AdS/CFT without Supersymmetry

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Work done in collaboration with

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Motivation:

- The only examples where AdS/CFT can be shown to work involve supersymmetry
- Nevertheless, the fact that any theory on AdS to the extend that it is well defined has to be dual to a conformal theory on the boundary seems to be quite general.

  (Closest to S-Matrix one can define)
- Also it is believed that small (even non-supersymmetric) deformations of known dual pairs give rise to "good" duals.

Motivation:

BUT HOW DO WE KNOW FOR SURE?

- For the QCD/String Program to have a hope for success we would like a non-supersymmetric example of AdS/CFT for which one can CHECK that both sides still agree quantitatively.
- Look for non-supersymmetric deformations that inherit some of the non-renormalization theorems of the N=4 theory.

Motivation:

- In two dimensional conformal field theories a powerful tool is conformal perturbation theory:

$$S = S_{CFT}(\lambda) + \gamma \int d^d z O_{ma.}(z)$$

$$\langle A_1(x_1) A_2(x_2) \ldots \rangle = \int \mathcal{D}X e^{-S} A_1(x_1) A_2(x_2) \ldots =$$

$$= \int \mathcal{D}X e^{-S_{CFT}} \left(1 - \gamma \int d^d z O_{ma.}(z) + \ldots\right) A_1(x_1) A_2(x_2) \ldots =$$

$$\langle A_1(x_1) A_2(x_2) \ldots \rangle_{CFT} - \gamma \int d^d z \langle A_1(x_1) A_2(x_2) O_{ma.}(z) \ldots \rangle_{CFT} + \ldots$$

Motivation:

- Conformal perturbation theory nicely complements standard Feynman diagrams.
- As long as we know the correlation functions in the original theory to all orders in the coupling λ, conformal perturbation theory, while being order by order in the perturbation γ, will sum up all orders in λ.
- For simple perturbations (like the ones I am going to consider) Feynman diagrams can incorporate the perturbation to all orders in γ while being only perturbative in λ.

Motivation:

- Can conformal perturbation theory be used to derive non-renormalization theorems (in λ) in non-supersymmetric, conformal field theories that are perturbations of the supersymmetric N=4?
- How does regularization work?
- Stumbling block: "small" set of known examples of non-supersymmetric CFTs in 4d with exactly marginal couplings (approximately zero)

Example -- Janus Solution:

(Bak, Gutperle, Hirano)

The Field Theory:

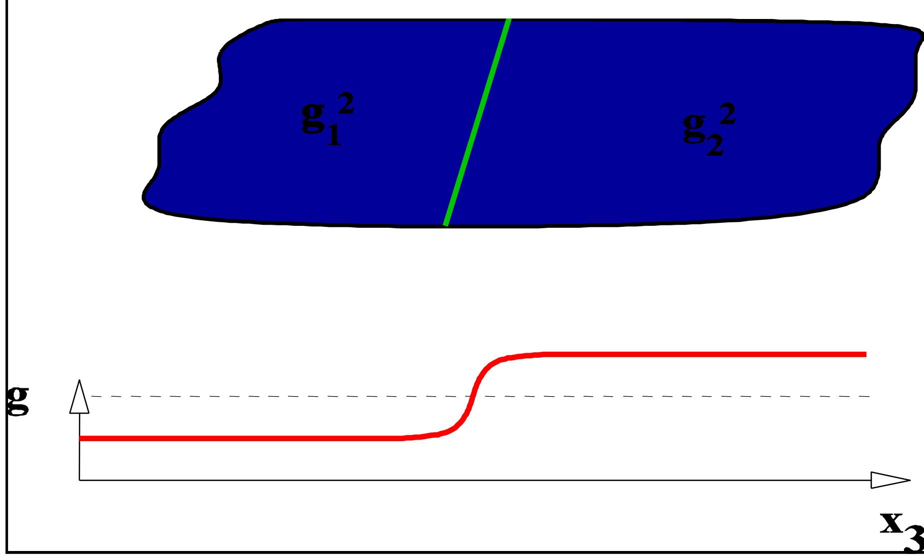
- Consider the following field theory action:

$$S = \int d^4x \frac{1}{g^2(x_3)} \mathcal{L}_{\mathcal{N}=4}$$

where we are mostly interested in

$$g^2(x) = \begin{cases} g_1^2 & \text{for } x_3 < 0 \\ g_2^2 & \text{for } x_3 > 0 \end{cases}$$

The Field Theory:



The Field Theory:

- Treat the jump as conformal perturbation:

$$S = \int d^4x \frac{\mathcal{L}_{\mathcal{N}=4}}{\bar{g}^2} - \gamma \int d^4x \epsilon(x_3) \mathcal{L}_{\mathcal{N}=4}$$

where

$$\epsilon(x) = \begin{cases} -1 & \text{for } x_3 < 0 \\ +1 & \text{for } x_3 > 0 \end{cases}$$

The Field Theory:

- The unperturbed theory is given in terms of the average coupling

$$2\bar{g}^{-2} = g_1^{-2} + g_2^{-2}$$

- The perturbation parameter is

$$\gamma = \frac{g_1^2 - g_2^2}{g_1^2 + g_2^2}$$

The Field Theory:

- The dynamics associated with this interface is the non-abelian generalization of the textbook scenario of E & B fields at the interface of two different materials

- One far-fetched application one could envision would be the analysis of domain walls across which α_s jumps

The Field Theory:

- Translation invariance broken ! at best 8 of the 16 supersymmetries preserved
- Studying the transformation properties explicitly one finds that actually all 16 supersymmetries are broken in the FT described above.
- 4 supercharges can be restored by adding counterterms (new Yukawa couplings) on the interface, but they break SO(6) to SU(3)
- Janus has SO(6) and no supersymmetry

SUMMARY – The field theory:

- The Janus field theory describes N=4 SYM close to an interface across which the coupling constant jumps
- The situation is analogous to the usual interfaces in E&M between two different materials
- Supersymmetry is completely broken, but the field theory is free of diseases. Conformal invariance is preserved.

The Supergravity:

- The Janus field theory is a particular example of a defect conformal field theory.

  (AK & Randall; DeWolfe, Freedman &Ooguri)
- Since there are no degrees of freedom localized on the defect for Janus, we often refer to it as an interface CFT.
- As such it fairly straight forward to embed it into supergravity via AdS slicings of AdS

The Supergravity:

- Some basic AdS/CFT lore: The bulk metric does not uniquely determine the boundary metric, only its conformal structure.
- Recipe: Pick function f which vanishes linearly as you approach the boundary (which is the place at which the metric diverges quadratically as z ! z_b).
- Define boundary metric:

$$ds^2_{bound.} = \lim_{z \to z_b} f^2(z) ds^2$$

The Supergravity:

- Different choices for f give different answers for the boundary metric
- They are related by a conformal transformation

$$ds^2_{boundary,2} = \frac{f_2^2}{f_1^2} ds^2_{boundary,1}$$

  - The corresponding field theory has to be conformal, the operators transform as $(f_1/f_2)^\Delta$

The Supergravity:

- Different slicings of AdS:

$$ds^2_{AdS_5} = e^{2A(r)} ds^2_{4d} + dr^2$$

- These are just different coordinate systems on AdS, but they lead to different "natural" choices of f. For Minkowski at r ! 1 chose:

$$ds^2 = e^{2r/L}(-dt^2 + d\vec{x}^2) + dr^2$$

$$f = e^{-r/L}$$

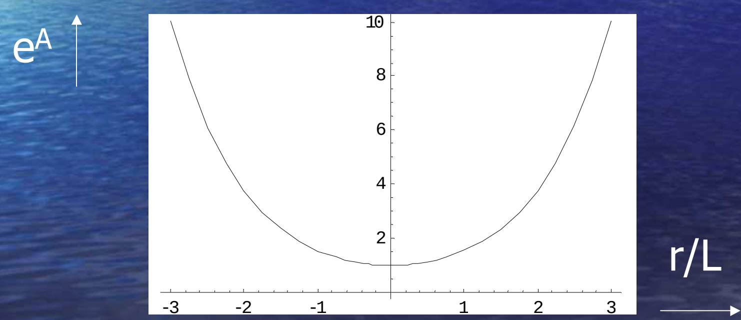
The Supergravity:

- A particular interesting slicing is the slicing of AdS_5 in terms of AdS_4 slices.
- The manifest SO(3,2) symmetry in this slicing can be interpreted as the isometry of AdS_4. It is however also the subgroup of the SO(4,2) conformal symmetry of the boundary CFT that leaves the line $X_3=0$ in the boundary CFT invariant.
- Note that this is the symmetry of an ICFT or DCFT.

The Supergravity:

- In this AdS_4 slicing the AdS_5 metric reads

$$ds^2_{AdS_5} = \cosh(r/L)^2 ds^2_{AdS_4} + dr^2$$



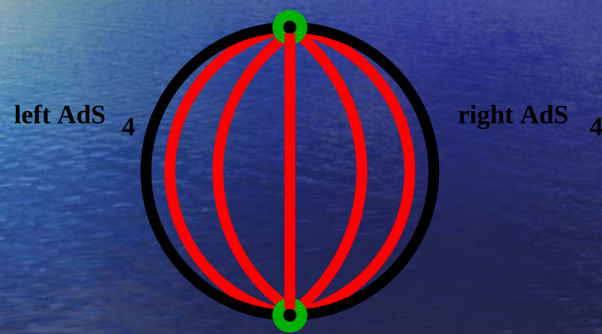
The Supergravity:

- Note that this metric approaches the boundary both for r ! § 1!!
- Using again f=e^{-A}! e$^{¨ r/L}$ as the natural choice, the boundary metric in this slicing appears to be two copies of AdS_4.

AdS_4 has a boundary.
A boundary of a boundary?
HOW IS THIS POSSIBLE?

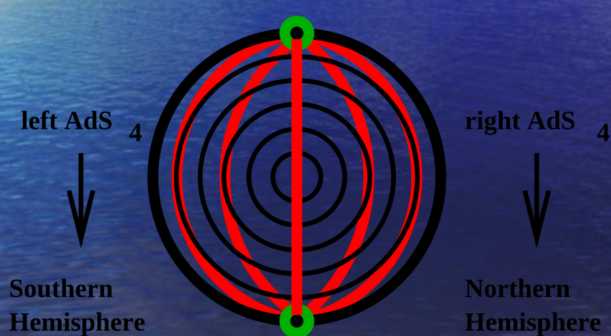
The Supergravity:

- The two boundary AdS_4's communicate across their common boundary!



The Supergravity:

- The system can be conformally mapped to a CFT on the sphere, where
  - the r ! 1 boundary maps to the northern hemisphere
  - the r ! -1 boundary maps to the southern hemisphere

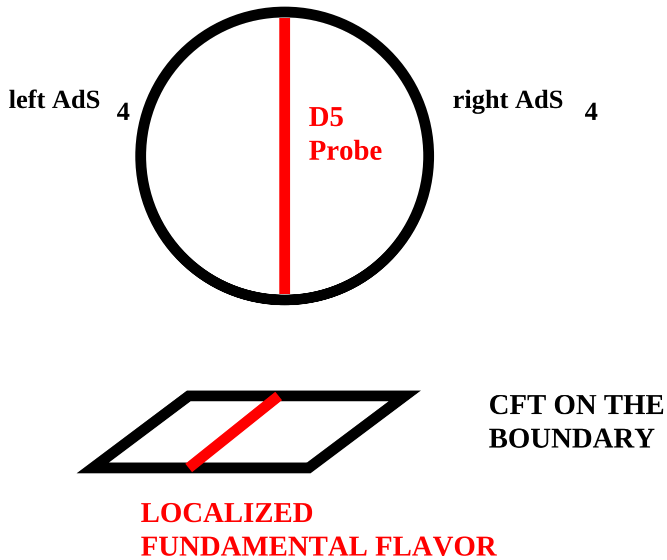


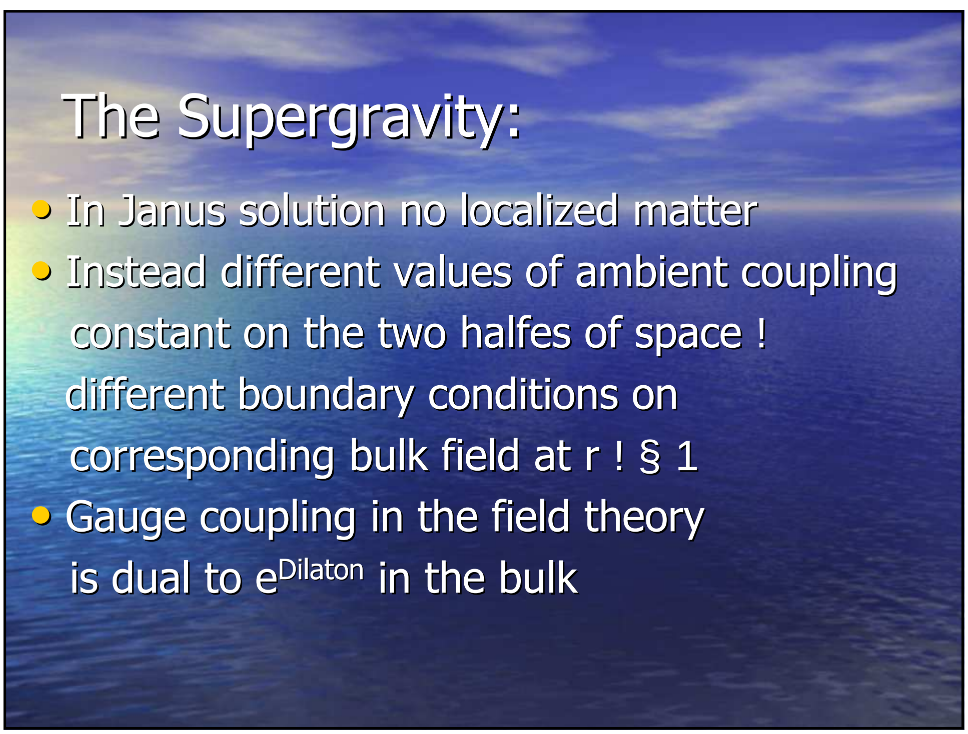
The Supergravity:

- The system can also be conformally mapped to a CFT on the Minkowski space, where
  - the r ! 1 boundary maps to the $x_3>0$ halfspace
  - the r ! -1 boundary maps to the $x_3<0$ halfspace
- This is just the right setup to capture the physics of an ICFT or DCFT!
- Boundary conditions at r ! § 1 determine couplings of the left and right CFT. Degrees of freedom on the defect live at r=0

The Supergravity:

- Example of a DCFT: The D3-D5 system.
- Toy model of flavor: a fundamental 3d hypermultiplet localized at $x_3=0$
- No contribution to 4d beta function. Asymptotic freedom is not lost.
- In the supergravity dual D5 brane probe lives on central r=0 slice


left AdS 4
D5
Probe
right AdS 4
CFT ON THE
BOUNDARY
LOCALIZED
FUNDAMENTAL FLAVOR


The Supergravity:
In Janus solution no localized matter
Instead different values of ambient coupling constant on the two halfes of space !
different boundary conditions on corresponding bulk field at r ! § 1
Gauge coupling in the field theory is dual to e^Dilaton in the bulk

The Supergravity:

- Look for solution of IIB SUGRA of the form

$$ds^2 = e^{2A(r)} ds^2_{AdS_4} + dr^2 + d\Omega^2_{S^5}$$

$$\Phi = \Phi(r)$$

with boundary behaviour as $r \to \pm\infty$

$$e^{\Phi} \to g_{1,2} \quad , \qquad e^{A} \to \frac{1}{2} e^{|r|/L}$$

The Supergravity:

- JANUS SOLUTION:

$$r = \int_{A_0}^{A} \frac{dA}{\sqrt{e^{2A} - 1 + \frac{c^2 L^2}{24} e^{-6A}}}$$

$$\Phi(r) = \Phi_0 + c \int_0^r e^{-3A}$$

c measures jump in coupling constant

The Supergravity:

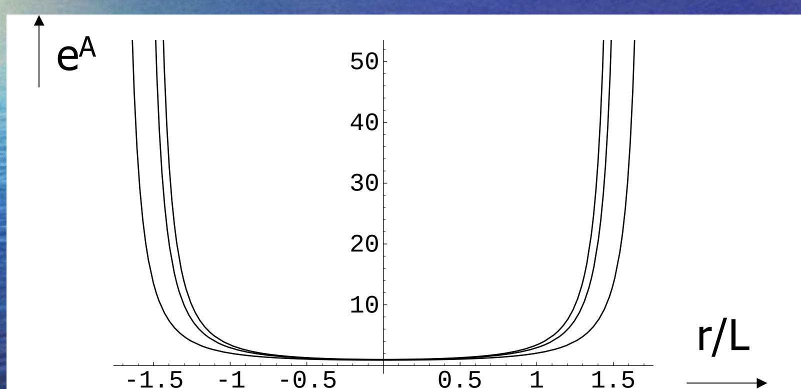
$$\Phi_{\pm\infty} = \Phi_0 \pm \frac{2}{3}c + \mathcal{O}(c^3)$$

$$\gamma = \tanh\left(\frac{\Phi_{+\infty} - \Phi_{-\infty}}{2}\right) = \frac{2}{3}c + \ldots$$

Expanding SUGRA in powers of c maps to Conformal perturbation theory

The Supergravity:

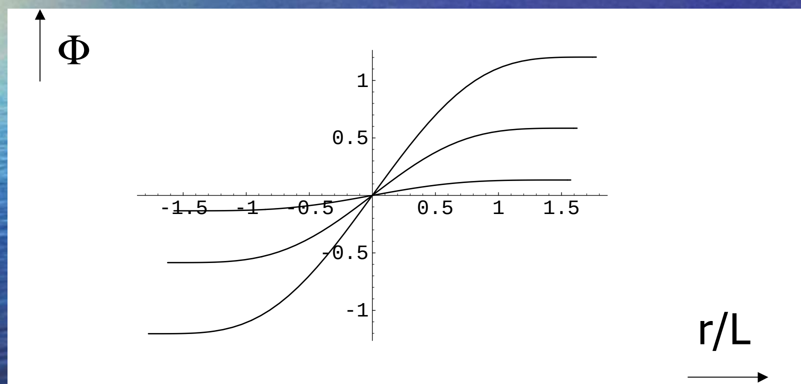
- Warpfactor e^A of the Janus solution:



From inner to outer curve c takes values 0.2, 0.8, 1.3

The Supergravity:

- Dilaton Φ of the Janus solution:



From flat to steep curve c takes values 0.2, 0.8, 1.3

The Supergravity:

- For $c < c_{crit} = 1.59$ the Janus solution is free of curvature singularities
- The curvature actually remains small in string units as long as the 't Hooft coupling is large on both sides
- Feynman diagrams are a good description as long as the coupling is small on both sides
- For protected correlators conformal perturbation theory is good for small γ

The Supergravity:

- Supersymmetry is broken by Janus; dilaton gradient can only be compensated by 3-form
- SO(6) is manifestly preserved, S^5 untouched
- We saw in the field theory that 4 supercharges can be restored by adding defect localized counterterms, at the cost of breaking the global SO(6) down to SU(3)
- Can we find the supergravity dual for this?

(currently under investigation with A. Clark)

The Supergravity:

- The existence of a well defined field theory dual which can serve as a non-perturbative definition of IIB string theory on Janus suggests that the Janus solution is stable
- Indeed stability can be proven also from the gravity point of view using Witten-Nester like positive energy theorems under mild assumptions

(Freedman, Nunez, Schnabl, Skenderis)

SUMMARY -- Supergravity:

- ICFT and DCFTs are dual to AdS_4 sliced backgrounds that asymptote to AdS_5
- Janus is dual to a smooth, stable, NON-SUPERSYMMETRIC, SO(6) invariant background of this type in IIB gravity that is supported by only dilaton and 5-form.
- Analytic expression valid to all orders in c can be given for A(r), but it is convenient to express the result order by order in c

Correlation Functions:

- Having set up both Field theory Lagrangian and Supergravity Solution we now want to calculate correlation functions using
  - Supergravity
  - Conformal Perturbation Theory
  - Feynman Diagrams
- Argue for non-renormalization properties inherited from the N=4 and compare weak and strong coupling results

Correlation Functions:

- Where to start? Usually simplest non-trivial correlation function is 2-pt function.
- One point function vanishes in CFTs!

$$\langle O_\Delta(x)\rangle = \frac{C}{|x|^\Delta}$$

- This form is fixed by rotation invariance and scale transformations, but |x| is not invariant under translations, so C has to be 0.

Correlation Functions:

- In DCFT or ICFT translation invariance is broken, so one-point function is allowed.

$$\langle O_\Delta(\vec{x}, x_3)\rangle = \frac{C}{x_3^\Delta}$$

(McAvity and Osborn)

- Higher n-point functions are also less constrained; e.g. 2-pt function contains a free function of $\xi=(x-y)^2/(x_3\ y_3)$

Correlation Functions - Aside:

- Curiously only scalar operators can have 1-pt functions, in particular

$$\langle T_{\mu\nu}(\vec{x}, x_3)\rangle = 0$$

- For CFT with large central charge the normalization constant C is large, so that for a far away domain wall one can have approximately constant vacuum expectation values (for, say, the Higgs) while the cc is still forced to vanish

Correlation Functions – Gravity:

- One point functions on the gravity side can be read off directly from the geometry!

$$S \sim S_{lead.} e^{(\Delta - d)r} + S_{subl.} e^{-\Delta r}$$

- For any field this is the asymptotic form
- $S_{lead.}$ gives the value of the corresponding coupling with which the dual operator is added to the Lagrangian
- $S_{subl.}$ determines the 1-pt function

Correlation Functions – Gravity:

- In Janus only one scalar field turned on, the dilaton. Dilaton is dual to the full N=4 Lagrangian

$$\langle \mathcal{L}_{\mathcal{N}=4} \rangle = \epsilon(x_3) \frac{N^2}{2\pi^2} \frac{c}{4 x_3^4} + \mathcal{O}(c^3)$$

- Corrections to metric start at order $e^{-6r/L}$, so the $e^{-4r/L}$ term that gives $<T_{\mu\nu}>$ vanishes as it should

Correlation Functions – CPT:

- In Conformal Perturbation theory we have to lowest non-trivial order:

$$\langle \mathcal{L}(x) \rangle = -\gamma \int d^4 z \epsilon(z_3) \left\langle \mathcal{L}(x) \mathcal{L}(z) \right\rangle_0$$

- The zeroth order term $<L>_{N=4}$ vanishes
- The correlator on the rhs is in the unperturbed N=4 theory
- <LL> is a protected 2-pt functions. Its value is exact too all orders in the 't Hooft coupling

Correlation Functions – CPT:

A few subtelties:

- ✓ The integral is divergent and needs to be regulated. After absorbing the quartic divergence by a constant shift in L, the only divergent terms left are irrelevant contact terms.
- ✓ L is defined as the top-component of the supermultiplet containing Tr(X^2). The scalar kinetic term obtained this way is $- X \Box X$, not the usual $(dX)^2$

Correlation Functions – CPT:

- Final answer:

VICTORY!!!!!!!!!!!!!!

$$\langle \mathcal{L}(x) \rangle = \epsilon(x_3) \frac{3}{16\pi^2} \frac{\gamma}{x_3^4} + \mathcal{O}(\gamma^2)$$

- Completely agrees with SUGRA after taking into account that γ=2/3 c + O(c^2).

SUMMARY – Correlation Functions:

- One point function of L can be calculated in two different regimes.
- Gravity calculates at large 't Hooft coupling λ to all orders in γ
- Conformal Perturbation Theory calculates order by order in γ. The O(γ) and O(γ²) contribution are exact to all orders in λ, since <LL> and <LLL> are protected in the full N=4

PERFECT AGREEMENT

Feynman Graphs:

- In standard Feynman graph perturbation theory, the interface just modifies the (Feynman gauge) propagator
- Like in E&M, the interface introduces mirror charge at $(Ry)_{mu}=(y_0,y_1,y_2,-y_3)$

$$\Delta_{\mu\nu}(x-y) = g_2^2 \frac{1}{4\pi^2}\frac{\delta_{\mu\nu}}{(x-y)^2} + \\ + \gamma \quad g_2^2 \frac{1}{4\pi^2}\frac{R_{\mu\nu}}{(x-Ry)^2}$$

Feynman Graphs:

- One point function from leading bubble

- Direct Propagator gives divergent contact term ----- vanishes in N=4 SYM against scalar and fermion bubbles.

Feynman Graphs:

- Mirror term gives finite contribution

$$\langle \mathcal{L}(x) \rangle = \epsilon(x_3) \frac{3}{16\pi^2} \frac{\gamma}{x_3^4}$$

SUMMARY --- Feynman Graphs:

- Standard perturbation theory agrees with both gravity and CPT.
- It gives the result order by order in the 't Hooft coupling, but to all orders in γ
- From the point of view of Feynman diagrams the exact agreement between the free theory and the strongly coupled gravity result looks like a miracle

THE BOTTOM LINE:

- ANDREAS LEARNED HOW TO USE POWERPOINT…….

THE BOTTOMLINE:

- Janus is a stable, non-supersymmetric background of string theory
- It's dual is an interface conformal field theory, that is under control and can be analyzed either via Feynman diagrams or CPT
- Certain one-point functions are protected and yield perfect agreement between weak and strong coupling results