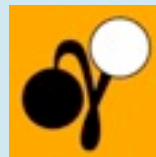


# Time-Dependent Calculations of Strong Field Ionization and High Harmonics Generation

**Anatoli Kheifets** and Igor Ivanov



Igor Bray



Centre for Antimatter-Matter Studies

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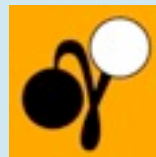
# Time-Dependent Calculations of Strong Field Ionization and High Harmonics Generation

Interplay between strong field effects  
and many-electron correlations

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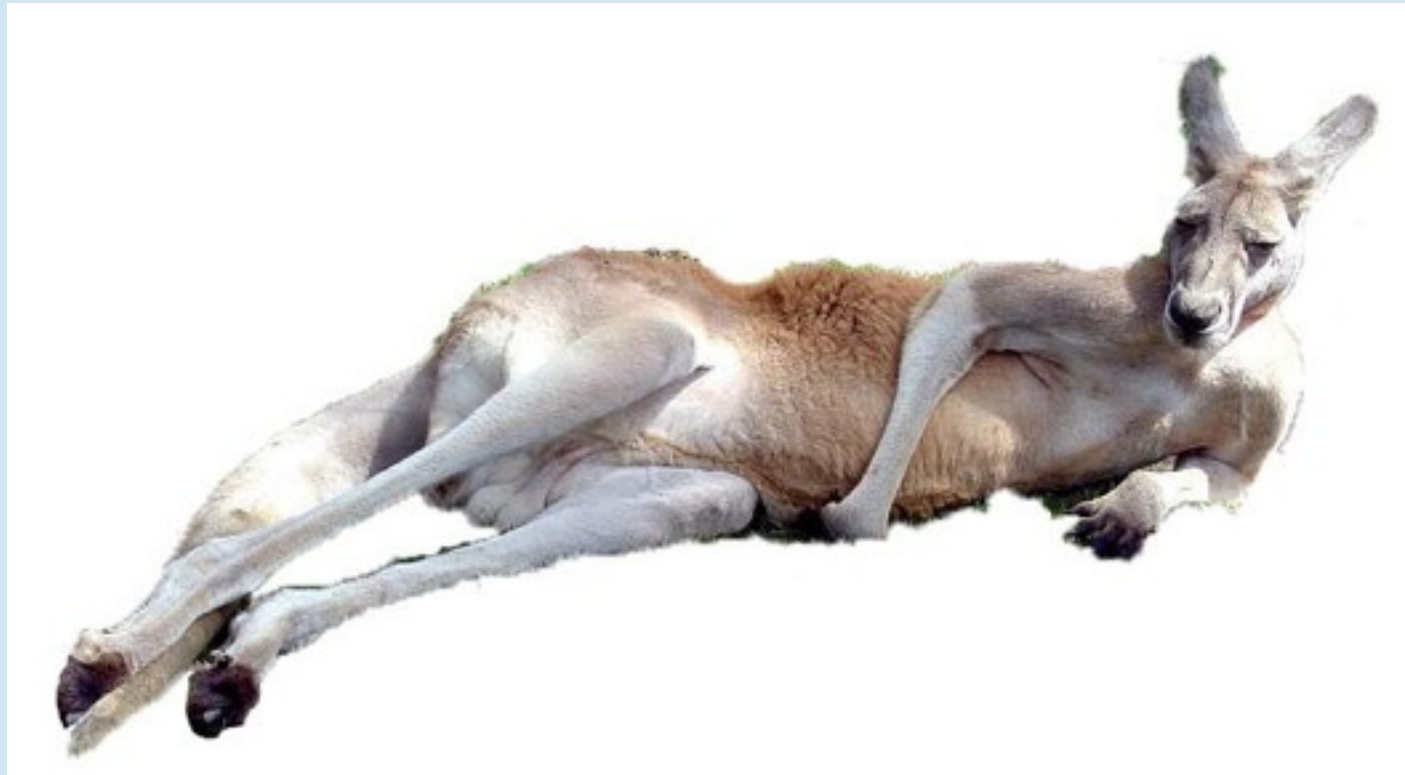


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# Time-Dependent Calculations of Strong Field Ionization and High Harmonics Generation

Interplay between strong field effects  
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# Motivation

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# Motivation

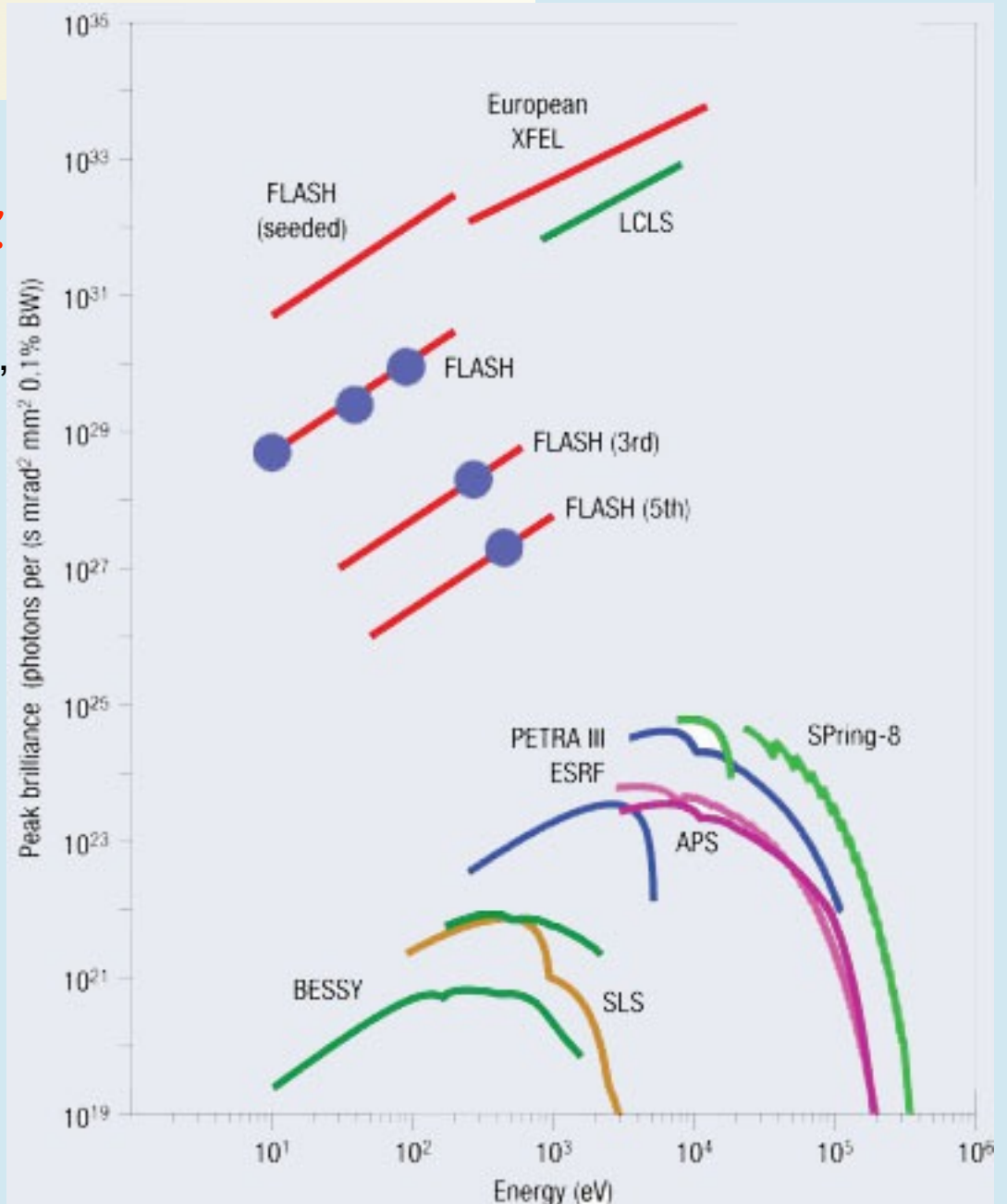
- Strong Field Ionization

# Motivation

- **Strong Field Ionization**
  - New XUV sources
    - FEL's (FLASH, SCSS, LCLS etc)

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    - FEL's (FLASH, SCSS, LCLS etc)
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  - Fundamental importance
    - Interplay of correlations and strong field

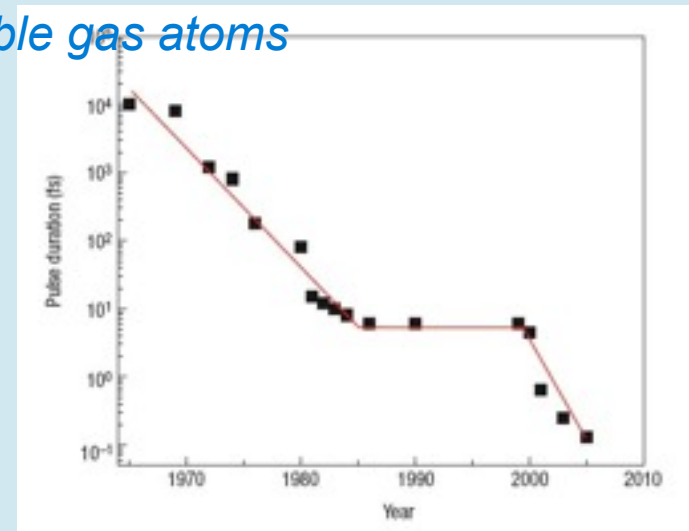
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  - Race for the “table-top synchrotron”
    - *Coherent, compact, intense, tunable, source of XUV radiation*
    - *Neutral gases or ionized plasmas of noble gas atoms*

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  - *Steering of coherent wave packets.*
  - *Time-resolved atomic physics*

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# Time-dependent theory road map



*Number of cycles*



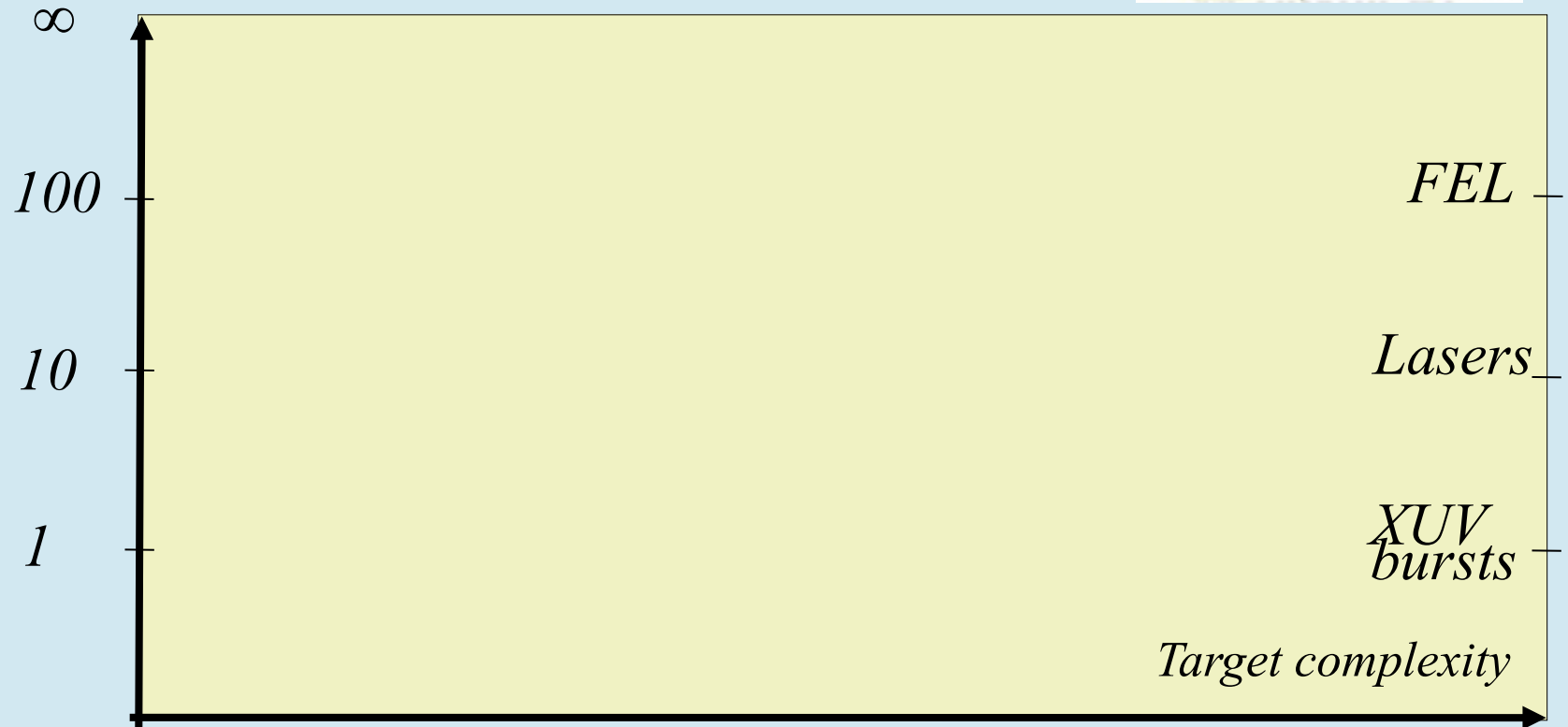
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# Time-dependent theory road map



*Number of cycles*



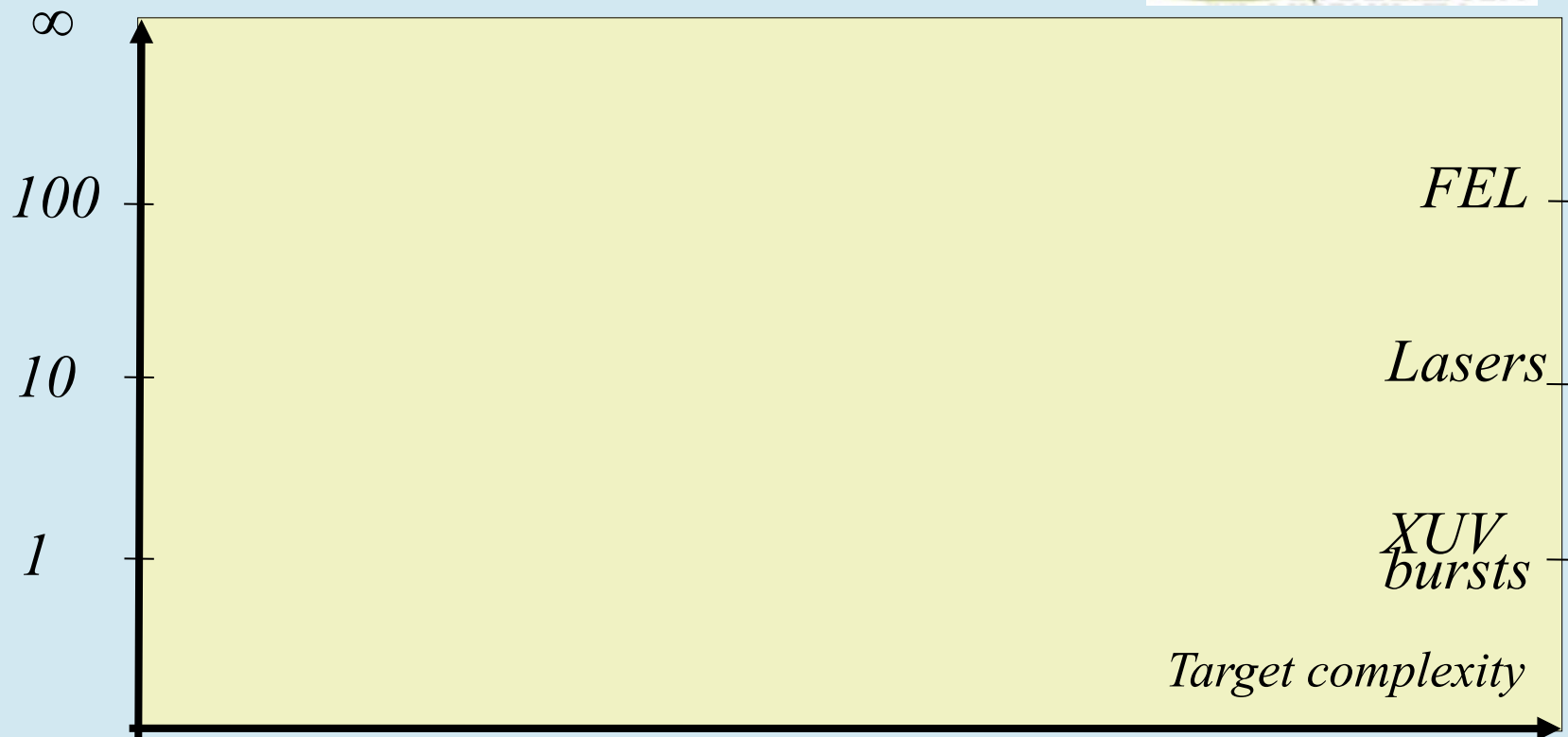
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# Time-dependent theory road map



*Number of cycles*



*One-electron targets*  
*Li, K, Rb*

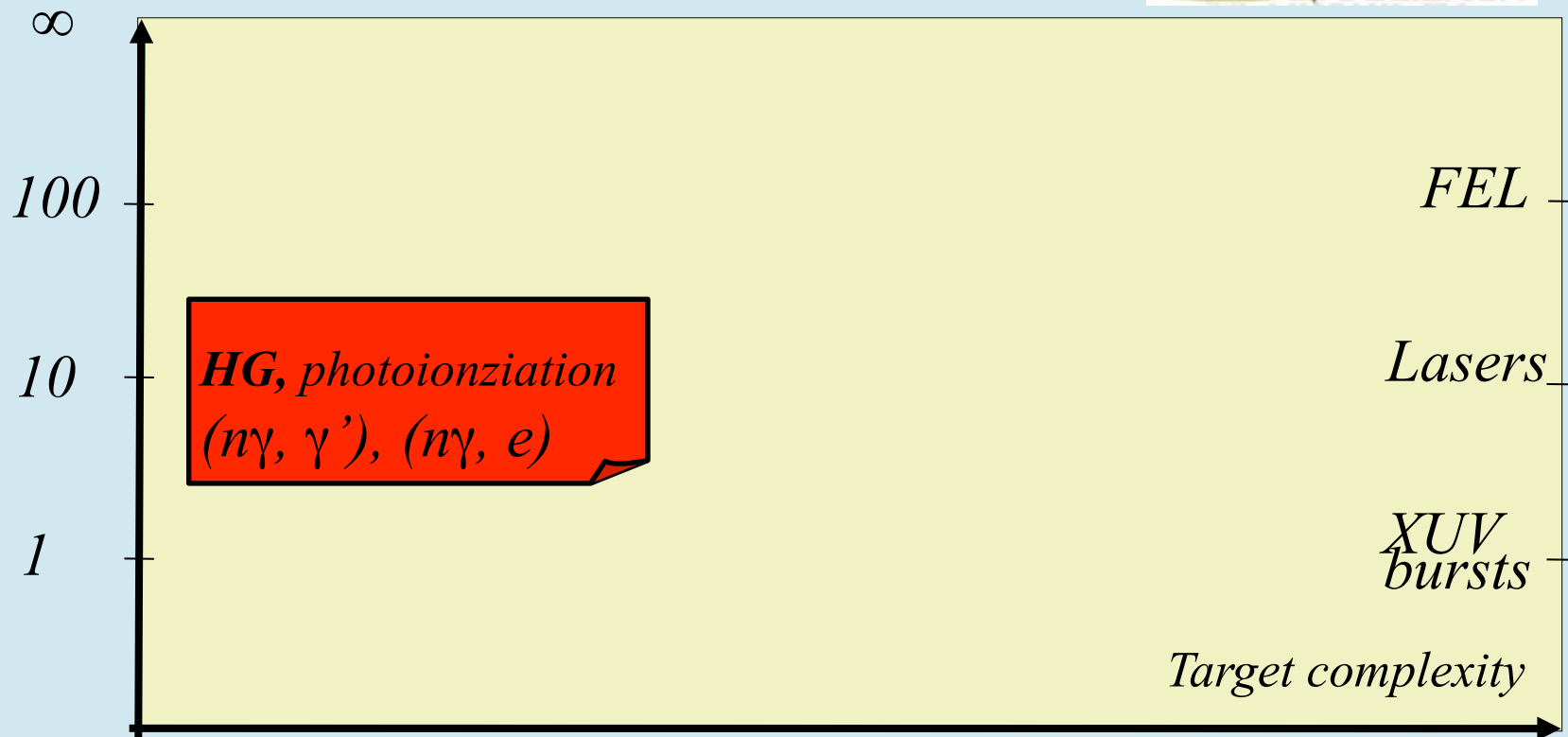
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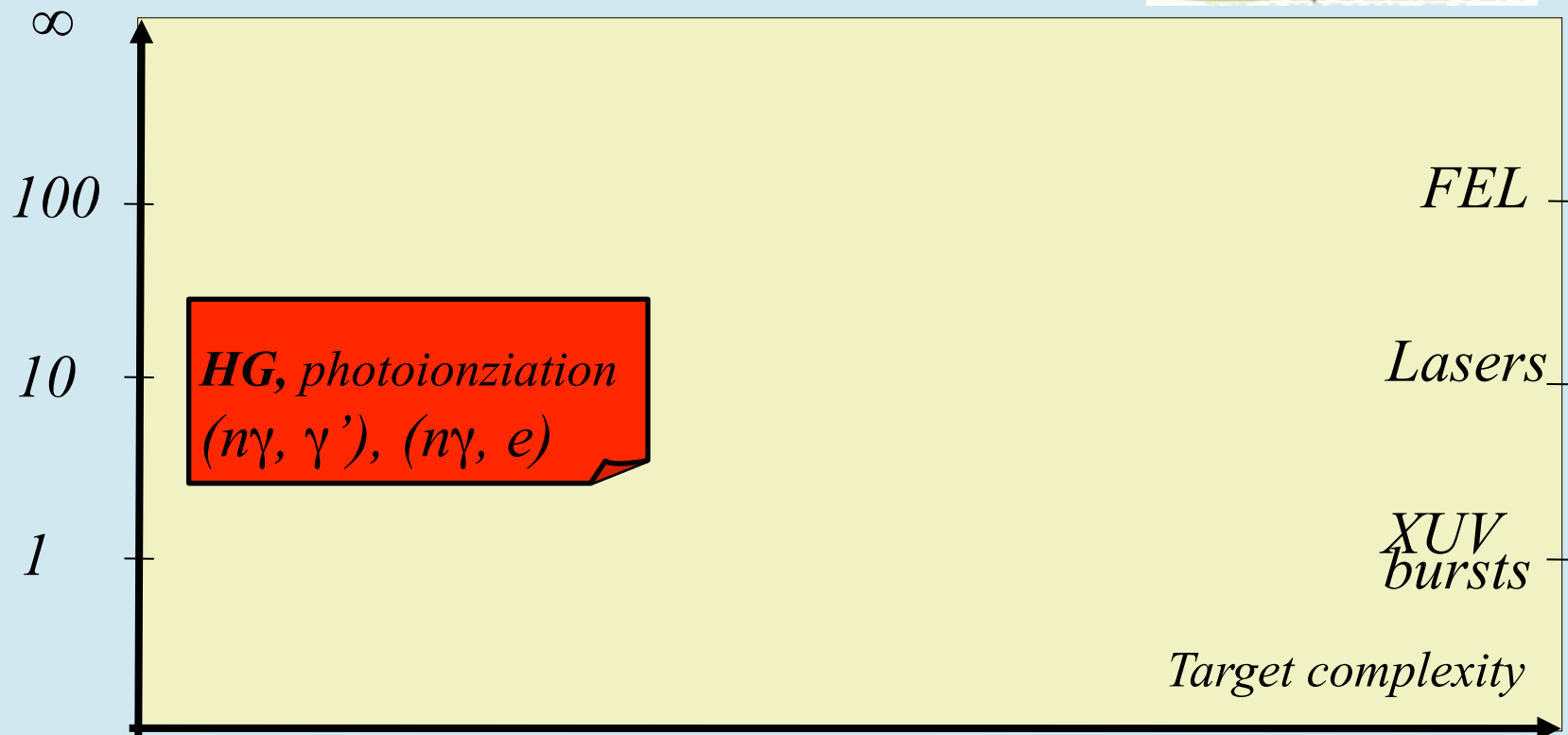
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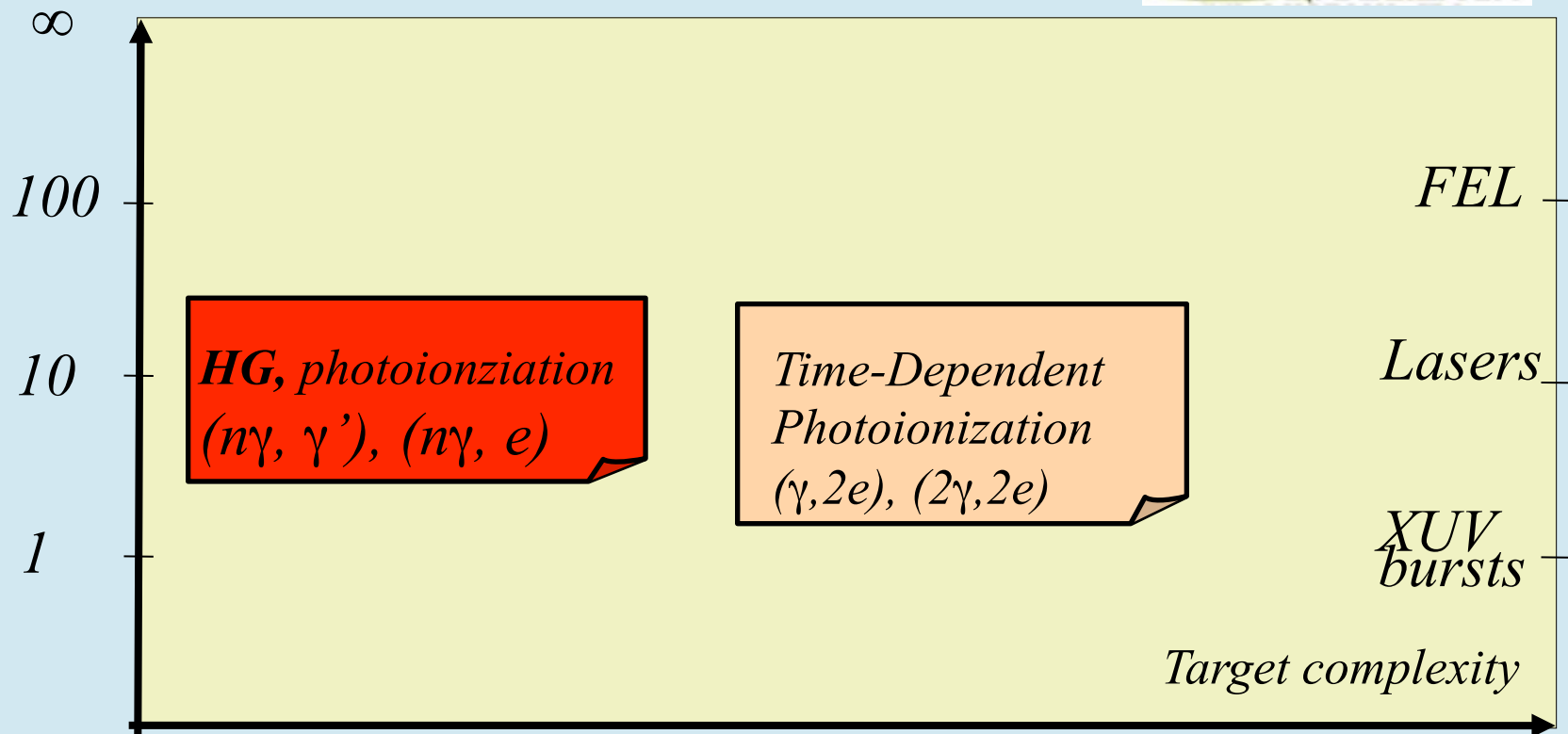
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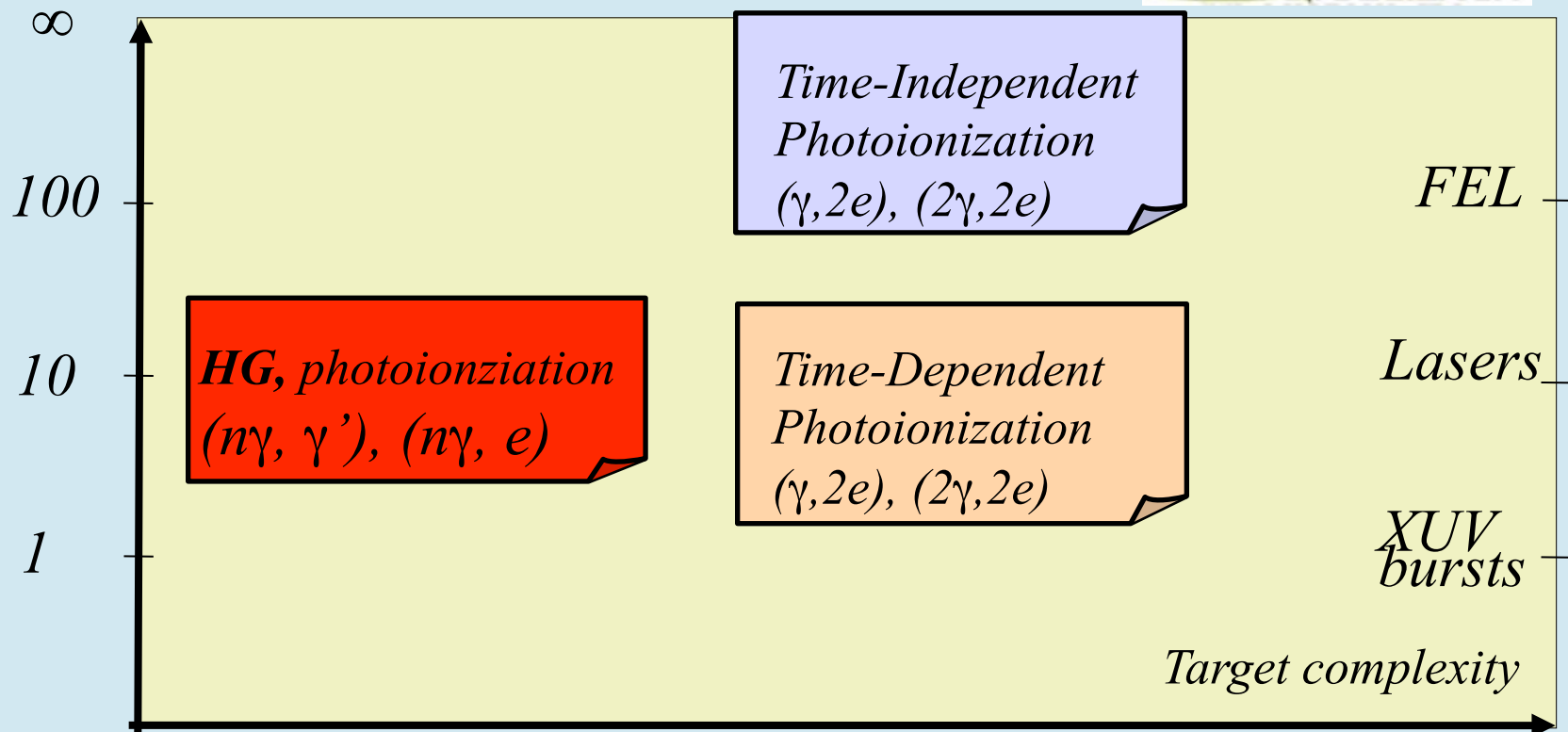
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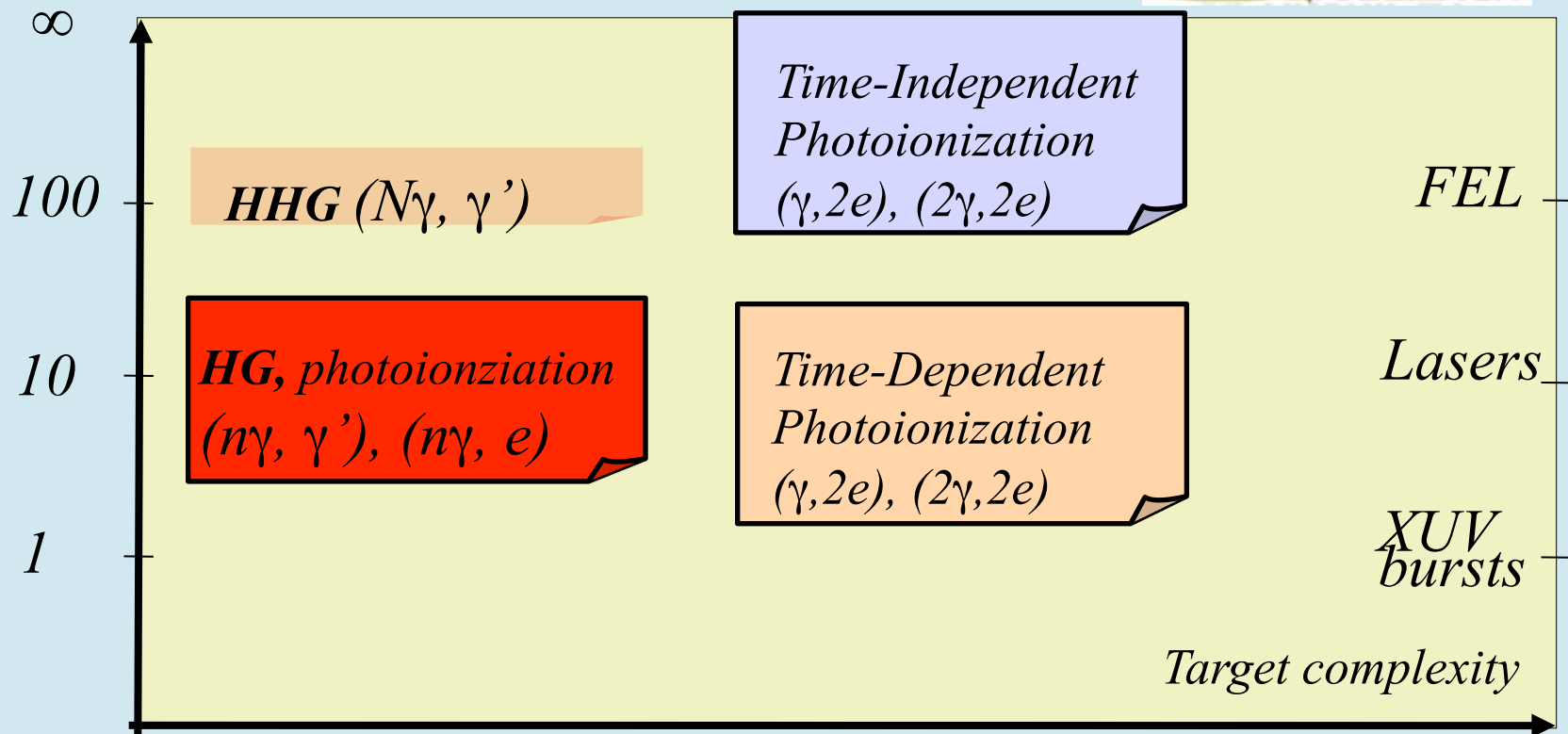
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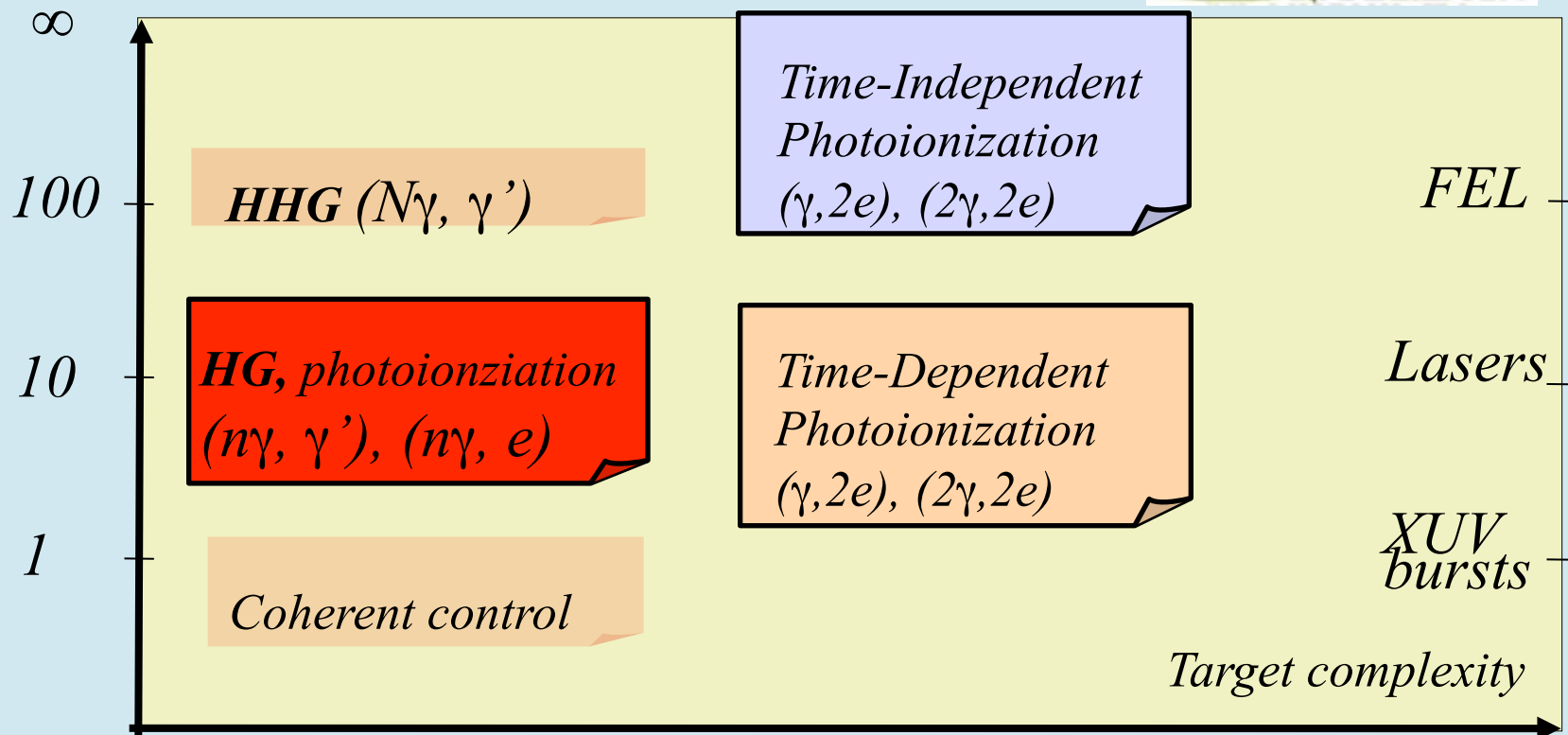
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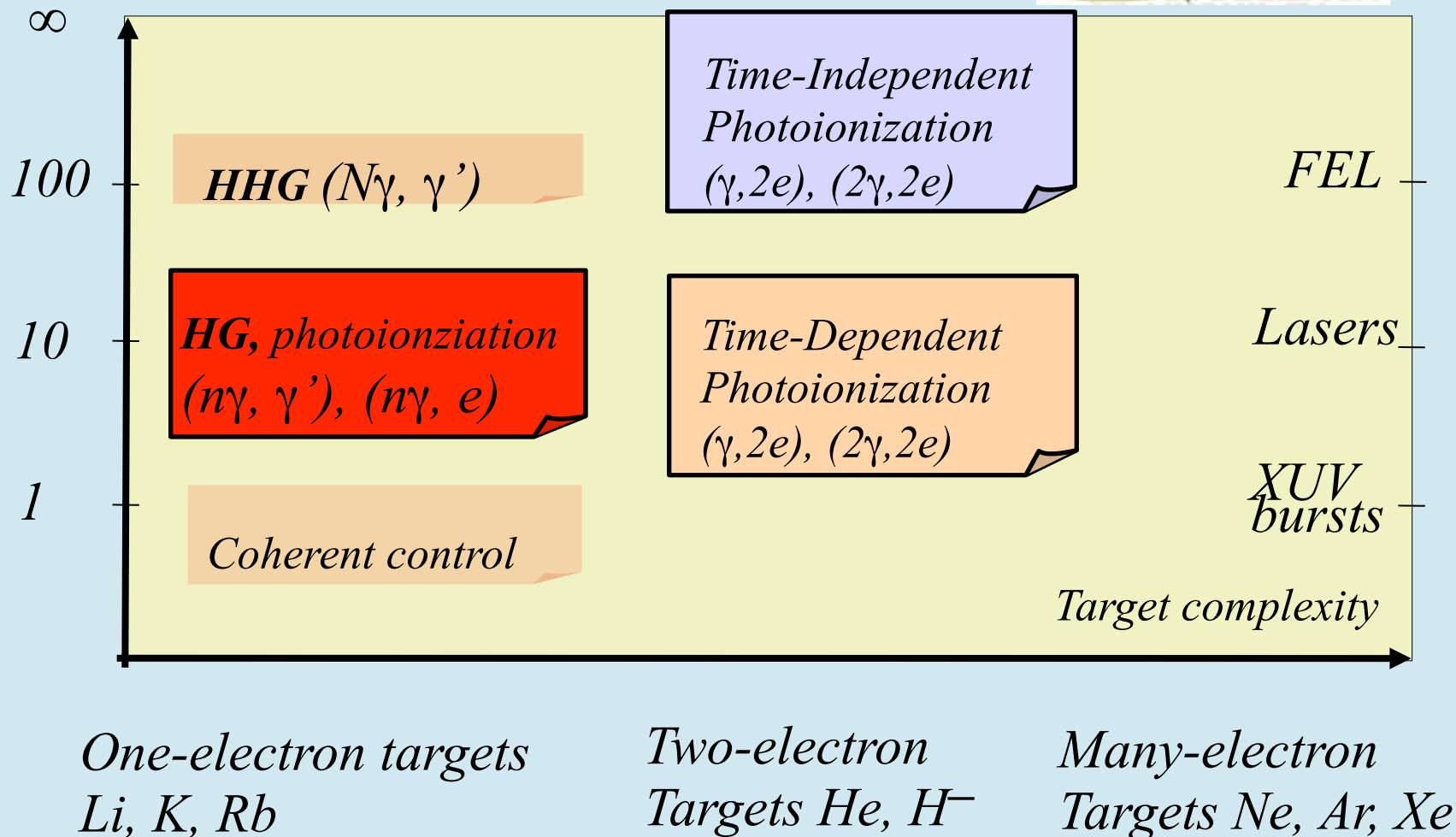
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# Time-dependent theory road map



*Number of cycles*



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# Outline

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# Outline

- Theoretical model
  - Time-dependent Schrödinger equation

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# Outline

- **Theoretical model**
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    - *One-electron HF states*
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  - Discretization of continuum
    - *Pseudostates vs continuum states*

# Outline

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# Outline

- Applications

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# Outline

- Applications
  - Multi-photon ionization

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# Outline

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- *Optimal control*

# Theoretical Model

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# Theoretical Model

Field on:



# Theoretical Model



Field on:

Time-Dependent Schrödinger equation  $i\frac{\partial\Psi}{\partial t} = \hat{H}\Psi$

$$\hat{H} = \frac{\mathbf{p}_1^2}{2} + \frac{\mathbf{p}_2^2}{2} - \frac{2}{r_1} - \frac{2}{r_2} + \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} + D \cdot F_{AC} \cos \omega t$$

# Theoretical Model



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Solution:

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, t) = \sum_{j \equiv \left\{ \begin{array}{c} n_1 n_2 N \\ l_1 l_2 L \end{array} \right\}} a_j(t) \phi_{n_1 l_1}^N(\mathbf{r}_1) \phi_{n_2 l_2}^N(\mathbf{r}_2) |l_1(1)l_2(2)L\rangle$$

Pseudostate basis:  $\langle \phi_i^N | \hat{H} | \phi_j^N \rangle = E_i \delta_{ij}$

# Theoretical Model



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Matrix notations:

$$i\mathbf{R}\cdot\dot{\mathbf{a}}(t) = \mathbf{H}\cdot\mathbf{a}(t), \quad \text{with initial condition } \Psi(\mathbf{r}_1, \mathbf{r}_2, t=0) = \Psi_0(\mathbf{r}_1, \mathbf{r}_2)$$

# Theoretical Model

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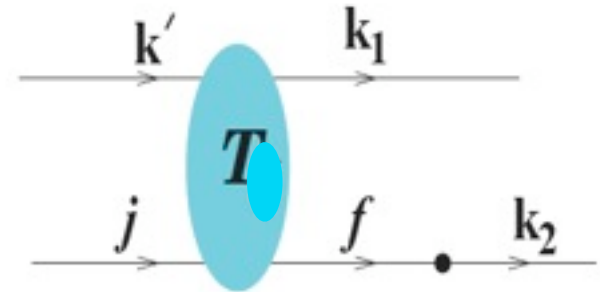
# Theoretical Model

Field off:

# Theoretical Model

Field off:

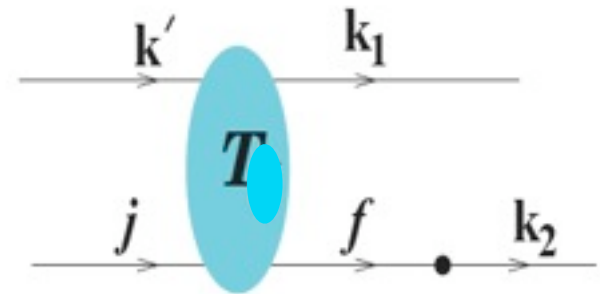
CCC expansion for two-electron continuum



# Theoretical Model

Field off:

CCC expansion for two-electron continuum



$$\Psi_{k_1 k_2}(\mathbf{r}_1, \mathbf{r}_2) = \Psi_{k_1 f}(\mathbf{r}_1, \mathbf{r}_2) \langle \mathbf{k}_2 | f \rangle, \quad \epsilon_f = k_2^2/2$$

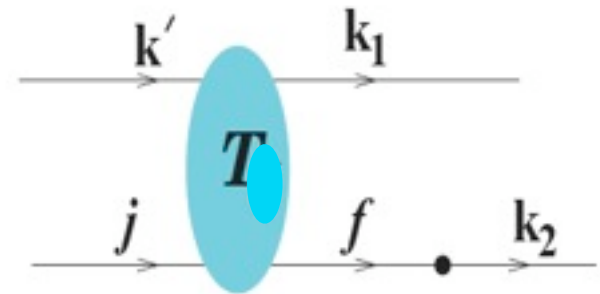
$$\Psi_{k f}(\mathbf{r}_1, \mathbf{r}_2) = \chi_{\mathbf{k}}(\mathbf{r}_1) \phi_f(\mathbf{r}_2) + \sum_{\mathbf{k}' j} \frac{\langle \mathbf{k} f | T_J | \mathbf{k}' j \rangle}{E - k'^2/2 - \epsilon_j + i0} \chi_{\mathbf{k}'}(\mathbf{r}_1) \phi_j(\mathbf{r}_2)$$



# Theoretical Model

## Field off:

CCC expansion for two-electron continuum



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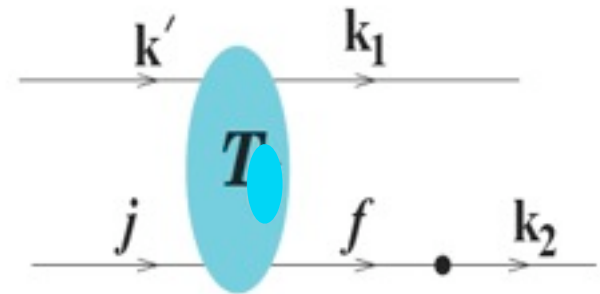
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Coulomb wave

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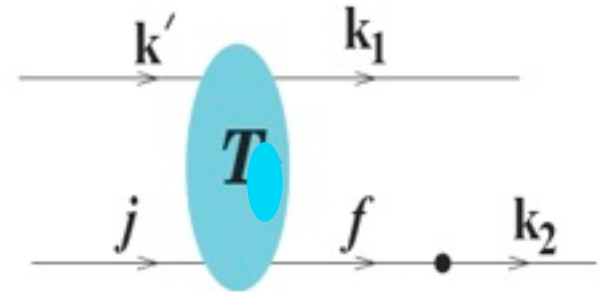
Coulomb wave

Pseudostate

# Theoretical Model

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Coulomb wave

Pseudostate

Fully differential cross-section

$$\frac{d\sigma}{d\mathbf{k}_1 d\mathbf{k}_2} = |\langle \Psi(\mathbf{r}_1, \mathbf{r}_2, t = T) | \Psi_{k_1 k_2}(\mathbf{r}_1, \mathbf{r}_2) \rangle|^2 \quad T = N \frac{2\pi}{\omega}, \quad N \gg 1$$

# Theoretical Model

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# Theoretical Model

Harmonics Generation:

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# Theoretical Model

## Harmonics Generation:

Harmonics spectrum

$$|d(\omega)|^2 = \left| \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} e^{-i\omega t} d(t) dt \right|^2$$

# Theoretical Model

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Expectation value of dipole operator

$$\begin{aligned} d(t) &= \langle \Psi(t) | z | \Psi(t) \rangle = \int d\mathbf{r} dt \Psi(\mathbf{r}, t) (\mathbf{r} \cdot \mathbf{e}) \Psi(\mathbf{r}, t) \\ &= \sum_{n,m} z_{mn} \exp[-i(m-n)\omega t] = \sum_N z_N \exp[-i(2M+1)\omega t] \end{aligned}$$

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## Harmonics Generation:

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Parity conservation  $\Pi = -1 = (-1)^{2M+1}$

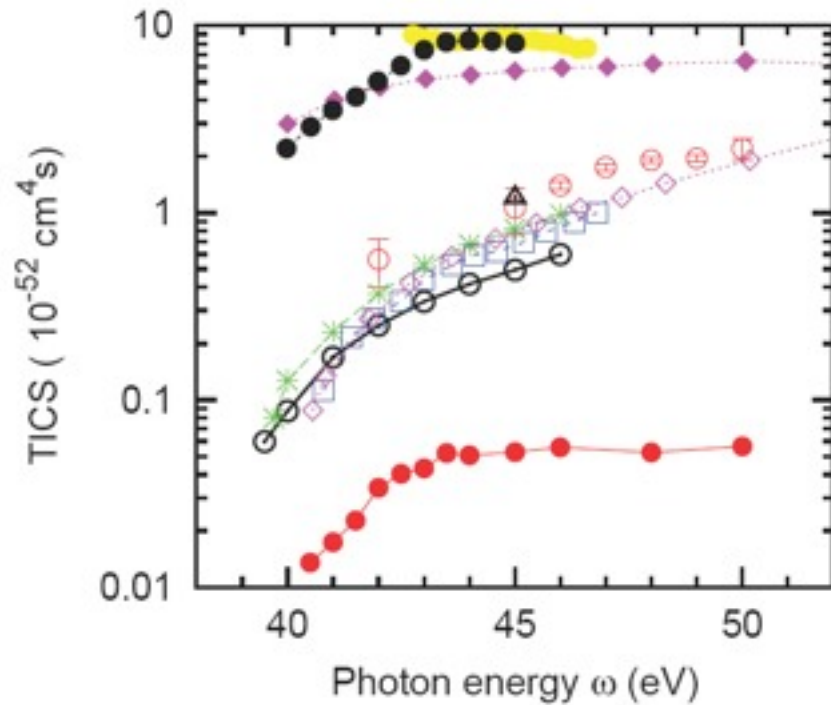
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# Integrated TPDI Cross-section of He



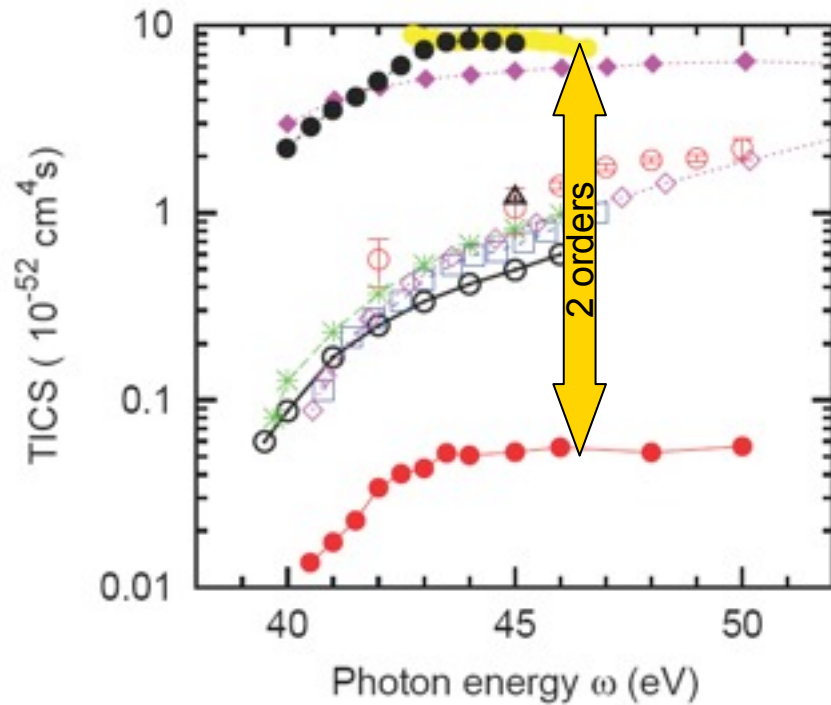
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TDSE x CCC  $\circ$   
Closure x CCC  $\bullet$

$3.5 \times 10^{14} \text{ W/cm}^2$

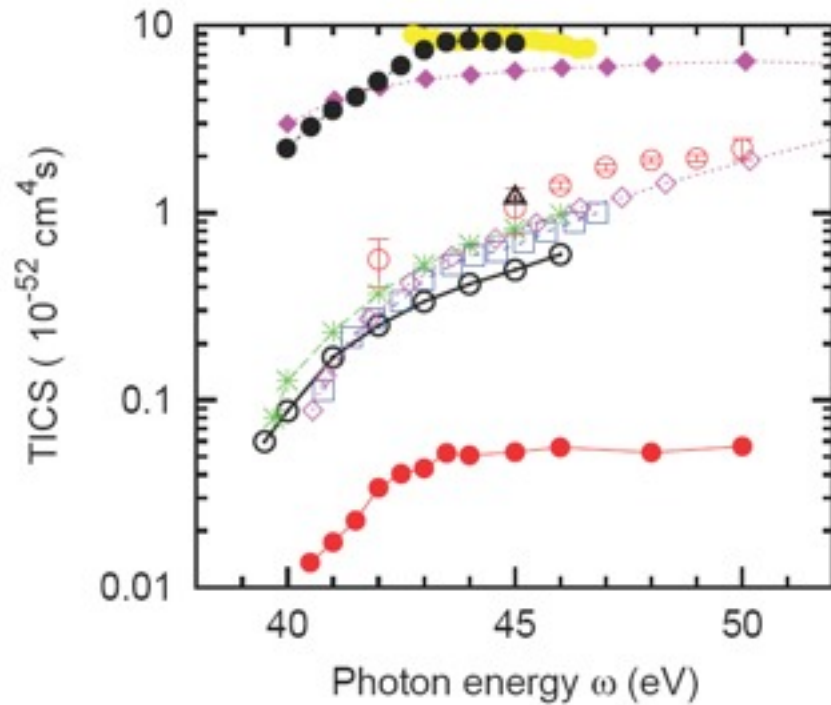
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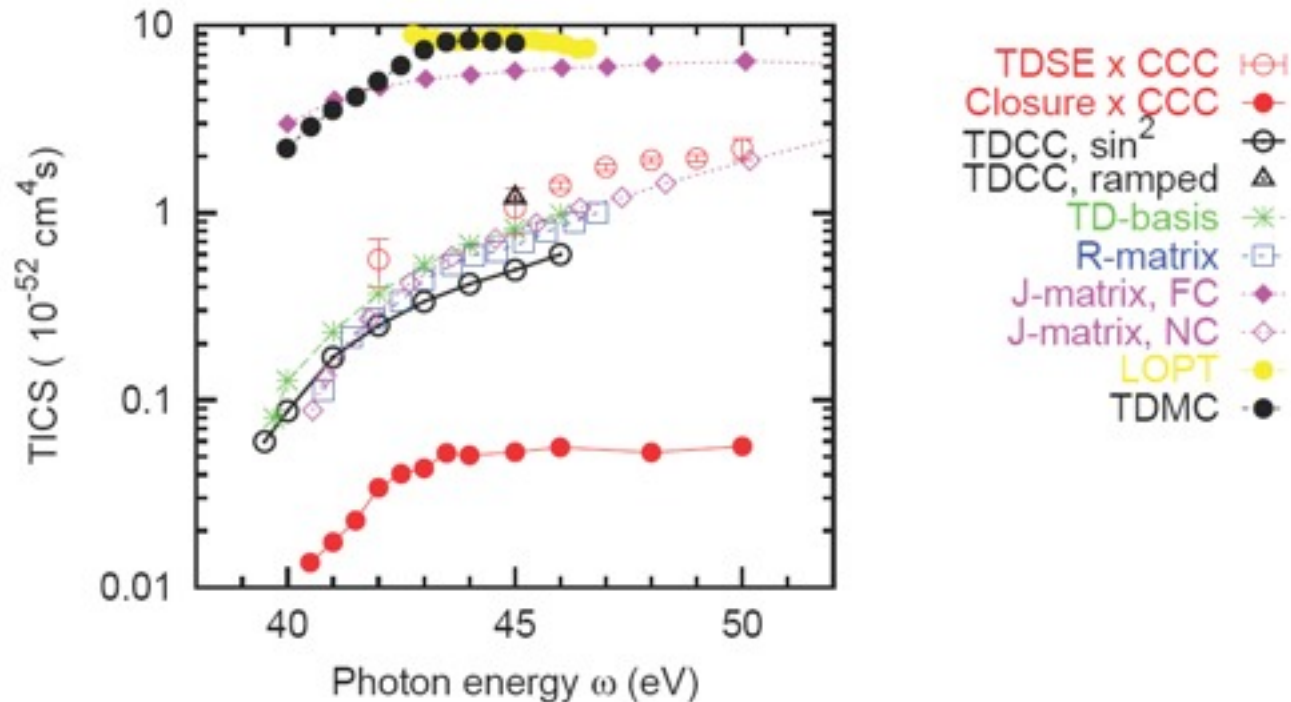
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TDSE x CCC (○)  
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# Integrated TPDI Cross-section of He



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TDCC $\sin^2$	Hu S X, Colgan J and Collins L A 2005 <i>J. Phys. B</i> <b>38</b> , L35
TDCC ramped	Colgan J and Pindzola L A 2002 <i>Phys. Rev. Lett.</i> <b>88</b> , 173002
TD-basis	Piroux B, Bauer J, Laulan S and Bachau H 2003 <i>Eur. Phys. J. D</i> <b>26</b> , 7
R-matrix	Feng L and van der Hart H W 2003 <i>J. Phys. B</i> <b>36</b> , L1
J-matrix	Foumouo et al. 2006 <i>Phys. Rev. A</i> <b>74</b> , 063409
LOPT	Nikolopoulos and Lambropoulos 2001 <i>J. Phys. B</i> <b>34</b> , 545
TDMC	Nikolopoulos and Lambropoulos 2007 <i>J. Phys. B</i> <b>40</b> , 1347

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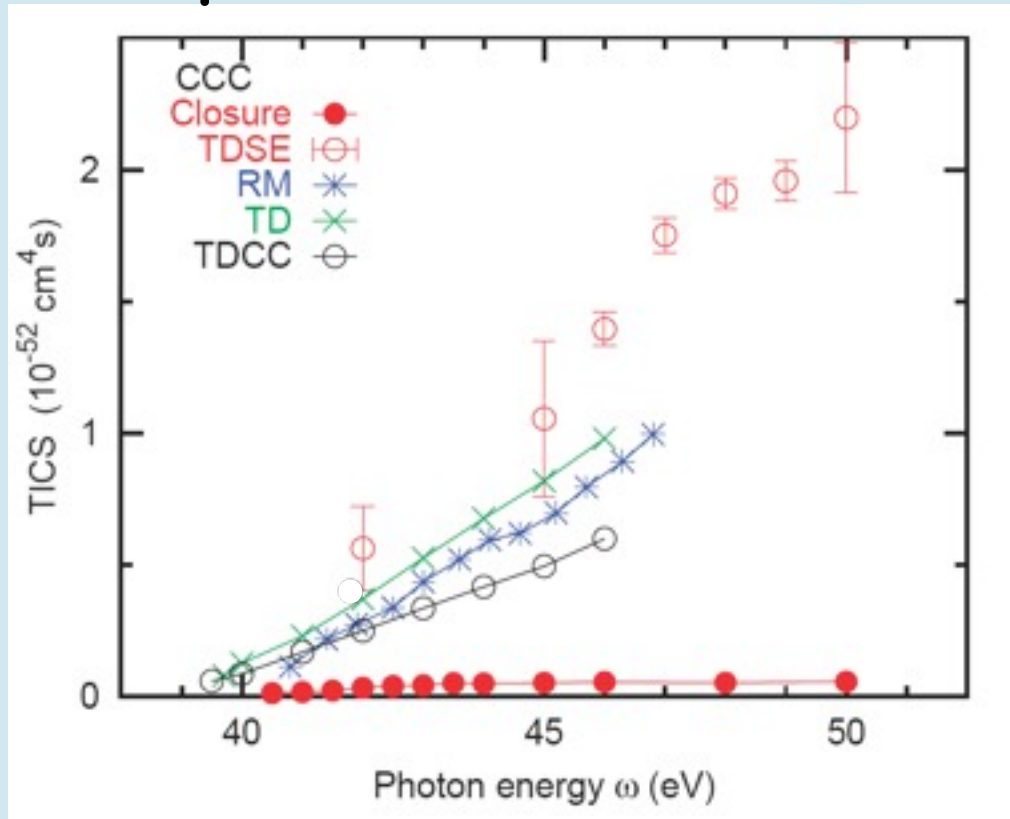
Theory vs. experiment:



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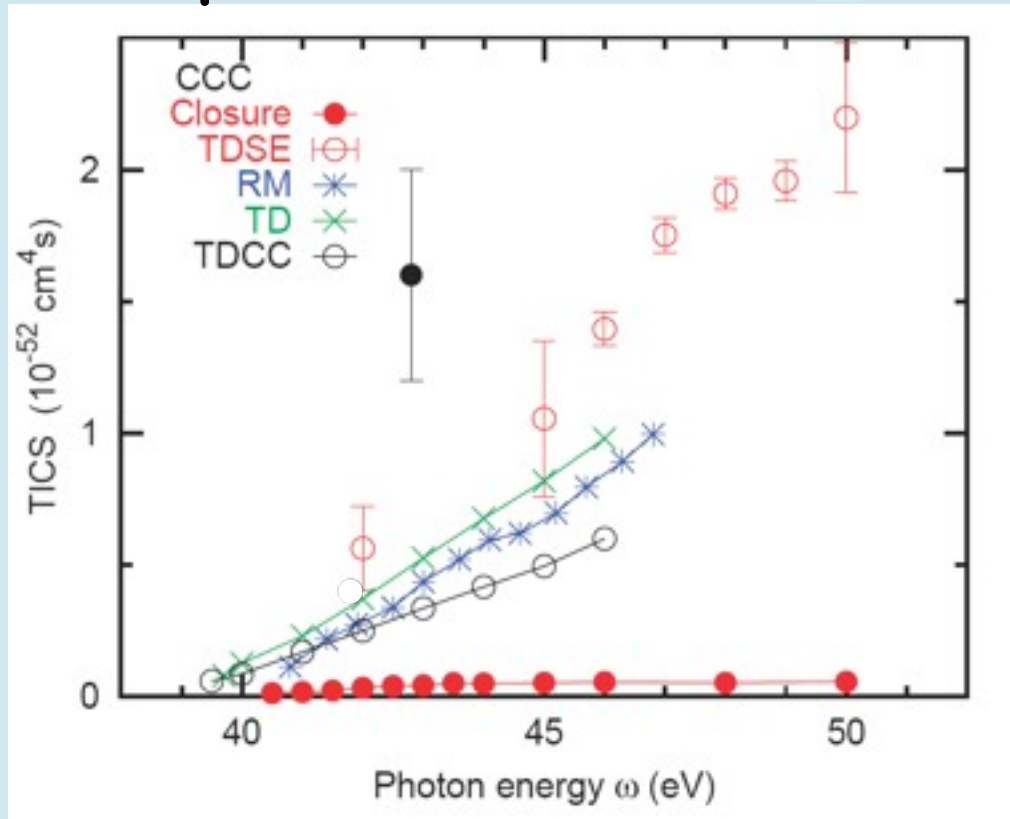
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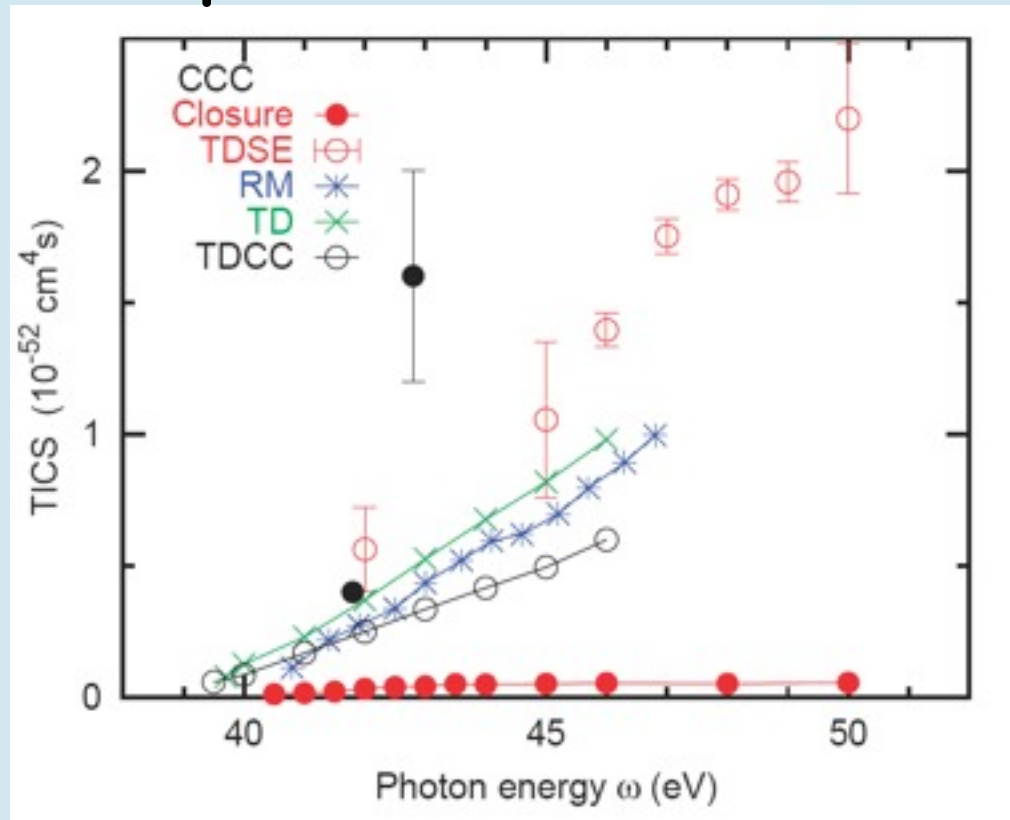
FEL @ 42.8 eV     $1.6 \pm 0.6 \times 10^{-52} \text{ cm}^4 \text{ s}$     Sorokin et al. 2007 PRA 75, 051402

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# Integrated TPDI Cross-section of He

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FEL @ 42.8 eV  $1.6 \pm 0.6 \times 10^{-52} \text{ cm}^4 \text{ s}$   
HHG @ 41.8 eV  $0.4 \times 10^{-52} \text{ cm}^4 \text{ s}$

Sorokin et al. 2007 PRA 75, 051402  
Hasegawa et al. 2005 PRA 71, 023407

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# Lithium atom in intense laser field



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Michael Schurike, Max-Planck-Institut für  
Kernphysik, Heidelberg, ICPEAC 2009

# Lithium atom in intense laser field

	1st IP [eV]	$I_{\text{OBI}}$ [W/cm <sup>2</sup> ]	$\gamma$	$U_p$ [eV]
Li(2s)	5.39	$3.38 \cdot 10^{12}$	5.29	0.20
Li(2p)	3.54	$6.28 \cdot 10^{11}$	9.76	0.037
He	24.59	$1.46 \cdot 10^{15}$	0.53	86.1
Ne	21.57	$8.65 \cdot 10^{14}$	0.65	50.5
Ar	15.76	$2.47 \cdot 10^{14}$	1.0	14.6
Xe	12.13	$8.66 \cdot 10^{13}$	1.5	5.11

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Michael Schurike, Max-Planck-Institut für  
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Low over-the-barrier intensities as compared to noble gases, absence of ponderomotive effects.

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Low over-the-barrier intensities as compared to noble gases, absence of ponderomotive effects.

High Keldysh-parameter  $\gamma$ , i.e. deep within multiphoton regime, but not perturbative.

# Lithium atom in intense laser field

	1st IP [eV]	$I_{\text{OBI}}$ [W/cm <sup>2</sup> ]	$\gamma$	$U_p$ [eV]
Li(2s)	5.39	$3.38 \cdot 10^{12}$	5.29	0.20
Li(2p)	3.54	$6.28 \cdot 10^{11}$	9.76	0.037
He	24.59	$1.46 \cdot 10^{15}$	0.53	86.1
Ne	21.57	$8.65 \cdot 10^{14}$	0.65	50.5
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Only 4 (2s) / 3 (2p) Ti:Sa-Photons required to ionize. Low non-linearity ( $\sigma \propto I^N$ ).

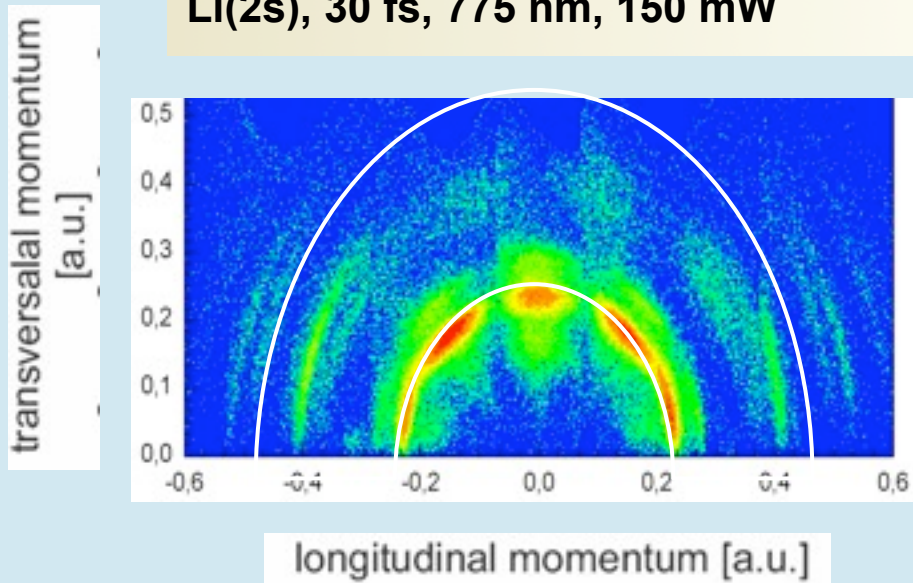
# Photoelectron momentum distribution

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**Experiment:**

**Li(2s), 30 fs, 775 nm, 150 mW**



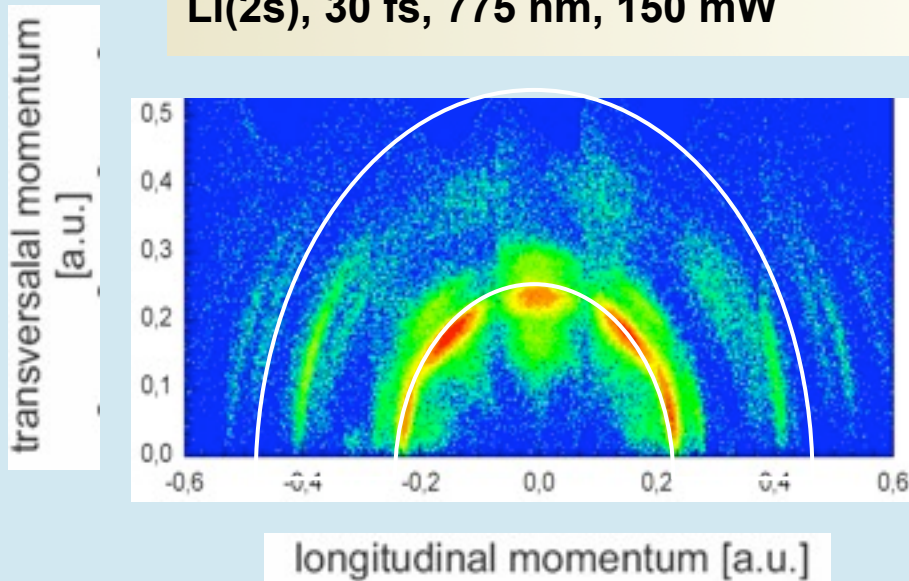
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**Momentum space multi-photon rings:**

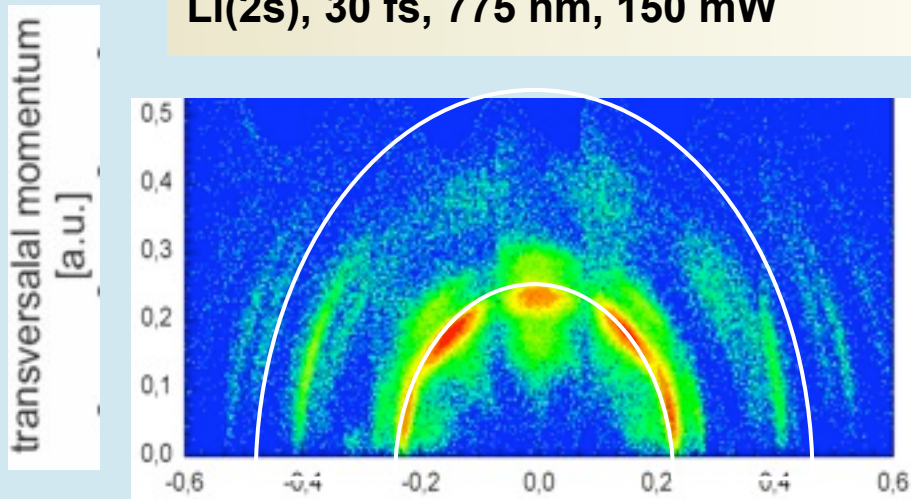
$$P_x^2 + P_z^2 = N\omega - IP, \quad N \geq 4 \text{ for Li } 2s$$

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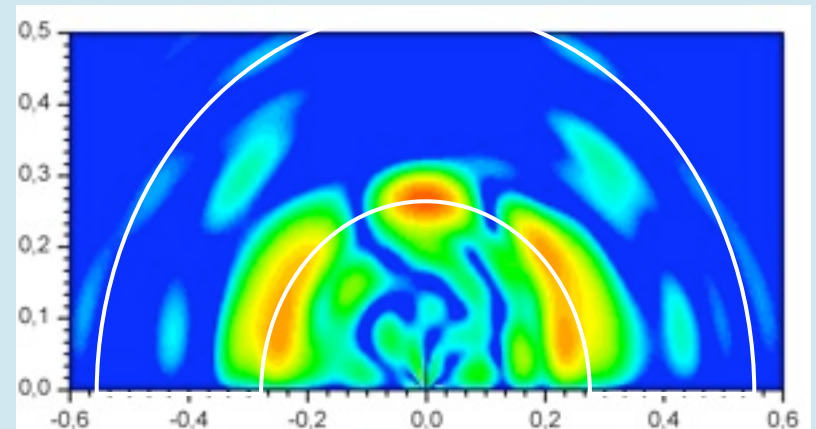
Li(2s), 30 fs, 775 nm, 150 mW



longitudinal momentum [a.u.]

**Calculation:**

Li(2s), 25 fs, 800 nm,  $3 \times 10^{11}$  W/cm<sup>2</sup>



longitudinal momentum [a.u.]

**Momentum space multi-photon rings:**

$$P_x^2 + P_z^2 = N\omega - IP, \quad N \geq 4 \text{ for Li } 2s$$

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# Resonant Enhancement of Harmonics Generation

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**Enhanced High Harmonic Generation from an Optically Prepared Excited Medium**

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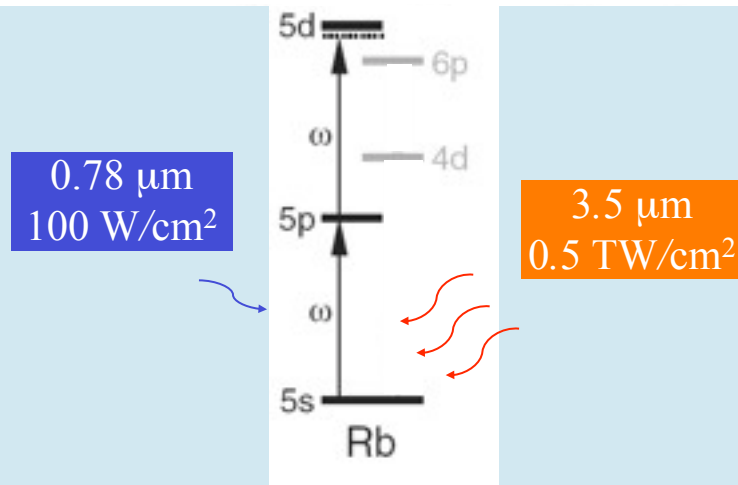
## Enhanced High Harmonic Generation from an Optically Prepared Excited Medium

We investigate high harmonics generated from rubidium atoms irradiated simultaneously by an intense  $3.5\ \mu\text{m}$  fundamental field and a weak cw diode laser. When  $5p$ ,  $5d$ , and  $4d$  excited states are populated through cascade excitation or deexcitation, orders-of-magnitude increases in harmonic yield as compared with the ground state are observed.

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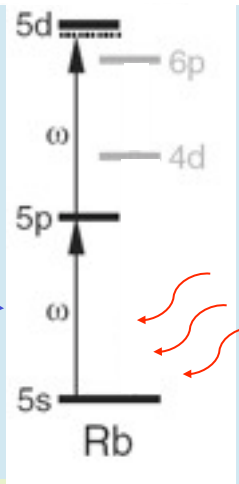
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$0.78 \mu\text{m}$   
 $100 \text{ W/cm}^2$



$3.5 \mu\text{m}$   
 $0.5 \text{ TW/cm}^2$

Scaled approach to HHG:

$$N_{\text{cut-off}} = (lp + 3.17Up)/\omega \gg 1$$

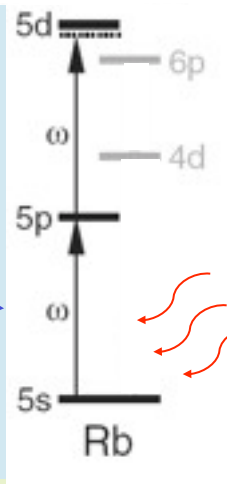
$$\gamma = (lp/Up)^{1/2} \approx 1$$

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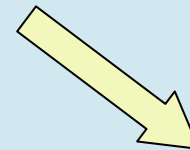
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From noble gases in NIR



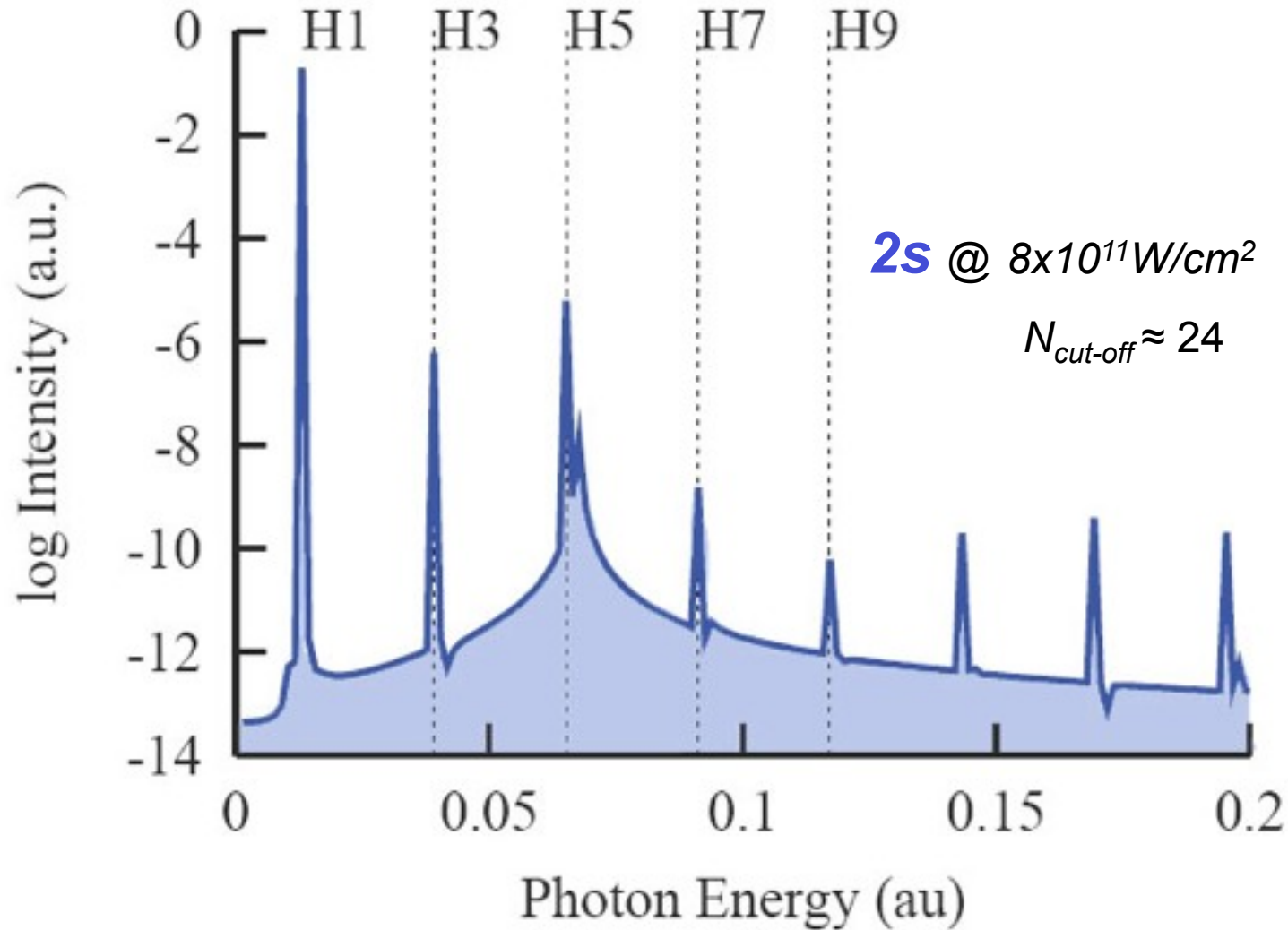
To alkaline metals in MIR

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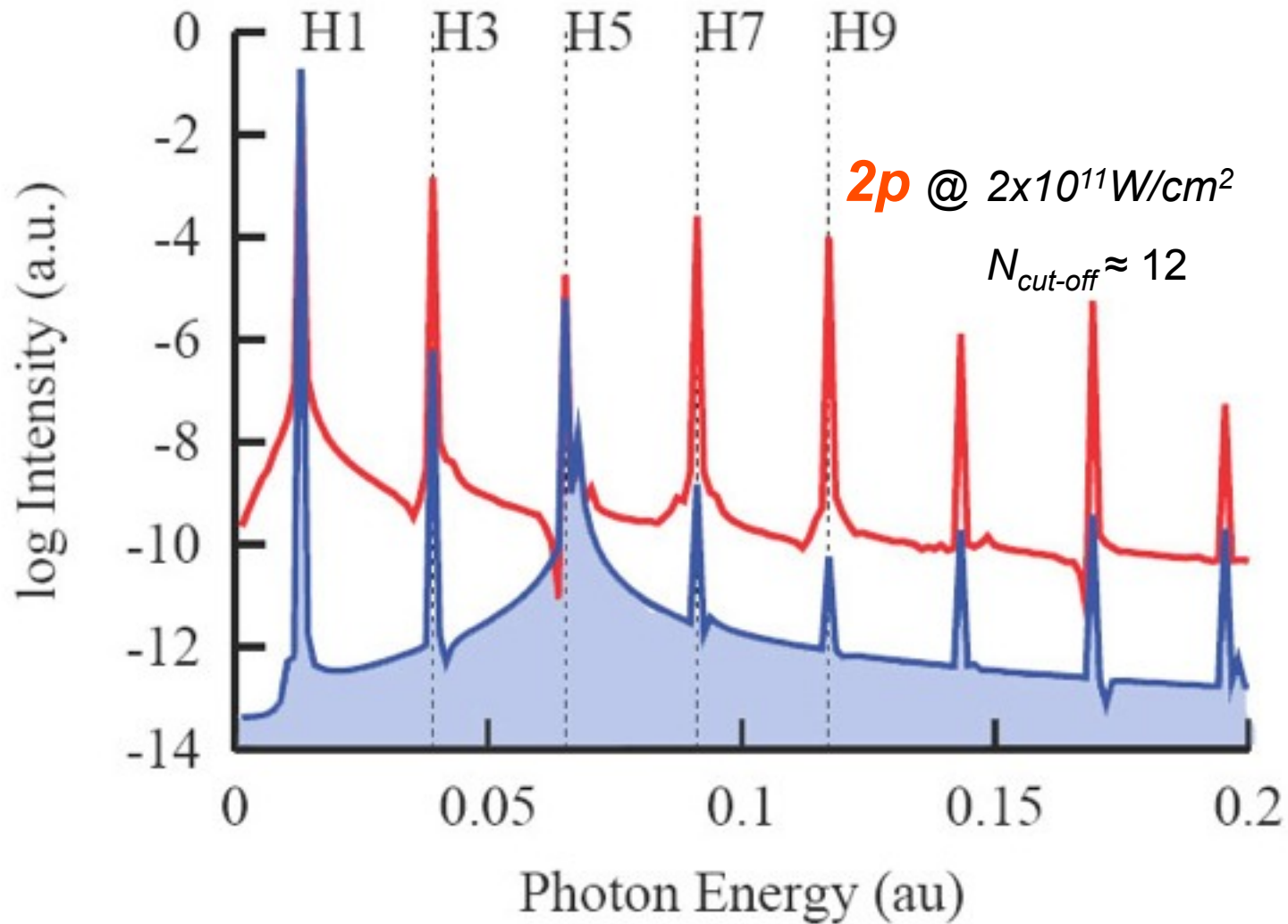


# Harmonic spectrum of Li @ 330 fs, 3500 nm

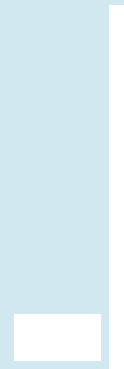
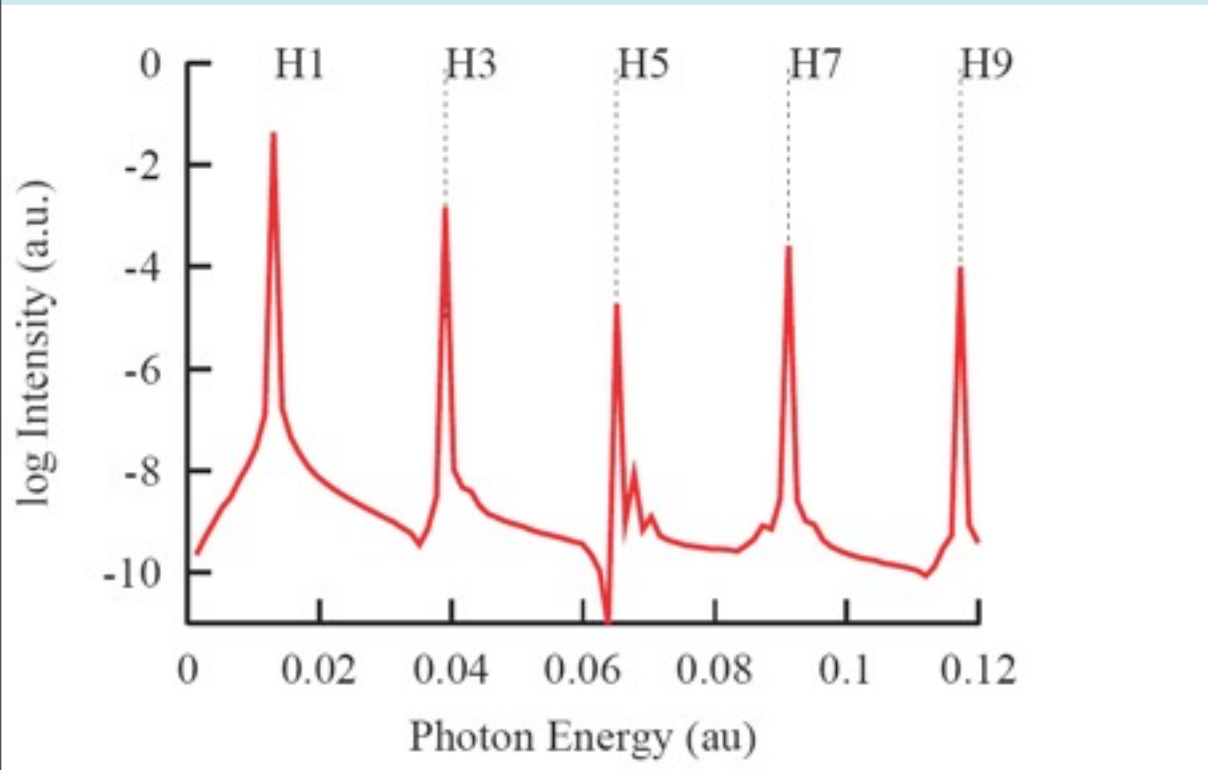
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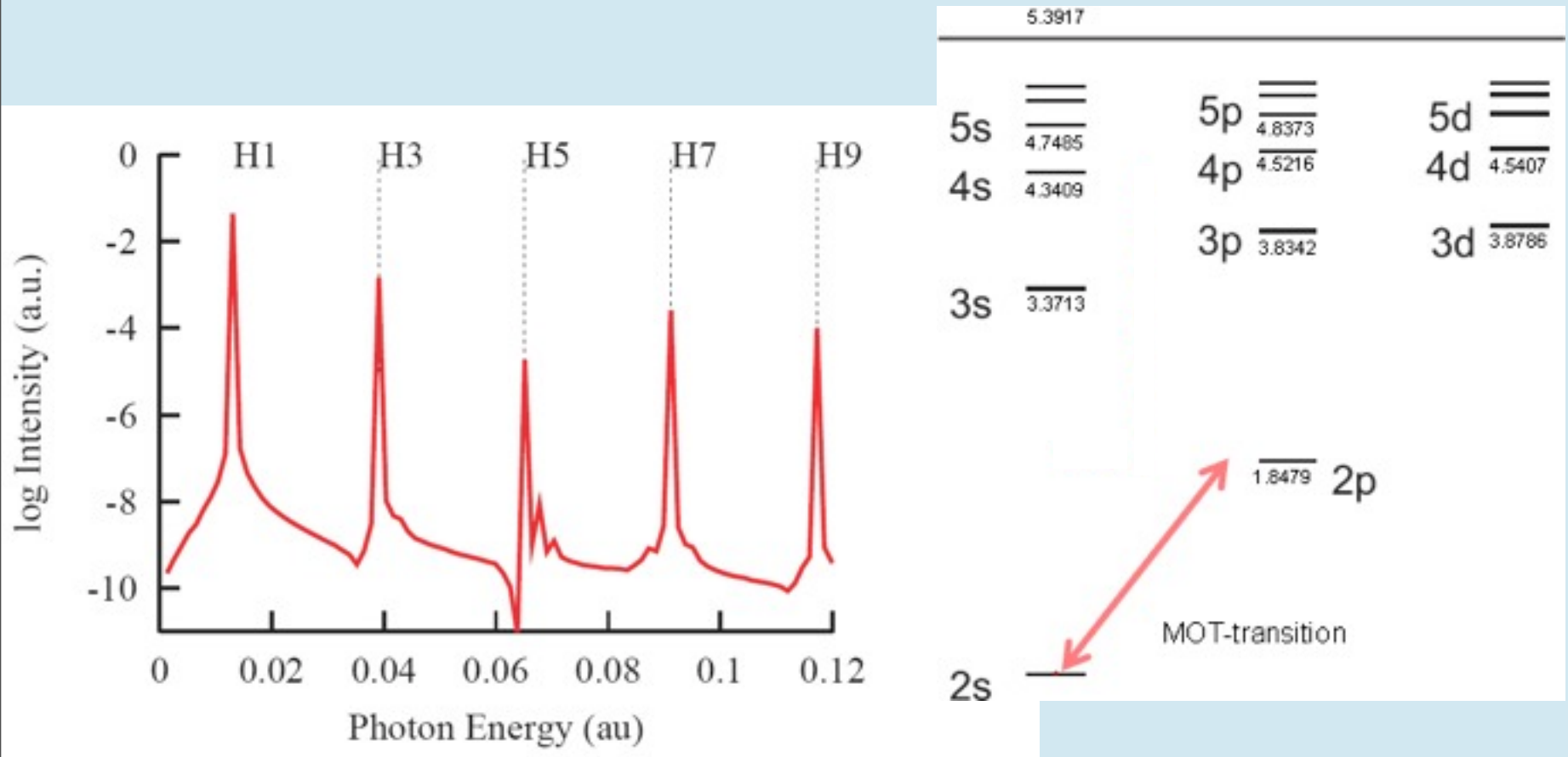


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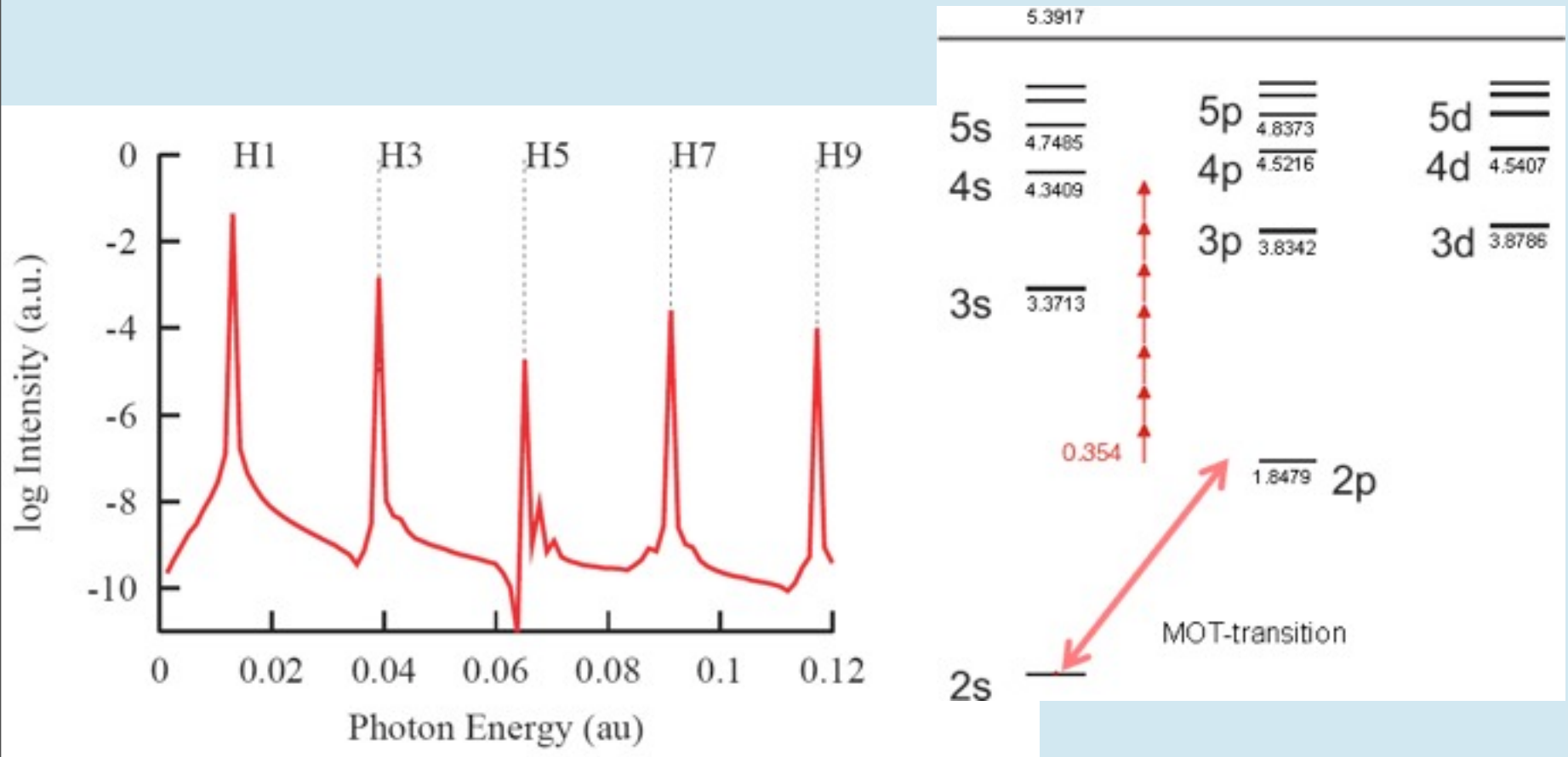
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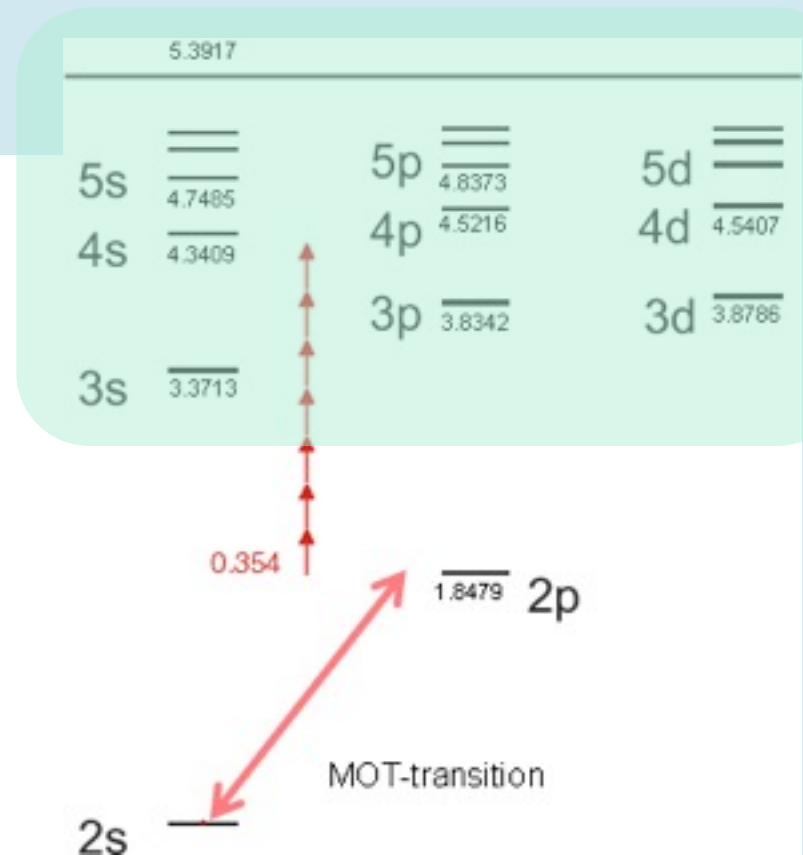
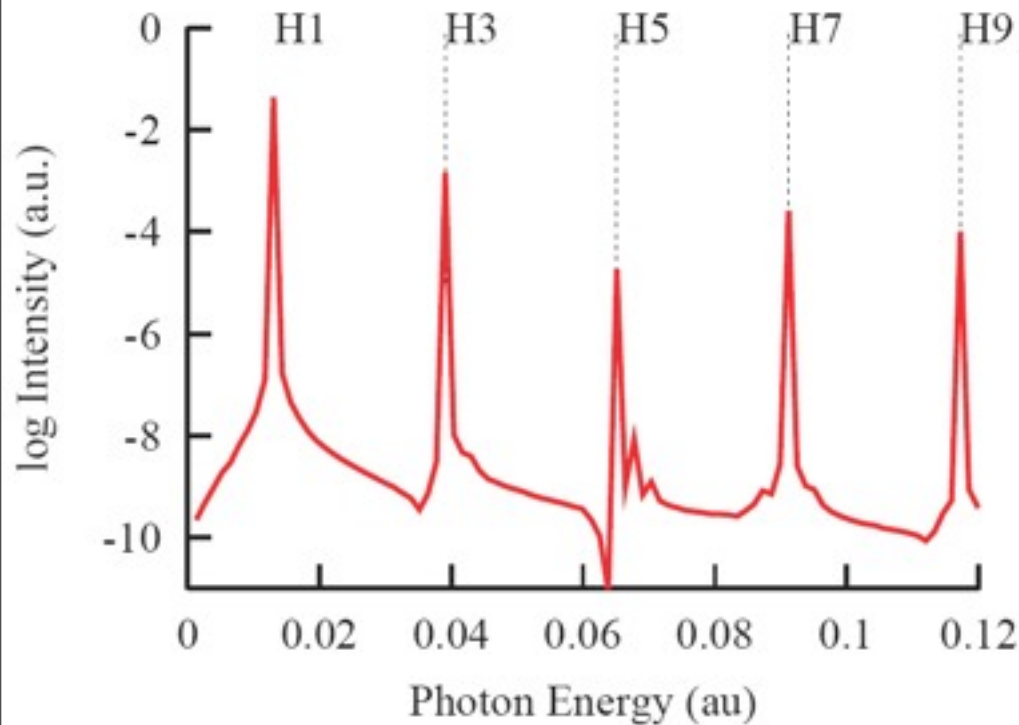
KITP 2009

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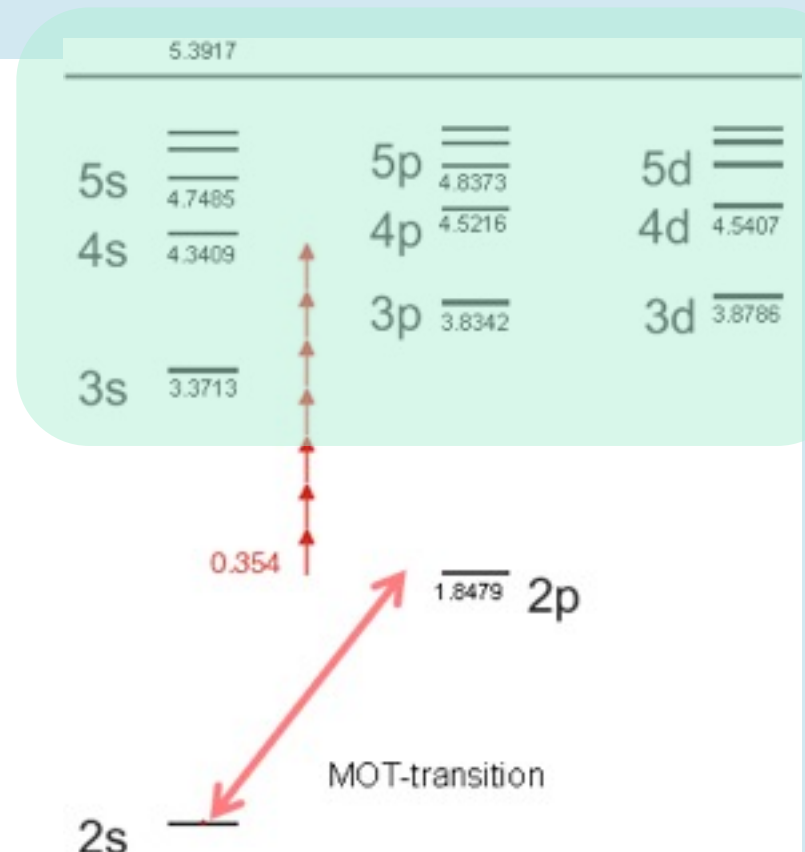
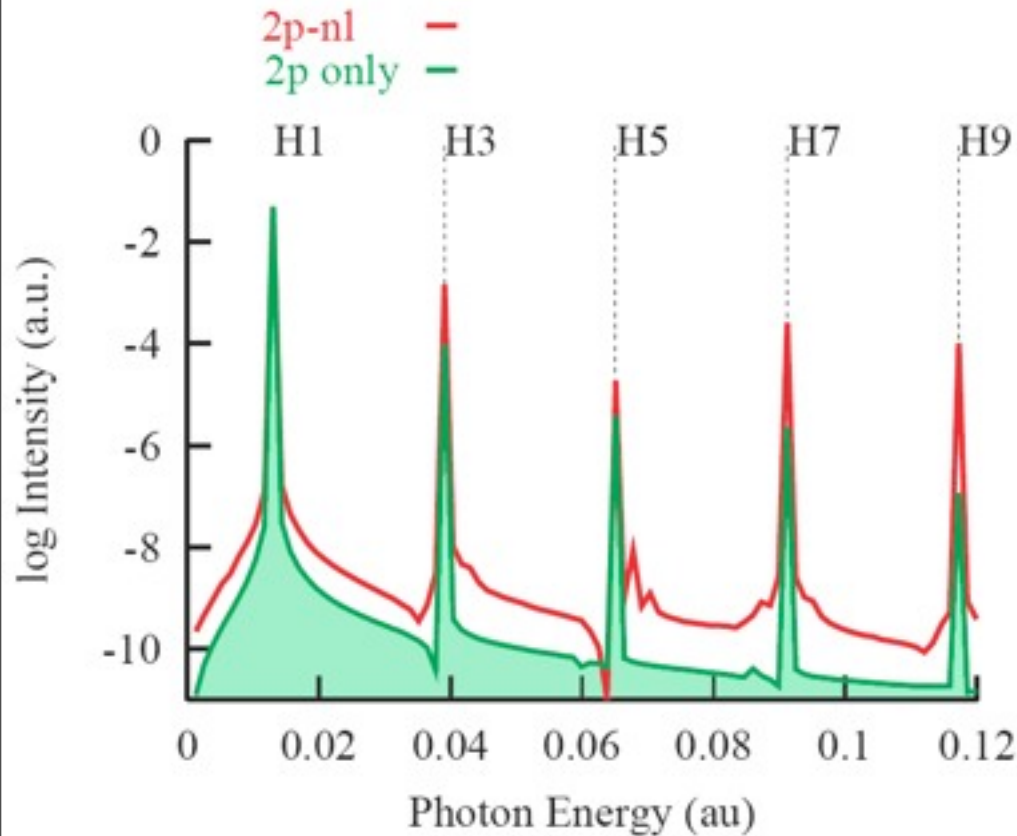
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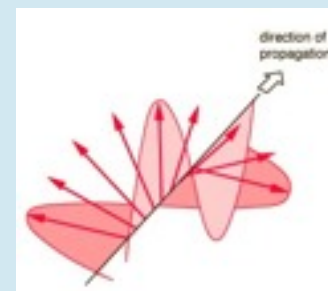
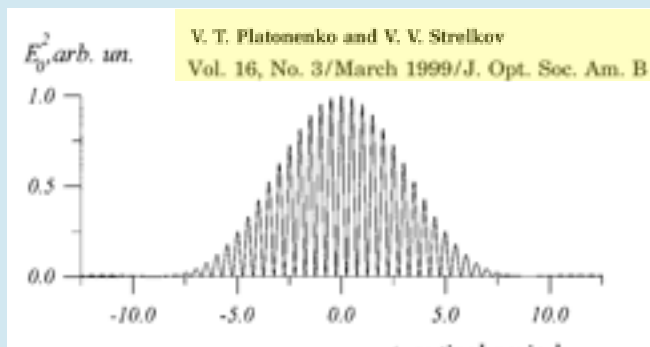


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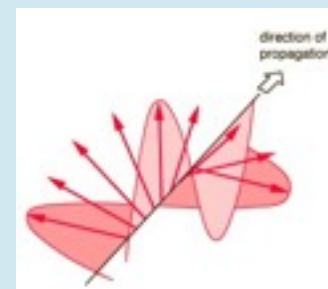
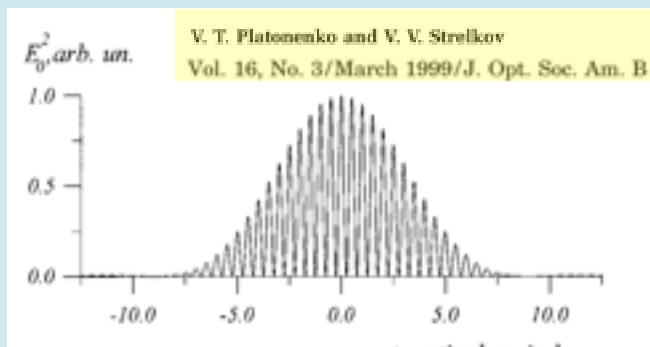


# Harmonics spectrum: effect of polarization

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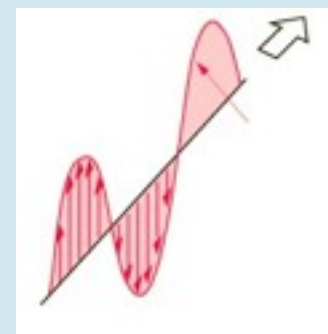
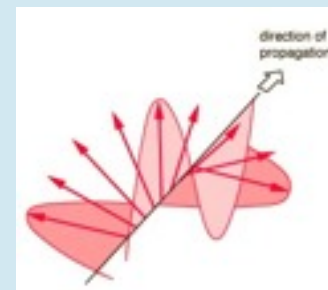
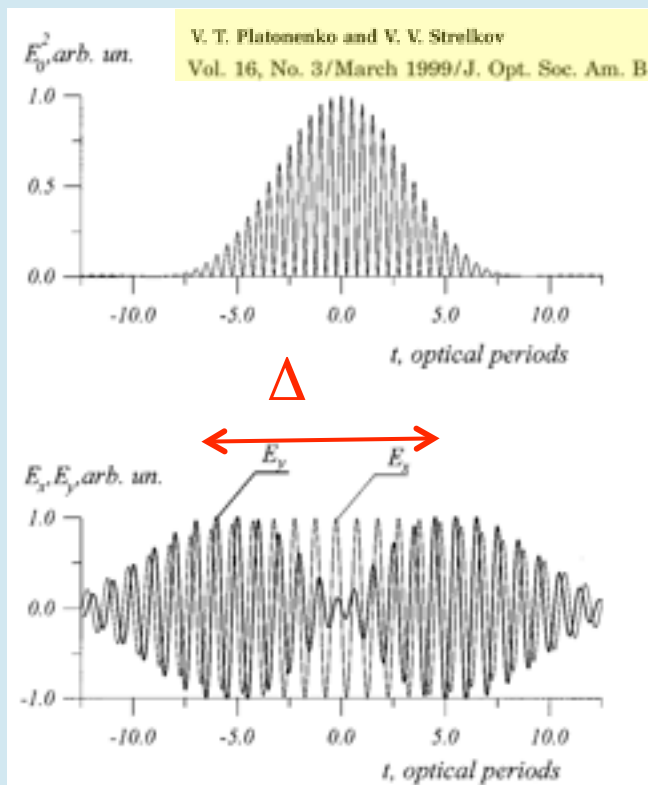
# Harmonics spectrum: effect of polarization



$$\hat{H}_{\text{int}}(t) = E_0 [(x f_1(t) \cos \omega t + y f_2(t) \sin \omega t)]$$

$$f_1(t) = f(t) + f(t - \Delta), f_2(t) = f(t) - f(t - \Delta).$$

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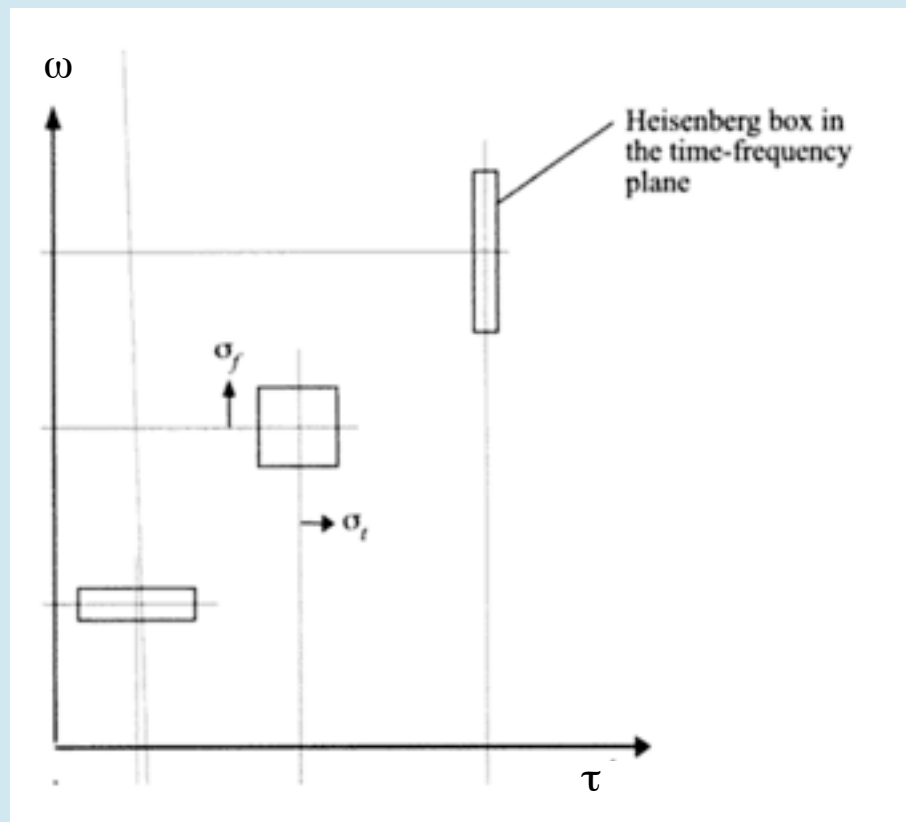
$$T_{\Psi}(\omega, \tau) = \int d(t) \sqrt{|\omega|} \Psi^*(\omega t - \omega \tau) dt$$

$$\Psi(x) = x_0^{-1} \exp(-ix) \exp[-x^2/(2x_0^2)].$$

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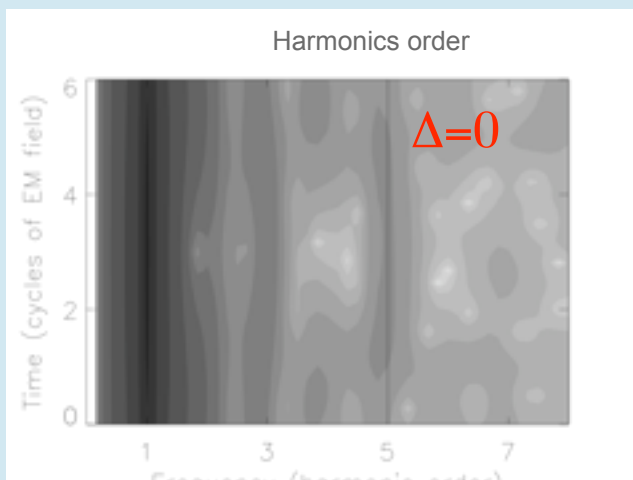
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Rb  $F = 0.002$  a.u.  $\omega = 0.2$  eV

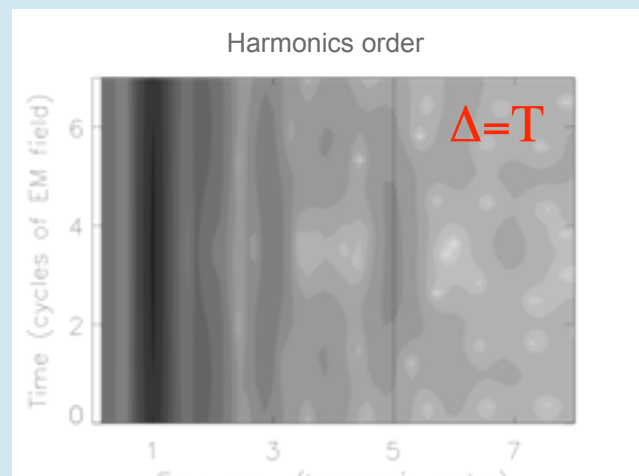
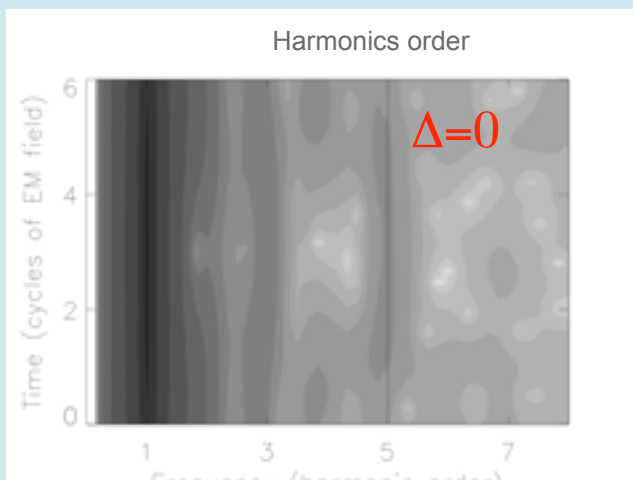
17



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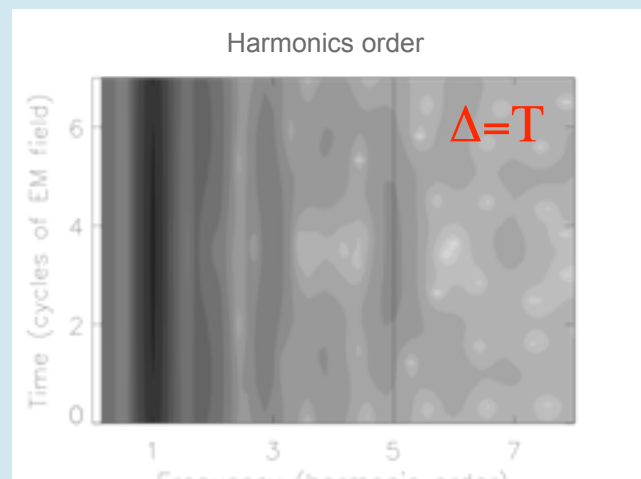
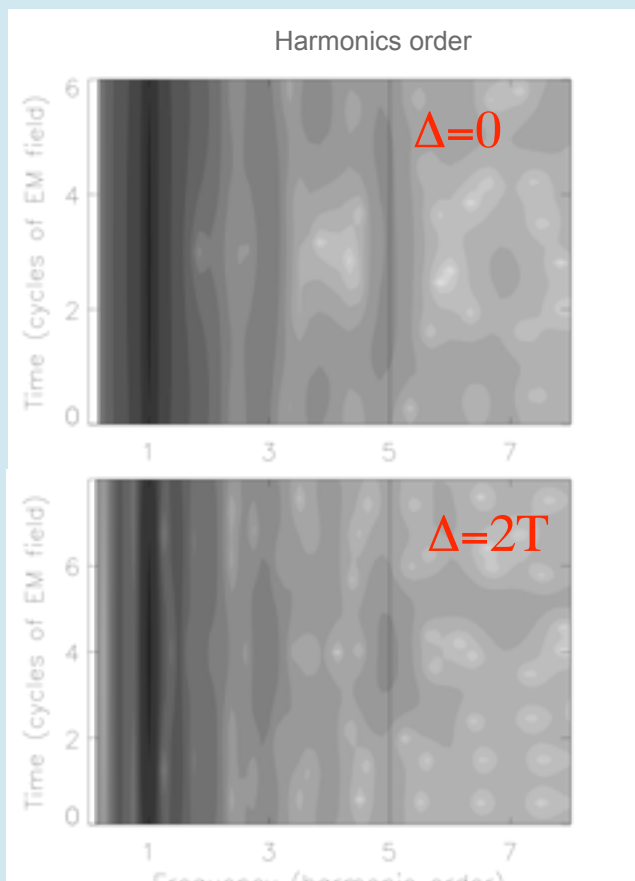
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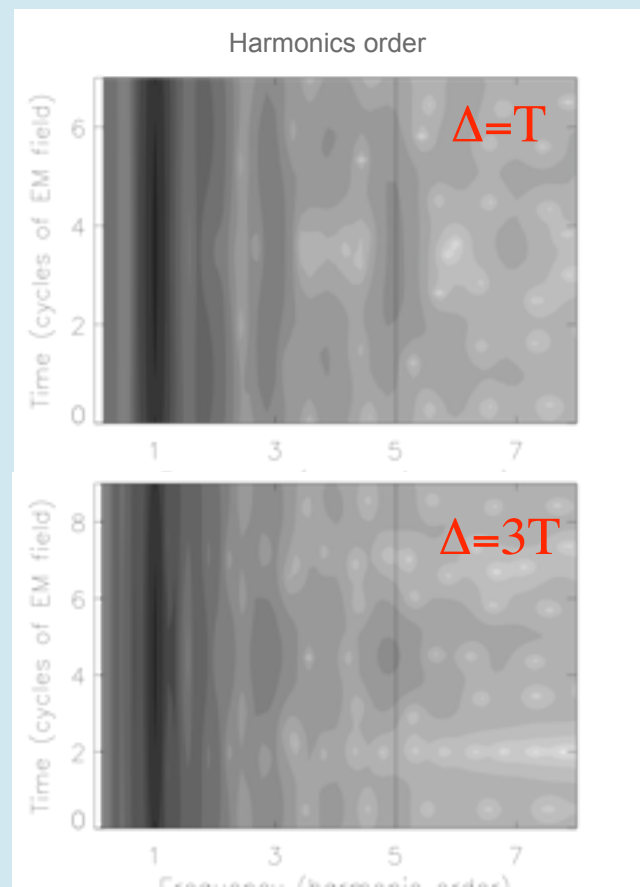
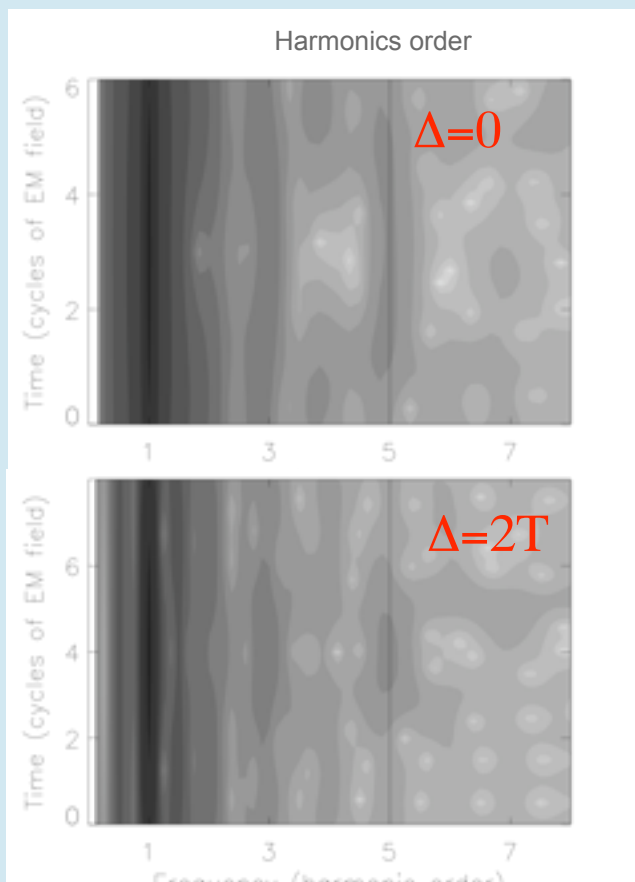
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17

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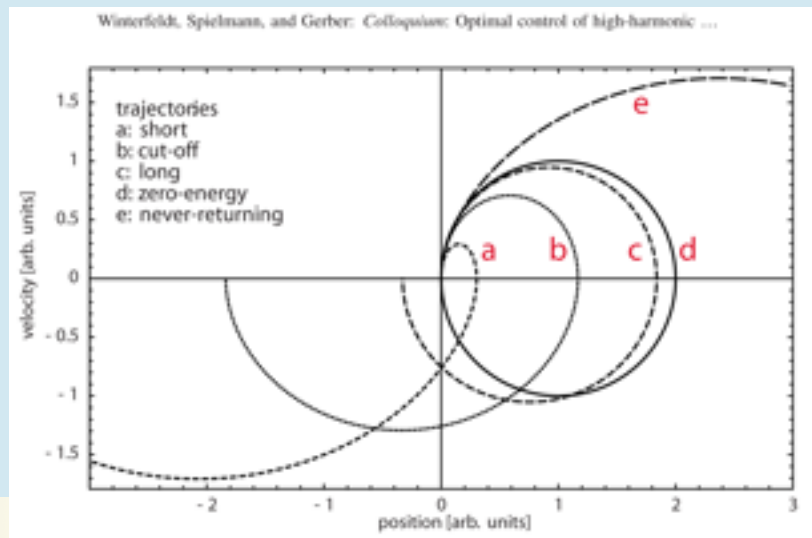
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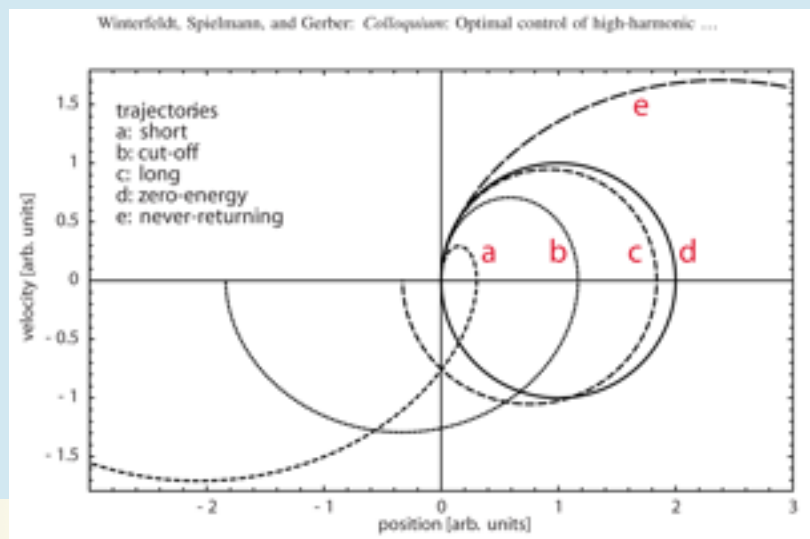
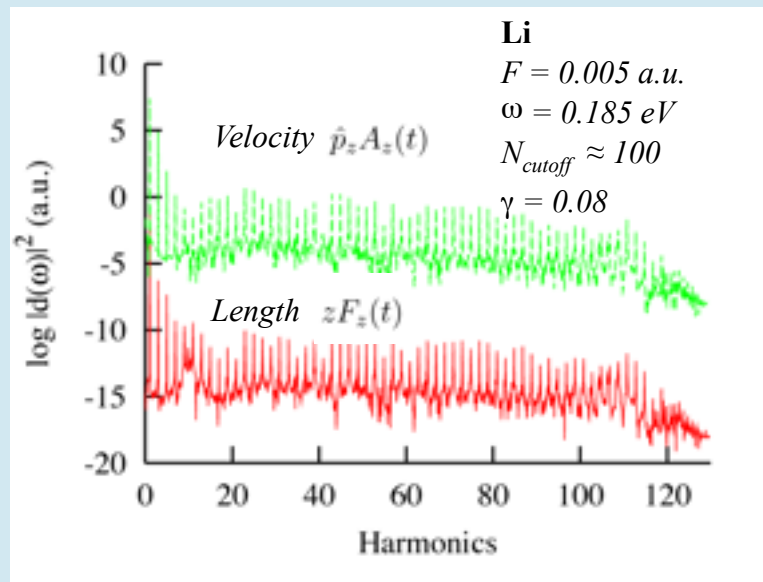
17

# Harmonics spectrum: wavelet-trajectory analysis

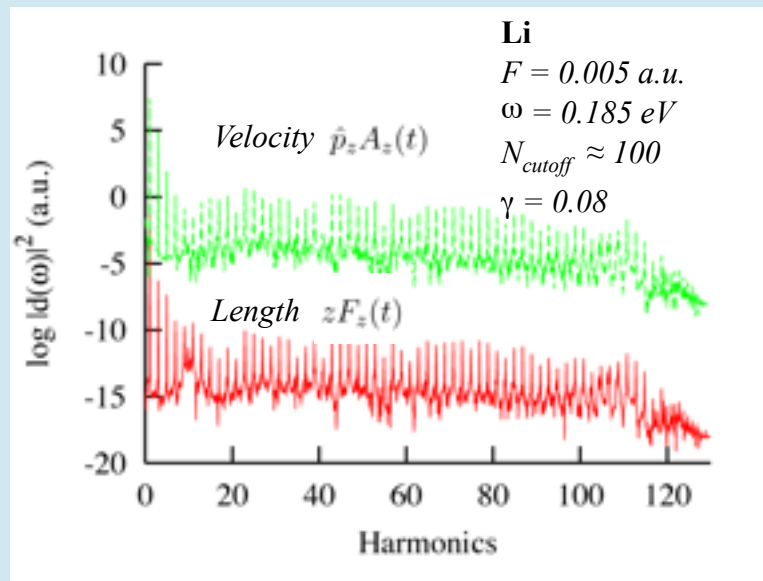
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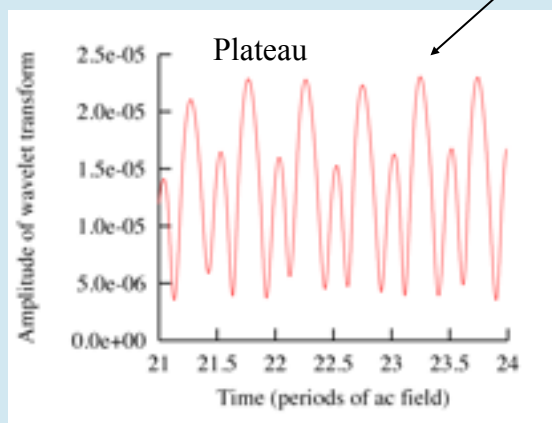
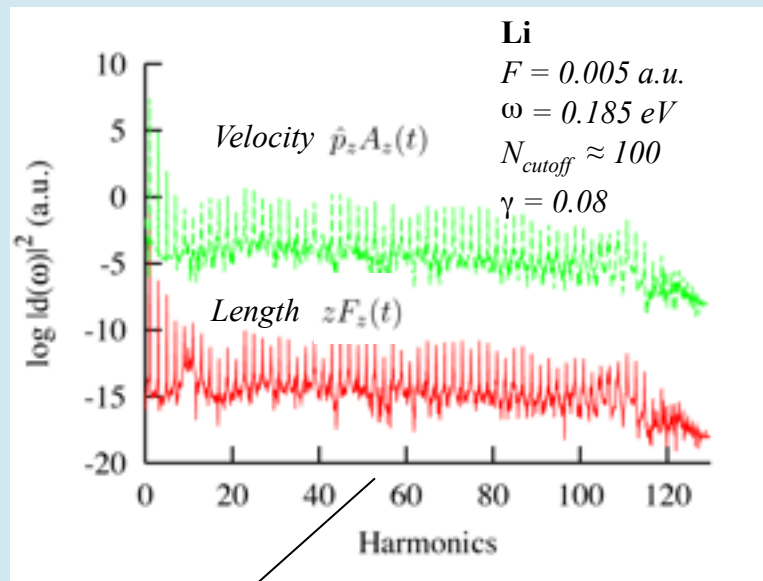
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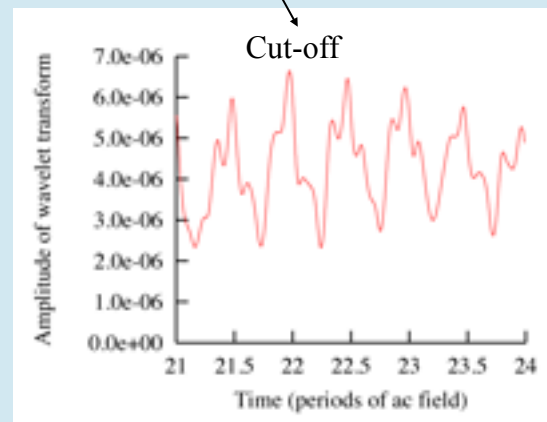
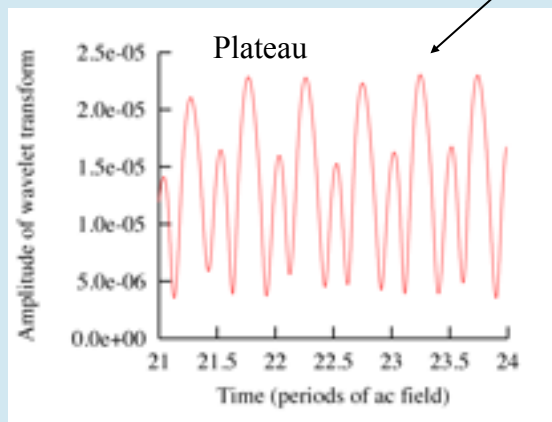
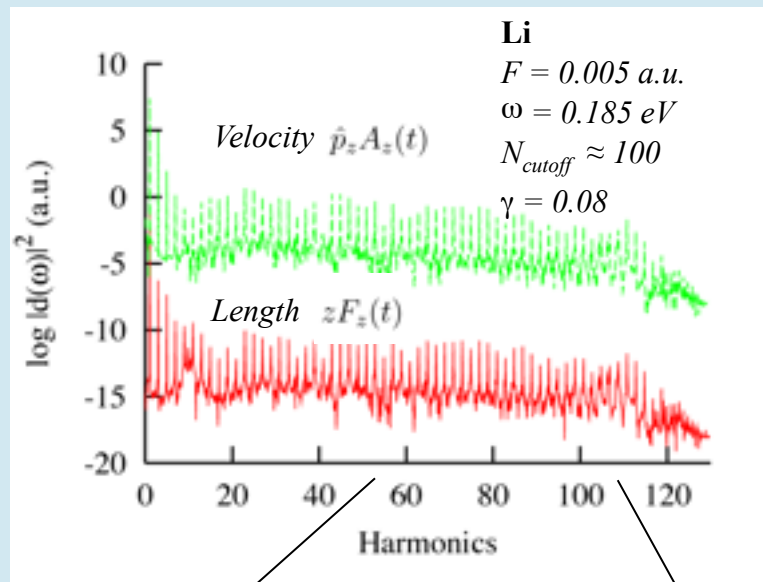


# Harmonics spectrum: wavelet-trajectory analysis





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# Harmonic spectrum: Effect of Chirp

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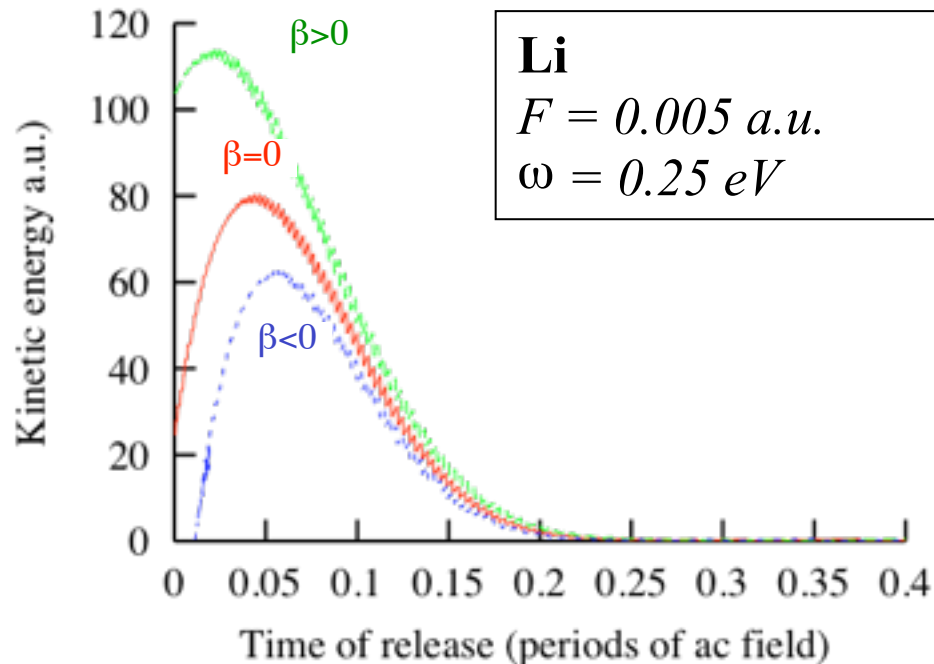
Juan J. Carrera and Shih-I Chu  
PHYSICAL REVIEW A **75**, 033807 (2007)

$$\hat{V}(\mathbf{r}, t) = -\mathbf{F} \cdot \mathbf{r}E(t) = -Fz f(t) \cos[\omega t + \delta(t)],$$

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# Harmonic spectrum: Effect of Chirp



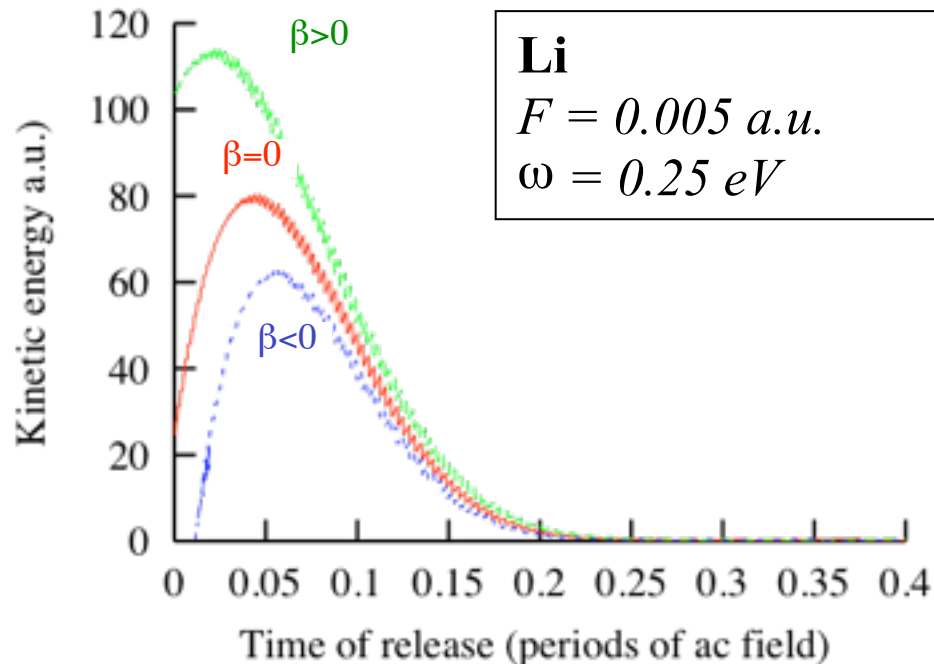
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*Kinetic energy at the moment of return as function of the time of release field without chirp (red), and positive and negative chirps*

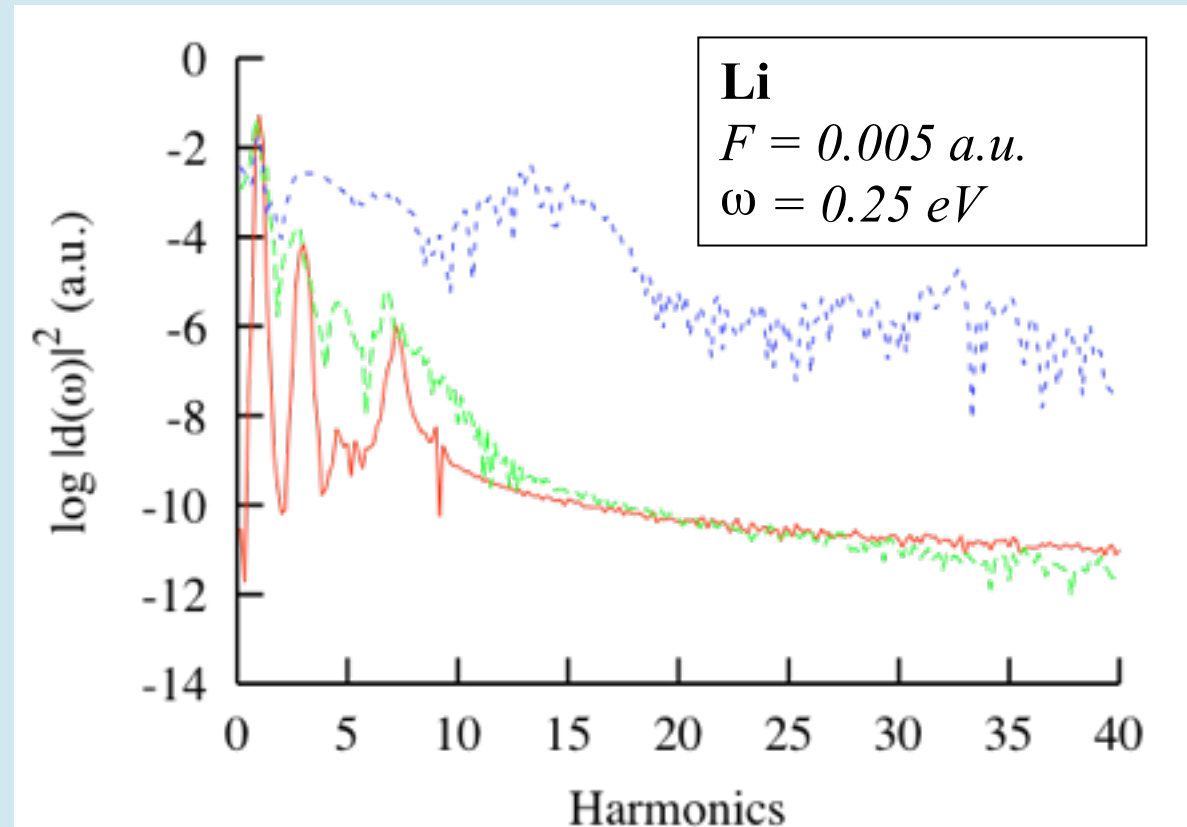
# Harmonic spectrum: Effect of Chirp

**Li**

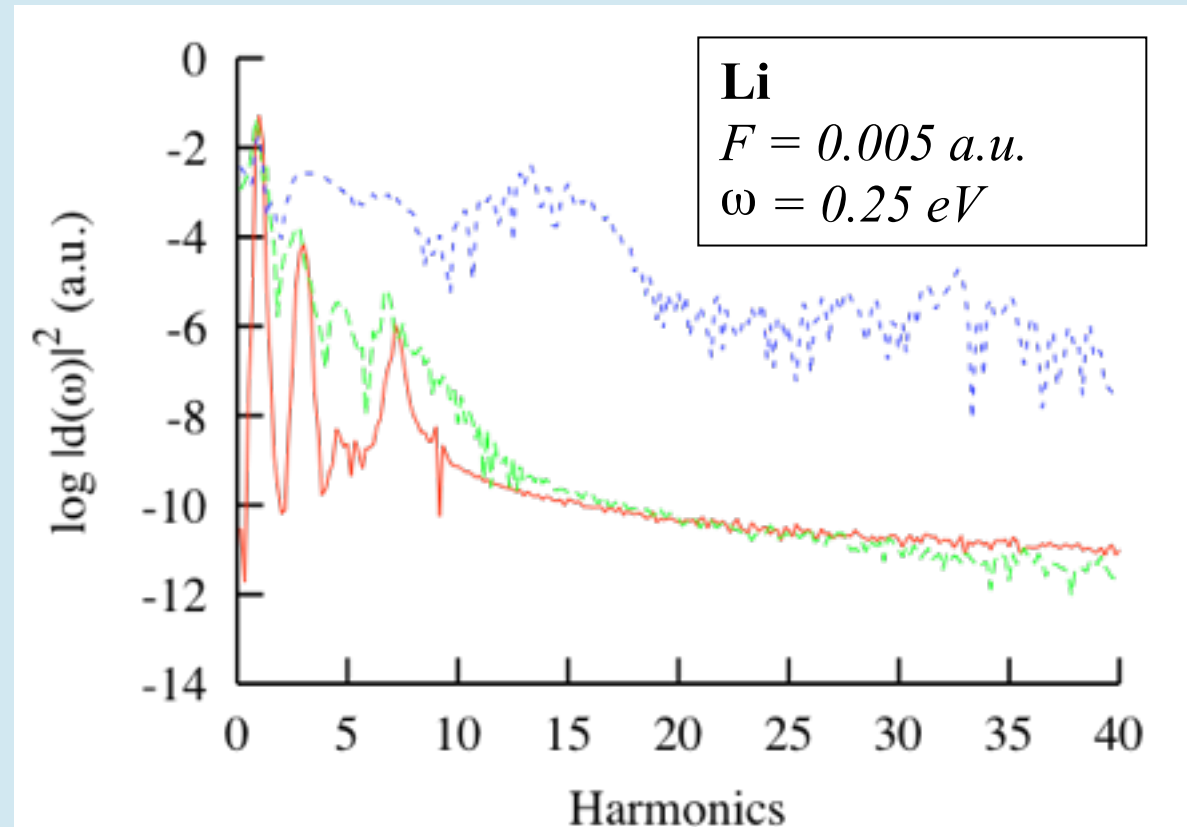
$$F = 0.005 \text{ a.u.}$$

$$\omega = 0.25 \text{ eV}$$

# Harmonic spectrum: Effect of Chirp



# Harmonic spectrum: Effect of Chirp



*Three calculations for Li for a short pulse with increasing chirp (red, green blue progressively)*



# Conclusion

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    - $(4\gamma, e)$  on Li
  - HHG
    - Li  $N_{\text{cut-off}} \approx 10$  optical regime
    - Li, Rb  $N_{\text{cut-off}} \approx 100$  XUV regime
    - Resonant enhancement
    - Polarization gating
    - Optimal control
- Further directions
  - Multi-photon many-electron processes on complex targets
  - More HHG control