Universal Dynamical Control of Decoherence / Decay and Zeno Cooling

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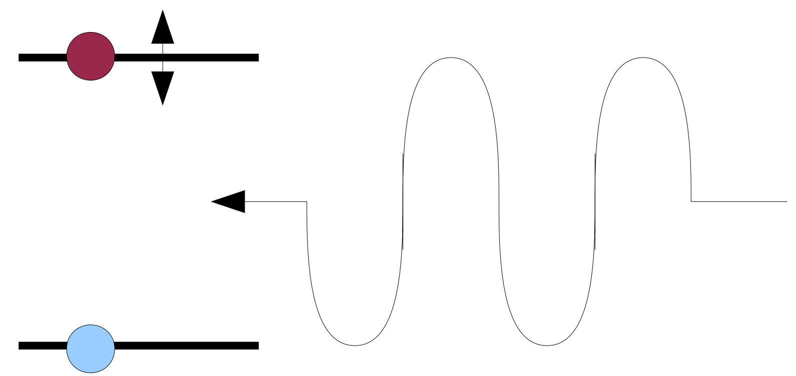
Basics of Decoherence



$$H=\frac{\omega_a}{2}\sigma_Z+\frac{\varepsilon(t)}{2}\sigma_Z$$

$$|\Psi_0\rangle=\frac{|0\rangle+|1\rangle}{\sqrt{2}}\quad,\quad|\Psi(t)\rangle=\frac{|0\rangle+e^{-i\int_0^t dt'\varepsilon(t')}|1\rangle}{\sqrt{2}}$$

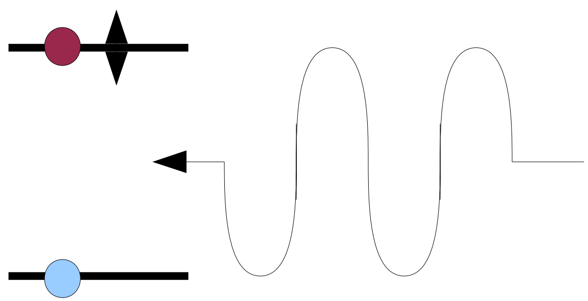
$$\rho(t)=\overline{|\Psi(t)\rangle\langle\Psi(t)|}=0.5\begin{pmatrix}1 & \overline{e^{i\int_0^t\varepsilon(t')dt'}}\\ \frac{1}{e^{i\int_{\cdot}^{t}\varepsilon(t')dt'}} & 1\end{pmatrix}$$

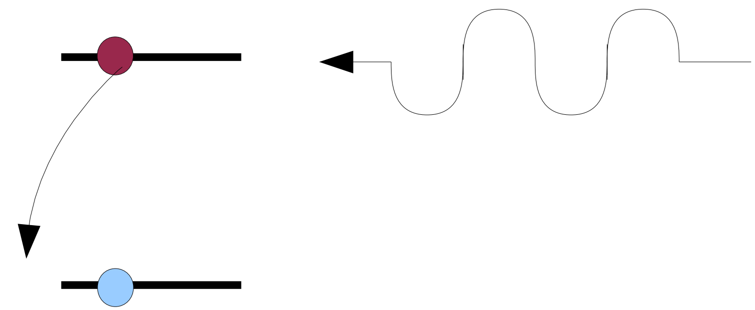
$$\overline{e^{i\int_0^t\varepsilon(t')dt'}}=1+i\int_0^t dt'\,\overline{\varepsilon(t')}-\iint_0^t dt'\,dt''\,\overline{\varepsilon(t')\varepsilon(t'')}+O(\varepsilon^3)$$

$$\approx e^{-R(t)t}\quad,\quad R(t)=t^{-1}\iint_0^t dt'\,dt''\,\Phi(t'-t'')$$

$$R(t)\rightarrow_{t\ll TC}\Phi(0)t\quad,\quad R(t)\rightarrow_{t\gg T_C}\int_0^\infty d\tau\,\Phi(\tau)\quad(\Phi(\tau\gg T_C)\rightarrow 0)$$

Decoherence Vs. Decay





$$H=\frac{\omega_a}{2}\sigma_Z+\frac{\varepsilon(t)}{2}\sigma_Z+\delta(t)e^{-i\sigma_Z\omega_a t}\sigma_X$$

$$|\pm\rangle\equiv\frac{|g\rangle\pm e^{-i\omega_a t}|e\rangle}{\sqrt{2}}$$

$$H=\text{٠}\,\sigma_{\pm Z}+\frac{\varepsilon(t)}{\text{٢}}\sigma_{\pm X}+\delta(t)\sigma_{\pm Z}$$

$$\rho_{++}=\frac{\rho_{ee}+\rho_{gg}+\rho_{eg}+\rho_{ge}}{2}=0.5+\Re\,\rho_{eg}$$

$$H=\frac{\omega_a}{\text{٢}}\sigma_Z+\frac{\varepsilon(t)}{\text{٢}}\sigma_X+\delta(t)\sigma_Z$$

$$H=\frac{\omega_a}{\text{٢}}\sigma_Z+\frac{\varepsilon(t)}{\text{٢}}\sigma_X+\delta(t)\sigma_Z$$

$$\rho_{ee}$$

System Coupled to Bath

We control → $H_S(t)$; Origin of noise → H_I

$$H = H_S(t) + H_B + H_I$$

$$\dot{\rho}(t) = -i[H_I(t), \rho(t)]$$

$$\dot{\rho}(t) = -i[H_I(t), \rho(0)] - \int_0^t dt' [H_I(t), [H_I(t'), \rho(t')]]$$

We assume the interaction to be of the form

$$H_I = S \cdot B$$

To second order in the interaction we get

$$\dot{\rho}_S(t) = -i\langle [H_I(t), \rho(0)] \rangle_B - \int_0^t dt' \phi(t-t') [S(t), S(t') \rho_S(t')] + H.C.$$

$$\phi(\tau) \equiv \langle e^{-iH_B \tau} B e^{iH_B \tau} B \rho_B \rangle_B \quad , \quad \rho_B \equiv Z^{-1} e^{-\beta H_B}$$

Decay Control

Let us look as a specific case –
Two level system decaying into a bath

$$S=\sigma_X \quad , \quad H_S(t)=\omega_a\frac{\sigma_Z}{2}+\delta(t)\frac{\sigma_Z}{2}$$

$$S(t)=e^{i(\omega_a t+\phi(t))\frac{\sigma_Z}{2}}\sigma_X e^{-i(\omega_a t+\phi(t))\frac{\sigma_Z}{2}} \qquad \phi(t)\equiv\int_0^t d\tau\,\delta(\tau)$$

$$\dot{\rho}_{ee}(t)\approx f(t)-r_e(t)\rho_{ee}(t)+r_g(t)\rho_{gg}(t)$$

$$r_e(t)\equiv\int_0^t dt'\,\Phi(t-t')e^{+i\omega_a(t-t')}e^{i\phi(t)}e^{-i\phi(t')}+C.C.$$

$$r_g(t)\equiv\int_0^t dt'\,\Phi(t-t')e^{-i\omega_a(t-t')}e^{-i\phi(t)}e^{i\phi(t')}+C.C.$$

$$f(t)\equiv -i\,e^{i(\omega_a t+\phi(t))}\langle B(t)\langle g|\rho(0)|e\rangle\rangle_B+C.C.$$

Average Decay Rates

$$R_e(t) \equiv t^{-1}\int_0^t r_e(t')\,dt'$$
$$= t^{-1}\iint_0^t dt'\,dt''\,\Phi(t'-t'')\,e^{+i\omega_a(t'-t'')}\,e^{i\phi(t')}\,e^{-i\phi(t'')}$$

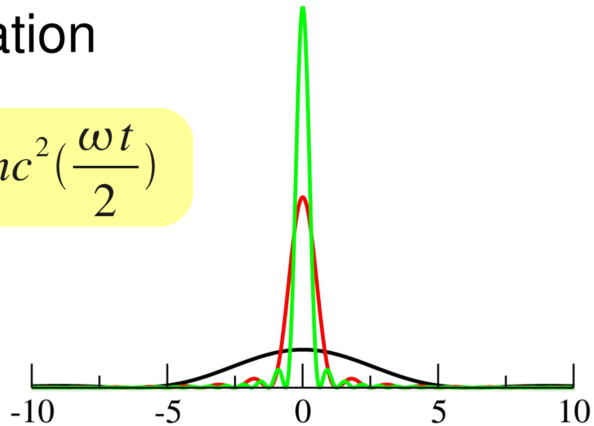
$$R_e(t) = (2\pi)^{-1}\int d\omega\, G(\omega)\, F_t(\omega+\omega_a)$$
$$R_g(t) = (2\pi)^{-1}\int d\omega\, G(\omega)\, F_t(-(\omega+\omega_a))$$

$$G(\omega) \equiv \int_{-\infty}^{\infty} d\tau\, \Phi(\tau)\, e^{-i\omega\tau}$$
$$F_t(\omega) \equiv \frac{|\varepsilon_t(\omega)|^2}{t}\ , \quad \varepsilon_t(\omega) \equiv \int_0^t d\tau\, e^{-i\phi(\tau)}\, e^{-i\omega\tau}$$

Common modulations

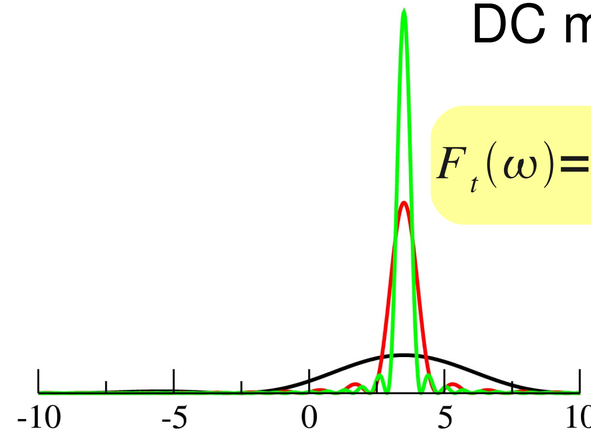
No modulation

$$F_t(\omega) = t\, sinc^2\left(\frac{\omega t}{2}\right)$$

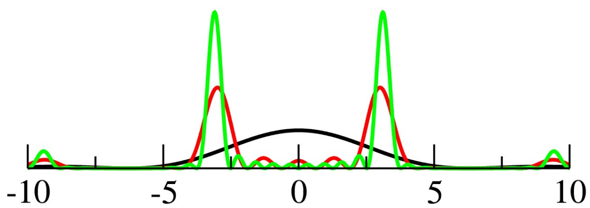


DC modulation

$$F_t(\omega) = t\, sinc^2\left(\frac{(\omega+\delta)t}{2}\right)$$

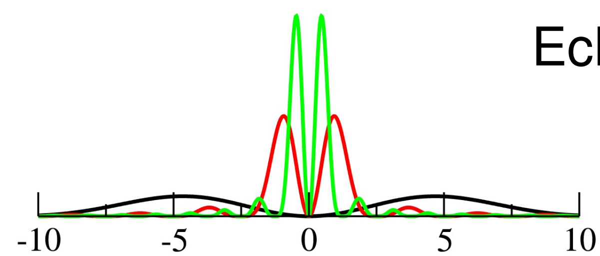


Bang bang



$$F_{n\tau}(\omega) = n\tau\, sinc^2\left(\frac{\omega\tau}{2}\right)\frac{\sin^2\left(n\frac{\omega\tau+\pi}{2}\right)}{n^2\sin^2\left(\frac{\omega\tau+\pi}{2}\right)}$$

Echo



$$F_t(\omega) = t\, sinc^2\left(\frac{\omega t}{4}\right)\frac{\sin^2\left(\frac{\omega t+2\pi}{2}\right)}{4\sin^2\left(\frac{\omega t+2\pi}{4}\right)}$$

Decay solution

$$\begin{aligned}\dot{\rho}_{ee}(t) &\approx f(t) - r_e(t)\rho_{ee}(t) + r_g(t)\rho_{gg}(t)\\ &= -(r_e(t)+r_g(t))\rho_{ee}(t) + r_g(t) + f(t)\end{aligned}$$

$$\rho_{ee}(t) = e^{-(R_e(t)+R_g(t))t}\left[\rho_{ee}(0) + \int_0^t dt'(r_g(t') + f(t'))e^{+(R_e(t')+R_g(t'))t'}\right]$$

$$\rho_{ee}(t) \underset{t\to\infty}{\to} \frac{R_g(t)}{R_e(t)+R_g(t)}$$

$$\begin{aligned}R_e(t) &= (2\pi)^{-1}\int d\omega\, G(\omega) F_t(\omega+\omega_a)\\ R_g(t) &= (2\pi)^{-1}\int d\omega\, G(\omega) F_t(-(\omega+\omega_a))\end{aligned}$$

Harmonic Thermal Bath

Let us choose a common example of bath systems –
The harmonic oscillator bath

$$H_B = \sum_\lambda \omega_\lambda \hat{n}_\lambda \quad , \quad B = \sum_\lambda \left(\mu_\lambda^* b_\lambda^+ + \mu_\lambda b_\lambda \right) \quad , \quad H_I = S \otimes B$$

The memory function of such a bath is:

$$\begin{aligned} \phi(\tau) &\equiv \left\langle e^{-iH_B\tau} B e^{iH_B\tau} B \rho_B \right\rangle \\ &= \sum_\lambda |\mu_\lambda|^2 \left\langle e^{-i\omega_\lambda \hat{n}_\lambda \tau} b_\lambda^+ e^{i\omega_\lambda \hat{n}_\lambda \tau} b_\lambda \rho_B \right\rangle \\ &\quad + \sum_\lambda |\mu_\lambda|^2 \left\langle e^{-i\omega_\lambda \hat{n}_\lambda \tau} b_\lambda e^{i\omega_\lambda \hat{n}_\lambda \tau} b_\lambda^+ \rho_B \right\rangle \\ &= \sum_\lambda |\mu_\lambda|^2 e^{-i\omega_\lambda} n(\omega_\lambda) + |\mu_\lambda|^2 e^{+i\omega_\lambda} \left(n(\omega_\lambda) + 1 \right) \end{aligned}$$

$$G(\omega) \equiv \int_{-\infty}^{\infty} d\tau\, \Phi(\tau) e^{-i\omega\tau} = G_0(-\omega)\, n(-\omega) + G_0(\omega) \left(n(\omega) + 1 \right)$$

$$G_0(\omega) = \sum_\lambda |\mu_\lambda|^2 \delta(\omega - \omega_\lambda)$$

Optimal Decoherence Control

Assuming symmetric bath,
decoherence dynamics take the form

$$\rho_{++}(t) = e^{-2R(t)t}\left[\rho_{++}(0) - 0.5\right] + 0.5$$

$$R(t) = (2\pi)^{-1}\int d\omega\, G(\omega)\, F_t(\omega)$$

$$F_t(\omega) \equiv \frac{|\varepsilon_t(\omega)|^2}{t}$$

$$\varepsilon_t(\omega) \equiv \int_0^t d\tau\, e^{-i\phi(\tau)} e^{-i\omega\tau}$$

$$\phi(t) \equiv \int_0^t \delta(t')\,dt'$$

We would like to find a control minimizing decoherence R(T) for a given time T, given constraint on modulation

Energy Constraint

$$\int_0^T \delta^2(t')\,dt' = E$$

Gate constraint

$$\int_0^T \delta(t')\,dt' = \Delta$$

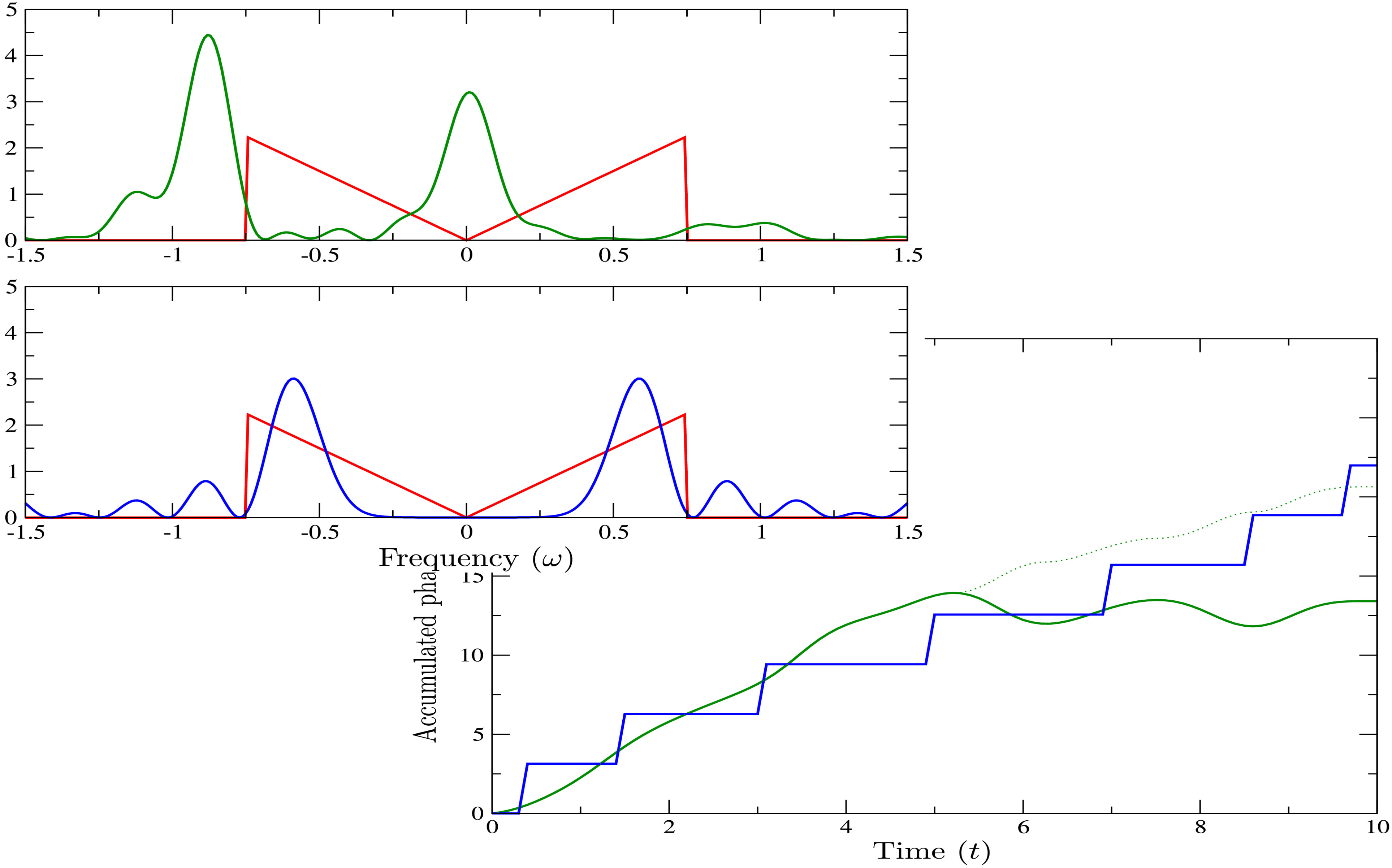
$$\int_0^T |\delta(t')|\,dt' = \Delta$$

Edge Constraint

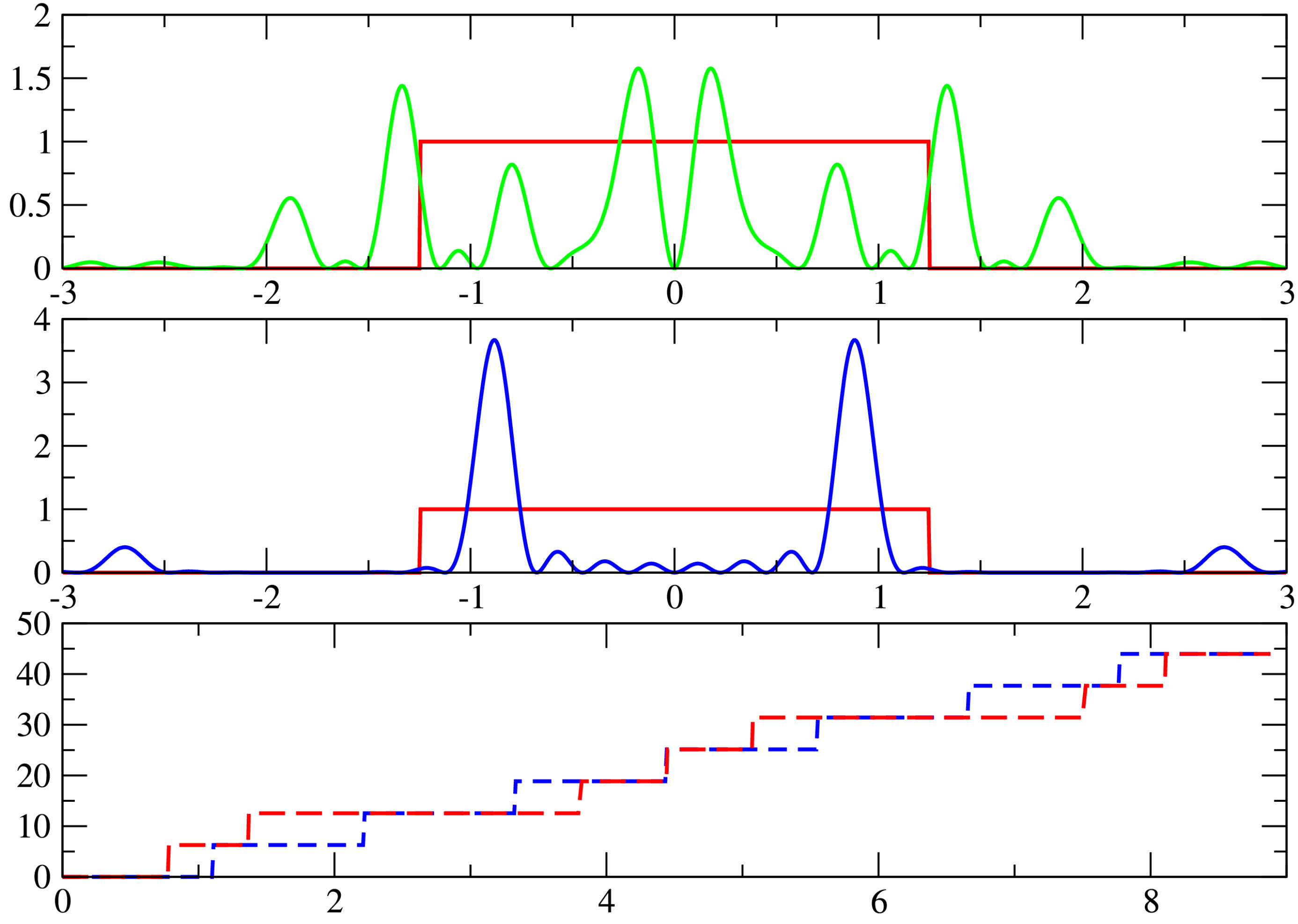
$$\delta(\cdot) = 0$$

$$\delta(T) = \cdot$$

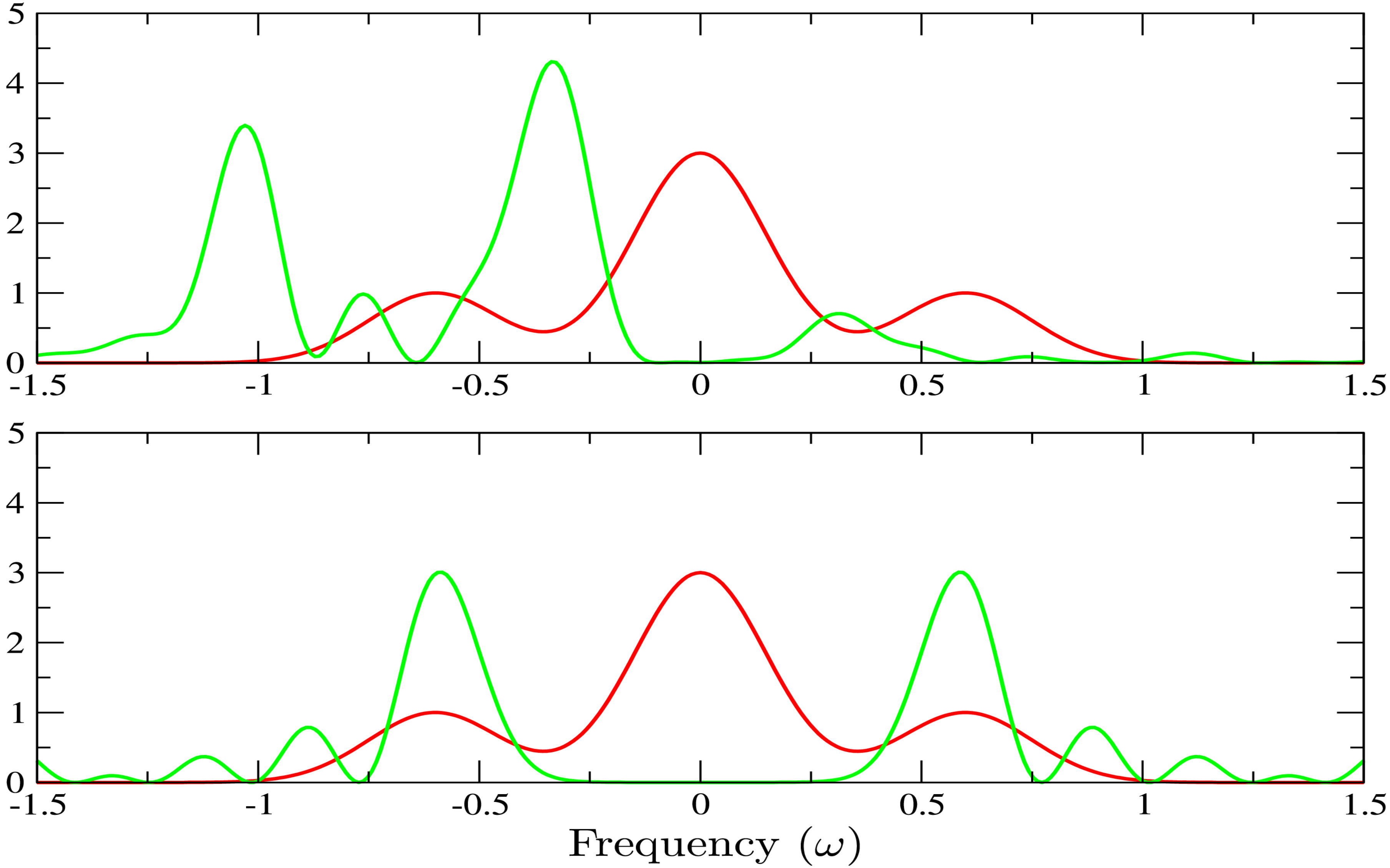
Optimal Decoherence Control Results



Optimal Decoherence Control Results

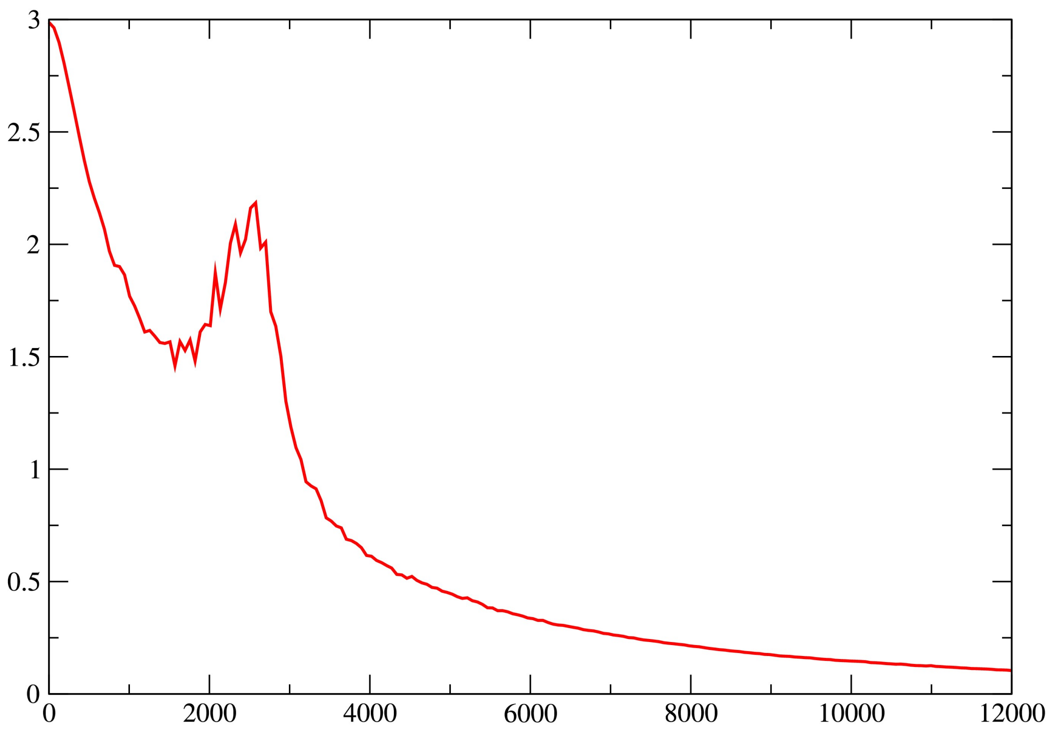


Optimal Decoherence Control Results



Bath Spectrum Extraction

- Choose “good” set of modulations → giving “good” set of F's
- Measure final fidelity for different modulations
- “Reverse engineer” bath spectrum G



Heating and Cooling

Start at thermal equilibrium system + bath
Perform measurement at t=0

$$\rho_{ee}(t) = e^{-(R_e(t)+R_g(t))t}\left[\rho_{ee}(\bullet) + \int_{\bullet}^{t} dt'\, r_g(t')\, e^{+(R_e(t')+R_g(t'))t'}\right]$$

$$R_e(t) \;=\; (2\pi)^{-1}\int d\omega\, G(\omega)\, F_t(\omega+\omega_a)$$

$$R_g(t) \;=\; (2\pi)^{-1}\int d\omega\, G(\omega)\, F_t(-(\omega+\omega_a))$$

Initially – system “heats up”

$$t \ll \frac{1}{\omega_a + T_C^{-1}}$$

$$R_e(t) \approx R_g(t) \propto t \int_{-\infty}^{\infty} d\omega\, G(\omega)$$

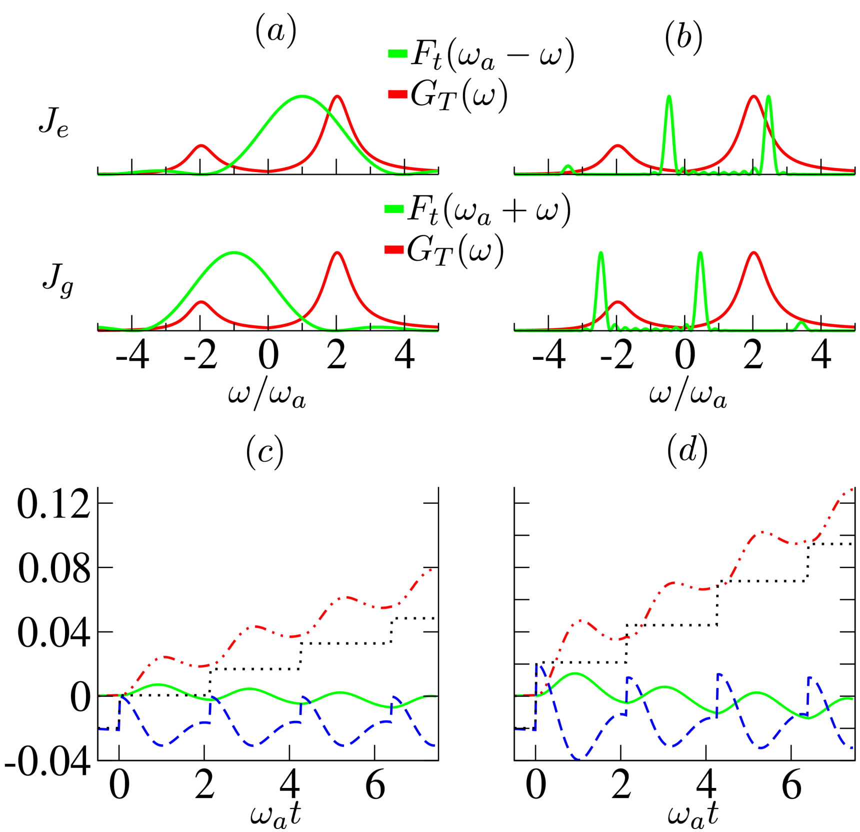
$$\rho_{ee}(t) \approx \rho_{ee}(0) + (0.5 - \rho_{ee}(0))(1 - e^{-2\mathrm{R}(t)t})$$

Eventually, system arrives at modulation-dependent population (temperature) bound

$$\rho_{ee}(t) \underset{t\to\infty}{\rightarrow} \frac{R_g(t)}{R_e(t)+R_g(t)}$$

Simulation Results

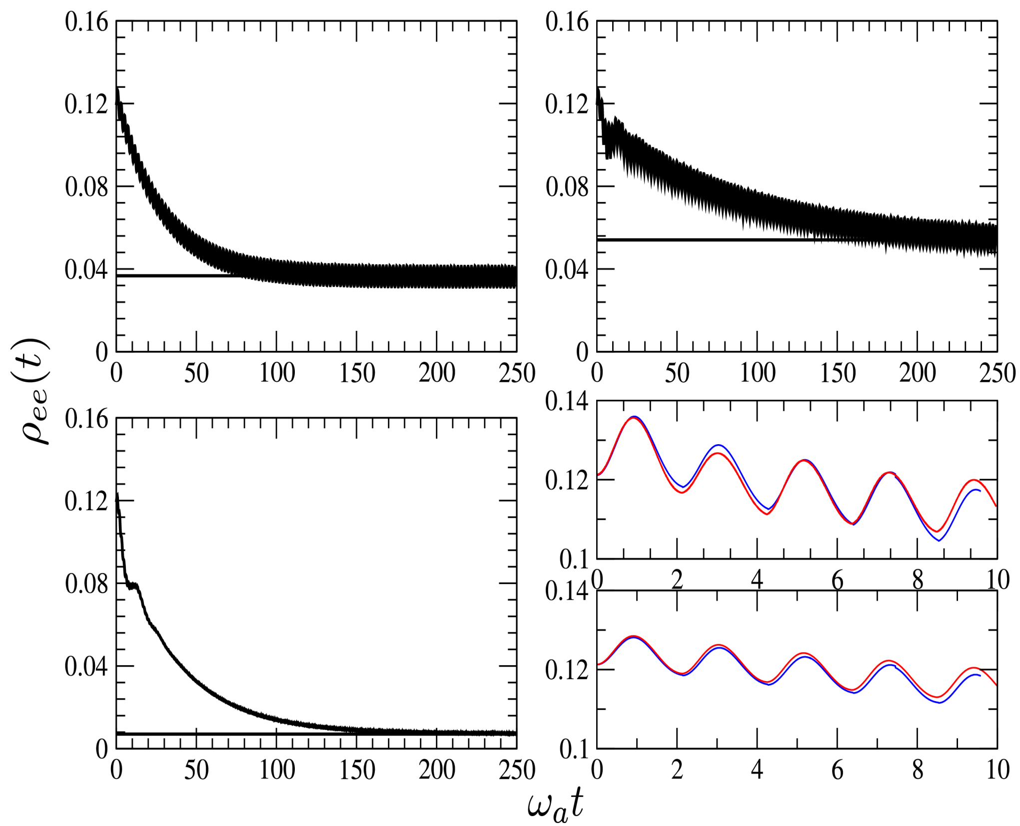
Measurements   Bang bang



Energy of each part
Of the Hamiltonian

$$H = H_B + H_S + H_I$$

Bang bang   Measurements



Small phase shifts

Theory Vs. Full simulation