UNIVERSITY OF INNSBRUCK

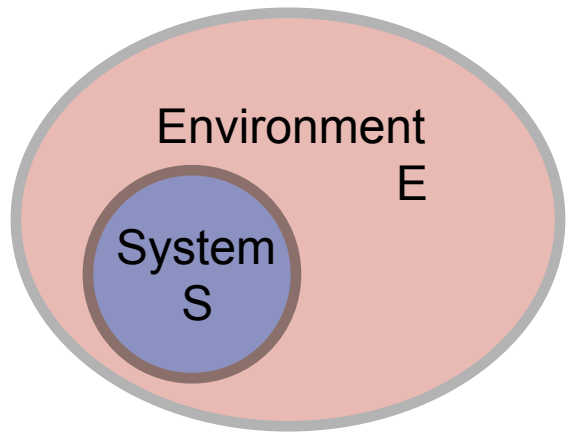
Quantum Reservoir Engineering (Discussion in the Quantum Coherent Control Program)

IQOQI
AUSTRIAN ACADEMY OF SCIENCES

Peter Zoller

€U AQUTE
€U COHERENCE
erc
UQUAM
European Research Council
Established by the European Commission
OLE
AFOSR
FWF Der Wissenschaftsfonds.

Quantum reservoir engineering?



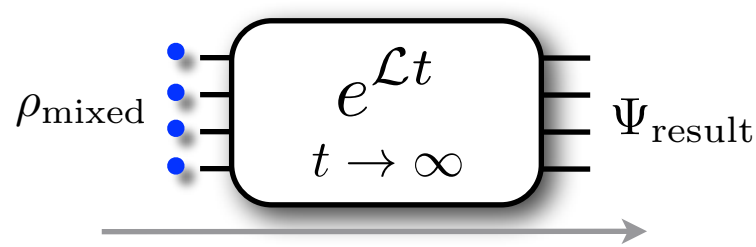
dissipation as
a resource

fundamentally
unevitable
coupling

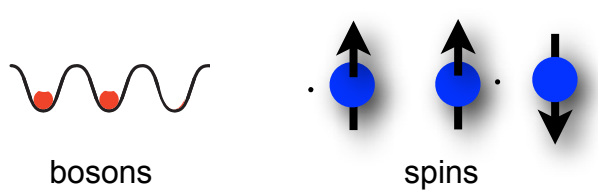
quantum information

diss. preparation of
multi-qubit quantum states

dissipative quantum computation



non-equilibrium dynamics
in open many-body quantum systems



Entanglement by Dissipation

"Optical Pumping"

theoretical concepts:

Review:

M. Müller, S. Diehl, G. Pupillo, P. Zoller, *Engineered Open Systems and Quantum Simulations with Atoms and Ions*, arXiv:1203.6595; published in Advances of Atomic, Molecular and Optical Physics 2012

F. Verstraete, M.M. Wolf, J.I.Cirac, Nature Physics (2009)

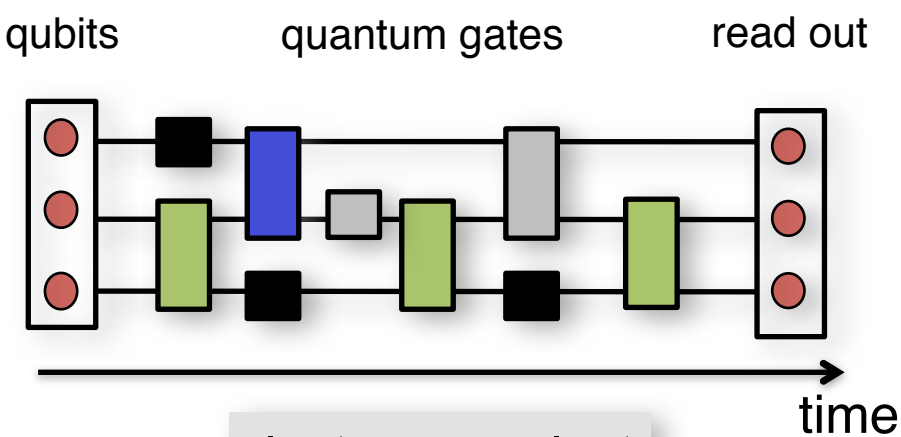
first experiments:

J. Barreiro, M. Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller & R. Blatt
Nature 470, 486 (2011)

H. Krauter, E. Polzik, I. Cirac et al. PRL 2011.

Entanglement via Unitary Evolution

- **quantum logic network model**

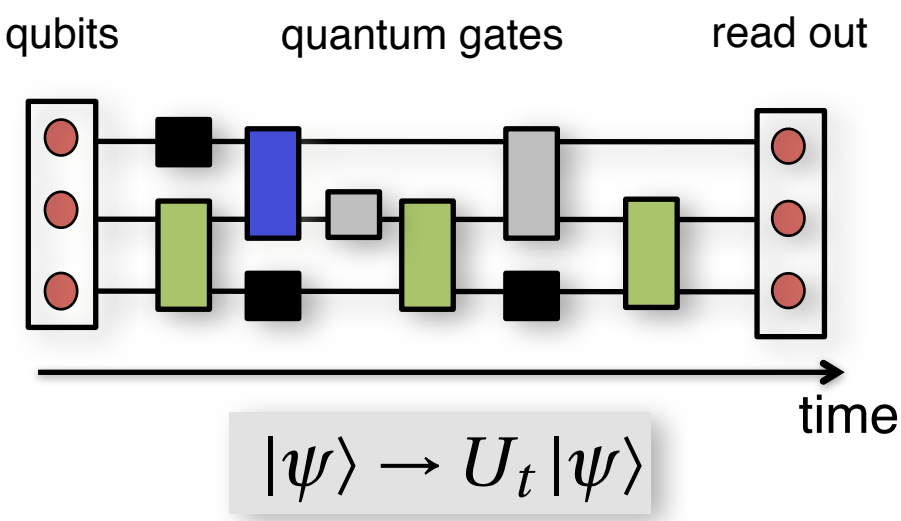


$$|\psi\rangle \rightarrow U_t |\psi\rangle$$

coherent Hamiltonian evolution
- quantum gates
- deterministic

Entanglement via Unitary Evolution

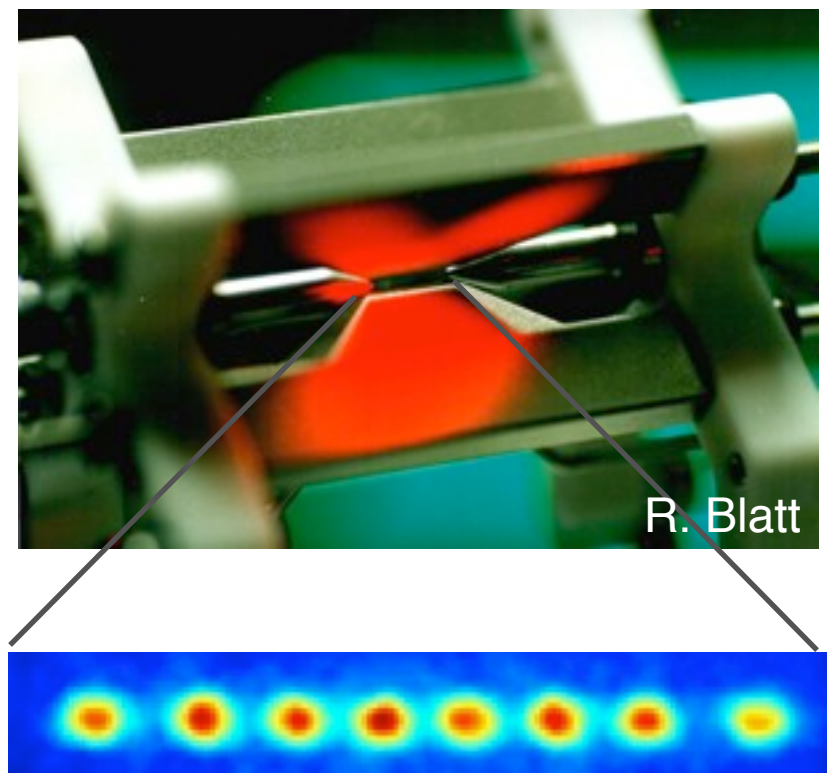
- **quantum logic network model**



$$|\psi\rangle \rightarrow U_t |\psi\rangle$$

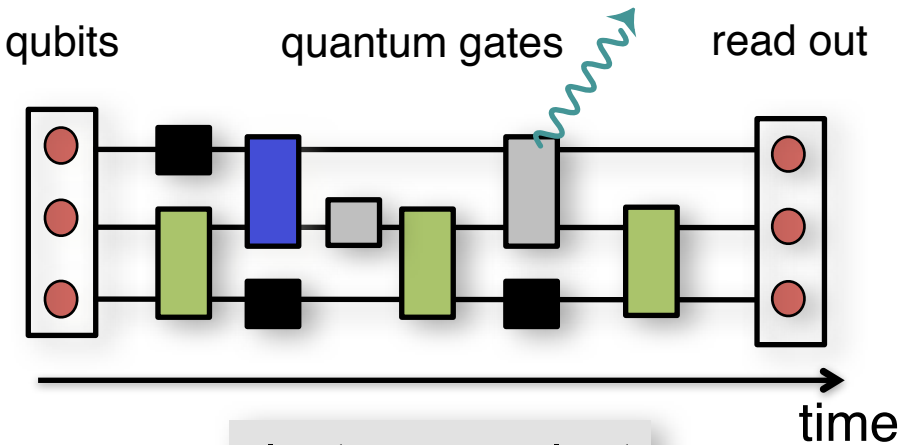
coherent Hamiltonian evolution
- quantum gates
- deterministic

- **atomic physics: trapped ions**



Entanglement via Unitary Evolution

- **quantum logic network model**



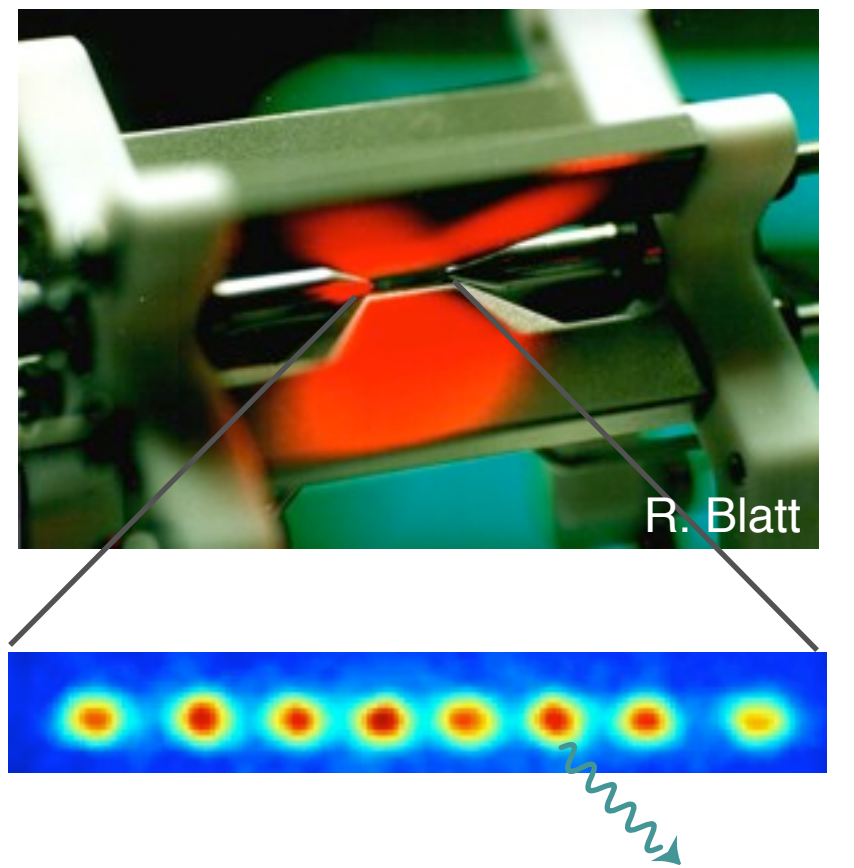
$$|\psi\rangle \rightarrow U_t |\psi\rangle$$

coherent Hamiltonian evolution
- quantum gates
- deterministic

- **decoherence ☹**

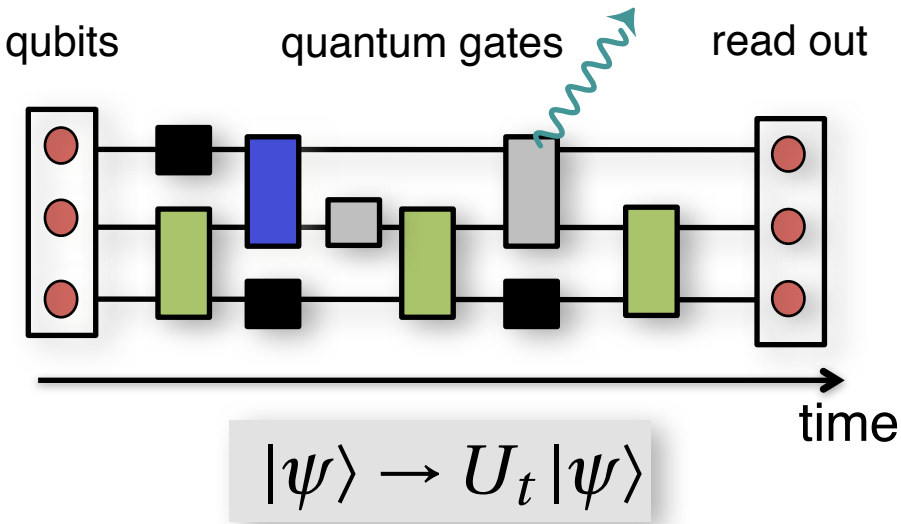
dissipation by coupling to environment
- spontaneous emission etc.

- **atomic physics: trapped ions**



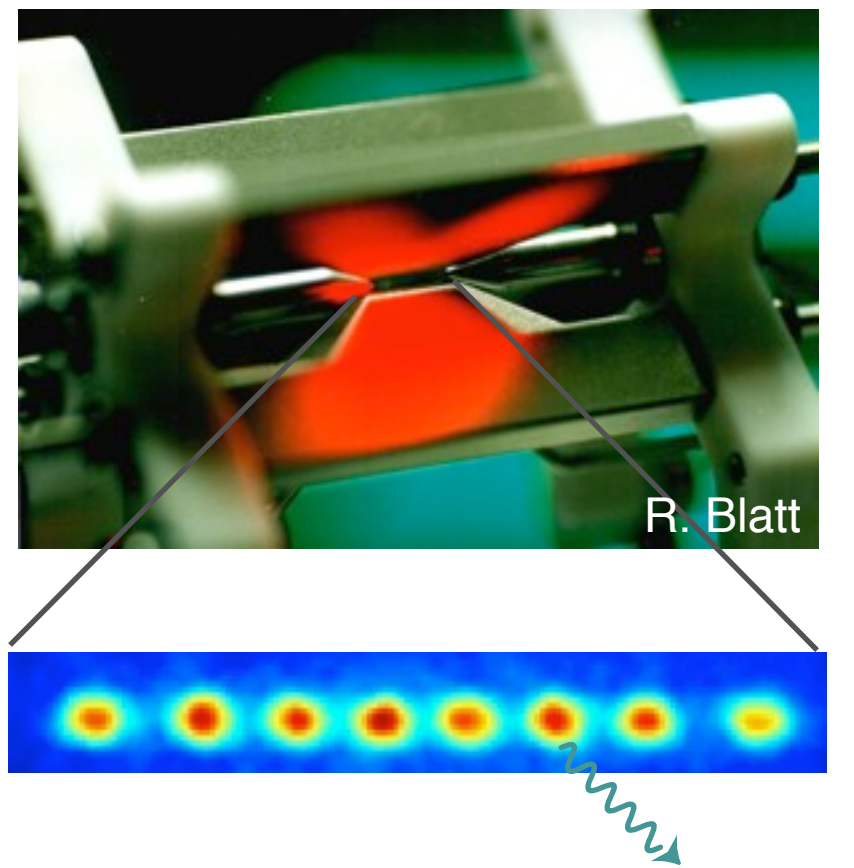
Entanglement via Unitary Evolution

- **quantum logic network model**



coherent Hamiltonian evolution
- quantum gates
- deterministic

- **atomic physics: trapped ions**



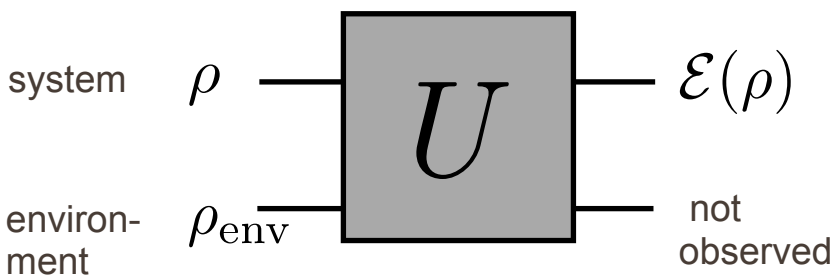
- **decoherence ☹**

dissipation by coupling to environment
- spontaneous emission etc.

Q.: Dissipation as an engineering tool? For entanglement?

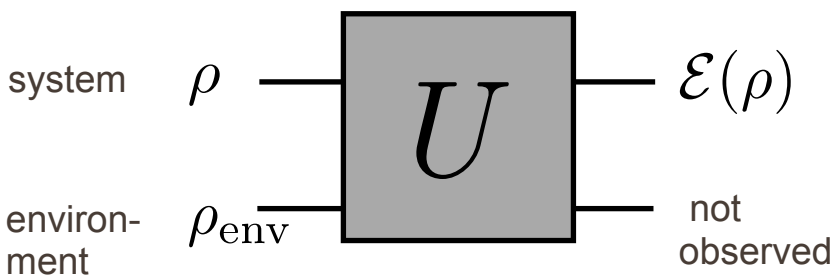
Open System Dynamics [& Decoherence ☹]

- **open system dynamics**



Open System Dynamics [& Decoherence ☹]

- **open system dynamics**



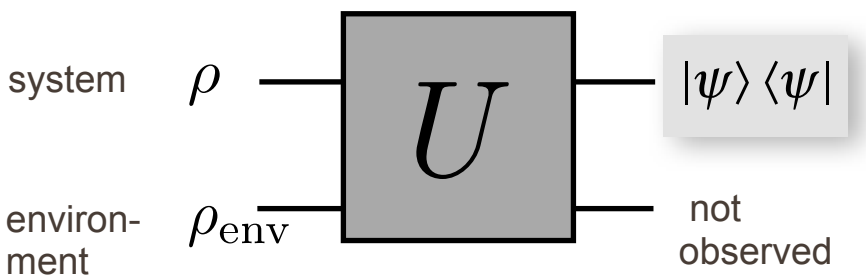
completely positive maps:

$$\rho \to \mathscr{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

Kraus operator

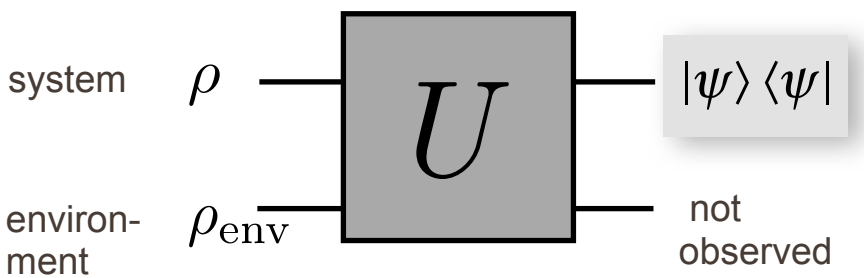
Entanglement from (Engineered) Dissipation

- **open system dynamics**



Entanglement from (Engineered) Dissipation

- **open system dynamics**



engineering Kraus operators:

$$\rho \to \mathscr{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

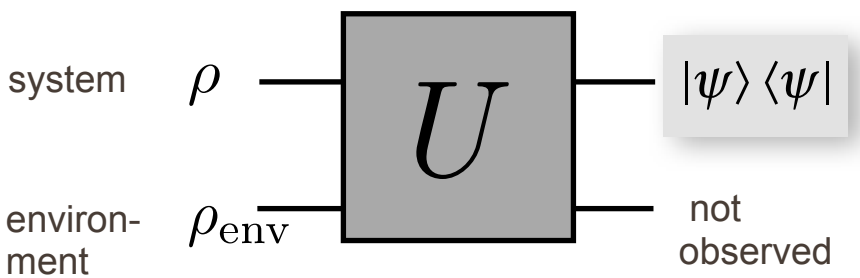
$$\overset{!}{=} |\psi\rangle\langle\psi|$$

desired (pure)
quantum state

"cooling" into a pure state
- non-unitary
- deterministic

Entanglement from (Engineered) Dissipation

- **open system dynamics**



engineering Kraus operators:

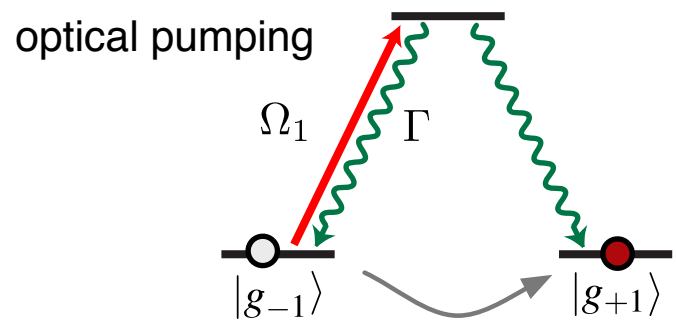
$$\rho \to \mathscr{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

$$\overset{!}{=} |\psi\rangle\langle\psi|$$

desired (pure)
quantum state

"cooling" into a pure state
- non-unitary
- deterministic

- **atomic physics: single particle**

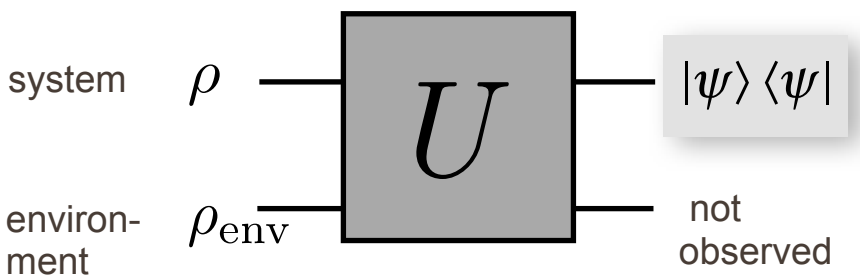


$$\rho(t) \xrightarrow{t\to\infty} |D\rangle\langle D|$$

pumping into a pure "dark state"

Entanglement from (Engineered) Dissipation

- **open system dynamics**



engineering Kraus operators:

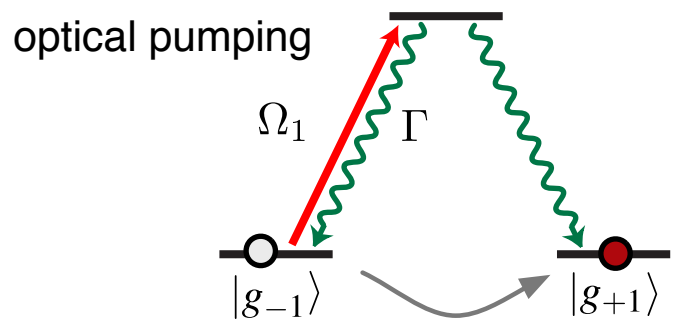
$$\rho \to \mathscr{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

$\overset{!}{=} |\psi\rangle\langle\psi|$

desired (pure)
quantum state

"cooling" into a pure state
- non-unitary
- deterministic

- **atomic physics: single particle**



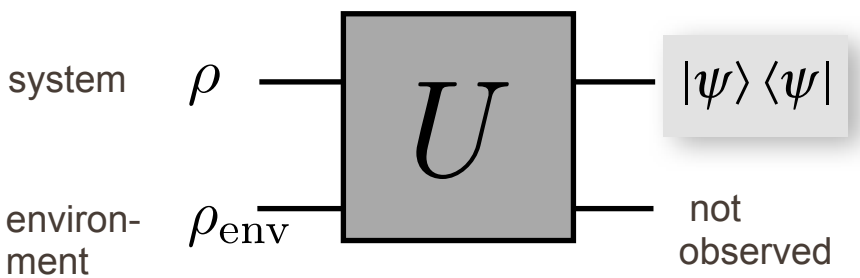
$$\rho(t) \xrightarrow{t\to\infty} |D\rangle\langle D|$$

pumping into a pure "dark state"

Q.: generalize to entangled states?

Entanglement from (Engineered) Dissipation

- **open system dynamics**



engineering Kraus operators:

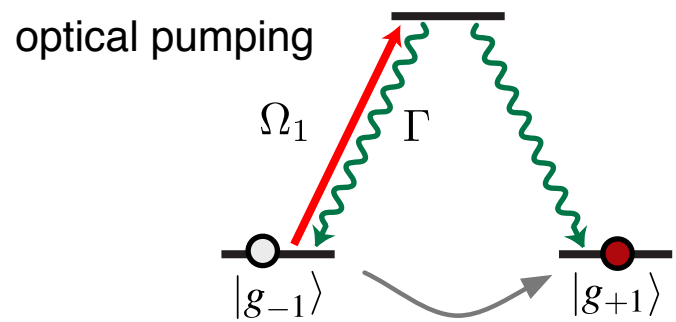
$$\rho \to \mathscr{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

$$\stackrel{!}{=} |\psi\rangle\langle\psi|$$

desired (pure)
quantum state

"cooling" into a pure state
- non-unitary
- deterministic

- **atomic physics: single particle**



$$\rho(t) \xrightarrow{t\to\infty} |D\rangle\langle D|$$

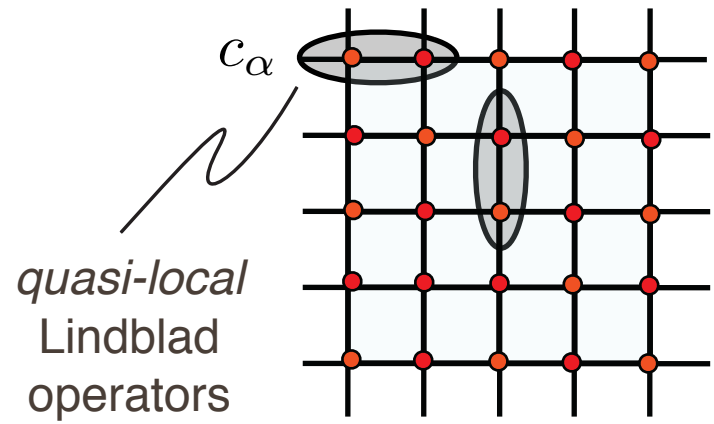
pumping into a pure "dark state"

Q.: generalize to entangled states?

see also: D Bacon et al. PRA 2001; S. Lloyd & L. Viola, PRA 2001; D. Lidar, et al. PRL 1998; G. Baggio, et al. arxiv:1209.5568 (2012).

Dark States: Many Particle

qubits or particles on a lattice



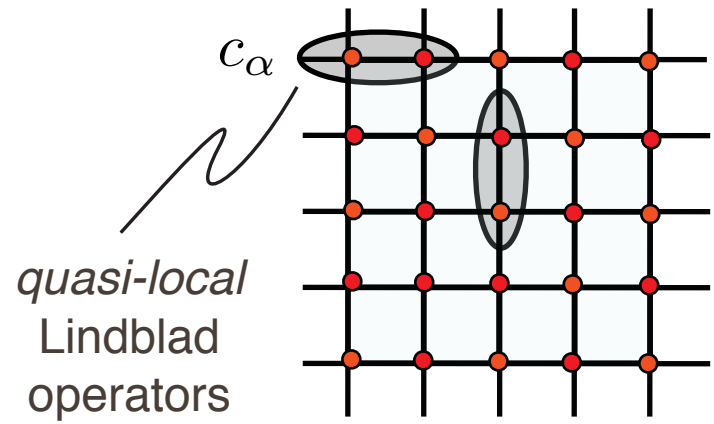
- **master equation**

quantum jump operator (nonhermitian)

$$\dot{\rho} = -i[H,\rho] + \sum_\alpha \gamma_\alpha \left(c_\alpha \rho c_\alpha^\dagger - \frac{1}{2} c_\alpha^\dagger c_\alpha \rho - \rho \frac{1}{2} c_\alpha^\dagger c_\alpha \right)$$

Dark States: Many Particle

qubits or particles on a lattice



- **master equation**

quantum jump operator (nonhermitian)

$$\dot{\rho} = -i[H,\rho] + \sum_\alpha \gamma_\alpha \left(c_\alpha \rho c_\alpha^\dagger - \frac{1}{2} c_\alpha^\dagger c_\alpha \rho - \rho \frac{1}{2} c_\alpha^\dagger c_\alpha \right)$$

- **desired state as "dark state"**

$$H|D\rangle = E|D\rangle$$
$$\forall \alpha \quad c_\alpha |D\rangle = 0$$

desired state

construct a *parent* Liouvillian

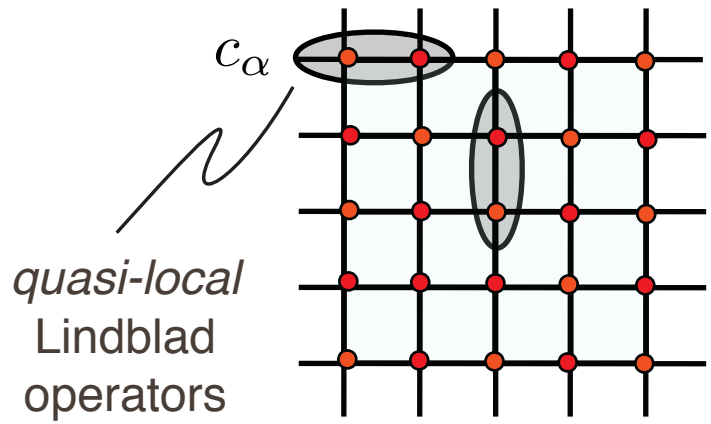
$$\rho(t) \overset{t\to\infty}{\longrightarrow} |D\rangle\langle D|$$

Dark States: Many Particle

qubits or particles on a lattice

Questions:

- ✓ given resources → states (?)
- ✓ uniqueness
- ✓ implementation



- **master equation**

quantum jump operator (nonhermitian)

$$\dot{\rho} = -i[H,\rho] + \sum_\alpha \gamma_\alpha \left(c_\alpha \rho c_\alpha^\dagger - \frac{1}{2} c_\alpha^\dagger c_\alpha \rho - \rho \frac{1}{2} c_\alpha^\dagger c_\alpha \right)$$

- **desired state as "dark state"**

$$H|D\rangle = E|D\rangle$$
$$\forall \alpha \quad c_\alpha |D\rangle = 0$$

desired state

construct a *parent* Liouvillian

$$\rho(t) \overset{t\to\infty}{\longrightarrow} |D\rangle\langle D|$$

Bell State Pumping

- **Bell States**

two spins / qubits

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell State Pumping

two spins / qubits

- **Bell States**

$X_1 X_2$ \ $Z_1 Z_2$	$+1$	-1
$+1$	Φ^+	Ψ^+
-1	Φ^-	Ψ^-

Bell states as eigenstates of (commuting) **stabilizer** operators $X_1 X_2$ and $Z_1 Z_2$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell State Pumping

two spins / qubits

- **Bell States**

$X_1 X_2$ \ $Z_1 Z_2$	$+1$	-1
$+1$	Φ^+	Ψ^+
-1	Φ^-	Ψ^-

Bell states as eigenstates of (commuting) **stabilizer** operators $X_1 X_2$ and $Z_1 Z_2$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

- **Goal: Bell state pumping** $\rho(t) \longrightarrow |\Psi^-\rangle\langle\Psi^-|$

Bell State Pumping

two spins / qubits

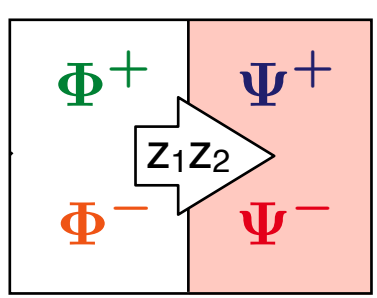
- **Bell States**

X_1X_2 \ Z_1Z_2	$+1$	-1
$+1$	Φ^+	Ψ^+
-1	Φ^-	Ψ^-

Bell states as eigenstates of (commuting) **stabilizer** operators X_1X_2 and Z_1Z_2

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

- **Goal: Bell state pumping** $\rho(t) \longrightarrow |\Psi^-\rangle\langle\Psi^-|$



Bell State Pumping

two spins / qubits

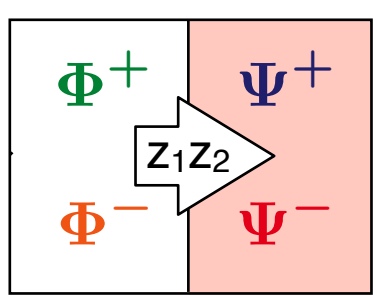
- **Bell States**

$X_1 X_2$ \ $Z_1 Z_2$	$+1$	-1
$+1$	Φ^+	Ψ^+
-1	Φ^-	Ψ^-

Bell states as eigenstates of (commuting) **stabilizer** operators $X_1 X_2$ and $Z_1 Z_2$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

- **Goal: Bell state pumping** $\rho(t) \longrightarrow |\Psi^-\rangle\langle\Psi^-|$



quantum jump operators

$$c_1 = X_1(1 + Z_1 Z_2)$$

Bell State Pumping

two spins / qubits

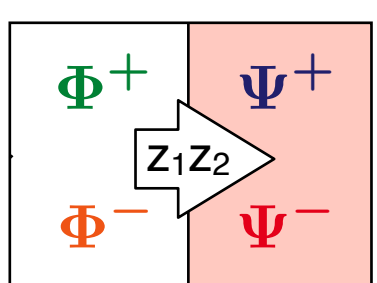
- **Bell States**

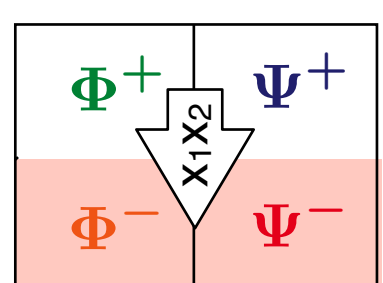
X_1X_2 \ Z_1Z_2	$+1$	-1
$+1$	Φ^+	Ψ^+
-1	Φ^-	Ψ^-

Bell states as eigenstates of (commuting) **stabilizer** operators X_1X_2 and Z_1Z_2

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

- **Goal: Bell state pumping** $\rho(t) \longrightarrow |\Psi^-\rangle\langle\Psi^-|$





quantum jump operators

$$c_1 = X_1(1 + Z_1Z_2)$$

Bell State Pumping

two spins / qubits

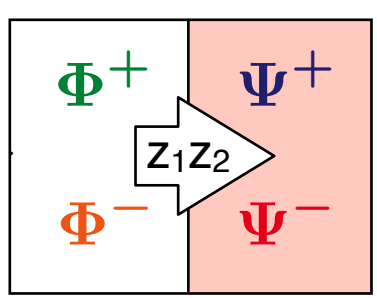
- **Bell States**

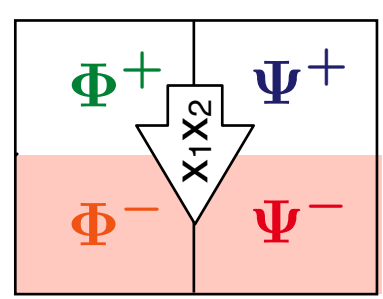
X_1X_2 \ Z_1Z_2	$+1$	-1
$+1$	Φ^+	Ψ^+
-1	Φ^-	Ψ^-

Bell states as eigenstates of (commuting) **stabilizer** operators X_1X_2 and Z_1Z_2

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

- **Goal: Bell state pumping** $\rho(t) \longrightarrow |\Psi^-\rangle\langle\Psi^-|$





quantum jump operators

$c_1 = X_1(1 + Z_1Z_2)$

$c_2 = Z_1(1 + X_1X_2)$

Bell State Pumping

two spins / qubits

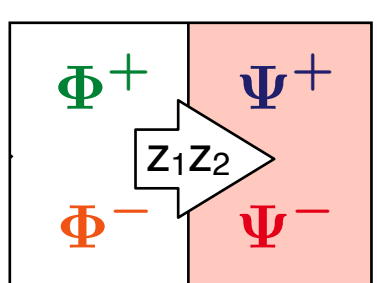
- **Bell States**

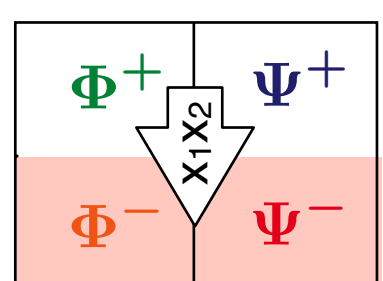
X_1X_2 \ Z_1Z_2	$+1$	-1
$+1$	Φ^+	Ψ^+
-1	Φ^-	Ψ^-

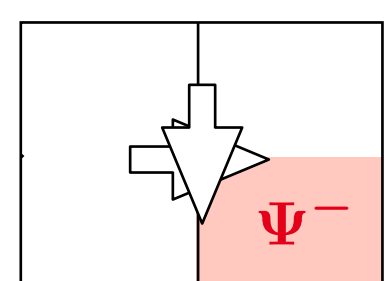
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell states as eigenstates of (commuting) **stabilizer** operators X_1X_2 and Z_1Z_2

- **Goal: Bell state pumping** $\rho(t) \longrightarrow |\Psi^-\rangle\langle\Psi^-|$







quantum jump operators

$c_1 = X_1(1 + Z_1Z_2)$

$c_2 = Z_1(1 + X_1X_2)$

Bell State Pumping

two spins / qubits

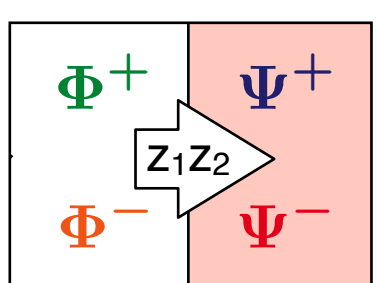
- **Bell States**

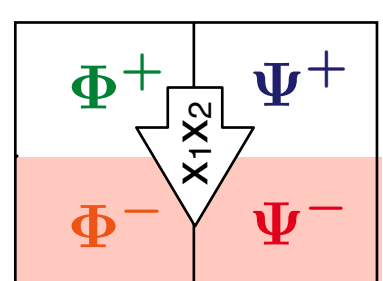
X_1X_2 \ Z_1Z_2	$+1$	-1
$+1$	Φ^+	Ψ^+
-1	Φ^-	Ψ^-

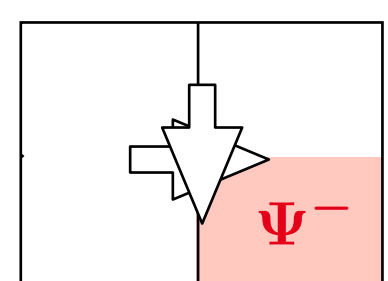
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell states as eigenstates of (commuting) **stabilizer** operators X_1X_2 and Z_1Z_2

- **Goal: Bell state pumping** $\rho(t) \longrightarrow |\Psi^-\rangle\langle\Psi^-|$





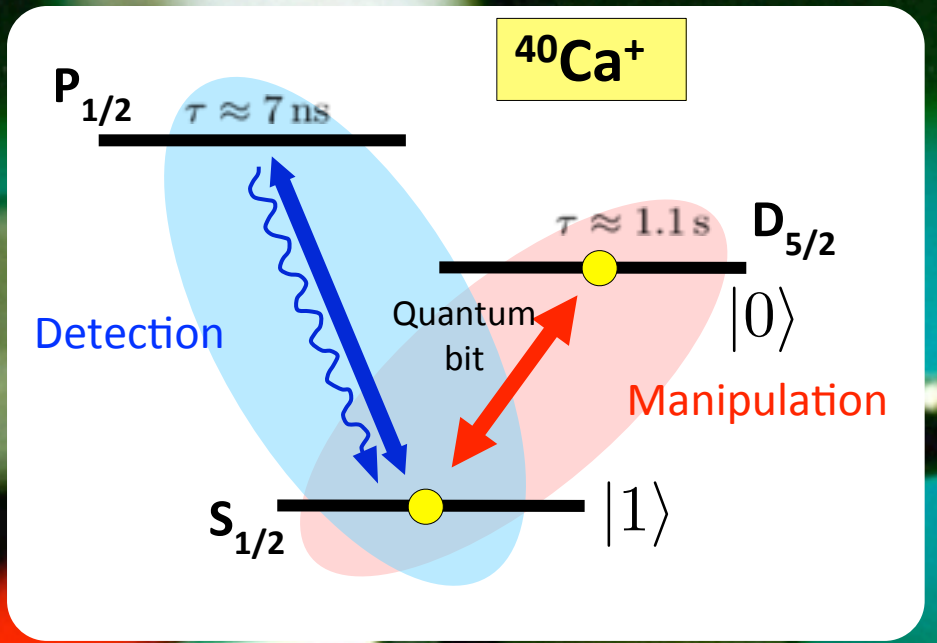


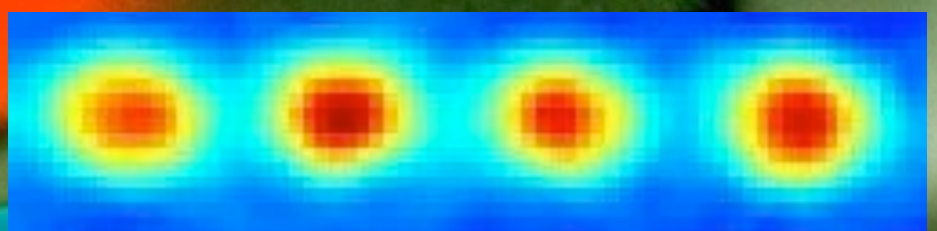
quantum jump operators

$c_1 = X_1(1 + Z_1Z_2)$

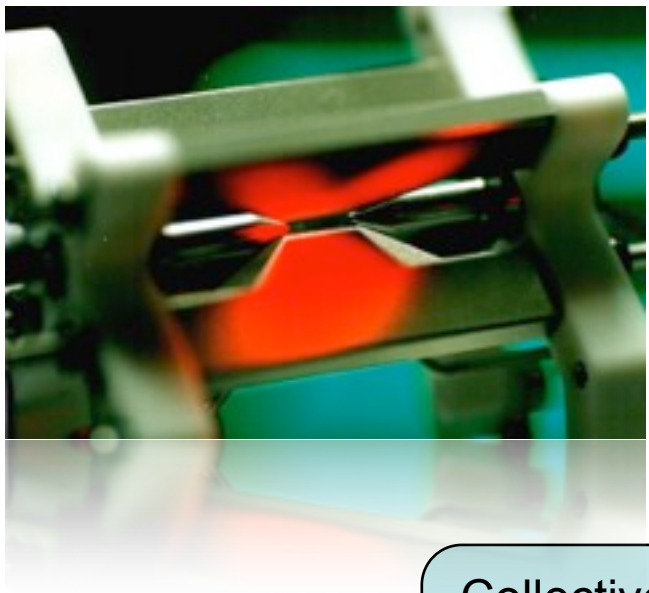
$c_2 = Z_1(1 + X_1X_2)$ ← 3-particle operators ☹

Ions confined in a string by a Paul trap





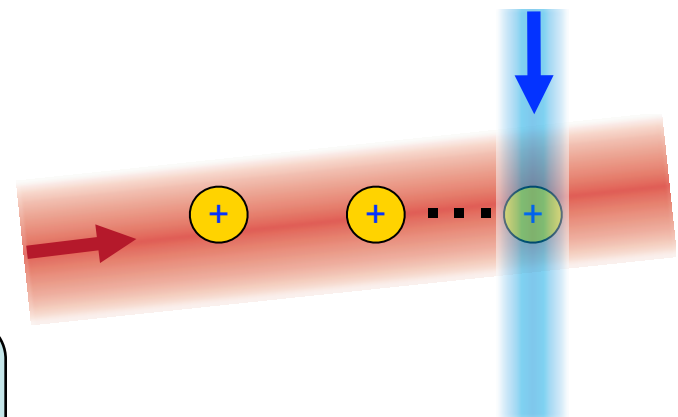
Innsbruck ion trap



Quantum operations on Innsbruck ion-trap quantum computer

Individual light-shift gates

$$\sigma_z^{(0)}, \sigma_z^{(1)}, \sigma_z^{(2)}$$

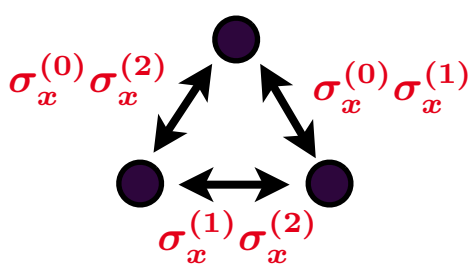


Collective spin flips

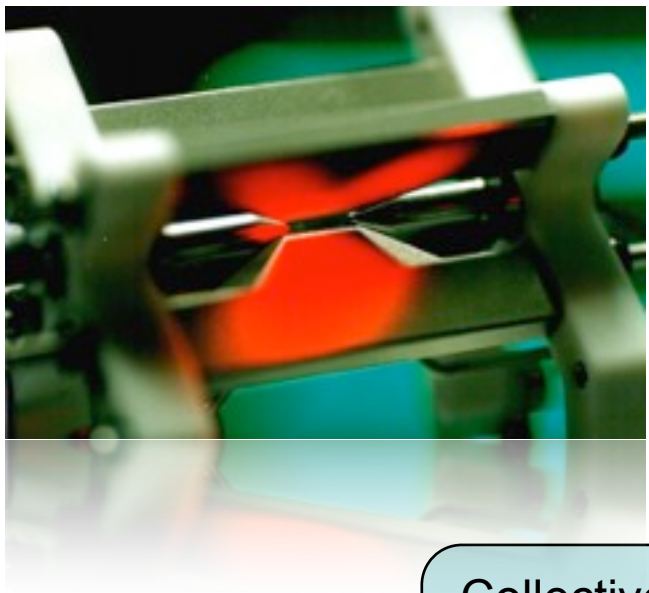
$$S_x, S_y$$

Mølmer-Sørensen gate

$$S_x^2 = \sigma_x^{(0)}\sigma_x^{(1)} + \sigma_x^{(1)}\sigma_x^{(2)} + \sigma_x^{(0)}\sigma_x^{(2)}$$



... 14-qubit entanglement, T. Monz et al. PRL (2011) talk U6.4



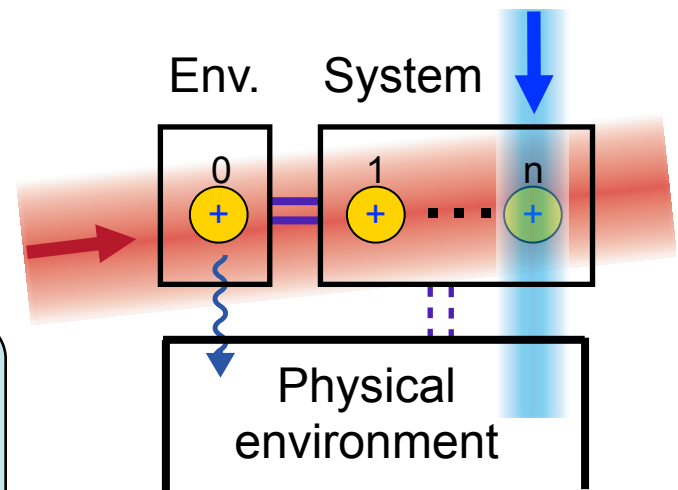
Quantum operations on Innsbruck ion-trap quantum computer

Individual light-shift gates

$$\sigma_z^{(0)}, \sigma_z^{(1)}, \sigma_z^{(2)}$$

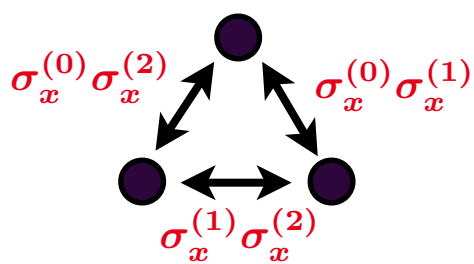
Collective spin flips

$$S_x, S_y$$



Mølmer-Sørensen gate

$$S_x^2 = \sigma_x^{(0)}\sigma_x^{(1)} + \sigma_x^{(1)}\sigma_x^{(2)} + \sigma_x^{(0)}\sigma_x^{(2)}$$



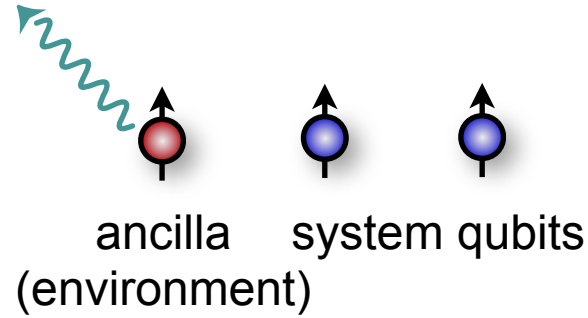
Coupling of environment with physical environment

... 14-qubit entanglement, T. Monz et al. PRL (2011) talk U6.4

Optical pumping of „environmental“ ion

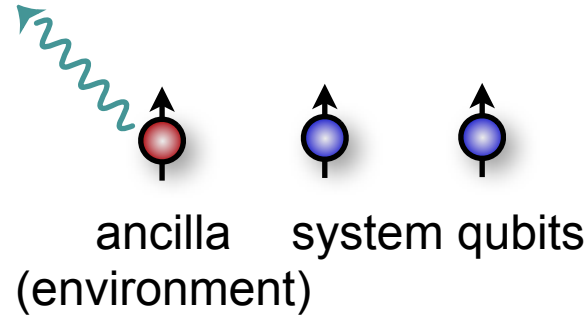
Bell State Pumping

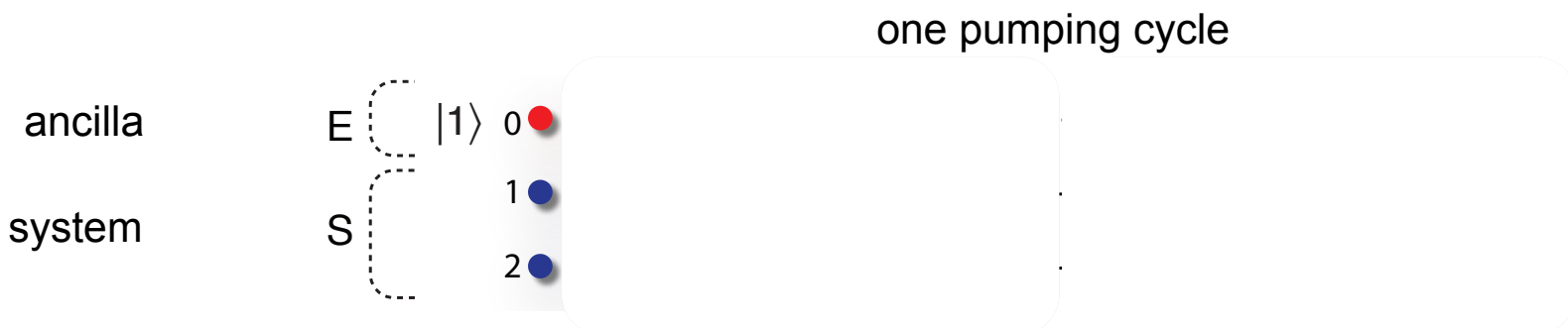
- **quantum circuit**



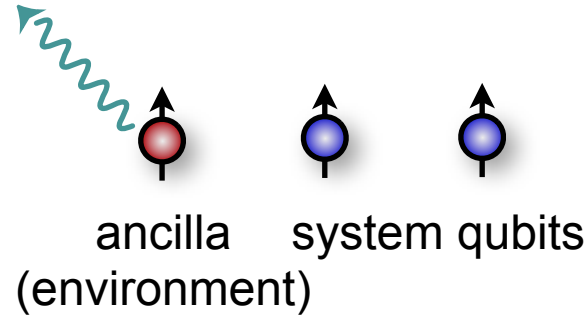
Bell State Pumping

- **quantum circuit**

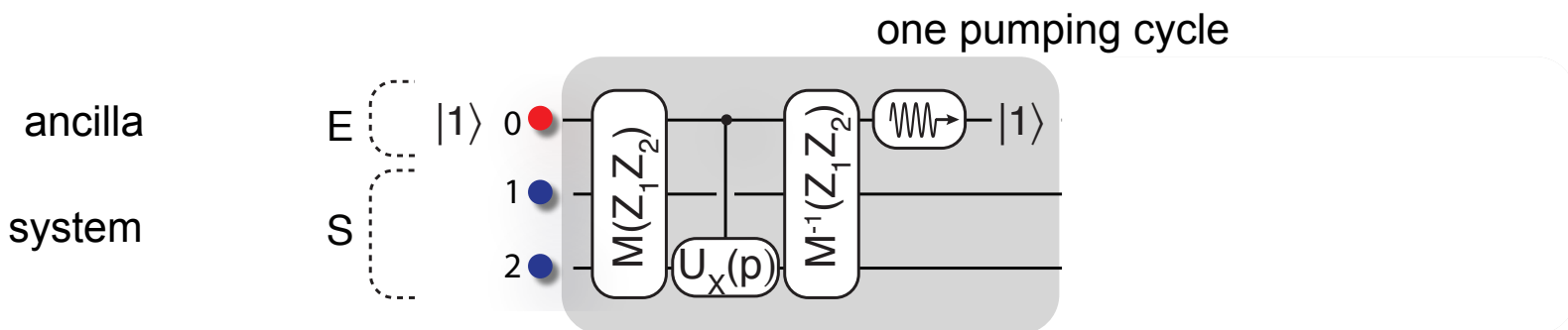




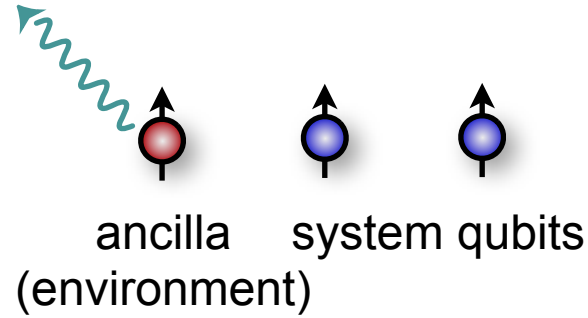
Bell State Pumping



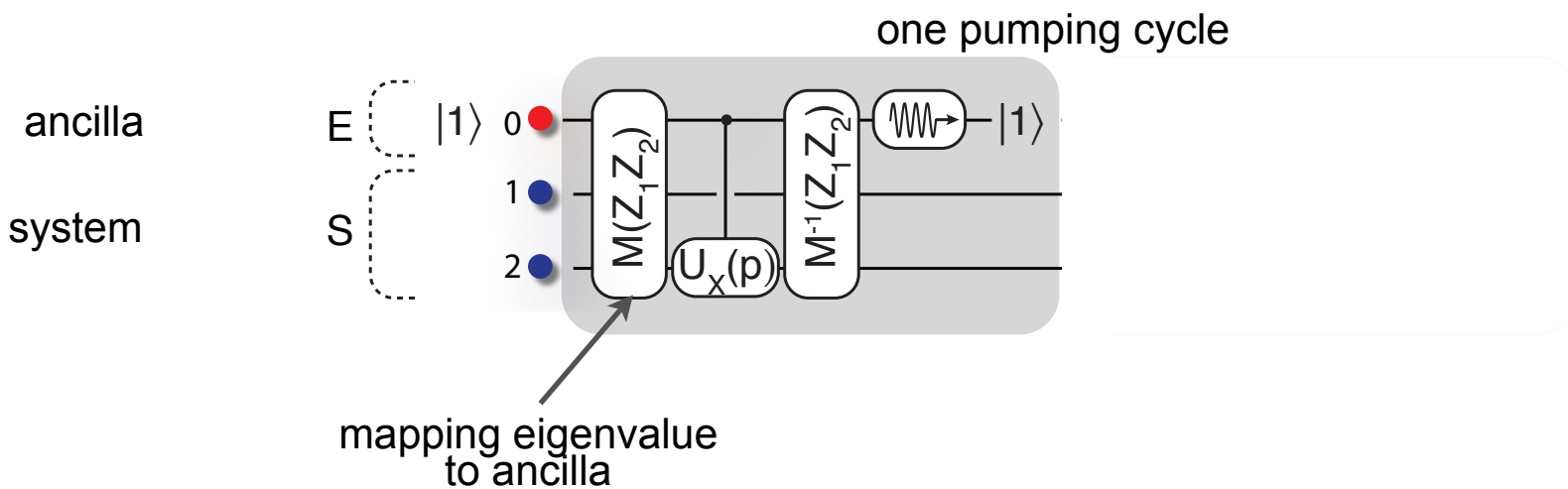
- **quantum circuit**



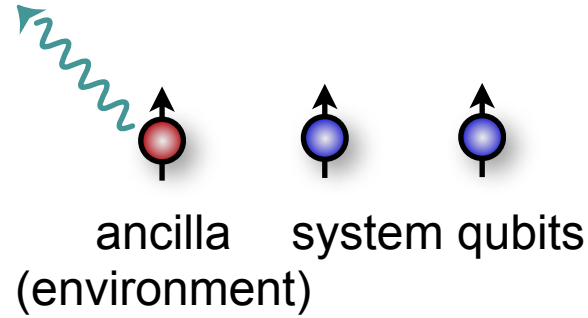
Bell State Pumping



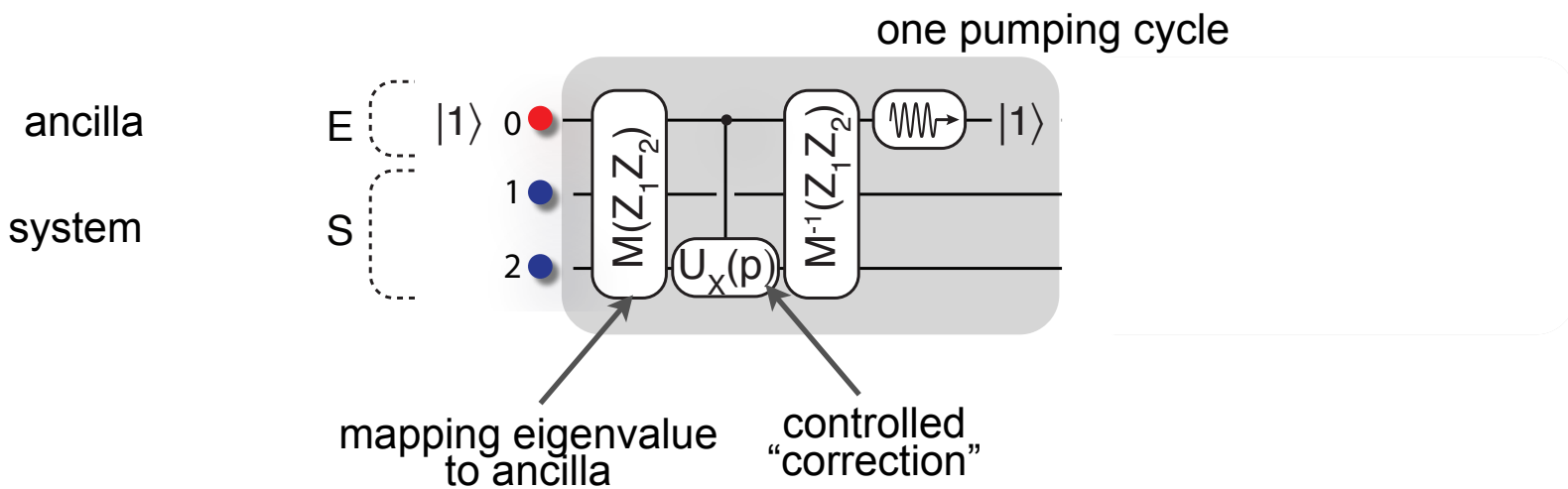
- **quantum circuit**



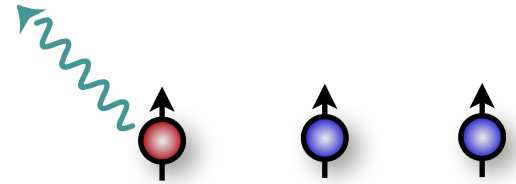
Bell State Pumping



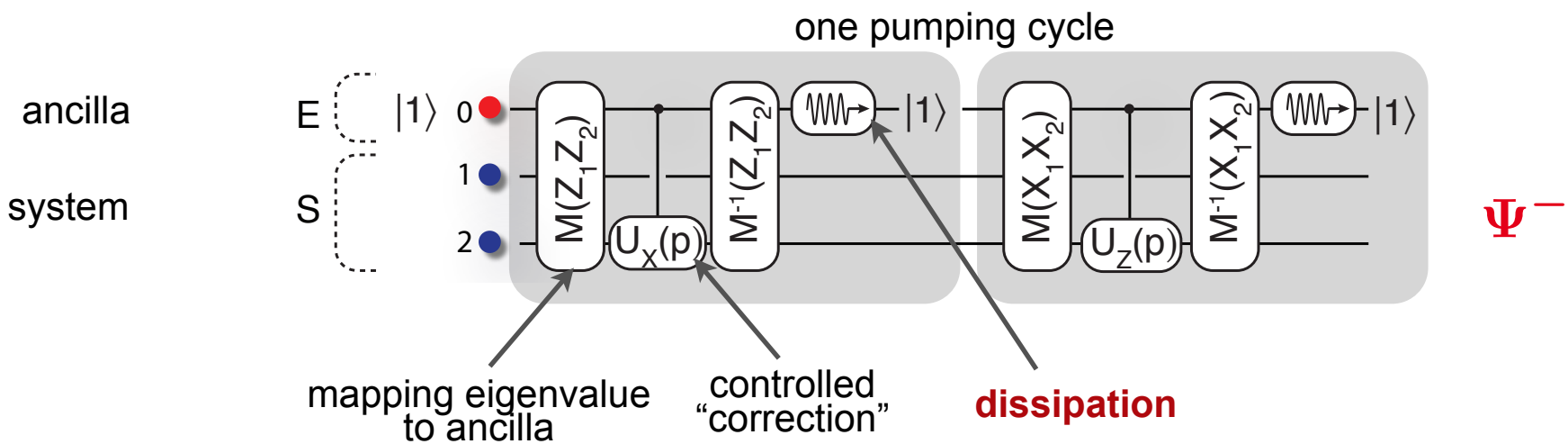
- **quantum circuit**



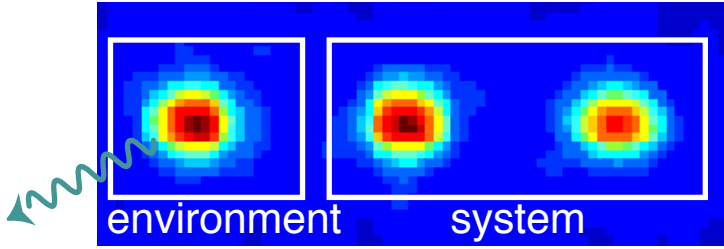
Bell State Pumping



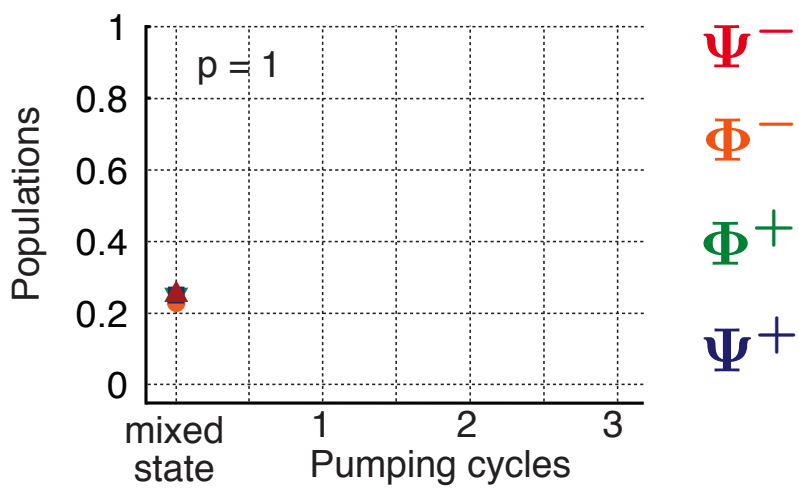
- **quantum circuit**



Bell State Pumping: Ion Experiment

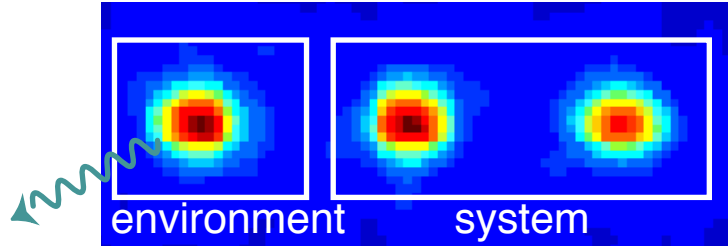


- **two step deterministic pumping**

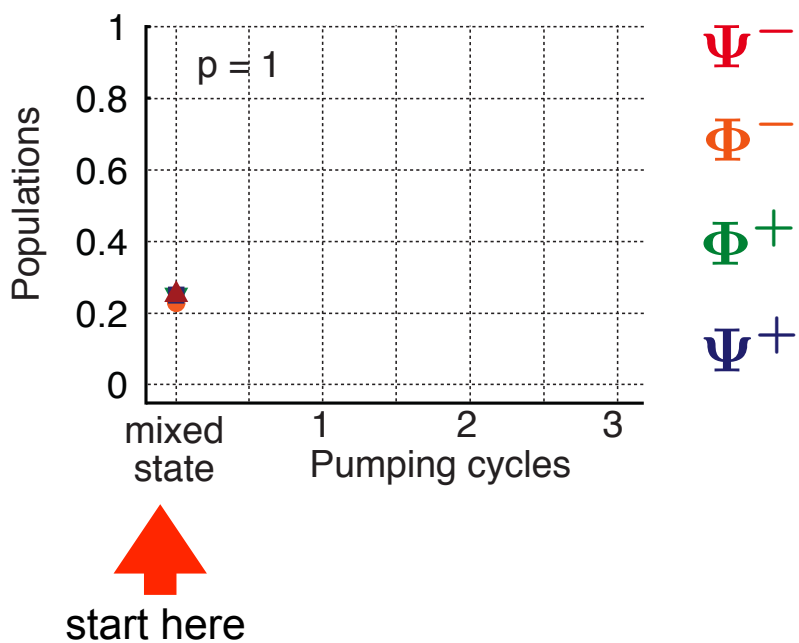


J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment

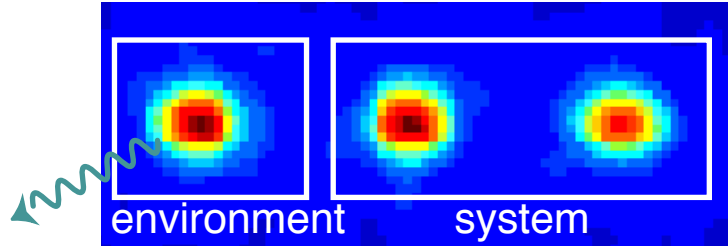


- **two step deterministic pumping**

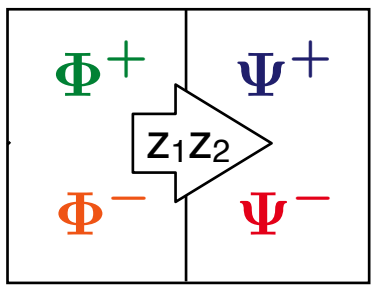


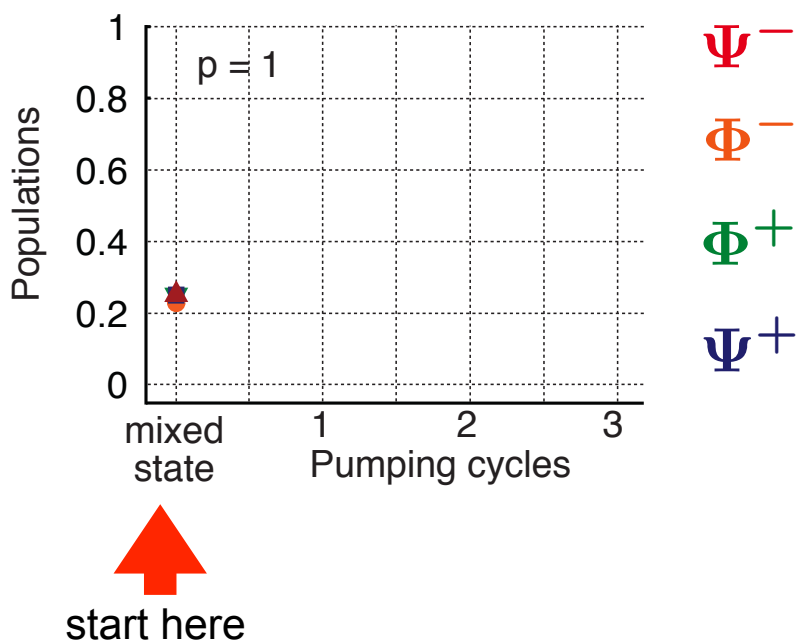
J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment



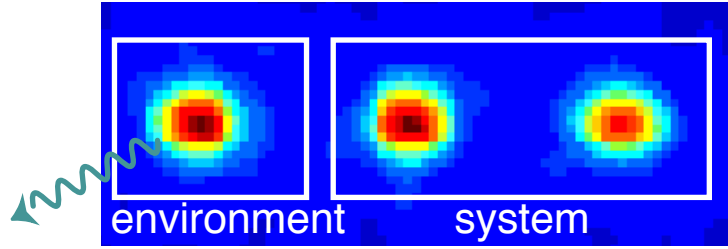
- **two step deterministic pumping**



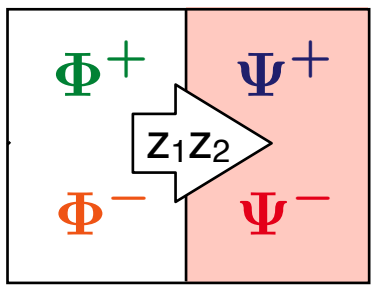


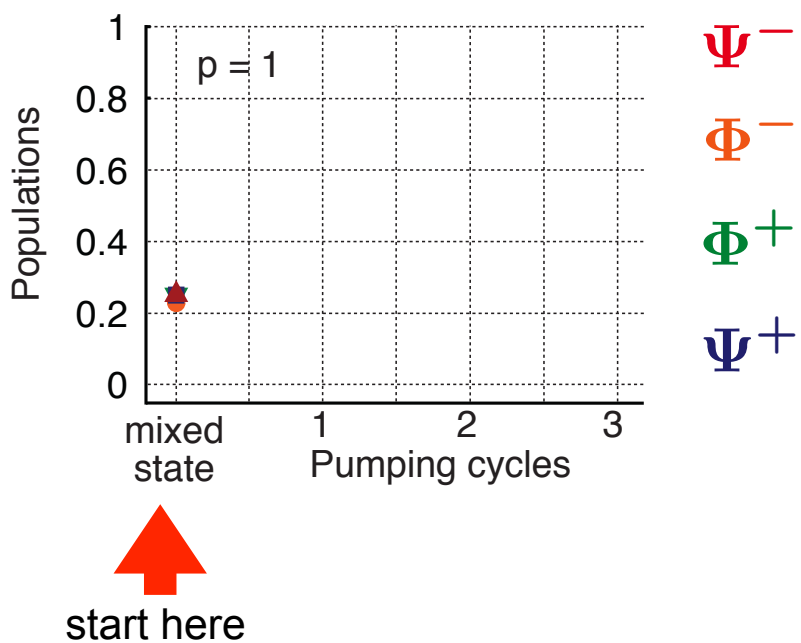
J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment



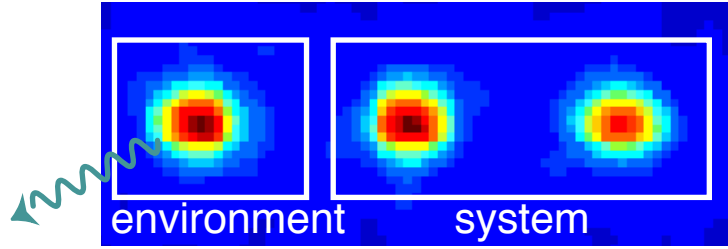
- **two step deterministic pumping**



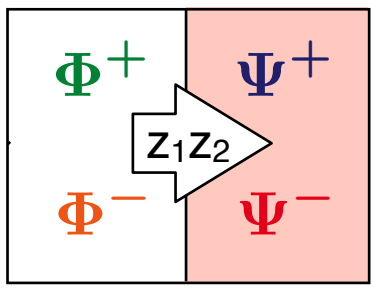


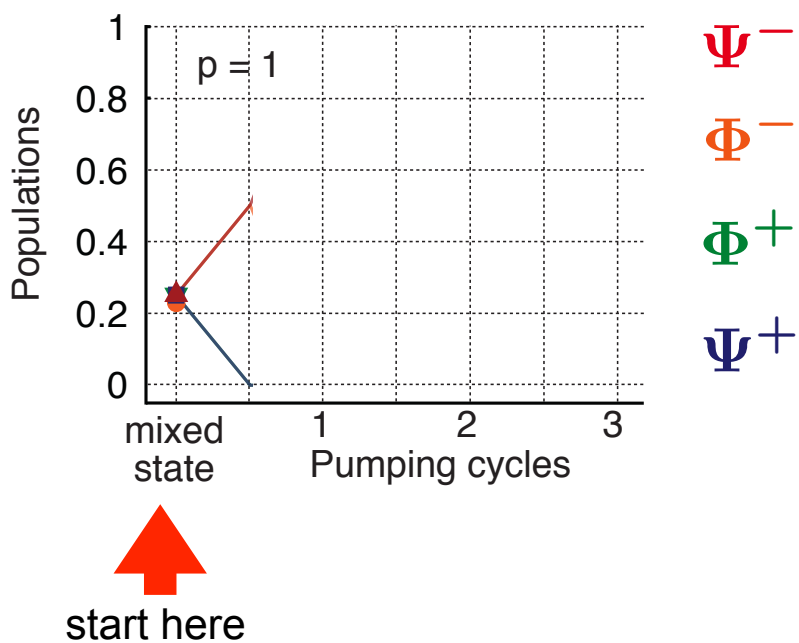
J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment



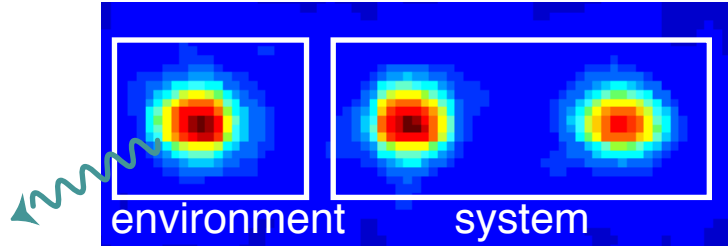
- **two step deterministic pumping**



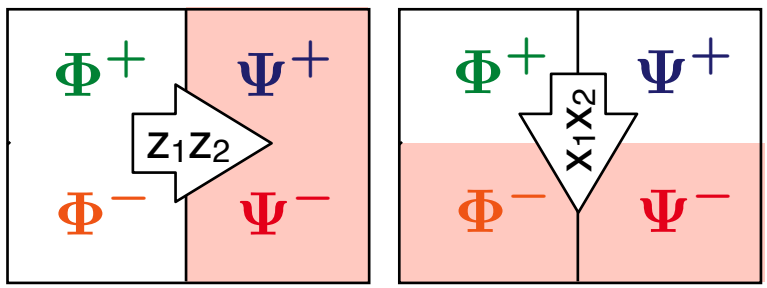


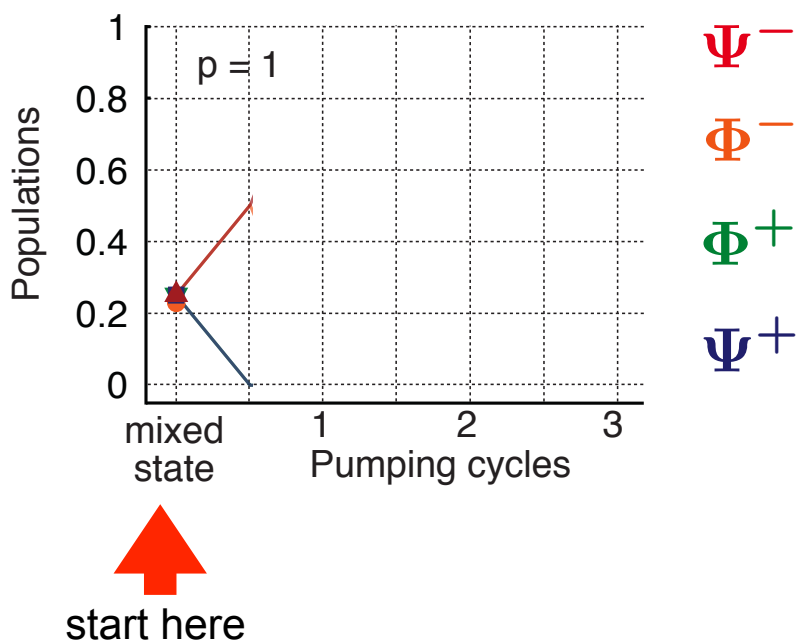
J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment



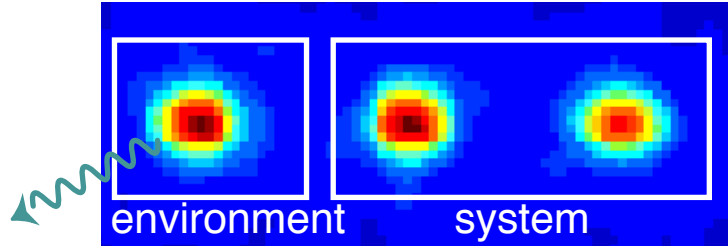
- **two step deterministic pumping**



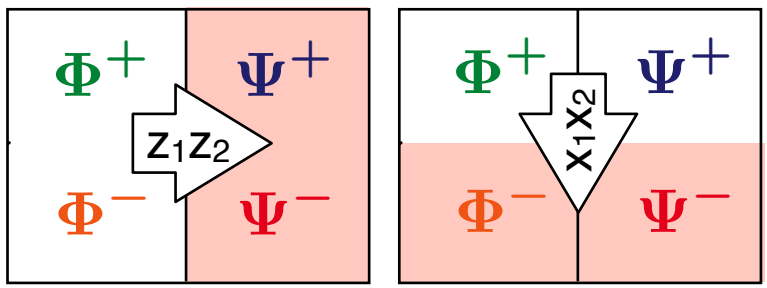


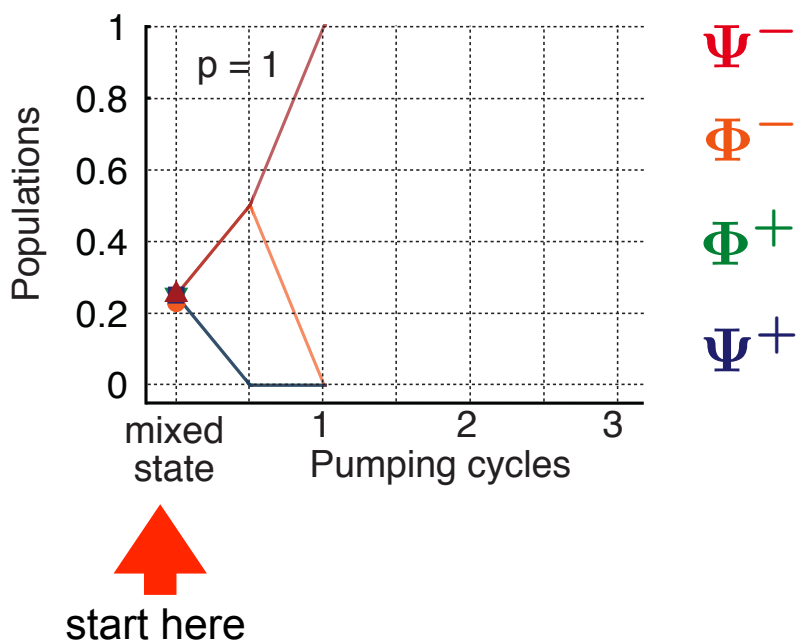
J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment



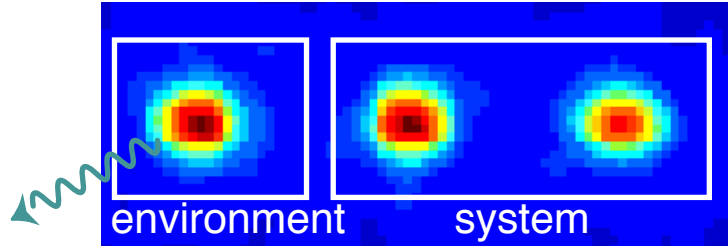
- **two step deterministic pumping**



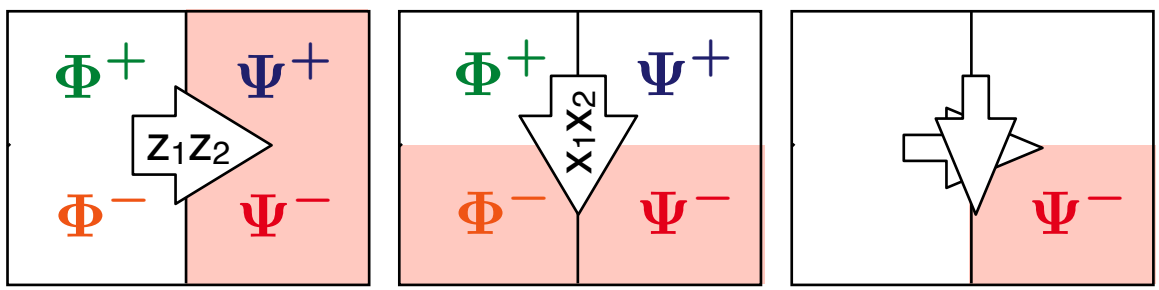


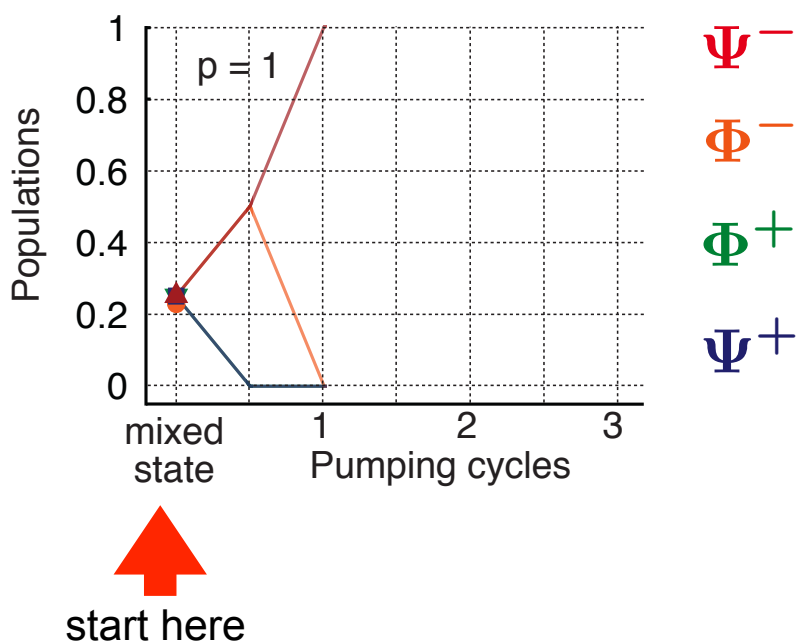
J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment



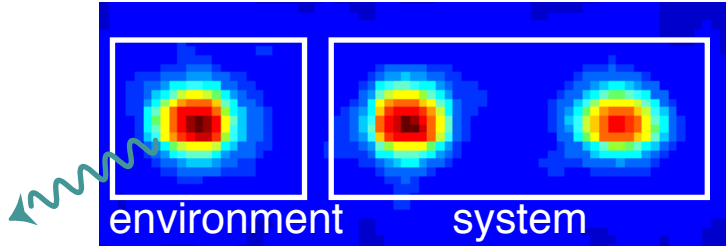
- **two step deterministic pumping**



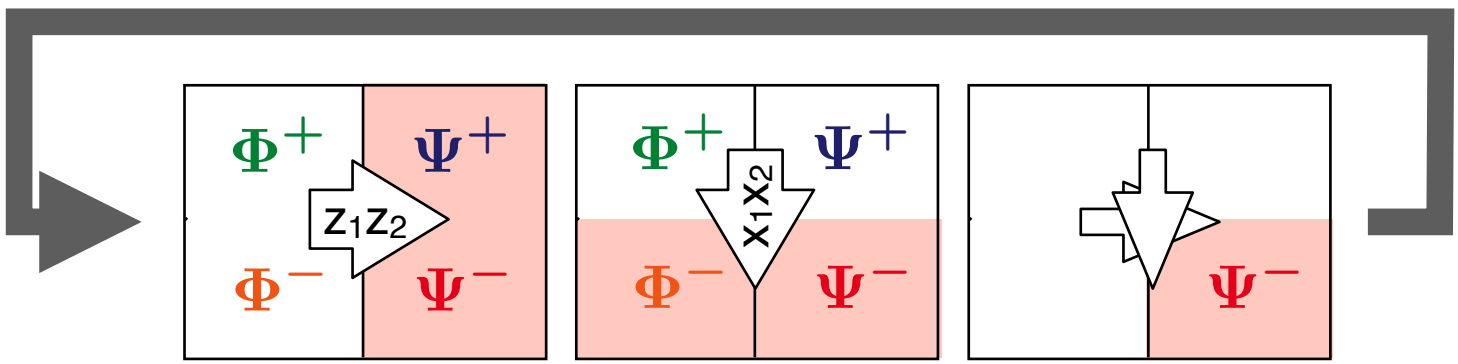


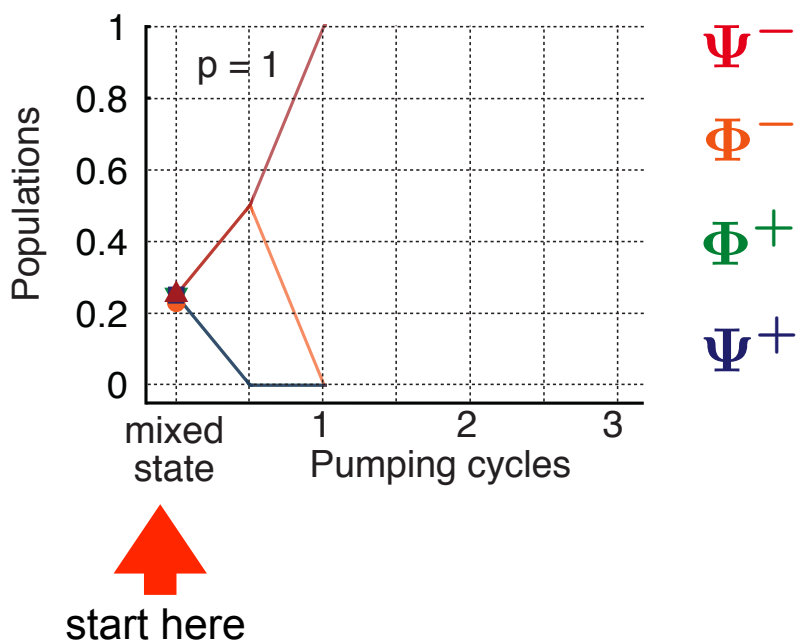
J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment



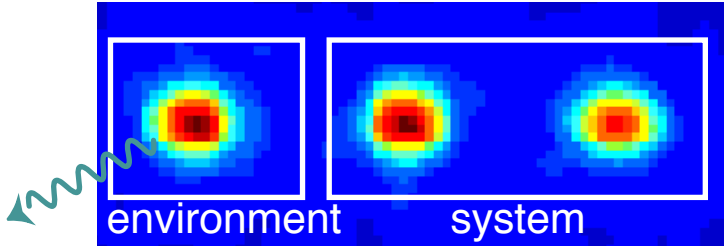
- **two step deterministic pumping**



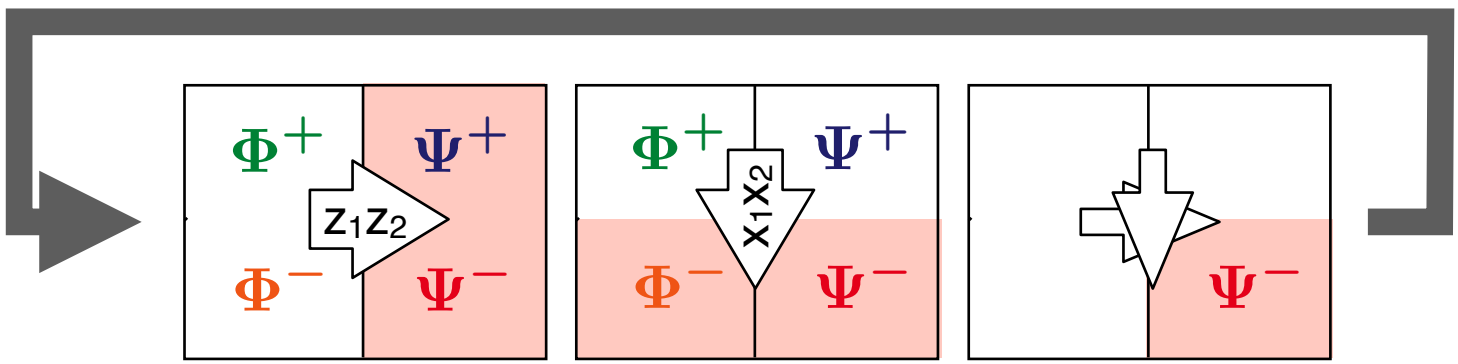


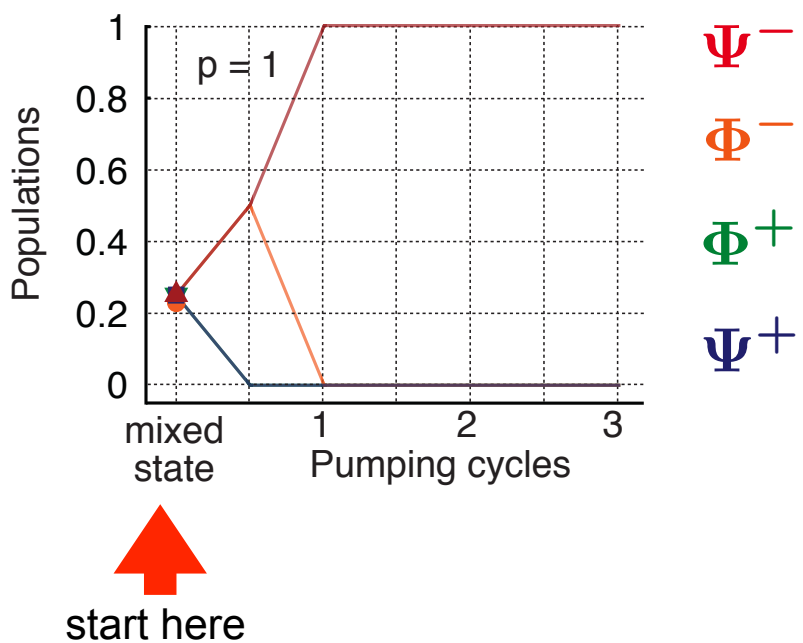
J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment



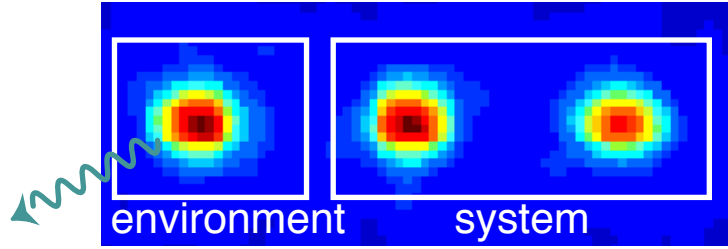
- **two step deterministic pumping**



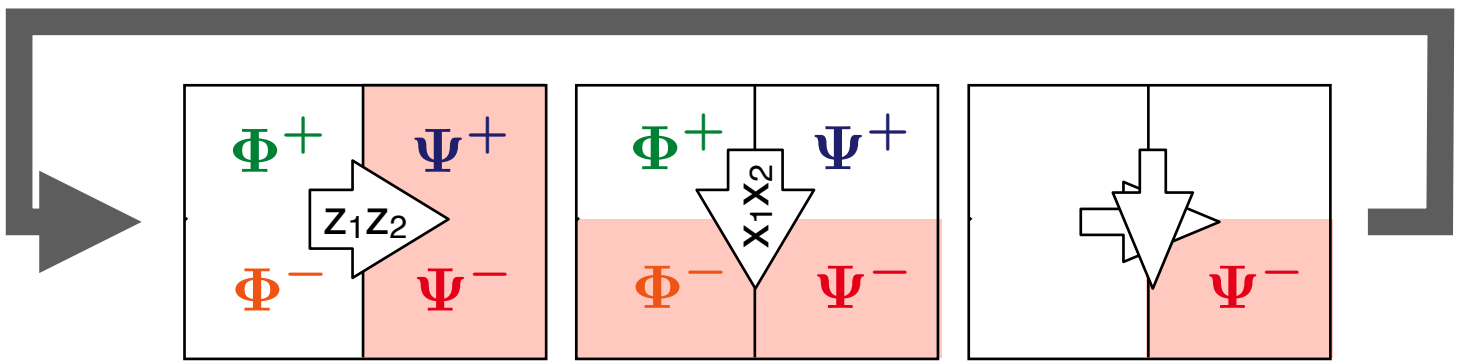


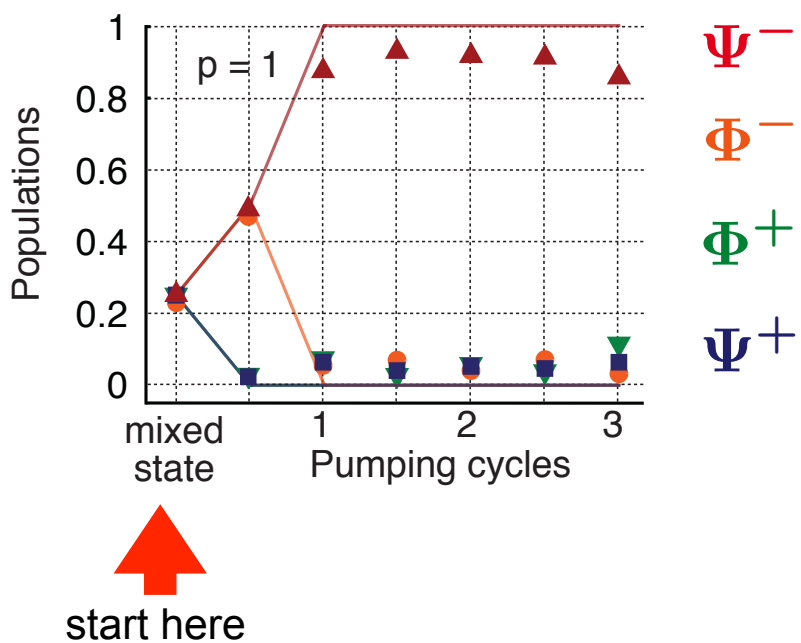
J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment



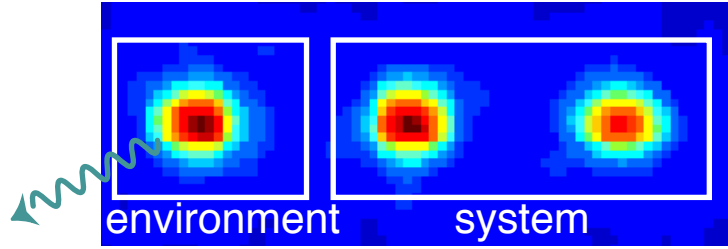
- **two step deterministic pumping**



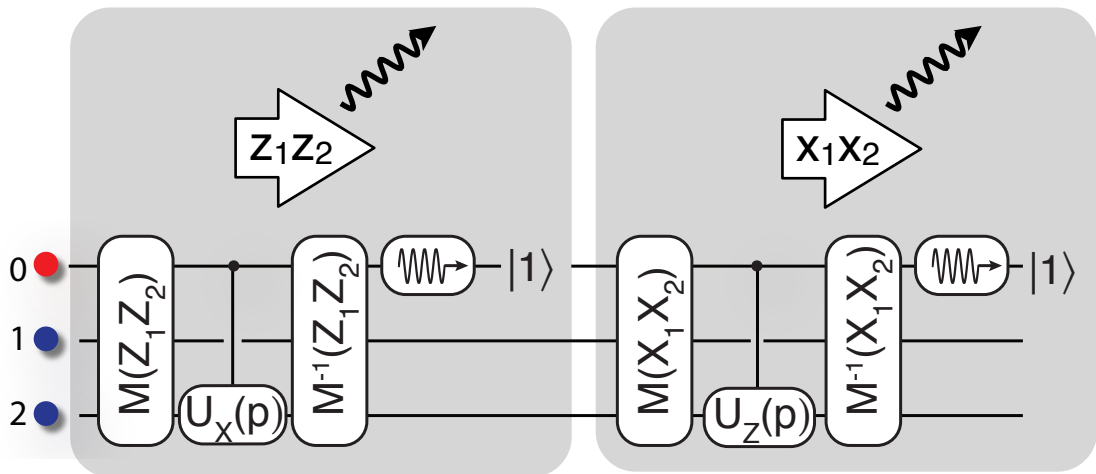


J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Bell State Pumping: Ion Experiment

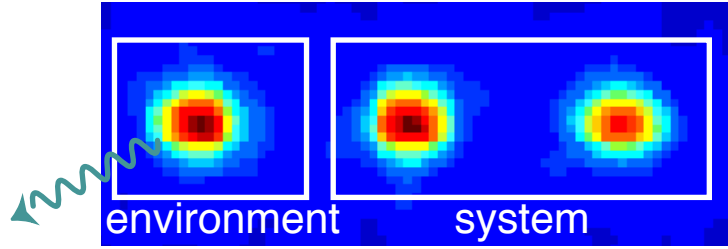


- **master equation limit:** probabilistic pumping

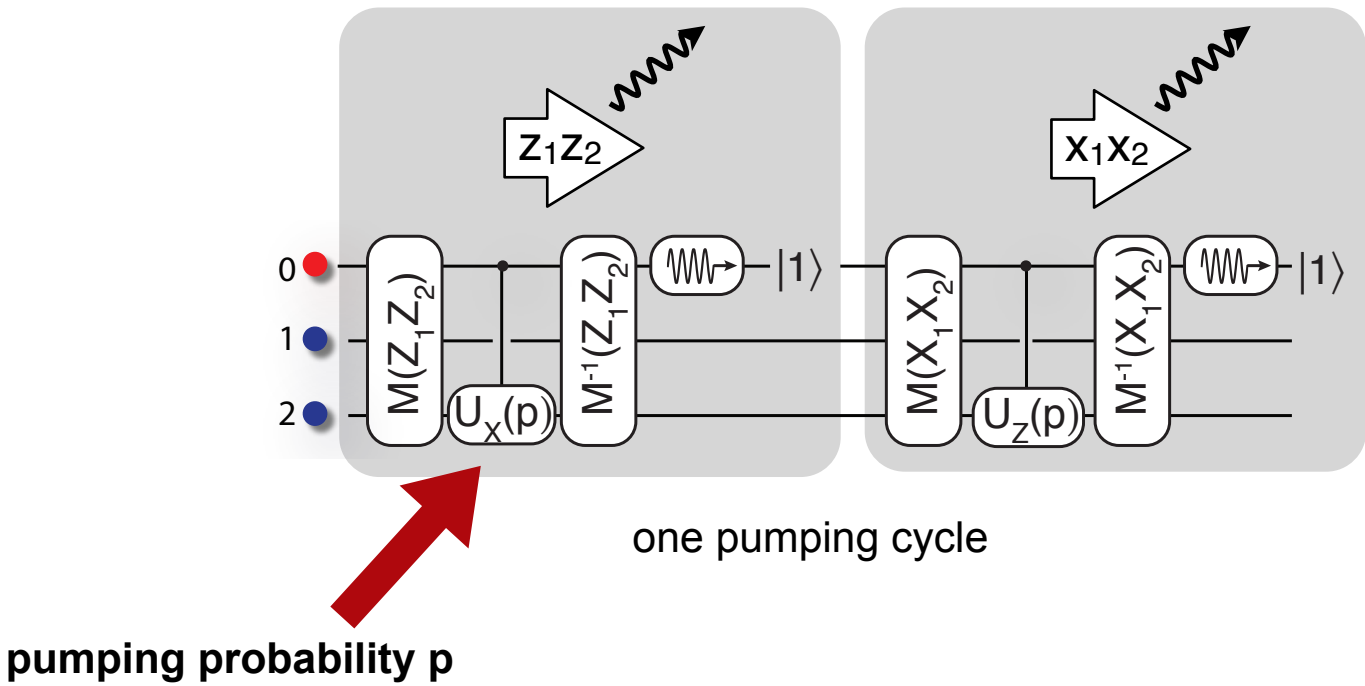


one pumping cycle

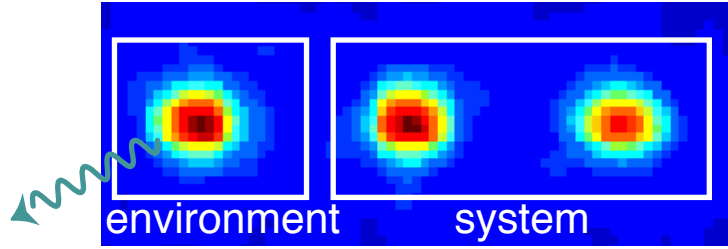
Bell State Pumping: Ion Experiment



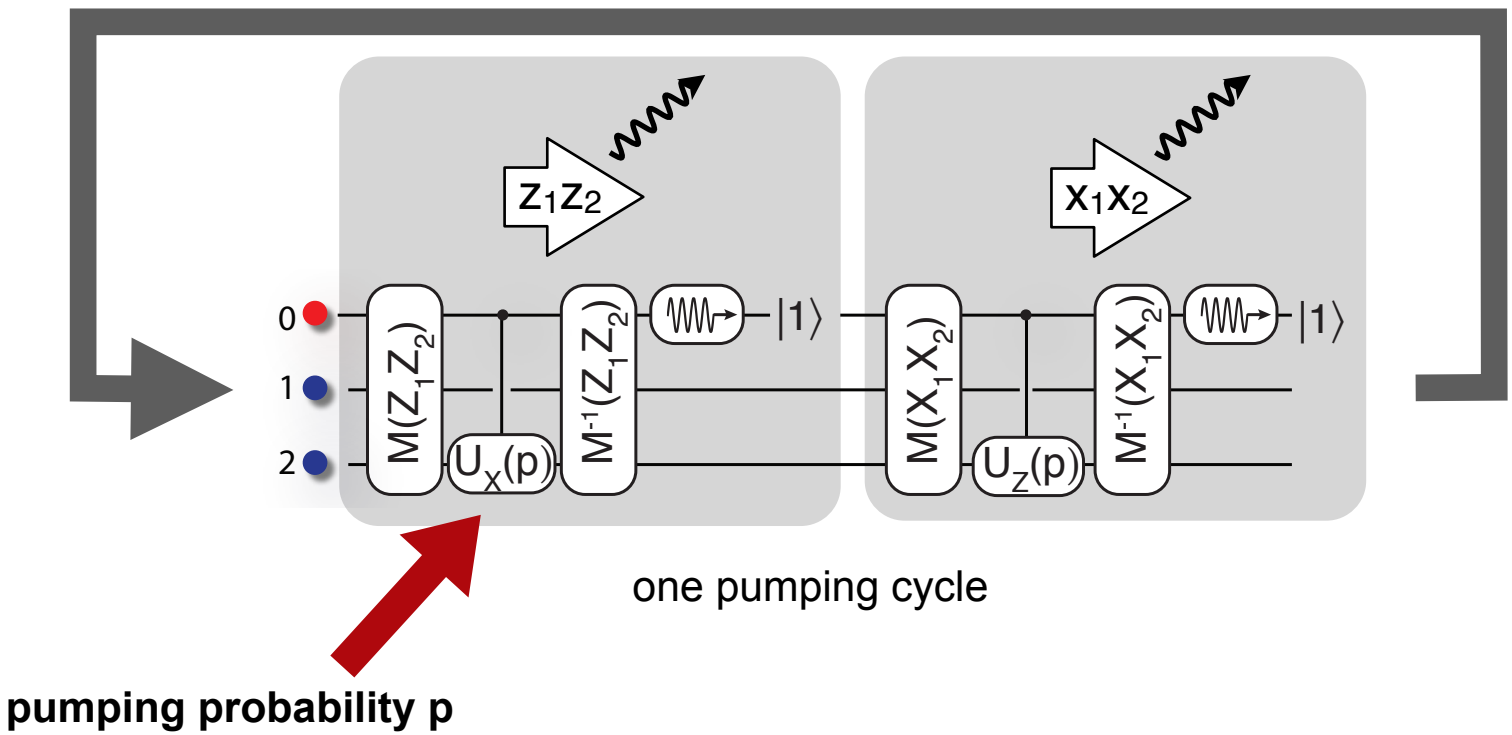
- **master equation limit:** probabilistic pumping



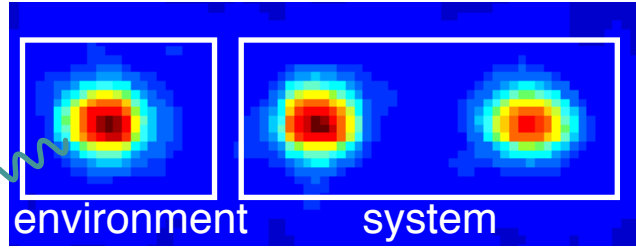
Bell State Pumping: Ion Experiment



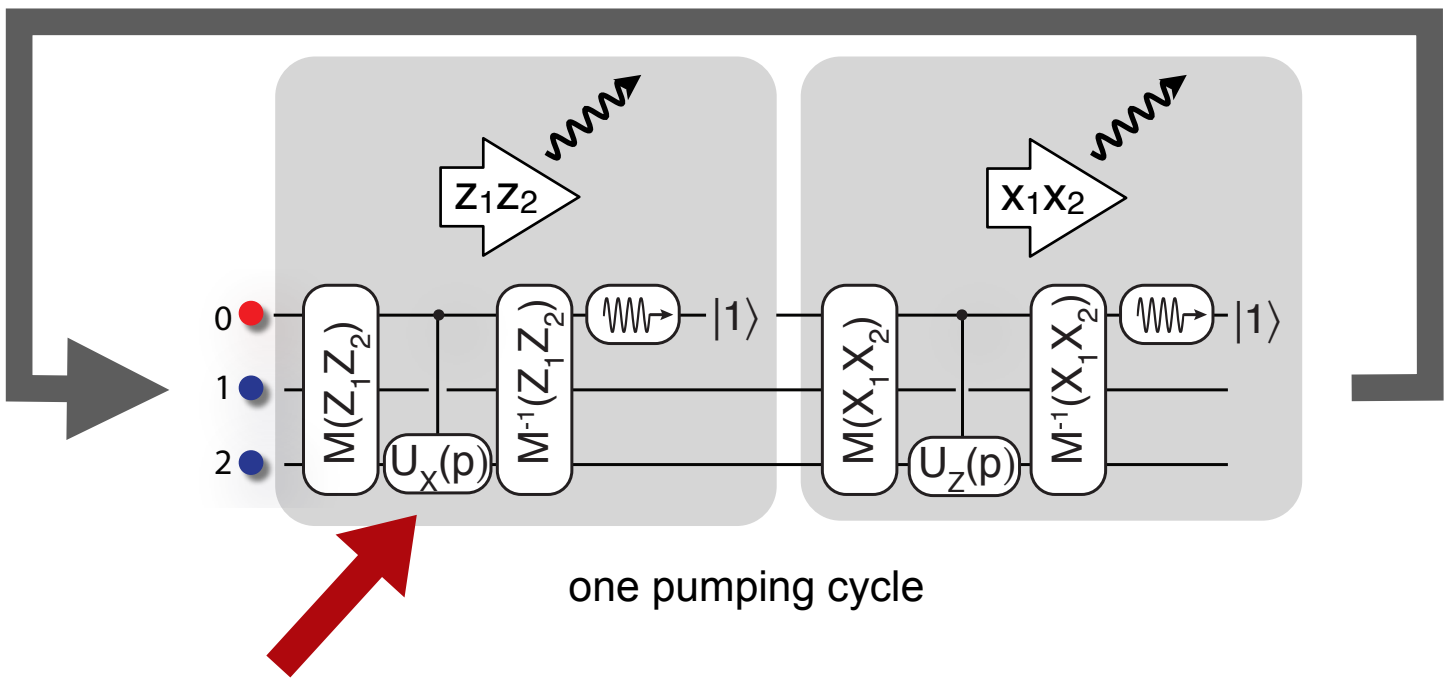
- **master equation limit:** probabilistic pumping

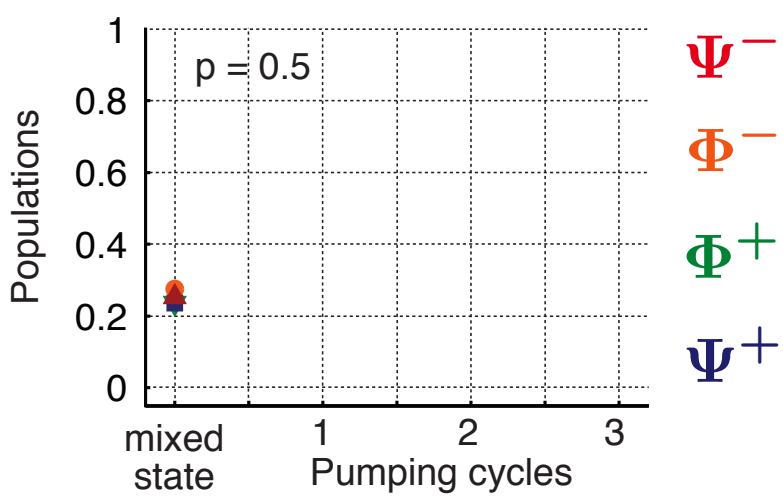


Bell State Pumping: Ion Experiment

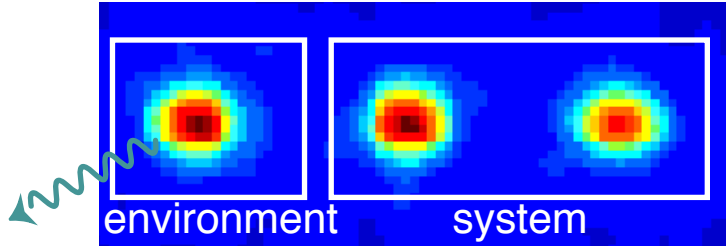


- **master equation limit:** probabilistic pumping

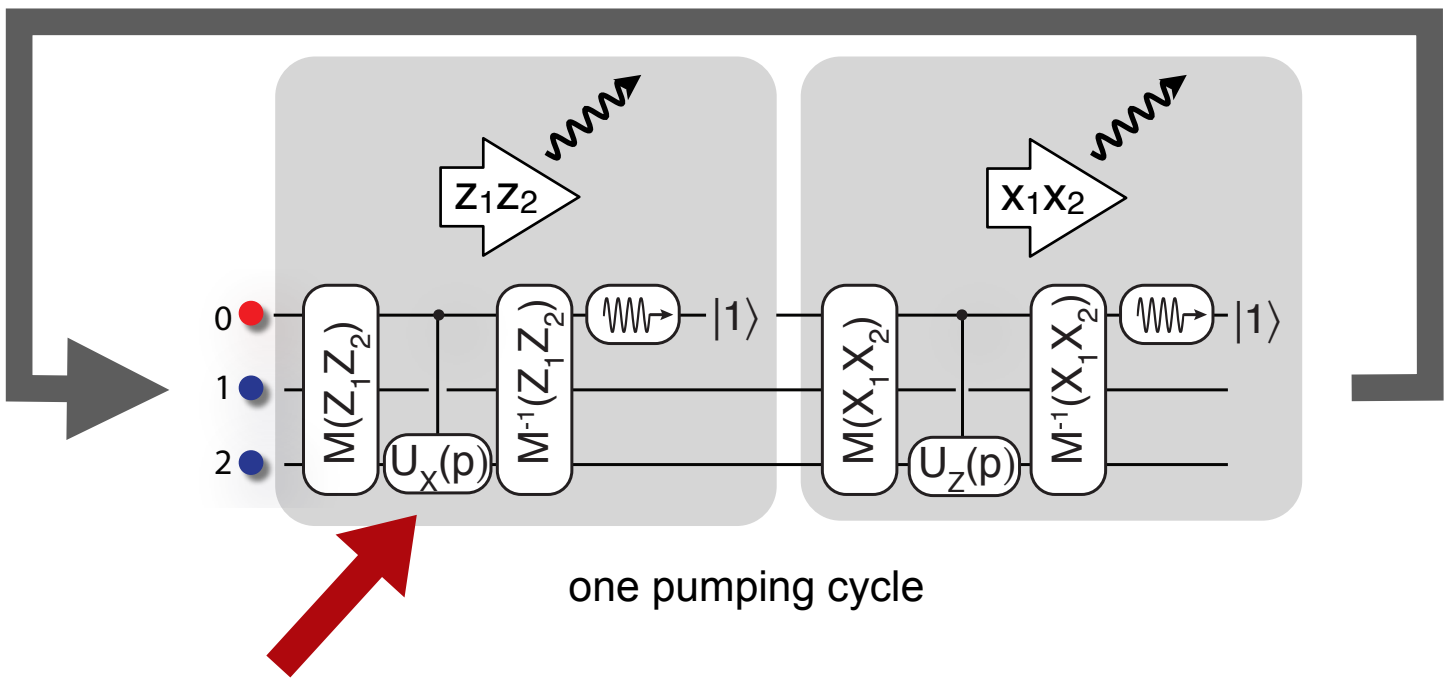


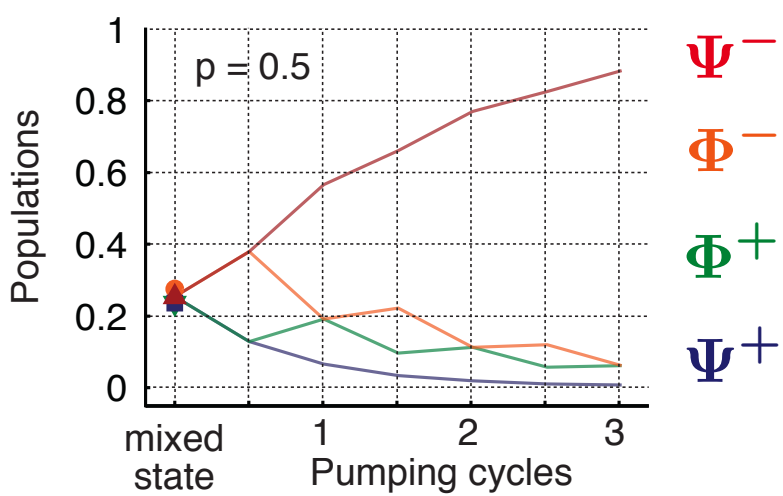


Bell State Pumping: Ion Experiment

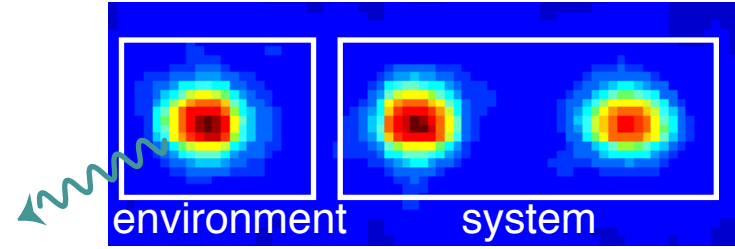


- **master equation limit:** probabilistic pumping

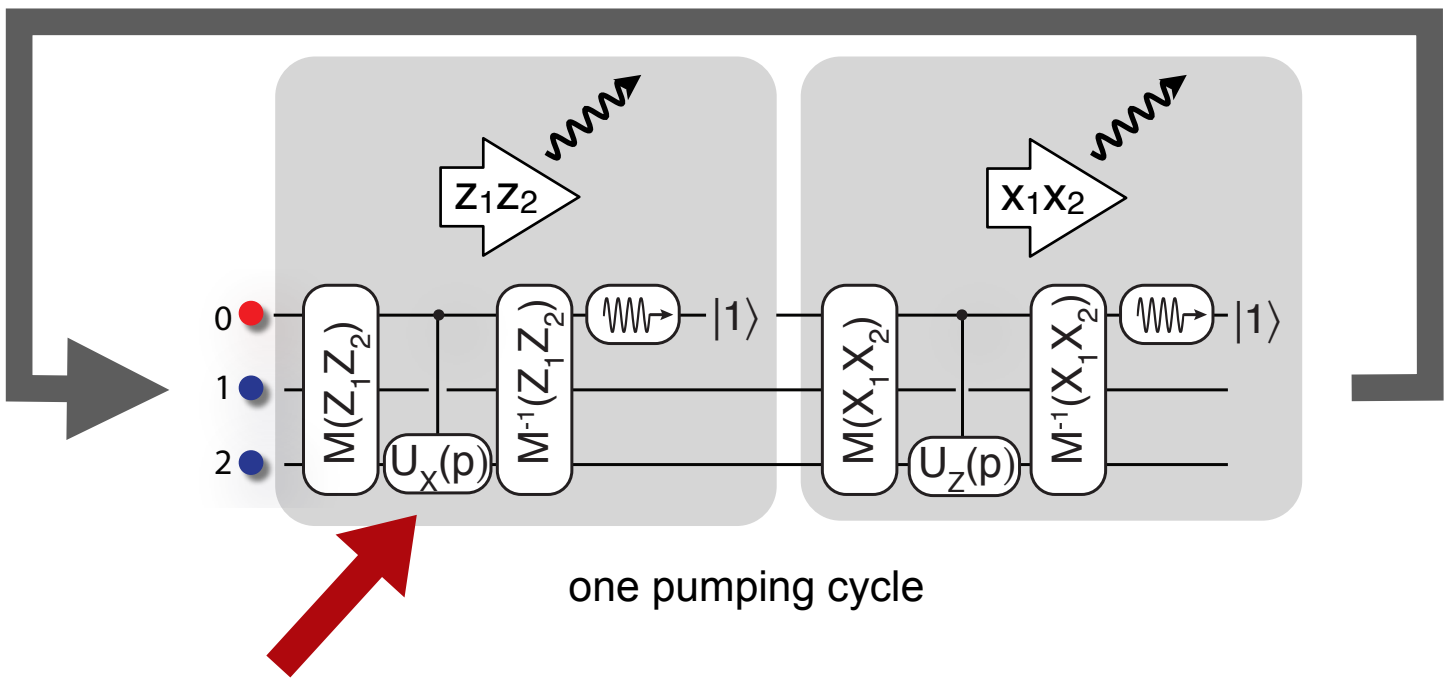


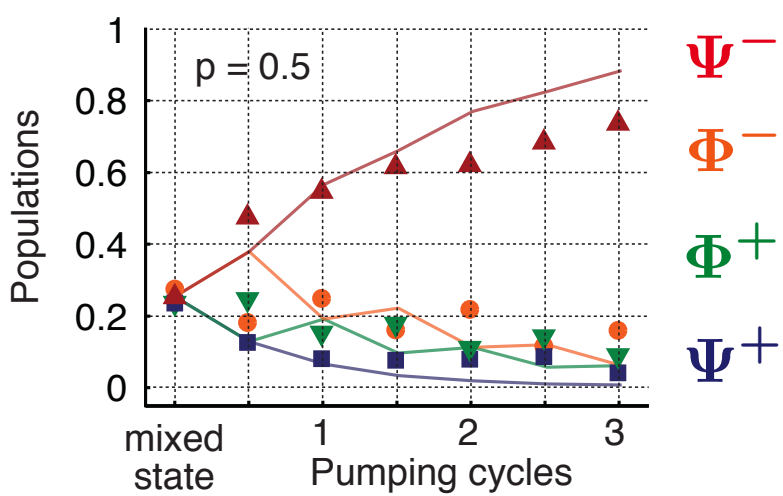


Bell State Pumping: Ion Experiment

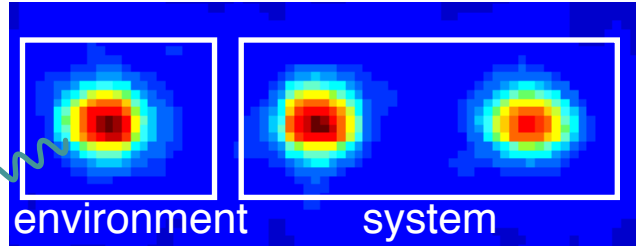


- **master equation limit:** probabilistic pumping

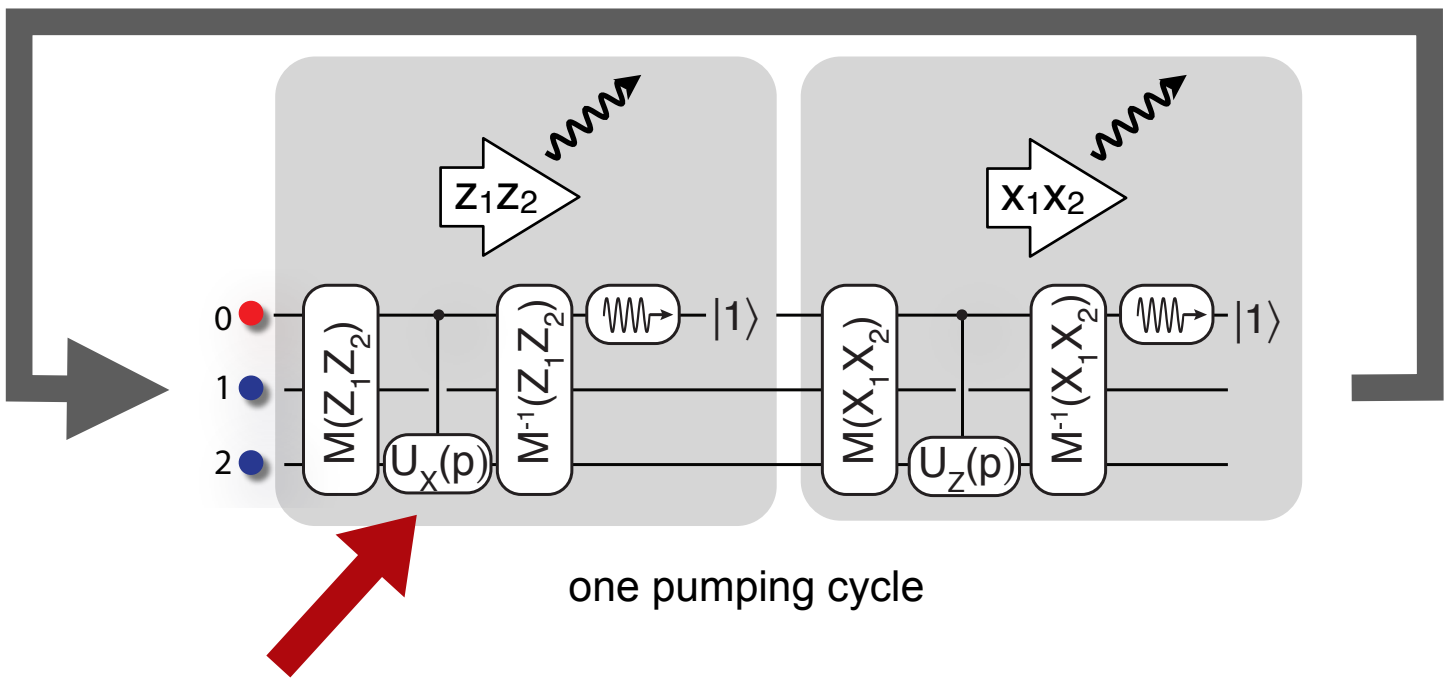




Bell State Pumping: Ion Experiment

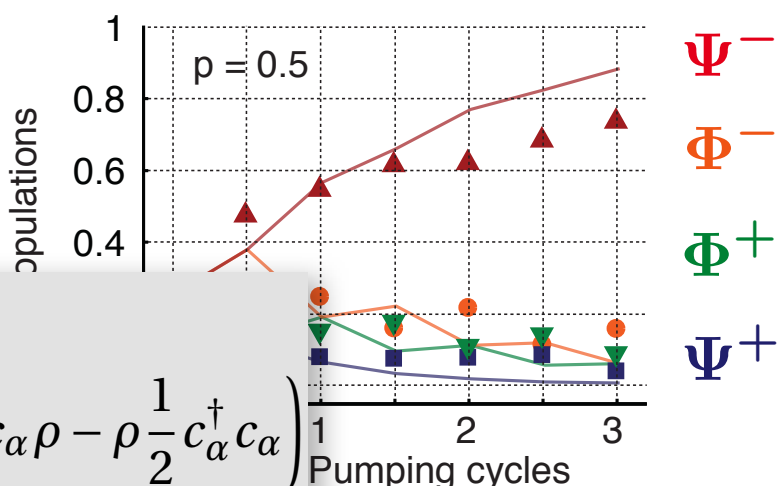


- **master equation limit:** probabilistic pumping



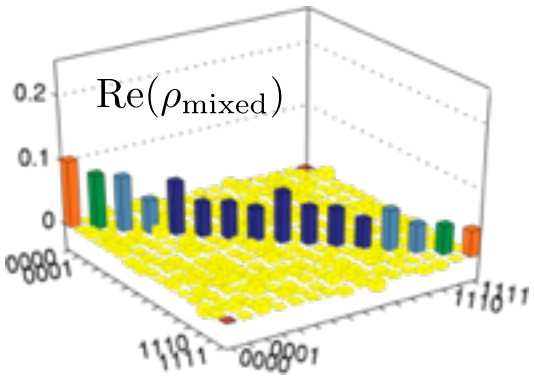
one pumping cycle

pumping probability p



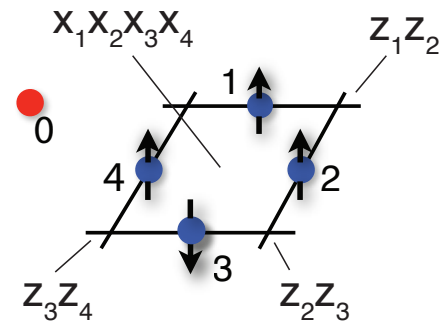
$$\dot{\rho} = -i[\cancel{H},\rho] + \sum_{\alpha=1}^{2}\gamma_\alpha\left(c_\alpha\rho c_\alpha^\dagger - \frac{1}{2}c_\alpha^\dagger c_\alpha\rho - \rho\frac{1}{2}c_\alpha^\dagger c_\alpha\right)$$

Stabilizer pumping: 1+4 ions

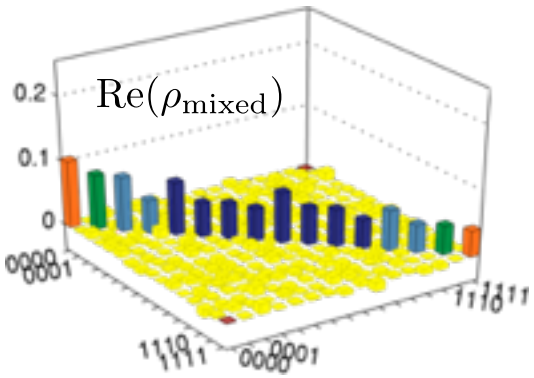


Populations

Ion:	1	2	3	4
	0	0	0	0
	0	0	0	1
	0	0	1	0
	0	0	1	1
	0	1	0	0
	0	1	0	1
	0	1	1	0
	0	1	1	1
	1	0	0	0
	1	0	0	1
	1	0	1	0
	1	0	1	1
	1	1	0	0
	1	1	0	1
	1	1	1	0
	1	1	1	1



Stabilizer pumping: 1+4 ions

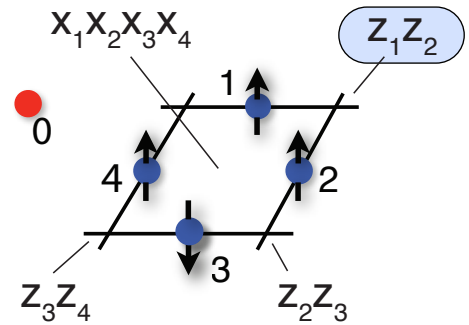


Ion: 1 2 3 4

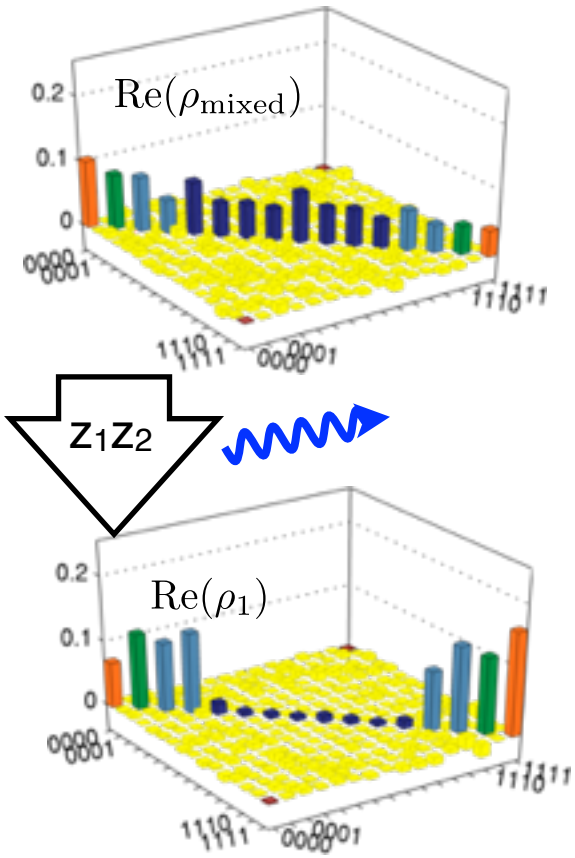
Z_1Z_2

Populations

0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1



Stabilizer pumping: 1+4 ions

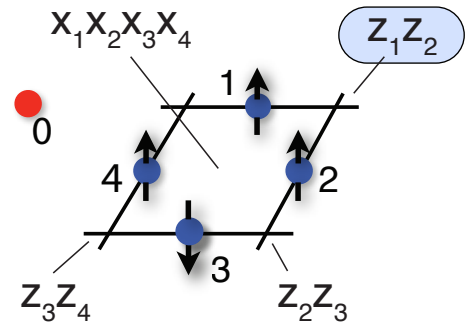


Ion: 1 2 3 4

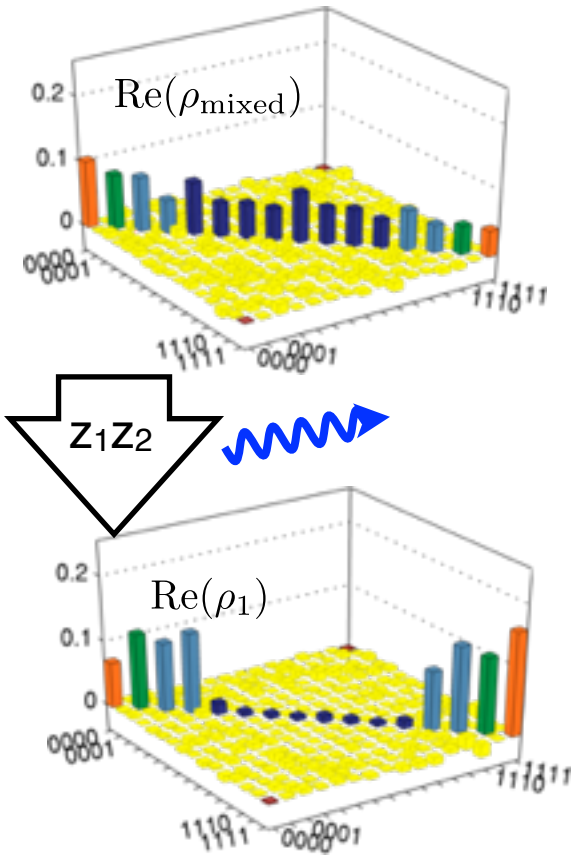
Z_1Z_2

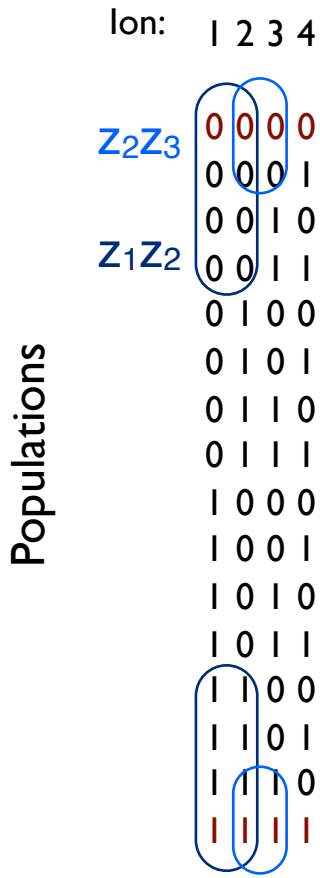
Populations

0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1



Stabilizer pumping: 1+4 ions



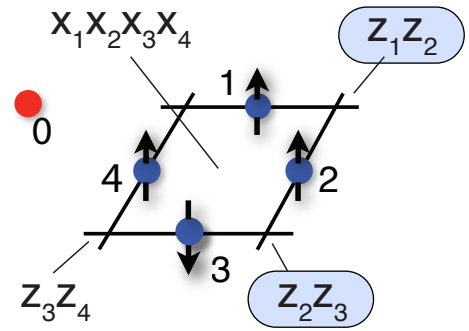


Ion: 1 2 3 4

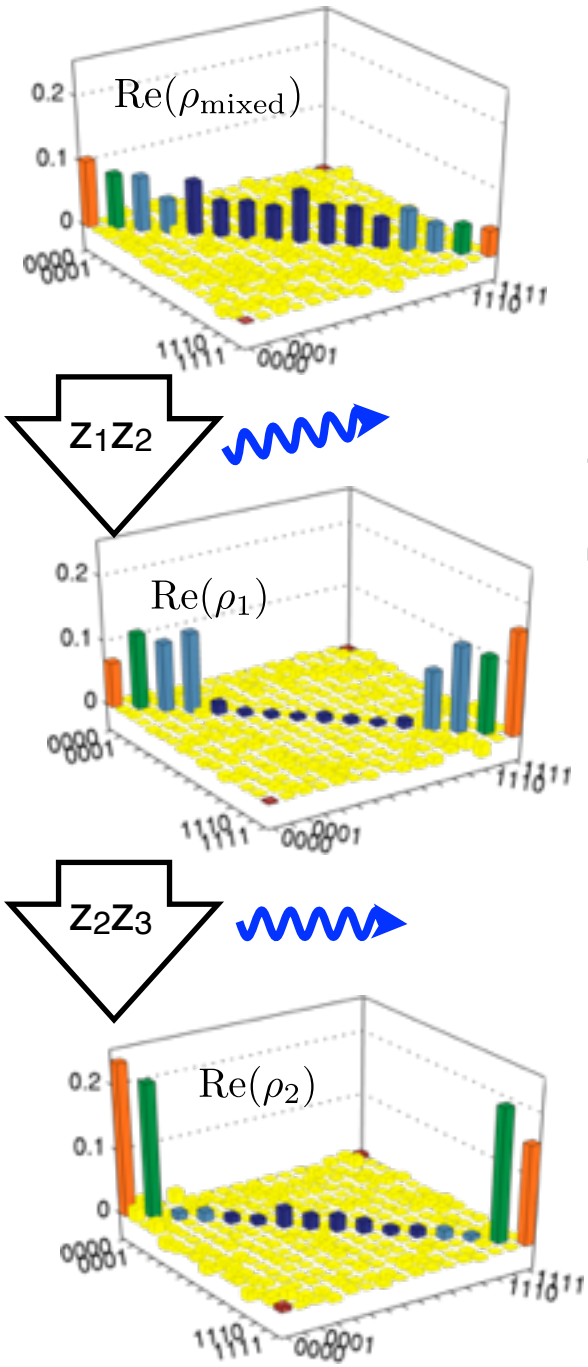
Z_2Z_3

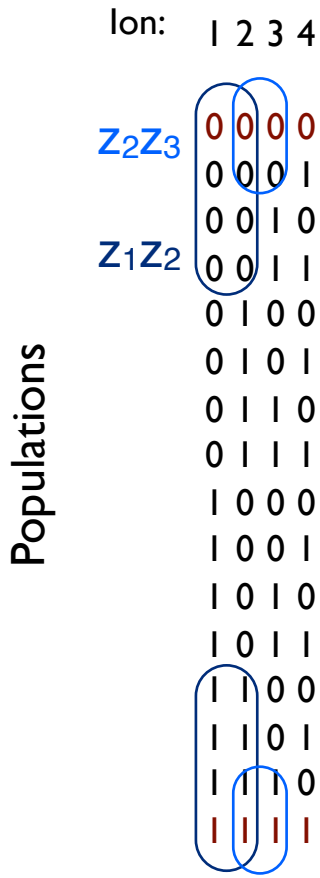
Z_1Z_2

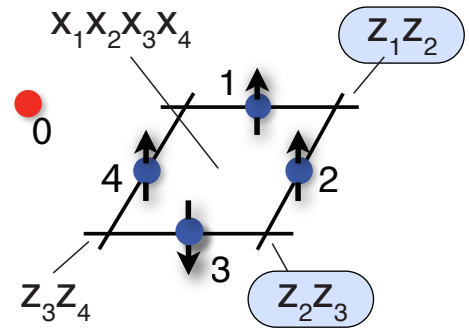
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1



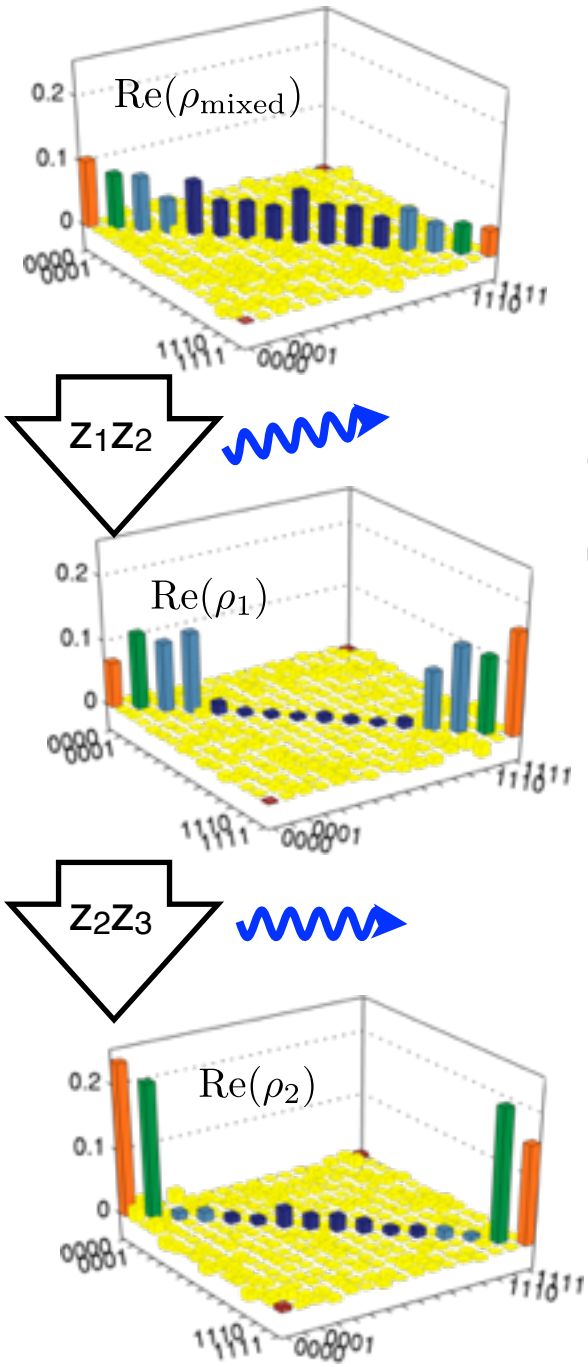
Stabilizer pumping: 1+4 ions

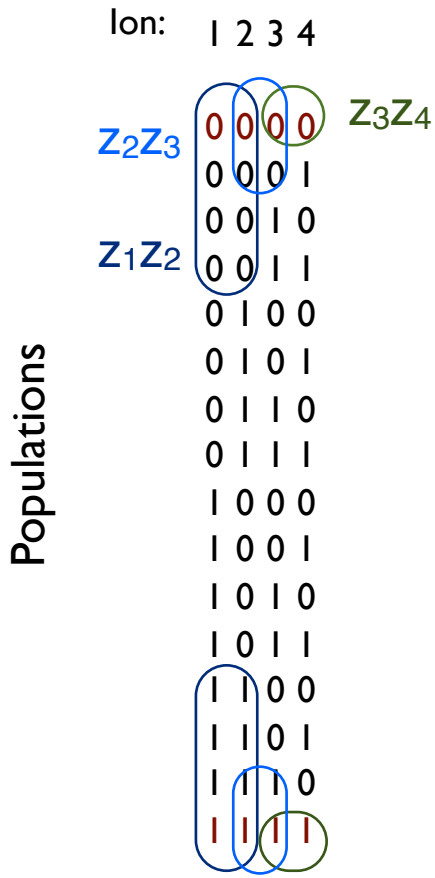


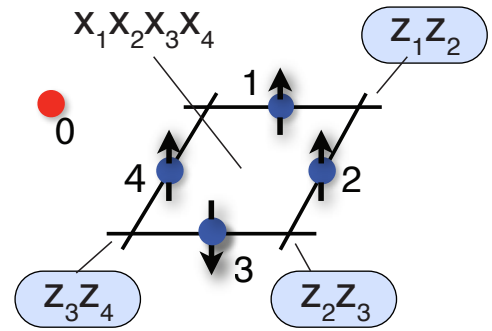




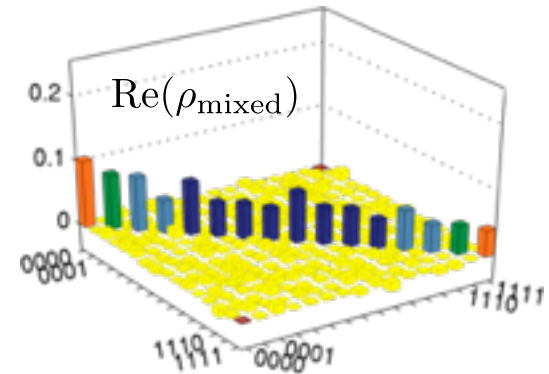
Stabilizer pumping: 1+4 ions



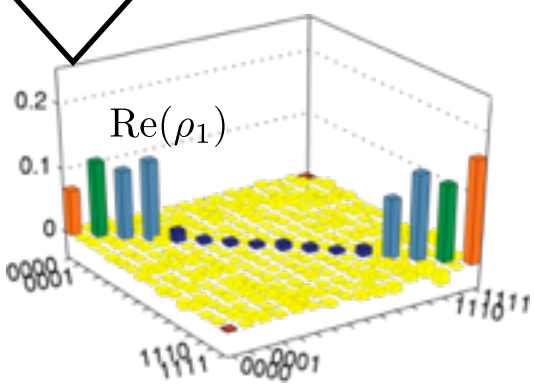




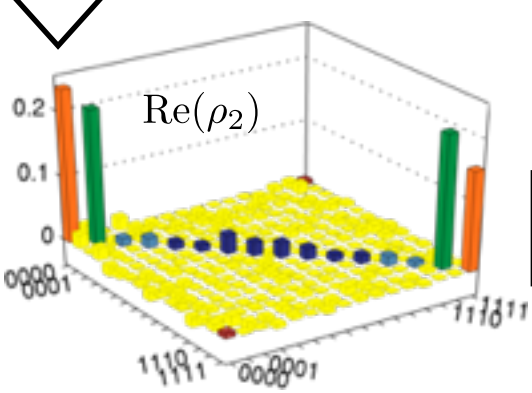
Stabilizer pumping: 1+4 ions



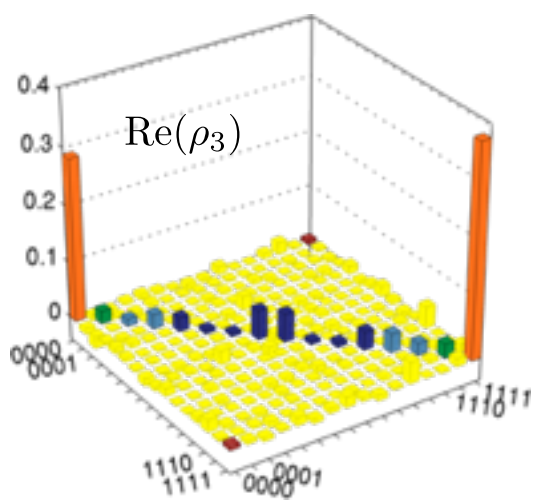
Z_1Z_2



Z_2Z_3

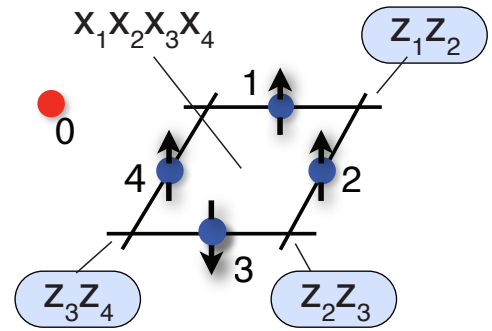


Z_2Z_3

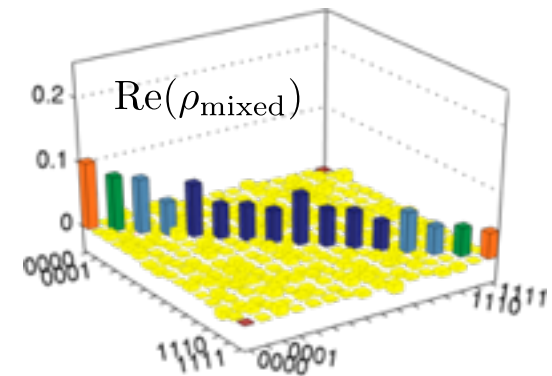


Populations

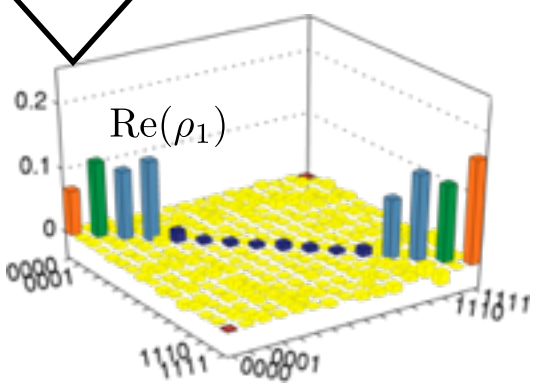
Ion:	1	2	3	4	
Z_2Z_3	0	0	0	0	Z_3Z_4
	0	0	0	1	
	0	0	1	0	
Z_1Z_2	0	0	1	1	
	0	1	0	0	
	0	1	0	1	
	0	1	1	0	
	0	1	1	1	
	1	0	0	0	
	1	0	0	1	
	1	0	1	0	
	1	0	1	1	
	1	1	0	0	
	1	1	0	1	
	1	1	1	0	
	1	1	1	1	



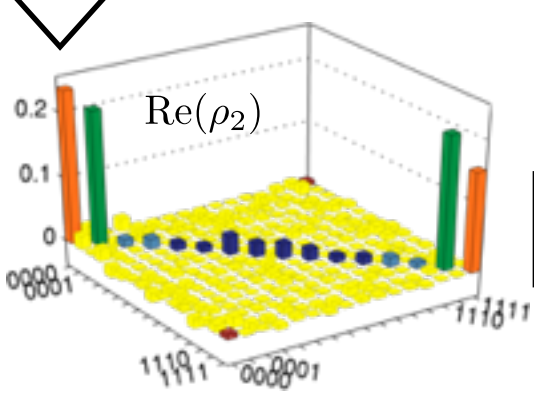
Stabilizer pumping: 1+4 ions



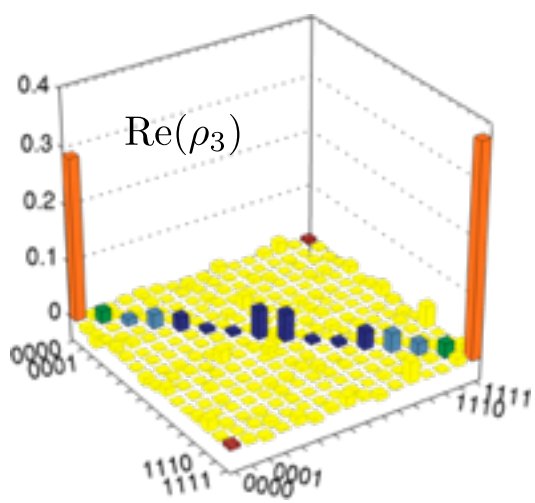
Z_1Z_2



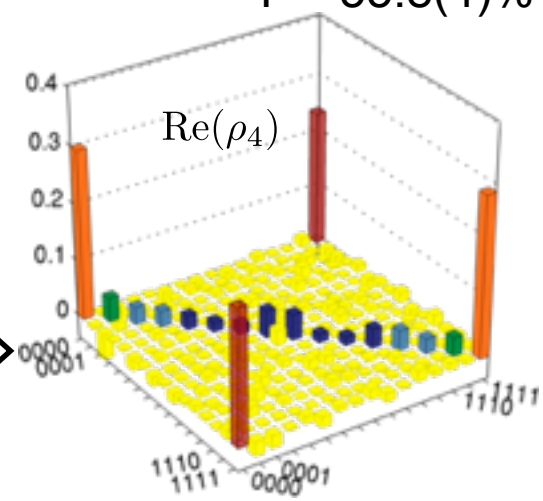
Z_2Z_3



Z_2Z_3

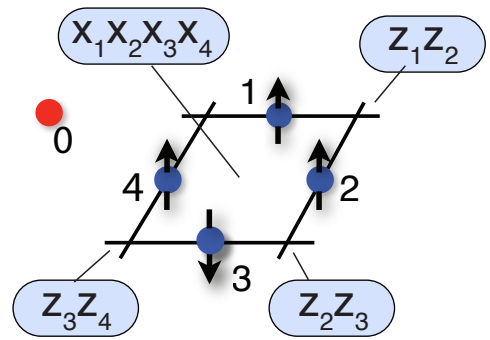


$X_1X_2X_3X_4$



Populations

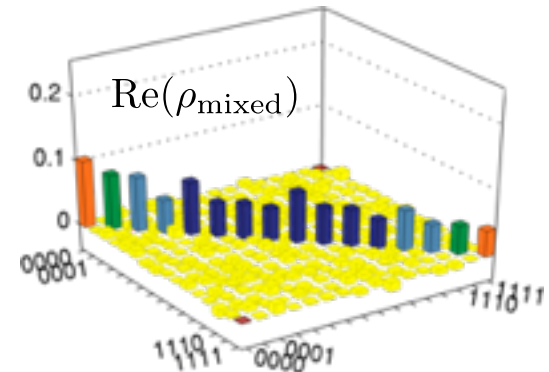
Ion:	1	2	3	4	
Z_2Z_3	0	0	0	0	Z_3Z_4
	0	0	0	1	
	0	0	1	0	
Z_1Z_2	0	0	1	1	
	0	1	0	0	
	0	1	0	1	
	0	1	1	0	
	0	1	1	1	
	1	0	0	0	
	1	0	0	1	
	1	0	1	0	
	1	0	1	1	
	1	1	0	0	
	1	1	0	1	
	1	1	1	0	
	1	1	1	1	



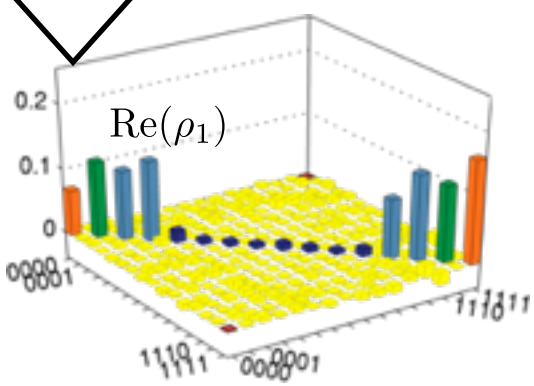
$$|0000\rangle + |1111\rangle$$

F = 55.8(4)%

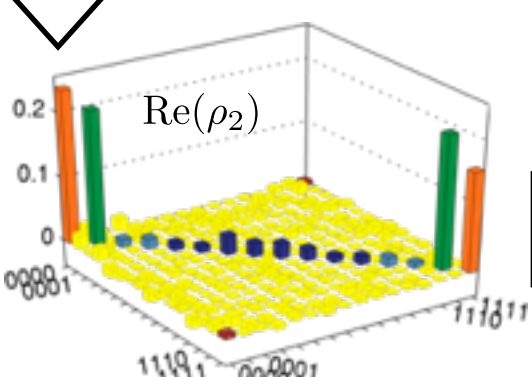
Stabilizer pumping: 1+4 ions



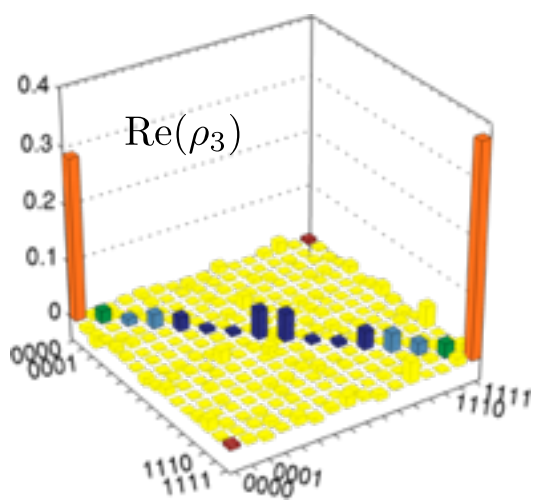
Z_1Z_2



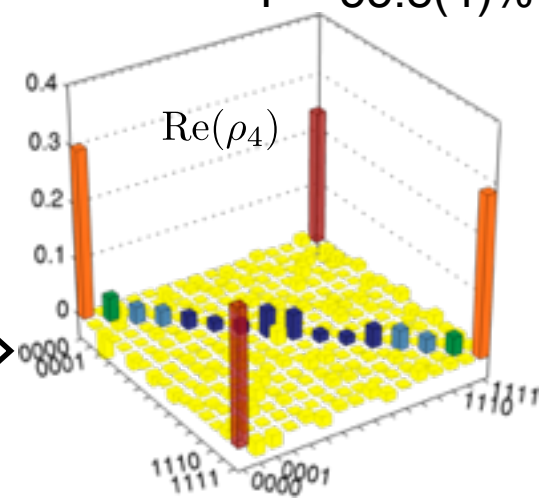
Z_2Z_3



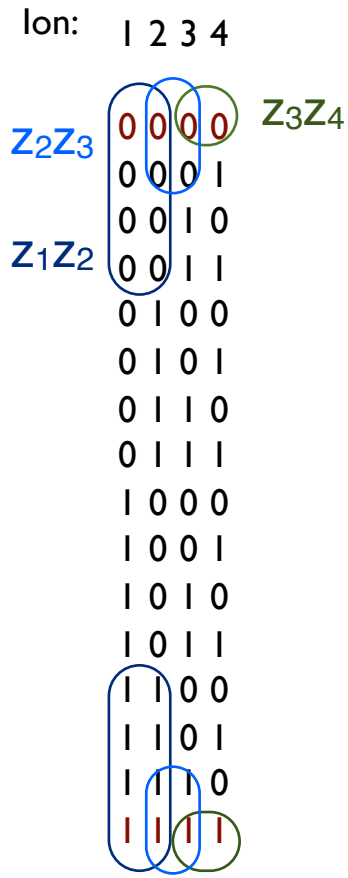
Z_2Z_3

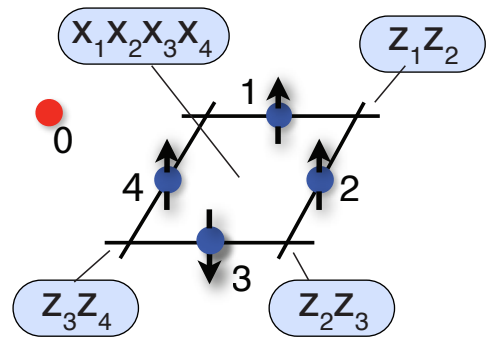


$X_1X_2X_3X_4$



Populations

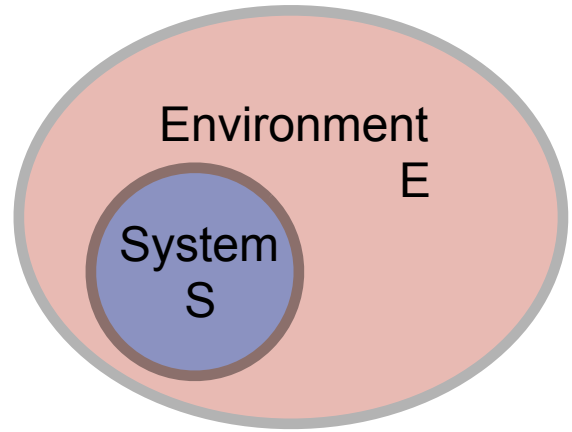




16 five-ion entangling gates
20 collective rotations
34 single-ion gates
+ hiding and optical pumping
+ state preparation
+ readout pulses

$$|0000\rangle + |1111\rangle$$

F = 55.8(4)%



Non-Equilibrium Condensed Matter Physics with Engineered Reservoirs

- **open-system many-body dynamics & quantum phase transitions**

  Review:

  M. Müller, S. Diehl, G. Pupillo, P. Zoller, *Engineered Open Systems and Quantum Simulations with Atoms and Ions*, arXiv:1203.6595; published in Advances of Atomic, Molecular and Optical Physics 2012

  Exp.:

  P. Schindler, M. Müller, D. Nigg, J. T. Barreiro, E. A. Martinez, M. Hennrich, T. Monz, S. Diehl, P. Zoller, R. Blatt, *Quantum simulation of open-system dynamical maps with trapped ions*, arXiv:1212.2418

- **topology via (engineered) dissipation**

  Review:

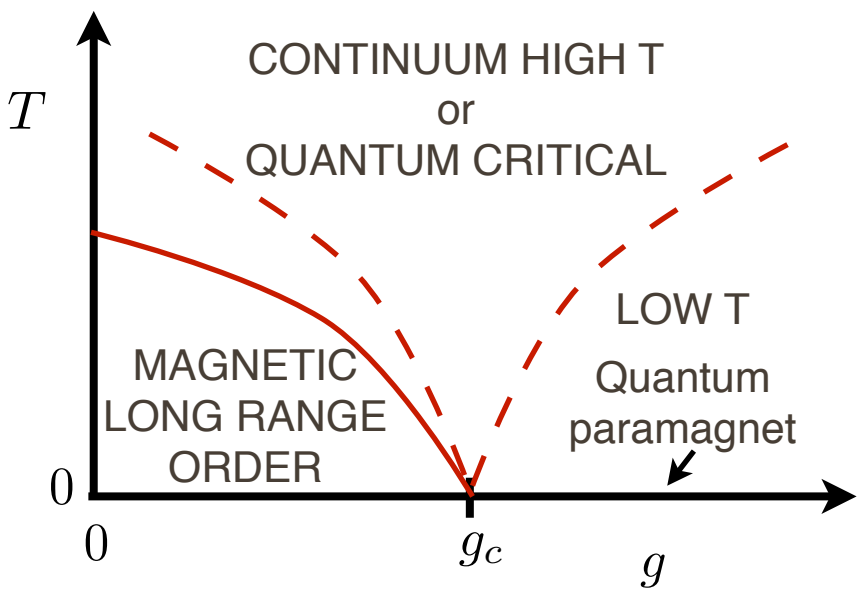
  C.-E. Bardyn, M. A. Baranov, C. V. Kraus, E. Rico, A. Imamoglu, P. Zoller, S. Diehl, *Topology by dissipation [with cold atoms]*, arXiv:1302.5135

S. Diehl et al., Nat Phys 2008
S. Diehl et al., PRL 2010

Non-equilibrium cond mat

- **thermodynamic equilibrium**

O(N) Rotor Model Phase Diagram



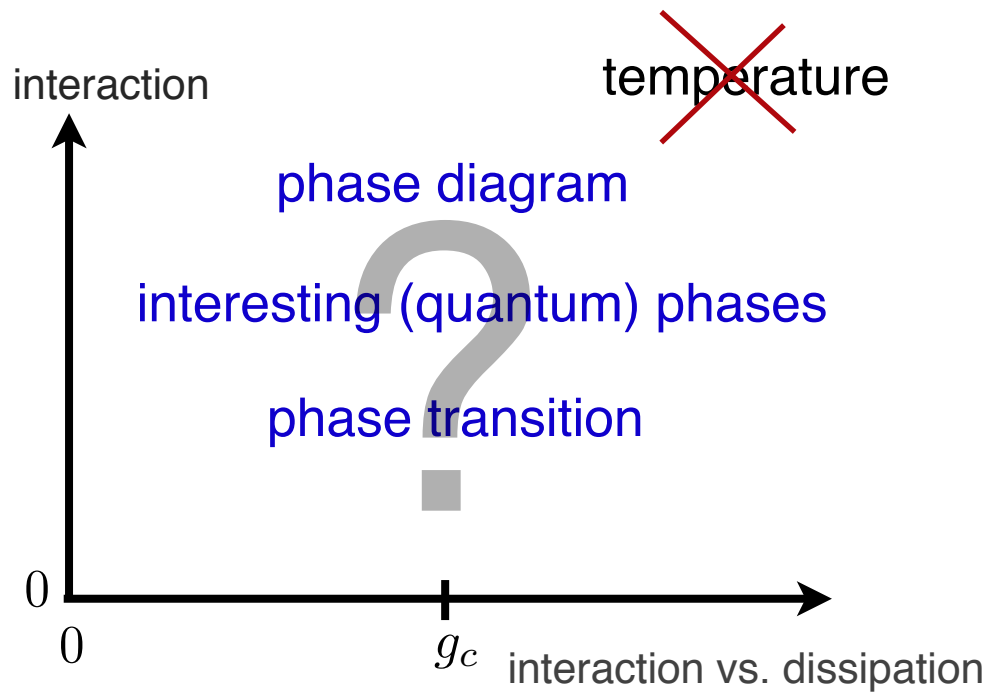
quantum phase transition

$$H\,|E_g\rangle = E_g\,|E_g\rangle$$

$$H = H_0 + gH_1$$

Hamiltonian (many body)

- **non-equilibrium**



dynamical quantum phase transition

Superfluid
(pure state)

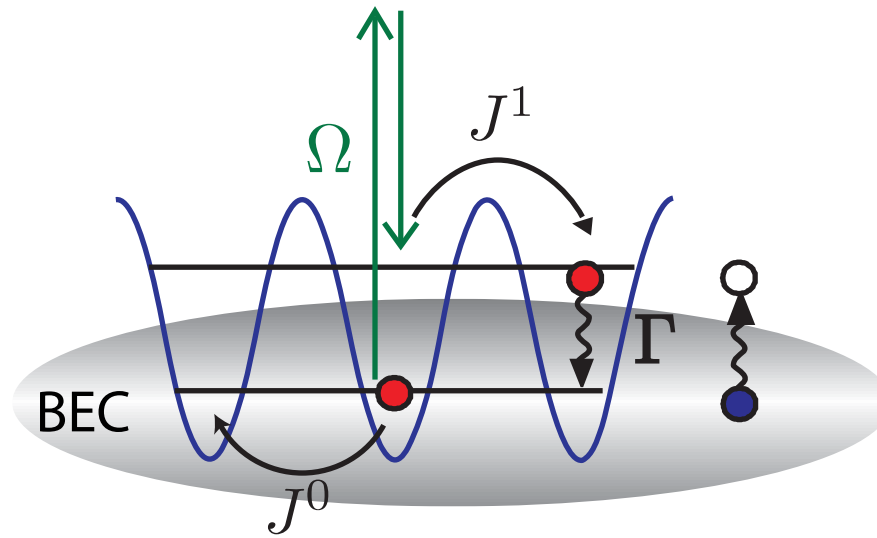
"thermal"
(mixed state)

$$\frac{d\rho}{dt} = -i\,[H,\rho] + \mathcal{L}\rho$$

A. Griessner et al.,
PRL 2006

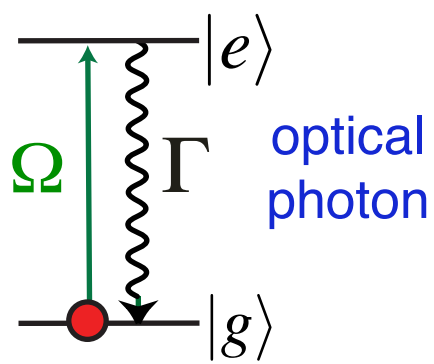
Driven Dissipative Hubbard Models

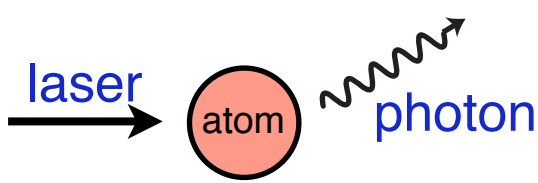
- BEC as a “phonon reservoir”
  - quantum reservoir engineering



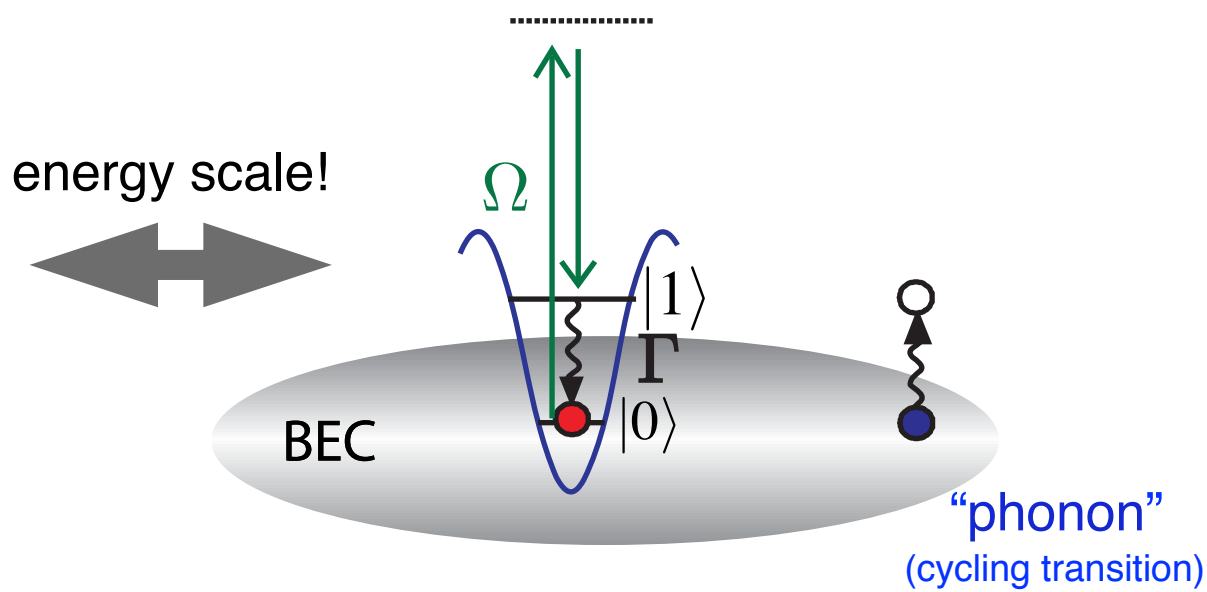
“think quantum optics”

- driven two-level atom + spontaneous emission





- trapped atom in a BEC reservoir

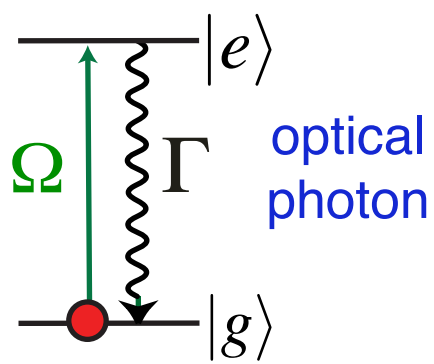


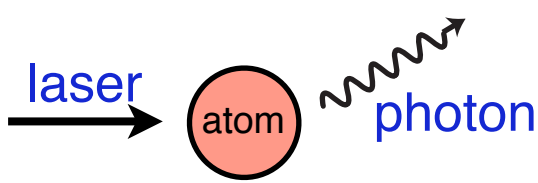
laser assisted atom + BEC collision

- reservoir: vacuum modes of the radiation field (T=0)

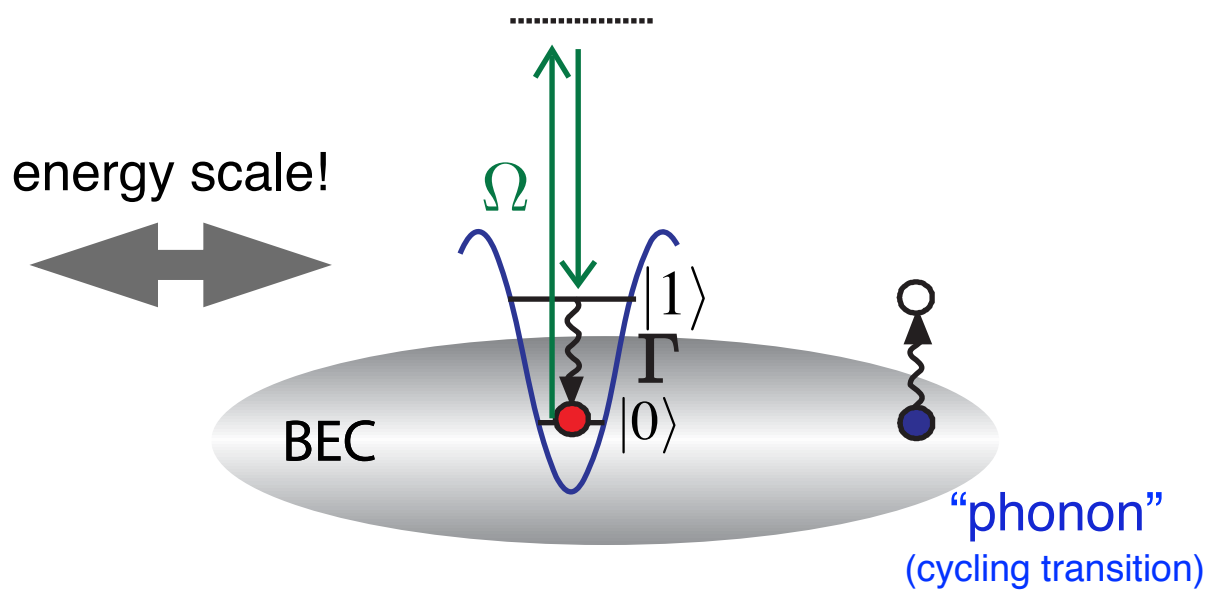
“think quantum optics”

- driven two-level atom + spontaneous emission





- trapped atom in a BEC reservoir

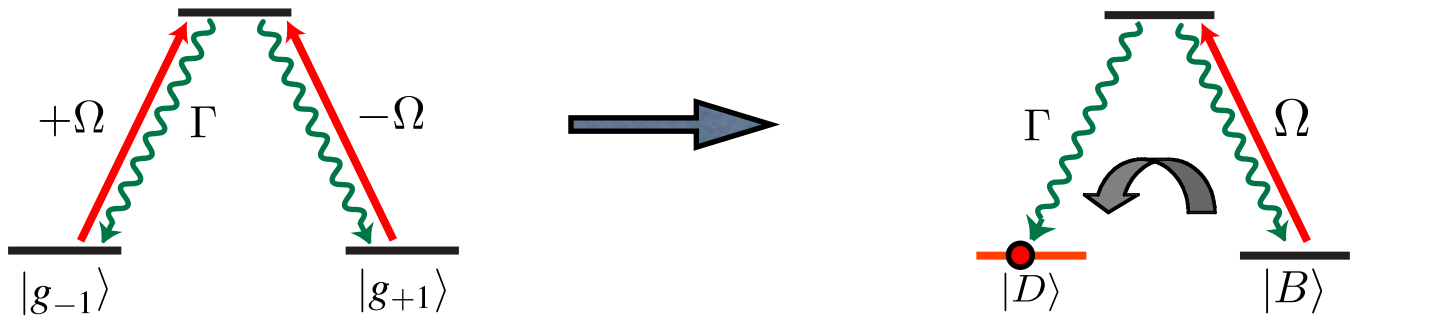


laser assisted atom + BEC collision

- reservoir: vacuum modes of the radiation field (T=0)
- reservoir: Bogoliubov excitations of the BEC (@ temperature T)

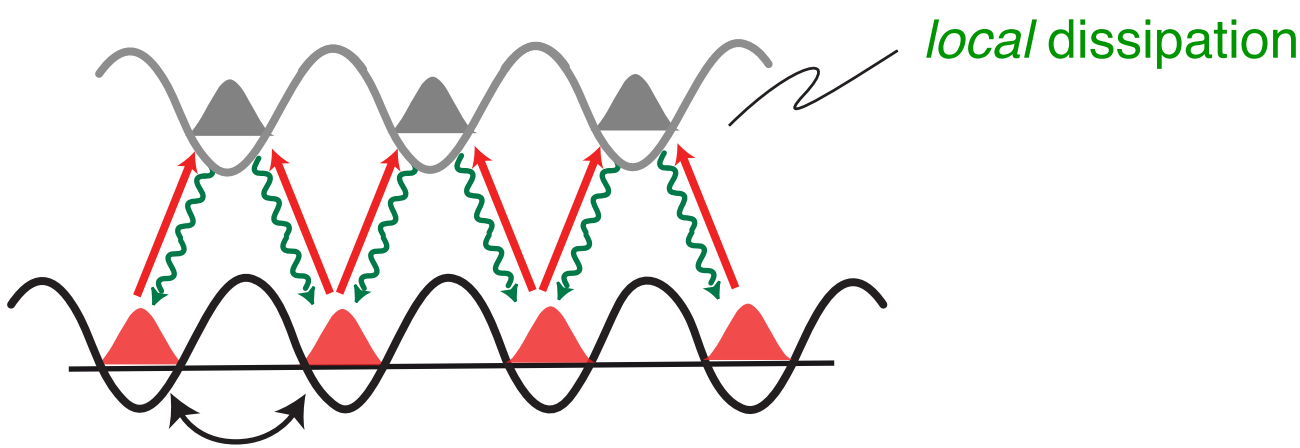
“think quantum optics”

- Λ-system



three electronic levels dark state bright state

- N atom on M sites

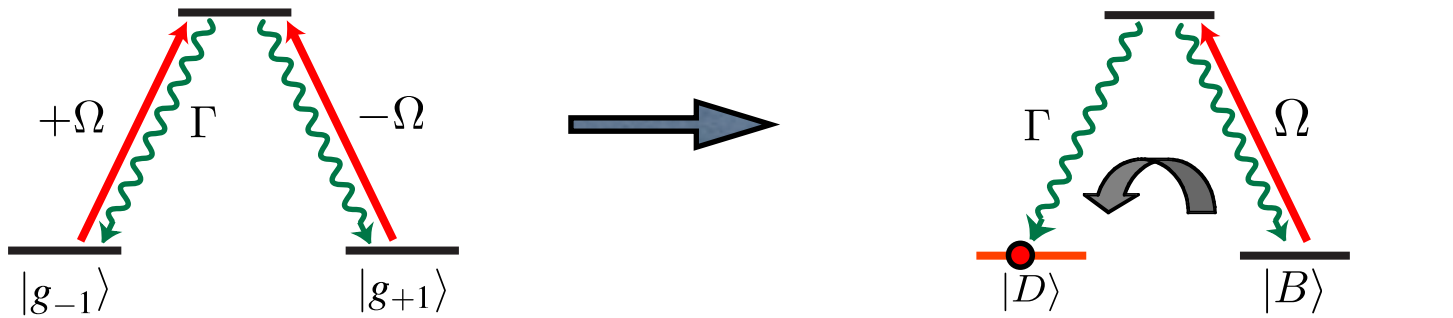


cond mat: ~ array of dissipative Josephson junctions

quantum optics: spatial chain of Λ-systems

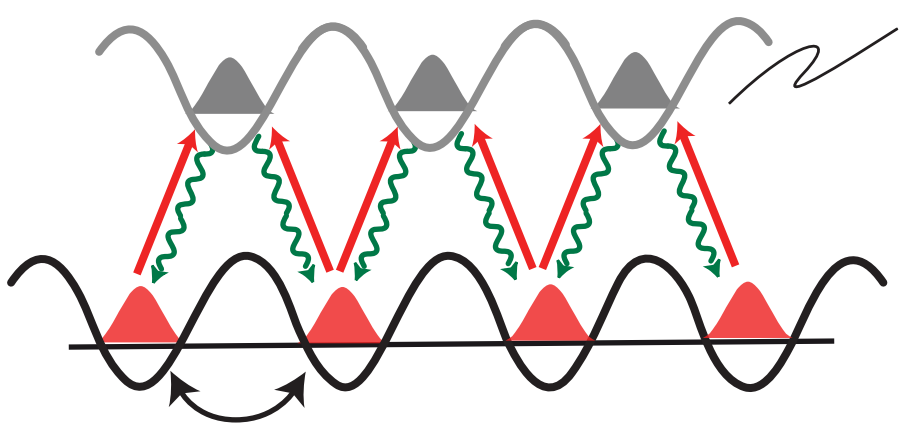
“think quantum optics”

- Λ-system



three electronic levels dark state bright state

- N atom on M sites



local dissipation

$$\rho(t) \xrightarrow{t\to\infty} |\mathrm{BEC}\rangle\langle\mathrm{BEC}|$$

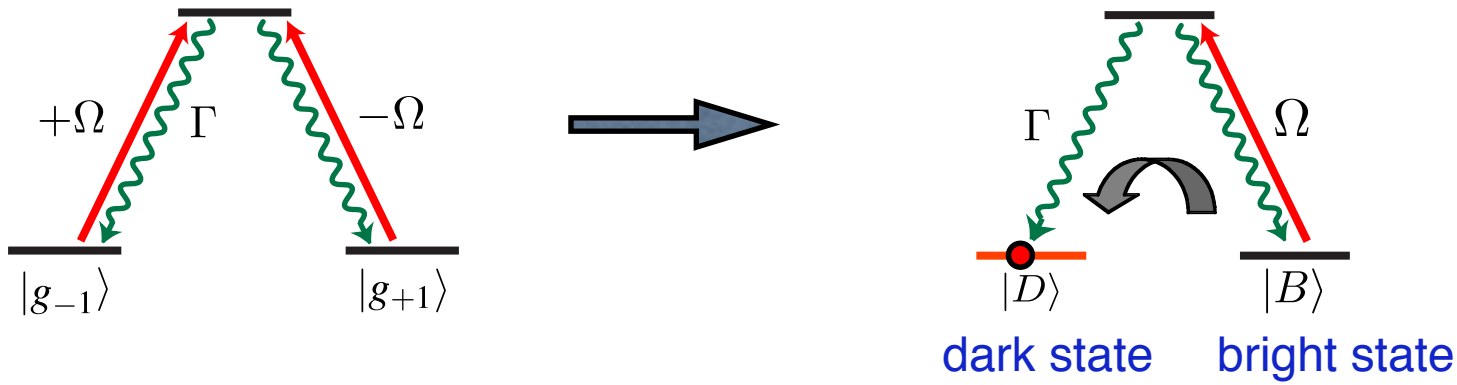
driven dissipative BEC

cond mat: ~ array of dissipative Josephson junctions

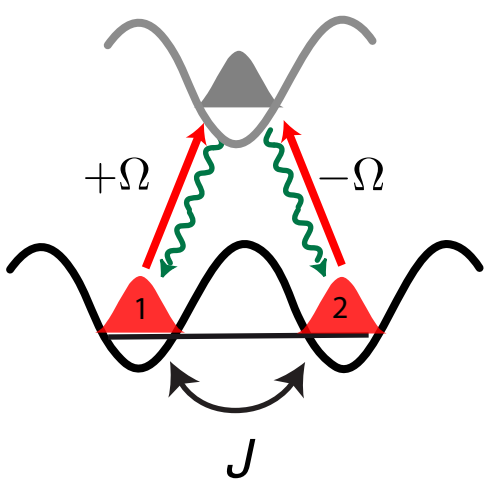
quantum optics: spatial chain of Λ-systems

“think quantum optics”

- Λ-system



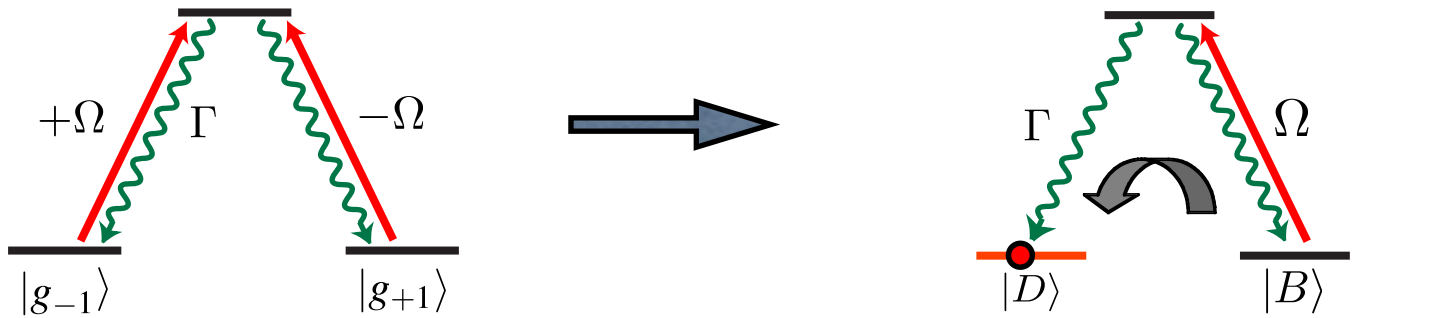
- 1 atom on 2 sites



~ dissipative Josephson junction

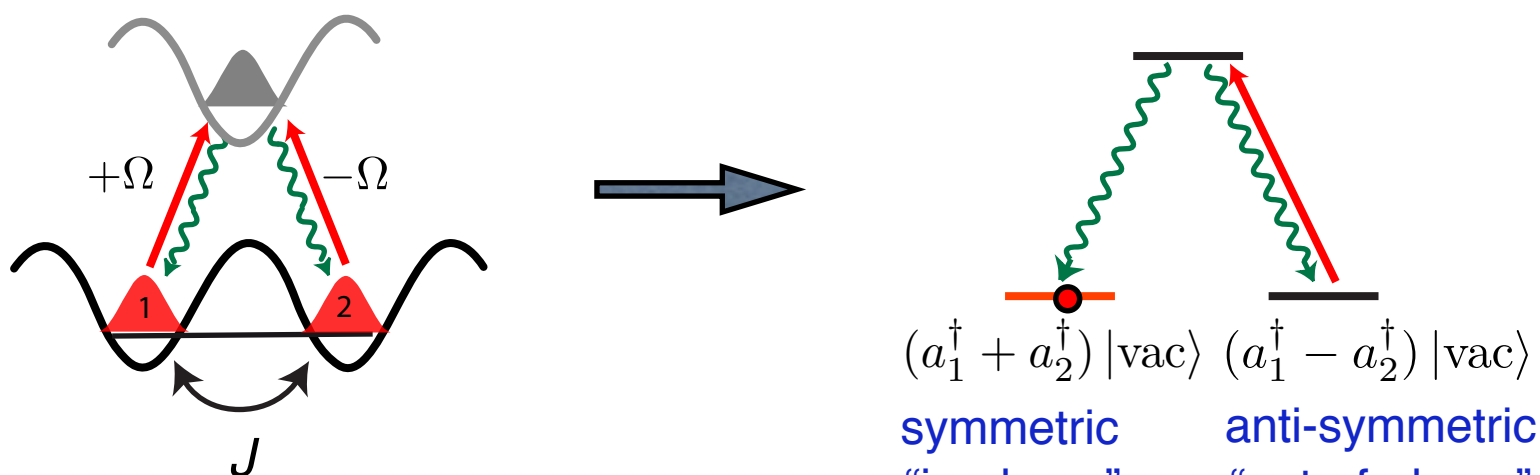
“think quantum optics”

- Λ-system



dark state bright state

- 1 atom on 2 sites



symmetric “in-phase”

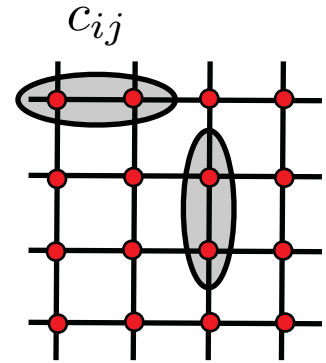
anti-symmetric “out-of-phase”

~ dissipative Josephson junction

pumping into symmetric state

“phase locking”

Driven Dissipative Bose Hubbard Model



- master equation

$$\frac{d}{dt}\rho = -i\left[H,\rho\right] + \kappa \sum_{\langle ij \rangle} \left[2c_{ij}\rho c_{ij}^{\dagger} - c_{ij}^{\dagger}c_{ij}\rho - \rho c_{ij}^{\dagger}c_{ij} \right]$$

adjacent sites

1. Bose Hubbard Hamiltonian

$$H = -J \sum_{<i,j>} a_i^{\dagger} a_j + \tfrac{1}{2} U \sum_i a_i^{\dagger 2} a_i^2$$

SF - Mott insulator physics

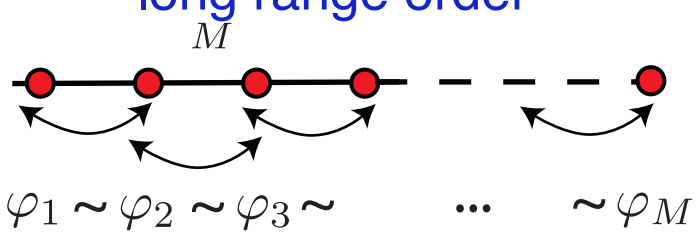
2. Liouvillian

$$c_{ij} = \left(a_i^{\dagger} + a_j^{\dagger}\right)\left(a_i - a_j\right)$$

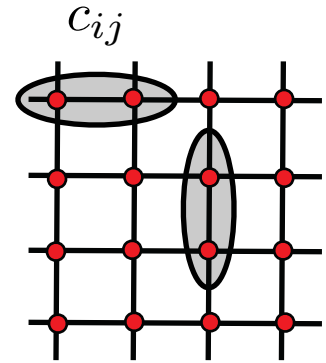
jump operator (quasi-local)

"cooling" to a BEC

local "driving/dissipation" causes long range order



Driven Dissipative Bose Hubbard Model



- master equation

$$\frac{d}{dt}\rho = -i\left[H,\rho\right] + \kappa \sum_{\langle ij \rangle} \left[2c_{ij}\rho c_{ij}^{\dagger} - c_{ij}^{\dagger}c_{ij}\rho - \rho c_{ij}^{\dagger}c_{ij} \right]$$

adjacent sites

1. Bose Hubbard Hamiltonian

$$H = -J \sum_{<i,j>} a_i^{\dagger} a_j + \tfrac{1}{2} U \sum_i a_i^{\dagger 2} a_i^2$$

SF - Mott insulator physics

Rem.: we have a microscopic justification for this model

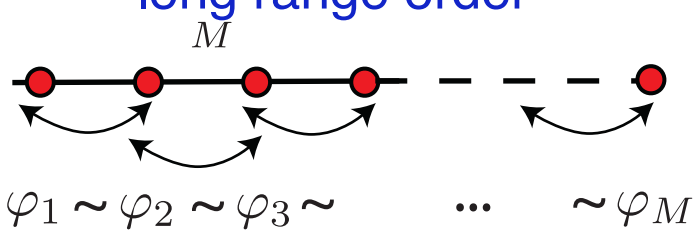
2. Liouvillian

$$c_{ij} = \left(a_i^{\dagger} + a_j^{\dagger}\right)\left(a_i - a_j\right)$$

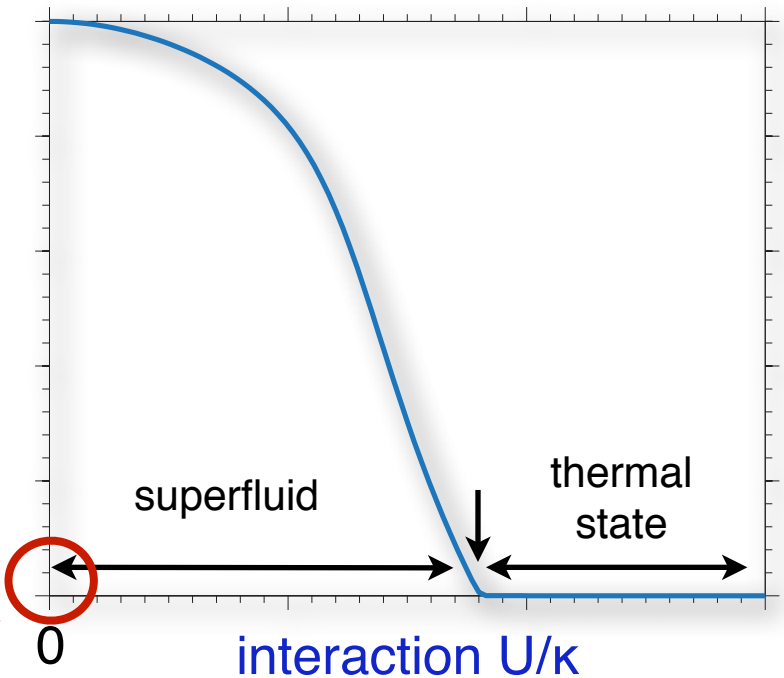
jump operator (quasi-local)

"cooling" to a BEC

local "driving/dissipation" causes long range order



Steady State Quantum Phases & Phase Transition



superfluid order parameter

interaction U/κ

~ interaction + "temperature"

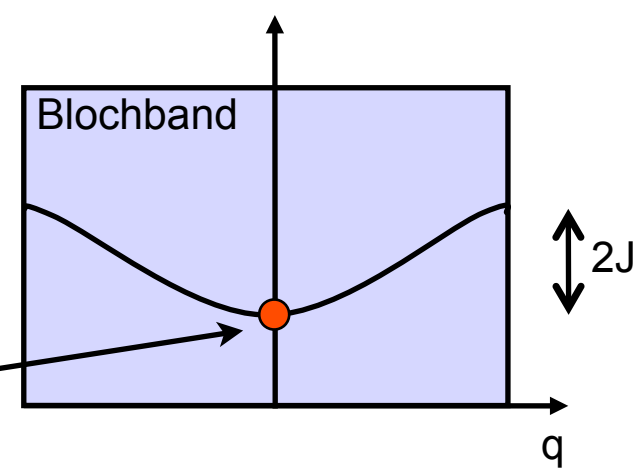
What happens when we switch on interactions?

for NO interactions

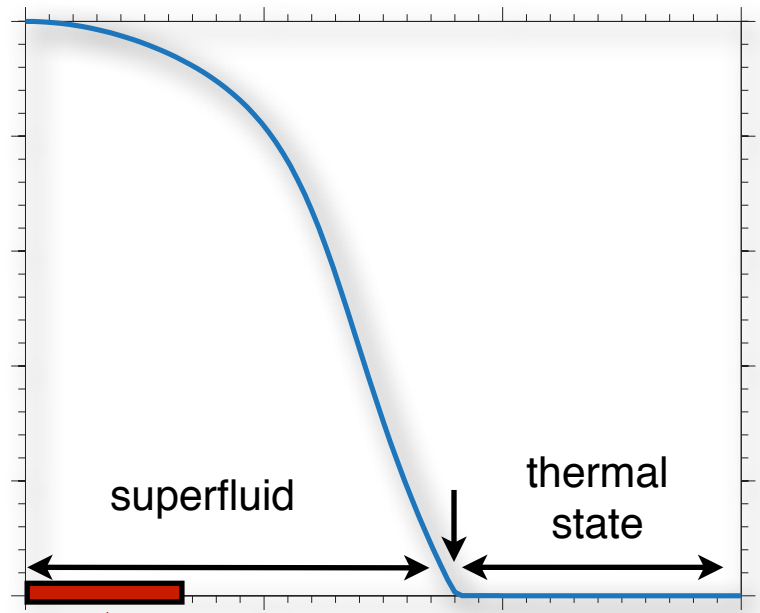
- perfect (driven dissipative) BEC

$$|\mathrm{BEC}\rangle \sim (\sum_i a_i^\dagger)^N |\mathrm{vac}\rangle \equiv (a_{q=0}^\dagger)^N |\mathrm{vac}\rangle$$

$$\rho(t) \xrightarrow{t\to\infty} |\mathrm{BEC}\rangle \langle \mathrm{BEC}|$$



Steady State Quantum Phases & Phase Transition



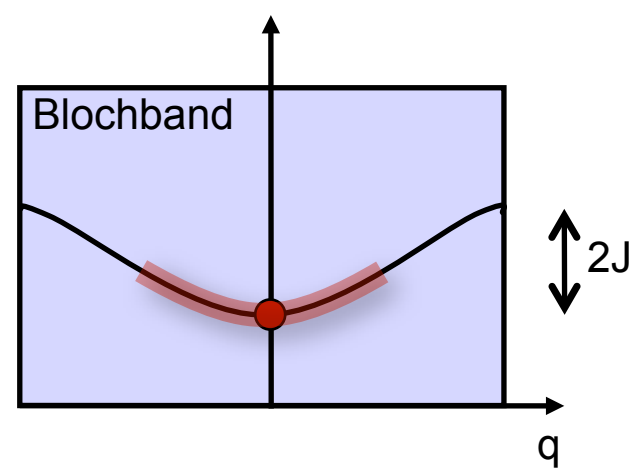
superfluid order parameter

interaction U/κ

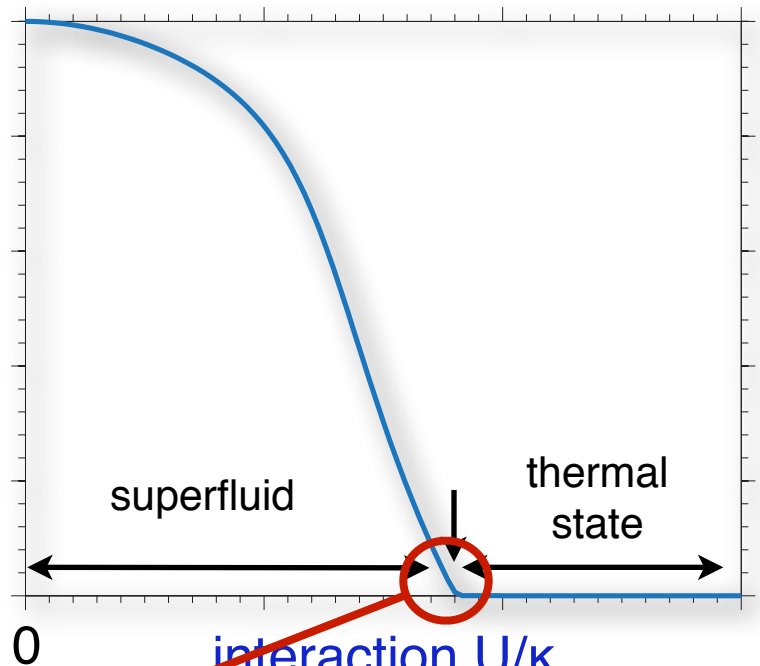
~ interaction + ”temperature”

for WEAK interactions

- 3D: depleted condensate (Bogoliubov)
- 1D/2D critical Kosterlitz-Thouless / Luttinger

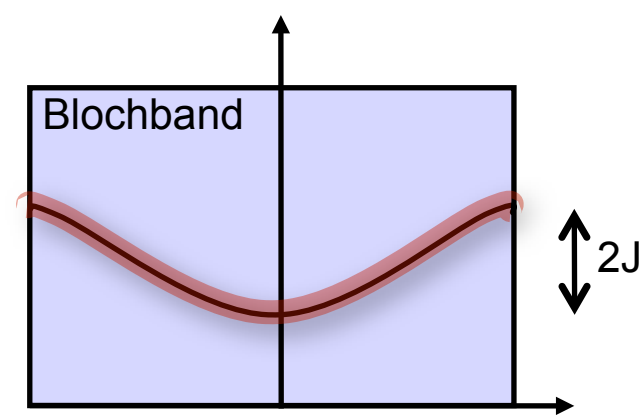


Steady State Quantum Phases & Phase Transition



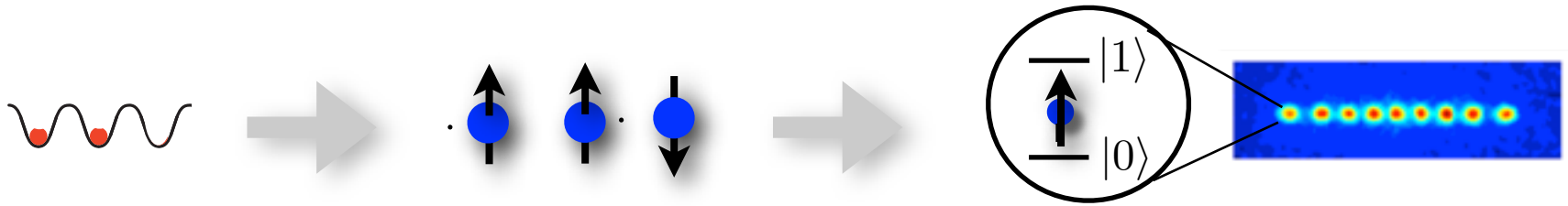
for STRONG interactions

- quantum phase transition
  (here: in mean field for 3D)



$$\rho_{n,n} = \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}$$

thermal state:
“*fixed temperature*”



bosons

spins

ion trap experiment

✓in the present form not scalable

✓a very small system

Quantum Simulation of Dynamical Maps with Trapped Ions

P. Schindler*,[1] M. Müller*,[2] D. Nigg,[3,4] J. T. Barreiro†,[3,4] E. A. Martinez,[3,4] M. Hennrich,[3,4] T. Monz,[3,4] S. Diehl,[3,4] P. Zoller,[4] and R. Blatt[4]

http://arxiv.org/abs/1212.2418

Non-equilibrium dynamics for bosons and spins - a sketch

- dissipation: delocalizes particles over the lattice
- Hamiltonian: „asks“ for particle number, tends to localize particles

$$0 \stackrel{!}{=} \partial_t \rho = -i\,[H,\rho] + \mathcal{L}[\rho]$$

translation table	**bosons** (cold atoms)
jump operators in $\mathcal{L}(\rho)$	$c_k = (a_k^\dagger + a_{k+1}^\dagger)(a_k - a_{k+1})$
dissipative target dark state	$\|\mathrm{BEC}\rangle(N) \sim (\sum_k a_k^\dagger)^N \|\mathrm{vac}\rangle$
competing Hamiltonian	$H = U \sum_k n_k(n_k-1)$ with $n_k = a_k^\dagger a_k$

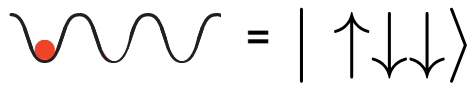
boson scenario: S. Diehl S. Diehl, A. Tomadin, A. Micheli, R. Fazio, P. Zoller, PRL 2010

Non-equilibrium dynamics for bosons and spins - a sketch

- dissipation: delocalizes particles over the lattice
- Hamiltonian: „asks“ for particle number, tends to localize particles

$$0 \stackrel{!}{=} \partial_t \rho = -i\,[H,\rho] + \mathcal{L}[\rho]$$

- a spin analog (hard-core bosons)

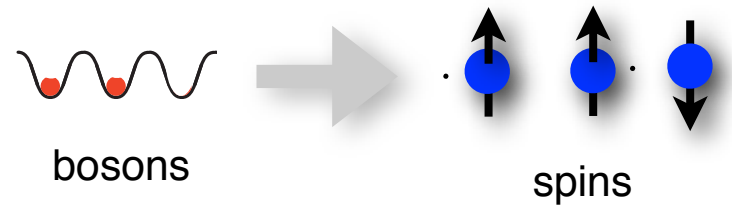


translation table	**bosons** (cold atoms)	**spins / hardcore bosons** (trapped ions)
jump operators in $\mathcal{L}(\rho)$	$c_k = (a_k^\dagger + a_{k+1}^\dagger)(a_k - a_{k+1})$	$c_k = (\sigma_k^+ + \sigma_{k+1}^+)(\sigma_k^- - \sigma_{k+1}^-)$ $c\,\lvert\uparrow\downarrow\rangle \rightarrow \lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle$
dissipative target dark state	$\lvert\mathrm{BEC}\rangle(N) \sim (\sum_k a_k^\dagger)^N \lvert\mathrm{vac}\rangle$	$\lvert\mathrm{Dicke}\rangle(N) \sim (\sum_k \sigma_k^+)^N \lvert\downarrow\ldots\downarrow\rangle$
competing Hamiltonian	$H = U\sum_k n_k(n_k-1)$ with $n_k = a_k^\dagger a_k$	$H = U'\sum_k n_k n_{k+1}$ with $n_k = \frac{1}{2}(1+\sigma_k^z)$

boson scenario: S. Diehl S. Diehl, A. Tomadin, A. Micheli, R. Fazio, P. Zoller, PRL 2010

Dynamical Maps as Iteration of Kraus Maps

Kraus map:



$$\rho(t_\ell) \mapsto \rho(t_{\ell+1}) = \mathcal{E}^{(l)}[\rho(t_\ell)] = \sum_{k=1}^{K} E_k^{(l)} \rho(t_\ell) E_k^{(l)\dagger}$$

- **Dissipative maps**

two-body Kraus operators acting on pairs of neighboring spins

$$E_{i,1} = c_i, \qquad E_{i,2} = 1 - c_i^\dagger c_i.$$

$$c_i = (\sigma_i^+ + \sigma_{i+1}^+)(\sigma_i^- - \sigma_{i+1}^-),$$

dark state

$$|\psi\rangle_D = |D(m,N)\rangle \sim \Big(\sum_{i=1}^{N} \sigma_i^+\Big)^m |\downarrow\rangle^{\otimes N}$$

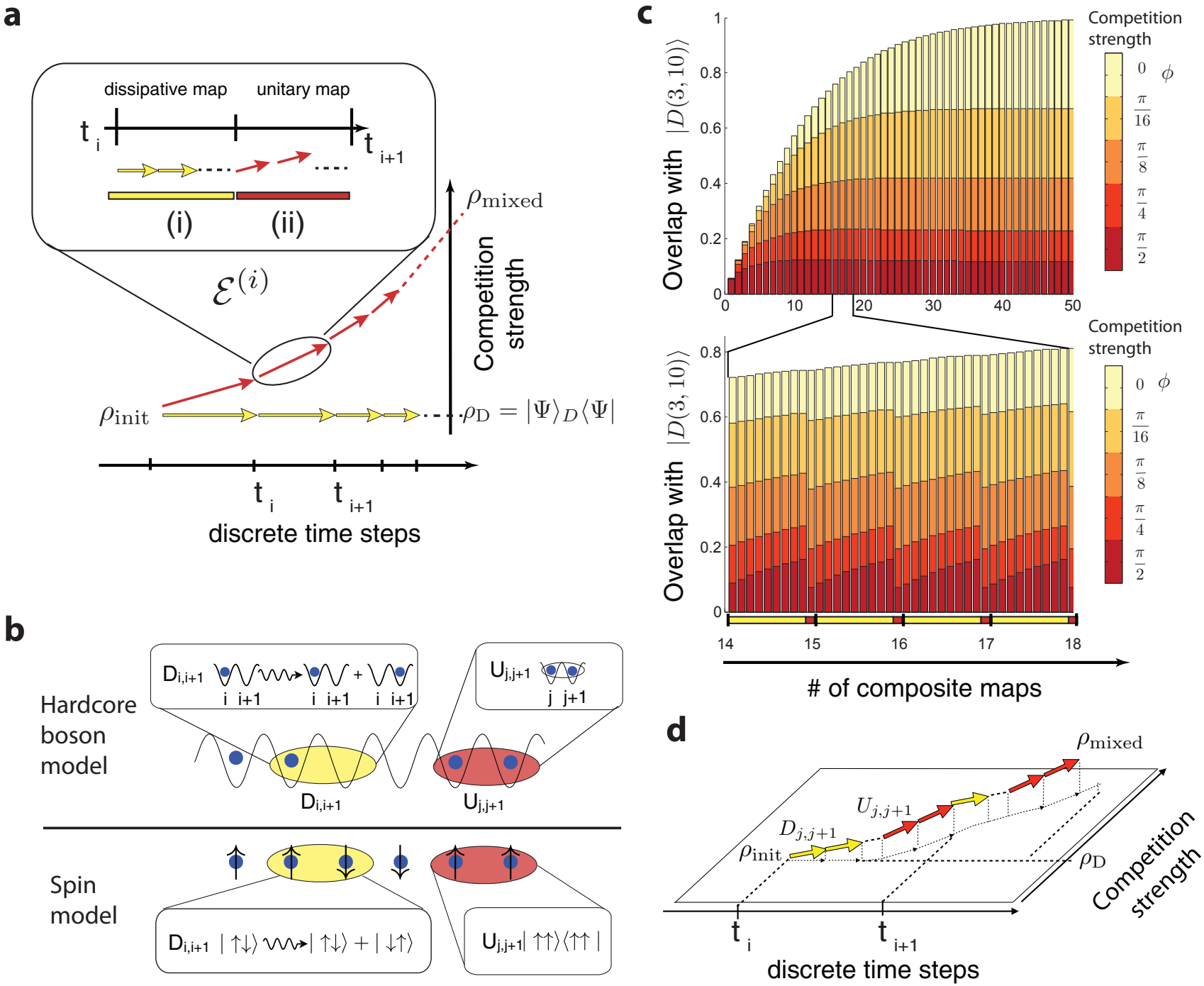
- **Unitary**

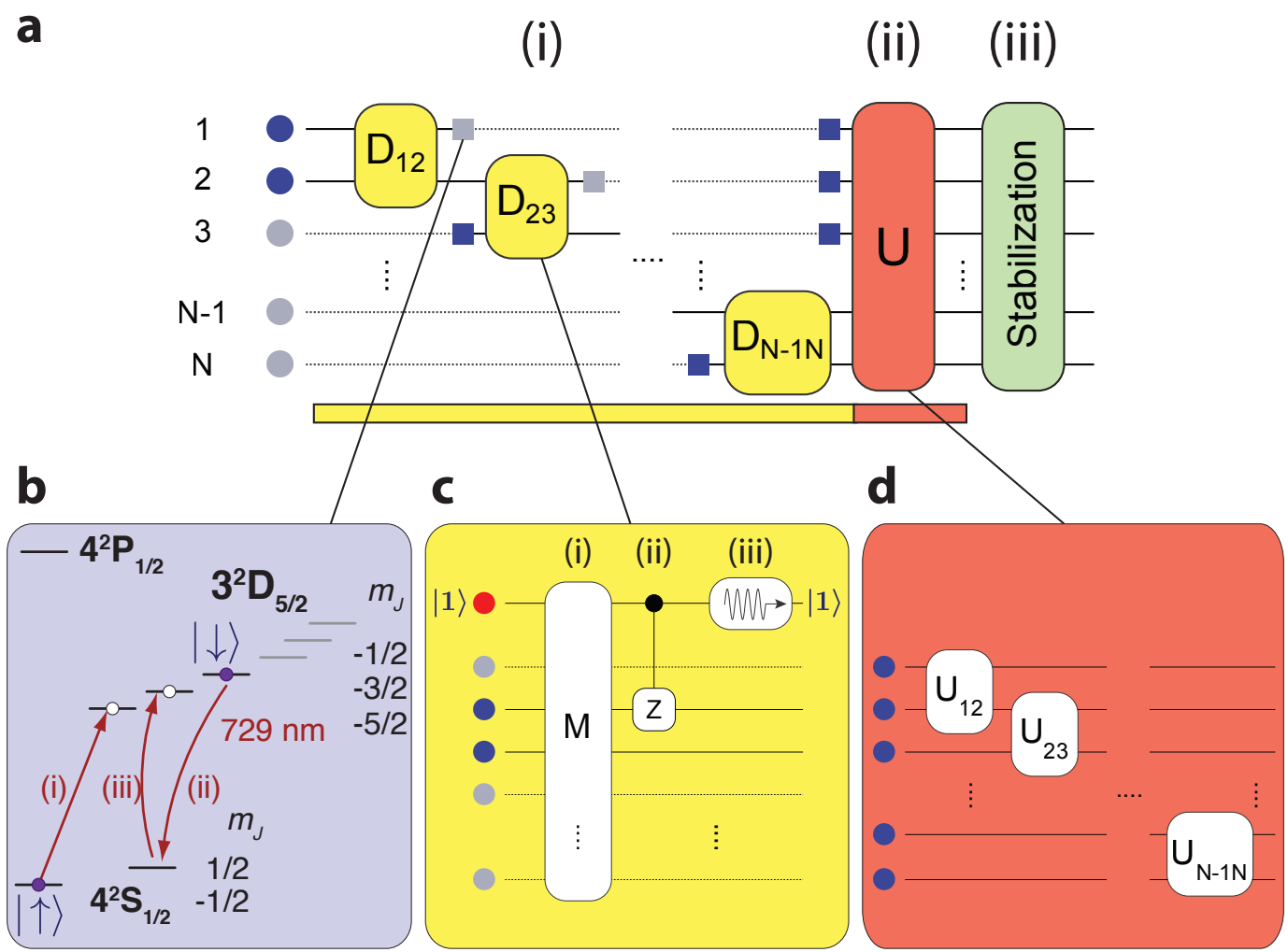
Hamiltonian

$$U = \exp(-\mathrm{i}\phi H)$$

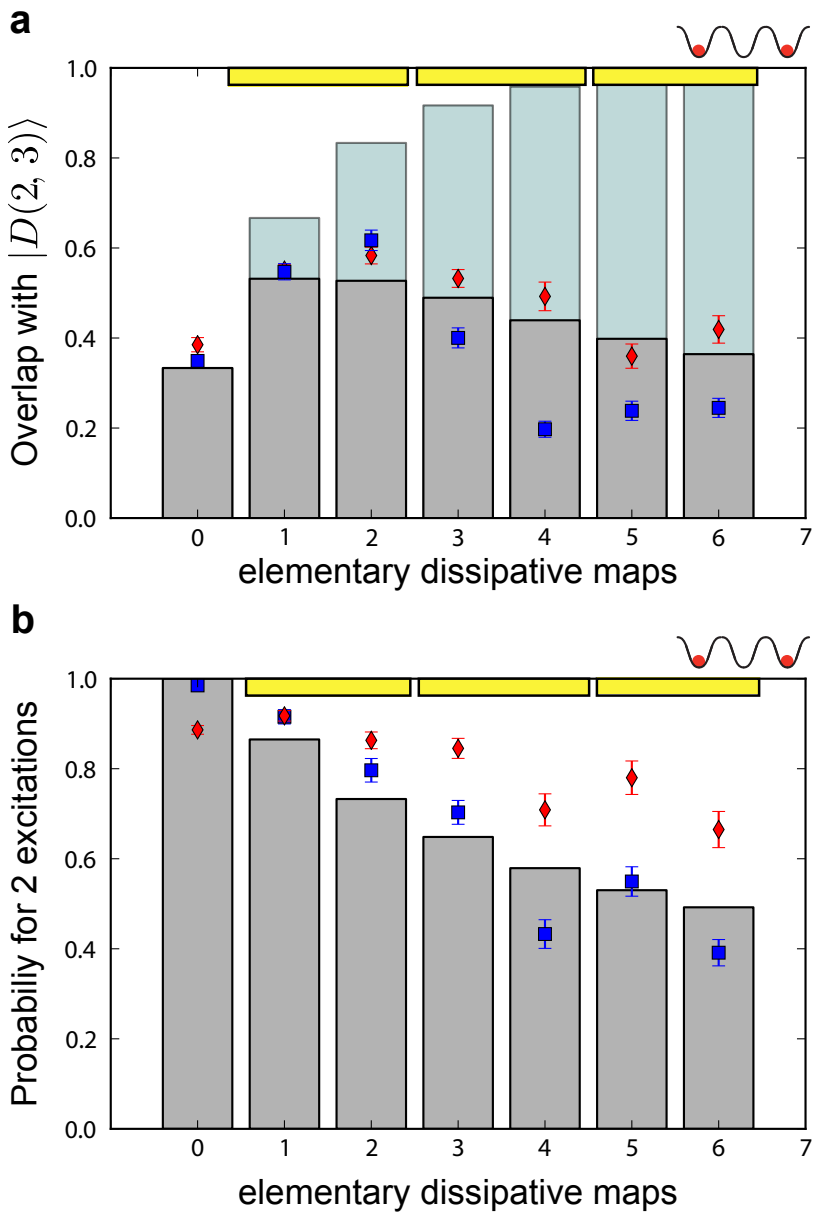
nearest neighbor interaction

$$H = \sum_i |\uparrow\rangle\langle\uparrow|_i \otimes |\uparrow\rangle\langle\uparrow|_{i+1} = \sum_i (1+\sigma_i^z)(1+\sigma_{i+1}^z)/4.$$

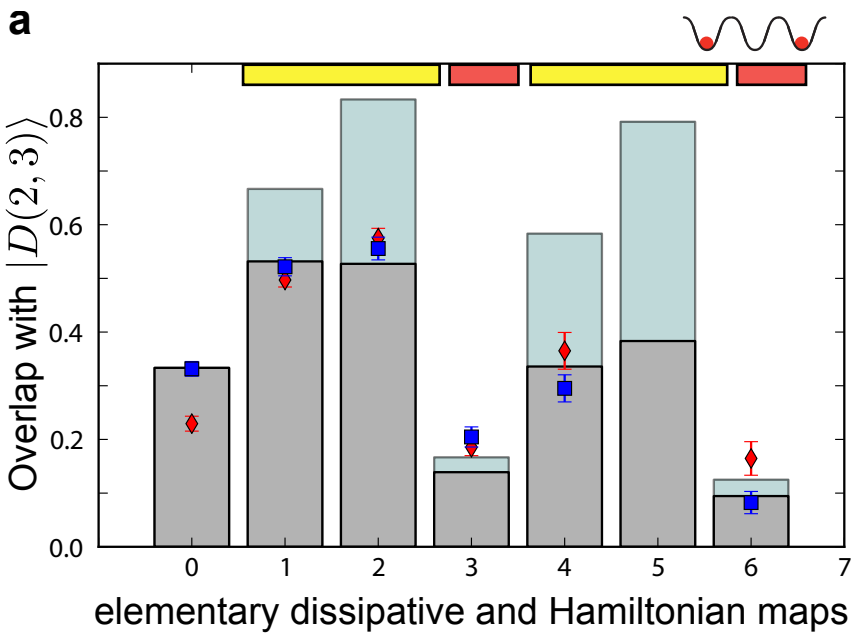

a
dissipative map
unitary map
t i
t i+1
(i)
(ii)
$\mathcal{E}^{(i)}$
ρ_{mixed}
Competition strength
ρ_{init}
$\rho_{\mathrm{D}} = |\Psi\rangle_D\langle\Psi|$
t i
t i+1
discrete time steps
b
Hardcore boson model
$D_{i,i+1}$
i i+1
$U_{j,j+1}$
j j+1
Spin model
$D_{i,i+1}$ $|\uparrow\downarrow\rangle \rightsquigarrow |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$
$U_{j,j+1}|\uparrow\uparrow\rangle\langle\uparrow\uparrow|$
c
Overlap with $|D(3,10)\rangle$
Competition strength
ϕ
0
$\frac{\pi}{16}$
$\frac{\pi}{8}$
$\frac{\pi}{4}$
$\frac{\pi}{2}$
# of composite maps
d
ρ_{mixed}
$U_{j,j+1}$
$D_{j,j+1}$
ρ_{init}
ρ_{D}
Competition strength
t i
t i+1
discrete time steps

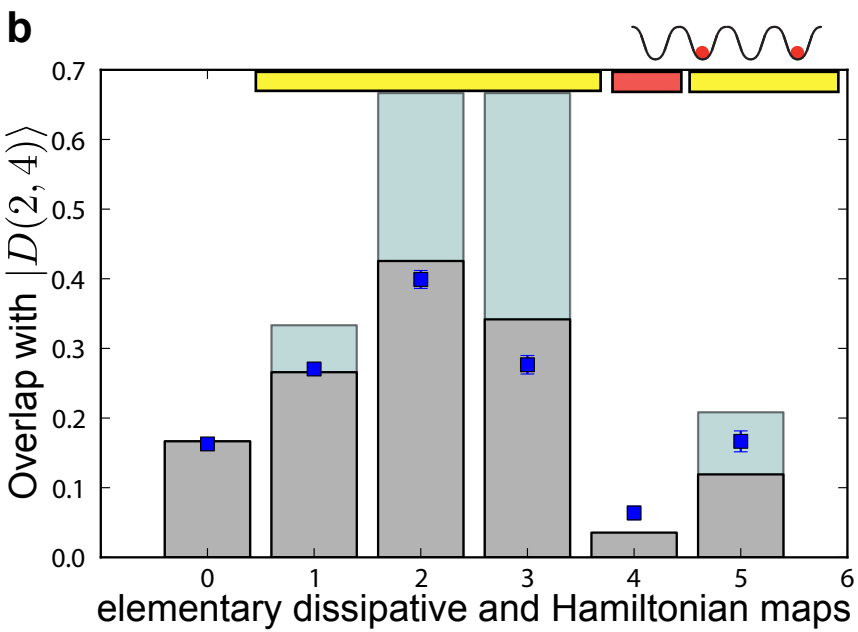


Experimental procedure to implement open-system dynamical maps.

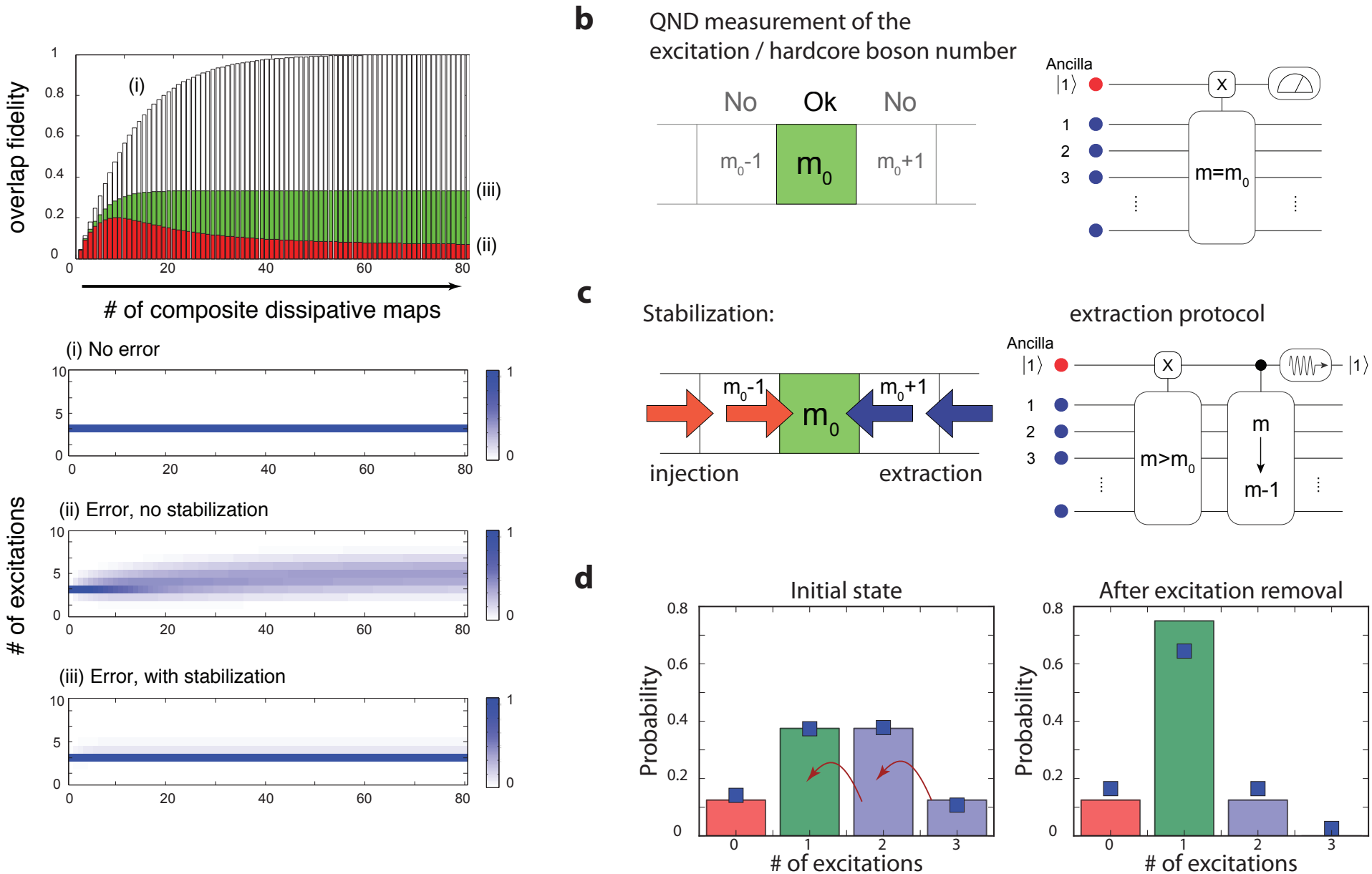


Experimental results of dissipatively in- duced delocalization through composite dynamical maps with 3+1 ions.





Experimental results for competing dissipative and coherent dynamics with 3+1 and 4+1 ions.



Experimental error detection and reduction techniques.

Algorithm type	Nr. of MS gates	Nr. of global rotations R	Nr. of AC-Stark shifts S_Z	Nr. of resets	Total number of operations	Described in section
Elementary dissipative map	6	7	9	1	23	IIb
Hamiltonian competing dynamics with 3+1 ions	2	3	2	0	7	IIc
Hamiltonian competing dynamics with 4+1 ions	3	4	4	0	11	IIc
QND post-selective error detection	4	8	6	0	18	IV
Mapping for the spin excitation removal	2	9	4	0	15	V
Mapping for the spin excitation injection	3	12	7	0	22	V
Excitation injection / removal step (single site)	4	0	2	0	6	V
Spectroscopic decoupling	0	5	4	0	9	Ic
Composite dissipative map (3 spins)	12	26	24	2	64	
Composite diss. and coh. dyn. map (3 spins)	14	29	26	2	71	
Composite diss. and coh. map + QND (3 spins)	18	40	27	3	88	
Composite dissipative map (4 spins)	18	33	35	3	89	
Composite diss. and coh. dyn. map (4 spins)	21	44	44	3	112	

TABLE I Summary of the required resources for the elementary and composite dynamical maps and additional tools used in the quantum simulation. The required operations for the composite maps do not strictly match the sum of the required elementary operations, since in the implementation of composite maps synergy effects in the resources for the spectroscopic decoupling operations can be exploited.