Large N_c gauge theories:

old & new

Pavel Kovtun
Mithat Ünsal
L. G. Y.

Old:

$N_c \to \infty$ limit = classical limit

Construct $N_c = \infty$ dynamics via gauge invariant coherent states

$$|u\rangle = \hat{\mathcal{U}}\,|0\rangle = e^{\hat{\Lambda}}\,|0\rangle$$

pure YM:

$\hat{\Lambda}$ = arbitrary (anti-Hermitian) sum of spatial Wilson loops + loops with one electric field insertion

$\{e^{\hat{\Lambda}}\}$ = "coherence group" $\mathcal{G}$

show: $\mathcal{G}$ acts irreducibly

$$\langle u|\hat{\vartheta}|u\rangle = 0 \quad \forall\ |u\rangle \Rightarrow \hat{\vartheta} = 0$$

$$\langle u|u'\rangle = e^{-O(N^2)}$$

$\Rightarrow$ classical limit:

phase space $\sim \{|u\rangle\}$ = coadjoint orbit of $\mathcal{G}$

action $S_{cl} = \lim_{N\to\infty} \frac{1}{N^2} \langle u(t)| i\partial_t - \hat{H} |u(t)\rangle$

classical observables

$$\lim_{N\to\infty} \langle u|\hat{A}|u\rangle = a(\xi)$$

$$\lim_{N\to\infty} \langle u|\hat{A}\hat{B}|u\rangle = a(\xi)\, b(\xi)$$

$$\lim_{N\to\infty} i N^2 \langle u|[\hat{A},\hat{B}]|u\rangle = \{a(\xi), b(\xi)\}_{PB}$$

classical $N=\infty$ dynamics determines:

$\lim_{N\to\infty}$ $\begin{cases} \langle\text{Wilson loops}\rangle \\ \text{glueball, meson masses} \\ N \cdot \text{ two body decay amplitudes} \\ N^{p+q-2} \cdot\ p \leftrightarrow q \text{ scattering amplitudes} \\ N^{2(k-1)} \cdot \text{ connected } k\text{-pt. correlators} \\ \vdots \end{cases}$

Use classical dynamics to:

1) Solve $N=\infty$ theory

hard! $\rightsquigarrow$ minimize $h_{cl}(\xi) = \lim_{N\to\infty} \frac{1}{N^2} \langle u|\hat{H}|u\rangle$

expand S_{cl} about minimum

2) Compare $N\to\infty$ limits of different theories

Coinciding coherence group structure plus appropriate mapping of observables

$\Rightarrow$ coinciding $N=\infty$ dynamics

$\Rightarrow$ exact $N=\infty$ duality — w/o solving either theory

(Better method than comparing loop equations)

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Newer: Orbifold projection

"Parent" theory

symmetry group $\supset$ discrete subgroup P

$\Big\downarrow$ Remove degrees of freedom not invariant under P

"Daughter" theory

Kachru, Silverstein, Lawrence, Nekrasov, Vafa, Bershadsky, Kakushadze, Schmaltz, Armoni, Kol, Strassler, Gorsky, Shifman, Tong, Klebanov, Erlich, Naqvi, Dijkgraaf, Neitzke, Veneziano, ...

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Ex: Parent = $\mathcal{U}(2N)$ $\mathcal{N}=1$ supersymmetric YM

$\mathcal{U}(2N)$ gauge field + massless adjoint fermion

choose $P = \mathbb{Z}_2$ = global gauge trans $\begin{bmatrix} 1 & & & \\ & \ddots & & \\ & & -1 & \\ & & & \ddots \end{bmatrix}$

$\times \; (-1)^F$

adjoint rep. $A_\mu \rightsquigarrow \begin{bmatrix} \; & 0 \\ 0 & \; \end{bmatrix}$

2 indep. $\mathcal{U}(N)$ gauge fields A^1_μ, A^2_μ

adjoint rep. $\lambda \rightsquigarrow \begin{bmatrix} 0 & \; \\ \; & 0 \end{bmatrix}$

2 bifundamental Weyl fermions λ^1, λ^2

$\therefore$ Daughter = $\mathcal{U}(N) \times \mathcal{U}(N)$ gauge theory w. 2 bifundamental fermions — not supersymmetric

Generalizations: Multiple adjoint fermions or scalars

$\mathbb{Z}_{k_1} \times \mathbb{Z}_{k_2} \times \cdots \times \mathbb{Z}_{k_\ell}$ projections

w. $\mathcal{U}(k_1 k_2 \cdots k_\ell N) \rightsquigarrow \mathcal{U}(N)^{k_1 \cdots k_\ell}$

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Perturbation theory: Bershadsky + Johansen

planar diagrams of parent + daughter coincide provided

$$\lambda_{\text{parent}} = g_p^2 (2N)$$
$$\parallel$$
$$\lambda_{\text{daughter}} = g_d^2 N$$

"Non-perturbative orbifold conjecture": Strassler

Corresponding correlators coincide, exactly, as $N \to \infty$

If true:

Relates parent + daughter particle spectra, scattering amplitudes, thermodynamics, ... when $N \to \infty$

Powerful tool for gaining non-perturbative information in non-SUSY theories

Ex: bose - fermi mass degeneracies in large N limit of non-supersymmetric daughter w. SUSY parent.

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Previous consistency tests: mixed

daughters	equivalent?	
CFT	✓	Kachru + Silverstein
	✓	Lawrence, Nekrasov, Vafa
SUSY	✓	Erlich + Naqvi
~~SUSY~~	✓	Dijkgraaf, Neitzke, Vafa
	✗	Gorsky + Shifman
	✗	Tong
	✗	Armoni, Shifman, Veneziano

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New: P.K., M.Ü., L.Y.

Consider $Z_{k_1} \times Z_{k_2} \times \cdots \times Z_{k_\ell}$ projections

parent: $\mathcal{U}(mN)$ gauge theory – lattice regularized
$m = k_1 k_2 \cdots k_\ell$ w. N_f adjoint fermions,
N_s adjoint scalars

Choose embedding of $P = Z_{k_1} \times \cdots \times Z_{k_\ell}$ in
(global gauge group) × (flavor symmetry group)

"neutral" sector ≡ invariant under P

daughter: $\mathcal{U}(N)^m$ gauge theory with adjoint
and/or bifundamental matter

global symmetry ⊃ $Z_{k_1} \times \cdots \times Z_{k_\ell}$ permutations
of gauge group factors

"neutral" sector ≡ invariant under
this $Z_{k_1} \times \cdots \times Z_{k_\ell}$

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Construct appropriate coherence groups
of parent + daughter

Lie algebras =
{ Wilson loops w. scalar and/or fermion insertions
+ loops w. scalar + one scalar conjugate momentum
+ loops w. one electric field insertion }

Compare structure of coherence algebras

Compare gauge invariant observables

Find:

1-to-1 mapping between neutral
single trace observables

Ex: $\frac{1}{mN} \underset{mN \times mN}{\mathrm{tr}}\, \mathcal{U}[C] \longrightarrow \frac{1}{m} \sum_j \frac{1}{N} \underset{N \times N}{\mathrm{tr}}\, \mathcal{U}^j[C]$

Isomorphism between neutral
subalgebras of coherence algebras

$$\frac{\hat{\lambda}^p}{(mN)^2} \longrightarrow \frac{\lambda^d}{m N^2}$$

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$\Rightarrow$ Isomorphism between neutral subspaces of $N=\infty$ classical phase space

Isomorphism between classical dynamics in neutral sector, provided

$$\frac{\hat{H}^p}{(mN)^2} \longrightarrow \frac{\hat{H}^d}{mN^2}$$

$\therefore$ Non-perturbative large N equivalence within neutral sector if:

no spontaneous breaking of P symmetry in parent

no spontaneous breaking of $Z_{k_1} \times \cdots \times Z_{k_d}$ symmetry in daughter

Necessary and sufficient conditions for large N equivalence

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Example: Z_2 projection of $\mathcal{N}=1$ $\mathcal{U}(2N)$ SYM on $\mathbb{R}^4$

parent:

$$\mathcal{L}_p = \mathrm{tr}\Big\{ \frac{1}{2g_p^2} F_{\mu\nu}F_{\mu\nu} + \frac{\Theta_p}{16\pi^2} F_{\mu\nu}\tilde{F}_{\mu\nu} + \frac{1}{g_p^2}\big[\bar{\lambda}\sigma^\mu D_\mu \lambda + \tfrac{1}{2} m(\lambda\lambda + \bar{\lambda}\bar{\lambda})\big]\Big\}$$

Z_{4N} chiral symmetry $\xrightarrow[\langle\lambda\lambda\rangle]{}$ unbroken $\underset{\lambda\to-\lambda}{Z_2}$

vacuum energy

$$\mathcal{E}_p = \frac{m}{2g_p^2}\big[\langle \mathrm{tr}\,\lambda\lambda\rangle\, e^{i\Theta_p/2N} + \mathrm{c.c.}\big] + O(m^2)$$

daughter:

$$\mathcal{L}_d = \sum_{j=1}^{2} \mathrm{tr}\Big\{ \frac{1}{2g_d^2} F^j_{\mu\nu}F^j_{\mu\nu} + \frac{\Theta_d}{16\pi^2} F^j_{\mu\nu}\tilde{F}^j_{\mu\nu} + \frac{1}{g_d^2}\bar{\lambda}^j\sigma^\mu D_\mu\lambda^j\Big\} + \frac{m}{g_d^2}\mathrm{tr}(\lambda^1\lambda^2 + \bar{\lambda}^2\bar{\lambda}^1)$$

Z_{2N} chiral symmetry $\xrightarrow[\langle\lambda^1\lambda^2\rangle]{}$ unbroken Z_2

$$\mathcal{E}_d = \frac{m}{g_d^2}\big[\langle \mathrm{tr}\,\lambda^1\lambda^2\rangle\, e^{i\Theta_d/N} + \mathrm{c.c.}\big] + O(m^2)$$

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Gorsky + Shifman; Armoni, Shifman + Veneziano :

coinciding vacuum energies $\mathcal{E}_p = \mathcal{E}_\mathcal{Q}$

$\Rightarrow$ differing topological susceptibilities $\mathcal{T}_p \neq \mathcal{T}_\mathcal{Q}$

If $N_f > 1$: differing # Goldstone bosons

Correct mapping :

$$\mathrm{tr}\, FF \to \mathrm{tr}\,(F^1F^1 + F^2F^2)$$

$$\mathrm{tr}\,\lambda\lambda \to \mathrm{tr}\,(\lambda^1\lambda^2 + \lambda^2\lambda^1) = 2\,\mathrm{tr}\,\lambda^1\lambda^2$$

$$\mathcal{L}_p \to 2\,\mathcal{L}_\mathcal{Q}$$

$\therefore\ g_p^2 = \frac{1}{2} g_\mathcal{Q}^2$ equality of 't Hooft couplings

$\theta_p = 2\,\theta_\mathcal{Q}$ compatible θ dependence

$\Rightarrow \mathcal{E}_p = 2\,\mathcal{E}_\mathcal{Q}$ consistent w. $O(m)$ expressions

Orbifold equivalence must hold, unless necessary symmetry realization fails

No evidence for any failure

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Same theories, compactified on $\mathbb{R}^3 \times S^1$

Gorsky + Shifman:

instanton zero modes does not match

reliable for small radius

Tong:

Z_2 interchange symmetry breaks spontaneously for sufficiently small radius

$\therefore$ Orbifold equivalence fails for small radius, due to change in symmetry realization

Consistent w. necessary/sufficient conditions

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Other examples

$N_f > 1$, same Z_2 projection

$[\#\ \text{GBs}]_{\text{parent}} \neq [\#\ \text{GBs}]_{\text{daughter}}$ G+S

$[\#\ \text{neutral GBs}]_{\text{parent}} = [\#\ \text{neutral GBs}]_{\text{daughter}}$ ✓

Eguchi - Kawai reduction

parent YM on $(kL)^d$ lattice

$\downarrow$ $(Z_k)^d$ projection

daughter YM on L^d lattice

equivalent provided $L \geq L_c^{(d)}(\lambda)$

Inverse E - K reduction

parent YM on 1^d lattice = d matrix model

$\downarrow$ $(Z_L)^d$ projection

daughter YM on L^d lattice

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A.S.V.

Orientifold equivalence

parent USp(2N) gauge theory w/ antisymmetric tensor fermion

Z_2 projection

U(N) w. adjoint fermion — daughter 1

U(N) w. antisym. tensor fermion — daughter 2

parent / daughter equivalence $\Rightarrow$ daughter / daughter equivalence

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Conclusions:

Non-perturbative orbifold equivalence conjecture is valid — given necessary symmetry realizations.

Some dualities can be derived, not just conjectured.

Orbifold projections can yield useful relations between large N theories — numerous applications possible.

Open question:

Most general choice of projection yielding large N equivalence?