AdS/CFT correspondence for half-BPS states

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Outline

- 1/2 BPS states in field theory & Fermi liquid.
- Technique for constructing gravity solutions.
- Solutions of IIB SUGRA and Laplace equation.
- Solutions of 11D SUGRA and Toda equation.
- Summary.

Breaking supersymmetry in AdS/CFT

- Strings on $AdS_5 \times S^5 \leftrightarrow \mathcal{N}=4$ SYM
- Theories with fewer supersymmetries
  - field theory: adding superpotential
  - gravity: less supersymmetric backgrounds

    Klebanov, Tseytlin '00; Polchinski, Strassler '00;
    Klebanov, Strassler '00, Maldacena, Nunez '00
  - gravity solution $\leftrightarrow$ vacuum of field theory
- Breaking SUSY by looking at nontrivial states
  - no change in the Lagrangian
  - gravity solution $\leftrightarrow$ state in $\mathcal{N}=4$ theory
  - states preserving some supersymmetries correspond to regular geometries

Half–BPS states in $\mathcal{N}=4$ SYM

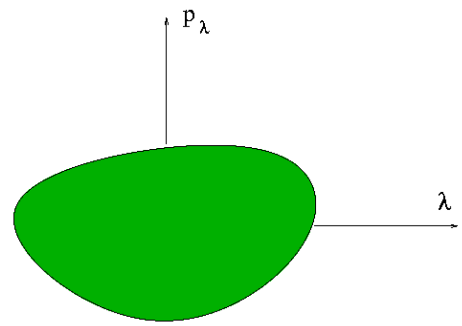
- $\mathcal{N}=4$ SYM on $S^3 \times R$:
  - chiral primaries:

    $$\mathrm{Tr}(Z^{n_1})\ldots\mathrm{Tr}(Z^{n_k}), \qquad Z=\phi_1+i\phi_2$$
  - symmetry: $S^3 \times SO(4)$
- Action in 1/2 BPS sector:

$$S=\int dt\mathrm{Tr}\left[\frac{1}{2}|D_tZ|^2-\frac{1}{2R^2}|Z|^2\right]$$

- Matrix model description: harmonic oscillator
  - set of harmonic oscillators: $\alpha_n^\dagger=\mathrm{Tr}[(a^\dagger)^n]$
  - eigenvalue basis & Fermi liquid

Berenstein '04



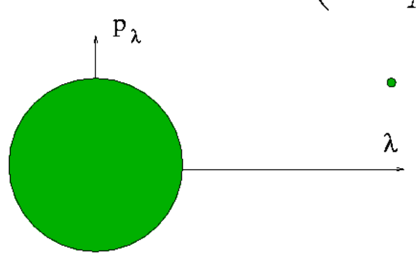
Branes and states in field theory

- States with small $\Delta = J$:
  - field theory: $\mathrm{Tr}(Z^{n_1})\ldots\mathrm{Tr}(Z^{n_k})$
  - small ripples on the Fermi sea
  - perturbative states in string theory
- Supersymmetric branes
  - giant gravitons expanding on S^3 or $\tilde{S}^3$.

    McGreevy, Susskind, Toumbas '00
  - field theory: excitation of a single eigenvalue:

$$Z \sim e^{i\omega t}\mathrm{diag}\left(\eta, -\frac{\eta}{N-1}, \ldots -\frac{\eta}{N-1}\right)$$

Hashimoto, Hirano, Itzhaki '00



- Our goal: exact solutions of SUGRA
- $AdS_7 \times S^4$: giant gravitons with $S^5 \times S^2$.

Technique for constructing gravity solutions

- Assumptions
  - bosonic symmetries: $SO(4) \times SO(4)$
  - bosonic fields: metric and $F^{(5)}$
  - existence of Killing spinor:

$$\nabla_M \eta + \frac{i}{480}\Gamma^{M_1M_2M_3M_4M_5}F^{(5)}_{M_1M_2M_3M_4M_5}\Gamma_M\eta = 0$$

- Reduction on $S^3 \times S^3$: spinor in 4D interacting with gauge field and 2 scalars
- Using bilinears of Killing spinor

Gauntlett, Gutowski, Martelli, Pakis, Reall, Sparks, Waldrum '02–'04

$$K_\mu = -\bar{\varepsilon}\gamma_\mu\varepsilon \qquad K \cdot L = 0, \quad L^2 = -K^2$$
$$L_\mu = \bar{\varepsilon}\gamma^5\gamma_\mu\varepsilon$$

  - L is an exact form, K^μ is a Killing vector

$$ds^2 = h^2 dy^2 - h^{-2}(dt + V_i dx^i)^2 + \tilde{h}_{ij}dx^i dx^j$$

1/2 BPS geometries in Type IIB SUGRA

- Explicit geometry and Laplace equation

$$ds^2 = -h^{-2}(dt + V_i dx^i) + h^2(dy^2 + dx^i dx^i) + y e^G d\Omega_3^2 + y e^{-G} d\tilde{\Omega}_3^2$$

$$F_{(5)} = F_{\mu\nu} dx^\mu \wedge dx^\nu \wedge d\Omega_3 + \tilde{F}_{\mu\nu} dx^\mu \wedge dx^\nu \wedge d\tilde{\Omega}_3$$

$$F = dB_t(dt + V) + B_t dV + d\hat{B}$$

– functions appearing in the solution:

$$h^{-2} = 2y\cosh G, \qquad y dV = *_3 dz$$

$$B_t = -\frac{1}{4}y^2 e^{2G}, \qquad d\hat{B} = -\frac{1}{4}y^3 *_3 d(\frac{z+\frac{1}{2}}{y^2})$$

$$\tilde{B}_t = -\frac{1}{4}y^2 e^{-2G} \qquad d\hat{\tilde{B}} = -\frac{1}{4}y^3 *_3 d(\frac{z-\frac{1}{2}}{y^2})$$

– solution is parameterized by one function z

$$z = \frac{1}{2}\tanh(G), \qquad \partial_i\partial_i z + y\partial_y(\frac{\partial_y z}{y}) = 0$$

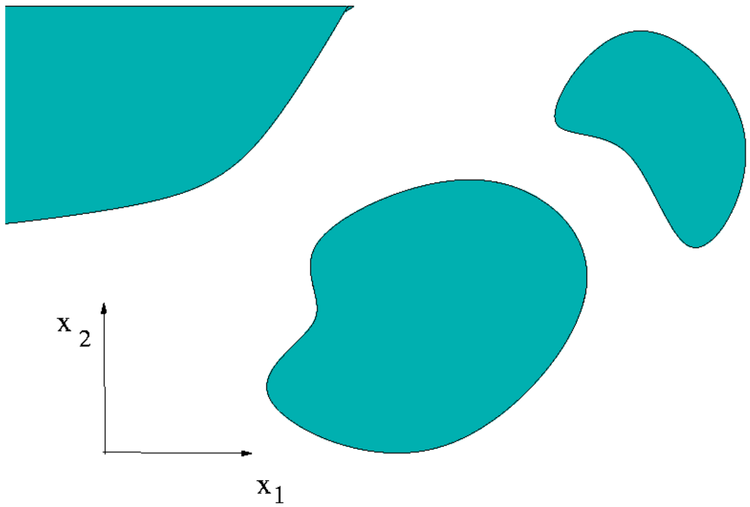
- potential singularities: $R\tilde{R} = y = 0$

Regular solutions: general description

- Laplace equation and boundary conditions
  – 6D Laplace equation for $\Phi = \frac{z}{y^2}$
  – regularity at $y = 0$: $z = \pm\frac{1}{2}$

$$h^2 dy^2 + y e^{-G} d\tilde{\Omega}_3^2 \sim \frac{1}{c(x)}(dy^2 + y^2 d\tilde{\Omega}_3^2)$$

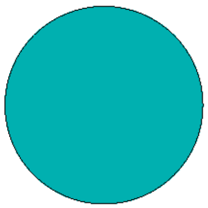
- Boundary condition for a generic state



- Plane $y = 0 \leftrightarrow$ phase space of the oscillator

Examples

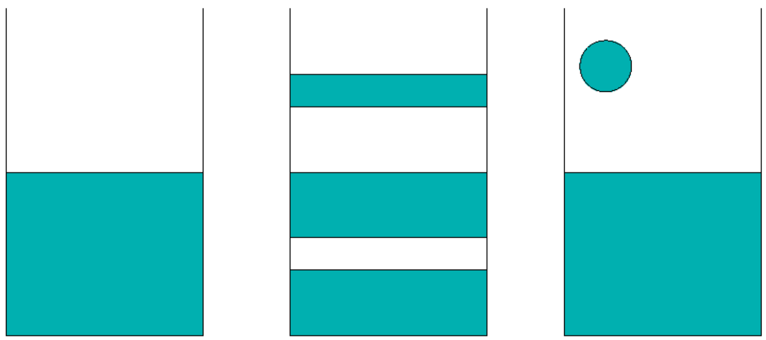
- $AdS_5 \times S^5$



$$z = \frac{r^2 - r_0^2 + y^2}{2\sqrt{(r^2+r_0^2+y^2)^2 - 4r^2 r_0^2}}$$

- (Giant) gravitons

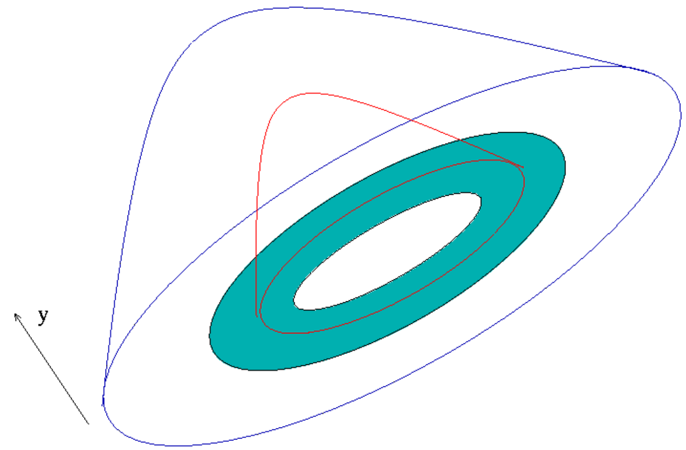
- PP wave and its excitations



$$z = \frac{x_2}{2\sqrt{x_2^2+y^2}}$$

Topology and fluxes

- Two types of closed five–manifolds



- Different topologies: non–contractible spheres
- Quantization of fluxes:

$$\tilde{N} = -\frac{1}{2\pi^2 l_p^4}\int d\hat{\tilde{B}} = \frac{(\text{Area})_{z=-\frac{1}{2}}}{4\pi^2 l_p^4}$$

- Energy and higher moments

$$\Delta = J = \int \frac{d^2x}{2\pi\hbar}\frac{\frac{1}{2}(x_1^2+x_2^2)}{\hbar} - \frac{1}{2}\left(\int_D \frac{d^2x}{2\pi\hbar}\right)^2$$

$1/2$ BPS geometries in M theory

- Bosonic symmetries: $SO(6) \times SO(2)$

$$
\begin{aligned}
ds_{11}^2 &= \frac{e^{2\lambda}}{m^2} d\Omega_5^2 + \frac{y^2 e^{-4\lambda}}{4m^2} d\tilde{\Omega}_2^2 \\
&- \frac{e^{2\lambda} h^2}{m^2} (dt + V_i dx^i)^2 + \frac{e^{-4\lambda}}{4m^2 h^2} \left(dy^2 + e^D d\mathbf{x}^2 \right)
\end{aligned}
$$

$$
h = 1 + y^2 e^{-6\lambda}, \qquad e^{-6\lambda} = \frac{\partial_y D}{y(1 - y\partial_y D)}
$$

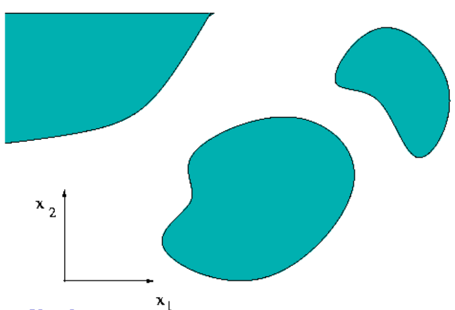
- Solution is parameterized by one function D which satisfies 3D Toda equation:

$$
\Delta D + \partial_y^2 e^D = 0
$$

- Boundary conditions at $y = 0$

$$
\partial_y D = 0, \quad R_2 \to 0
$$

$$
D \sim \log y, \quad R_5 \to 0
$$



- Boundary condition at infinity

Solutions of Toda equation

- $AdS_7 \times S^4$

$$
e^D = \frac{r^2 L^{-6}}{4 + r^2}, \quad x = (1 + \frac{r^2}{4}) \cos\theta, \quad 4y = L^{-3} r^2 \sin\theta
$$

- $AdS_4 \times S^7$

$$
e^D = 4L^{-6} \sqrt{1 + \frac{r^2}{4} \sin^2\theta}
$$

$$
x = \left(1 + \frac{r^2}{4}\right)^{1/4} \cos\theta, \quad 2y = L^{-3} r \sin^2\theta
$$

- PP wave

$$
e^D = \frac{r_5^2}{2}, \quad y = \frac{1}{4} r_5^2 r_2, \quad x_2 = \frac{r_5^2}{4} - \frac{r_2^2}{2}
$$

- Translational invariance in x_1: linear equation

$$
e^D = \rho^2, \quad \rho \partial_\rho V = y, \quad \partial_\eta V = x_2
$$

$$
\frac{1}{\rho} \partial_\rho (\rho \partial_\rho V) + \partial_\eta^2 V = 0
$$

Ward '90

- Compactification of x_1: type IIA gravity duals of the BMN matrix model

Solution of gauged SUGRA

- M theory on S^4: gauged SUGRA in 7D
  - field content: $SL(5,R)/SO(5)$ coset, $SO(5)$ gauge field, five 3–forms
    Perini, Pilch, van Nieuwenhuizen '84
  - 1/2 BPS black hole: symmetry group $SO(6)\times SO(3)\times SO(2)\times U(1)$ Liu, Minasian '99
- Our goal: regular supersymmetric solution
  - symmetry $SO(6)\times SO(3)\times U(1)$
  - excited fields:

$$V_I{}^i=\left[\begin{array}{cc} e^{-3\chi}g & \mathbf{0}_{2\times 3}\\ \mathbf{0}_{3\times 2} & e^{2\chi}\mathbf{1}_{3\times 3}\end{array}\right],\quad A_{\mu I}{}^J=\left[\begin{array}{cc} iA_\mu\sigma_2 & \mathbf{0}_{2\times 3}\\ \mathbf{0}_{3\times 2} & 0_{3\times 3}\end{array}\right]$$

$$g=\exp(i\theta\sigma_2)\exp(-\rho\sigma_3)\in SL(2,R)/U(1)$$

- Regular solution exists and gives e^D

Relation to known solutions

- $\mathcal{N}=2$ superconformal field theories
  - 16 supercharges, $SO(4,2)\times SU(2)\times U(1)$
  - double analytic continuation of 11D solutions:

$$d\Omega_5^2\to -ds^2_{AdS_5},\qquad t\to\psi$$

  - different boundary conditions
  - example of a solution

$$e^D=\frac{1}{x_2^2}(\frac{1}{4}-y^2)$$

    Maldacena, Nunez '00
  - new feature: space ends at $y=1/2$:

$$\psi\sim\psi+2\pi$$

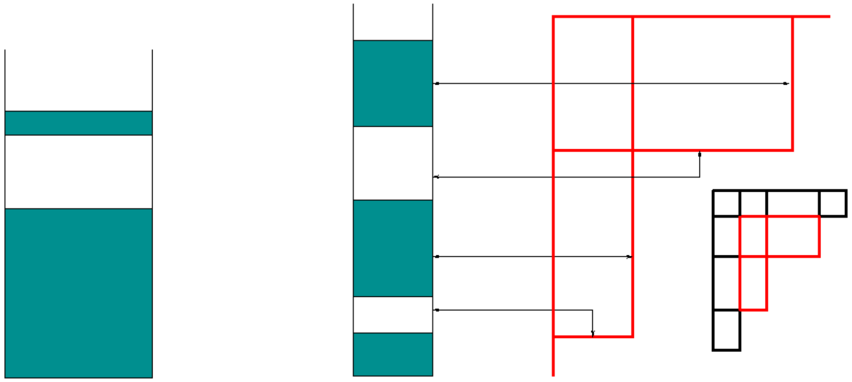
- M2 brane with mass deformation
  - IIB with x_1 isometry $\longrightarrow$ IIA $\longrightarrow$ M theory
  - Bena & Warner solutions with regular BC

M2 brane with mass deformation

- M2–M5 solution with 16 supercharges
  - gauged SUGRA: $SO(4) \times SO(4)$ Pope, Warner '03
  - M2 branes polarized into M5 Bena, Warner '04
- Relation to our solution
  - IIB with x_1 isometry $\longrightarrow$ IIA $\longrightarrow$ M theory
  - harmonic function of Bena & Warner:

$$h = \frac{g}{y^2}, \qquad \partial_x g = -\frac{z}{16}$$

- Dielectric M5 branes and Young tableaux



- Asymptotic geometry: $AdS_7 \times S^4$.

Summary

- Geometries dual to chiral primaries: no singularities or horizons
- All $1/2$ BPS gravity solutions for type IIB
  - reduction to 3D Laplace equation
  - boundary conditions and free fermions
  - explicit solutions in terms of integrals
  - fluxes and topologies
- All $1/2$ BPS solutions of 11D SUGRA
  - reduction to 3D Toda equation
  - specific boundary conditions
  - examples: AdS, pp wave
  - new regular solution of gauged SUGRA
- Future directions
  - properties of the new geometries
  - $1/4$ BPS states