From Strings to One-Flavor QCD

- Summary: SUSY gluodynamics at large N is equivalent to nonsupersymmetric orientifold daughter which at N=3 => one-flavor QCD!

Genesis of the idea:

S. Kachru & E. Silverstein, 4-D CONFORMAL THEORIES AND STRINGS ON ORBIFOLDS, 1998

R6 orbifolds + AdS/CFT; started from $\mathcal{N}$=4 => distinct (perturbatively) conformal daughters with $\mathcal{N}$<4

A.Lawrence, N.Nekrasov & C.Vafa, ON CONFORMAL FIELD THEORIES IN FOUR-DIMENSIONS, 1998

M.Bershadsky, Z.Kakushadze, Vafa, STRING EXPANSION AS LARGE *N* EXP. OF GAUGE THEORIES, '98

M.Bershadsky, a. Johansen, LARGE *N* LIMIT OF ORBIFOLD FIELD THEORIES, 1998

M.Schmaltz, DUALITY OF NONSUPERSYMMETRIC LARGE *N* GAUGE THEORIES, 1998

M.Strassler, ON METHODS FOR EXTRACTING EXACT NONPERTURBATIVE RESULTS IN NONSUPERSYMMETRIC GAUGE THEORIES, 2001

Tools:

- Orientifolding;
- Large N (planar) limit;
- Supersymmetry.

SUSY gluodynamics | Orientifold daughter

$$\mathcal{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu a} + \frac{i}{g^2} \bar\lambda^a_{\dot\alpha} D^{\dot\alpha\alpha} \lambda^a_\alpha$$

Dirac

$$\mathcal{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu a} + \frac{1}{g^2} \bar\psi_{[ij]} (i \not{D}) \psi^{[ij]}$$

Weyl

At N=3, orienti A = one-flavor QCD

$$\lambda^a \to \lambda^i_j \longrightarrow \eta^{ij} + \xi_{ij} \quad S \text{ or } A$$

$\lambda \longrightarrow N^2 - 1$ *dof*

$\psi^{[ij]} \to \frac{N^2 - N}{2}$ *dof*

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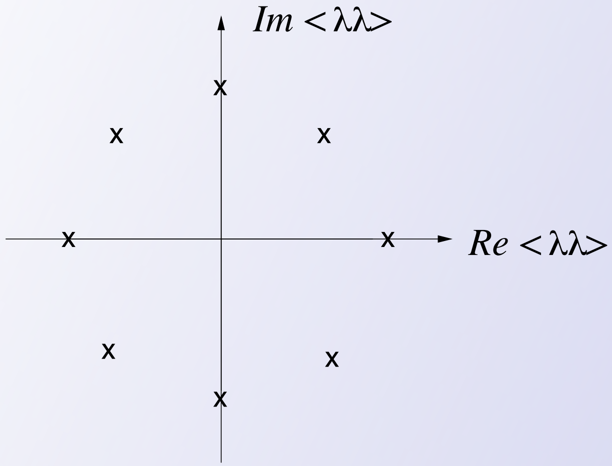
	SU(N)	U$_V$(1)	U$_A$(1)
$\eta_{\{ij\}}$	symmetric (two boxes, horizontal)	1	1
$\xi^{\{ij\}}$	conjugate symmetric (two boxes, horizontal, barred)	−1	1
A_μ	Adj	0	0

	SU(N)	U$_V$(1)	U$_A$(1)
$\eta_{[ij]}$	antisymmetric (two boxes, vertical)	1	1
$\xi^{[ij]}$	conjugate antisymmetric (two boxes, vertical, barred)	−1	1
A_μ	Adj	0	0

SUSY gluodynamics

✳ N vacua labeled by

$\langle\lambda\lambda\rangle = -6N\Lambda^3 \exp(2\pi i k/N)$



N vacua for SU(N)

Orientifold daughter

✳ N-2 vacua labeled by

$\langle\bar{\Psi}_R\, \Psi_L\rangle = -6(N\text{-}2)\Lambda^3\, e^{2\pi i k/(N-2)} + \ldots$

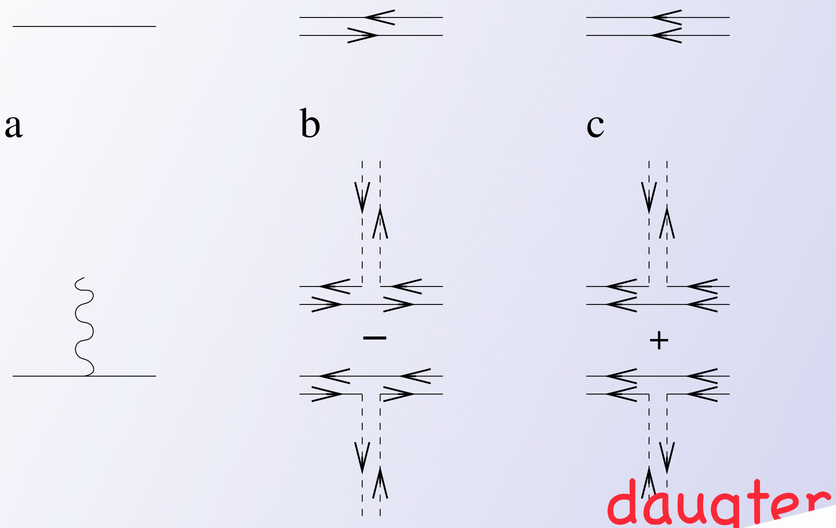
At N=3 the vacuum is unique (at □=0)

✳ Both theories confine; only composite color-singet hadrons in the spectra.

✳✳ Orientifold daughter is NOT supersymmetric: m_B(parent)=$O(N^0)$ while m_B(daughter)=$O(N^1)$.

Common Sector: SUSY Orienti □Glueballs+bifermions+...

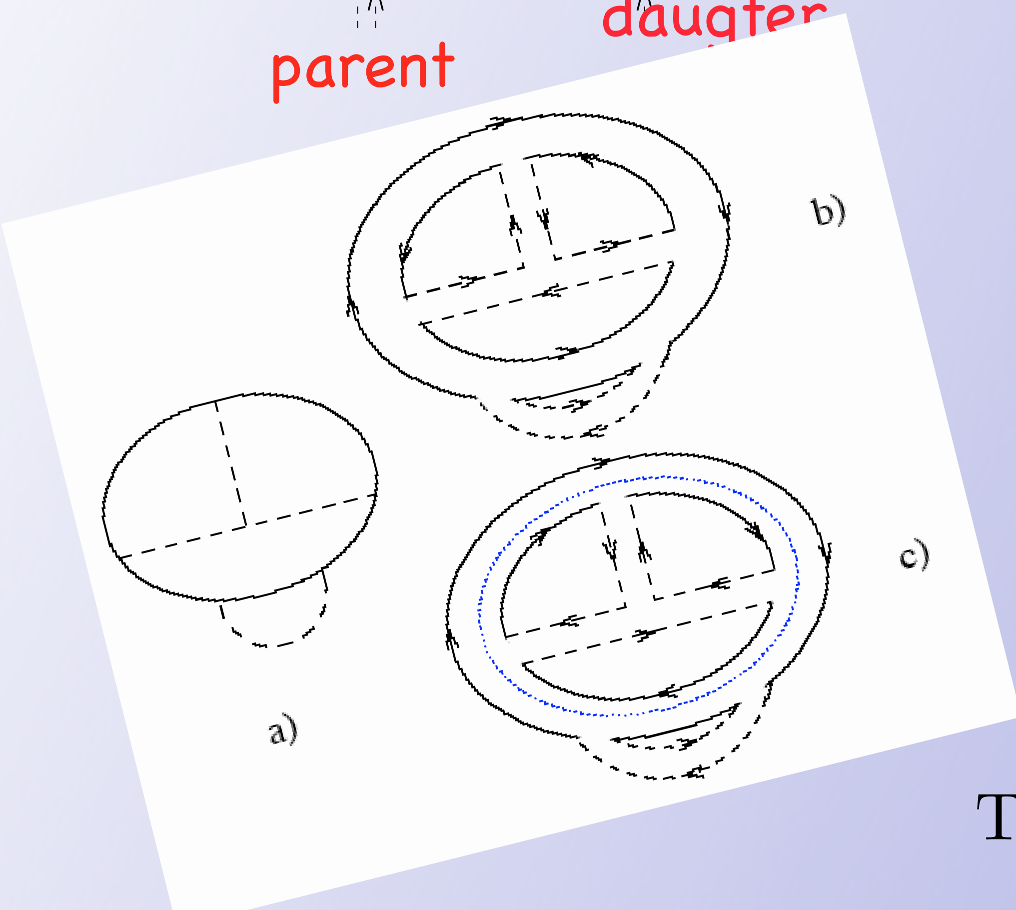
Perturbative Planar Equivalence



$$T^a_{\mathrm{Adj}} \sim T^a_{\square\times\bar{\square}} = T^a_{\square}\otimes 1 + 1\otimes T^a_{\bar{\square}} \equiv T^a\otimes 1 + 1\otimes \bar{T}^a$$

$$T^a_{\mathrm{two-index}} = T^a_{\square}\otimes 1 + 1\otimes T^a_{\square} \equiv T^a\otimes 1 + 1\otimes T^a$$

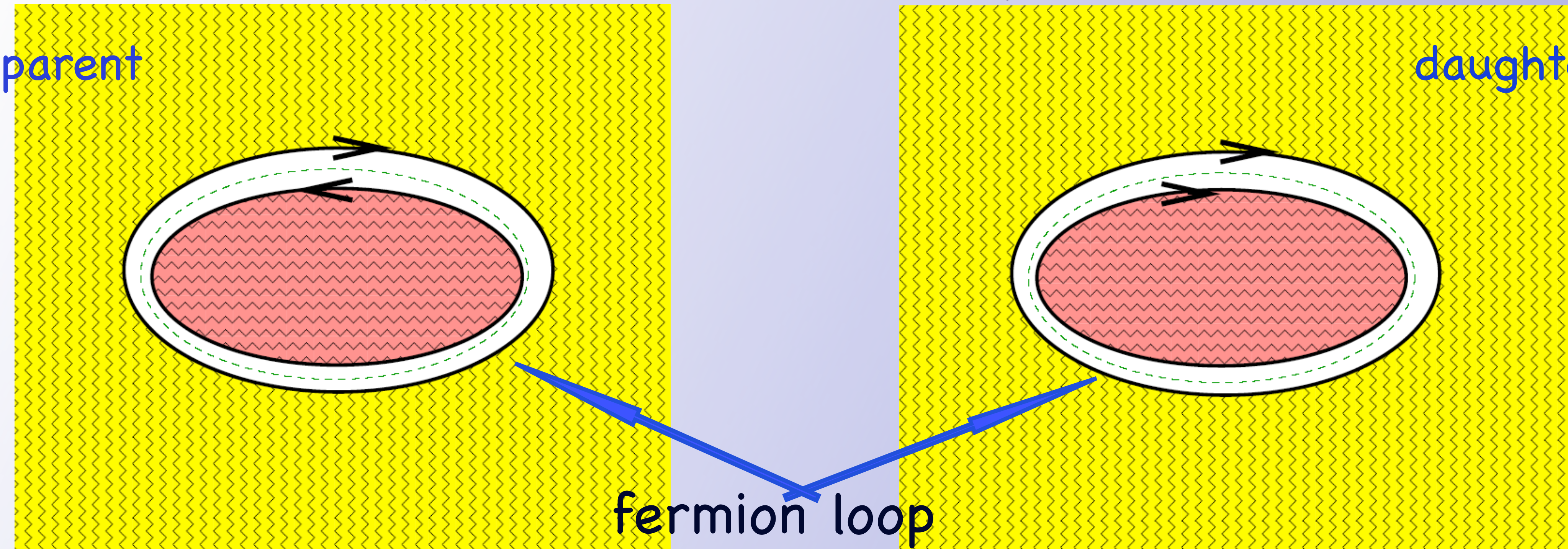
$$\text{or}\quad \bar{T}^a\otimes 1 + 1\otimes \bar{T}^a\,.$$



$$\bar{T} = -\tilde{T} = -T^*$$

$\mathrm{Tr}\,(T^aT^bT^c)\, f^{abc}$ □ $\mathrm{Tr}\,(\bar{T}^a\bar{T}^b\bar{T}^c)\, f^{abc}$

NONperturbative Planar Equivalence



Gauge field background is the same!

$$D \equiv \det\left(i\,\partial\!\!\!/ + A\!\!\!/^a T^a_{\rm Adj} - m\right) = \sum_{\mathcal{C}} \alpha_{\mathcal{C}}\, \mathcal{W}_{\mathcal{C}}[A_{\rm Adj}]$$

$$\mathcal{W}_{\mathcal{C}}[A_{\rm Adj}] = {\rm Tr}\, P\, \exp\left(i\int_{\mathcal{C}} A^a_\mu T^a_{\rm Adj}\, dx^\mu\right)$$

$$= \sum_{\mathcal{C}} \alpha_{\mathcal{C}}\, {\rm Tr}\, P\, \exp\left(i\int_{\mathcal{C}} A^a_\mu T^a\, dx^\mu\right) {\rm Tr}\, P\, \exp\left(i\int_{\mathcal{C}} A^a_\mu \bar{T}^a\, dx^\mu\right)$$

$$[(T^a \oplus 1),\ (1 \oplus \bar{T}^a)] = 0$$

Here comes Planarity

$$\sum_{\mathcal{C}} \alpha_{\mathcal{C}} \, \langle \, \mathcal{W}_{\mathcal{C}}[A_{\Box}] \; \mathcal{W}^*_{\mathcal{C}}[A_{\Box}] \, \rangle$$

$$= \sum_{\mathcal{C}} \alpha_{\mathcal{C}} \, \langle \, \mathcal{W}_{\mathcal{C}}(A_{\Box}) \, \rangle \, \langle \, \mathcal{W}^*_{\mathcal{C}}(A_{\Box}) \, \rangle = \sum_{\mathcal{C}} \alpha_{\mathcal{C}} \, \langle \, \mathcal{W}_{\mathcal{C}}(A_{\Box}) \, \rangle^2$$

det $(iD_{\Box}\Box^{\Box})_{\text{susy}}$ =det $(iD_{\Box}\Box^{\Box})_{\text{orienti}}$ at $N=\infty$

Consequences for orienti A at $N = \infty$:

- Infinite number of degeneracies: e.g. 0^+ & 0^- □1^- & 0^+ □...;
- "BPS" domain walls;
- Lighness of □; $m_{\Box}{}^2 = m_{\Box'}{}^2\,(1+O(1/N))$;
- Exact β function; calculable quark condensate.

Quark condensate at N=3 (1-flavor QCD)

$$\langle \overline{\Psi}_R \Psi_L \rangle = -6(N-2)\Lambda^3 e^{2\pi i k/(N-2)} (1+O(1/N))$$

$$\Rightarrow -6\Lambda^3 (1\pm(1/3)) \Rightarrow -(0.6 \text{ to } 1.1)\Lambda^3_{\overline{MS}}$$

estimate

"Experimental" => $-(0.4 \text{ to } 0.9)\Lambda^3_{\overline{MS}}$

lattice or extrapolation to n_f=1

Vacuum energy density (cosmological constant)

Usually in non-SUSY ϵ_{vac} □ N^2;

in orienti ~~ϵ_{vac} □ N^2~~

ϵ_{vac} □ N^1

More generally:

- Parent: k "flavors" of adjoint Majoranas
  - Daughter: k flavors of $\Psi^{[ij]}$'s

A new "orientifold" large N expansion

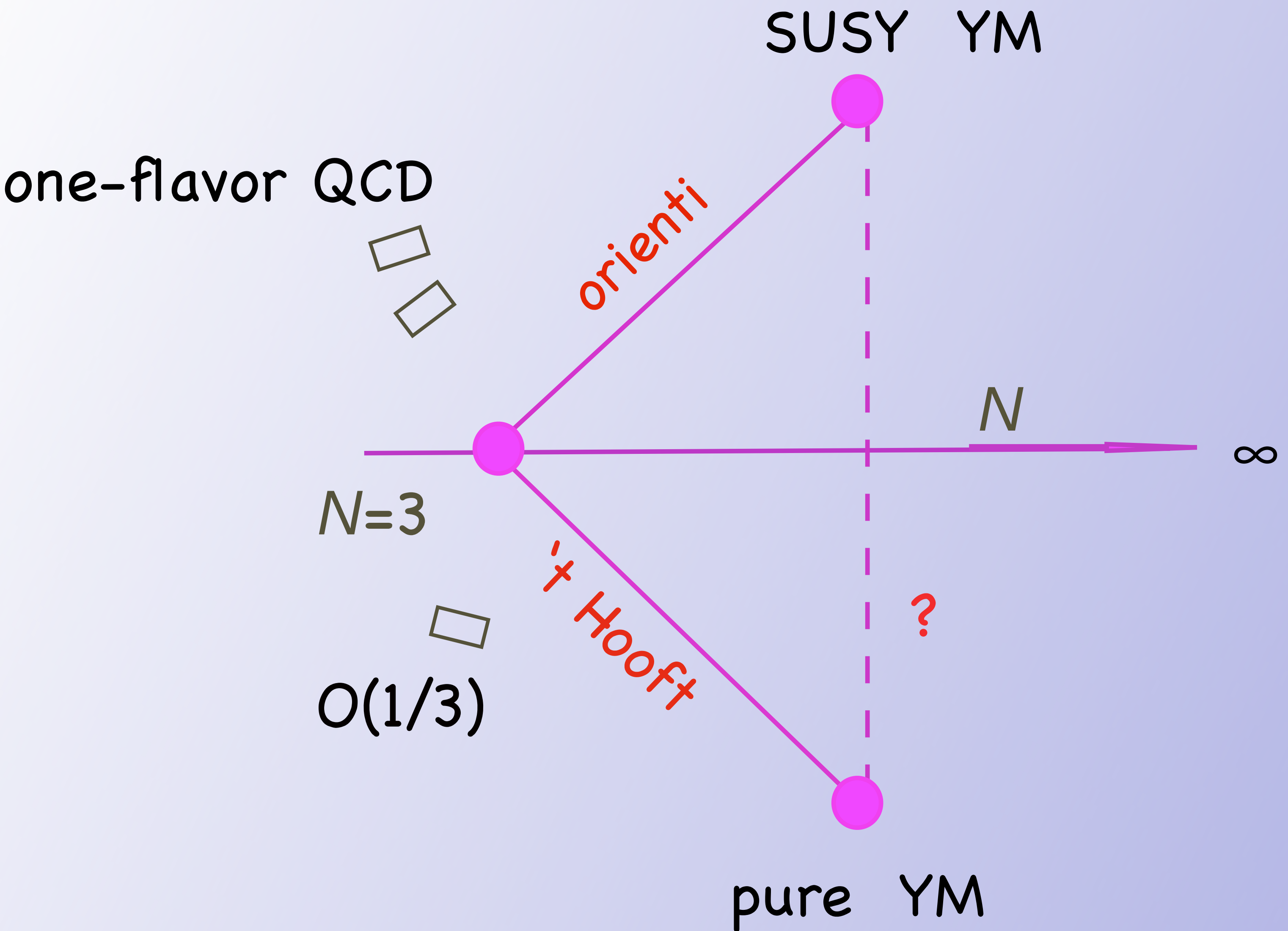
The same at N=3!

't Hooft: fundamental Dirac quarks at all N

orineti: Dirac $\psi[ij]$ at all N

$\Box_{gl}/\Box_{qu}\Box N^{-1}$

$\Box_{gl}/\Box_{qu}\Box N^{0}$



Remnants of SUSY in pure Yang-Mills?

SUSY in pure Yang-Mills (with ~30% accuracy):

$$\left\langle G^a_{\mu\nu} G^{\mu\nu a} + i\, G^a_{\mu\nu} \tilde{G}^{\mu\nu a} \right\rangle_{vac} = \mu^4 \exp\left\{ -\frac{1}{N}\left(\frac{8\pi^2}{g^2} + i\theta \right) \right\}$$

NSVZ, '80's

$$4 \cdot \frac{8\pi^2}{(11/3)\, N\, g^2} \longrightarrow 4 \cdot \frac{8\pi^2}{(12/3)\, N\, g^2}$$

Holomorphic coupling

Accuracy 1/11 — not so bad!

Conclusions:

- SUSY gluodynamcs is planar equivalent to non-SUSY orienti;
- At N=3 we get one-flavor QCD;
- Analytic predictions: spectral degeneracies, condensates,...

  ϵ_{vac} □ N^{1};

- Orientifold large-N expansion; Remnants of SUSY in pure Yang-Mills;

<u>Problems: Major => calculating 1/N corrections</u>

- ??? $[\text{Det}\,(iD\ \ \gamma\ \ -m)_A \times \text{Det}\,(iD\ \ \gamma\ \ -m)_S]^{1/2} \sim 1 + 1/N^{2}$???