Optimally controlling Osteoarthritis and Tumors with Spin 3/2

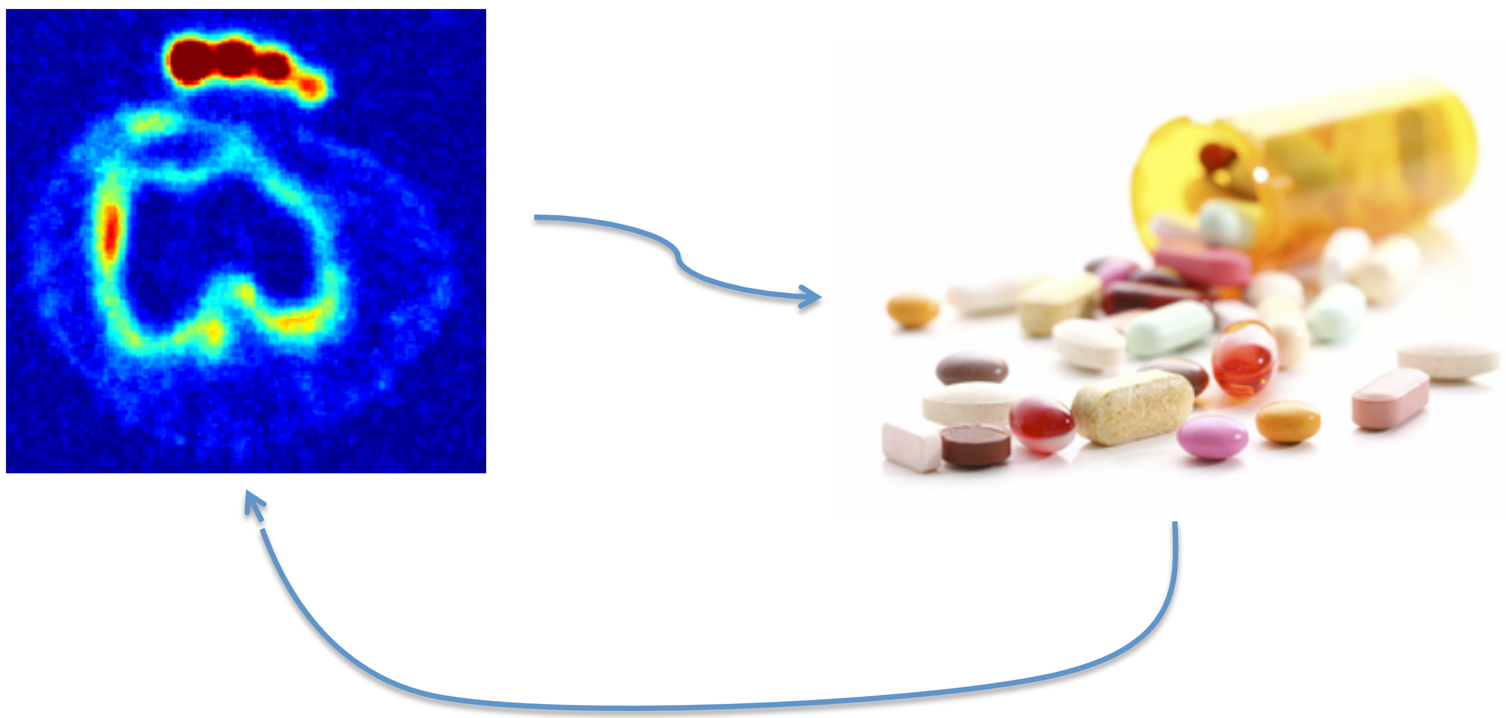
J.-S. Lee, S. C. Shekar, R. Regatte,
A. Jerschow
NYU

Overview

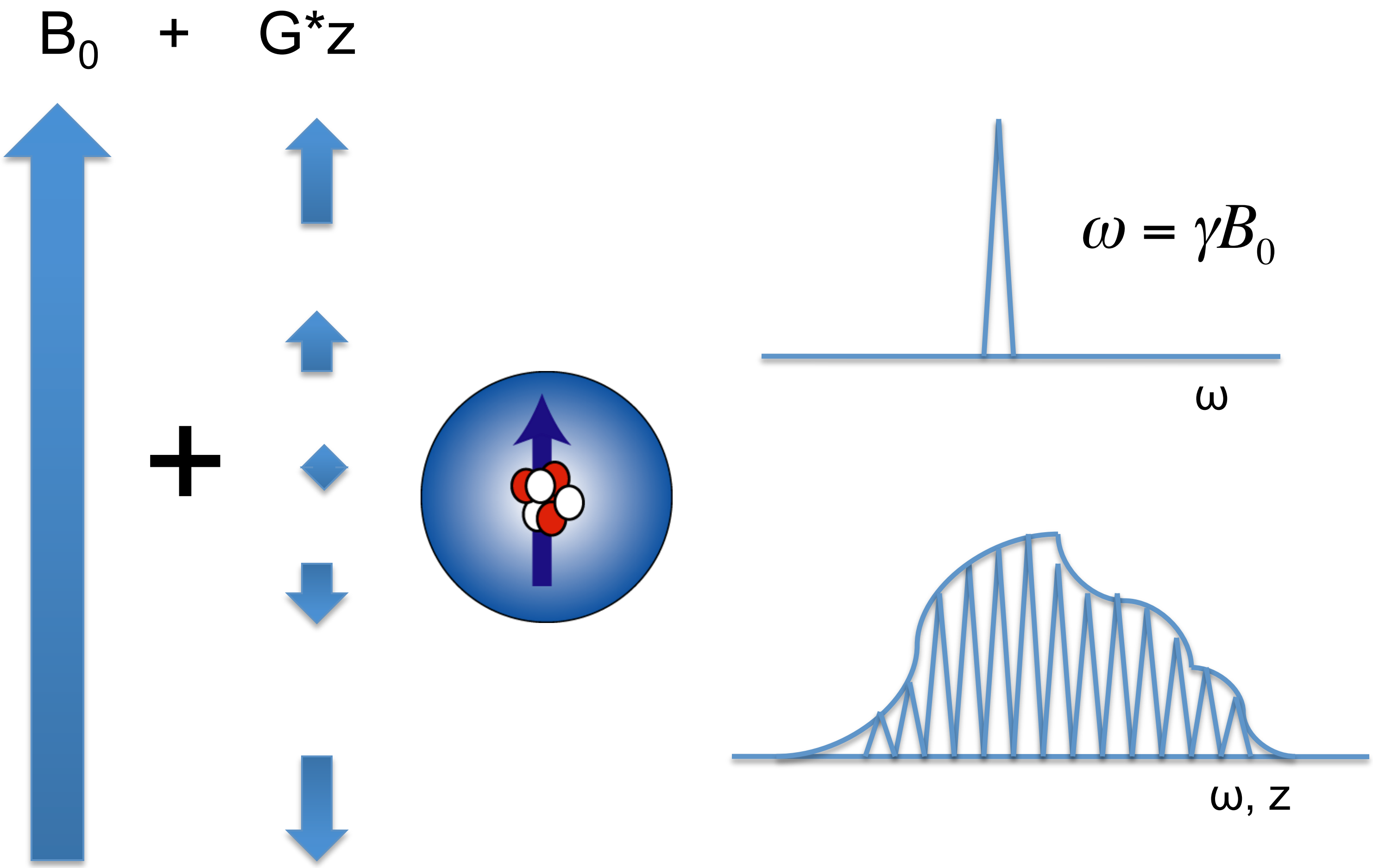
- Intro to MRI
- Intro to Spin>3/2
- Quadrupolar interaction + OCT results
- Quadrupolar relaxation + OCT results
- Quantum state tomography

NMR - MRI

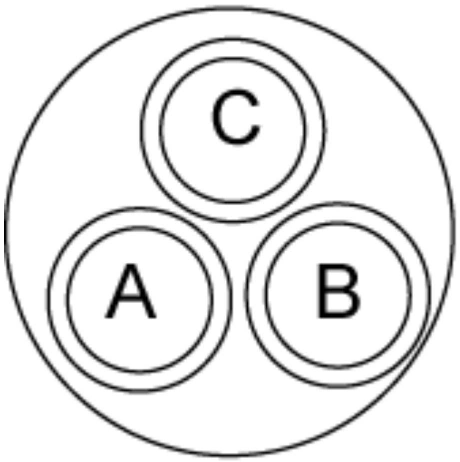
Optimally controlling osteoarthritis



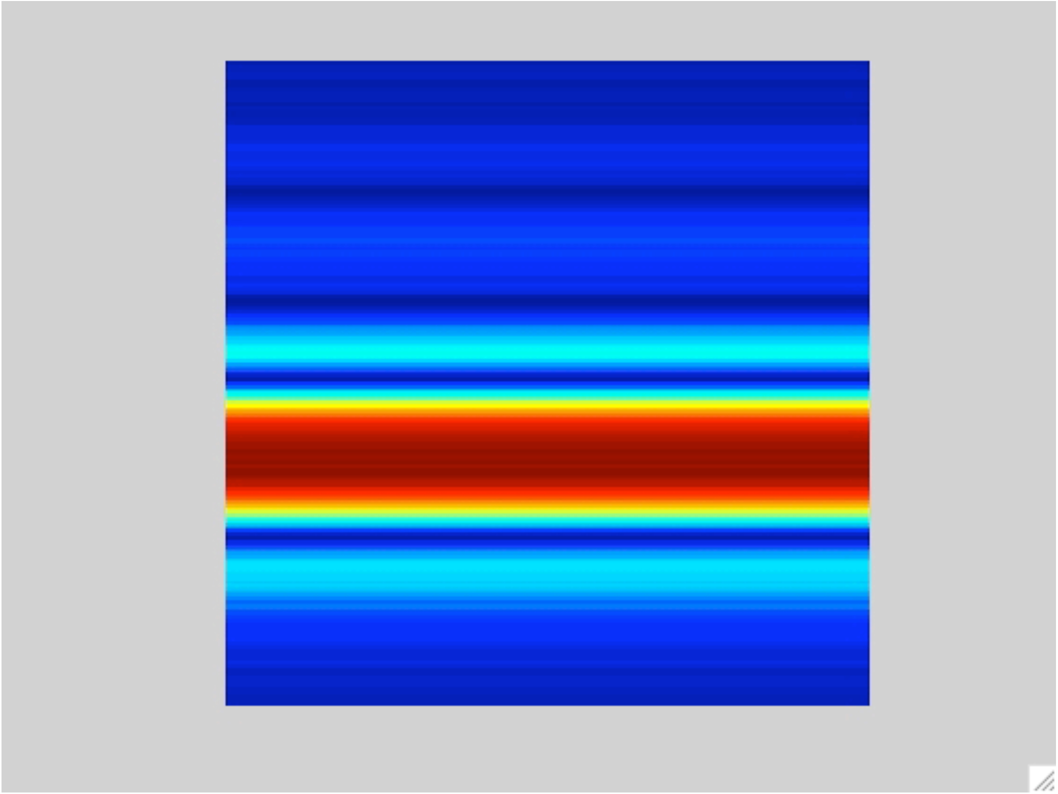
MRI



2D/3D Image Reconstruction



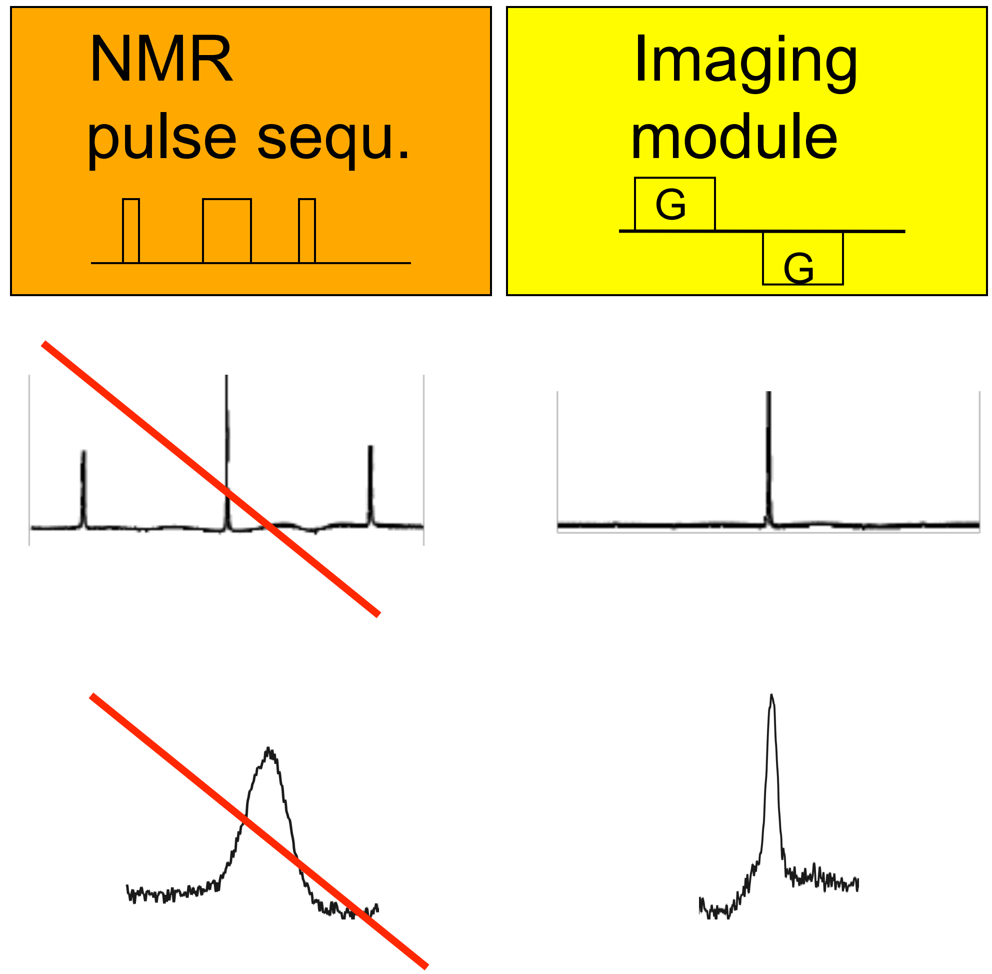
A: 150
B: 100
C: 50 mM NaCl



filtered backproj
twisted projection

How to modify MRI contrast?

- Add imaging *module* to NMR experiment
- One resonance
- Sharp



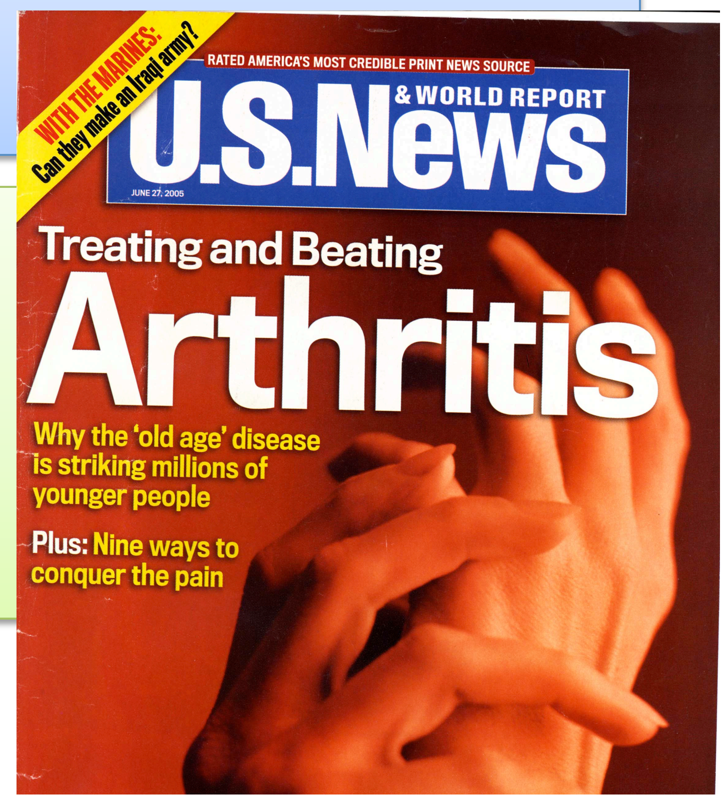
Oste … rit…
+ Degenera … Dis … sease.

1 in 10

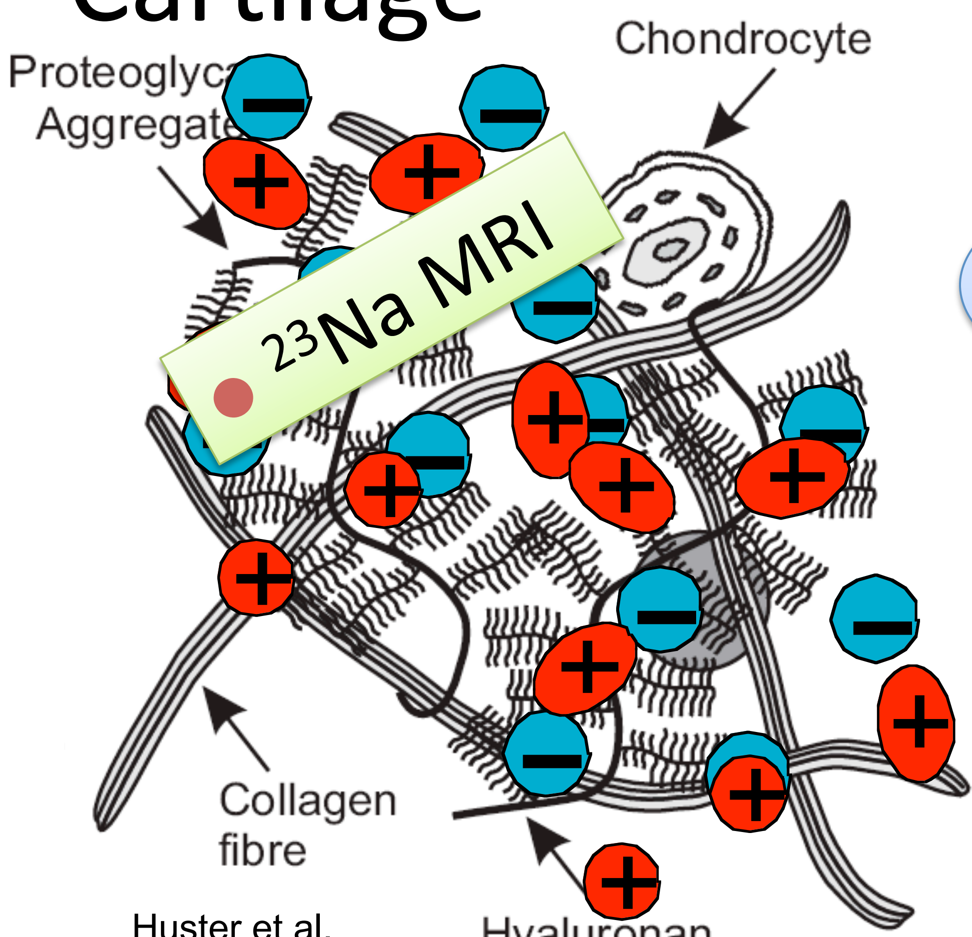
50% above 65 yrs

- Cartilage / disc does not heal!
- Physical therapy, surgery

- Detecting *early* changes.
- Determine effectiveness of preventive treatment.



Cartilage

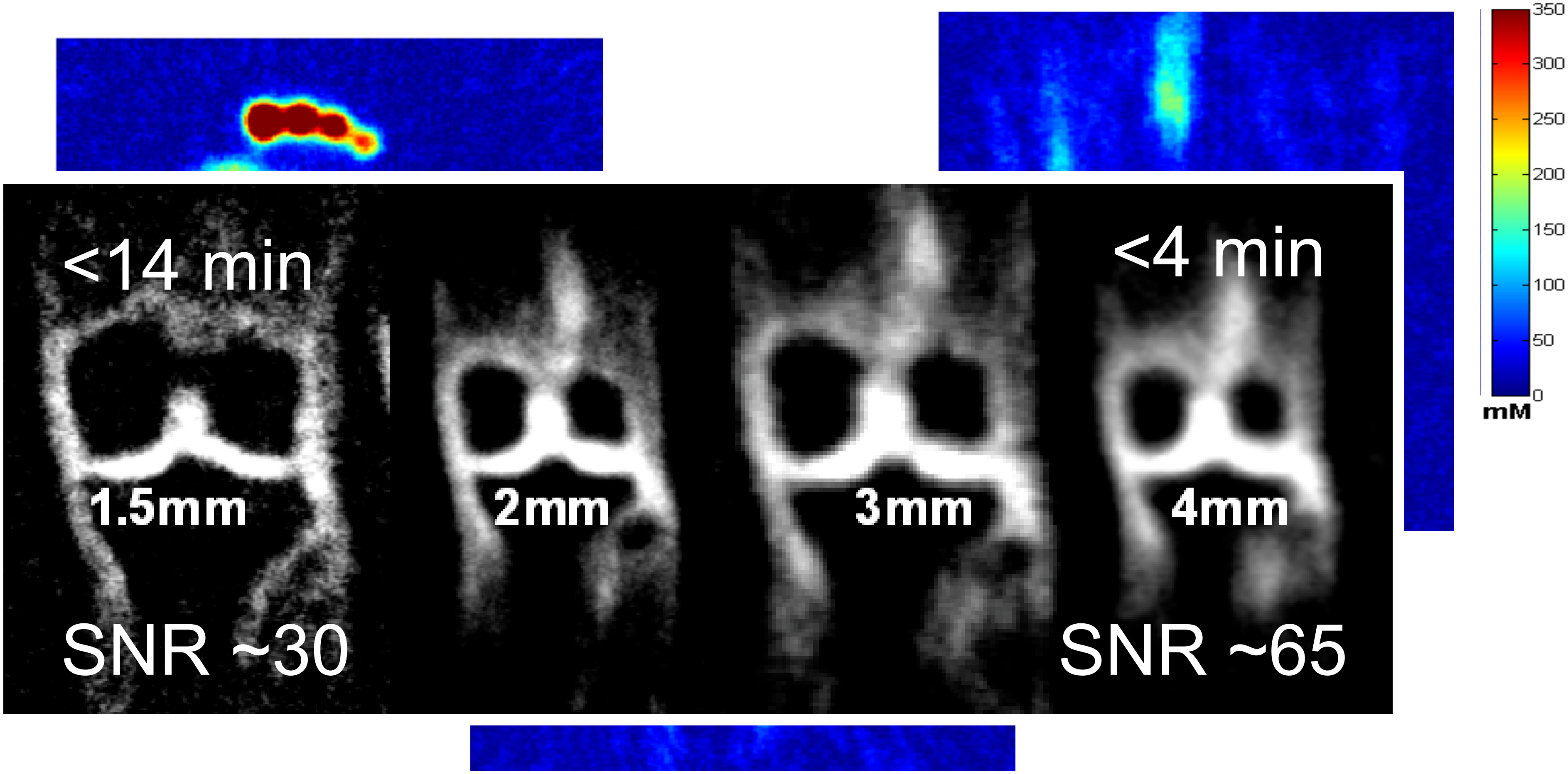


Huster et al.
Lect. Notes Phys. 634 (2004) 465

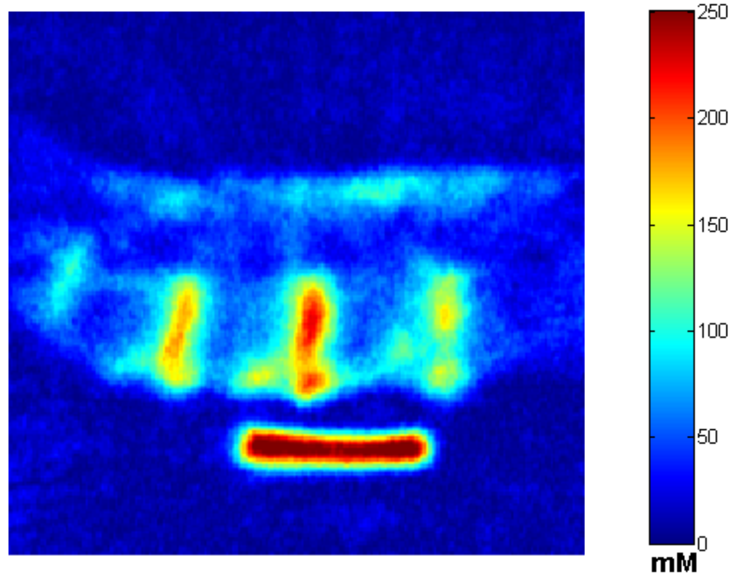
GAGs → negat charge

Osteoarthritis: loss of [GAG]

7 T ^{23}Na of knee
3D-radial

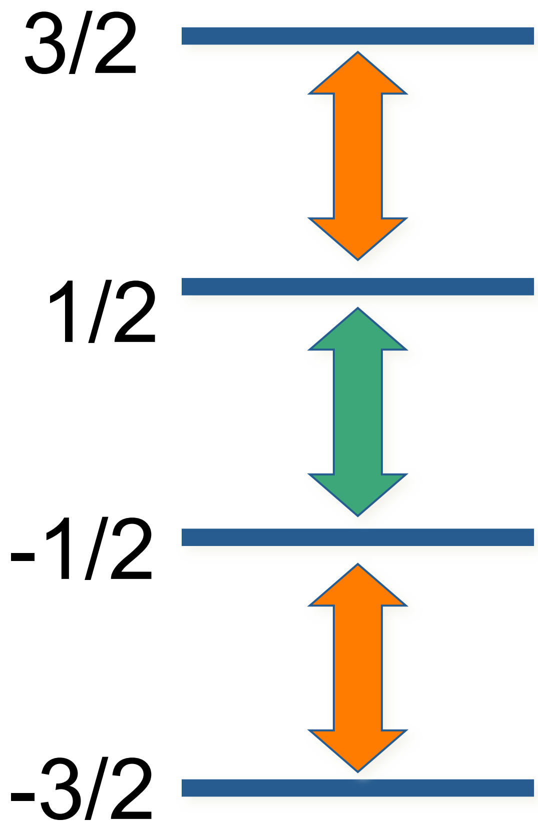


^{23}Na MRI @ 7T, spine



ex vivo

23Na – Spin 3/2



$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix}$$

$$= \sum_{k,l=1}^{4} c_{k,l} |k\rangle\langle l| = \sum_{L=0}^{3} \sum_{m=-L}^{L} a_{L,M} T_{L,M}$$

23Na – Spin 3/2

$$T_{0,0} = \mathbb{1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad T_{1,0} = I_z = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}$$

$$T_{1,1} = -\frac{1}{\sqrt{2}} I_+ = \begin{pmatrix} 0 & -\sqrt{\frac{3}{2}} & 0 & 0 \\ 0 & 0 & -\sqrt{2} & 0 \\ 0 & 0 & 0 & -\sqrt{\frac{3}{2}} \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad T_{1,-1} = \frac{1}{\sqrt{2}} I_- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt{\frac{3}{2}} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}} & 0 \end{pmatrix}$$

$$T_{2,0} = \frac{1}{\sqrt{6}} \left[3I_z^2 - I(I+1) \right] = \begin{pmatrix} \sqrt{\frac{3}{2}} & 0 & 0 & 0 \\ 0 & -\sqrt{\frac{3}{2}} & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{3}{2}} & 0 \\ 0 & 0 & 0 & \sqrt{\frac{3}{2}} \end{pmatrix}$$

Spin > ½: Quadrupolar interaction

$$E = \frac{1}{4\pi\epsilon_0} \iint \mathrm{d}r_\mathrm{e}\mathrm{d}r_\mathrm{n} \frac{\rho_\mathrm{e}(r_\mathrm{e})\rho_\mathrm{n}(r_\mathrm{n})}{|\mathbf{r}_\mathrm{e} - \mathbf{r}_\mathrm{n}|}$$

Electrostatic energy

Through moment expansion

$$H = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{l,m} B_{l,m}^{\dagger} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} (-1)^m A_{l,m} B_{l,-m}$$

electronic ($A_{l,m}$) nuclear ($B_{l,-m}$)

Quadrupolar interaction in NMR

$$H_Q = \omega_Q \sum_{m=-2}^{2} (-1)^m R_{2,-m} T_{2,m}$$

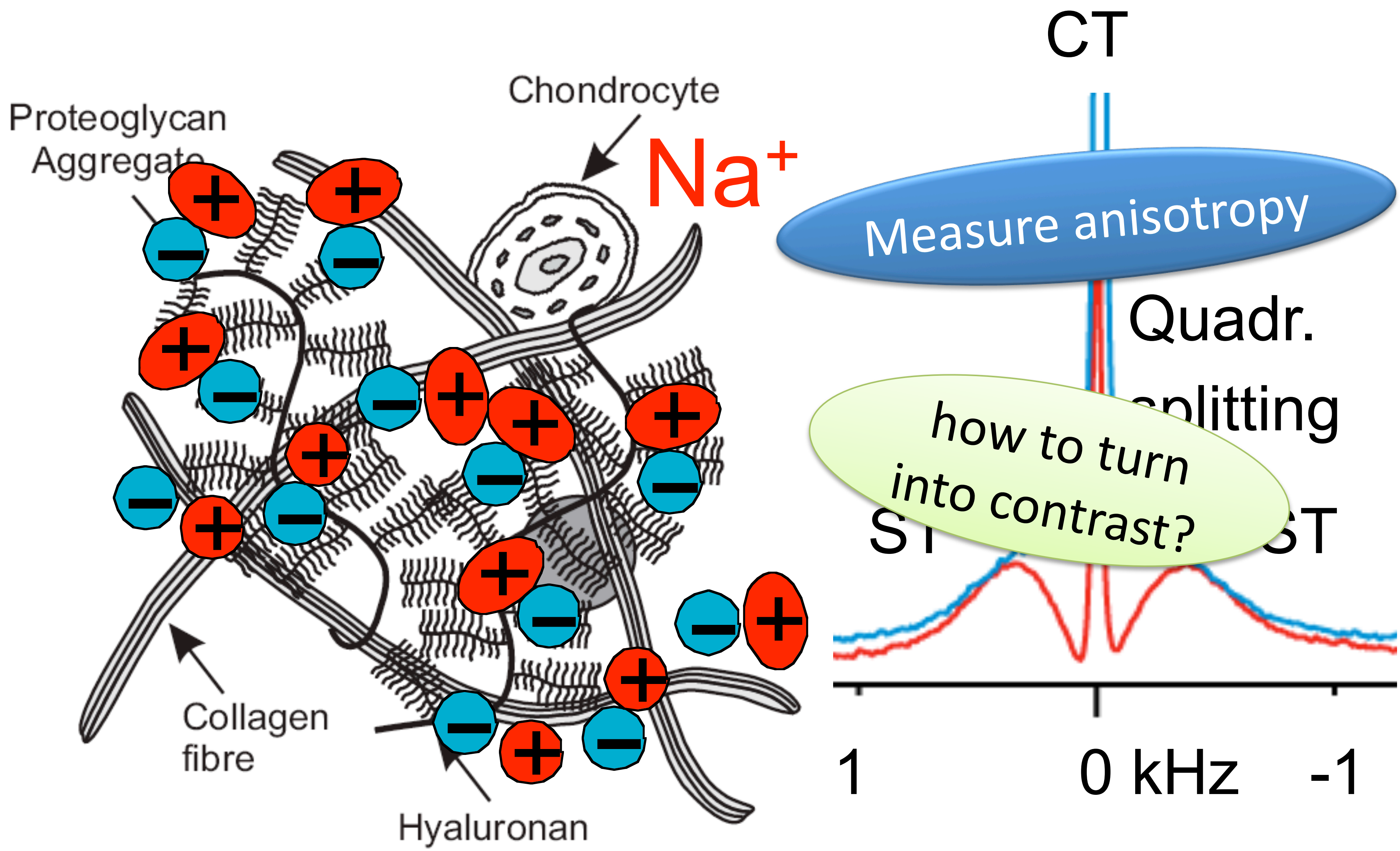
electric field gradient

nuclear spin op

If $H_Z >> H_Q$

$$H_Q = \omega_Q \left[I_z^2 - I(I+1)/3 \right] = \omega_Q \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$$

^{23}Na in cartilage



Which ^{23}Na method is best?

fast relaxation
low rf power

Optimal control

Rf shape in

Opt rf shape out

Optimal control

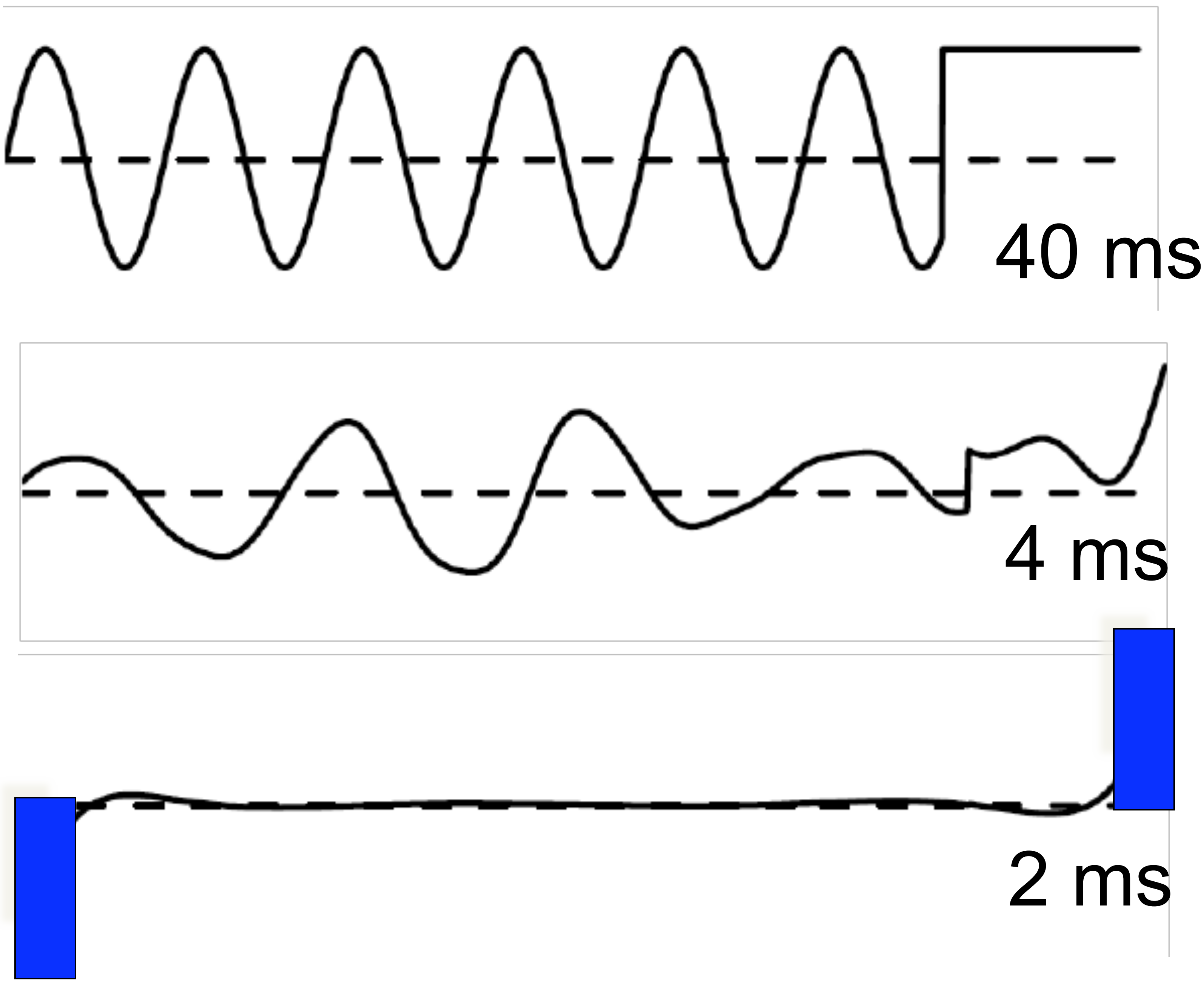
Rf shape in

Khaneja et al. JMR 172, 296 (2005)

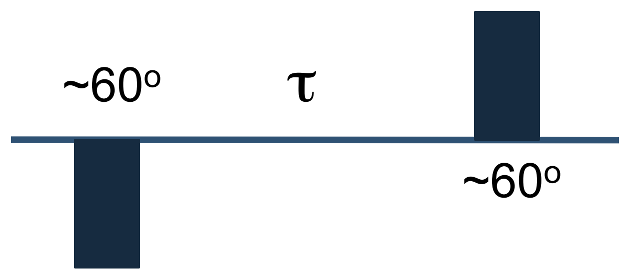
Opt rf shape out

Start $\omega_Q >> \omega_{rf}$
$1/T_2 << \omega_Q$

End $\omega_Q \sim \omega_{rf}$
$1/T_2 \sim \omega_Q$


40 ms
4 ms
2 ms

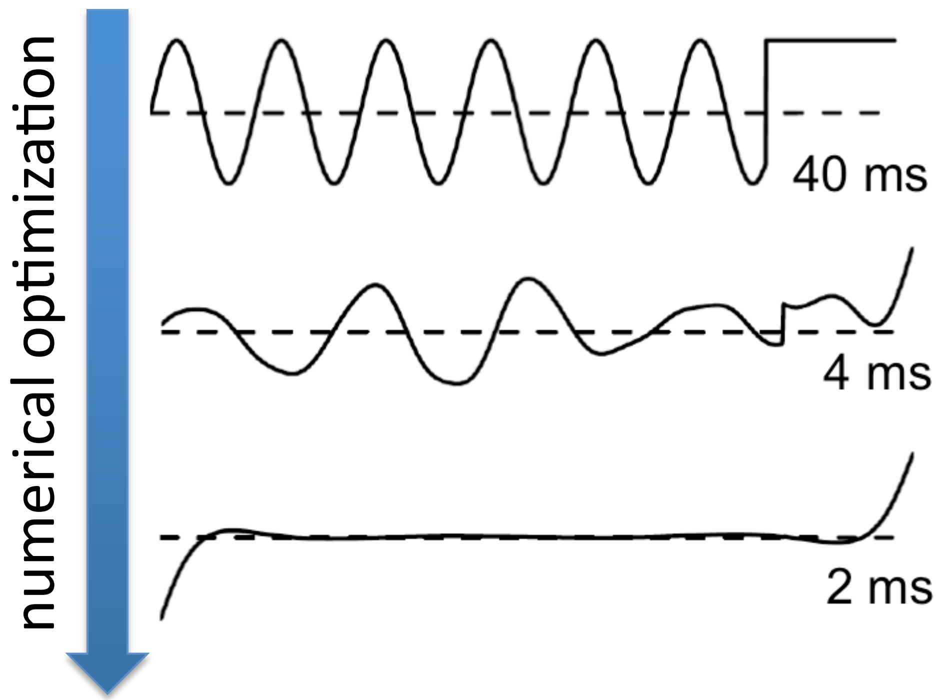
Two-pulse excitation



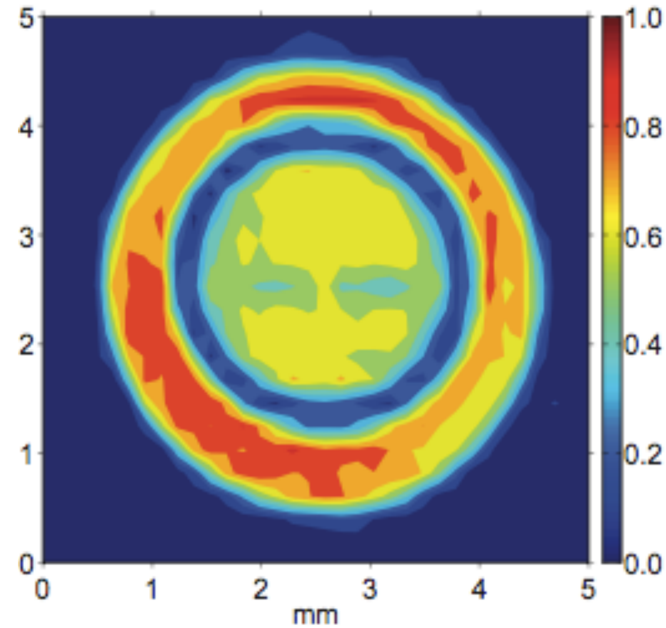
central peak

$$\frac{2}{5}\sin(\beta-\alpha)+\frac{9}{5}\sin\alpha\cos\beta\sin^2\beta\sin^2(\pi f_Q\tau)$$

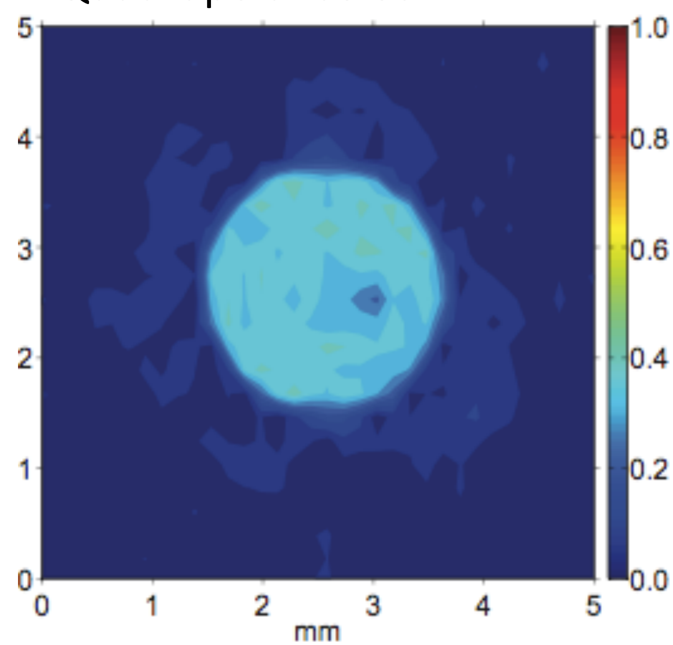
max at ~149% when $\tau = 1/(2f_Q)$ and α, $\beta \cong 60^o$



Simple pulse



Quadrupolar select



Lee, Regatte, AJ
JCP 2008, *129*, 224510

Quadrupolar relaxation

$$\frac{d}{dt}\rho(t) = -\imath[\mathcal{H}, \rho(t)] - \hat{\hat{\Gamma}}\{\rho(t) - \rho^{\text{th}}\}$$

$$\hat{\hat{\Gamma}} = \frac{1}{2}\sum_{m}\left[T_{2,-m},\left[T_{2,m},\bullet\right]\right]J(m\omega)$$

spectral density function

Hubbard, JCP, 53 (1970) 985
Sanctuary, JCP, 64 (1976) 4352
Jaccard, Wimperis, Bodenhausen, JCP 85 (1986) 6282

Write in terms of elements

$$\begin{aligned}\frac{d}{dt}\rho_{jk} &= -\imath\sum_m(\mathcal{H}_{jm}\rho_{mk}-\rho_{jm}\mathcal{H}_{mk})-\sum_{m,n}\Gamma_{jk,mn}(\rho_{mn}-\rho^{\mathrm{th}}_{mn})\\ &= \sum_{m,n}\big[-\imath(\mathcal{H}_{jm}\delta_{kn}-\mathcal{H}_{nk}\delta_{jm})-\Gamma_{jk,mn}\big]\rho_{mn}+\sum_{m,n}\Gamma_{jk,mn}\rho^{\mathrm{th}}_{mn}\\ &= \sum_{m,n}(-\imath H_{jk,mn}-\Gamma_{jk,mn})\rho_{mn}+\sum_{m,n}\Gamma_{jk,mn}\rho^{\mathrm{th}}_{mn},\end{aligned}$$

Homogeneous Master Equation

$$\frac{d}{dt}\begin{pmatrix}\vdots\\ \rho^{th}_{mn}\\ \vdots\\ \rho_{mn}\\ \vdots\end{pmatrix} = -\begin{pmatrix}\boxed{0} & \boxed{0}\\ \boxed{-\Gamma_{jk,mn}} & \boxed{\imath H_{jk,mn}+\Gamma_{jk,mn}}\end{pmatrix}\begin{pmatrix}\vdots\\ \rho^{th}_{mn}\\ \vdots\\ \rho_{mn}\\ \vdots\end{pmatrix}$$

$$\frac{d}{dt}\mu(t) = -P(t)\mu(t)$$

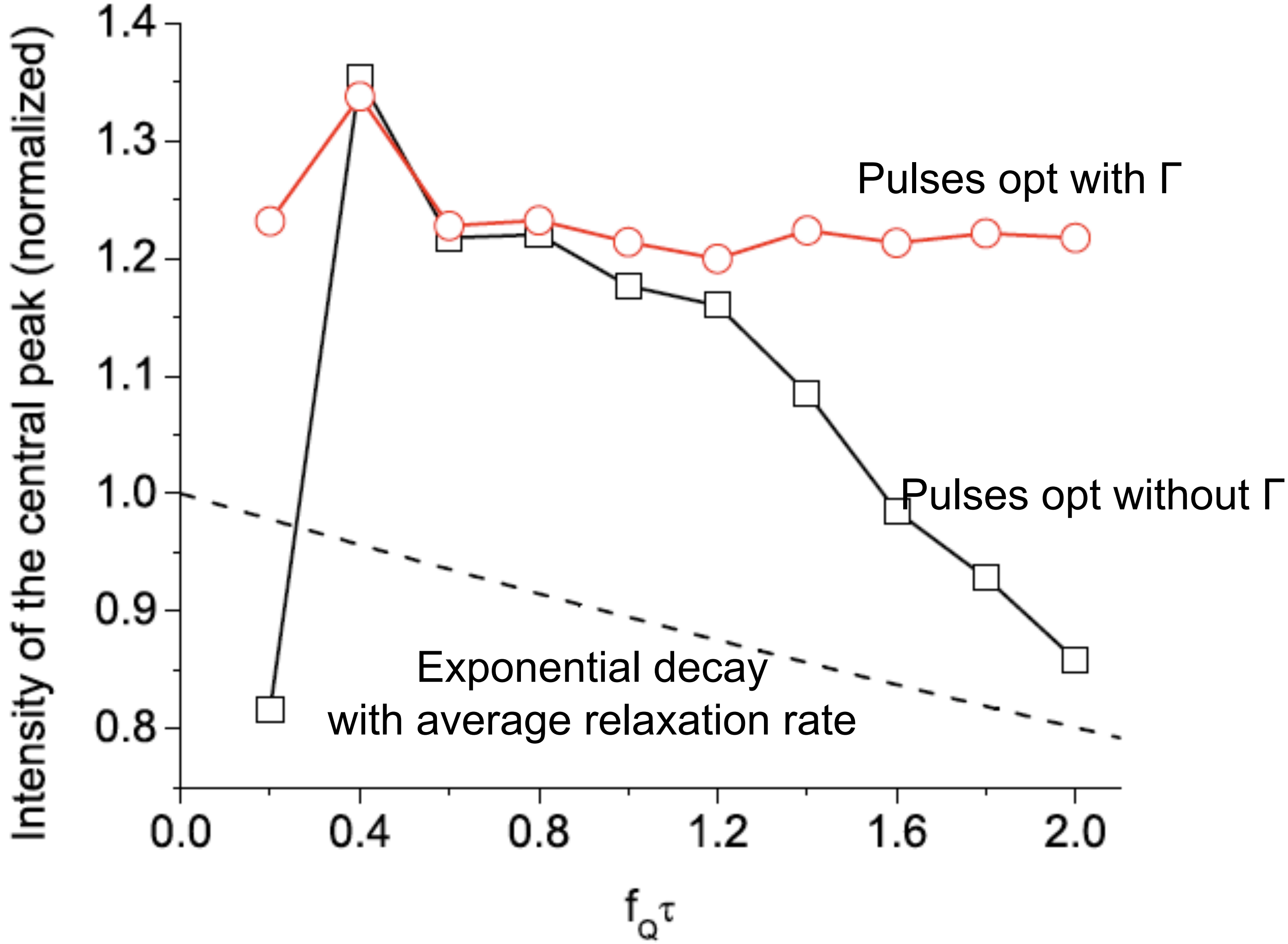
$$P(t) = P_0 + \sum_k P_k u_k(t)$$

$$h = \lambda^+ \frac{d\mu}{dt} + \frac{d\mu^+}{dt}\lambda = \lambda^+ P\mu + \mu^+ P^+ \lambda$$

$$\frac{d\lambda^+}{dt} = -\frac{\partial h}{\partial \mu} = \lambda^+ P$$

Using GRAPE

Experiment



Intensity of the central peak (normalized)
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.0
0.4
0.8
1.2
1.6
2.0
$f_Q\tau$
Pulses opt with Γ
Pulses opt without Γ
Exponential decay
with average relaxation rate

Tumor localization with 23Na

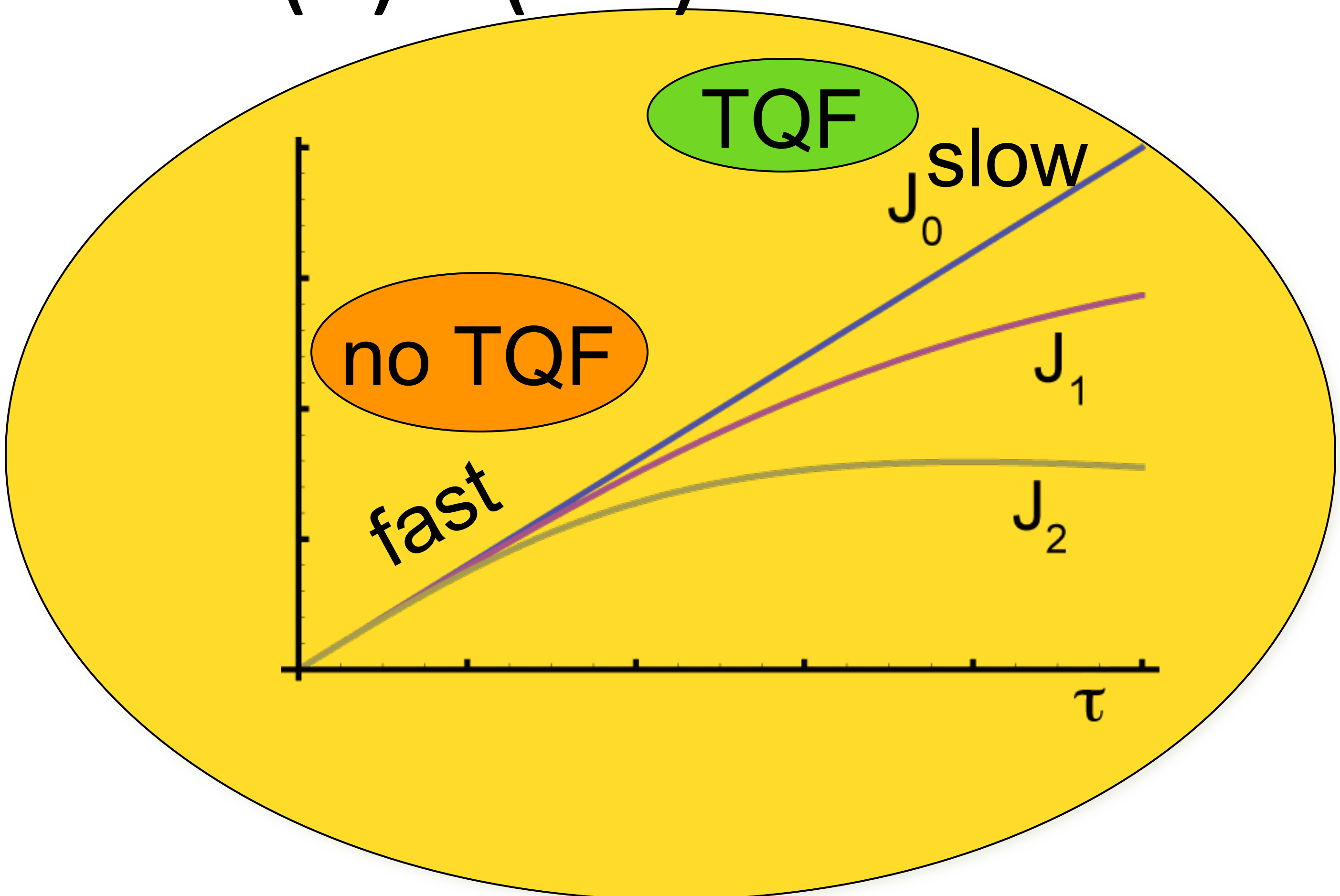
Assumption: **slow tumbling inside cell**

Tumor: **intracellular** Na builds up

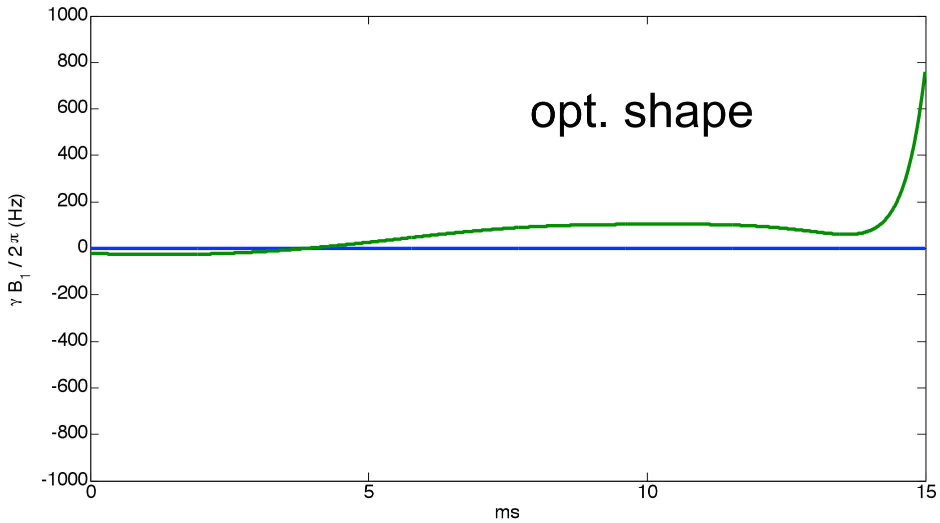
^{23}Na slow motion

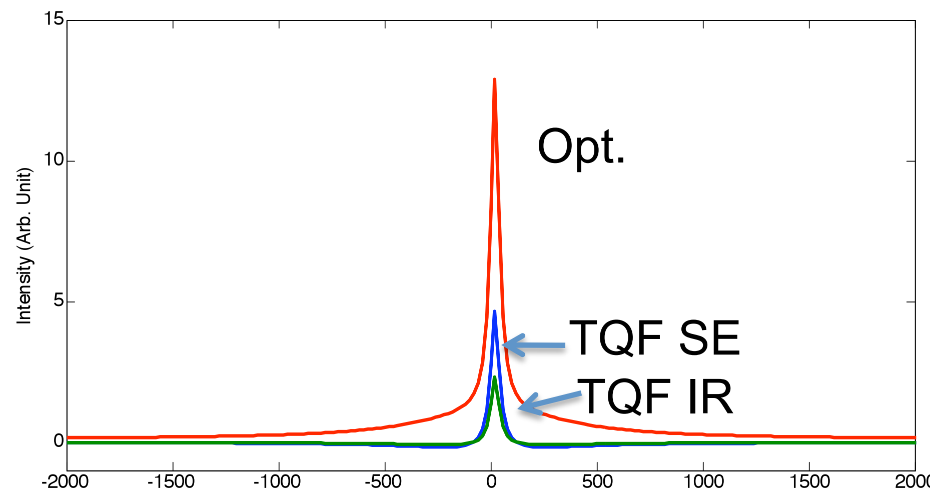
- $T_{11} \rightarrow T_{31}$ $\rightarrow T_{33}$, triple-quant filt (TQF) signal

buildup rate $J(0) - J(2\omega)$

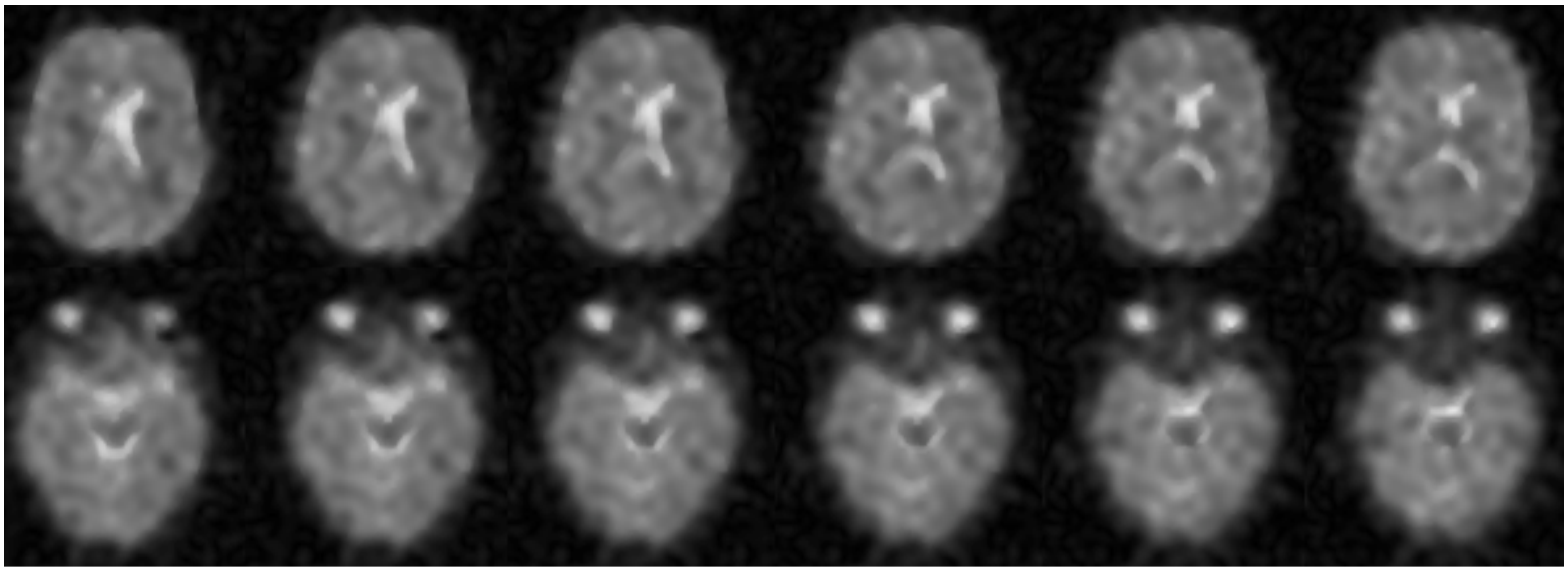


Opt slow motion Na



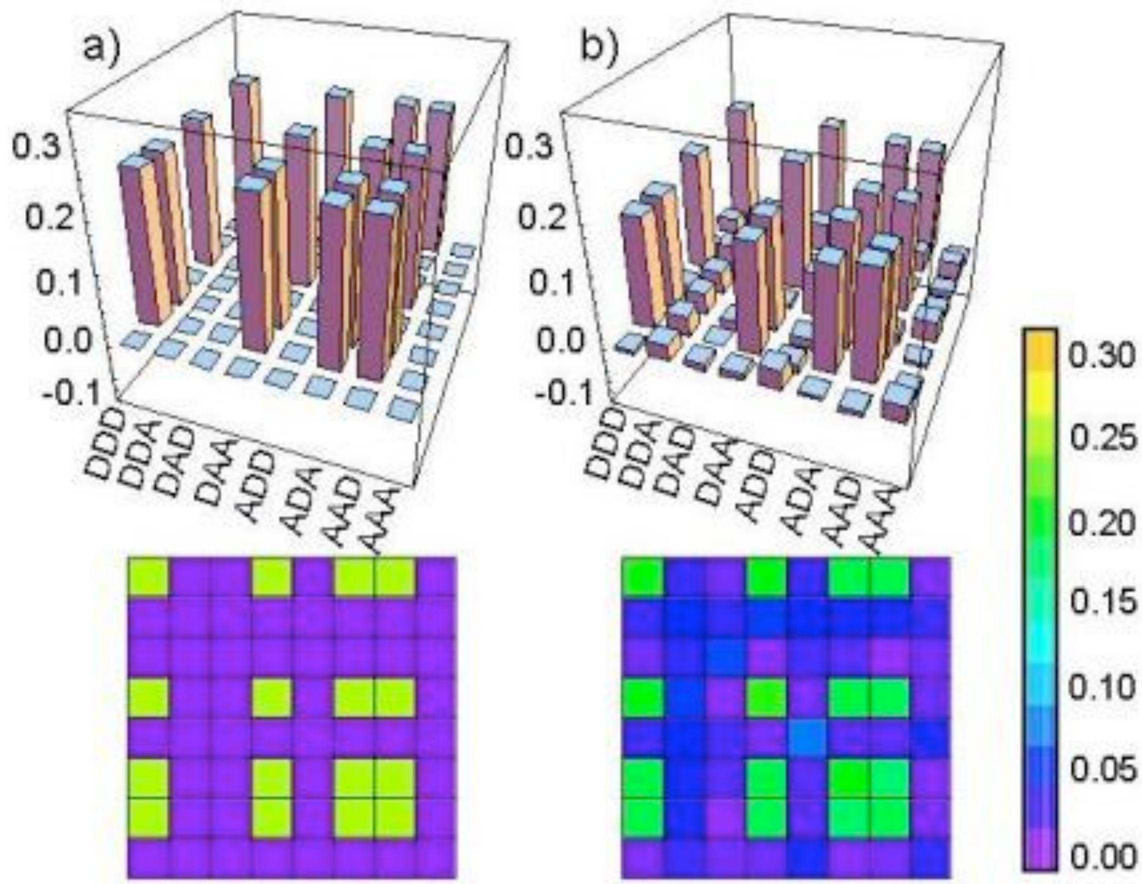


^{17}O (S=5/2) MRI/NMR

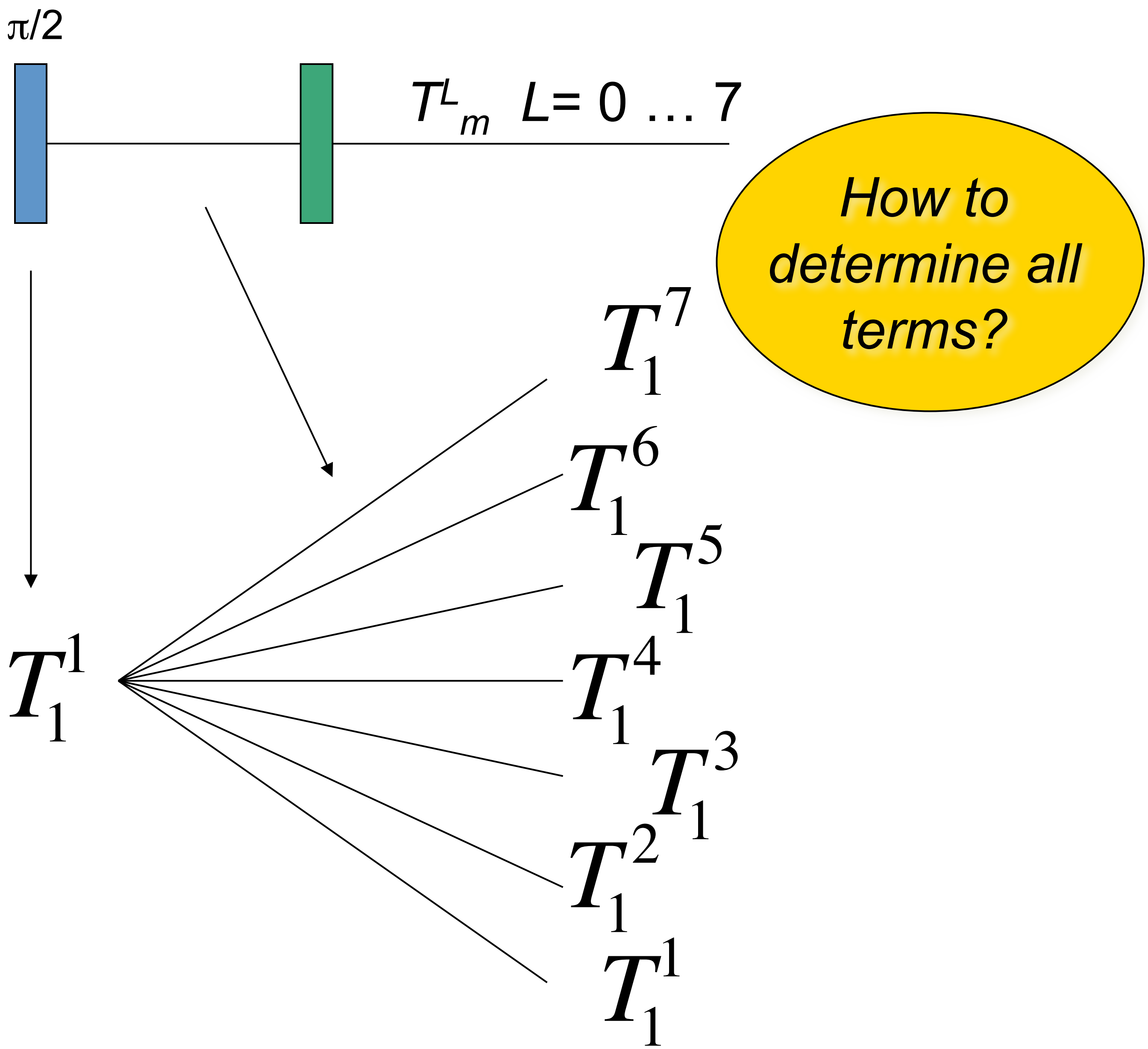


Atkinson, Thulborn, Lu, Haldar, Zhou, Claiborne,
Proc. Intl. Soc. Mag. Reson. Med. 16 (2008) 335

Quantum state tomography

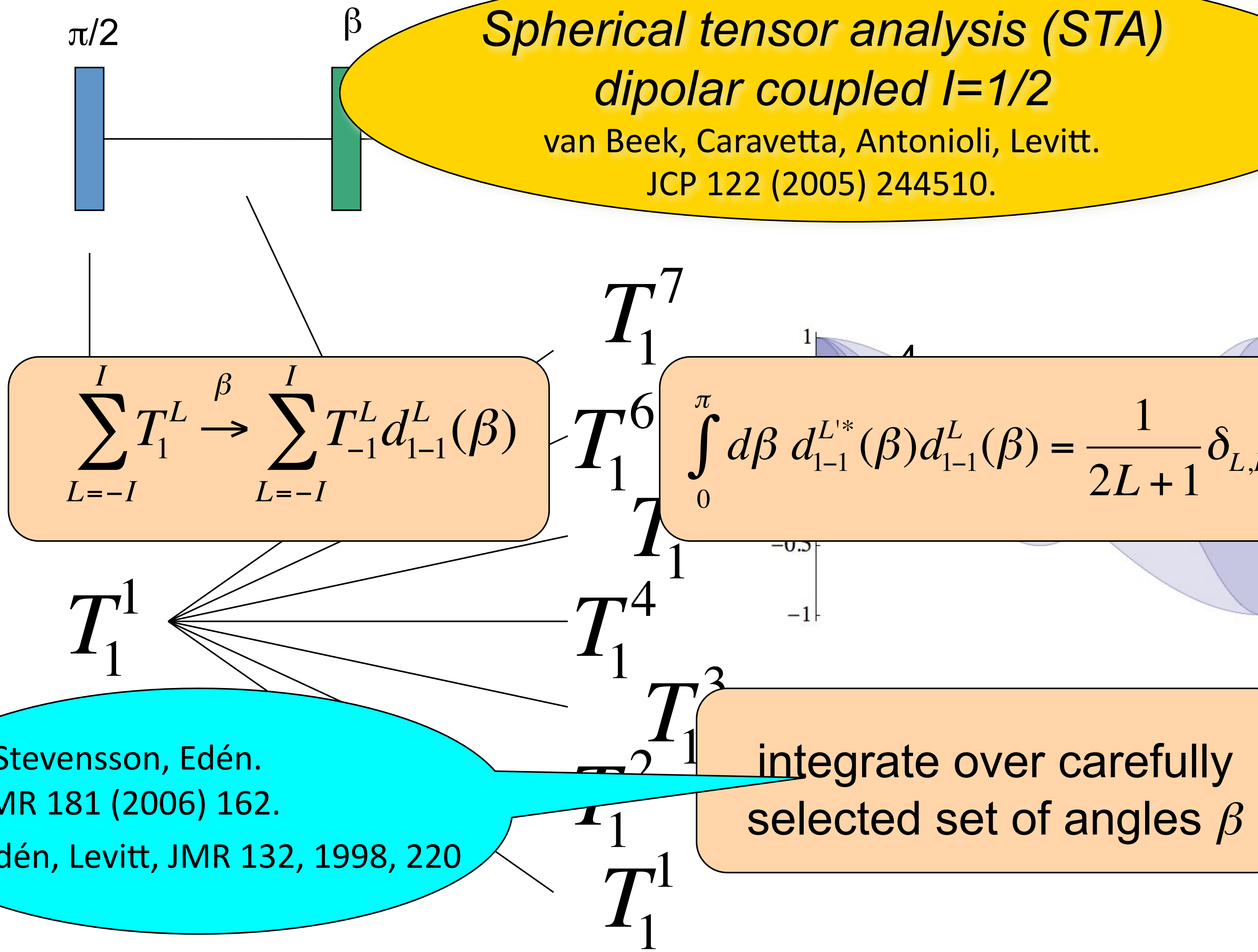


http://homepage.univie.ac.at/philip.walther/
Research_Dateien/DM_before.jpg


π/2
$T^L{}_m$ L= 0 … 7
How to determine all terms?
T_1^7
T_1^6
T_1^5
T_1^4
T_1^3
T_1^2
T_1^1
T_1^1

Spherical tensor analysis (STA) dipolar coupled I=1/2

van Beek, Caravetta, Antonioli, Levitt.
JCP 122 (2005) 244510.



$$\sum_{L=-I}^{I} T_1^L \xrightarrow{\beta} \sum_{L=-I}^{I} T_{-1}^L d_{1-1}^L(\beta)$$

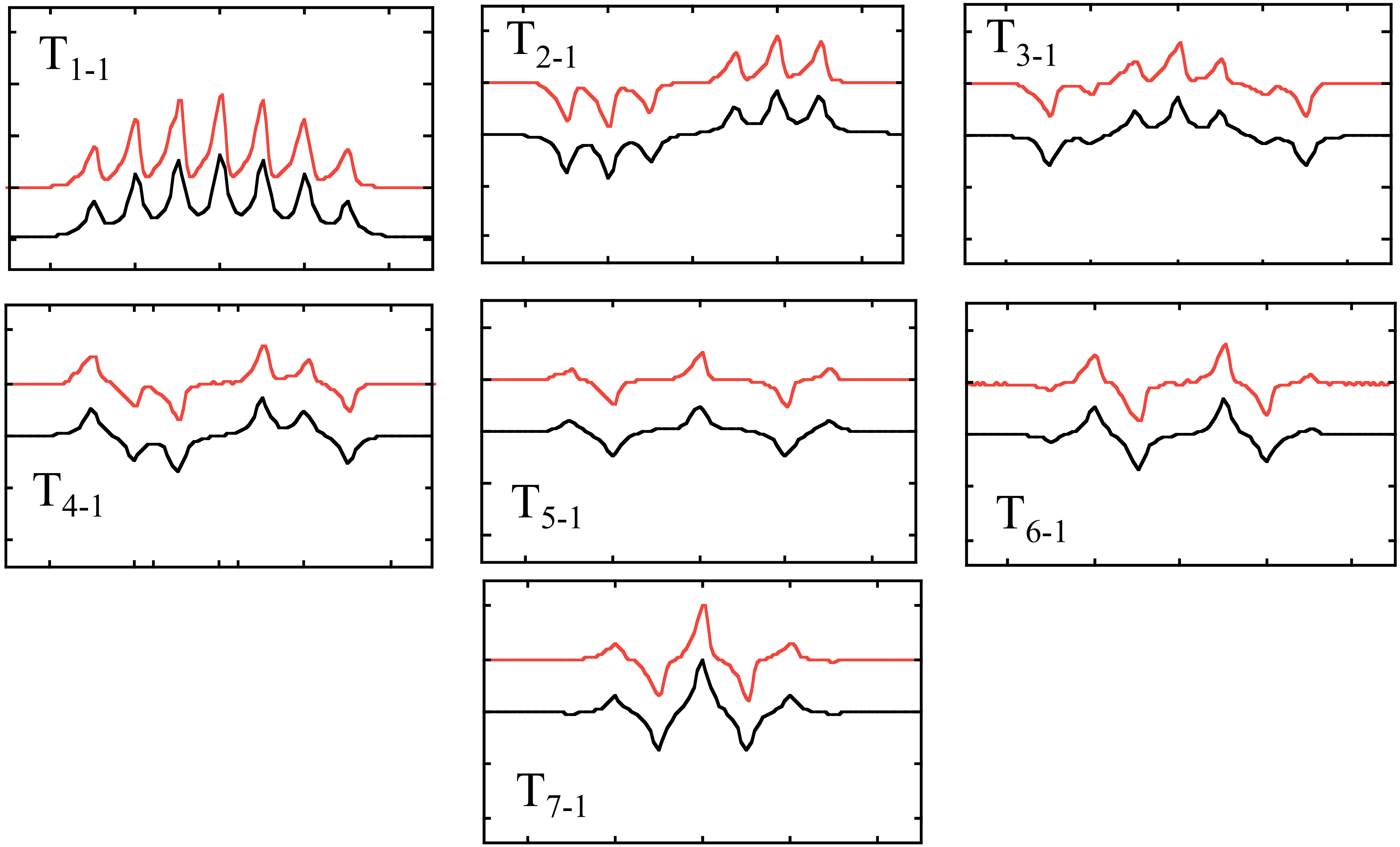
$$\int_0^{\pi} d\beta\; d_{1-1}^{L'*}(\beta) d_{1-1}^{L}(\beta) = \frac{1}{2L+1}\delta_{L,L'}$$

integrate over carefully selected set of angles β

Stevensson, Edén.
JMR 181 (2006) 162.

Edén, Levitt, JMR 132, 1998, 220

^{133}Cs (S=7/2) in liqu. cryst.
STA with 8 angles



Conclusions

- Optimally control spin 3/2 (23Na) NMR/MRI
- Select quadrupolar coupling (cartilage!)
- Select relaxation properties (tumor!)
- Minimize relaxation
- Systematic quantum state tomography for S>1/2
- OCT➔ Health

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$$Bayer/Schering

$$$NIH

$US-IL BSF

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