Optimal control for quantum computing

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Outline

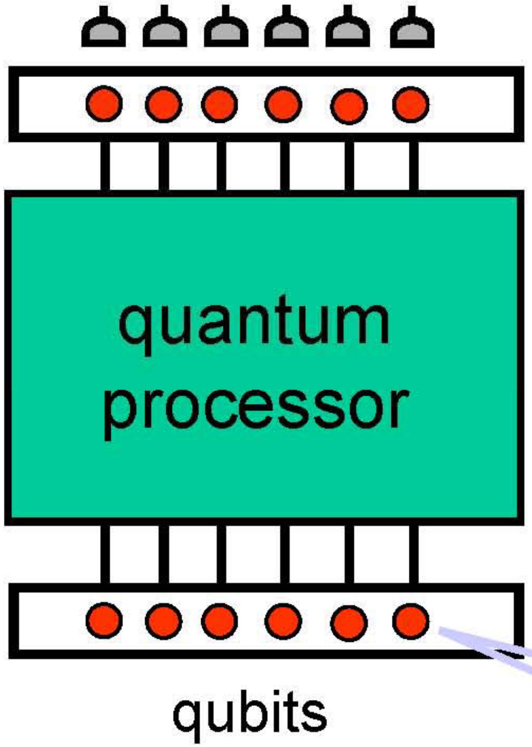
- Scalable quantum information systems
  - what do we need?
- Quantum control
  - what can we do?
- Entanglement generation
  - using different interactions
- What can go wrong
  - and how to fix it
- How much can we push?
  - the Quantum Speed Limit

Scalability

What do we need?

[TC, Grangier, Walraff, Zoller, Nature Phys. ’08]

quantum computing



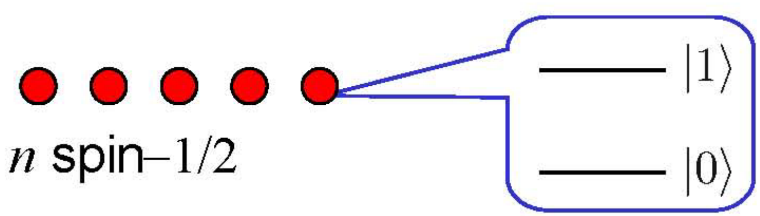
$$|\psi_{\text{out}}\rangle$$

$$|\psi_{\text{out}}\rangle = \hat{U}|\psi_{\text{in}}\rangle$$

$$|\psi_{\text{in}}\rangle = \sum_x c_x |x_{N-1}, \ldots, x_0\rangle$$

quantum register

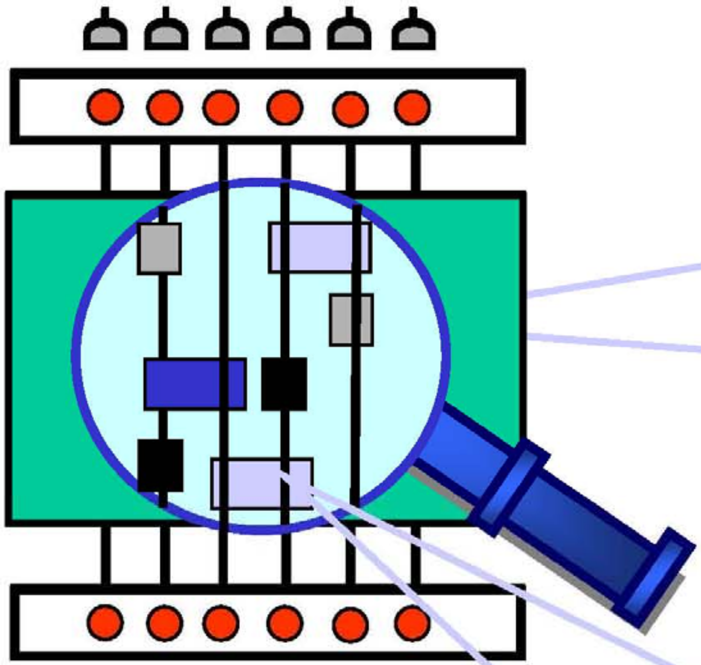
- quantum memory: qubits



example: two qubit entangled state

$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

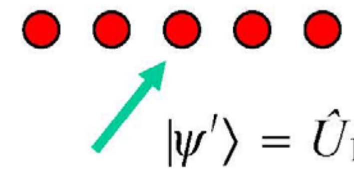
quantum computing



qubits

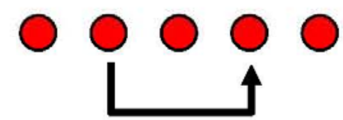
quantum gates

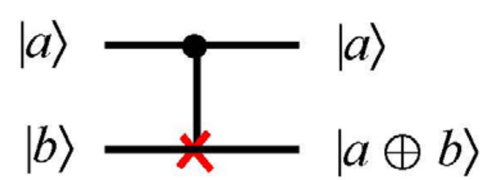
- single qubit gate



$$|\psi'\rangle = \hat{U}_1|\psi\rangle$$

- two qubit gate: entanglement

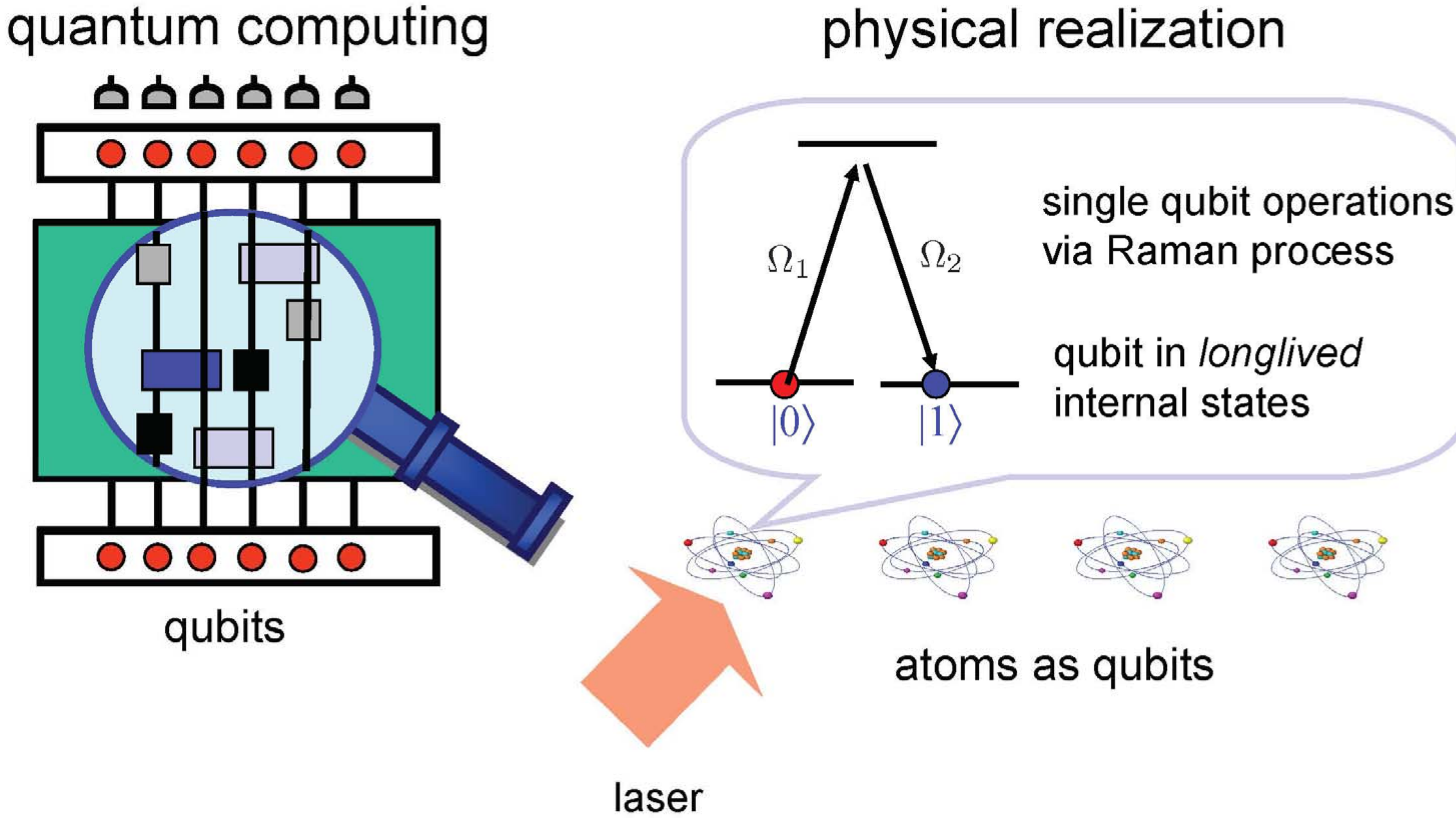




CNOT

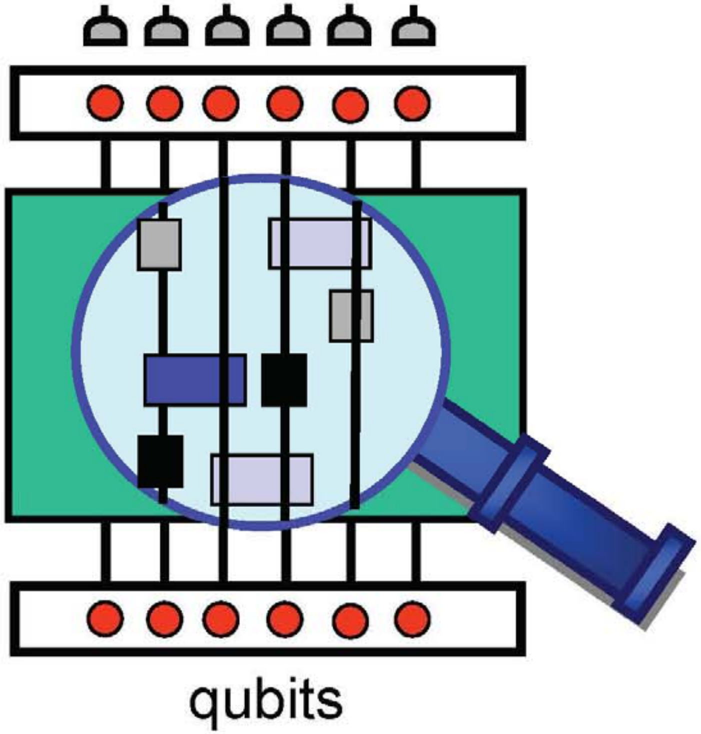
truth table

$\|0\rangle\|0\rangle$	$\rightarrow$	$\|0\rangle\|0\rangle$
$\|0\rangle\|1\rangle$	$\rightarrow$	$\|0\rangle\|1\rangle$
$\|1\rangle\|0\rangle$	$\rightarrow$	$\|1\rangle\|1\rangle$
$\|1\rangle\|1\rangle$	$\rightarrow$	$\|1\rangle\|0\rangle$

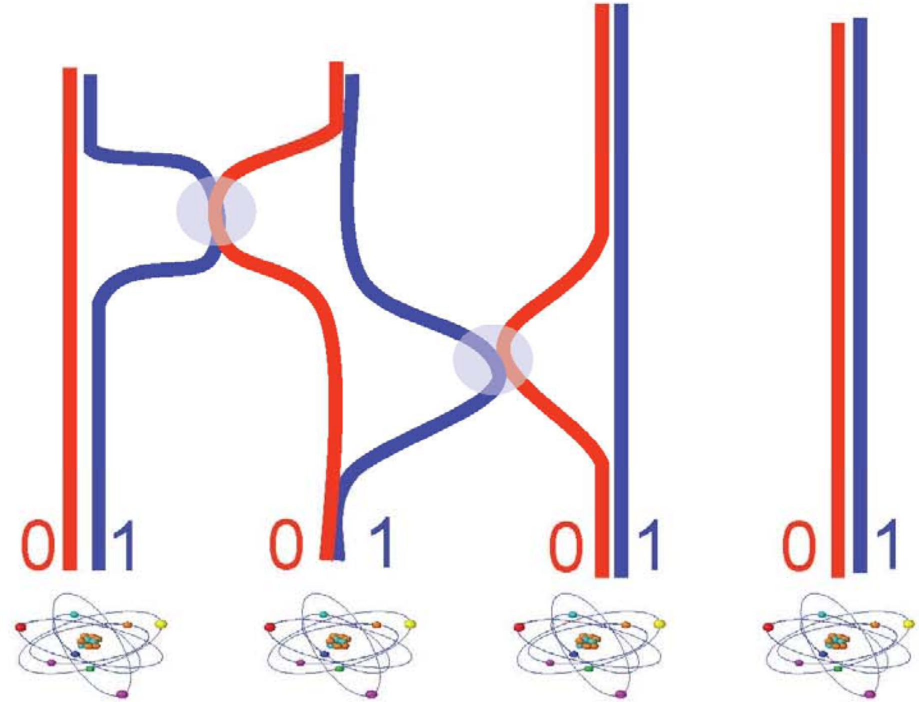


Requirements:

addressing
single qubit

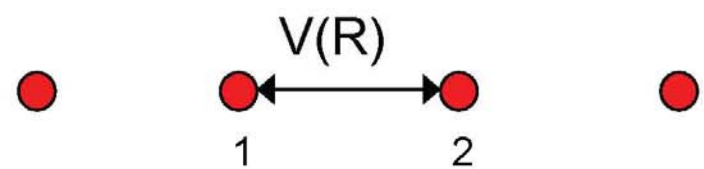


entangling qubits



controllable two body interactions:

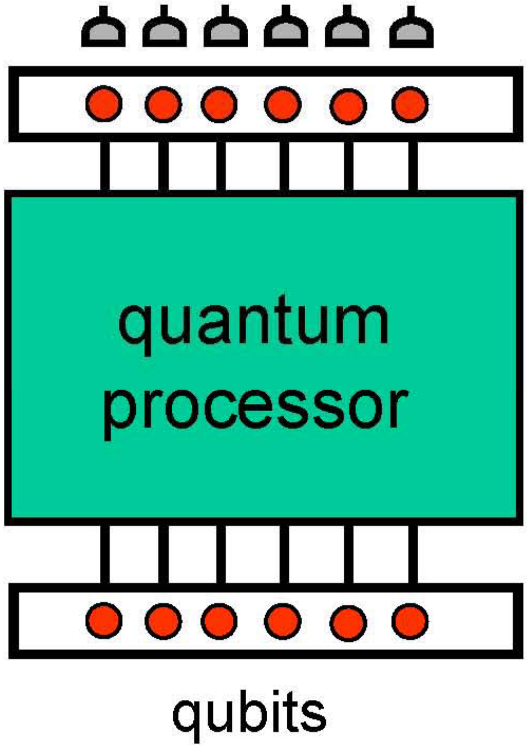
controlled collisions, ...



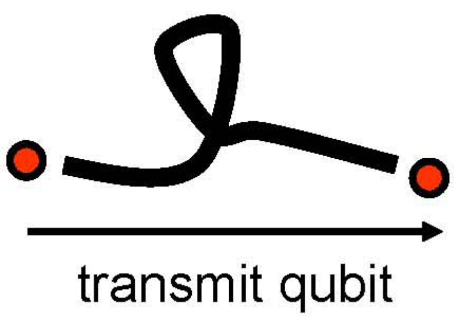
Hamiltonian $H = \Delta E(t) |1\rangle_1 \langle 1| \otimes |1\rangle_2 \langle 1|$

so that $|1\rangle_1 |1\rangle_2 \;\rightarrow\; e^{i\phi} |1\rangle_1 |1\rangle_2$

DiVincenzo Criteria



1. scalable system of well-characterized qubits
2. initialize qubits
3. long decoherence times
4. universal set of quantum gates
5. qubit readout

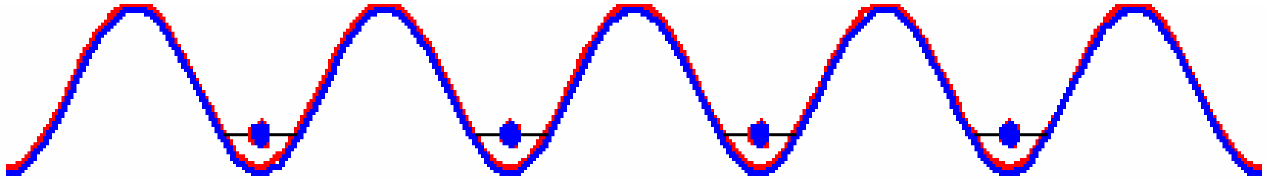


6. interconvert stationary and flying qubits
7. faithful transmission of qubits between specified locations

GOAL: satisfy requirements of fault tolerant quantum computing

Scalability desiderata

- Memory:
  - Quantum register with many qubits
  - Low decoherence rates
- Gates:
  - Fast operation
  - High fidelity
- …implementation with ultracold systems:
  - Good isolation from environment
  - Individual control
  - Periodic potentials



Scalability in practice

- Nobody really knows how to build a working QC
  - need to have quite fast AND ultra-accurate gates
- Specific practical problems – for instance
  - transport of particles in traps
  - strong coupling to control vs weak coupling to environment
- Quantum Optimal Control Theory
  - tailored answers to specific problems
  - …robustness to noise?

Control

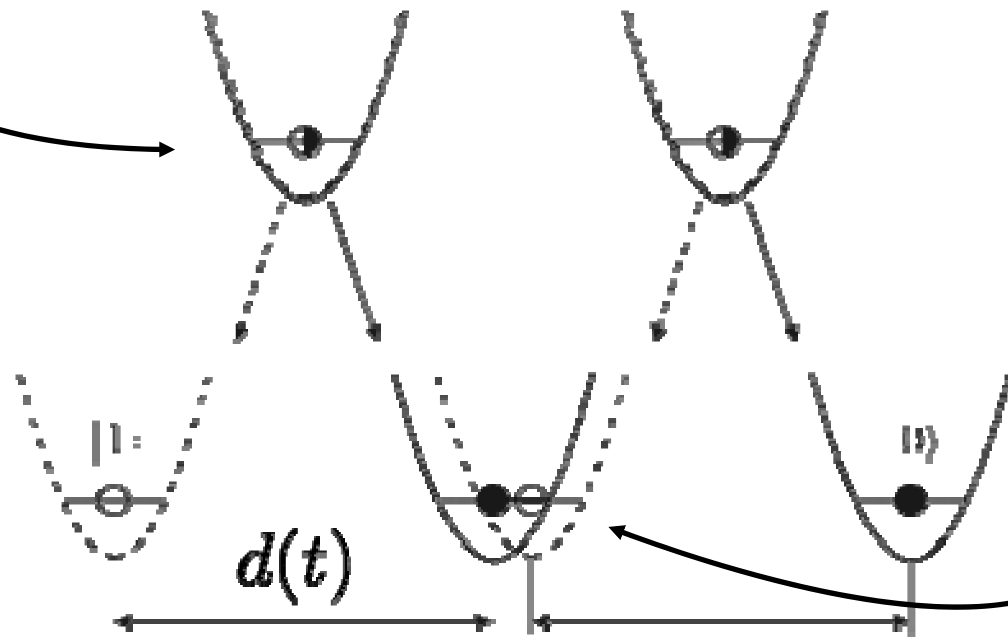
What can we do?

Example: transport in traps

Initial state — $|\psi_0\rangle$

Hamiltonian — $\mathcal{H}(d(t), t)$

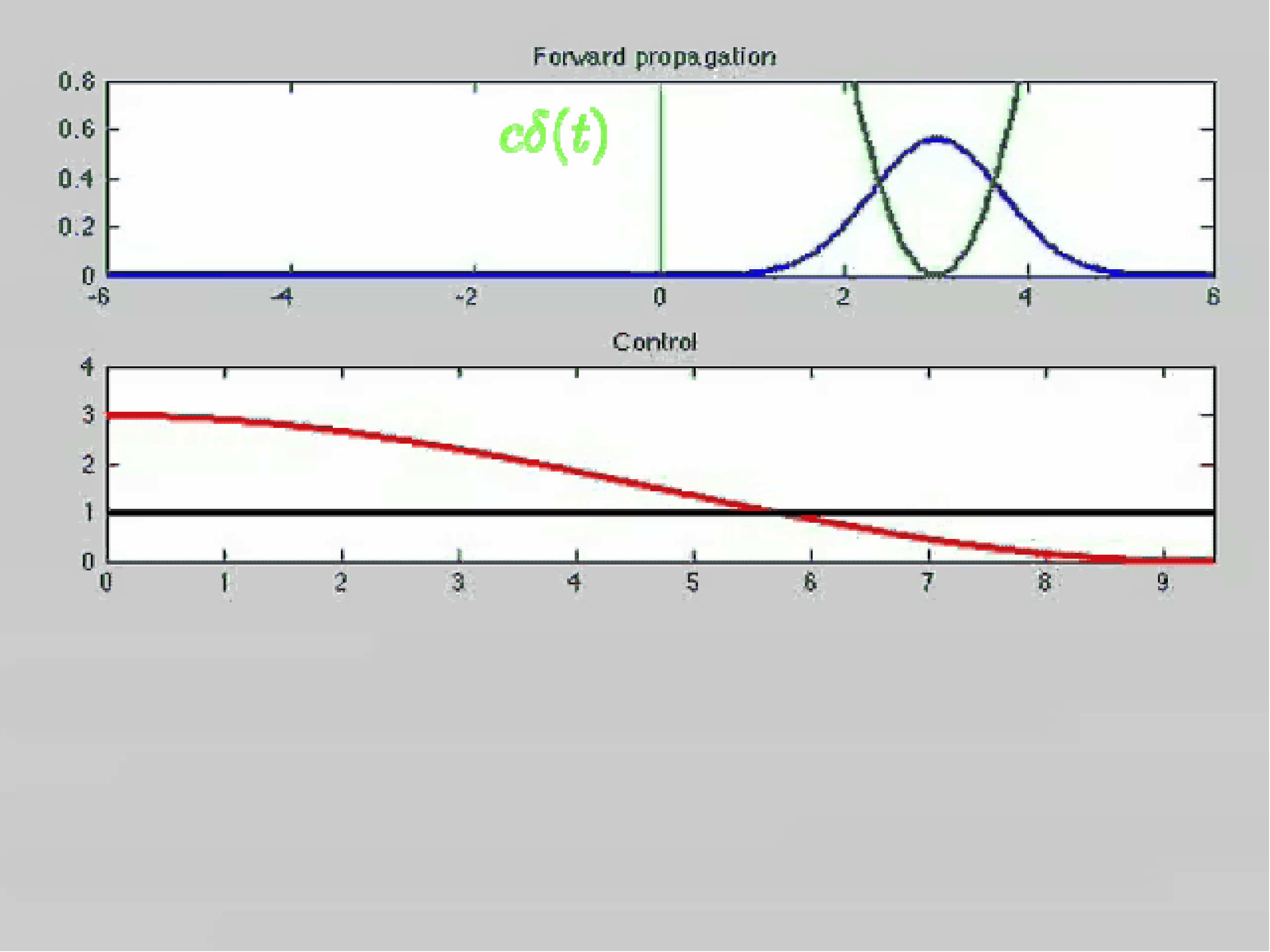
Evolved state — $|\psi(T)\rangle$



Task: minimize error $1 - \mathcal{F} = 1 - |\langle \psi_{\mathbf{goal}} | \psi(T) \rangle|^2$

Optimization via Krotov

$$\frac{m\omega^2(t)}{2}[x - d(t)]^2$$



$d(t)$ →

$\omega(t)$ →

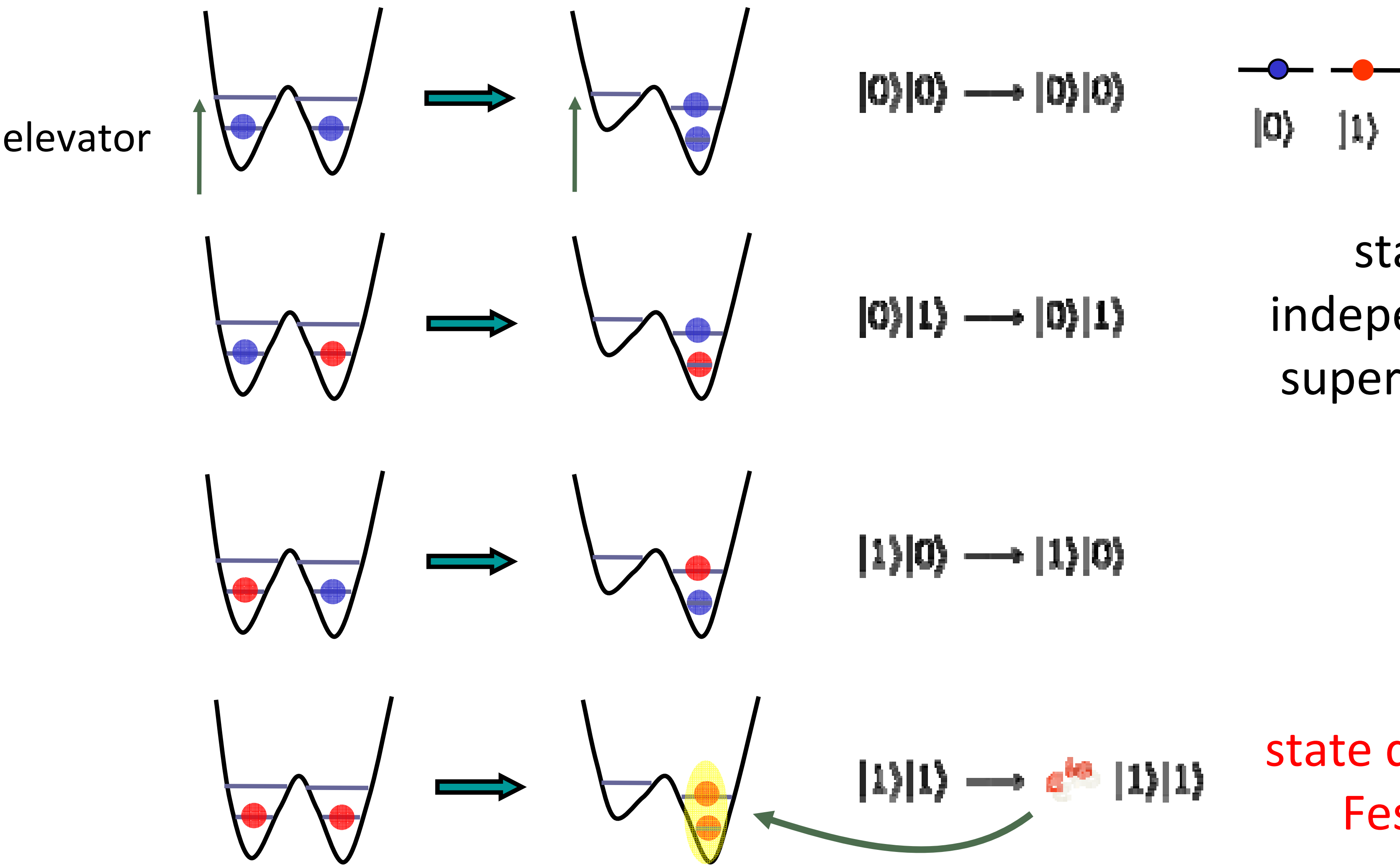
$\mathcal{F}$

Entanglement

using atomic interactions

Entangling Feshbach resonances

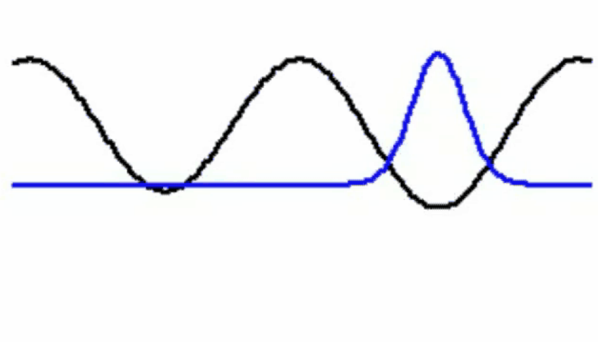
with P. Julienne, P. Zoller '04



state independent superlattice

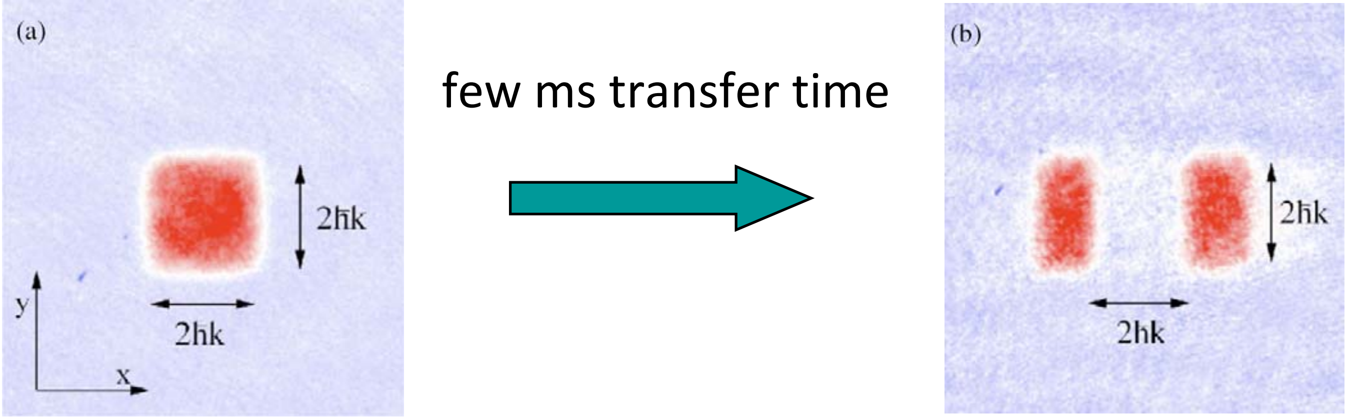
state dependent Feshbach resonances

Transport in dipole traps



© T. Porto, W. Phillips 2005

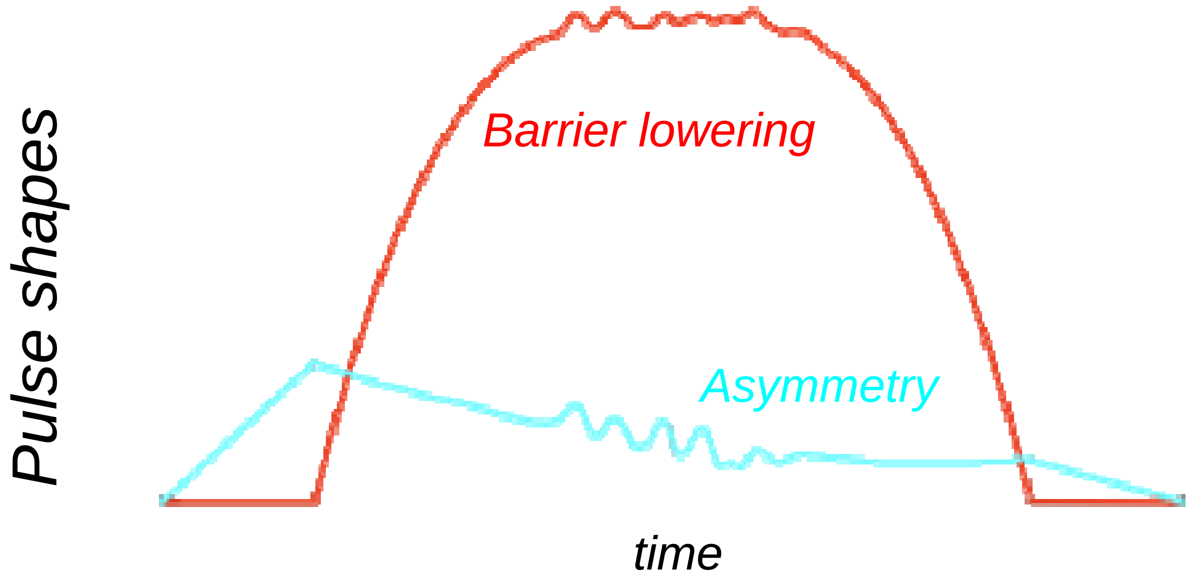
Realization of (not time-optimized) transport in an optical lattice



...two-qubit gate: W. Phillips, Nature 2007

Optimized pulses

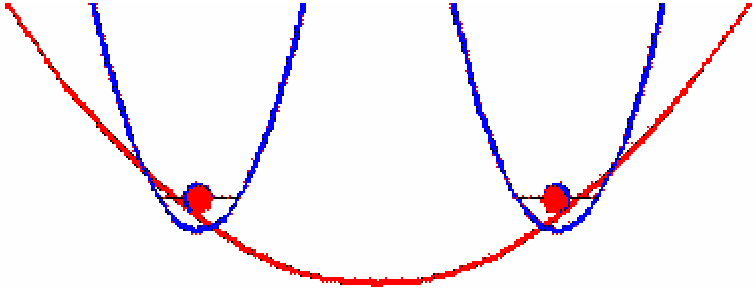
- Optimization algorithm introduces wiggles in pulse shapes
- “Shaking” helps exciting-deexciting
- Frequency higher than gate operation rate

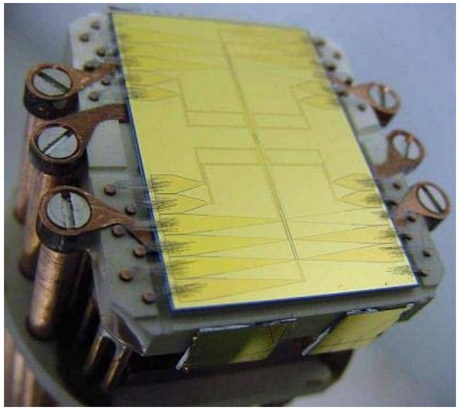


Switching gates on atom chips

with R. Folman, J. Schmiedmayer ‘00

- Modulated magnetic field yields state-dependent potential
- State-independent electrostatic attraction switches off the barrier
- Logical phase accumulated at each collision





© R. Folman, J. Schmiedmayer

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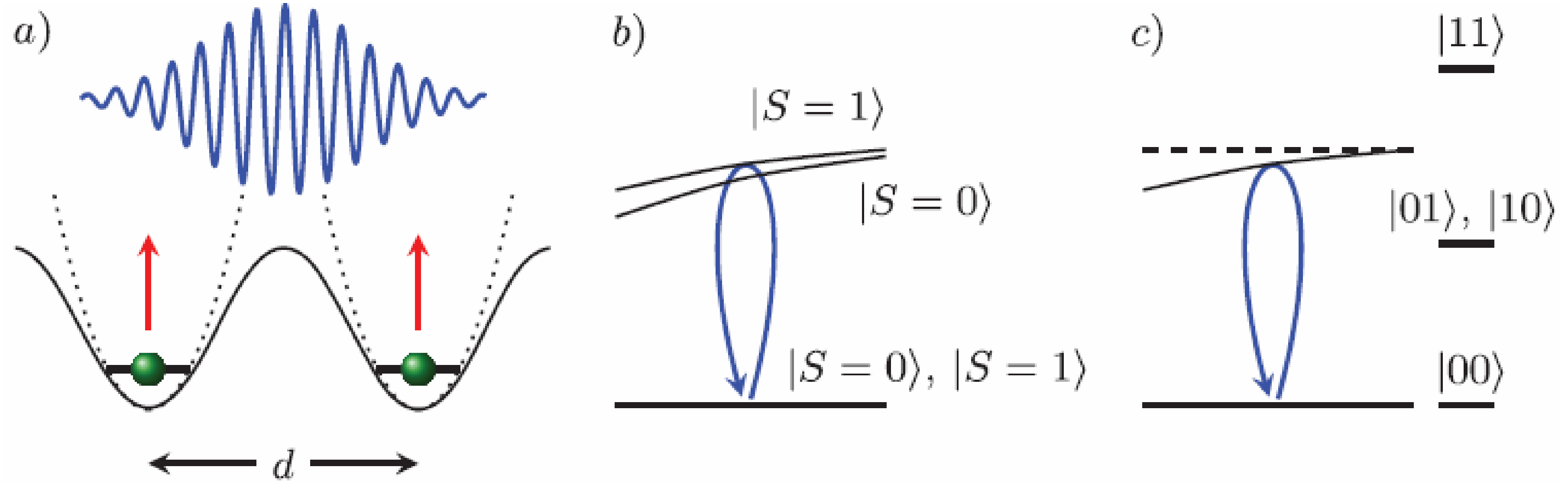
Ultrafast optical gate

ongoing collaboration with C. Koch



$$|\uparrow\rangle_1|\uparrow\rangle_2 = |S=1, m=1\rangle$$

$$|\uparrow\rangle_1|\downarrow\rangle_2 = \frac{|S=1, m=0\rangle + |S=0, m=0\rangle}{\sqrt{2}}$$

$$|\downarrow\rangle_1|\uparrow\rangle_2 = \frac{|S=1, m=0\rangle - |S=0, m=0\rangle}{\sqrt{2}}$$

$$|\downarrow\rangle_1|\downarrow\rangle_2 = |S=1, m=-1\rangle$$

$$\frac{1}{2}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1+e^{i\phi} & 1-e^{i\phi} & 0 \\ 0 & 1-e^{i\phi} & 1+e^{i\phi} & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$\phi = \frac{\pi}{2} \longrightarrow \sqrt{SWAP}$$

What can go wrong

and how to fix it

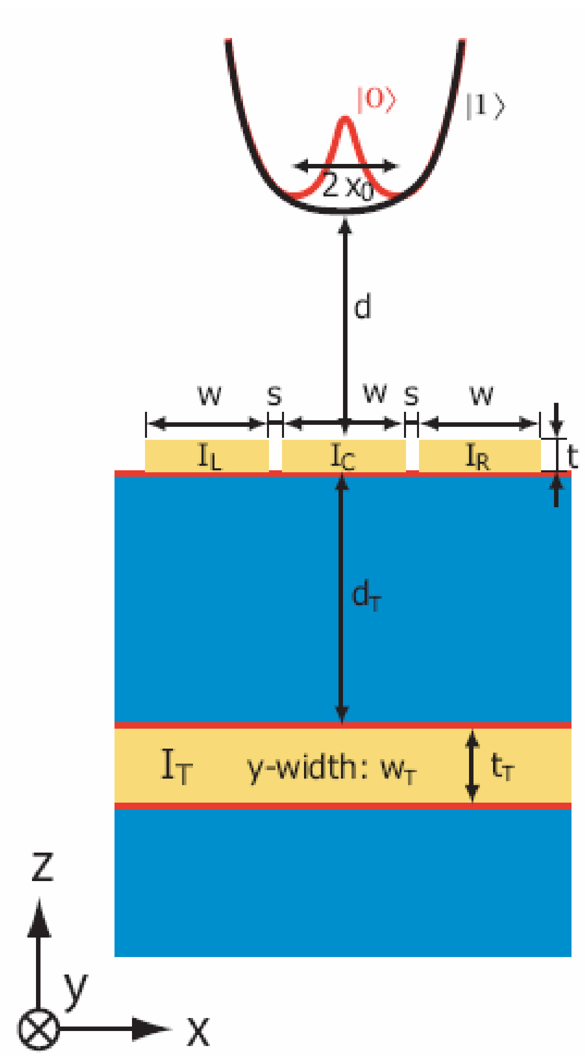
What can go wrong?

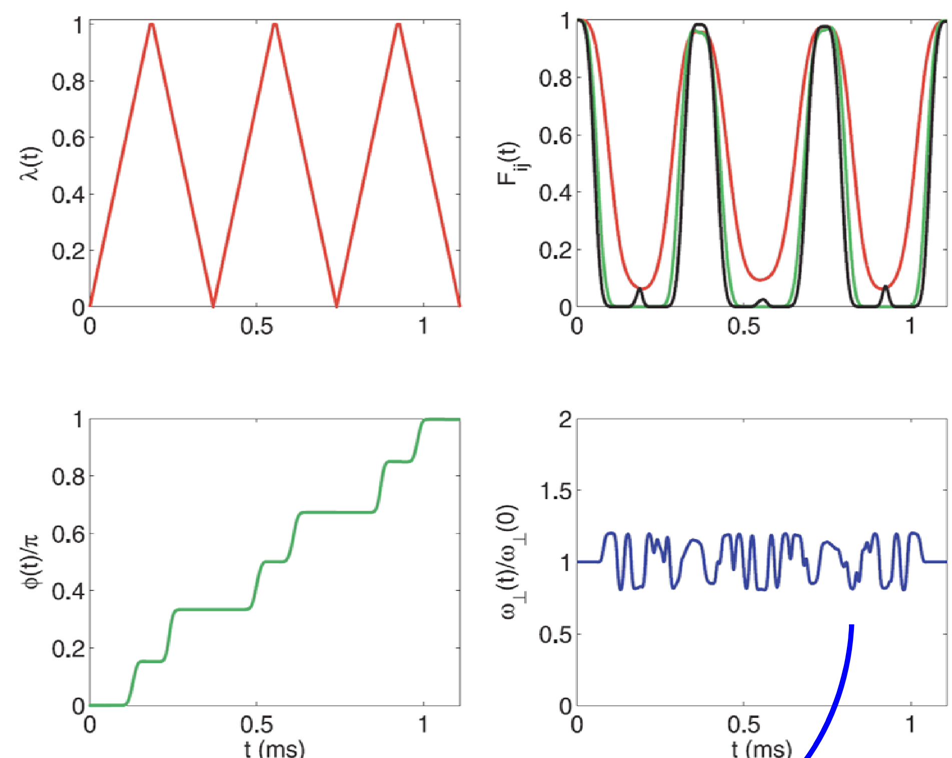
- Anharmonicity in the trapping potentials
- Noise in the control parameters
- Limited bandwidth
- Imperfect pulse calibration
- Leakage
- Finite temperature
- Inhomogeneous broadening
- ...decoherence

Anharmonicity
in the trapping potential

Microwave pulse shaping on atom chips

with J. Reichel, T. Hänsch '06





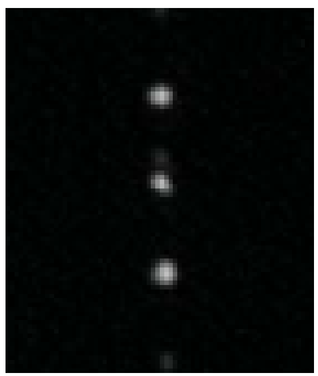
Transverse frequency modulation compensates for interaction-induced wavepacket distortion

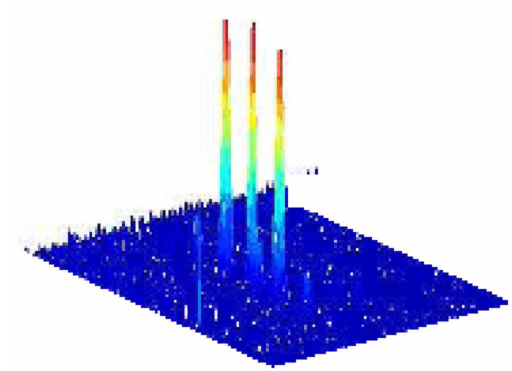
Noise in the control parameters

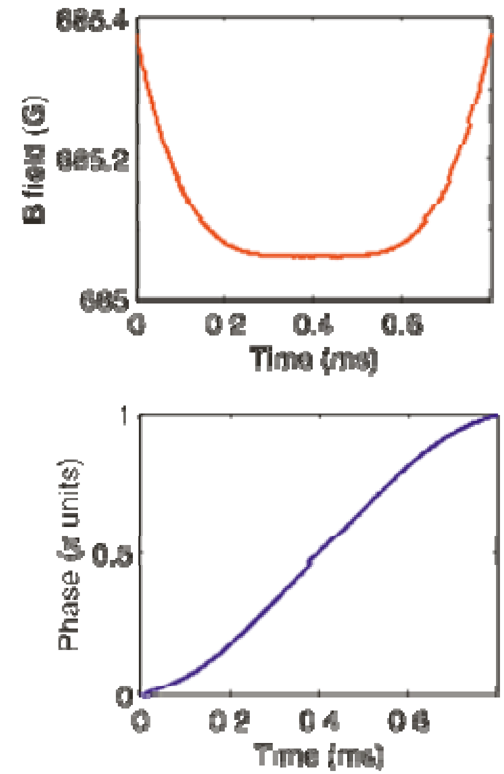
Noise in dipole traps

with P. Grangier ‘05

Holographic tweezer setup





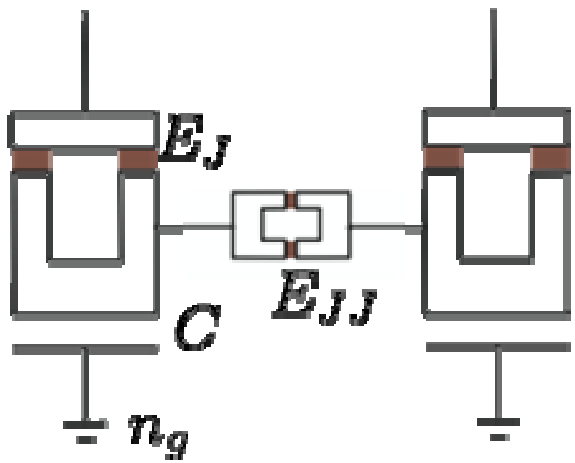


Typical noise for laser intensity and position around 10^{-2}

Fidelity decrease ~ 10^{-3} due to timescale separation between noise and control

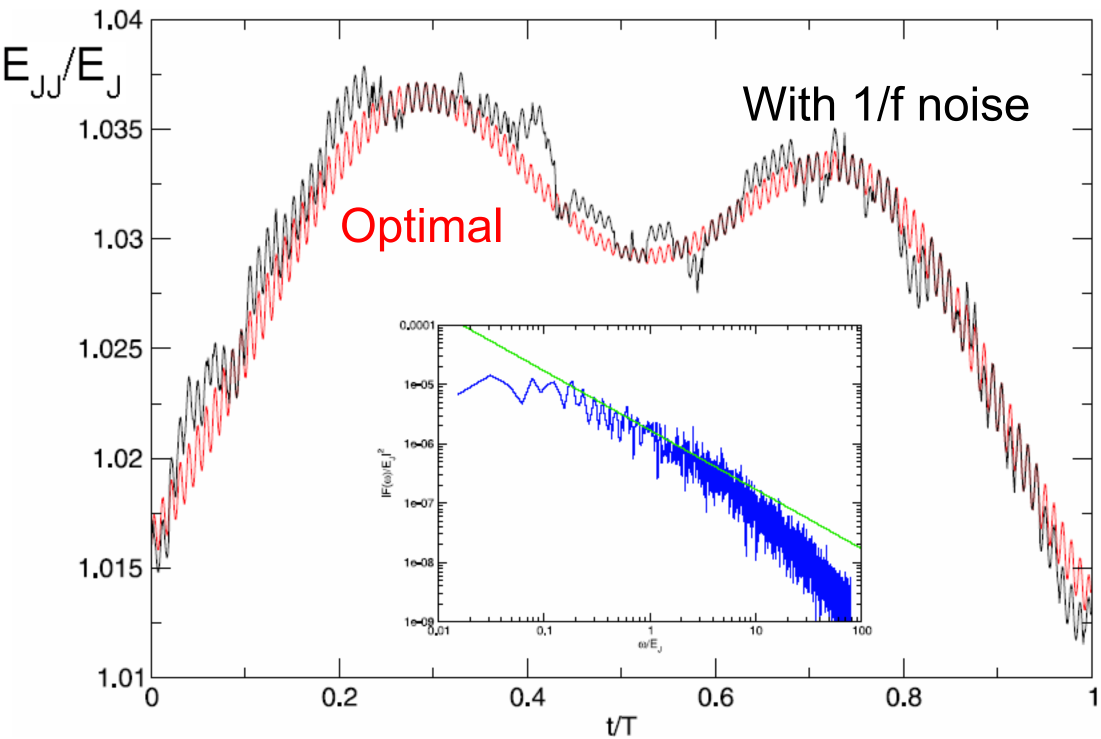
Noise in Josephson charge qubits

with R. Fazio '07



$$G_{JJ} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & \pm i & 0 & 0 \\ 0 & 0 & \pm i & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

- Qubit: 0 or 1 excess Cooper pair
- Control parameter: Josephson energy E_{JJ}



Error with/without control

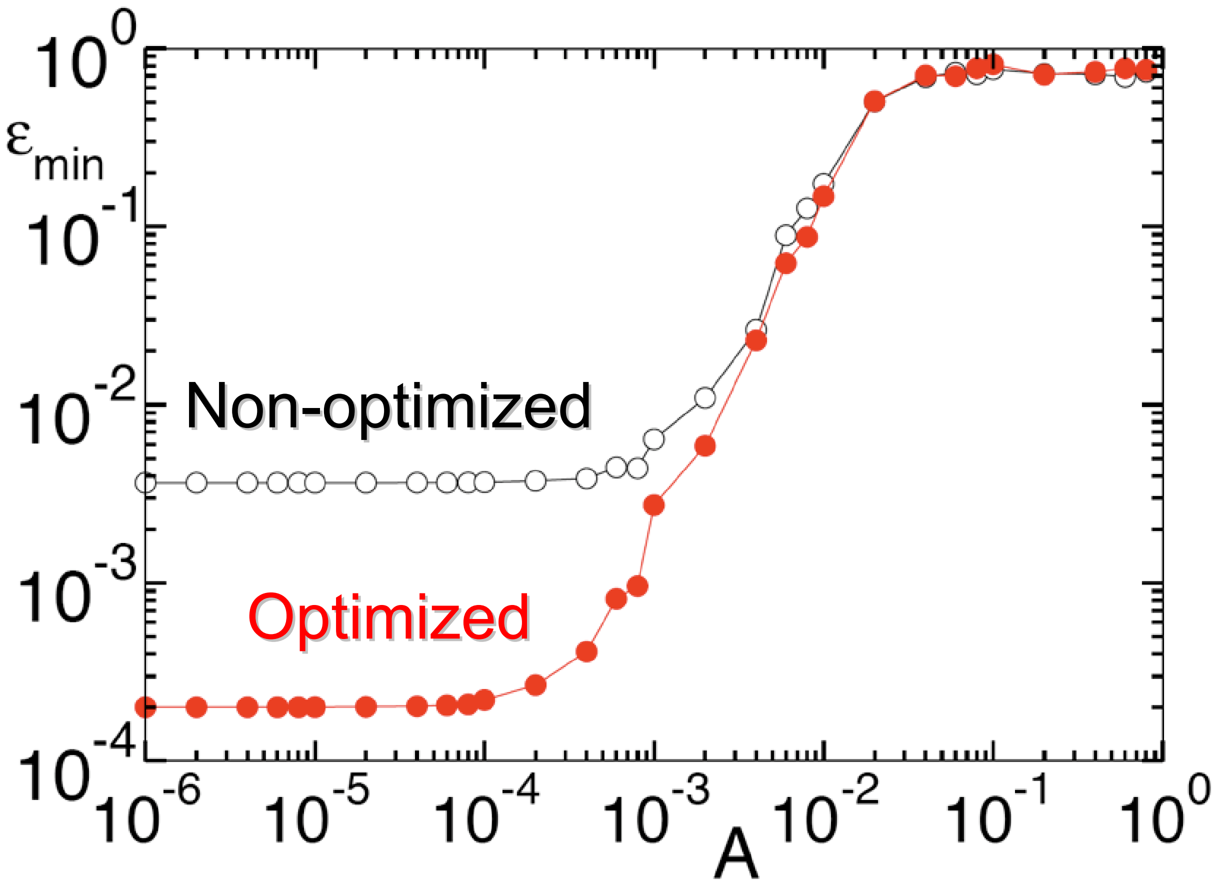
1/f noise

$$S(\omega) \propto A/\omega$$

Typical exp. values

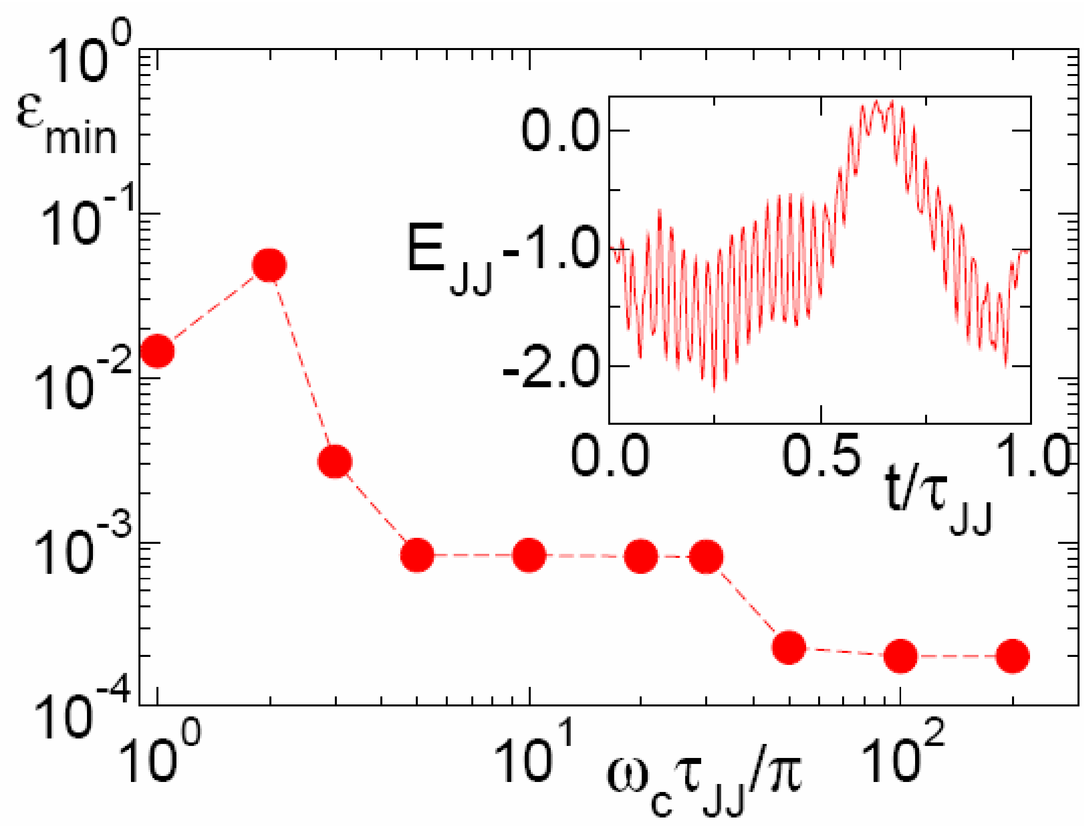
$$A \sim 10^{-5}$$

Fault tolerance with realistic noise?



Limited bandwidth

Limited bandwidth in JJ gates

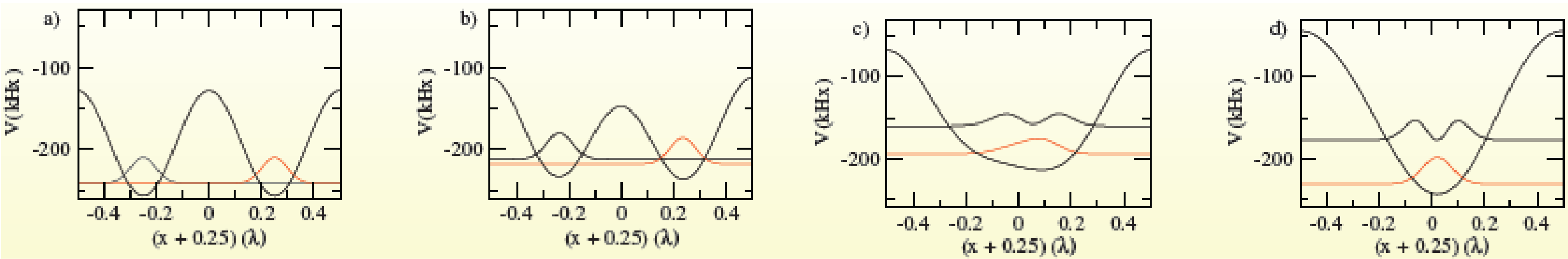


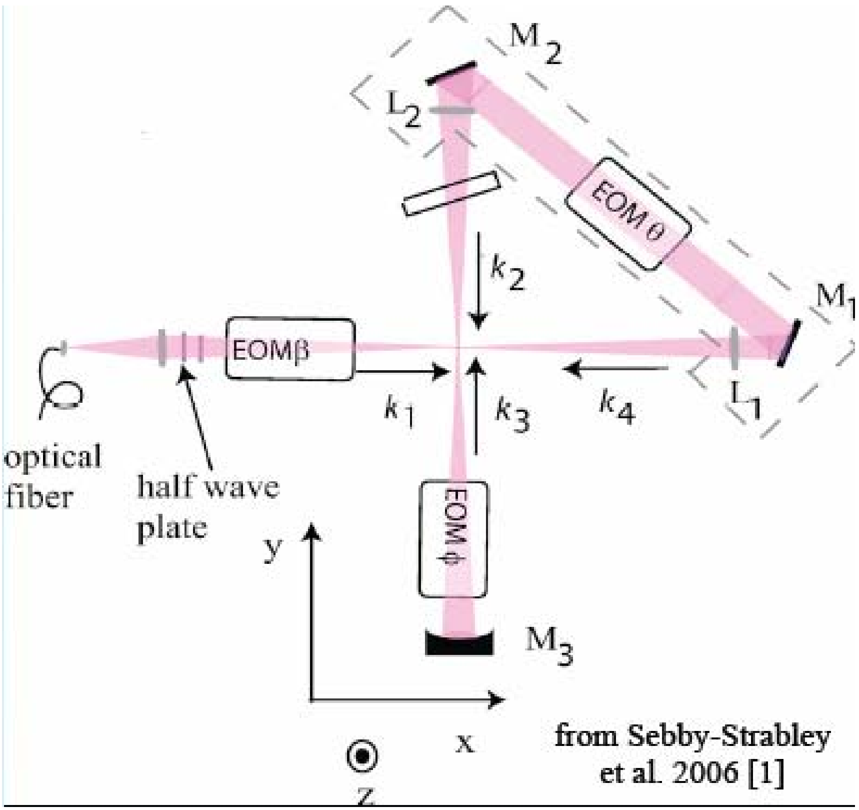
Pulse shape constrained in Fourier space

Imperfect pulse calibration

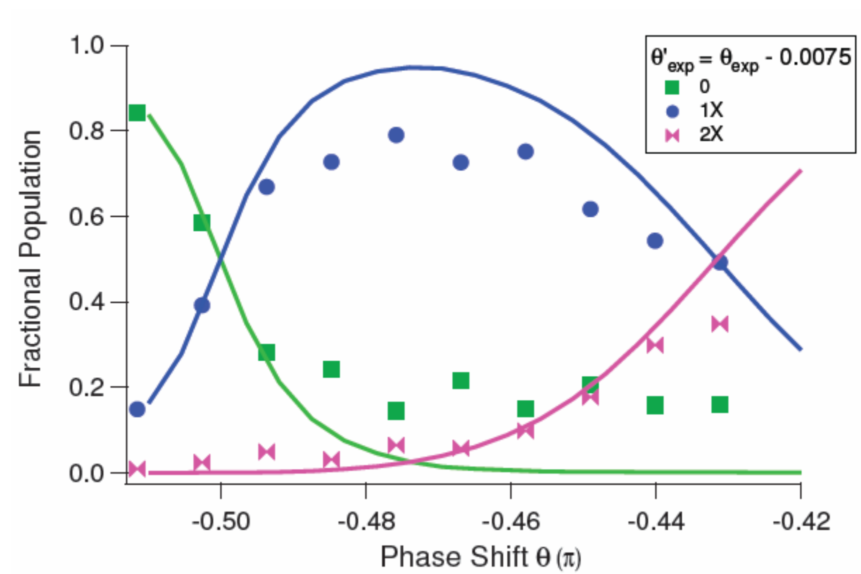
Transport in a real lattice

with T. Porto, W. Phillips ‘08



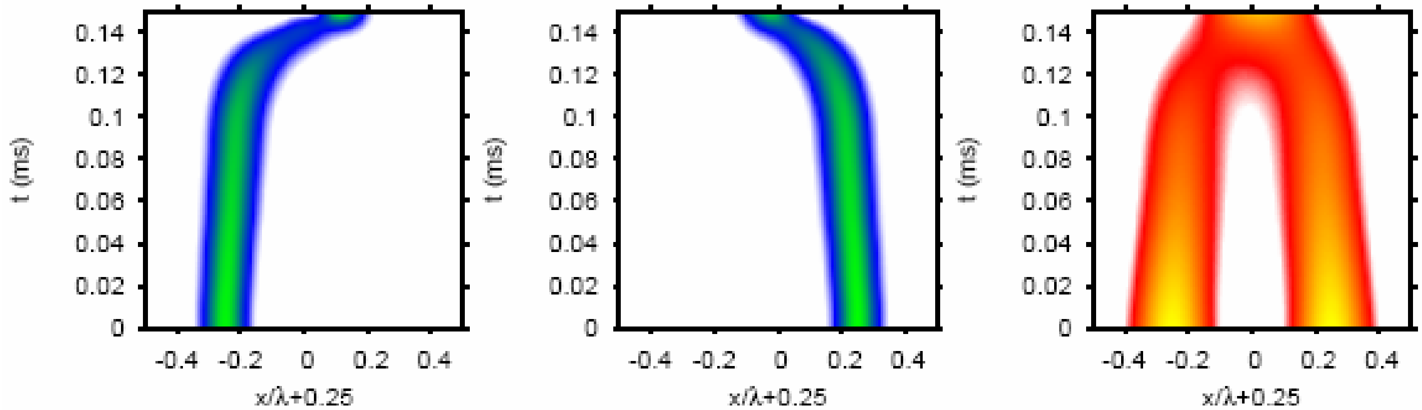


Calibration of the control simulation



Optimization results

Without optimization: F=0.22

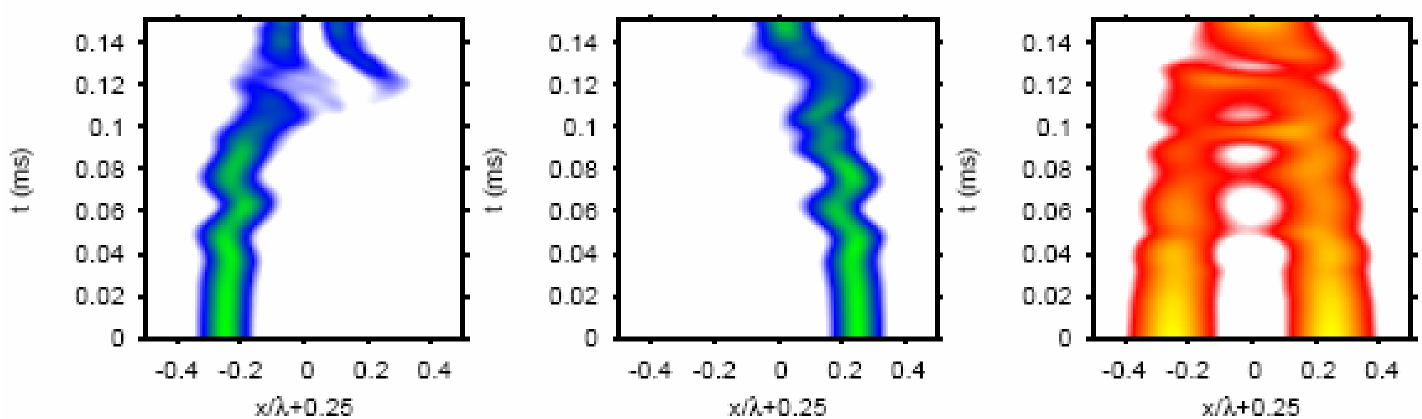


T=150 μs

Wavefunctions

Potential

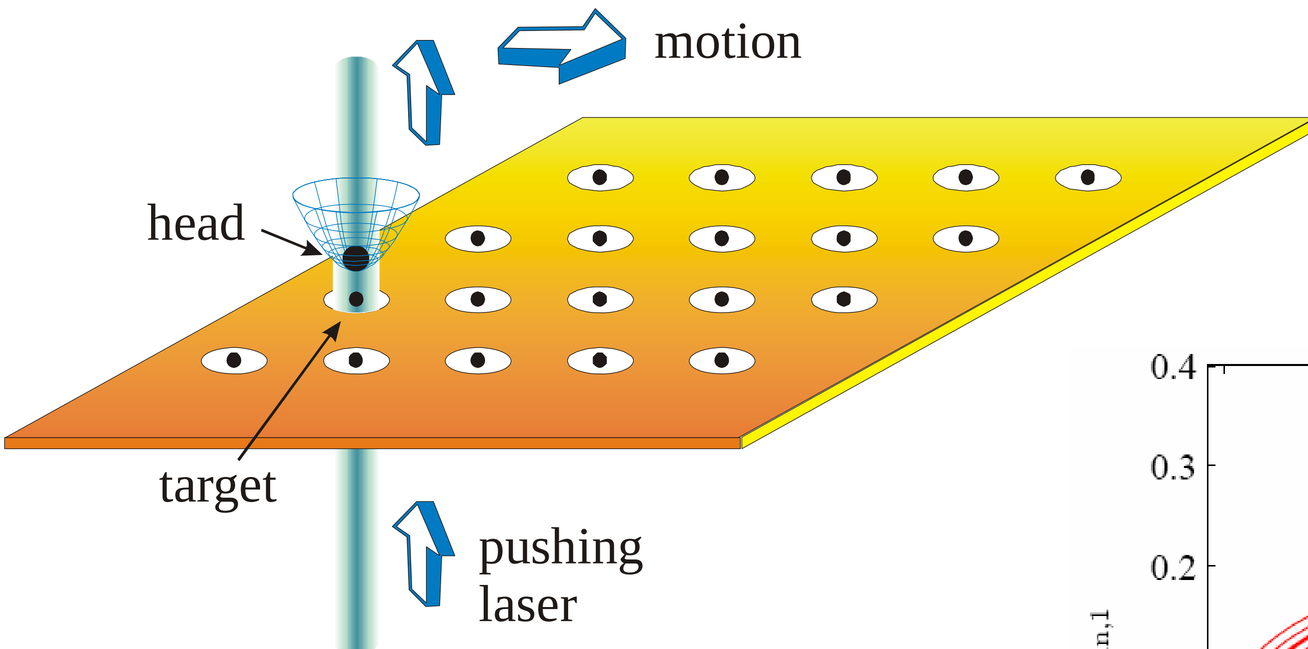
With optimization: F=0.97



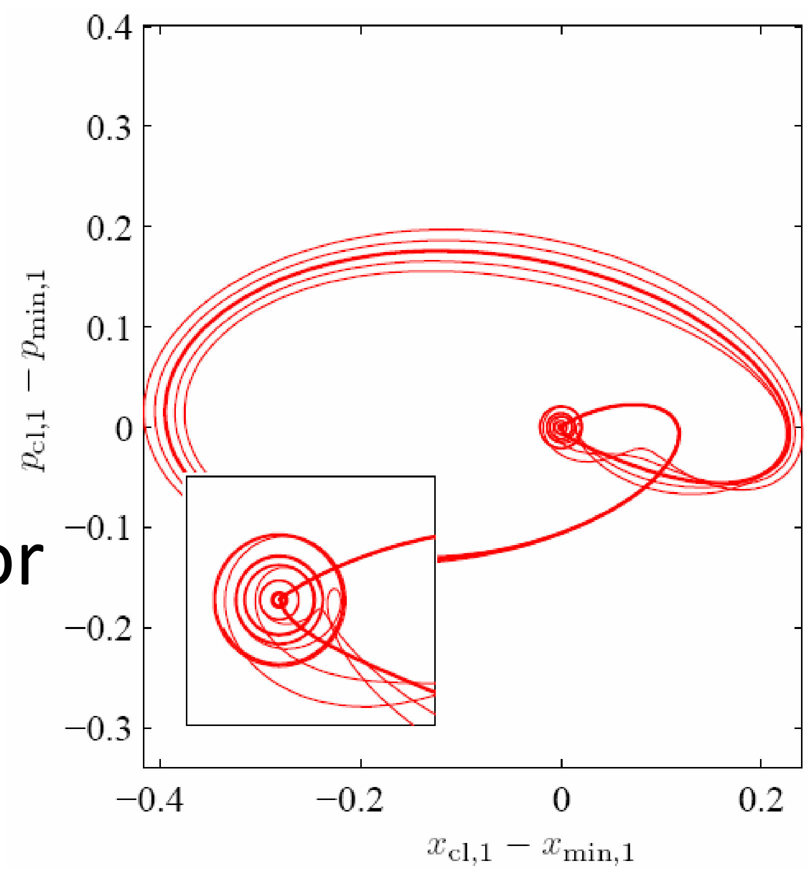
Leakage

Leakage in ion-pushing gates

with D. Tannor '09

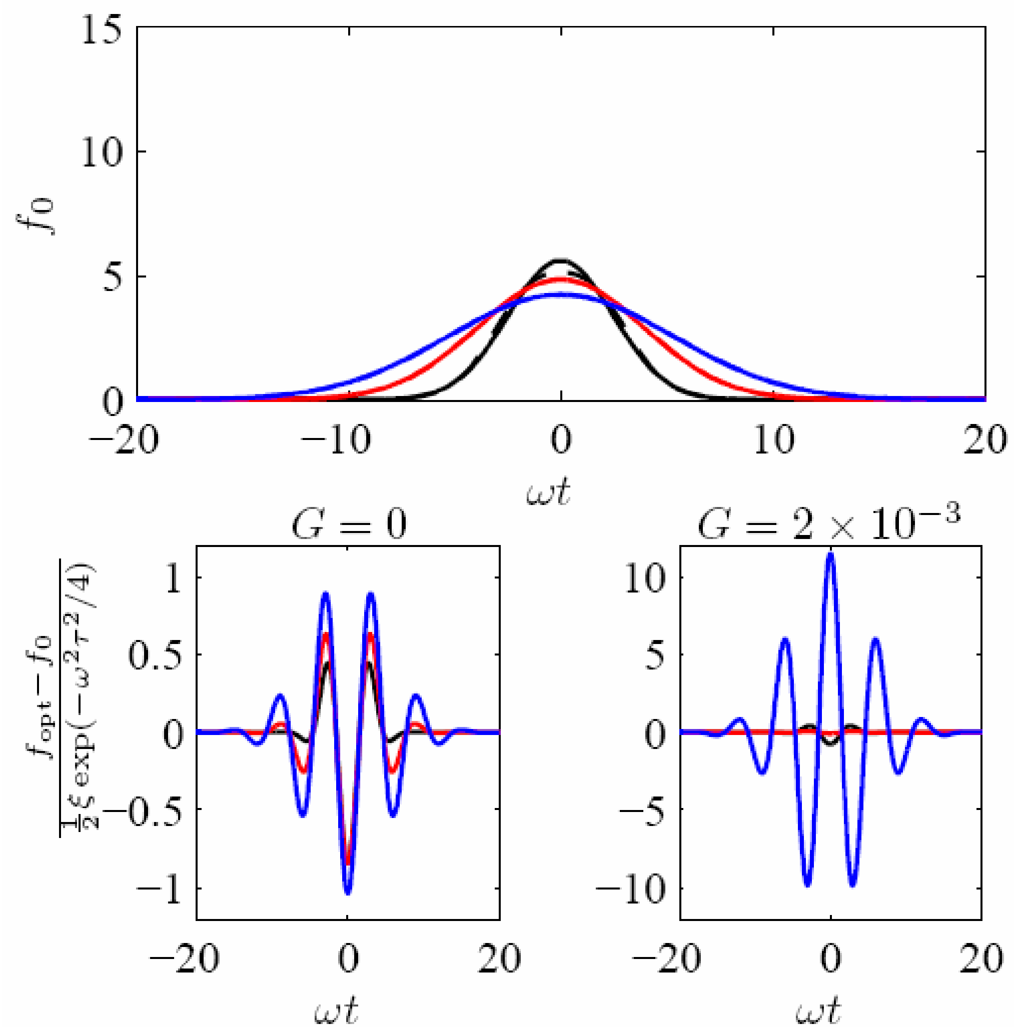


Phase-space ion trajectories for inhomogeneous pushing force

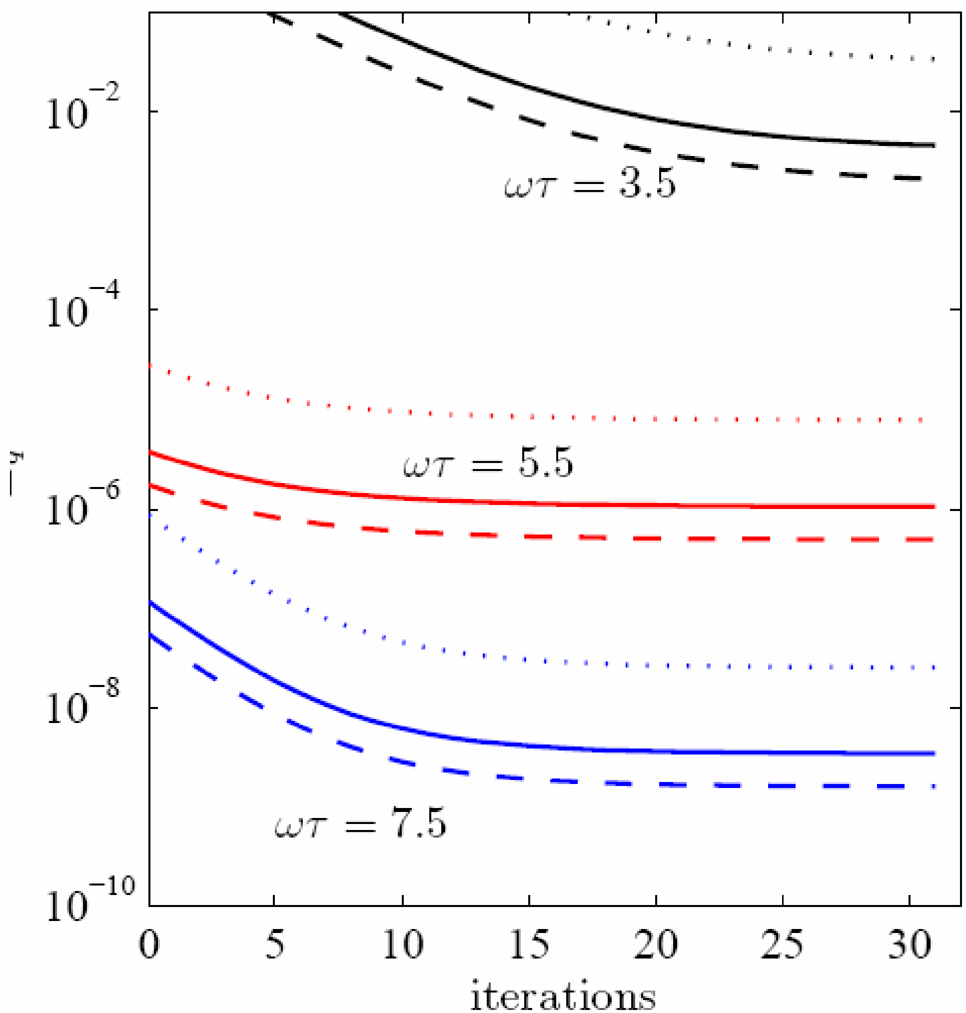


Krotov optimization

Pulses



Errors

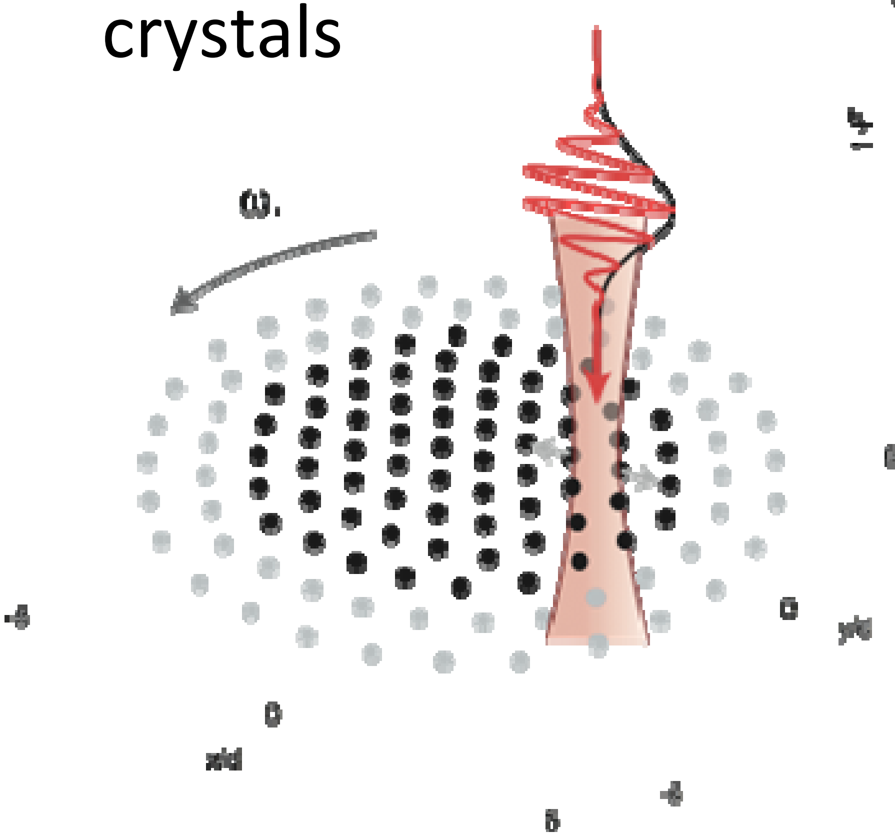


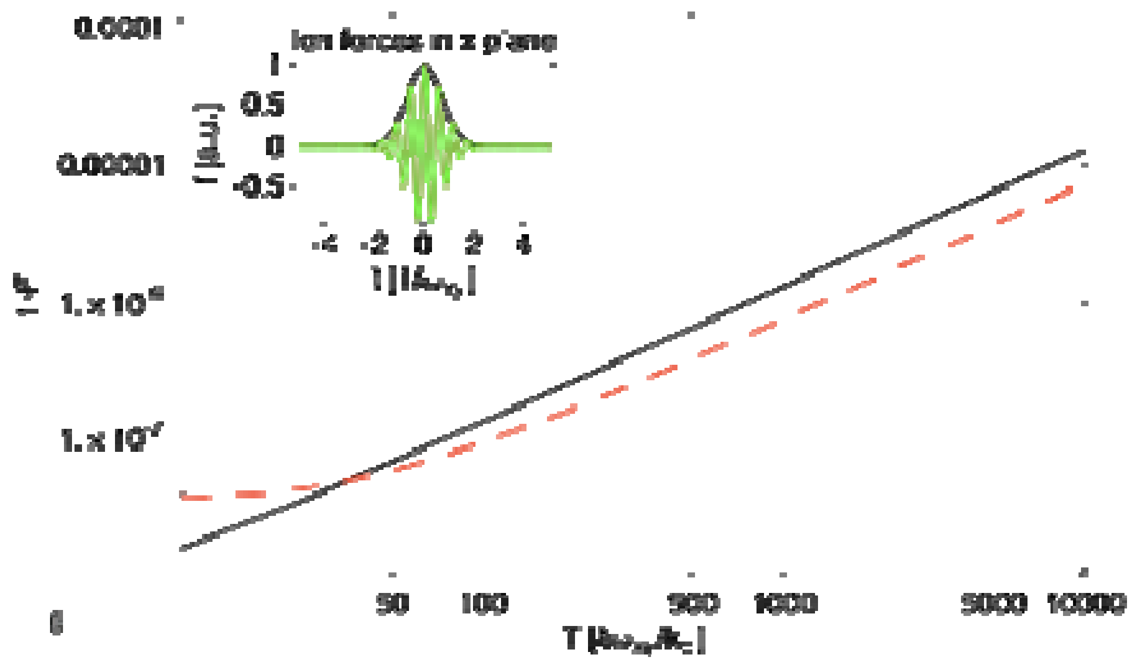
Finite temperature

Thermal noise in ion Wigner crystals

with J. Taylor ‘07

Quantum gates among ions in 2D crystals





Fast-carrier-modulation gate: decoupling from soft phonon modes

Inhomogeneous broadening

A broadening model

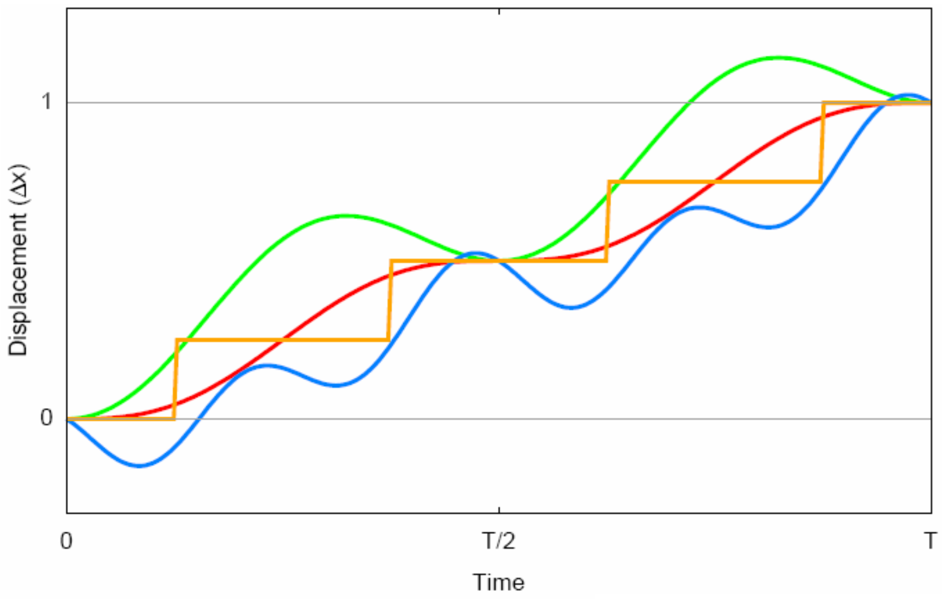
with N. Khaneja '08

Simple HO transport problem

$$V = \frac{m\omega^2}{2}[x - d(t)]^2$$

... when you move it, you make mistakes

$$d(t)$$



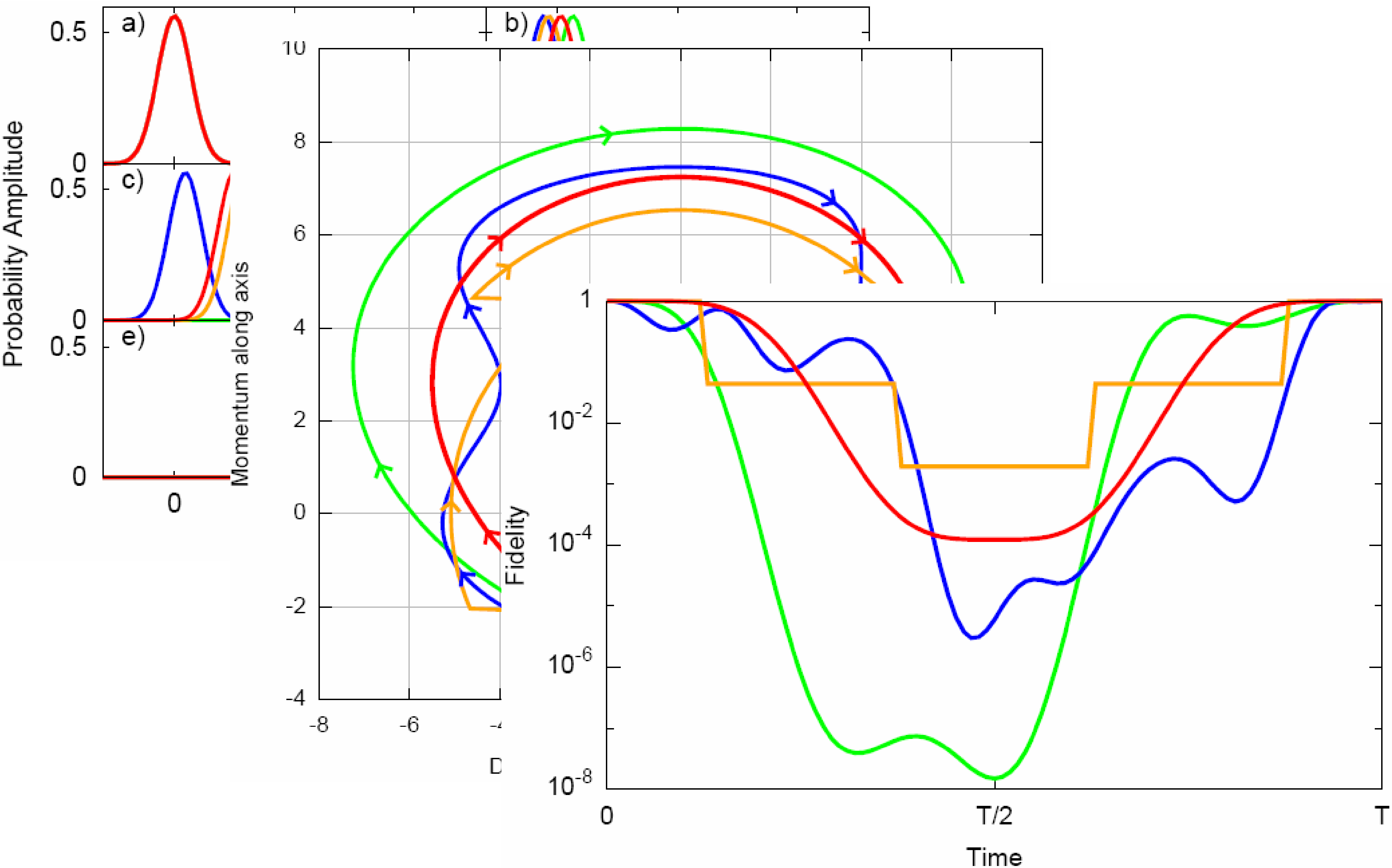
— Optimal path $d(t)$

— $d(t) + \dot{d}(t)$

— $d(t) + \alpha_0 + \sum_{n=1}^{2} \left(\alpha_1 \cos\left[2n\pi\frac{t}{T}\right] + \alpha_2 \sin\left[2n\pi\frac{t}{T}\right]\right)$

— $d(t)$ piecewise

Distorted transport results



How much can we push?

The Quantum Speed Limit

Quantum Speed Limit

The speed at which a quantum state evolves is linked to the dynamics of the Hamiltonian.

$$E = \langle \Psi | H | \Psi \rangle \qquad \Delta E = \sqrt{\langle \Psi | (H - E)^2 | \Psi \rangle}$$

Initial energy

Initial state

Energy variance

Minimum time required for a quantum state to evolve to an orthogonal state

$$T_{\min}(E, \Delta E) \equiv \max\left(\frac{\pi\hbar}{2E}, \frac{\pi\hbar}{2\Delta E}\right)$$

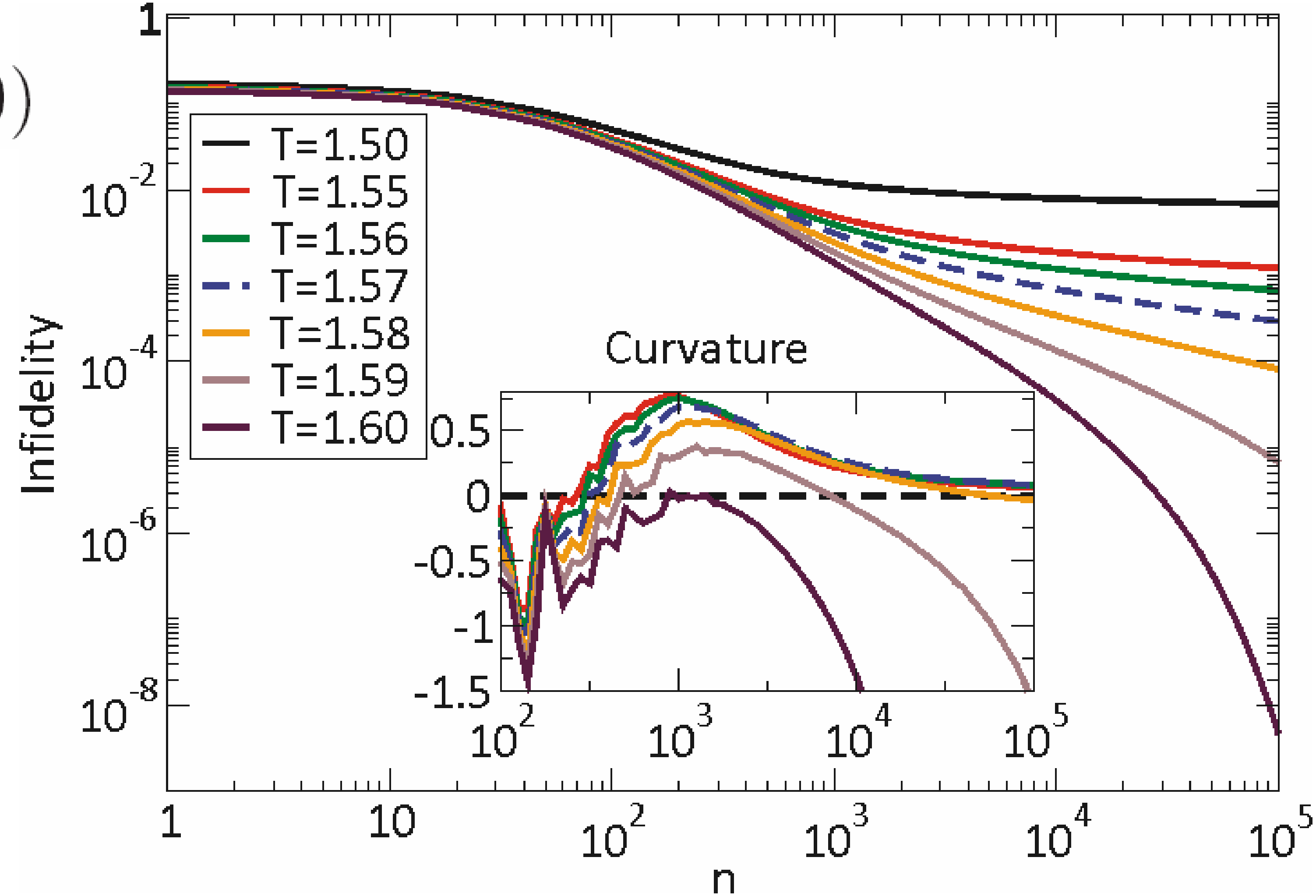
*V Giovannetti, S Lloyd, L Maccone, PRA **67**, 052109 (2003)*

Toy model: Landau-Zener crossing

with R. Fazio '09

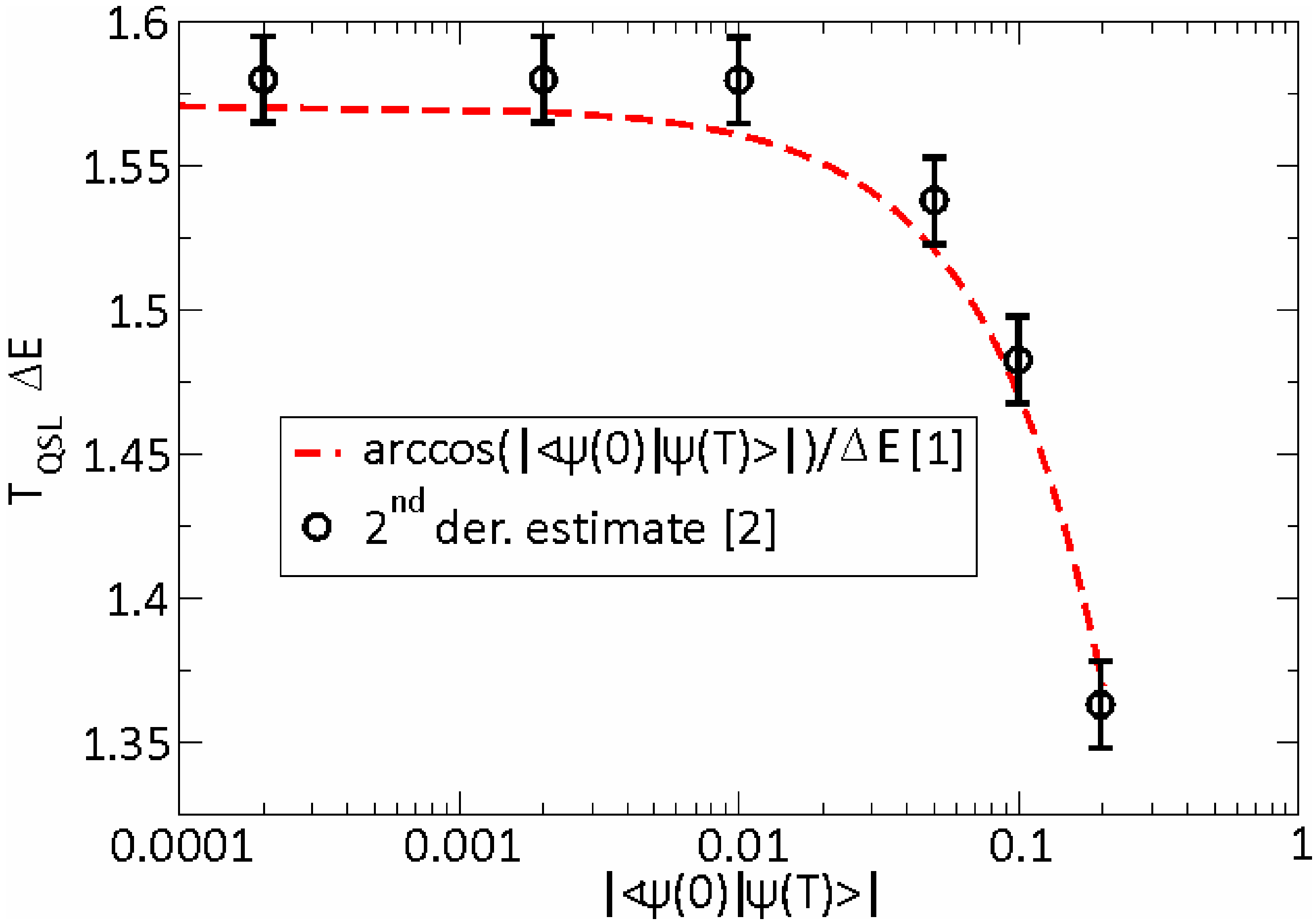
$$H[\Gamma(t)] = \begin{pmatrix} \Gamma(t) & \omega \\ \omega & -\Gamma(t) \end{pmatrix}$$

$$\Gamma(T) = -\Gamma(0)$$



Goal: back to ground state after crossing

Heuristic vs. analytic QSL

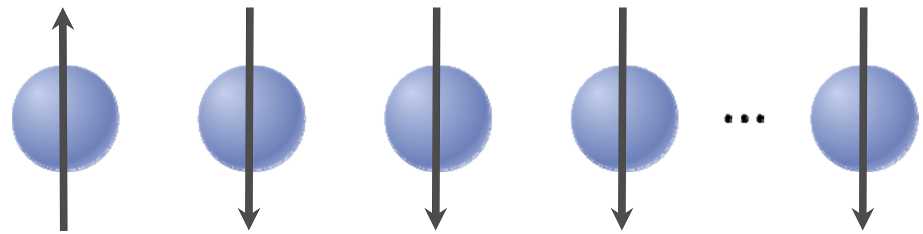


[1] K. Bhattacharyya, J. Phys. A: Math. Gen. 16, 2993 (1983).

[2] T. Caneva, M. Murphy, TC, R. Fazio, S. Montangero, V. Giovannetti, G. E. Santoro, arXiv:0902.4193

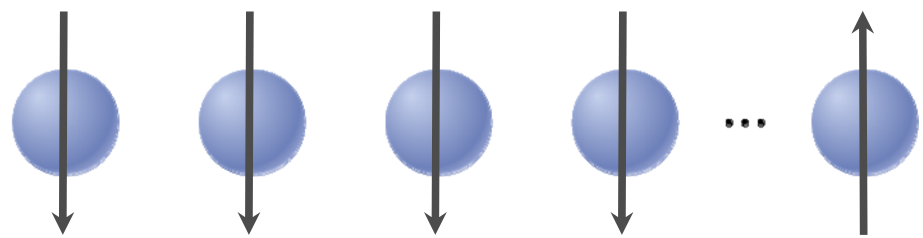
Spin chain transport

Initial state

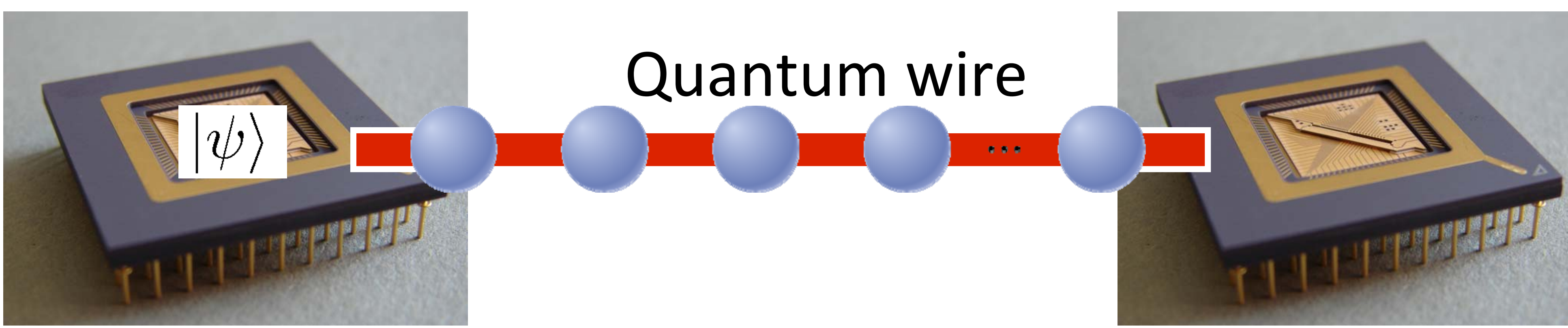


$$|\psi(0)\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |00\ldots\rangle$$

Final state



$$|\psi(0)\rangle = |00\ldots\rangle \otimes (\alpha|0\rangle + \beta|1\rangle)$$

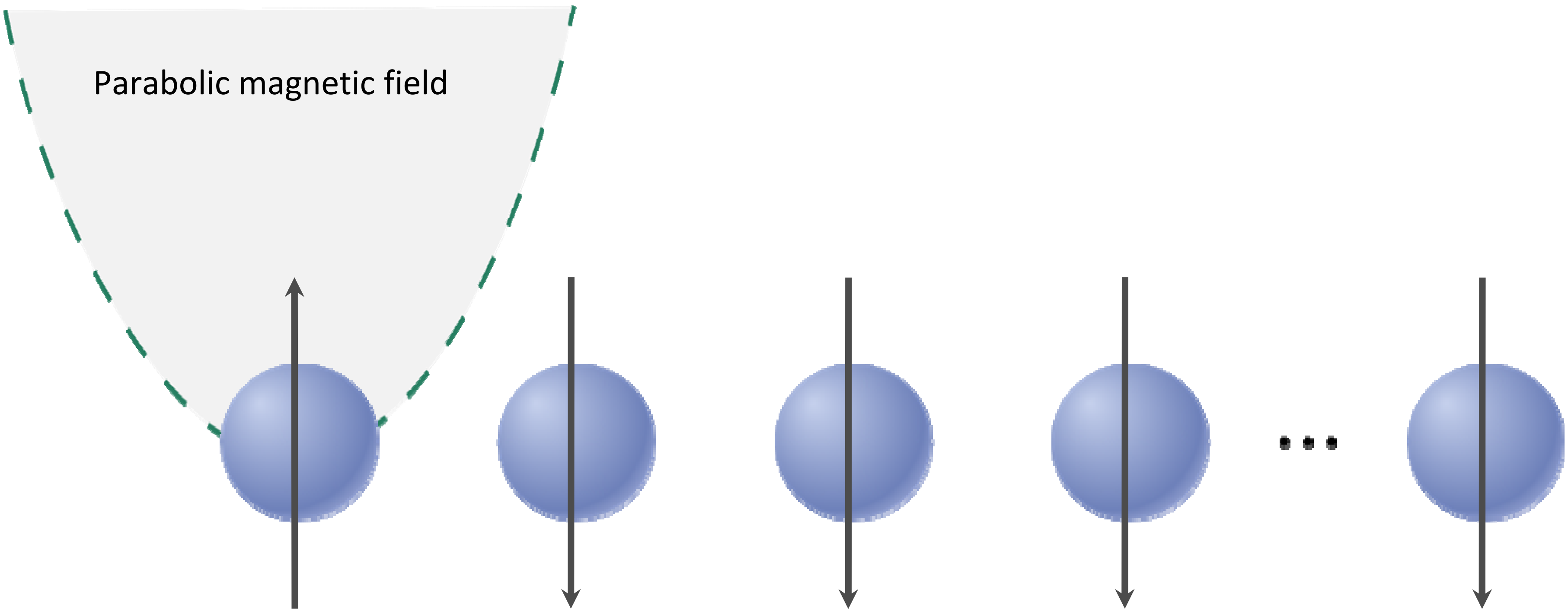


S. Bose, PRL **91** 207901 (2003)

The transport mechanism

$$H = -\frac{J}{2}\sum_{n=0}^{N-2} \vec{\sigma}_n \cdot \vec{\sigma}_{n+1}$$

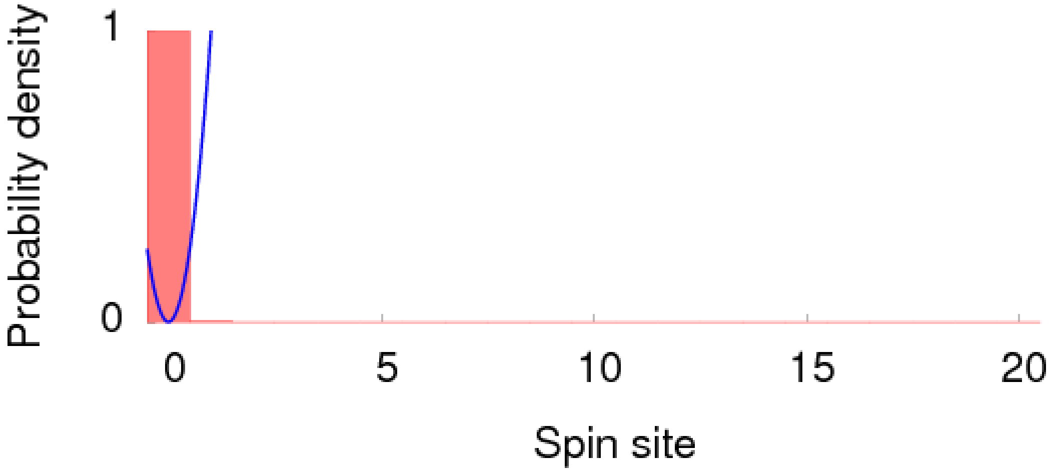
spin – spin coupling
(nearest neighbour)



Balachandran and Gong, Phys. Rev. A **77**, 012303 (2008)

The transport mechanism

No problem adiabatically,



But if we try naively to go faster...

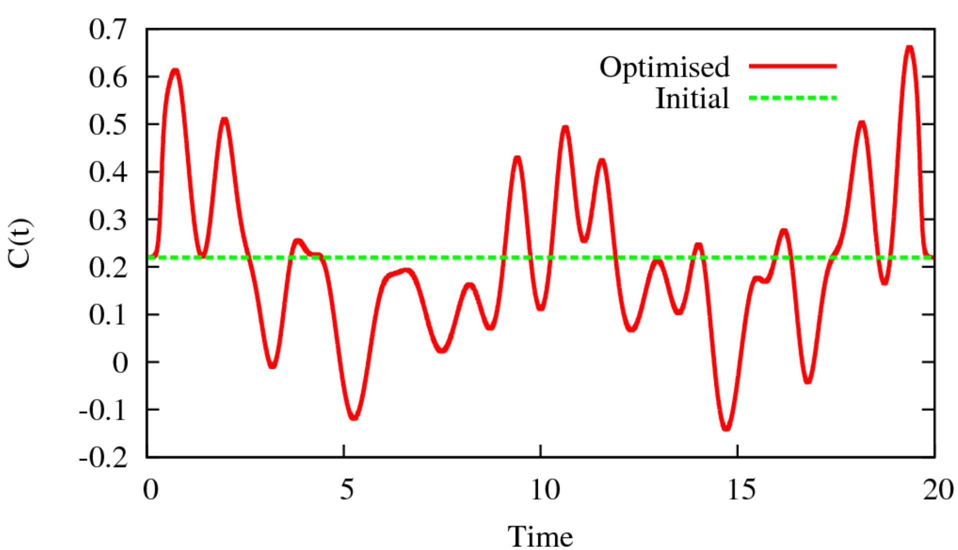
Optimal control formulation

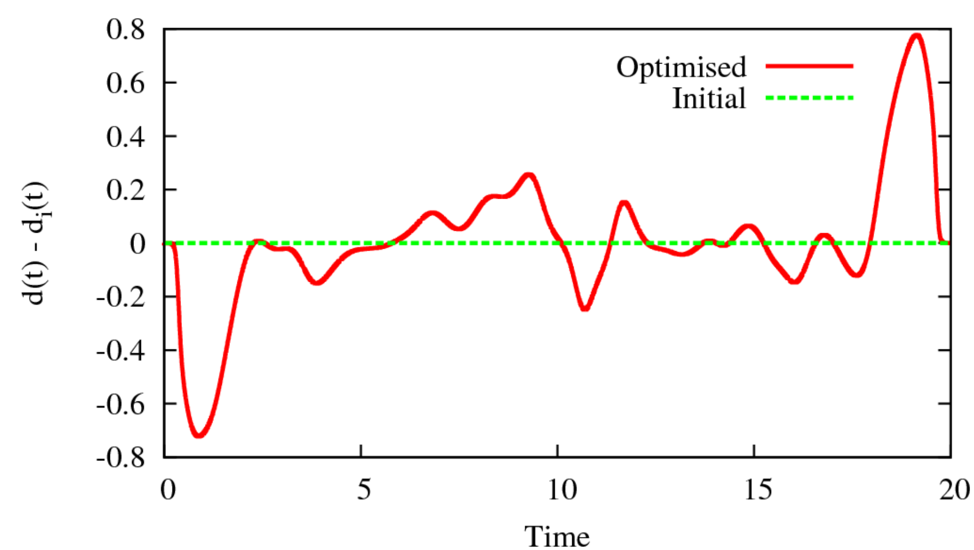
The Heisenberg Hamiltonian:

$$H = -\frac{J}{2}\sum_{n=0}^{N-2} \vec{\sigma}_n \cdot \vec{\sigma}_{n+1} + \sum_{n=0}^{N-1} \frac{C(t)}{2}\left(n - d(t)\right)^2 \sigma_n^z \ ,$$

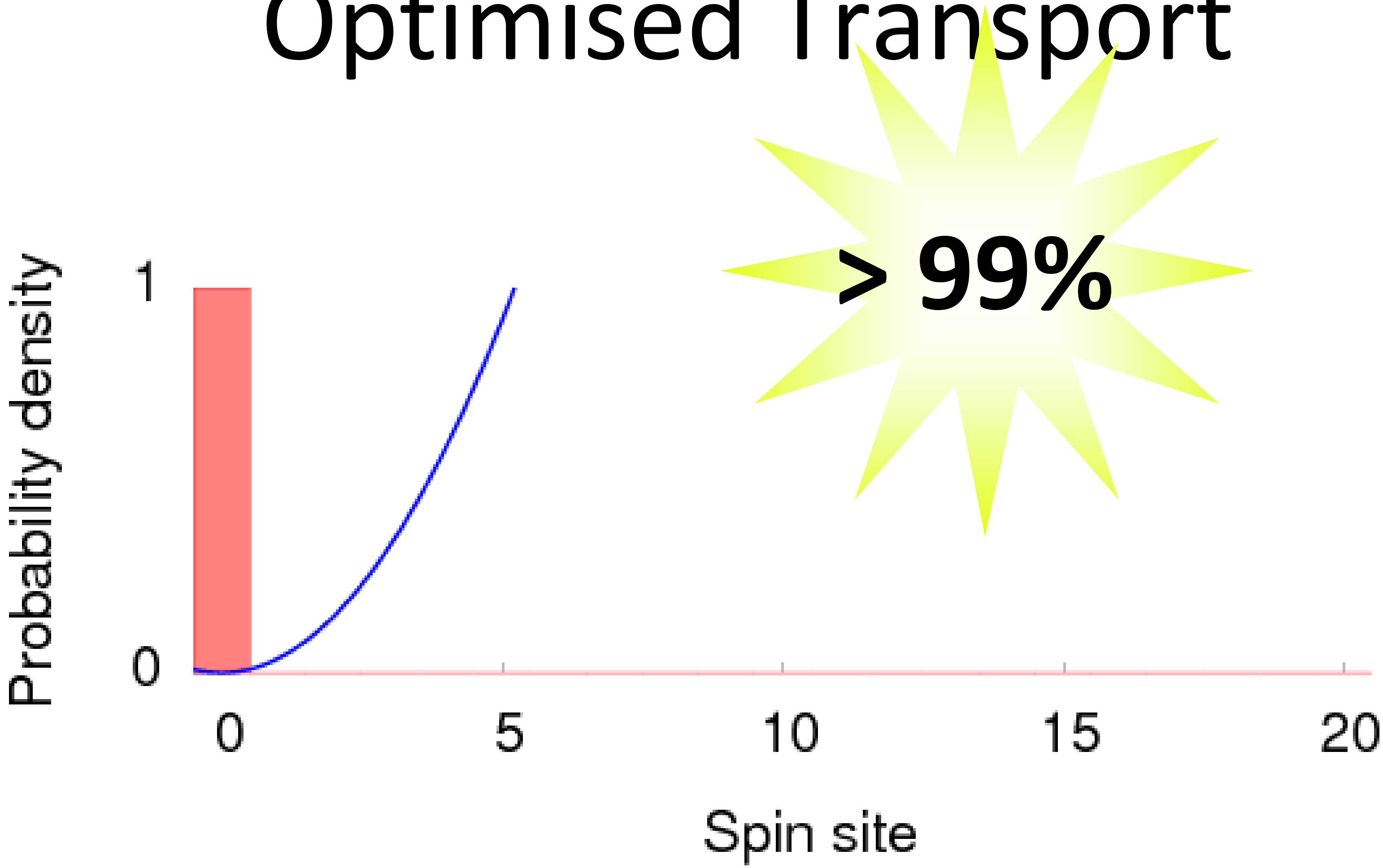
control parameters

Use the Krotov optimisation algorithm
to increase the transfer fidelity



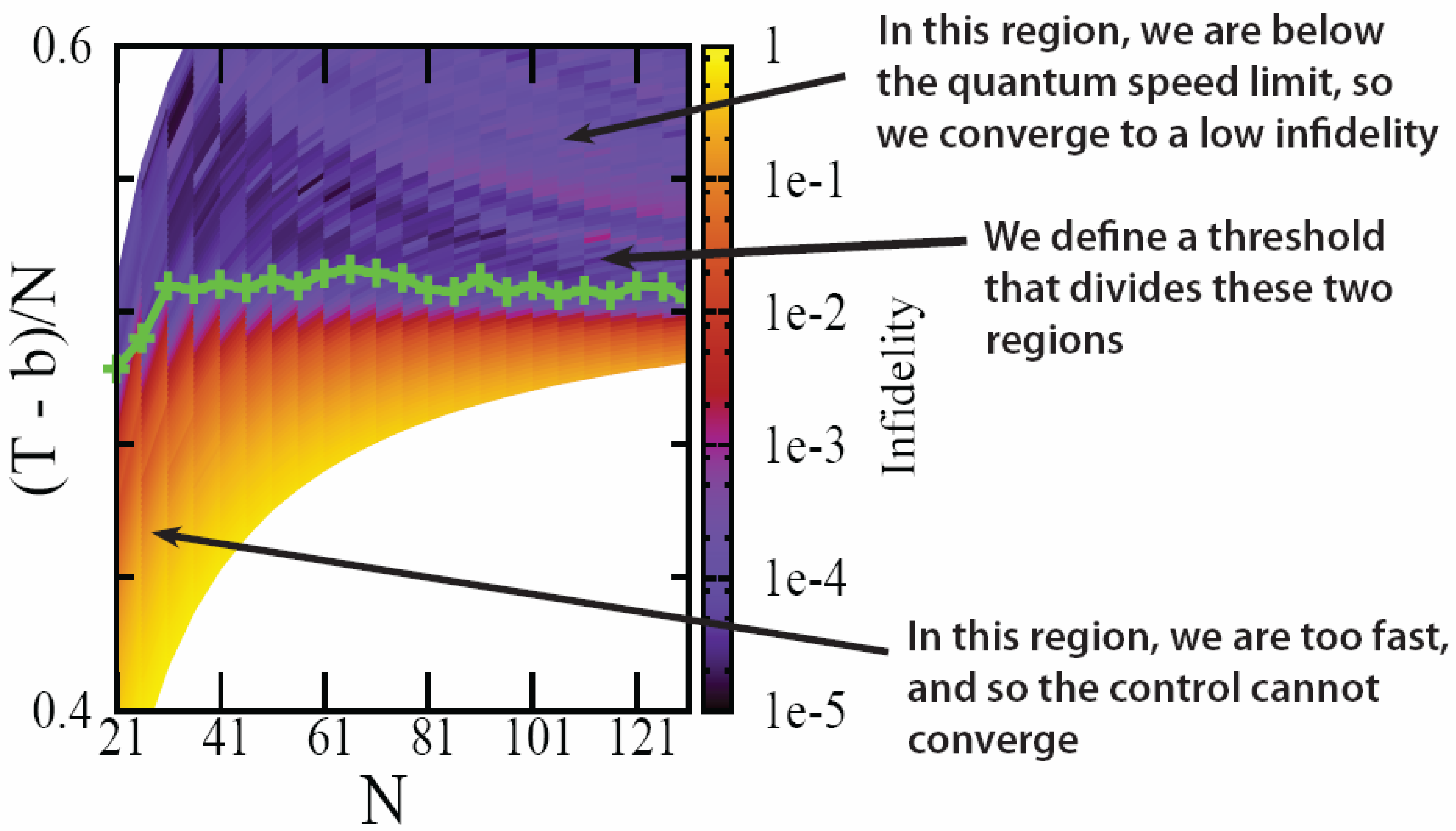


Optimised Transport

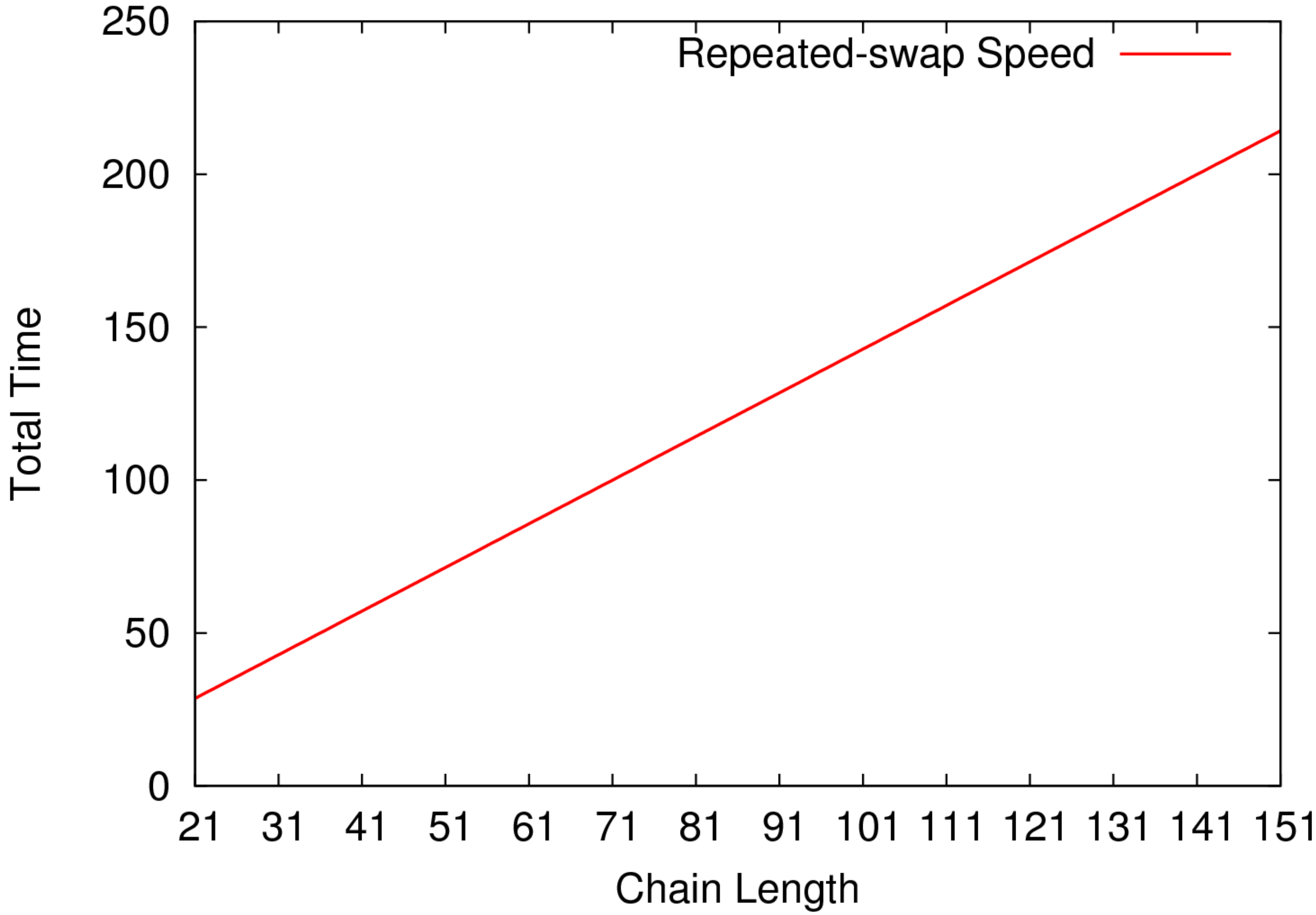


This is ~ 200 times faster

Final transport infidelity



Phenomenological Model



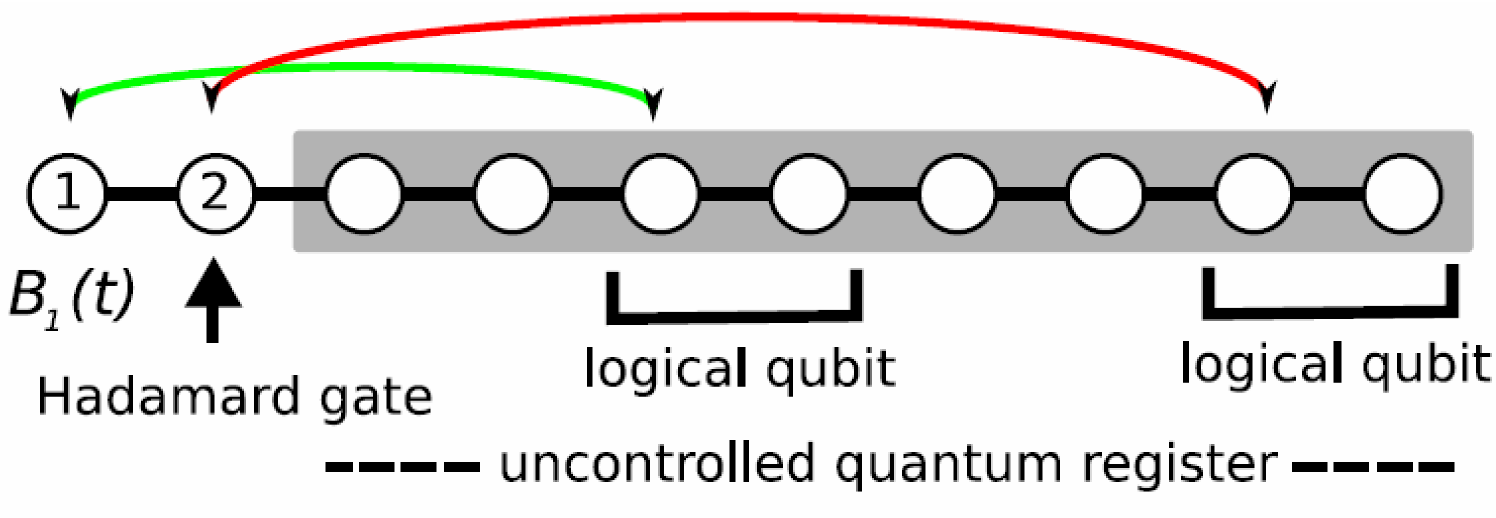
Theory:

$$\tau_{\mathrm{QSL}} \equiv \max\left\{\frac{\pi\hbar}{2J}, \frac{\pi\hbar}{2\Delta\mathcal{E}}\right\} \qquad \Delta\mathcal{E} = \frac{1}{T}\int_0^T \Delta E\,(t)\,\mathrm{d}t$$

Actual optimization speed limits is **lower**

Can this be used for computing?

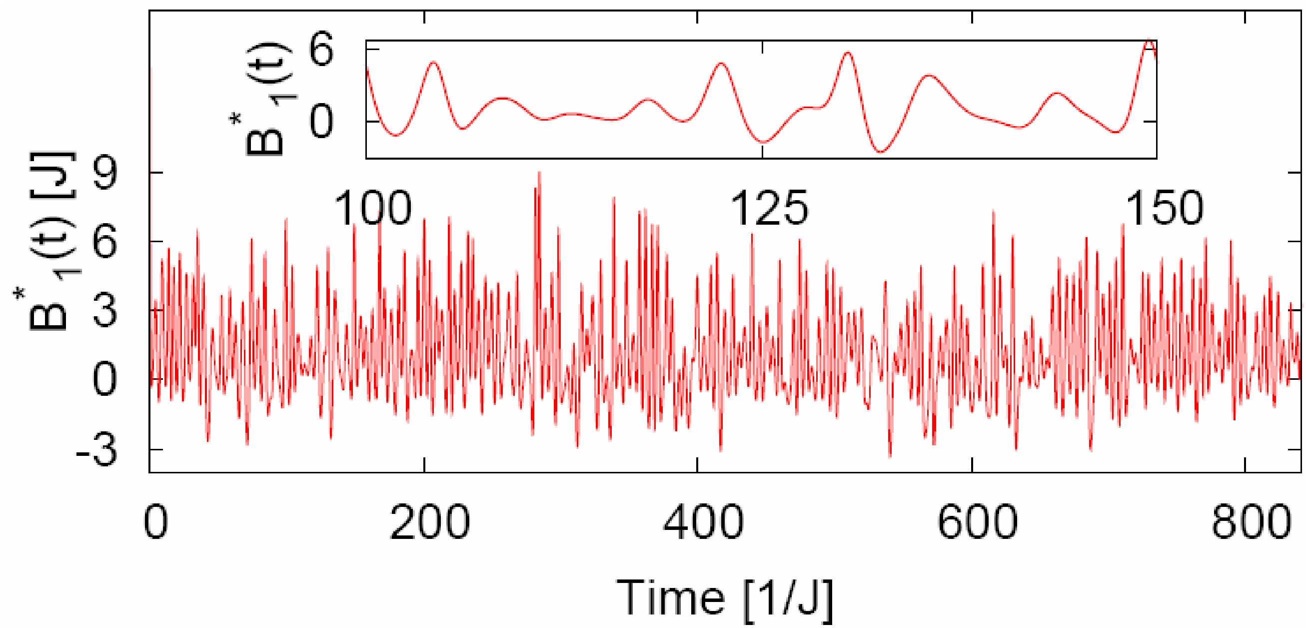
Scalable quantum computation via local control of only two qubits

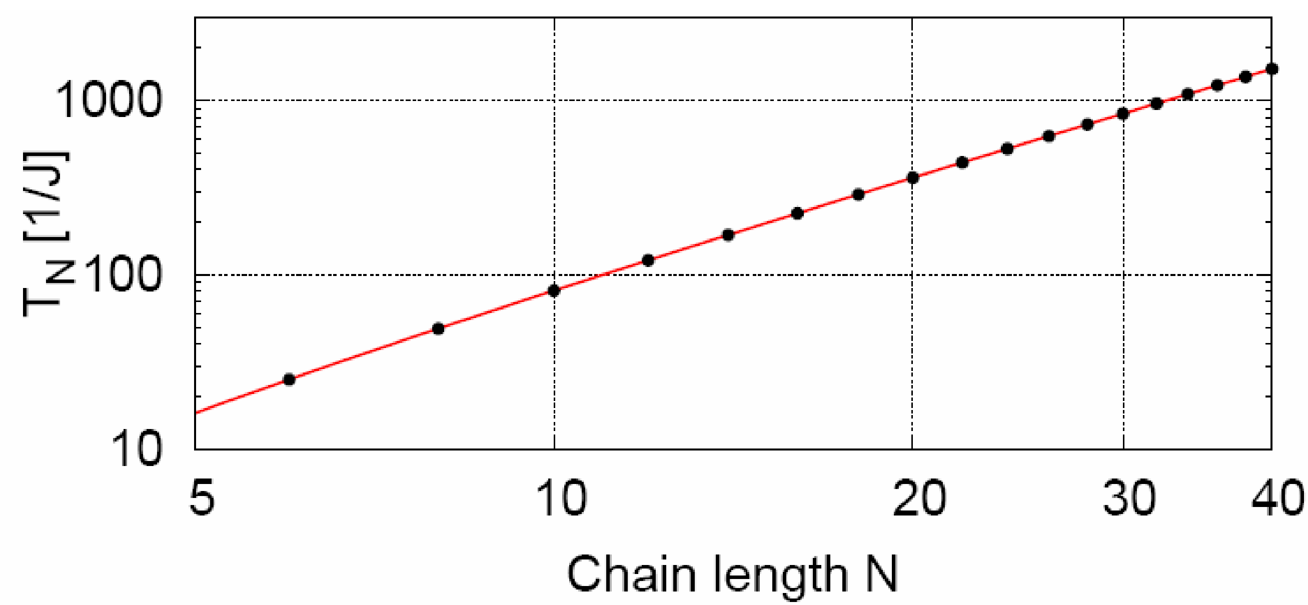


D. Burgarth, K. Maruyama, M. Murphy, S. Montangero, T. Calarco, F. Nori, M. Plenio, arXiv:0905.3373

Scaling of the operation time

Sample control pulse





$$T_N = (N-1)^2$$

Conclusions

- Quantum optimal control does work for quantum information processing
- It allows fixing a range of real issues
- Its limits deserve further exploration

Work done with...

- C. Koch (Berlin)
- R. Fazio (Pisa)
- P. Grangier (Orsay)
- T. Hänsch (Munich)
- P. Julienne, W. Phillips (Gaithersburg)
- M. Lukin (Harvard)
- Rei is)
- ch yer (Vienna)
- D. Tannor (Weizmann)
- P. Zoller (Innsbruck)

...and with no military funding.