_i, 0) | \uparrow \rangle$$

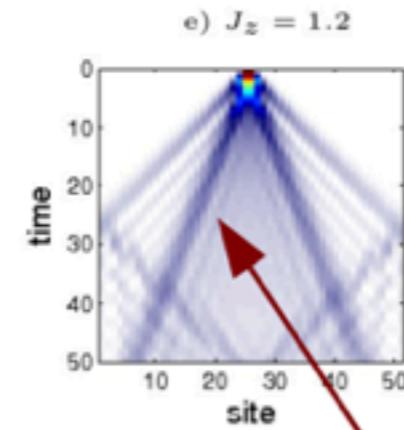
Two-spin excitation in FM



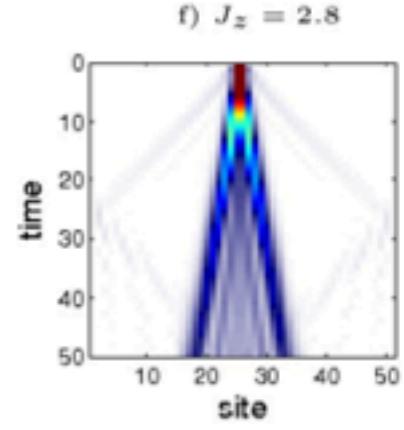
$J_z = 0.5$



$J_z = 0.9$

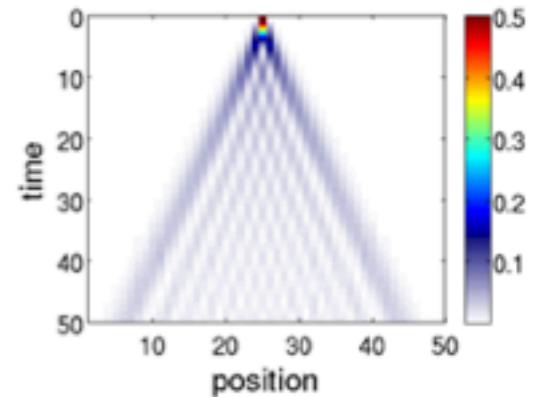


$J_z = 1.2$

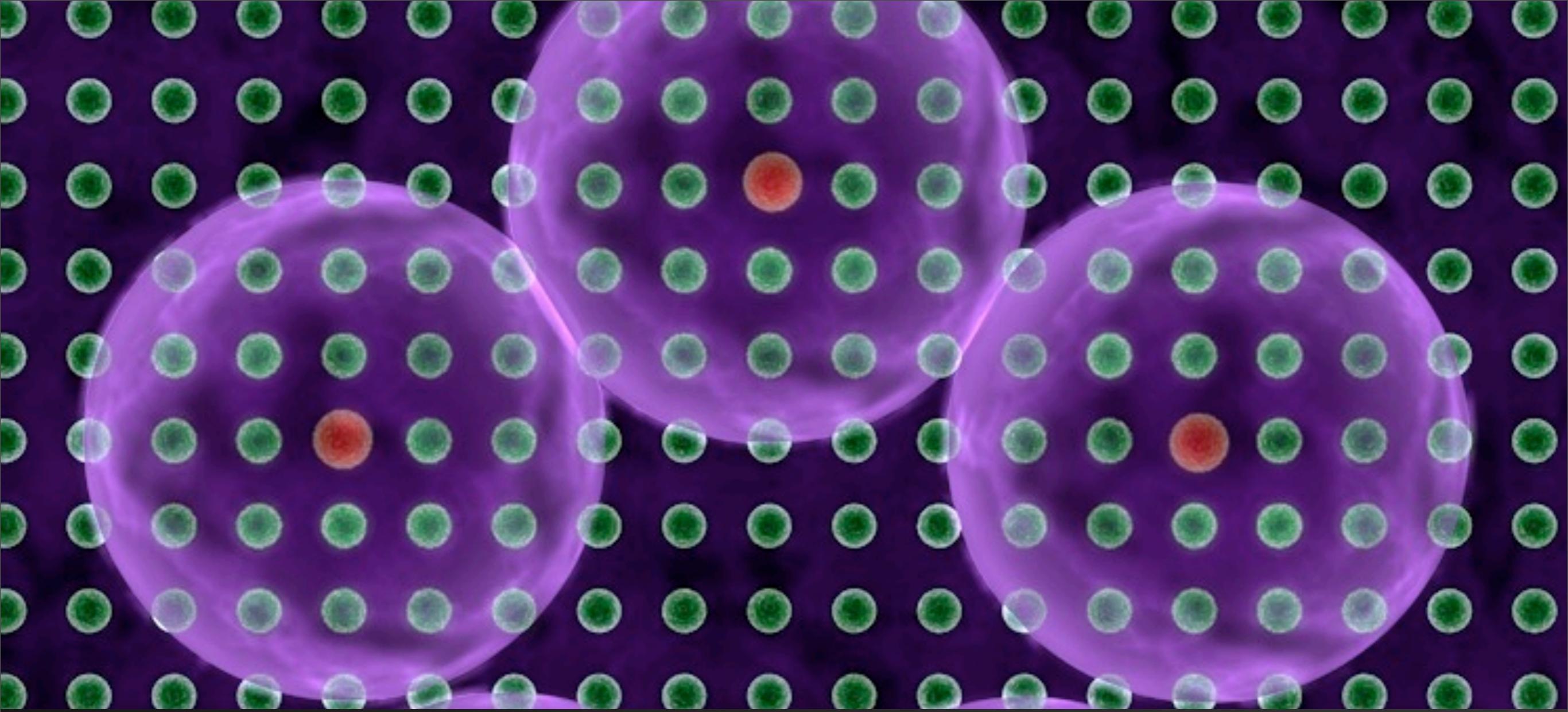


$J_z = 2.8$

- Two distinct propagation branches beyond $J_z = 0.7$
- New lower branch is *bound state*
- It dominates at large J_z , with decreasing velocity
- Low entanglement entropy (see below)

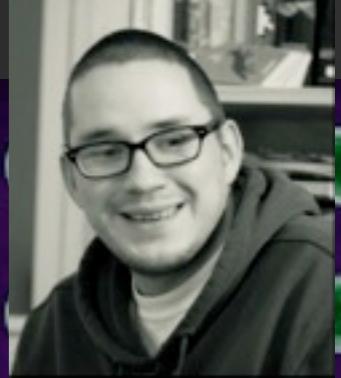


From: H.G. Evertz



Dynamical Formation of Rydberg Crystals

P. Schauss , M. Cheneau, M. Endres, T. Fukuhara, Th. Pohl, S. Kuhr & I.B.



(submitted)

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Rydberg atoms

- hydrogen-like wave function
 - quantum defect

$$E_{nlj} = - \frac{Ry}{[n - \delta_{lj}(n)]^2}$$

- Strong switchable interactions

$^{87}\text{Rb } 43\text{S}_{1/2}$

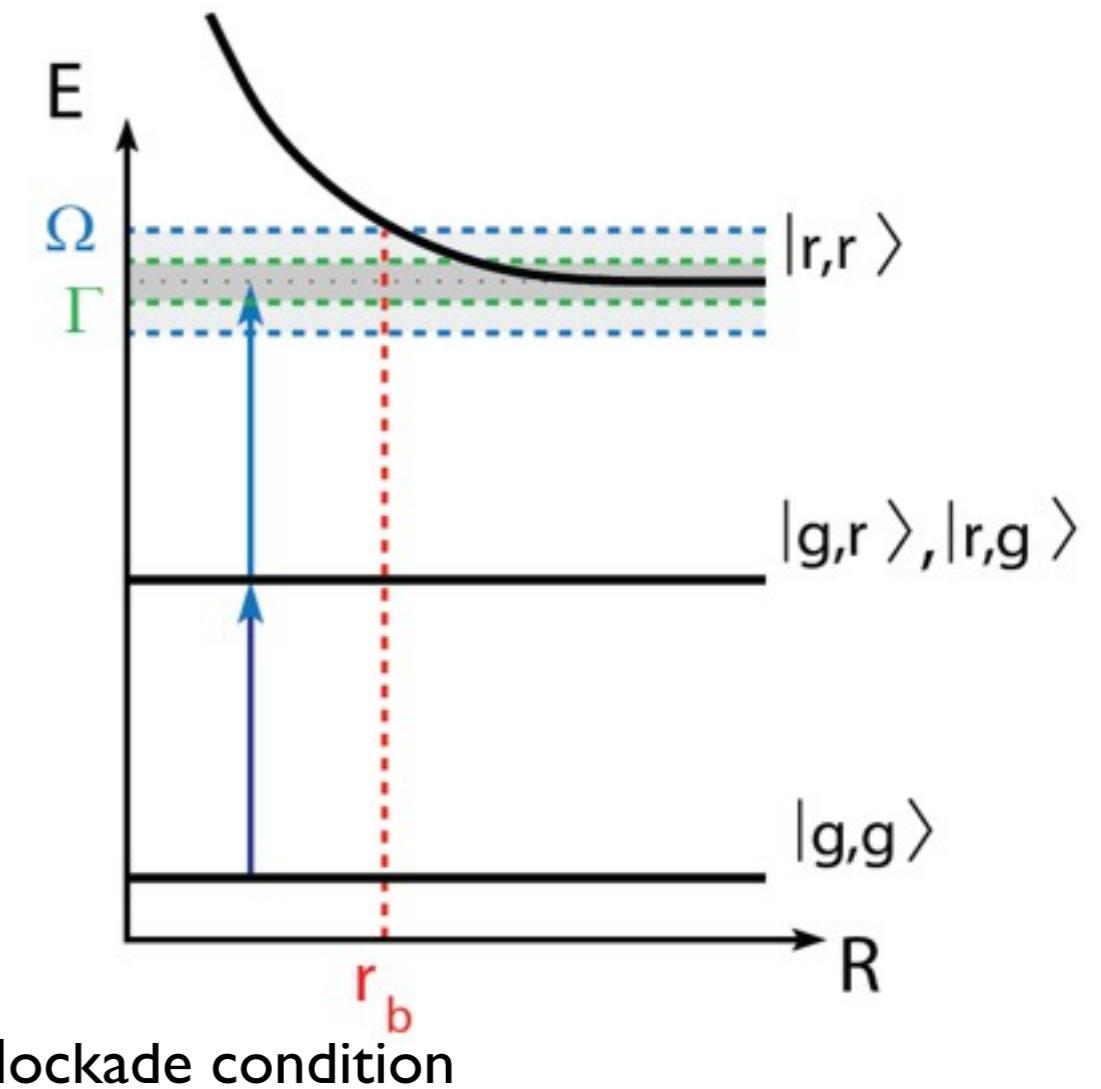
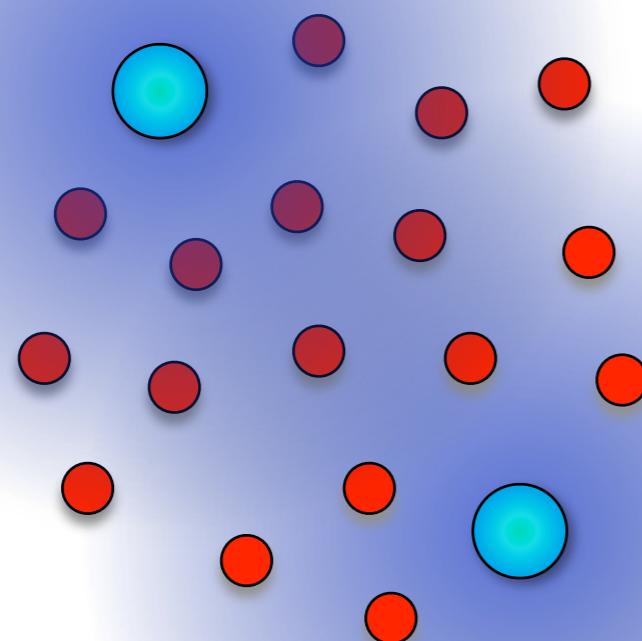
$^{87}\text{Rb } 5\text{S}_{1/2}$



$\varnothing 0.5\text{nm}$

$\varnothing 250\text{ nm}$

Property	Scaling	$^{87}\text{Rb } 43\text{S}$
Radius	$(n^*)^2$	$2400 a_0 = 127\text{nm}$
Lifetime (dominated by black body radiation for large n)	$(n^*)^2$	$45 \mu\text{s} @ 20^\circ\text{C}$
van der Waals coefficient	$(n^*)^{11}$	$C_6 = -1.7 \times 10^{19} \text{ a.u.}$
Blockade radius ($\Omega=2\pi 200 \text{ kHz}$)	$(n^*)^2$	$\sim 5 \mu\text{m}$



$$\mathcal{V}_{\text{vdW}} = \frac{C_6}{r^6} > \hbar \max(\Gamma, \Omega)$$

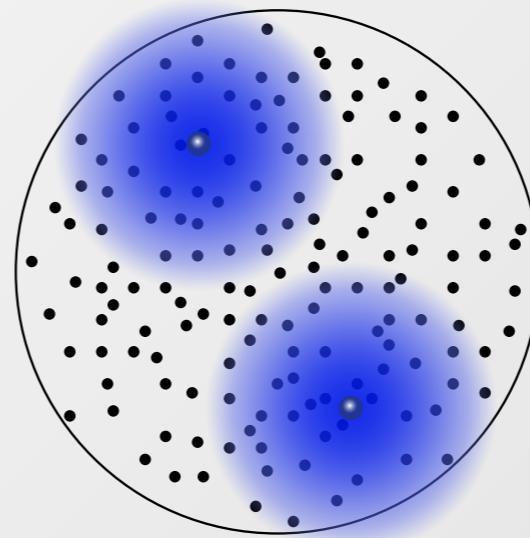
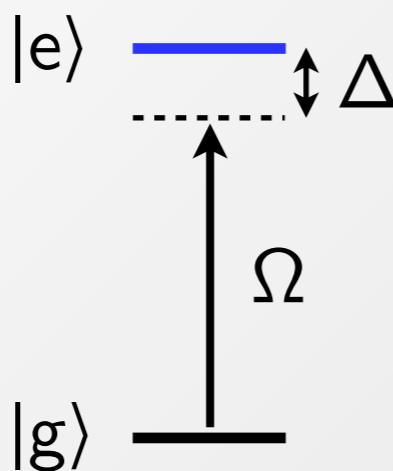
Each superatom:

$$\frac{1}{\sqrt{N}} (|r,0,0,0,\dots\rangle + |0,r,0,0,\dots\rangle + |0,0,0,\dots,r\rangle)$$

M. Lukin et al. PRL (2001)

$$r_b \equiv \sqrt[6]{\frac{C_6}{\hbar\Omega}}$$

The frozen Rydberg gas - long range QM



*no mechanical motion
on the timescale of the
internal dynamics*

$$H = \frac{\hbar\Omega}{2} \sum_i \left(\sigma_{eg}^{(i)} + \sigma_{ge}^{(i)} \right) + \sum_{i \neq j} \frac{V_{ij}}{2} \sigma_{ee}^{(i)} \sigma_{ee}^{(j)} - \Delta \sum_i \sigma_{ee}^{(i)}$$



coherent coupling



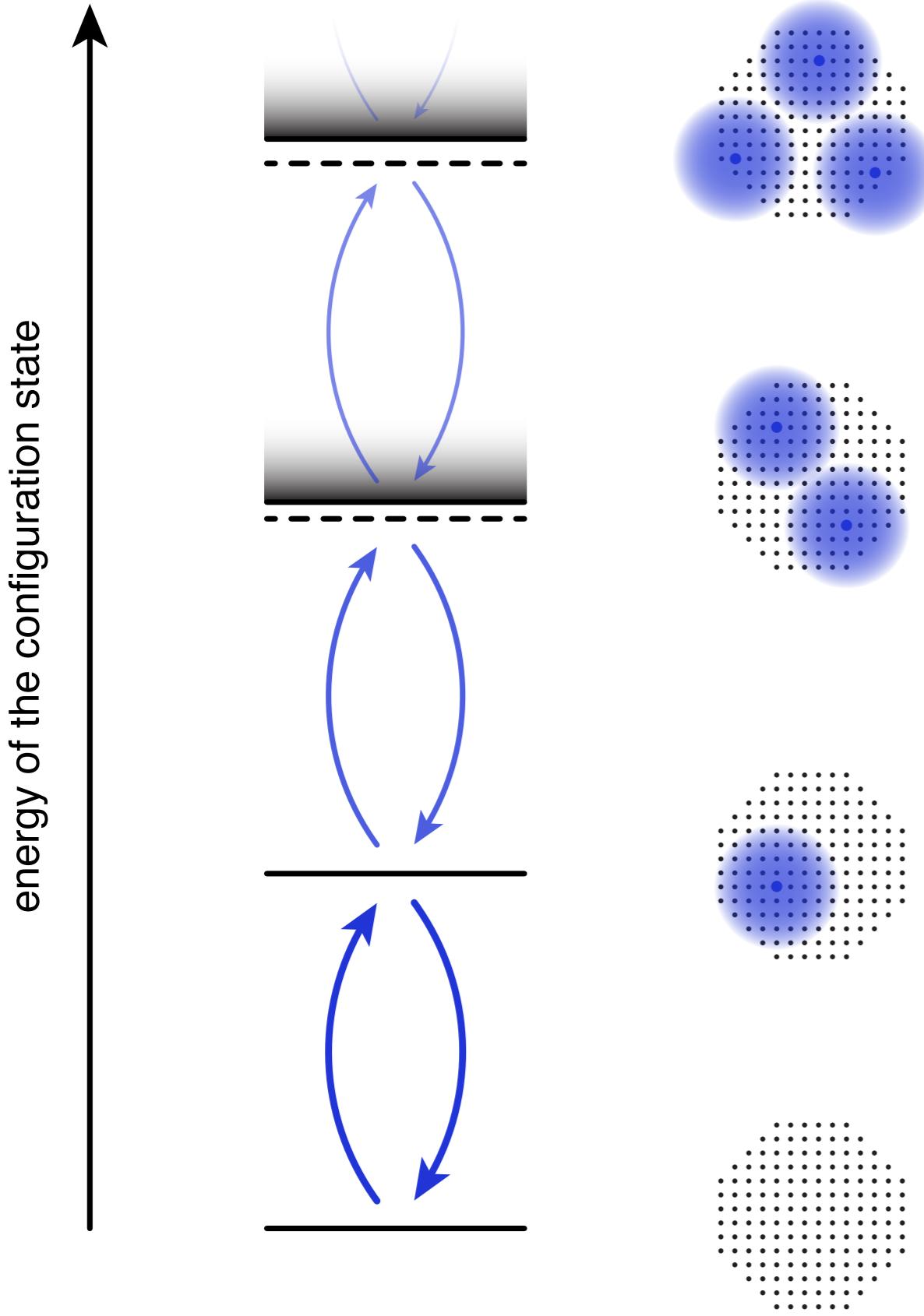
interaction between
Rydberg atoms

$$V_{ij} = C_\alpha |r_i - r_j|^{-\alpha}$$



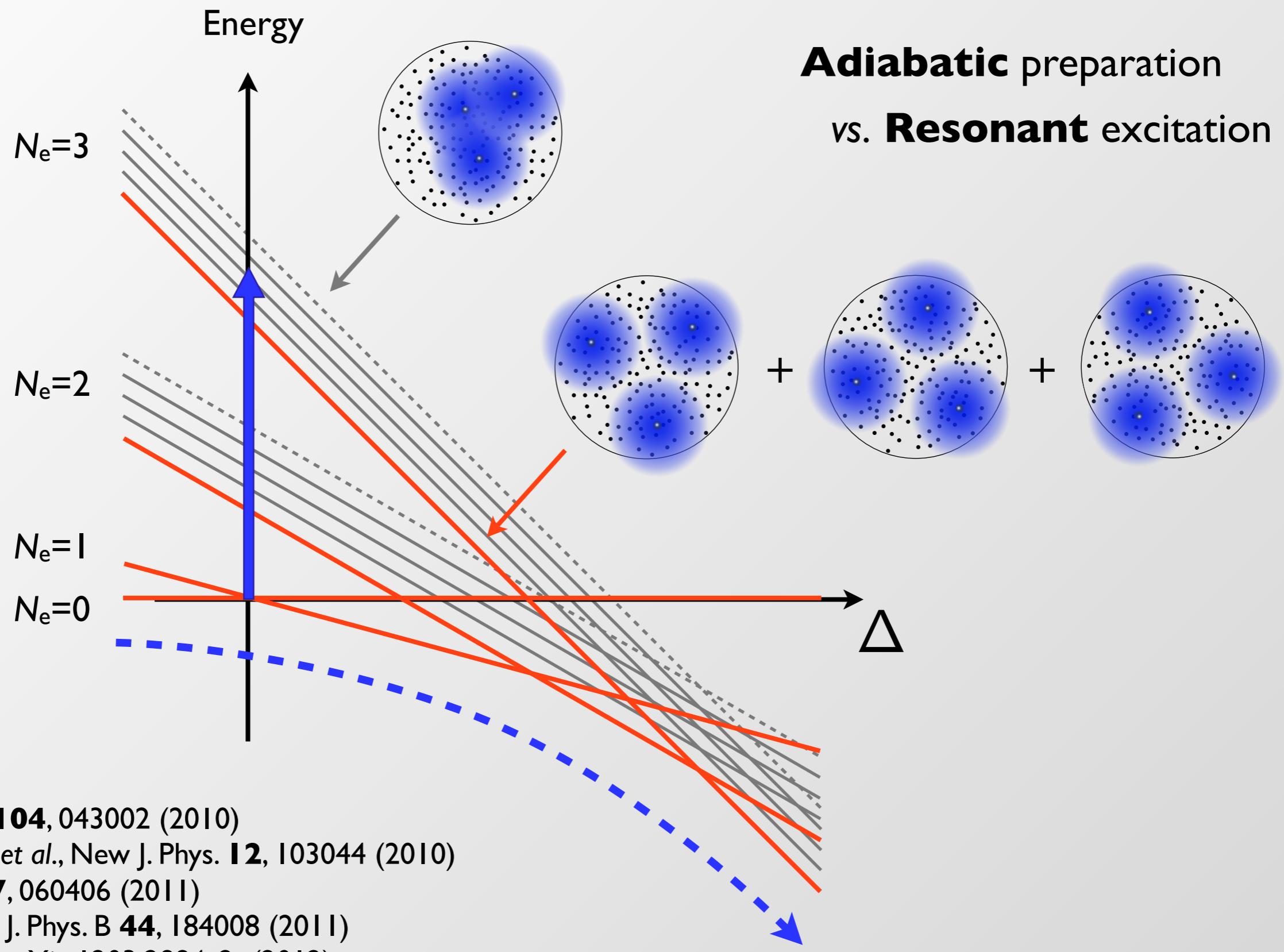
"chemical potential"

This work: $\alpha=6$, repulsive

**Theory see:**

- H. Weimer et al., PRL 2008
T. Pohl et al. PRL 2010
G. Pupillo et al. PRL 2010

Dynamical crystallization of the Rydberg gas



Dynamical Crystallization in the Dipole Blockade of Ultracold Atoms

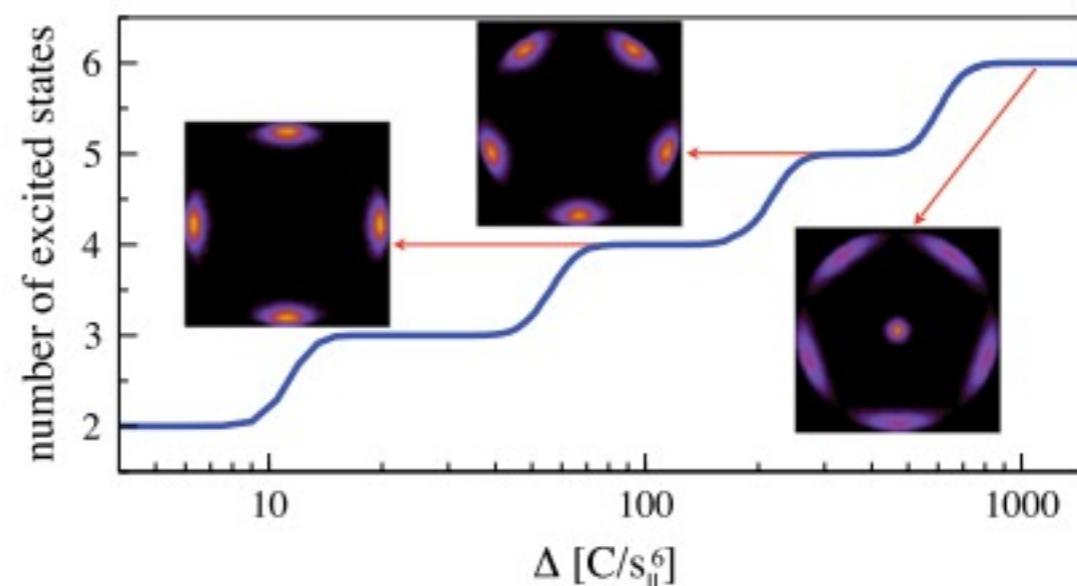
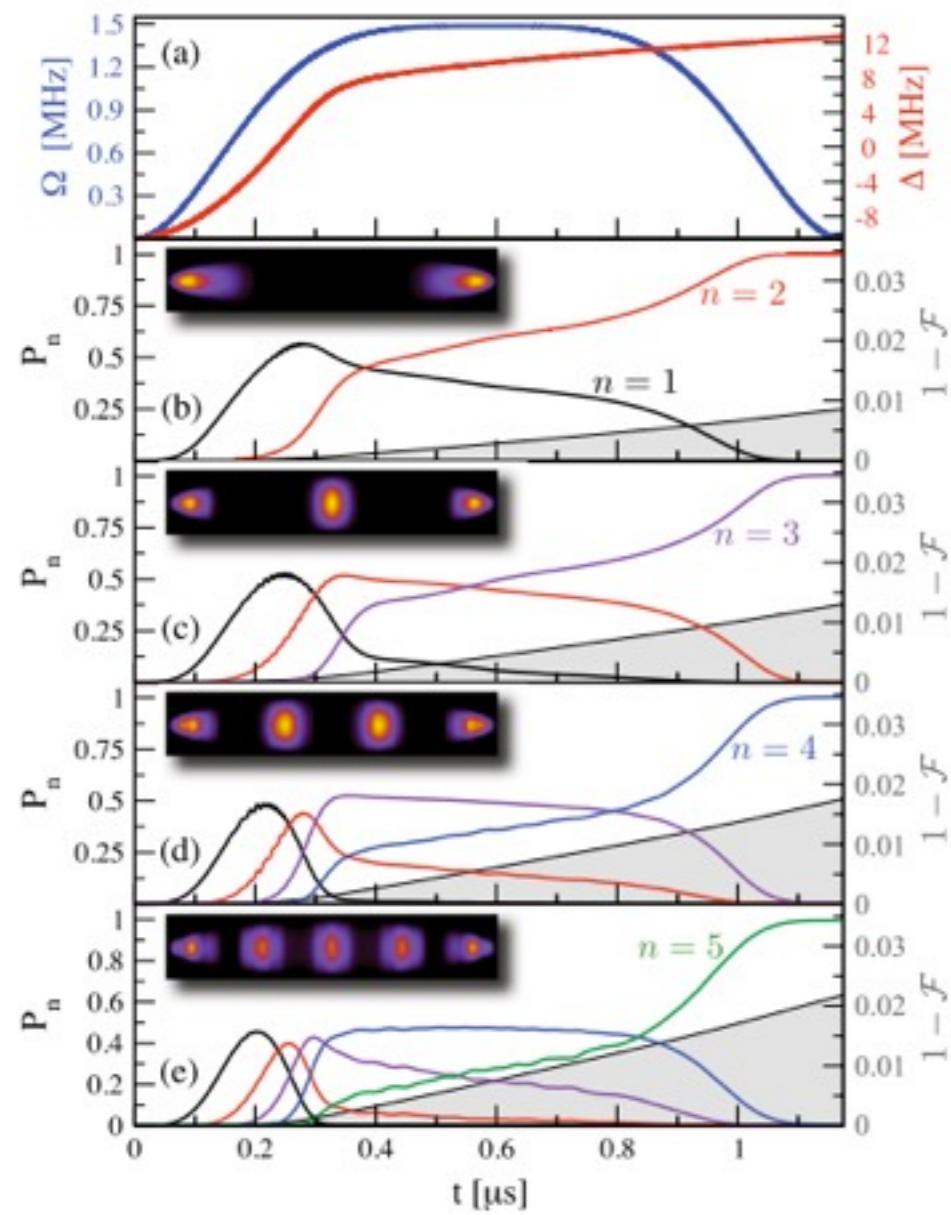
T. Pohl,^{1,2} E. Demler,^{2,3} and M. D. Lukin^{2,3}

¹*Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany*

²*ITAMP-Harvard-Smithsonian Center for Astrophysics, Cambridge Massachusetts 02138, USA*

³*Physics Department, Harvard University, Cambridge Massachusetts 02138, USA*

(Received 26 July 2009; revised manuscript received 23 October 2009; published 27 January 2010)

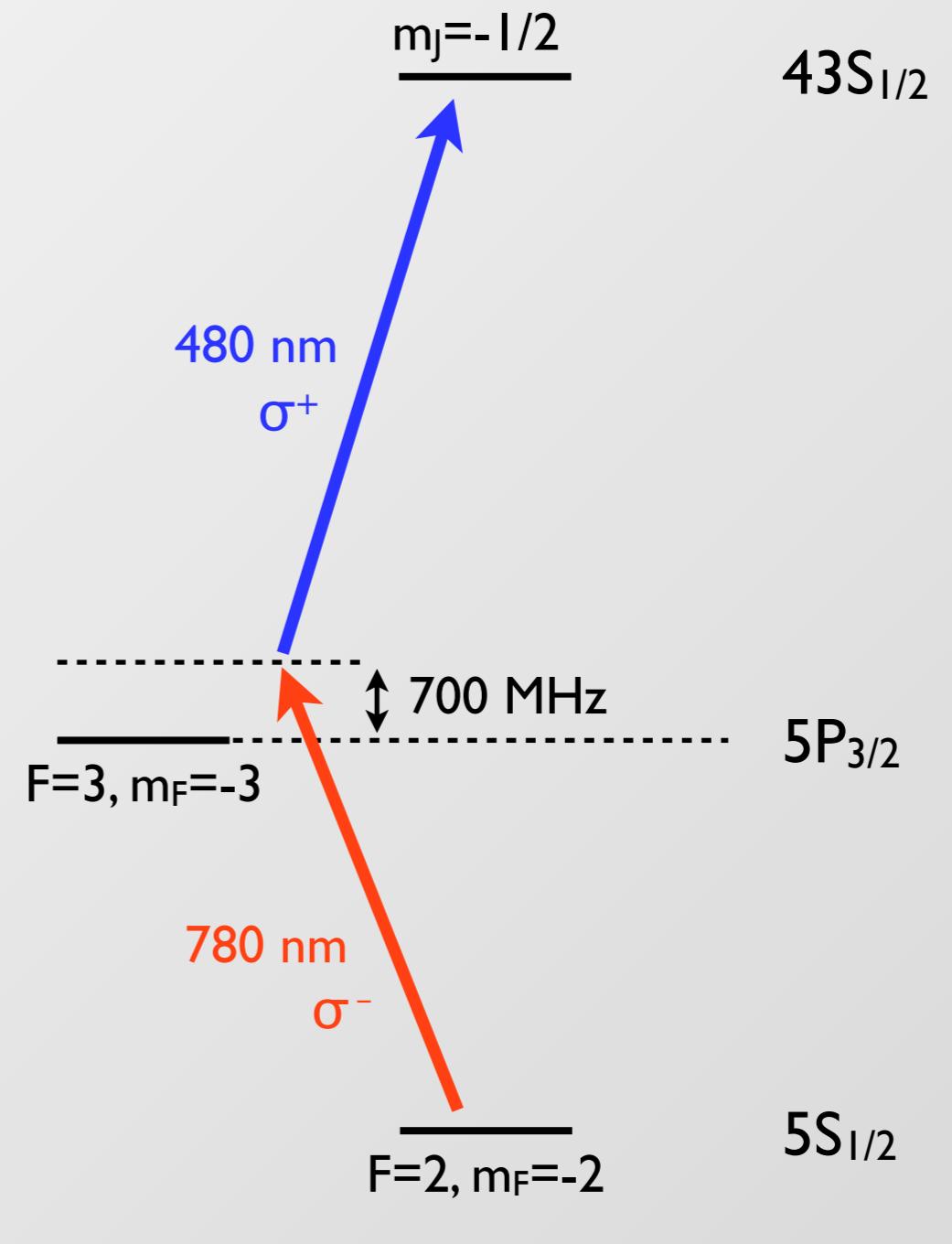
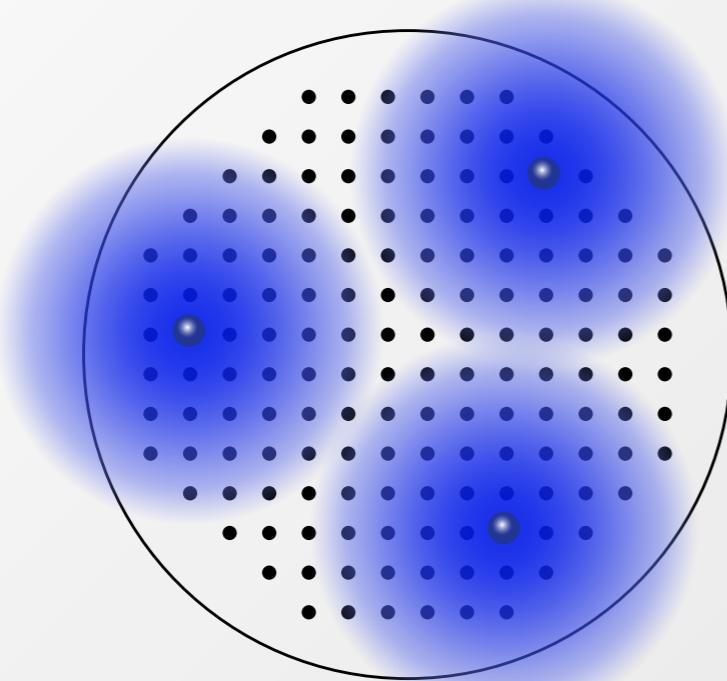


Rydberg atoms organize in crystalline excitations patterns!

Theory see:

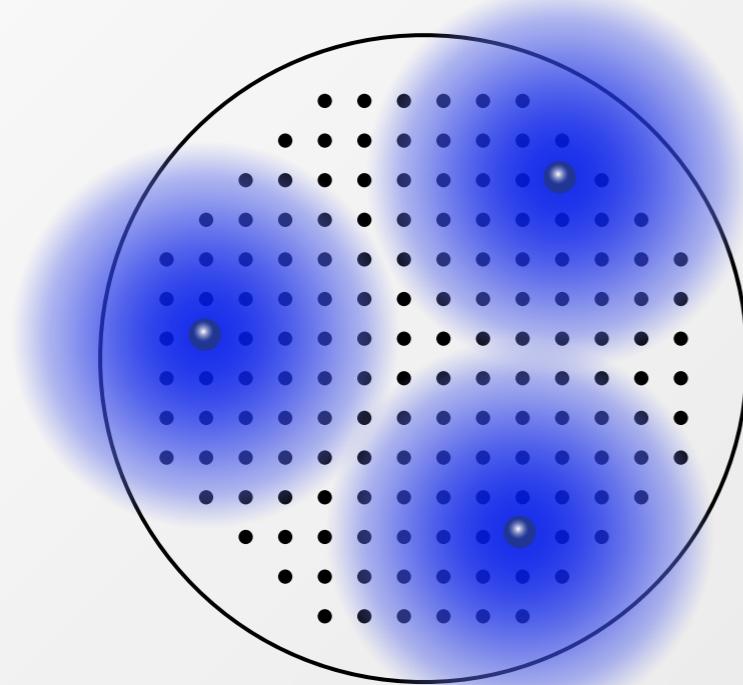
T. Pohl et al. PRL 2010; G. Pupillo et al. PRL 2010,
R.M.W van Bijnen et al. J. Phys. B: At. Mol. Opt. Phys. (2011)
see also: H. Weimer et al., PRL 2008

Excitation and detection of the Rydberg atoms



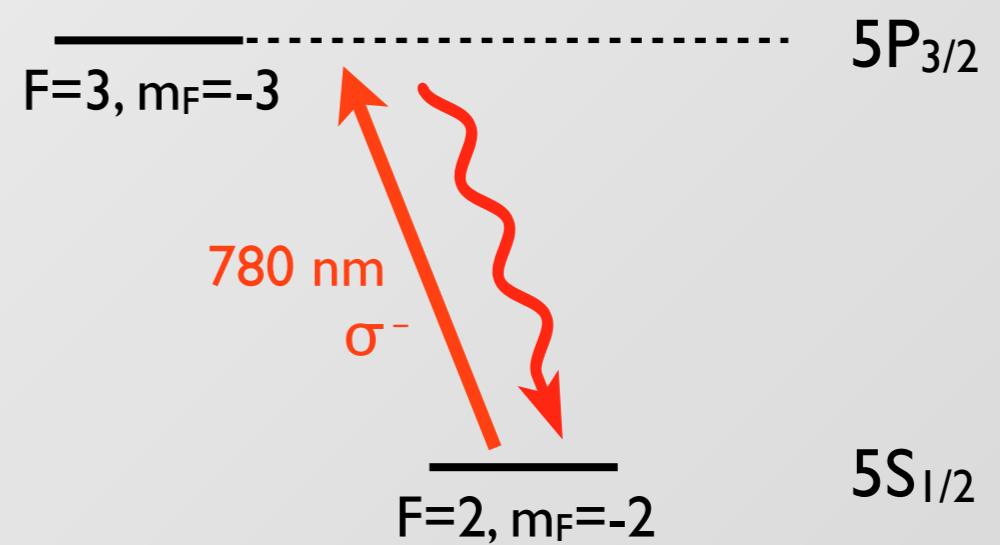
- two-photon Rabi frequency:
 $\Omega/2\pi = 170(20)$ kHz
- resonant excitation:
 $\Delta = 0$
- blockade radius:
 $R_b = 4.9(1)$ μm

Excitation and detection of the Rydberg atoms



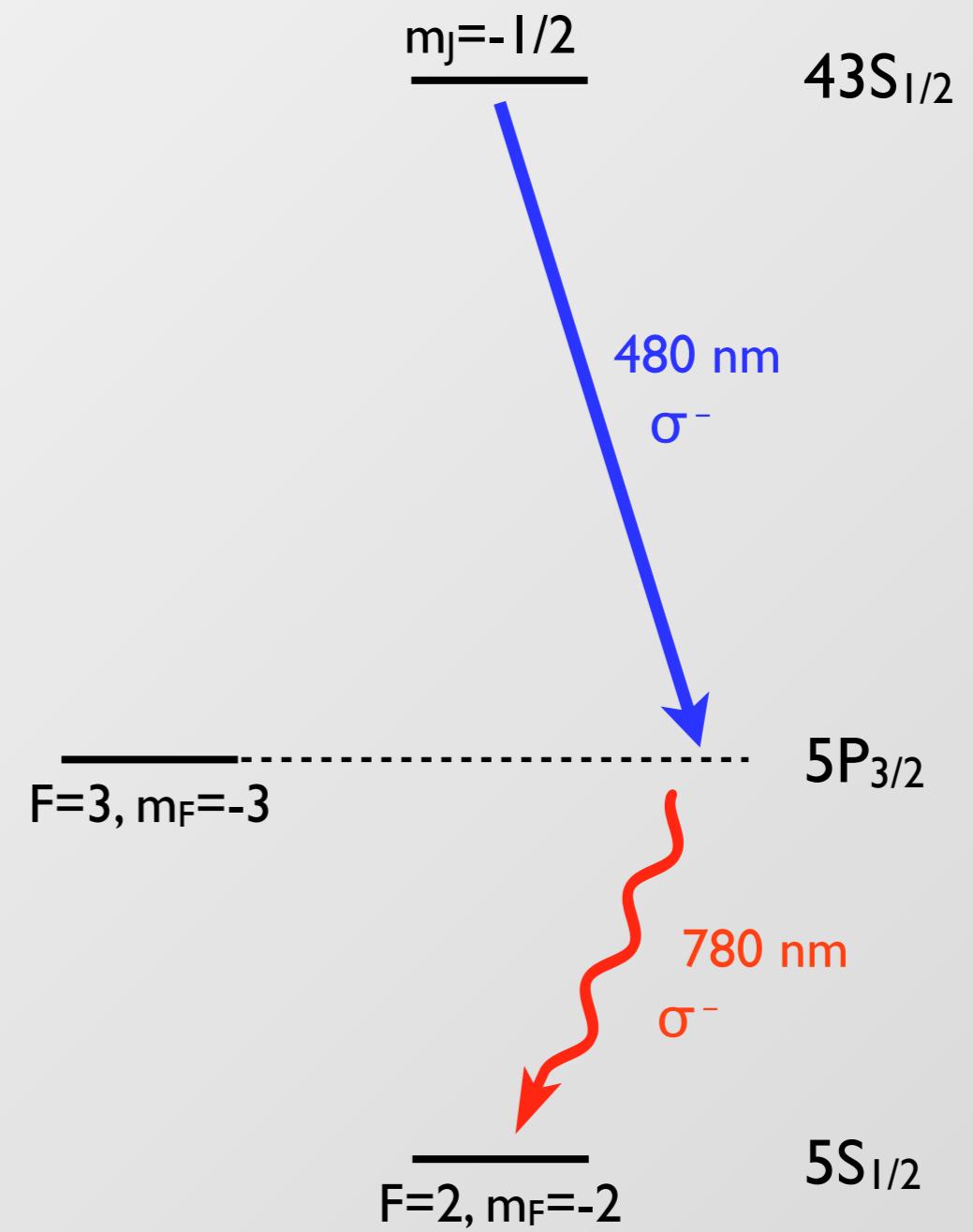
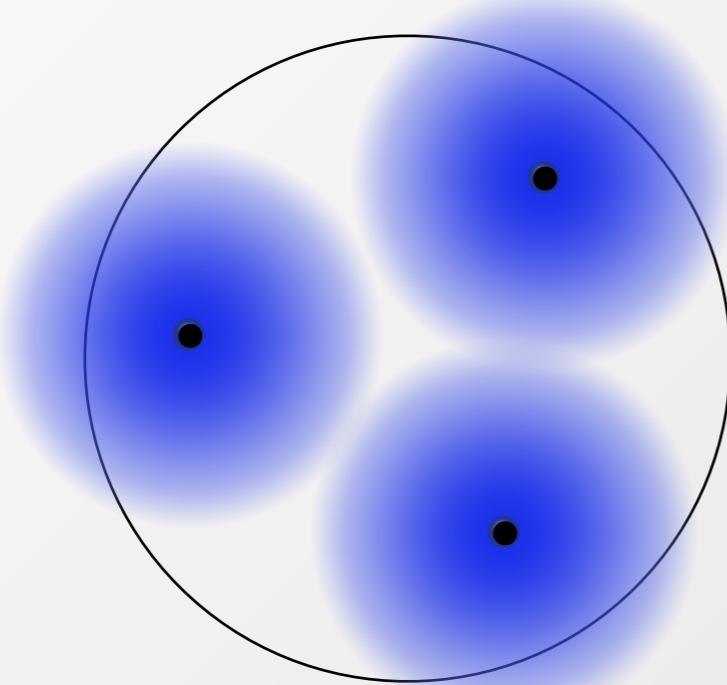
$m_J = -1/2$

$43S_{1/2}$



- removal pulse duration: 10 μ s
- survival probability: 0.1 %

Excitation and detection of the Rydberg atoms



- deexcitation pulse duration: $2 \mu\text{s}$
- detection efficiency: $75(10)\%$
- overall resolution: $\sim 500 \text{ nm}$

Snapshots of the excitation pattern

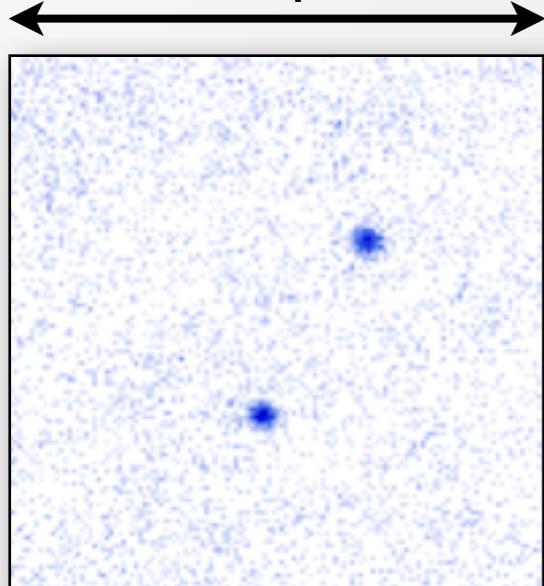
"Small" clouds:

150(30) atoms
diameter 7.2(8) μm
676 pictures

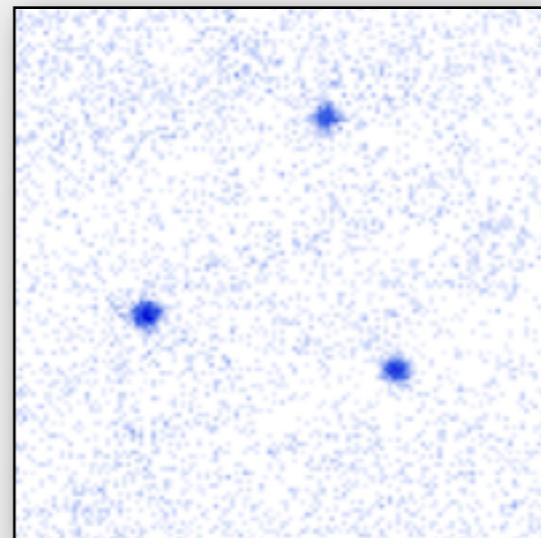
"Large" clouds:

390(30) atoms
diameter 10.8(8) μm
1654 pictures

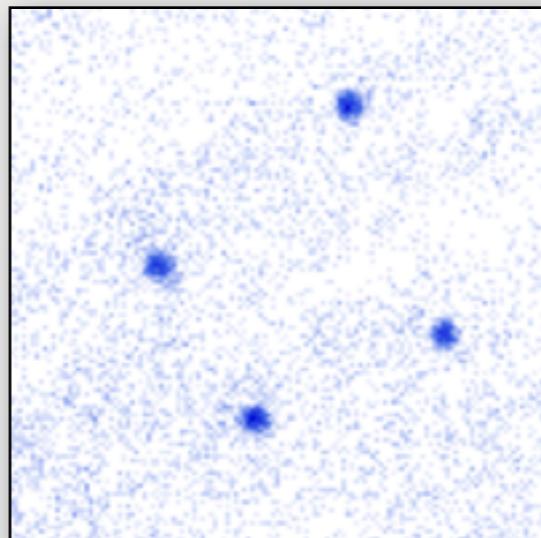
18 μm



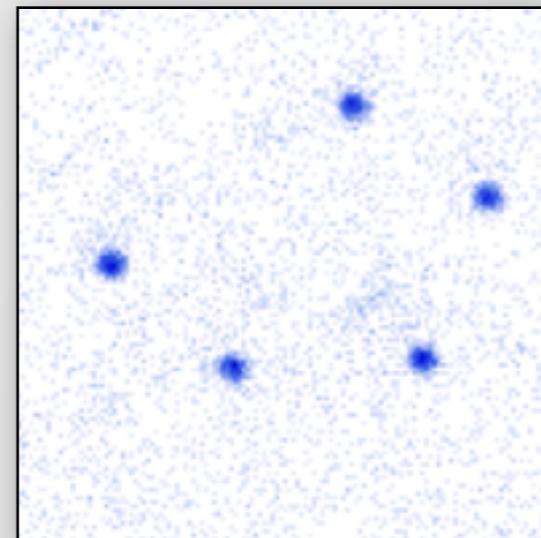
191 pictures



65 pictures

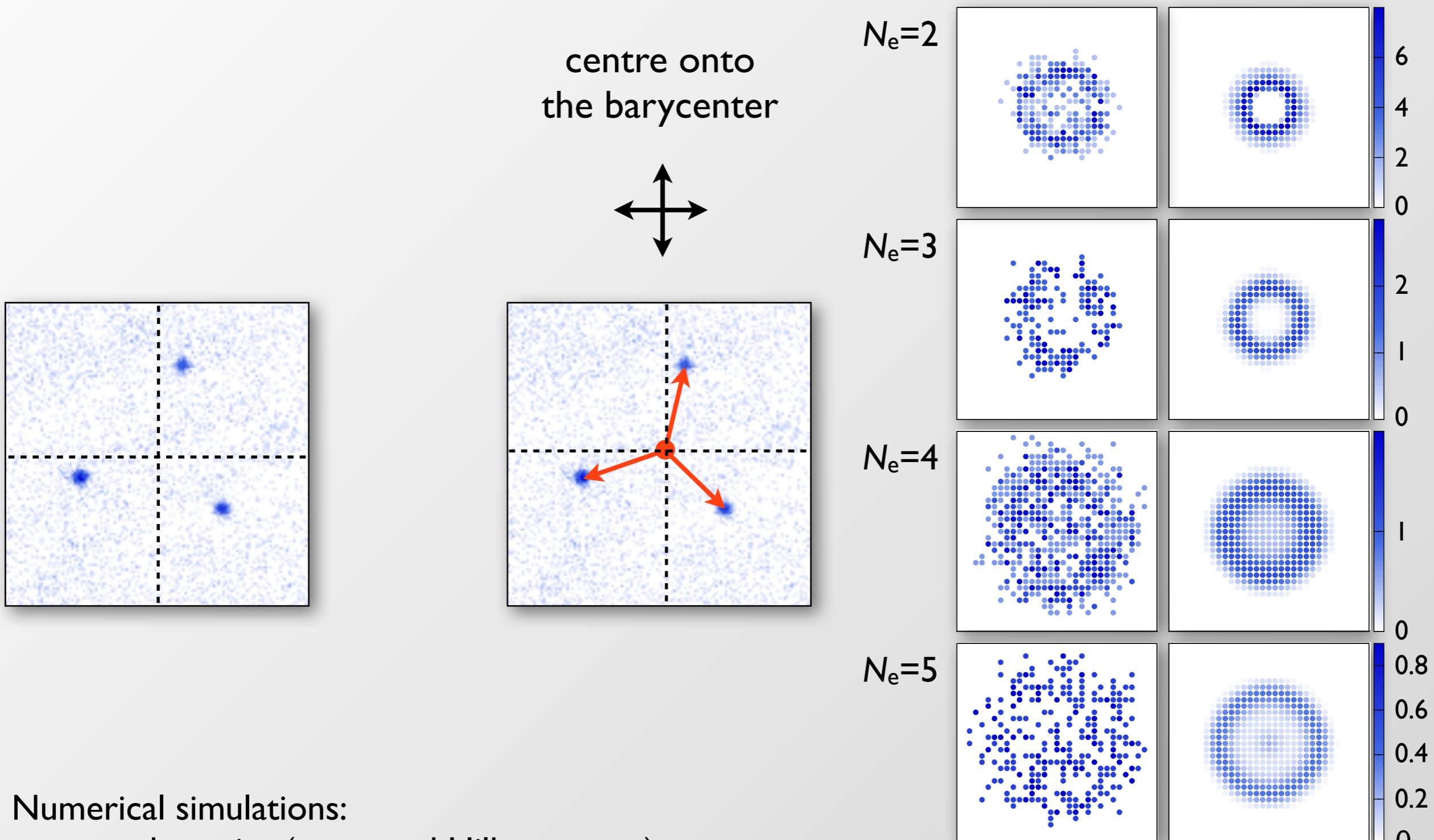


177 pictures



64 pictures

Revealing mesoscopic structures

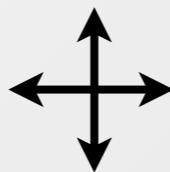


Numerical simulations:

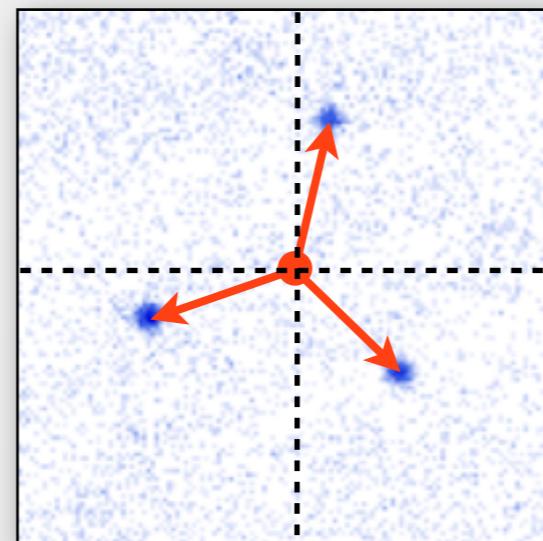
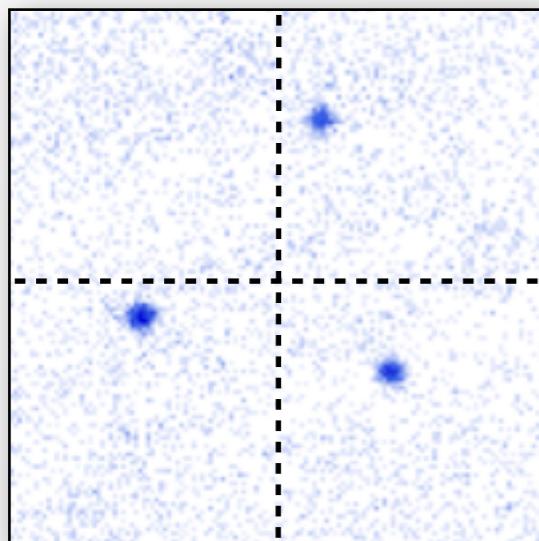
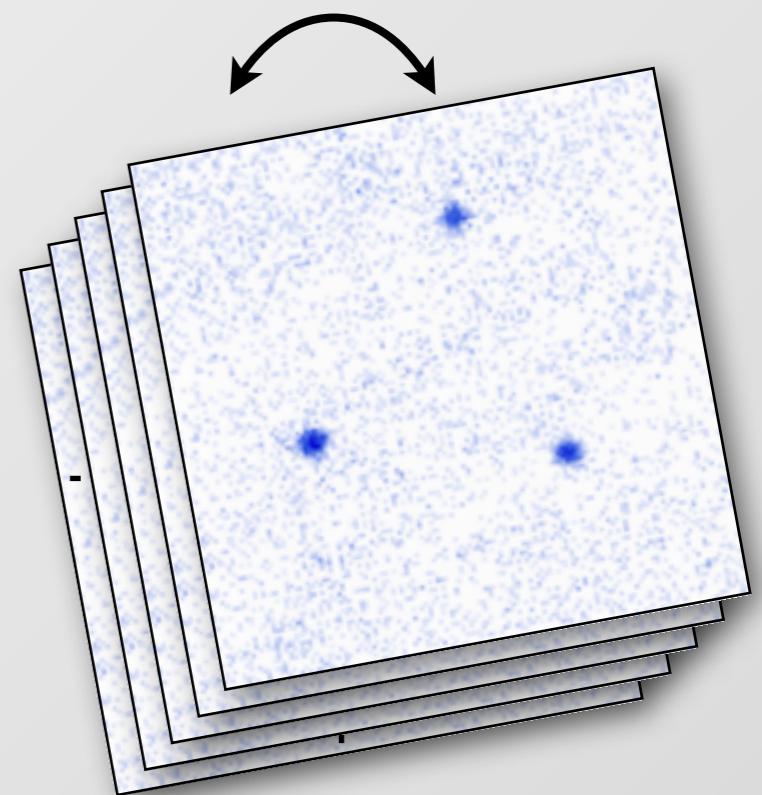
- exact dynamics (truncated Hilbert space)
- average over initial states from the grand canonical ensemble

Revealing mesoscopic structures

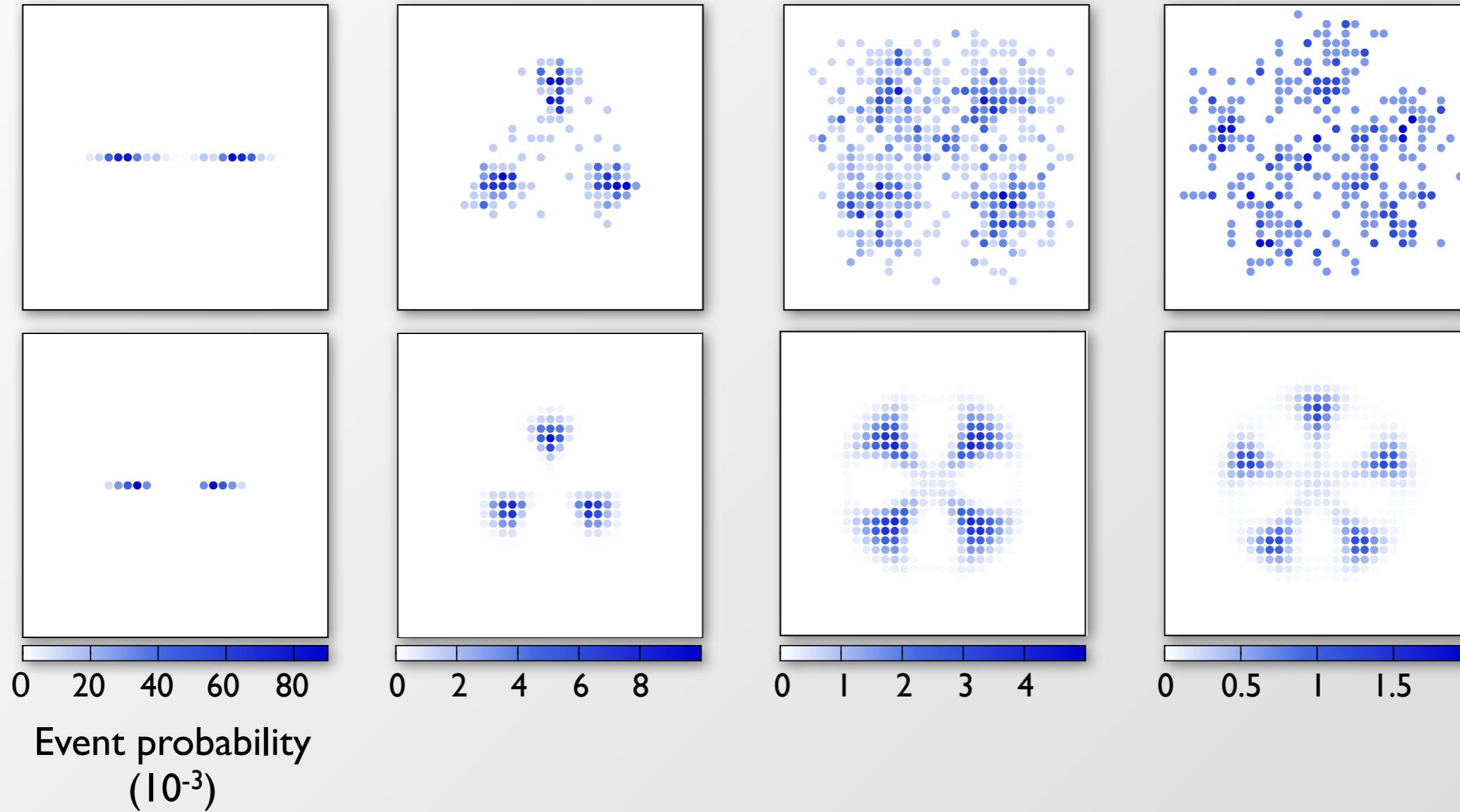
centre onto
the barycenter



align with a
reference axis



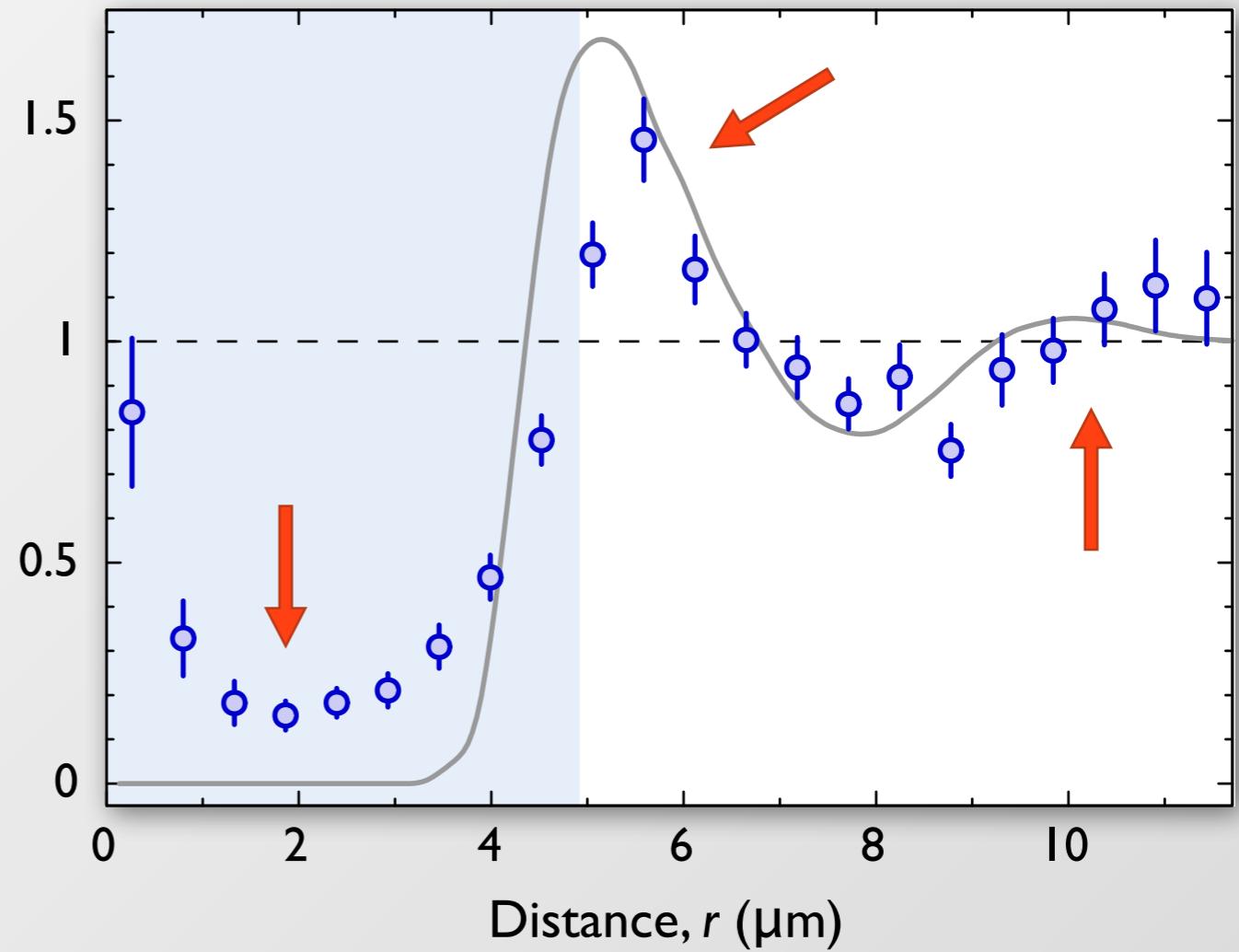
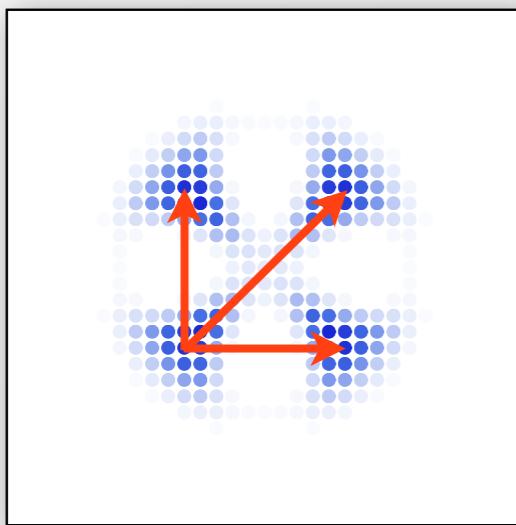
Revealing mesoscopic structures



Correlation functions

Pair correlation function:

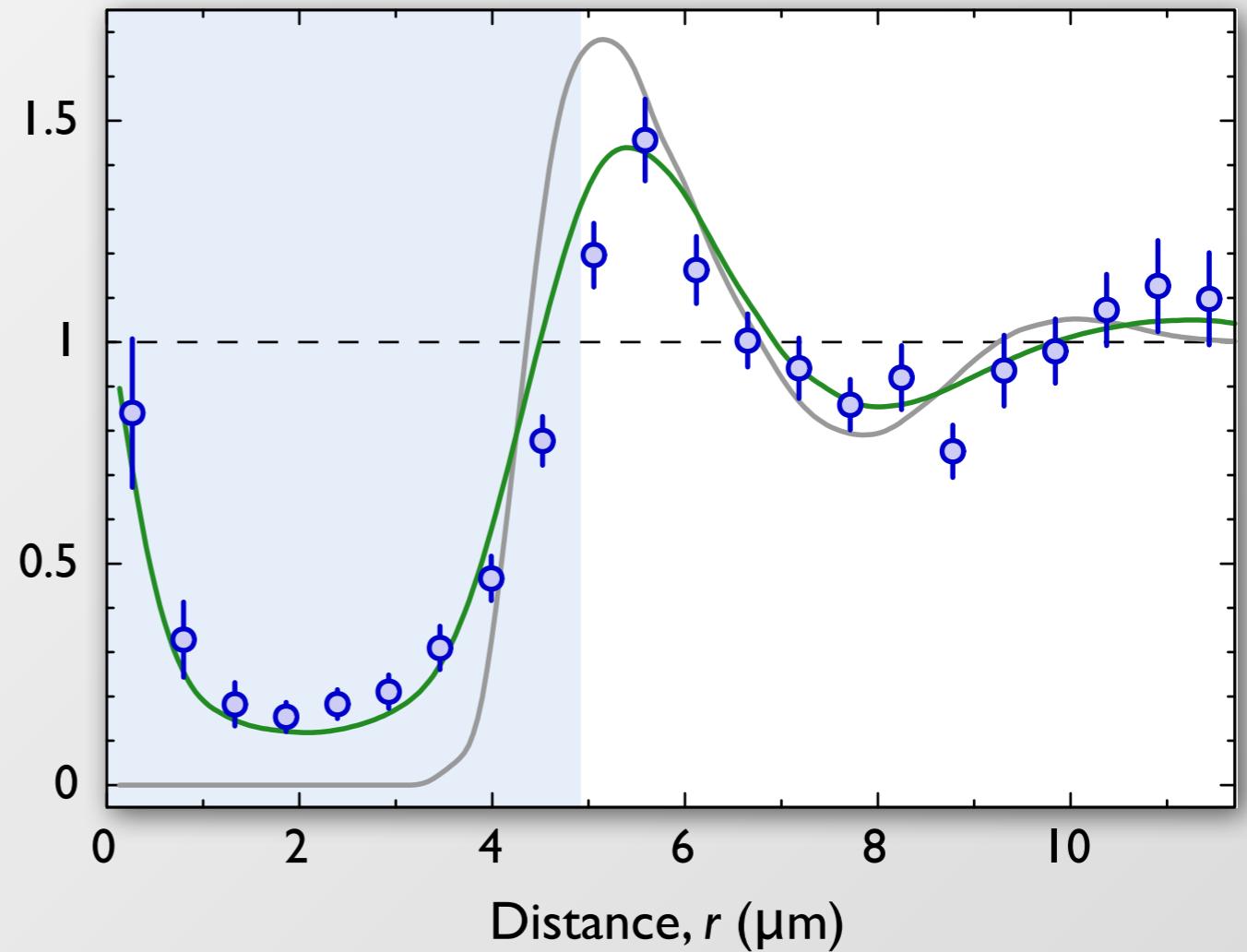
$$g^{(2)}(r) = \frac{\sum_{i \neq j} \delta_{r, r_{ij}} \langle \sigma_{ee}^{(i)} \sigma_{ee}^{(j)} \rangle}{\sum_{i \neq j} \delta_{r, r_{ij}} \langle \sigma_{ee}^{(i)} \rangle \langle \sigma_{ee}^{(j)} \rangle}$$



Correlation functions

Pair correlation function:

$$g^{(2)}(r) = \frac{\sum_{i \neq j} \delta_{r,r_{ij}} \langle \sigma_{ee}^{(i)} \sigma_{ee}^{(j)} \rangle}{\sum_{i \neq j} \delta_{r,r_{ij}} \langle \sigma_{ee}^{(i)} \rangle \langle \sigma_{ee}^{(j)} \rangle}$$



Deviations due to:

- hopping during the fluorescence imaging – *probability* $\sim 1\%$
- imperfect removal of the ground state atoms – *0.2 atoms per image*
- residual motion of the Rydberg atoms before imaging – $\pm 0.5\text{ }\mu\text{m}$

Blockade radius measurement, see also: Schwarzkopf et al. PRL (2011)

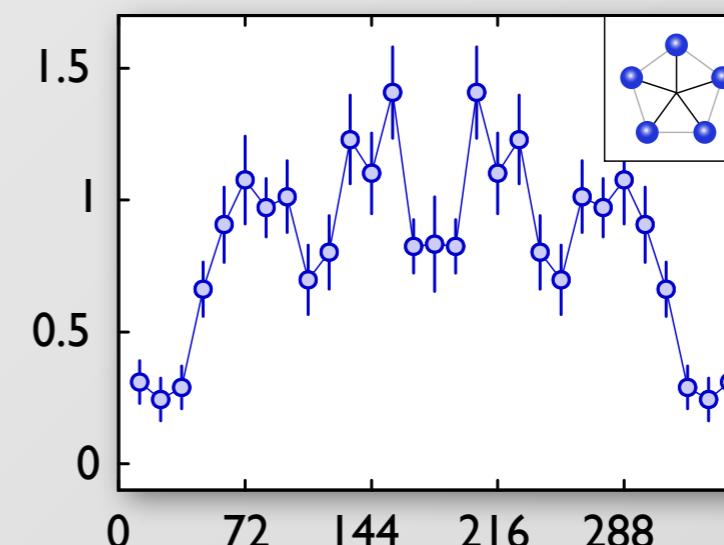
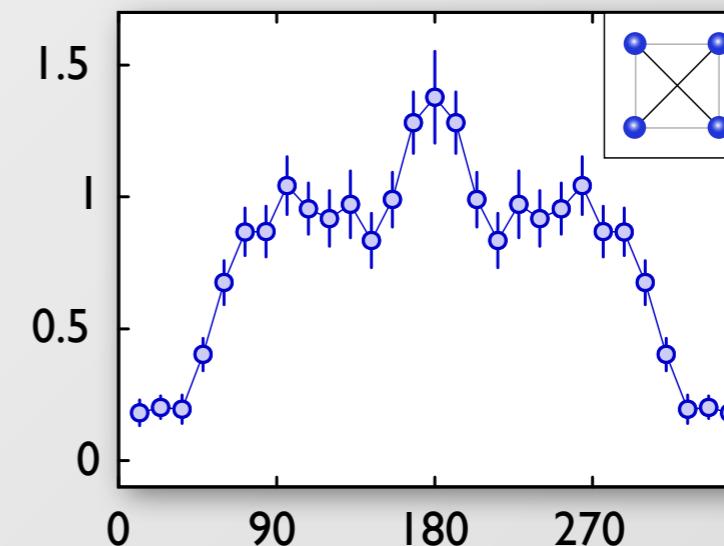
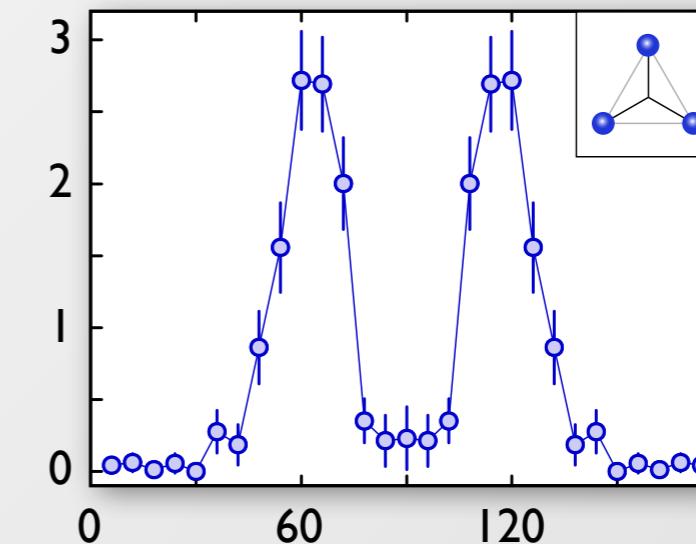
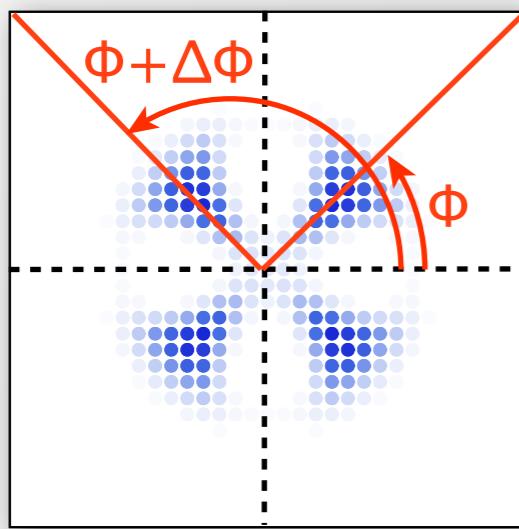
Correlation functions

effective 1D crystals along the circumference of the system

Azimuthal correlation function:

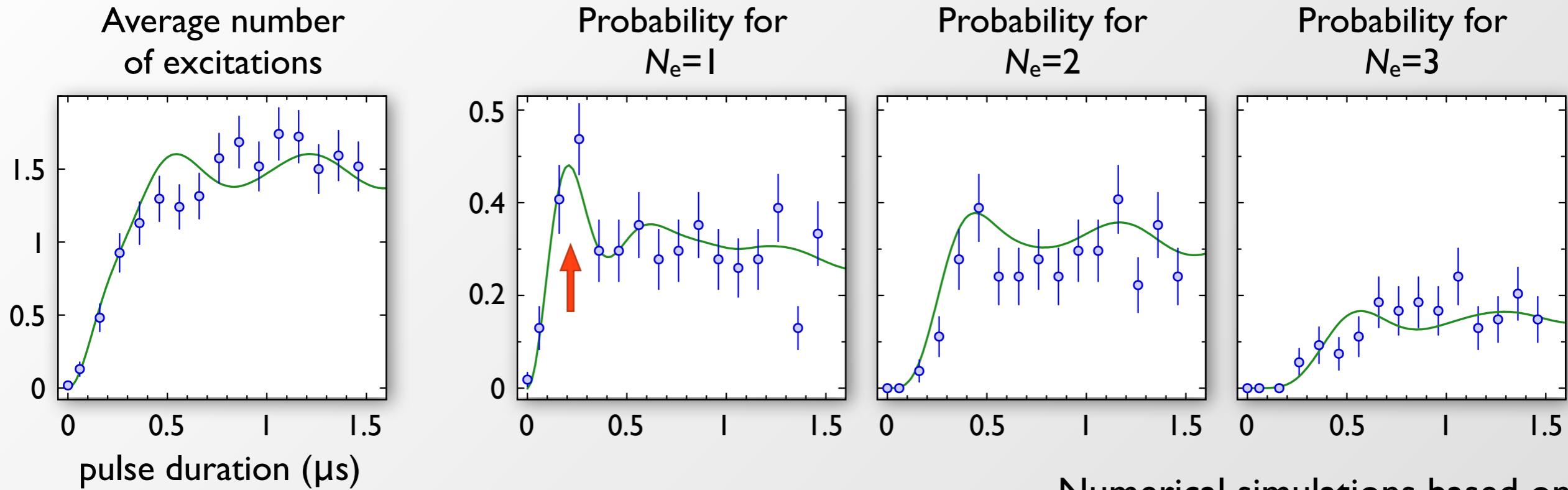
$$\tilde{g}^{(2)}(\Delta\phi) = \int \frac{d\phi}{2\pi} \frac{\langle n(\phi)n(\phi + \Delta\phi) \rangle}{\langle n(\phi) \rangle \langle n(\phi + \Delta\phi) \rangle}$$

with $n(\phi) = \sum_i \delta_{\phi, \phi_i} \sigma_{ee}^{(i)}$



Relative angle, $\Delta\Phi$ (degree)

Time evolution



Numerical simulations based on typical initial states observed in the experiment

- fast dynamics compared to $\pi/\Omega \approx 3 \mu\text{s}$
⇒ *collective enhancement of the coupling strength*
- different timescales for the different number of excitations
⇒ *reduced overlap between the states coupled by the optical radiation*



for $N_e = 1$:
 $\pi/(\sqrt{N_{\text{at}}} \Omega) = 220(40) \text{ ns}$

(some) evidence for the coherence of the many-body state

Smaller Blockade/Larger Cloud

- ✓ Larger Rydberg Crystals
- ✓ Larger Rydberg Atoms cp. to Lattice Spacing
- ✓ Adiabatic Sweeps to Deterministically Prepare Crystal Structures
- ✓ Show Coherence of Crystalline Superpositions! **a Quantum Crystal?**

T. Pohl et al. (2010), van Bijnen et al. (2011), Gärtner et al. (2012),...

Larger Blockade/Smaller Cloud

- ✓ Collectively enhanced Rabi oscillations
- ✓ Large Entangled states (e.g. EIT schemes)

M. Lukin et al. (2001), D. Moller et al. (2008), M. Müller et al. (2009), H. Weimer et al. (2009)...

Dressed Rydberg Atom Regime

- ✓ Admix controlled long range interactions

G. Pupillo et al. (2010), Henkel et al. (2010), Schachenmeyer et al. (2010), Honer et al. (2010), Cinti et al. (2010), Johnson & Rolston (2010)...

Measuring Zak-Berry Phase of Topological Bands

M. Aidelsburger, M. Atala, J. Barreiro & I.B.
D. Abanin, T. Kitagawa, E. Demler



www.quantum-munich.de

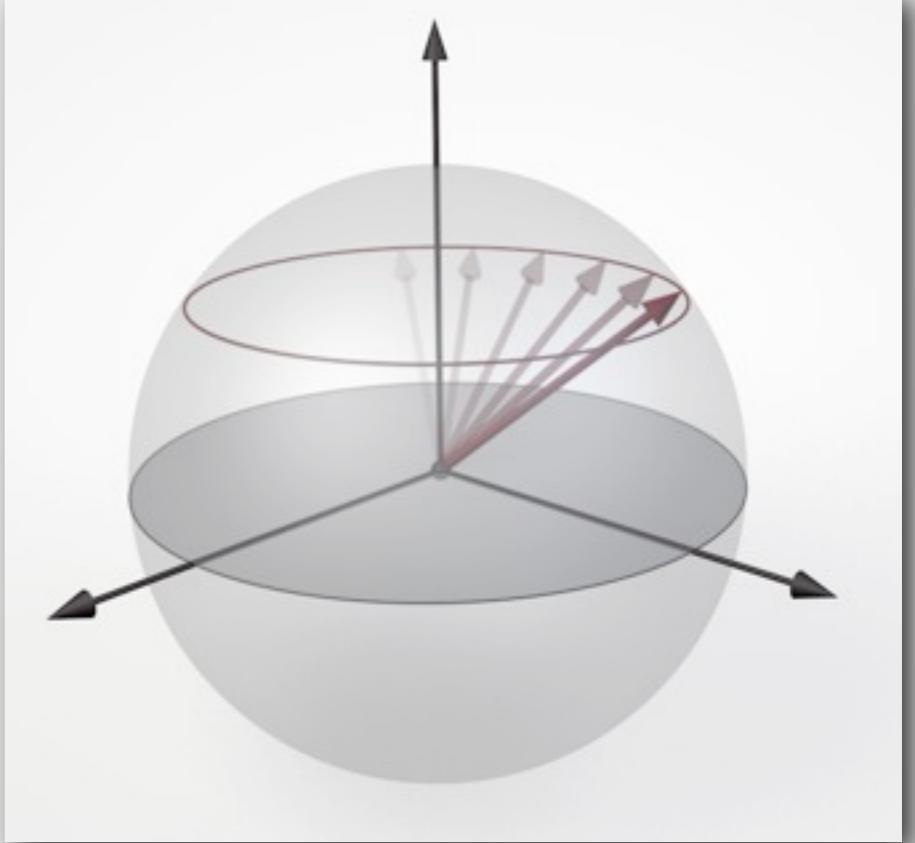
$$\Psi(R) \rightarrow e^{i(\varphi_{\text{Berry}} + \varphi_{\text{dyn}})} \Psi(R)$$

Adiabatic evolution through closed loop

$$\varphi_{\text{Berry}} = - \oint_C A_n(R) dR = -i \oint_C \langle n(R) | \nabla_R | n(R) \rangle dR$$

$$\varphi_{\text{Berry}} = \oint_A \Omega_n(R) dA$$

Berry Phase



M.V. Berry, Proc. R. Soc. A (1984)

Example: Spin-1/2 particle
in magnetic field

Berry connection

$$A_n(R) = i \langle n(R) | \nabla_R | n(R) \rangle$$

Berry curvature

$$\Omega_{n,\mu\nu}(R) = \frac{\partial}{\partial R^\mu} A_{n,\nu} - \frac{\partial}{\partial R^\nu} A_{n,\mu}$$

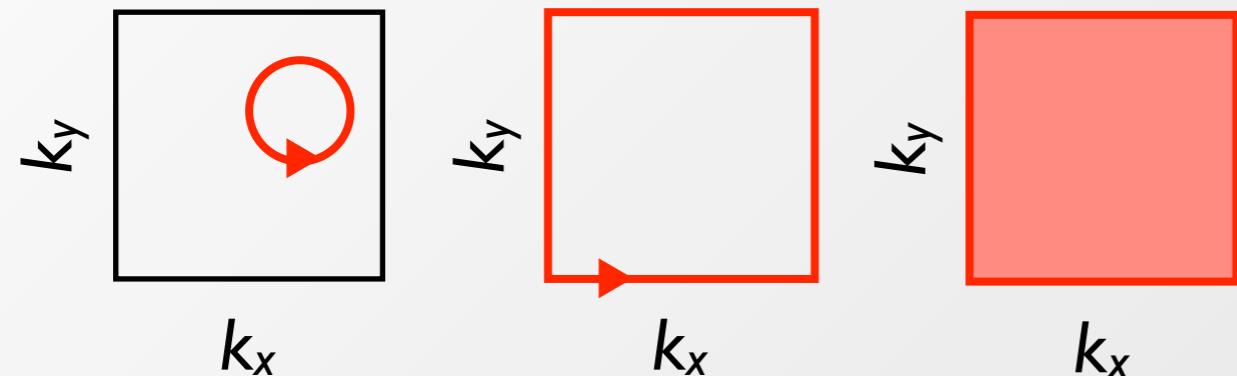


LMU

Berry Phase for Periodic Potentials

$$\Psi_k(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}} u_k(\mathbf{r}) \quad \text{Bloch wave in periodic potential}$$

Adiabatic motion in momentum space generates Berry phase!



Berry phase is fundamental to characterize topology of energy bands

$$n_{\text{Chern}} = \frac{1}{2\pi} \oint_{BZ} A_k dk = \frac{1}{2\pi} \int_{BZ} \Omega_k d^2k \quad \leftrightarrow \quad \sigma_{xy} = n_{\text{Chern}} e^2/h$$

Chern Number (Topological Invariant)

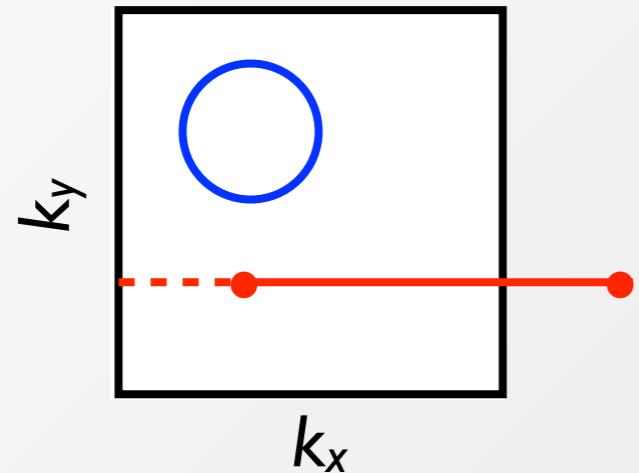
Thouless, Kohmoto, den Nijs, and Nightingale (TKNN), PRL
Kohmoto Ann. of Phys. 1985

Quantized Hall Conductance

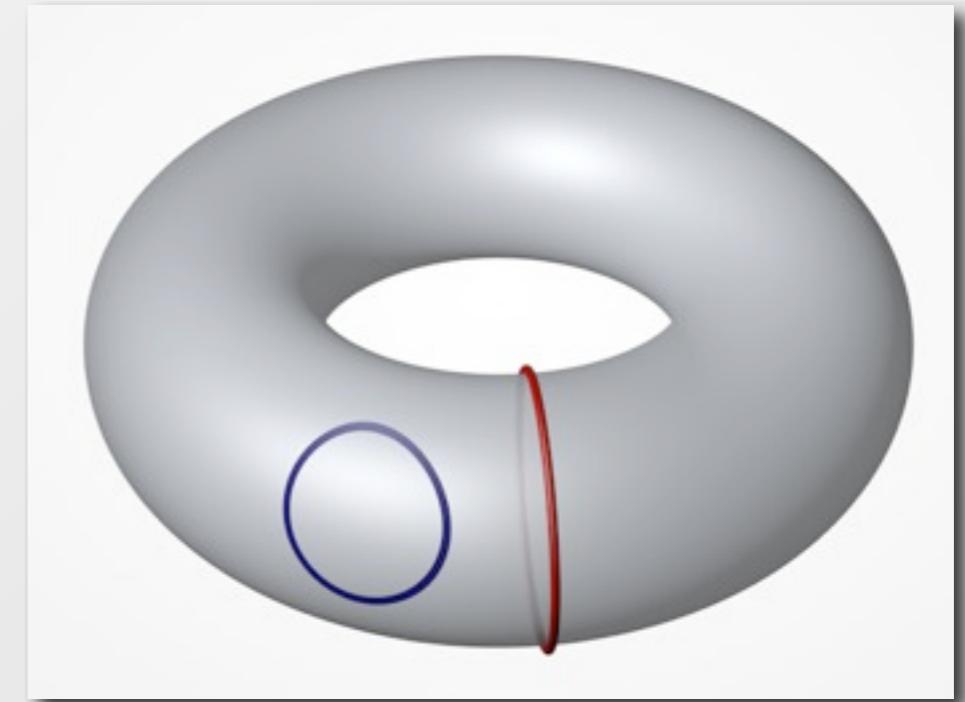
Mention Problem with going on a line is generally NOT A LOOP IN PARAMETER SPACE!

What is the extension to 1D

2D Brillouin Zone



going straight means going around!



Band structure has topology of a torus!

$$\varphi_{\text{Zak}} = \int_{k_0}^{k_0+G} A(k) dk = i \int_{k_0}^{k_0+G} \langle u_k | \partial_k | u_k \rangle dk$$



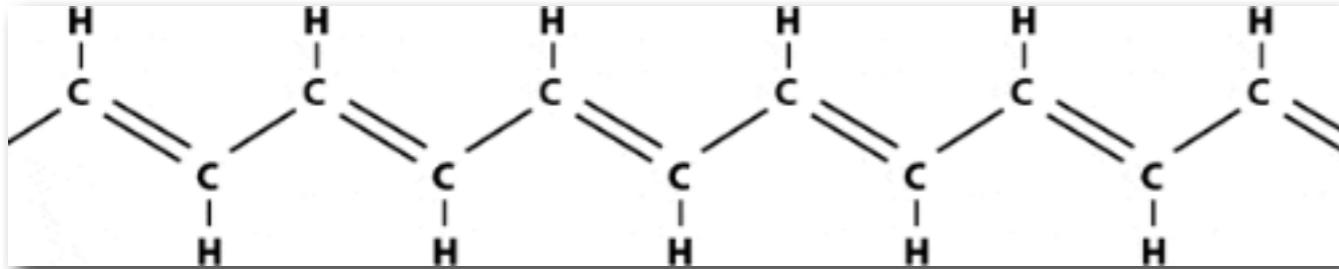
Zak Phase - the 1D Berry Phase

J. Zak, Phys. Rev. Lett. 62, 2747 (1989)

Non-trivial Zak phase indicates topological band.



Su-Shrieffer-Heeger Model (SSH)

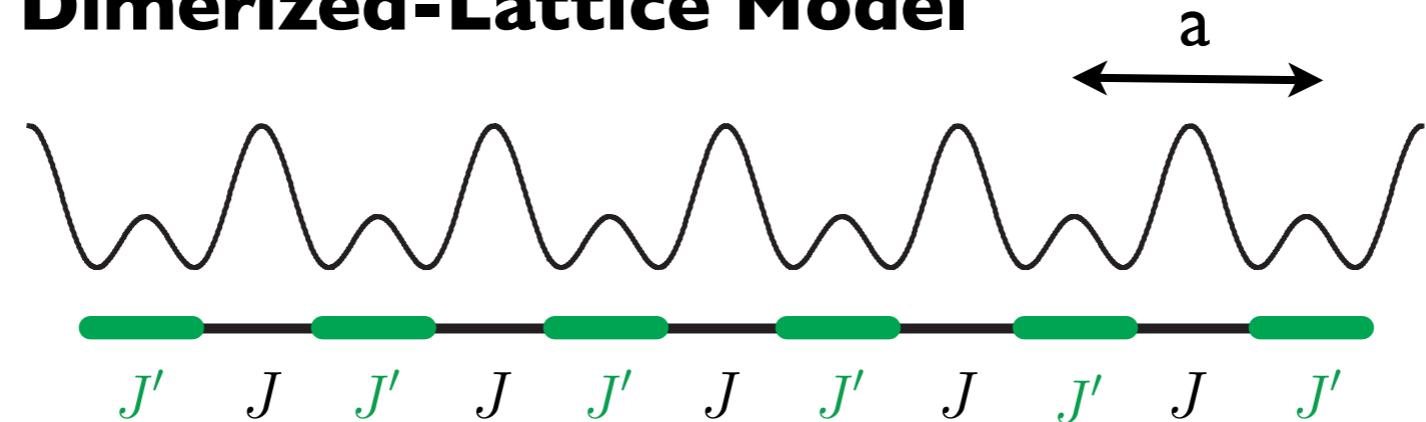


Polyacetylene

W. P. Su, J. R. Schrieffer & A. J. Heeger
Phys. Rev. Lett. 42, 1698 (1979).



Dimerized-Lattice Model

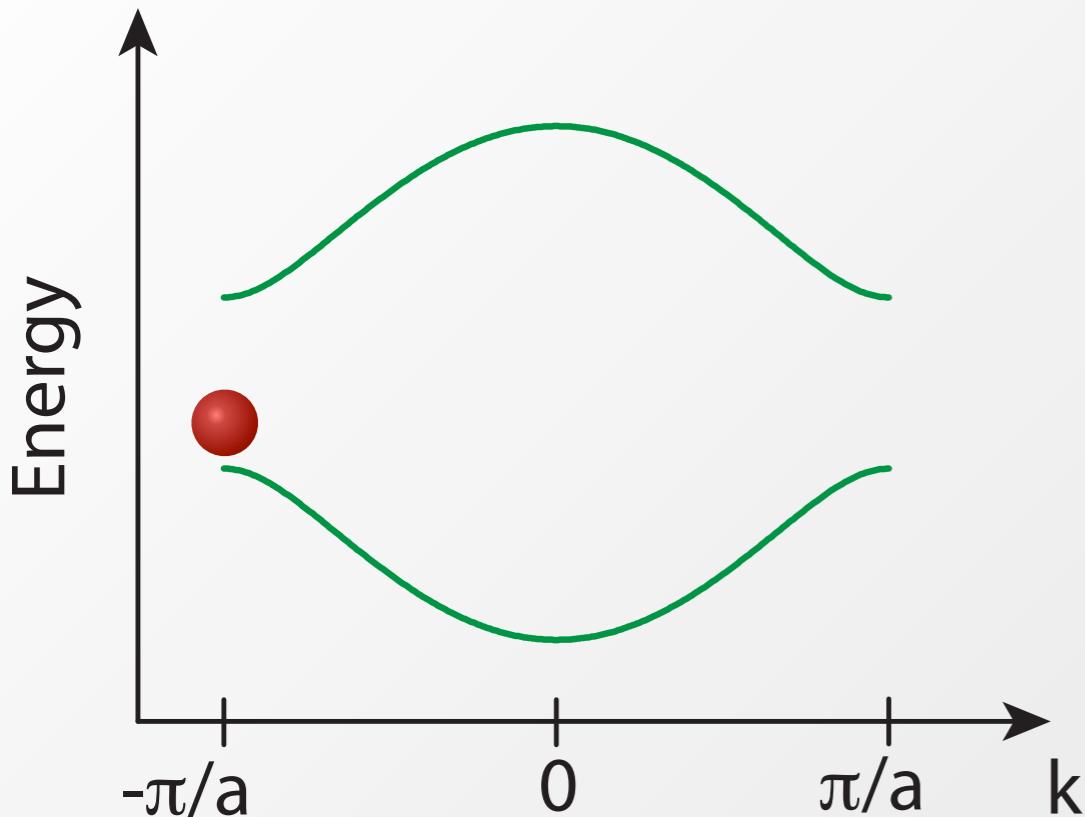


$$H_{\text{SSH}} = - \sum_n \left\{ J a_n^\dagger b_n + J' a_n^\dagger b_{n-1} + \text{h.c.} \right\}$$

- Topological Band
- Edge States (for finite system)
- Fractional Charge

R. Jackiw and C. Rebbi, Phys. Rev. D 13, 3398 (1976)
J. Goldstone and F. Wilczek, Phys. Rev. Lett. 47, 986 (1981)

SSH Energy Bands - Eigenstates



...ABABABAB... Lattice Structure...

$$\psi_k(x) = \sum_n \begin{cases} \alpha_k e^{ikx_n} \\ \beta_k e^{ik(x_n + a/2)} \end{cases}$$

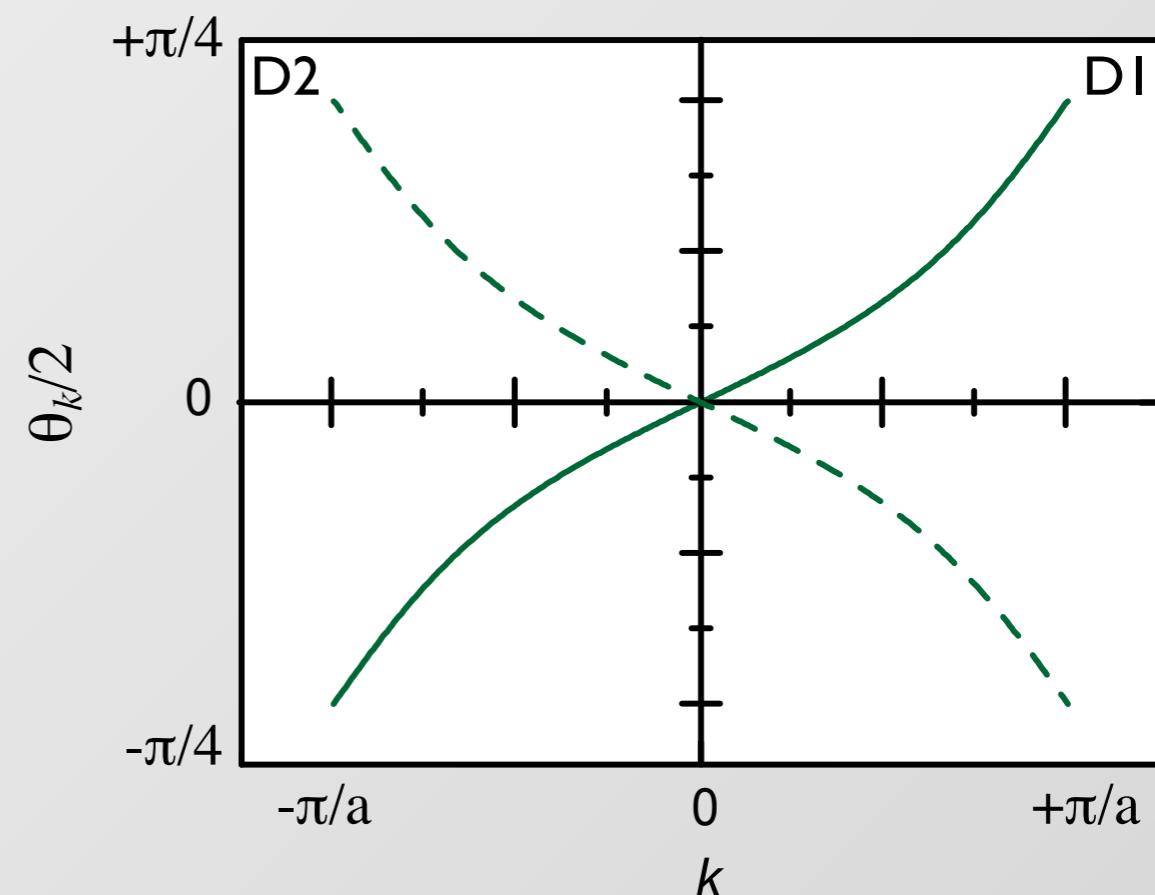
Eigenstates

$$\begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 \\ e^{i\theta_k} \end{pmatrix}$$

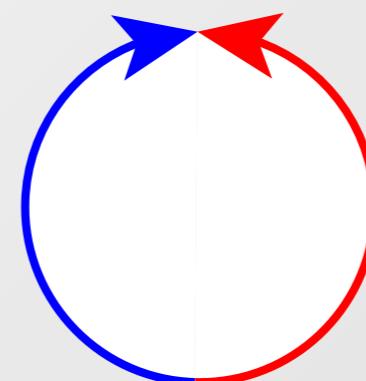
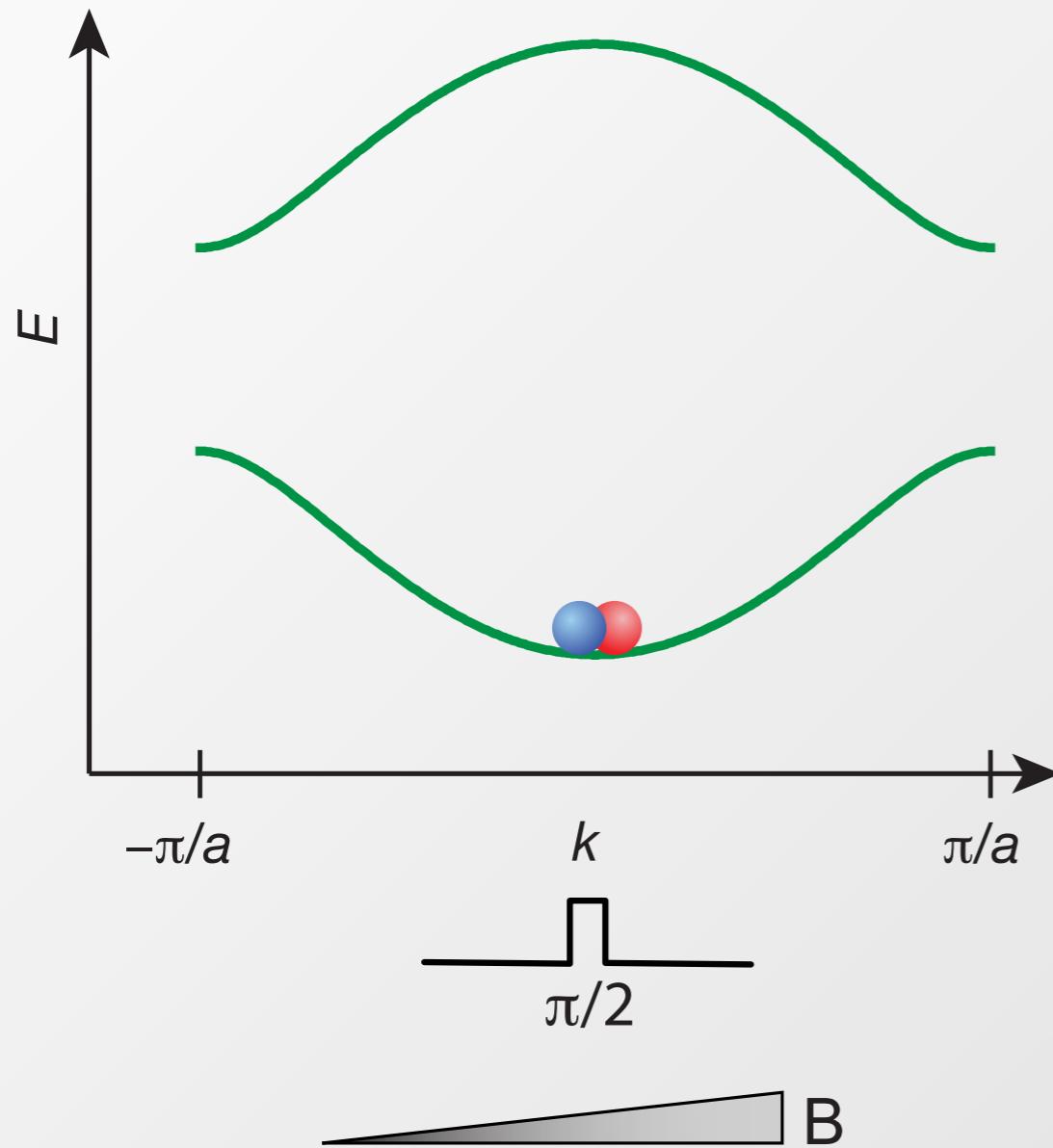
$$\varphi_{\text{Zak}} = \frac{i}{2} \int_0^G \partial_k \theta_k dk = \pm \frac{\pi}{2}$$

Zak-Berry Phase SSH Model

For the two dimerized configurations!



Measuring the Berry-Zak Phase (SSH Model)

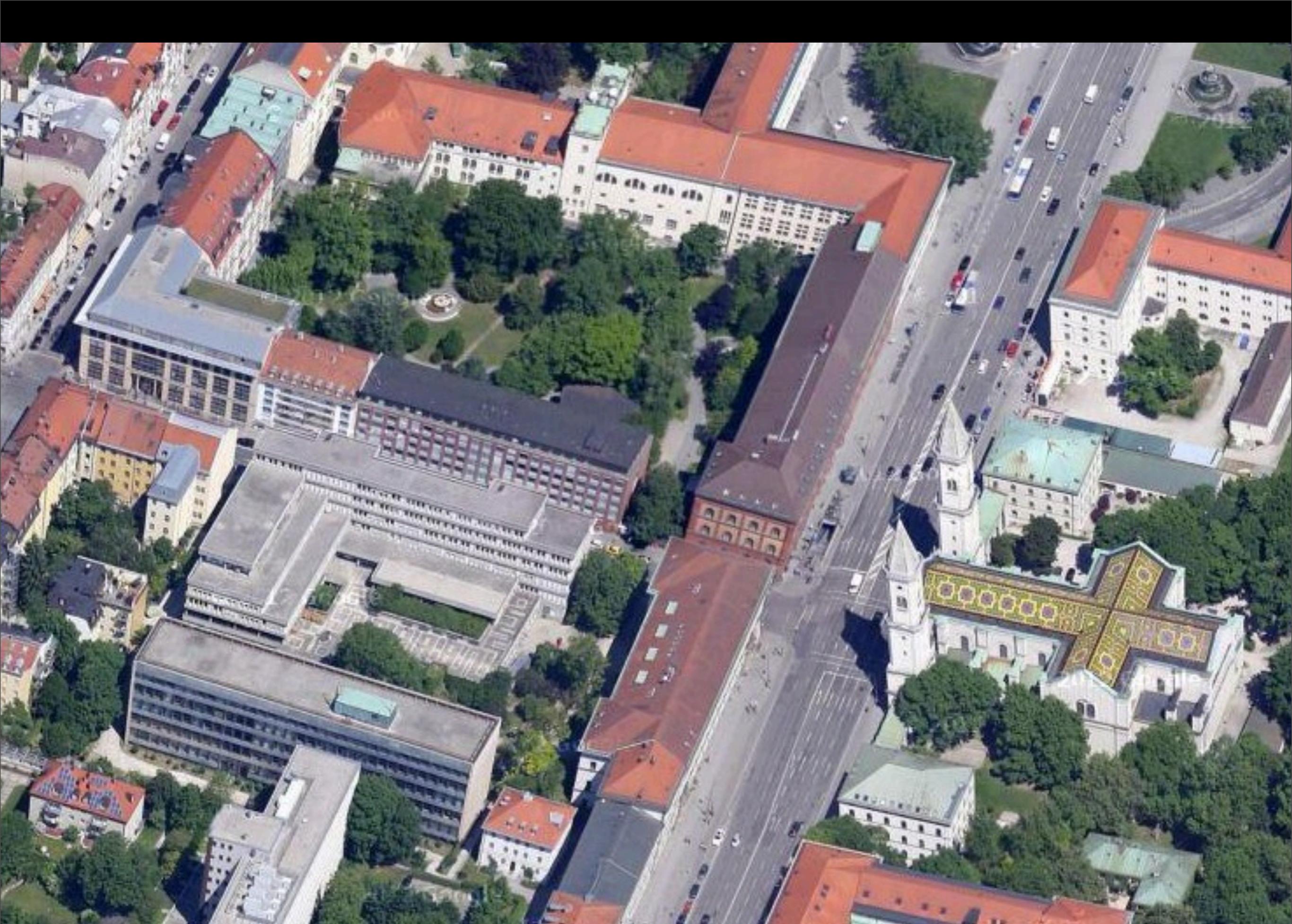


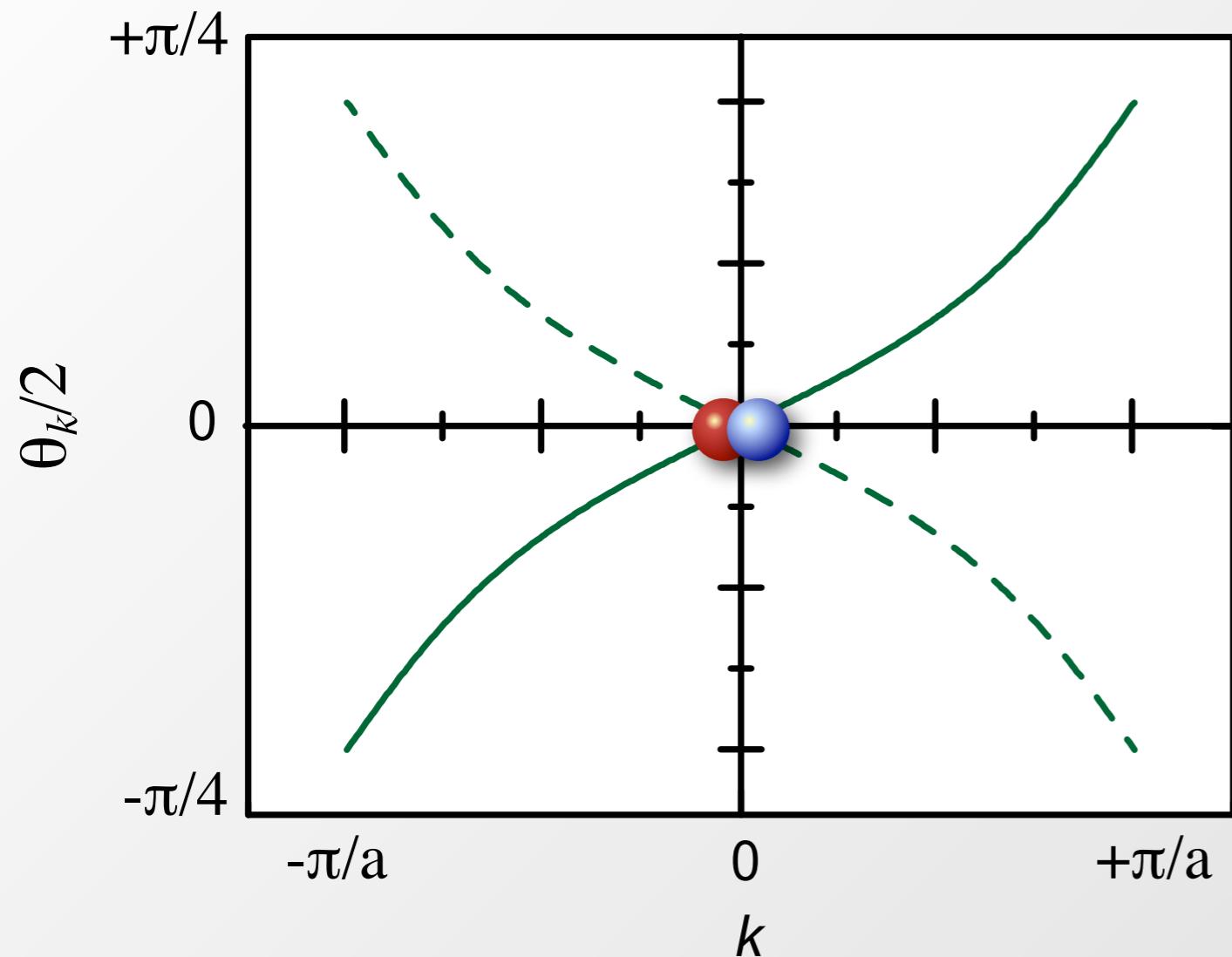
Closed Loop in
 k -Space

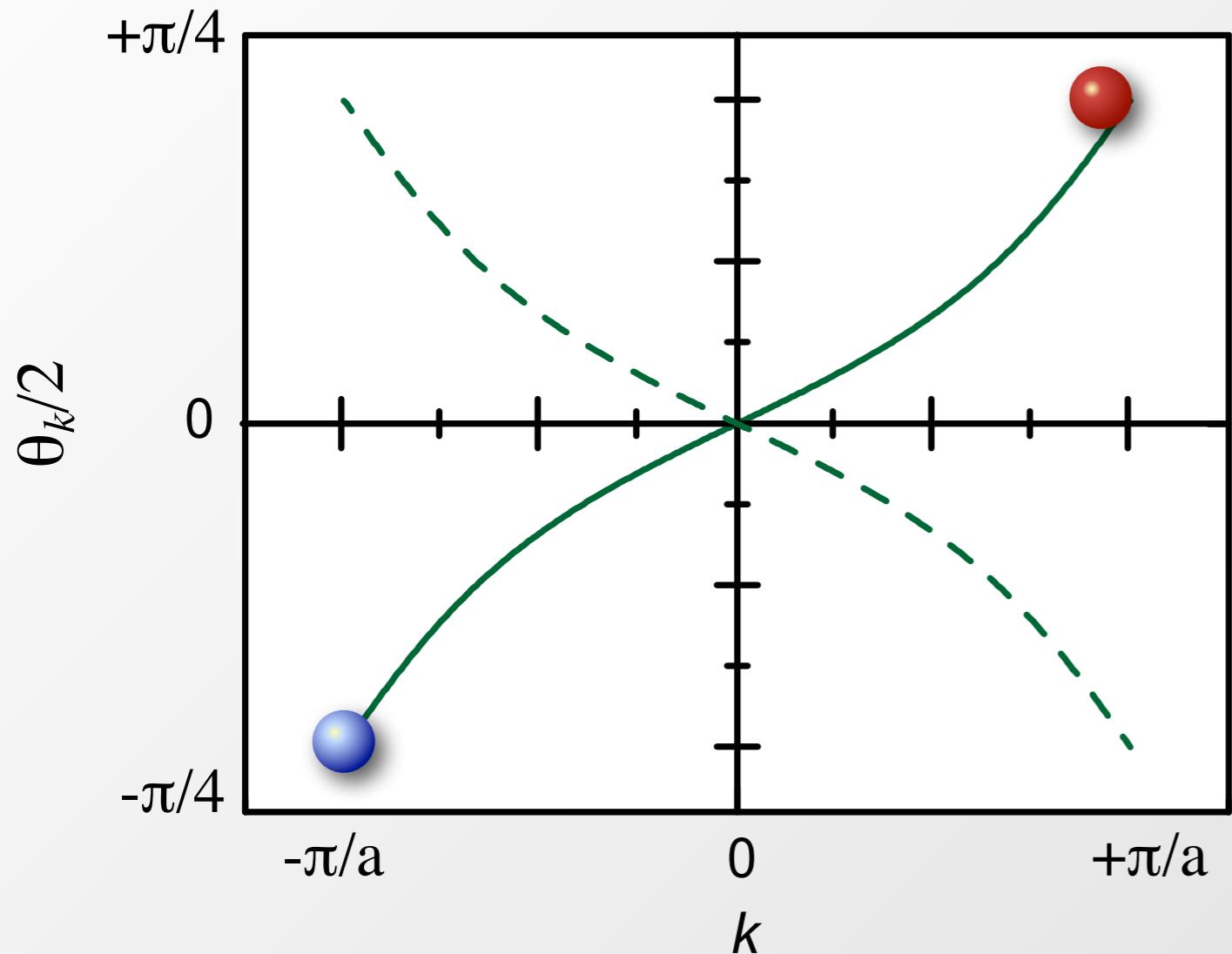
What have we measured:

$$\varphi_{\text{tot}} = \varphi_{\text{Zak}} + \varphi_{\text{dyn}} + \varphi_{\text{Zeeman}}$$



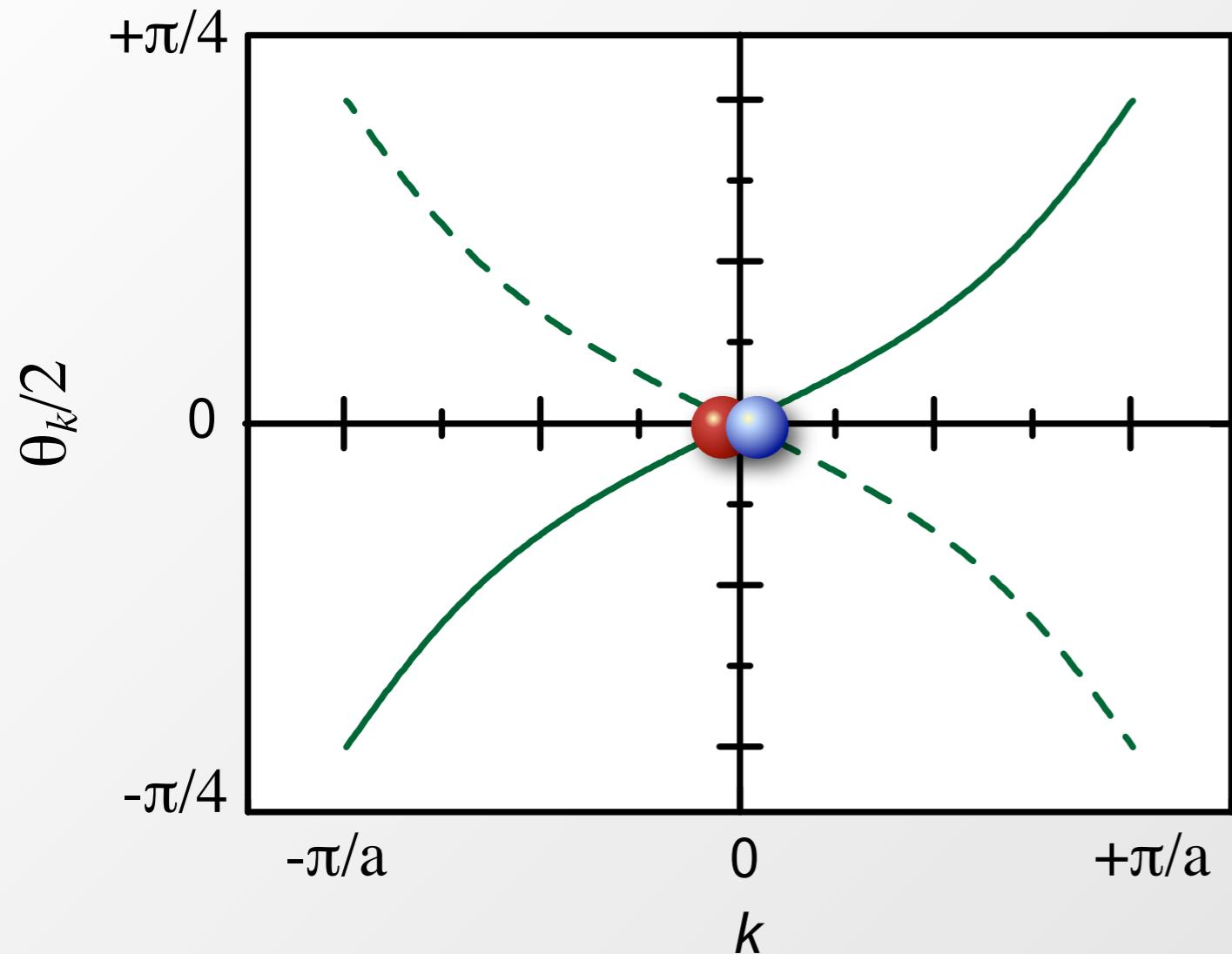


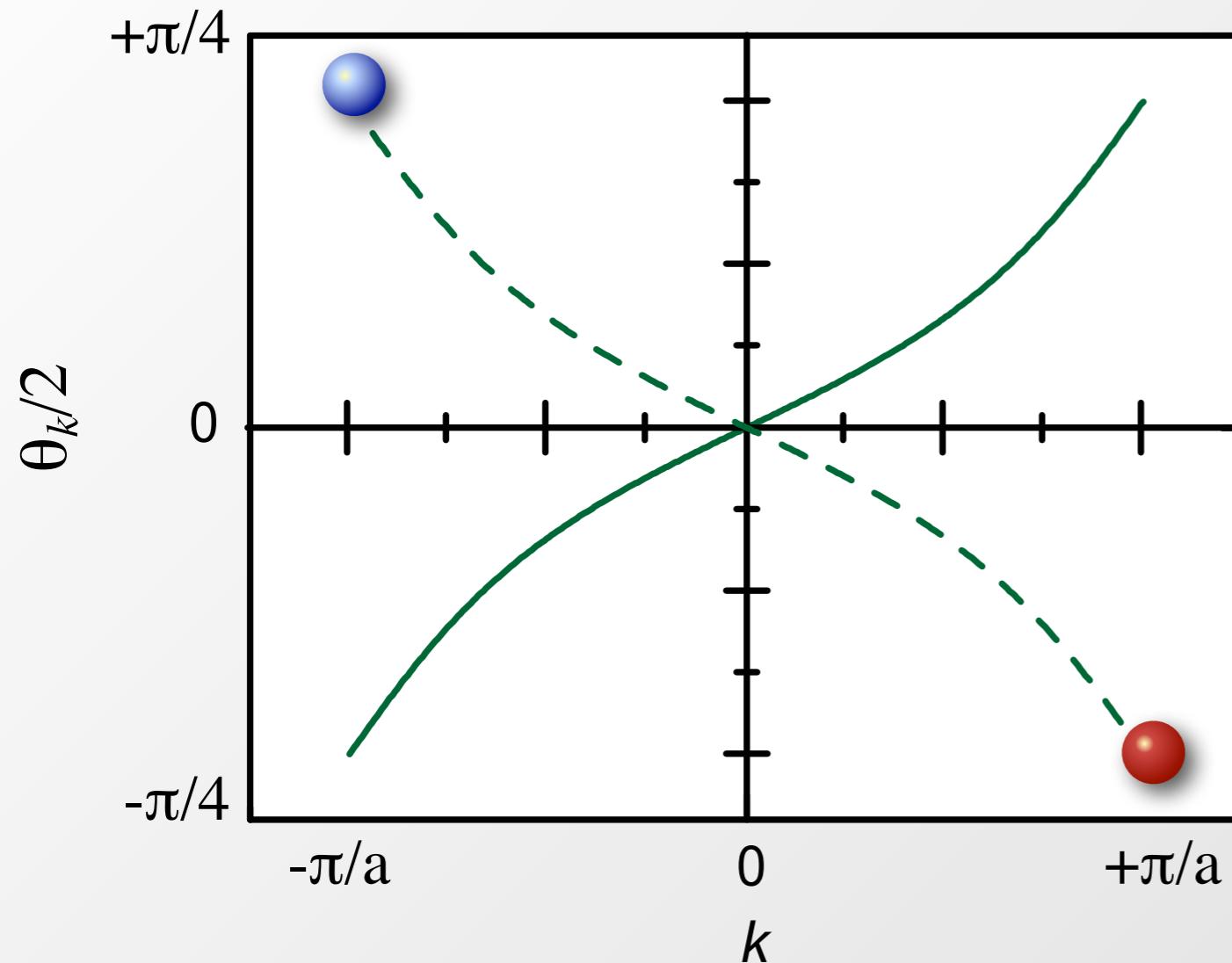




$$\varphi_{\text{tot}} = \varphi_{\text{ak}} + \varphi_{\text{vn}} + \varphi_{\text{Zeeman}}$$







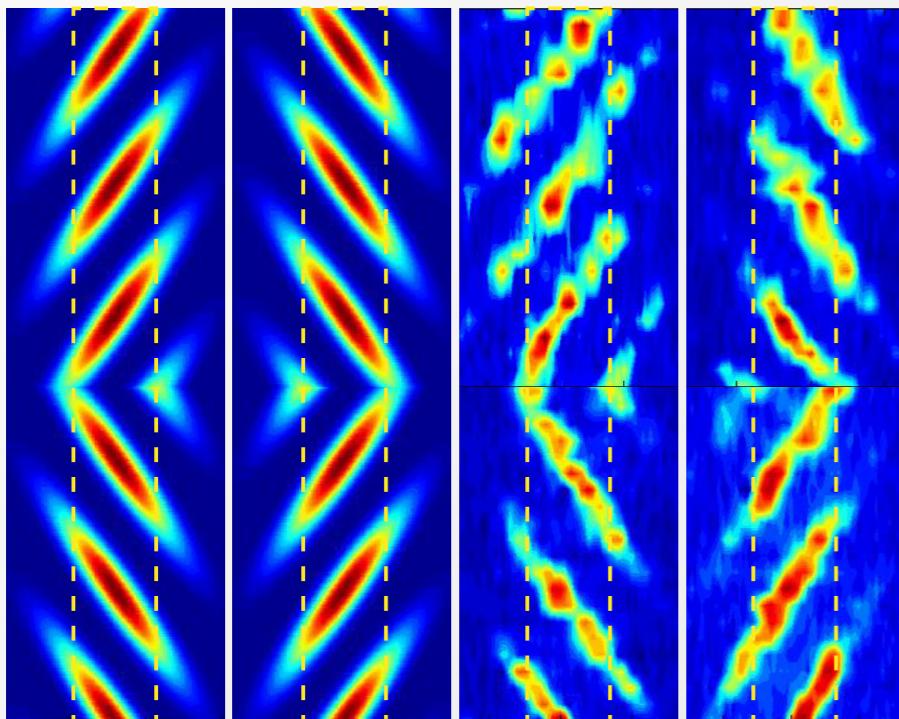
Photon Echo pi-Puls
+Change of Dimerization

$$\varphi_{\text{tot}} = (\varphi_{\text{Zak}}^{\text{D1}} - \varphi_{\text{Zak}}^{\text{D2}}) + \varphi_{\text{syn}} \cancel{+ \varphi_{\text{Zeeman}}}$$



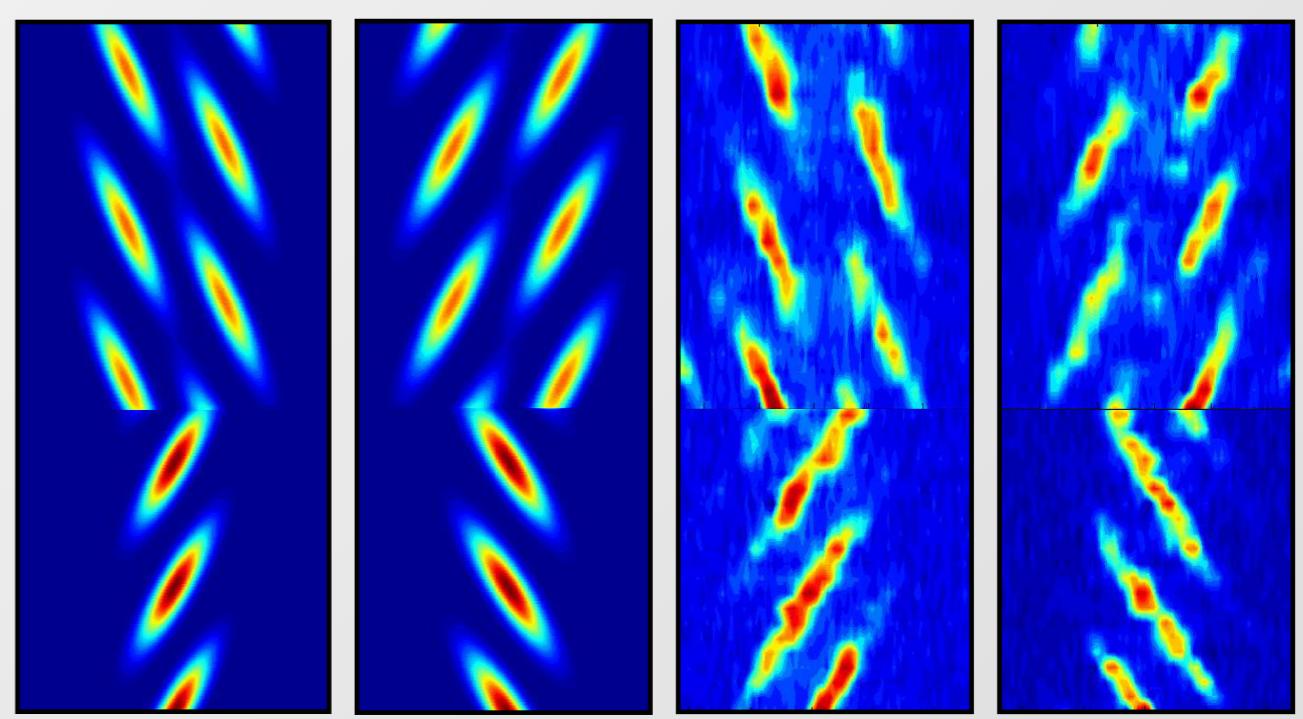
Bloch Oscillations - Experiment

Theory



Experiment

Theory



Experiment

Momentum k

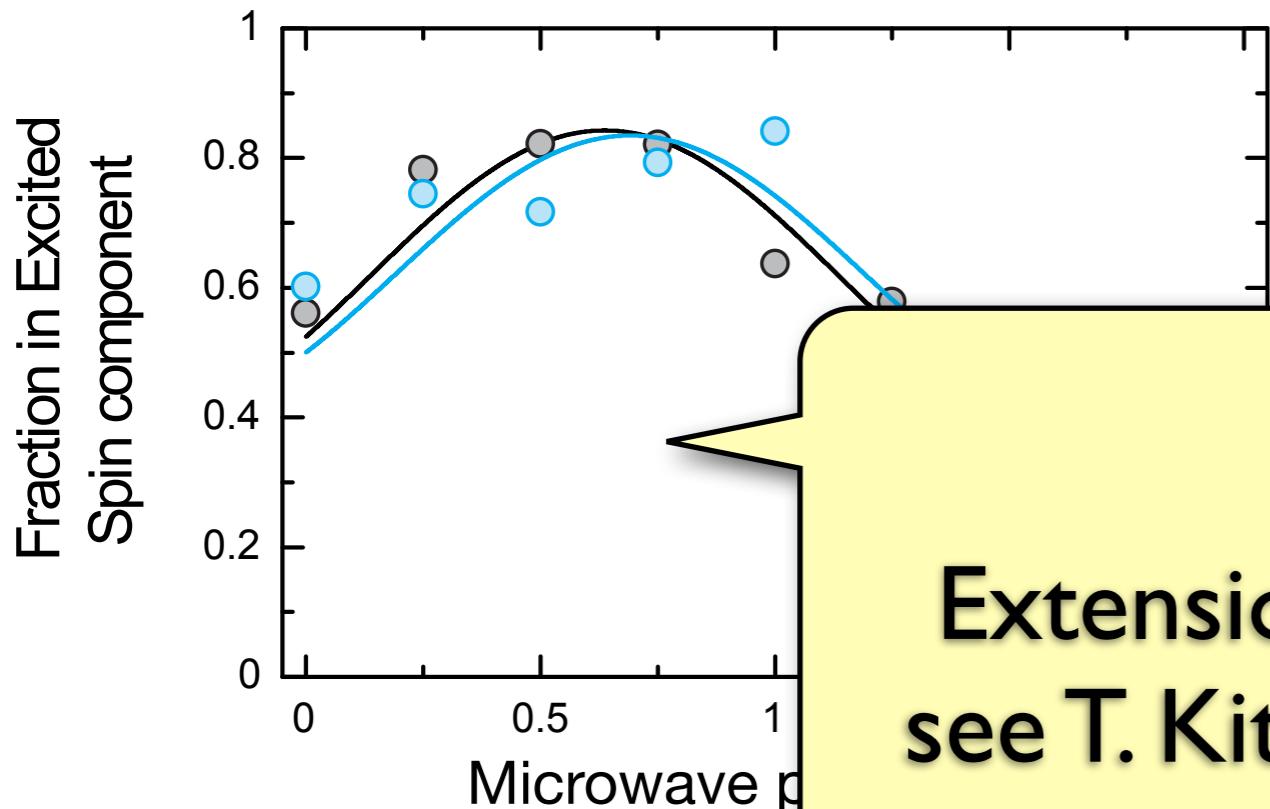
without dimerization swapping

Momentum k

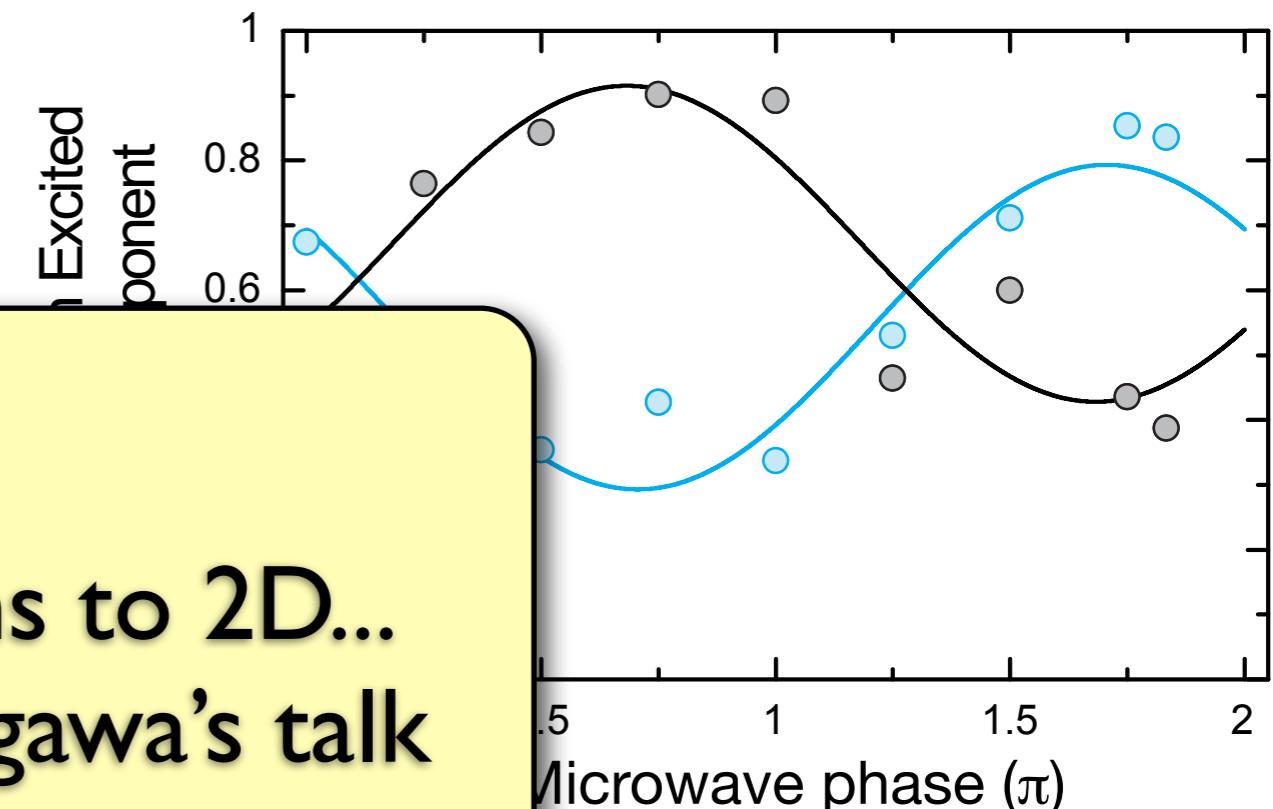
without dimerization swapping

Bloch oscillations in gradient field!





without dimerization



Extensions to 2D...
see T. Kitagawa's talk

merization swapping

Measured Topological Berry-Zak Phase:

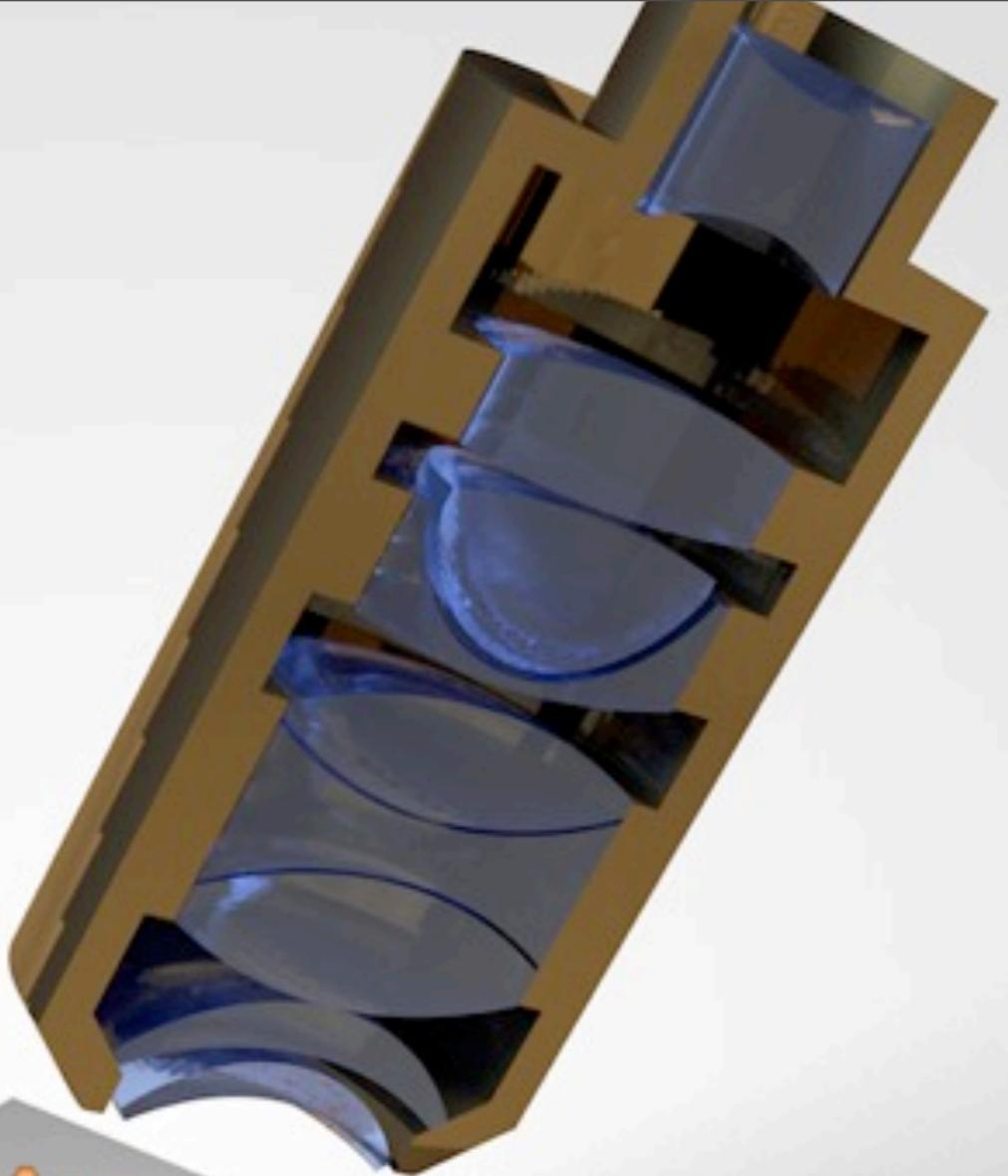
$$\Delta\varphi_{Zak} = 1.02(4)\pi$$



Outlook

- Rectified Flux, Hofstadter Butterfly
- Novel Correlated Phases in Strong Fields
- Novel Topological Insulators
- SU(N) Quantum Magnets
- Controlled Quasiparticle Manipulations
- Non-Equilibrium Dynamics (Universality?)
- Thermalization in Isolated Quantum Systems
- Lieb-Robinson in 2D?
- String Order in 2D?
- Dynamical Observation of Spin-Charge Separation
- Entanglement Measures in Dynamics
- Rydberg Quantum Crystals

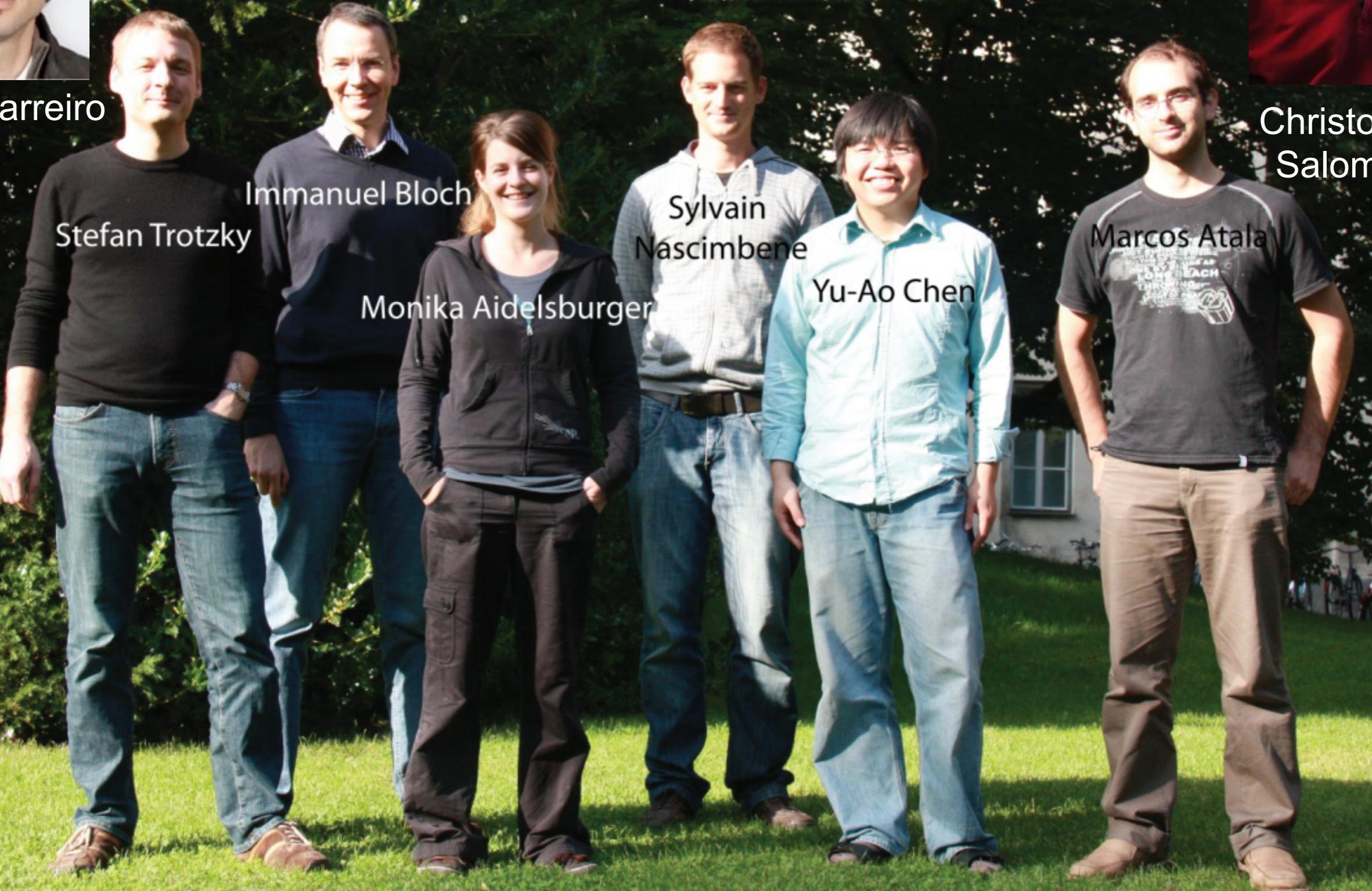
⋮



Gauge Field Team



Julio Barreiro



Single Atom Team

Takeshi
Fukuhara

Peter
Schauß

Ahmed
Omran

David Bellem

Manuel
Endres

Christof
Weitenberg

Sebastian
Hild

Jacob
Sherson

Rosa Glöckner
& Ralf Labouvie

Christian
Groß

Marc
Cheneau

Stefan
Kuhr

Immanuel
Bloch

Thank you!



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