Quantum Feedback Networks

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Outline

Some Engineering Perspectives

'Technology seems to advance in waves. Small advances in science and technology accumulate slowly ... until a critical level...

'Woven into the rich fabric of technological history is an invisible thread that has a profound effect on each of these waves...

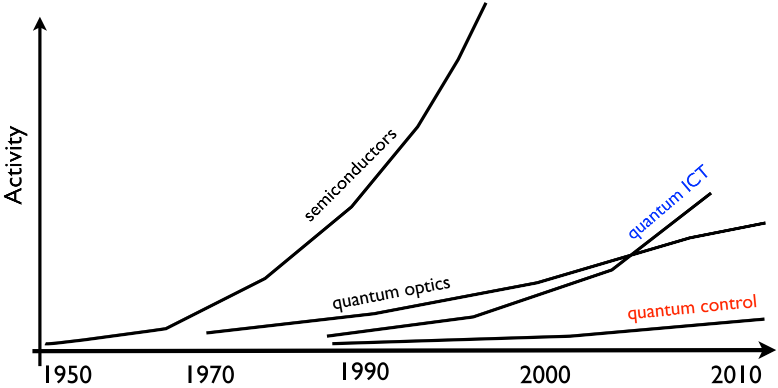
'This thread is the idea of feedback control.

Dennis Bernstein, *History of Control*, 2002 feed·back

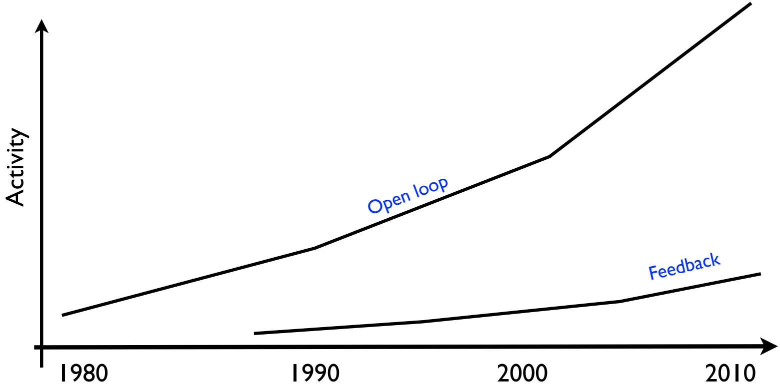
noun

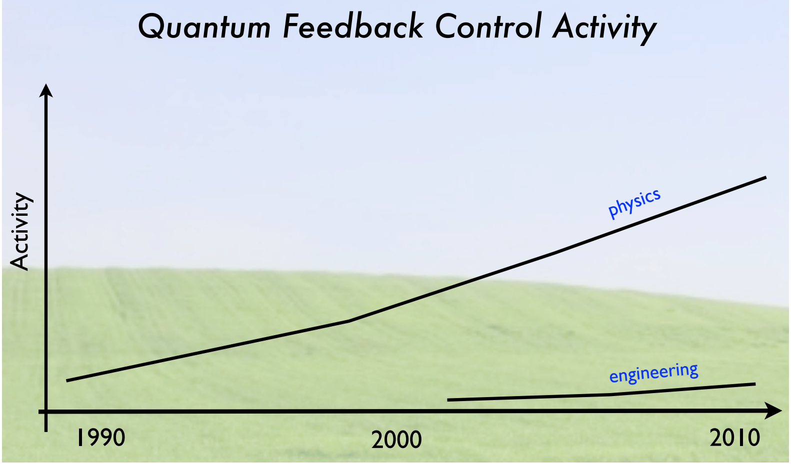
1. information about reactions to a product, a person's performance of a task, etc., used as a basis for improvement.
2. the modification or control of a process or system by its results or effects, e.g., in a biochemical pathway or behavioral response. See also negative feedback, positive feedback.
    - the return of a fraction of the output signal from an amplifier, microphone, or other device to the input of the same device; sound distortion produced by this.

Relative Maturity of Research Areas

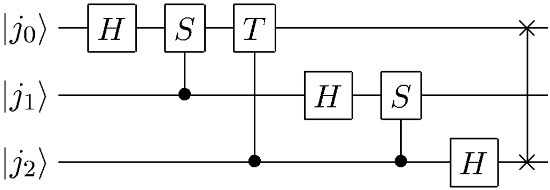


Quantum Control Activity

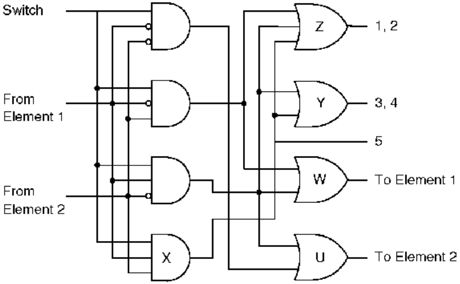



Quantum Feedback Control Activity
Activity
physics
engineering
1990
2000
2010

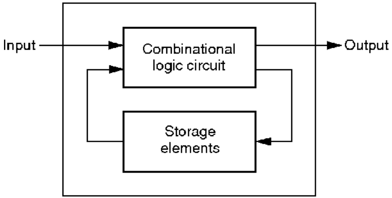
Quantum circuit



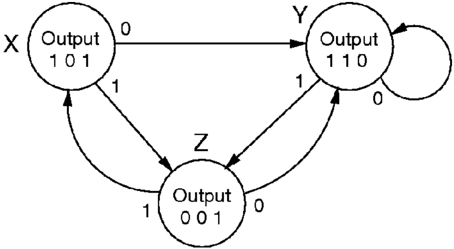
Classical combinational logic diagram



Classical sequential logic diagram

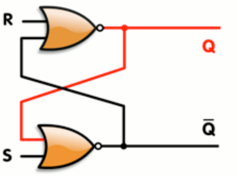


Classical state diagram

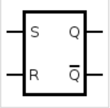


Flip Flop

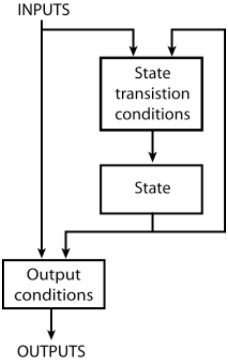
Simple *memory* elements basic to sequential logic (state machines).



SR latch operation		
S	**R**	**Action**
0	0	Keep state
0	1	Q = 0
1	0	Q = 1
1	1	Restricted combination



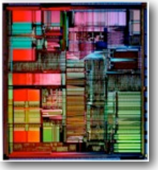
The symbol for an SR latch.



Dynamics and *feedback* are fundamental to the operation of flip-flops.

Development of Technology

1950



1970

2000+

Quantum technology is at about here?

Systems integration, hierarchy, component development, mid-level integration, large-scale integration, product

Engineering design process

Functional Level

Functional requirements/ specifications:
Purpose of the system.

↕

Circuit Level

Circuit design meeting requirements (may be abstract).

↕

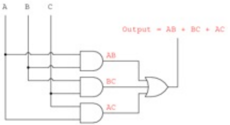
Hardware Level

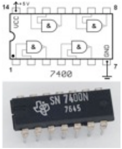
Practical specs include:
- voltage ranges
- noise margin
- fan in/out (*network* issue)
- rise/fall time (*dynamical* issue)

Important for system integration.

Secure communication: metrics for fidelity of transfer, safety from intrusion, etc

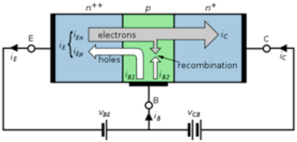
Efficient computation: metrics for computational complexity, memory and time requirements, etc

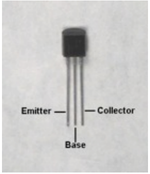




Need *appropriate* models for particular purposes at each level:

simple, but not too simple.

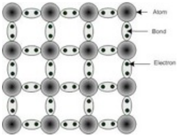


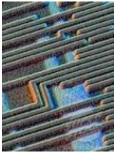


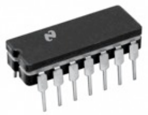
$$\frac{d^2 \Delta p_{\mathrm{B}}(x)}{dx^2} = \frac{\Delta p_{\mathrm{B}}(x)}{L_{\mathrm{B}}^2}$$

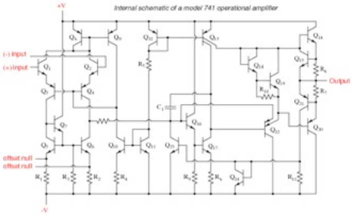
$$I_{\mathrm{C}} = qA\left(\left(\frac{D_{\mathrm{B}}}{W} p_{\mathrm{B0}}\right)(\exp(qV_{\mathrm{EB}}/kT) - 1) - \left(\frac{D_{\mathrm{C}} n_{\mathrm{C0}}}{L_{\mathrm{C}}} + \frac{D_{\mathrm{B}} p_{\mathrm{B0}}}{W}\right)(\exp(qV_{\mathrm{CB}}/kT) - 1)\right)$$

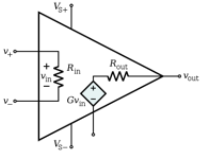
$$I_c = \beta I_b$$

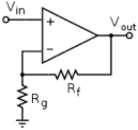






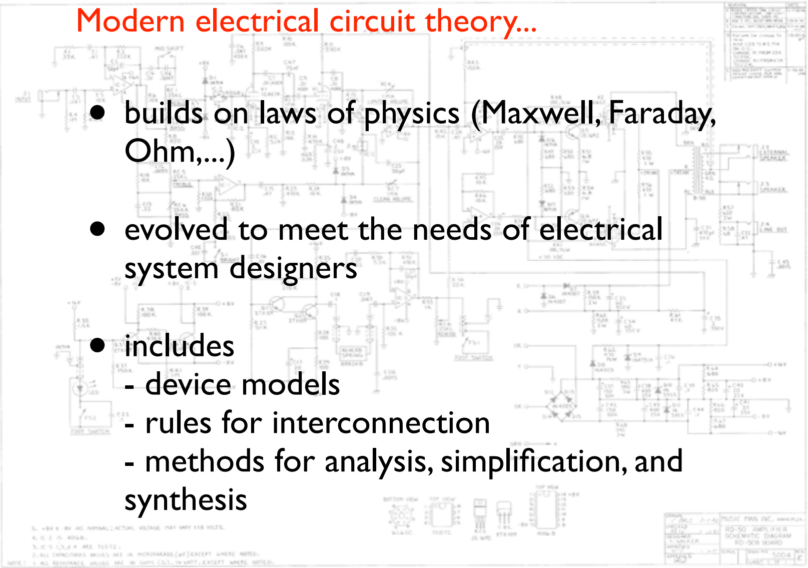






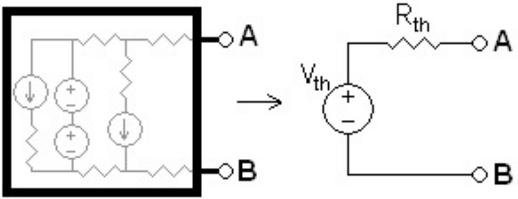
$$V_{out} = \frac{R_f + R_g}{R_f} V_{in}$$

Modern electrical circuit theory...



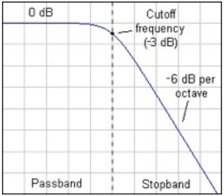
- builds on laws of physics (Maxwell, Faraday, Ohm,...)
- evolved to meet the needs of electrical system designers
- includes
  - device models
  - rules for interconnection
  - methods for analysis, simplification, and synthesis

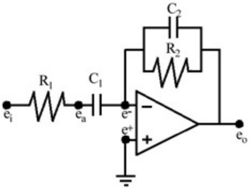
For example, Thevenin's theorem helps engineers simplify complex linear circuits.

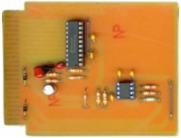


original circuit → equivalent circuit

For example, realisation techniques help engineers build devices from given specifications (synthesis).







specifications ⟶ circuit diagram ⟶ physical device

Quantum Feedback Networks

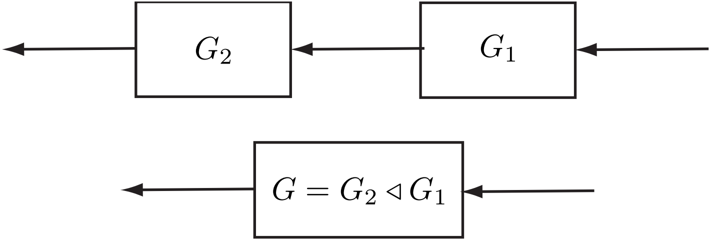
Basic ideas

Classical: series connection of resistors.

$$R = R_1 + R_2$$

The simple algebraic formula $R = R_1 + R_2$ is based on underling physics (electromagnetism).

Quantum: series connection of open systems.



The simple algebraic formula

$$G_2 \triangleleft G_1 = (S_2 S_1, L_2 + S_2 L_1, H_1 + H_2 + \mathrm{Im}[L_2^\dagger S_2 L_1])$$

is based on underlying physics (quantum mechanics).

Quantum stochastic equations

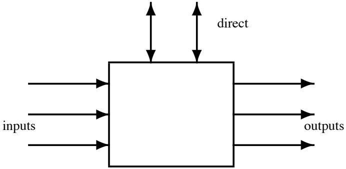
[Hudson-Parthasarathy, 1984]
[Gardiner-Collett, 1985]

$$\begin{aligned} dU(t) &= \Big\{ \mathrm{tr}[(\mathbf{S}-\mathbf{I})d\mathbf{\Lambda}] + d\mathbf{A}^{\dagger}\mathbf{L} - \mathbf{L}^{\dagger}\mathbf{S}d\mathbf{A} \\ &\quad - \frac{1}{2}\mathbf{L}^{\dagger}\mathbf{L}dt - iHdt \Big\} U(t) \end{aligned}$$

[Schrodinger equation]

- Boson field channels described using quantum noise
- Unitary evolution of physical variables (Heisenberg)
- Master equation for system states
- Measurements described using quantum conditional expectation and quantum filters (Belavkin)

Network components - open quantum systems



Parameters

interface

internal energy

$$\mathbf{G} = (\overbrace{\underbrace{\mathbf{S}, \mathbf{L}}_{\text{indirect field connections}}, \mathbf{Z}}^{\text{interface}}, H)$$

indirect field connections

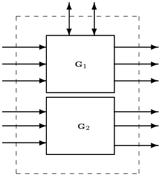
direct connections

Elementary network constructs

[Gough and James, 2008, 2010]

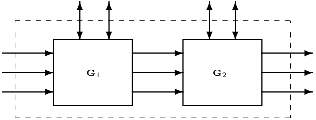
concatenation product

$$\mathbf{G}_1 \boxplus \mathbf{G}_2 = (\begin{pmatrix} \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2 \end{pmatrix}, \begin{pmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \end{pmatrix}, \begin{pmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{pmatrix}, H_1 + H_2)$$



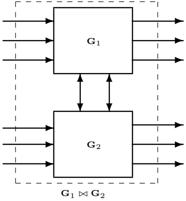
series product

$$\begin{aligned} \mathbf{G}_2 \triangleleft \mathbf{G}_1 &= \mathbf{S}_2\mathbf{S}_1, \mathbf{L}_2 + \mathbf{S}_2\mathbf{L}_1, \begin{pmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{pmatrix}, \\ & H_1 + H_2 + \frac{1}{2i}(\mathbf{L}_2^\dagger \mathbf{S}_2 \mathbf{L}_1 - \mathbf{L}_1^\dagger \mathbf{S}_2^\dagger \mathbf{L}_2)) \end{aligned}$$



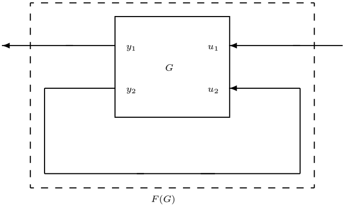
direct connection product

$$\mathbf{G}_1 \bowtie \mathbf{G}_2 = \left(\begin{pmatrix} \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2 \end{pmatrix}, \begin{pmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \end{pmatrix}, -, \right. \\ \left. H_1 + H_2 + \mathbf{Z}_2^\dagger \mathbf{Z}_1 + \mathbf{Z}_1^\dagger \mathbf{Z}_2 \right)$$



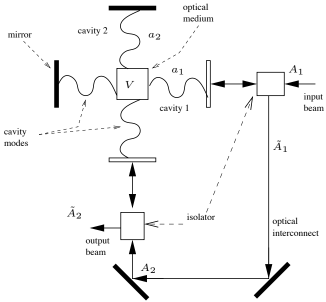
linear fractional transformation (LFT)

$$G = \left(\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}, \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}, H \right)$$

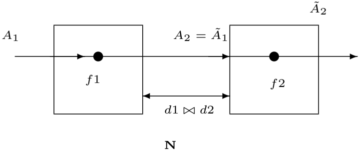


$$F(G) = (S_{11} + S_{12}(I - S_{22})^{-1} S_{21},\ L_1 + S_{12}(I - S_{22})^{-1} L_2, \\ H + \mathrm{Im}\{L_1^\dagger S_{12}(I - S_{22})^{-1} L_2\} + \mathrm{Im}\{L_2^\dagger S_{22}(I - S_{22})^{-1} L_1\})$$

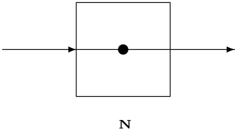
Reducible networks in quantum optics



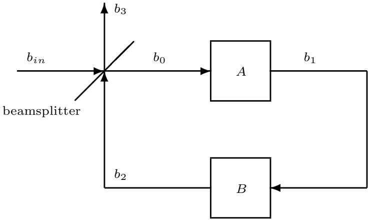
[Wiseman-Milburn, 1994]



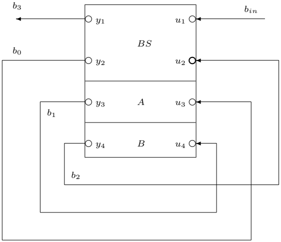
$$
\begin{aligned}
\mathbf{N} \quad &= \quad \mathbf{G}_1 \wedge \mathbf{G}_2 = (\mathbf{G}_{f2} \lhd \mathbf{G}_{f1}) \boxplus (\mathbf{G}_{d1} \bowtie \mathbf{G}_{d2}) \\
&= \quad (1, \sqrt{\gamma_2}\, a_2 + \sqrt{\gamma_1}\, a_1, \text{–}, \\
&\qquad \Delta_1 a_1^* a_1 + \Delta_2 a_2^* a_2 - ig(a_2 a_1^* - a_2^* a_1))
\end{aligned}
$$



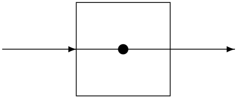
Non-reducible networks in quantum optics



[Yanagisawa-Kimura, 2003]



$$N = F\left((BS \boxplus A \boxplus B) \lhd T)\right)$$



Realistic detection

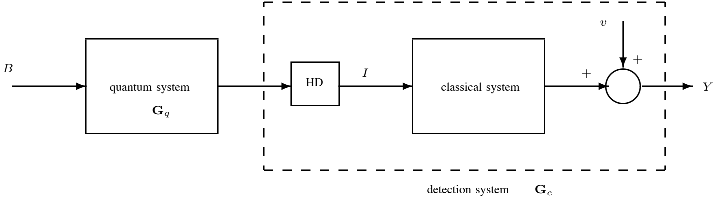
[Warszawski-Wiseman-Mabuchi, 2002]

The quantum system is given by

$$\mathbf{G}_q = (1, L_q, H_q),$$

and the classical detection system is given by the classical stochastic equations

$$\begin{aligned} dx(t) &= \tilde{f}(x(t))dt + g(x(t))dw(t), \\ dY(t) &= h(x(t))dt + dv(t), \end{aligned}$$



The classical system is equivalent to

$$\mathbf{G}_c = (1, L_{c1}, H_c) \boxplus (1, L_{c2}, 0)$$

where $L_{c1} = -ig^T p - \frac{1}{2}\nabla^T g$, $L_{c2} = \frac{1}{2}h$ and $H_c = \frac{1}{2}(f^T p + p^T f)$.

The complete cascade system is

$$\begin{aligned} \mathbf{G} &= ((1, L_{c1}, H_c) \triangleleft (1, L_q, H_q)) \boxplus (1, L_{c2}, 0) \\ &= (\mathbf{I}, \begin{pmatrix} L_1 + L_{c1} \\ L_{c2} \end{pmatrix}, H_q + H_c + \frac{1}{2i}(L_{c1}^* L_q - L_q^* L_{c1})) \end{aligned}$$

The unnormalized quantum filter for the cascade system is

$$\begin{aligned} d\sigma_t(X) &= \sigma_t(-i[X, H_q + H_c + \frac{1}{2i}(L_{c1}^* L_q - L_q^* L_{c1})] \\ &\quad + \mathcal{L}_{\begin{pmatrix} L_1 + L_{c1} \\ L_{c2} \end{pmatrix}}(X))dt + \sigma_t(L_{c2}^* X + X L_{c2})dy. \end{aligned}$$

For instance, $X = X_q \otimes \phi$, where ϕ is a smooth real valued function on $\mathbb{R}^n$.

Filtered estimate of quantum variables:

$$\pi_t(X_q) = \sigma_t(X_q)/\sigma_t(1)$$