SECULAR EVOLUTION OF COUPLED PLANET–DISC SYSTEMS

Gordon Ogilvie

IoA / DAMTP, University of Cambridge

with Steve Lubow

Space Telescope Science Institute

KITP, UCSB

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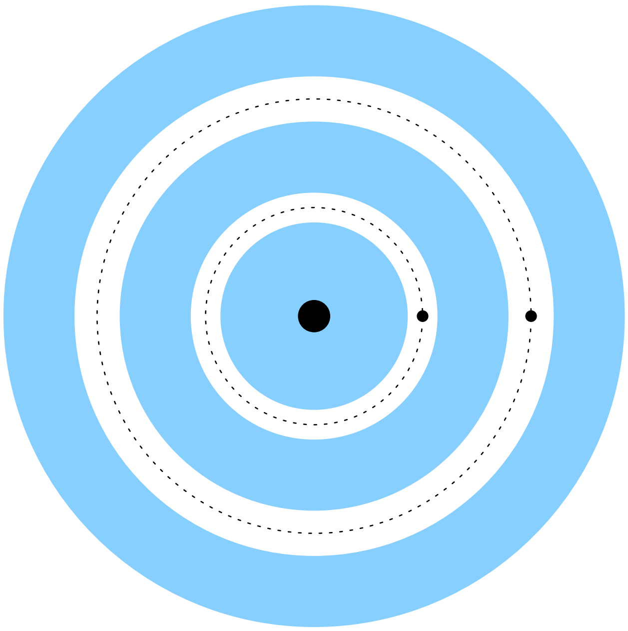
PLANET–DISC SYSTEMS

Protoplanetary system in intermediate phase

Disc partitioned into annuli



$$m_{\mathrm{p}} \sim m_{\mathrm{d}} \ll M_\star$$

Time-scales (e.g. for Jupiter):

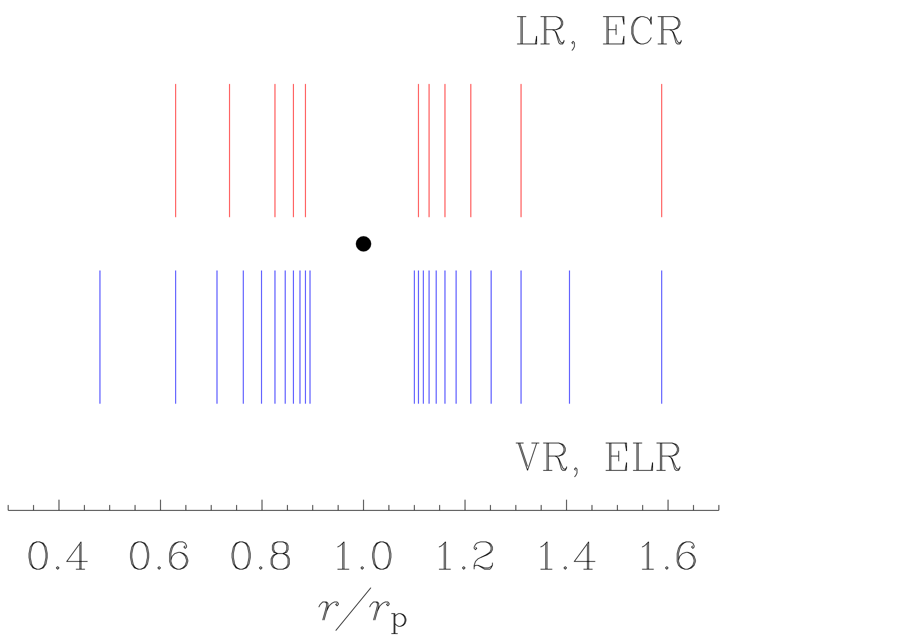
- dynamical $\qquad \Omega^{-1} \sim 10^1\,\mathrm{yr}$
- secular $\qquad \frac{M_\star}{m_{\mathrm{p}}}\Omega^{-1} \sim 10^4\,\mathrm{yr}$
- viscous / migration (?) $\qquad \frac{r^2}{\nu} \sim 10^7\,\mathrm{yr}$

Neglect accretion and migration: how do e and i evolve?

Focus on i for illustrative purposes (safer ground!)

EXISTING THEORIES

e and i of a gap-opening companion are both excited and damped by resonant interactions with the disc



- Goldreich & Tremaine (1980)
  - e is damped (delicate balance)
- Borderies, Goldreich & Tremaine (1984)
  - i is excited (delicate balance)
- Goldreich & Sari (2003)
  - e can be excited (ECR saturation: O & L 2003; Masset)

Can this be seen in planet-disc simulations?

PHILOSOPHY

Direct numerical simulations:

- particle dynamics, gas dynamics, MHD
- fully nonlinear
- increasingly powerful and important

(Semi-) analytic approaches:

- interpretative framework
- isolate different physical aspects
- suggest targeted experiments
- historical connections

Continuum celestial mechanics

- e and i for a disc

ECCENTRICITY AND INCLINATION IN A DISC

Generally, disturbances propagate through a gaseous disc

- inertia, pressure, buoyancy, self-gravitation, ...
- damped by linear or nonlinear mechanisms, or turbulence

Warped accretion discs (1975–)

- case III (Keplerian, $\alpha \lesssim H/r$): Papaloizou & Lin (1995)
- propagates as a non-dispersive wave
- damped by viscosity (or MHD turbulence)
- self-gravitation important when $Q \sim 1$

Eccentric accretion discs

- propagates as a dispersive wave
- damping / excitation very subtle (2D / 3D, relaxation, ...)
- self-gravitation important when $Q \sim r/H$!

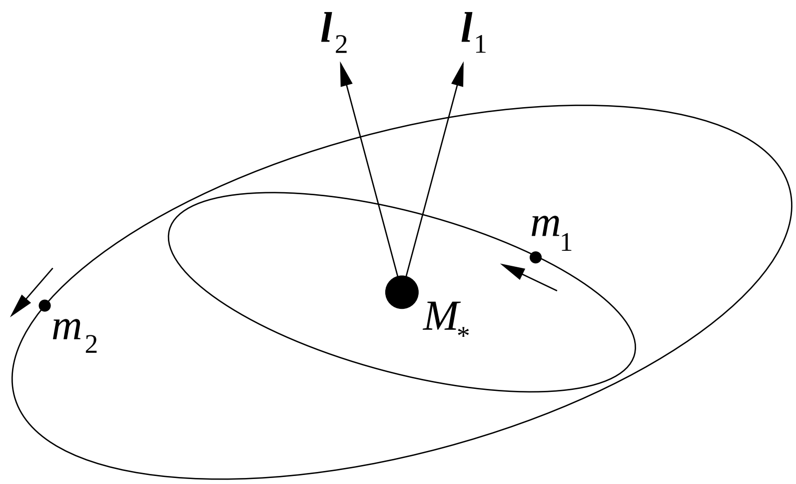
Asymptotic nonlinear theories (O 2001, 2004)

- 3D $\longrightarrow$ 1D evolutionary equations
- test problems

SUMMARY OF LAPLACE–LAGRANGE THEORY

Central star, mass $M_\star$

n planets, masses $m_i \ll M_\star$, in nearly Keplerian orbits



Complex inclination variable

$$W = l_x + \mathrm{i} l_y, \qquad |W| \ll 1$$

Secular inclination dynamics

$$J_i \frac{\mathrm{d}W_i}{\mathrm{d}t} = \mathrm{i} \sum_j C_{ij}(W_j - W_i)$$

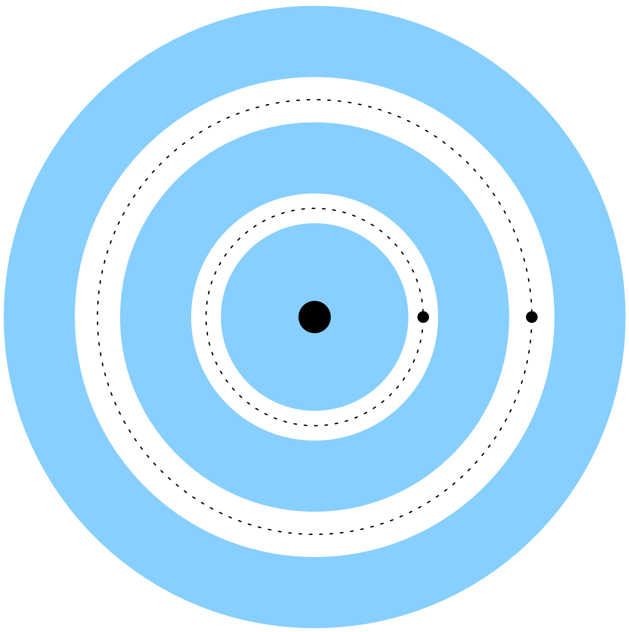
$$J_i = m_i r_i^2 \Omega_i, \qquad C_{ij} = G m_i m_j K(r_i, r_j)$$

Linear dynamical system, analyse into normal modes

PLANET–DISC SYSTEMS

Disc partitioned into annuli

Neglect accretion and migration: how does i evolve?



$W(r,t)$

Disc angular momentum equation

$$\Sigma r^2 \Omega \frac{\partial W}{\partial t} = \text{external torque density} + \frac{1}{2\pi r}\frac{\partial \mathcal{G}}{\partial r}$$

Internal torque (cf. Papaloizou & Lin 1995)

$$\frac{\partial \mathcal{G}}{\partial t} + \left(\frac{\kappa^2 - \Omega^2}{\Omega^2}\right)\frac{\mathrm{i}\Omega}{2}\mathcal{G} + \alpha\Omega\mathcal{G} = \frac{\pi}{2}\Sigma H^2 r^3 \Omega^3 \frac{\partial W}{\partial r}$$

Linear dynamical system

Planet i

$$J_{\mathrm{p}i}\frac{\mathrm{d}W_{\mathrm{p}i}}{\mathrm{d}t} = \mathrm{i}\sum_j Gm_{\mathrm{p}i}m_{\mathrm{p}j}K(r_{\mathrm{p}i},r_{\mathrm{p}j})(W_{\mathrm{p}j}-W_{\mathrm{p}i})$$

$$+\mathrm{i}\sum_k\int_{a_k}^{b_k}Gm_{\mathrm{p}i}\Sigma_k K(r_{\mathrm{p}i},r)(W_{\mathrm{d}k}-W_{\mathrm{p}i})\,2\pi r\,\mathrm{d}r$$

Disc k

$$\Sigma_k r^2\Omega\frac{\partial W_{\mathrm{d}k}}{\partial t} = \frac{1}{2\pi r}\frac{\partial\mathscr{G}_k}{\partial r}$$

$$+\mathrm{i}\sum_i Gm_{\mathrm{p}i}\Sigma_k K(r,r_{\mathrm{p}i})(W_{\mathrm{p}i}-W_{\mathrm{d}k})$$

$$+\mathrm{i}\sum_l\int_{a_l}^{b_l}G\Sigma_k\Sigma_l' K(r,r')(W_{\mathrm{d}l}'-W_{\mathrm{d}k})\,2\pi r'\,\mathrm{d}r'$$

Internal torque of disc k

$$\frac{\partial\mathscr{G}_k}{\partial t}+\left(\frac{\kappa^2-\Omega^2}{\Omega^2}\right)\frac{\mathrm{i}\Omega}{2}\mathscr{G}_k+\alpha\Omega\mathscr{G}_k = \frac{\pi}{2}\Sigma_k H^2 r^3\Omega^3\frac{\partial W_{\mathrm{d}k}}{\partial r}$$

Boundary conditions

$$\mathscr{G}_k(a_k)=\mathscr{G}_k(b_k)=0$$

Conservation laws

Horizontal angular momentum

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\sum_i J_{\mathrm{p}i}W_{\mathrm{p}i}+\sum_k\int_{a_k}^{b_k}\Sigma_k r^2\Omega W_{\mathrm{d}k}\,2\pi r\,\mathrm{d}r\right]=0$$

Vertical angular momentum

$$\frac{\mathrm{d}}{\mathrm{d}t}(-L_z)=-\sum_k\int_{a_k}^{b_k}\frac{2\alpha|\mathscr{G}_k|^2}{\pi\Sigma_k H^2r^3\Omega^2}\mathrm{d}r$$

with angular momentum deficit

$$\begin{aligned}-L_z&=\sum_i\frac{1}{2}J_{\mathrm{p}i}|W_{\mathrm{p}i}|^2\\&+\sum_k\int_{a_k}^{b_k}\left(\frac{1}{2}\Sigma_k r^2\Omega|W_{\mathrm{d}k}|^2+\frac{|\mathscr{G}_k|^2}{2\pi^2\Sigma_k H^2r^4\Omega^3}\right)2\pi r\,\mathrm{d}r\\&\geq 0\end{aligned}$$

Note

$$\cos i\approx 1-\frac{i^2}{2}=1-\frac{1}{2}|W|^2$$

Mean-motion resonances

Many resonances expected in a continuous disc

Local growth of inclination corresponds to a resonant torque

(Borderies, Goldreich & Tremaine 1984; Lubow 1992)

$$J_\mathrm{p}\frac{\mathrm{d}W_\mathrm{p}}{\mathrm{d}t} = \cdots + 2\pi s_\mathrm{res}\frac{Gm_\mathrm{p}^2}{M_\star}\Sigma r(W_\mathrm{p} - W)\bigg|_{r=r_\mathrm{res}}$$

$$\Sigma r^2\Omega\frac{\partial W}{\partial t} = \cdots + s_\mathrm{res}\frac{Gm_\mathrm{p}^2}{M_\star}\Sigma(W - W_\mathrm{p})\delta(r - r_\mathrm{res})$$

AMD can either grow or decay:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(-L_z) &= -\sum_k\int_{a_k}^{b_k}\frac{2\alpha|\mathscr{G}_k|^2}{\pi\Sigma_k H^2 r^3\Omega^2}\mathrm{d}r \\ &\quad + \sum_\mathrm{res} 2\pi s_\mathrm{res}\frac{Gm_\mathrm{p}^2}{M_*}\Sigma r|W - W_\mathrm{p}|^2\bigg|_{r=r_\mathrm{res}}\end{aligned}$$

Normal modes

Rigidly precessing patterns:

$$W_{\mathrm{p}i} = \hat{W}_{\mathrm{p}i}\,\mathrm{e}^{\mathrm{i}\omega t}, \qquad W_{\mathrm{d}k} = \hat{W}_{\mathrm{d}k}(r)\,\mathrm{e}^{\mathrm{i}\omega t}$$

Linear eigenvalue problem:

- integro-differential equations
- discretize and solve numerically

1 trivial mode

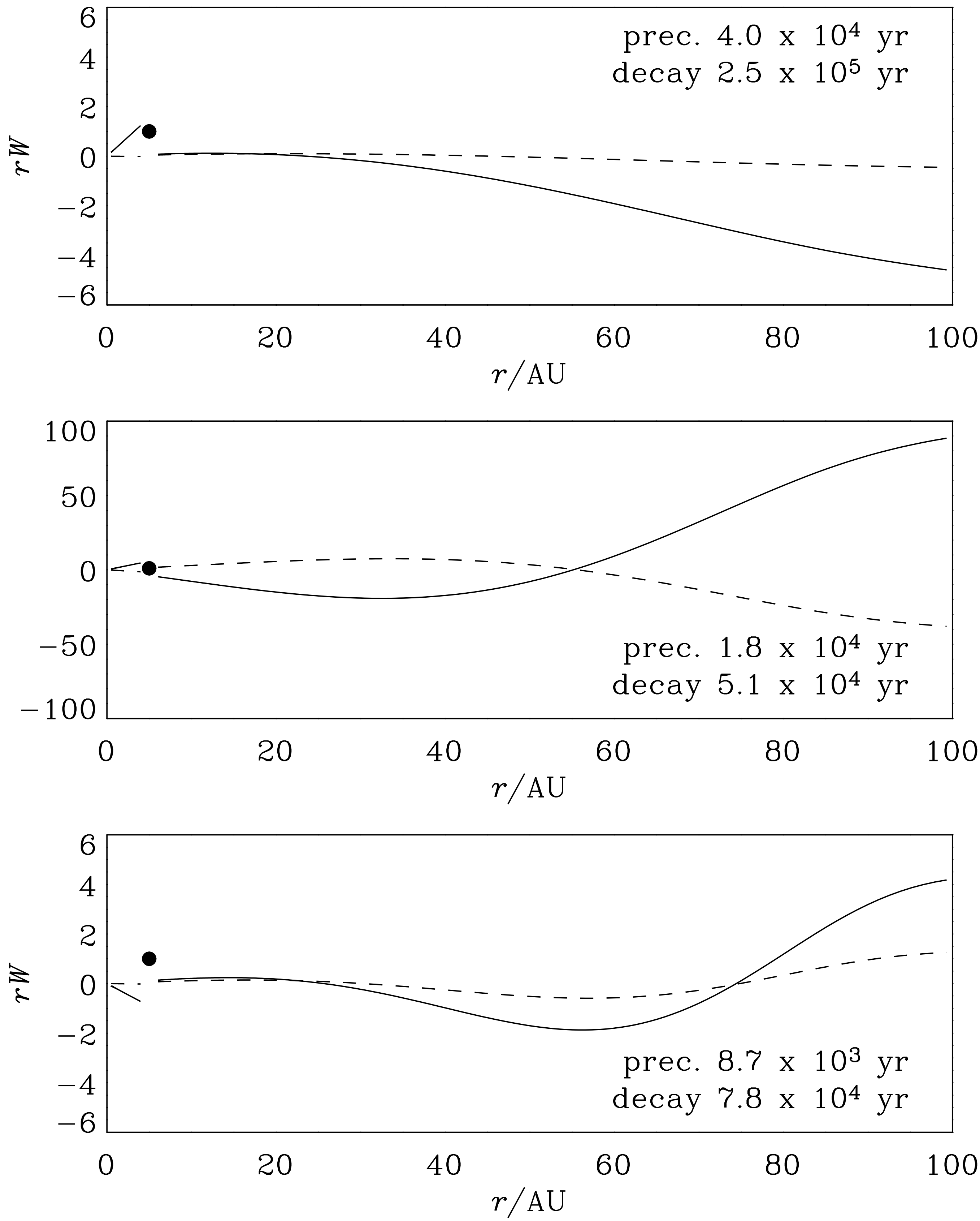
$$W_{\mathrm{p}i} = W_{\mathrm{d}k} = \mathrm{constant}$$

Infinite number of non-trivial modes, damped by viscosity

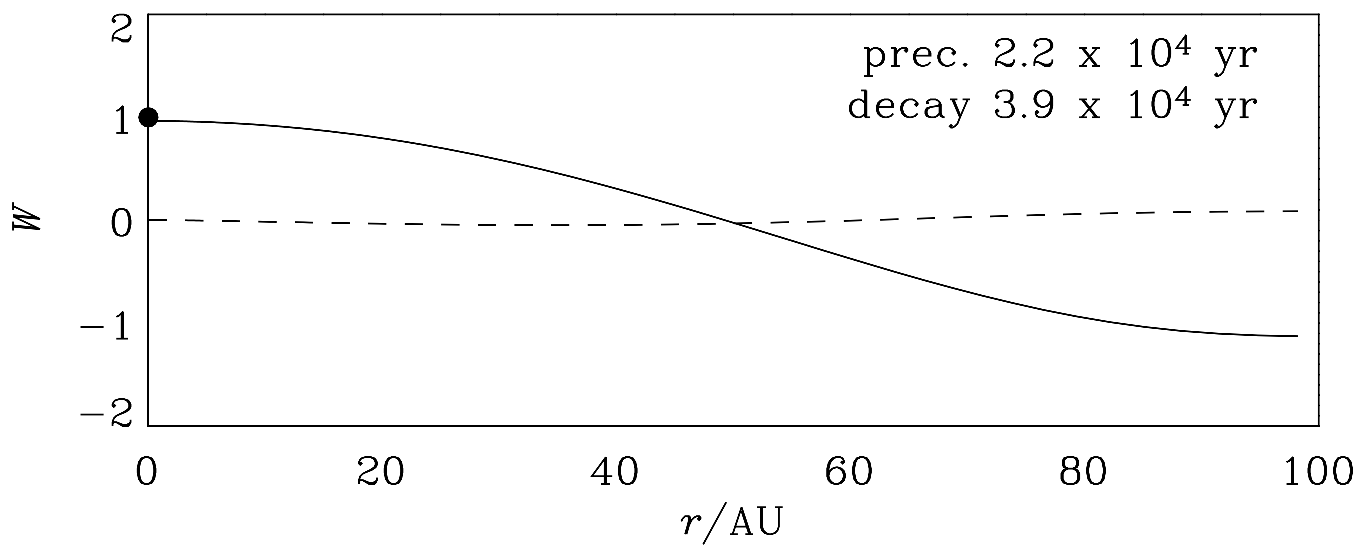
Low-order modes:

- nearly rigid behaviour if mode period $\gg$ sound travel time
- retrograde precession
- weakly damped

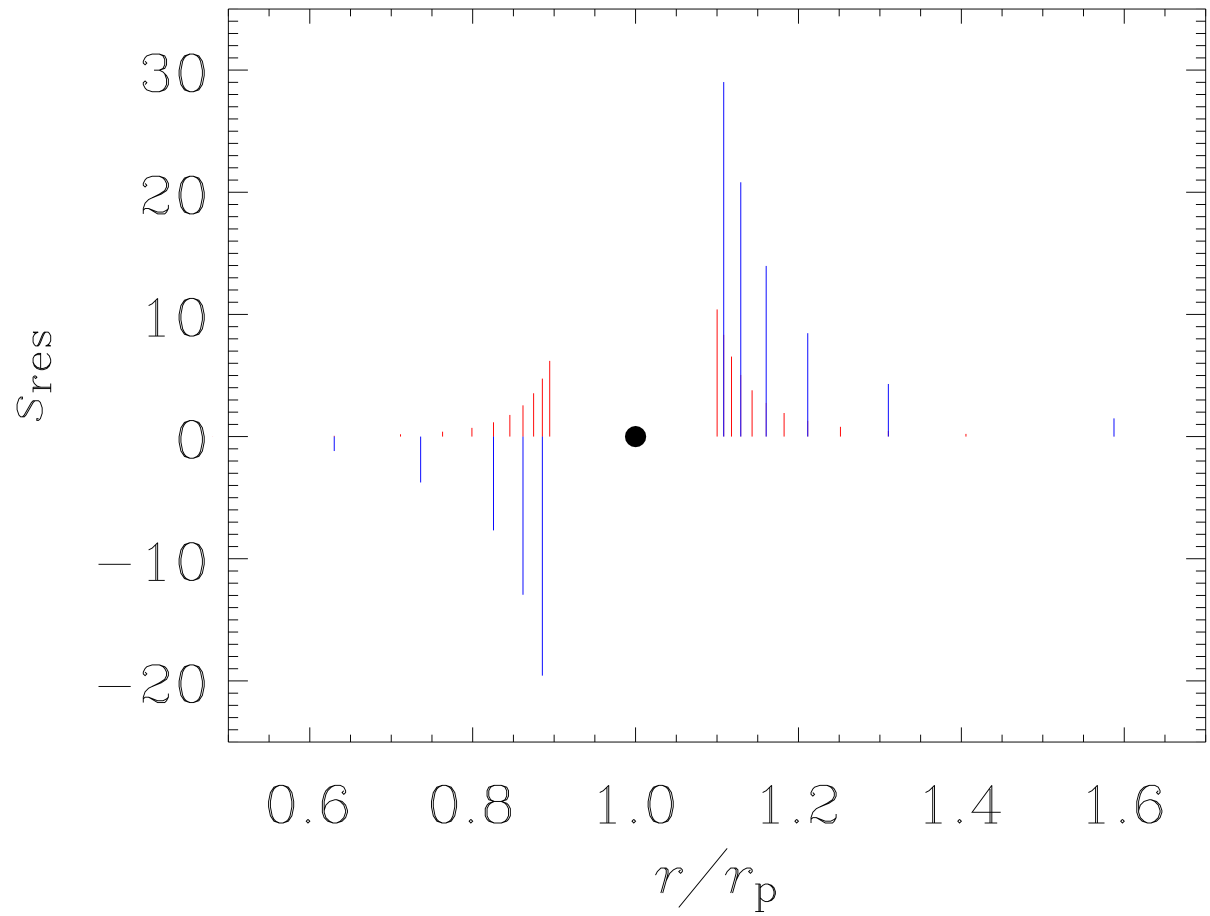
e.g. Jupiter in a 100 AU disc



e.g. Hot Jupiter (0.05 AU) in a 100 AU disc



Resonance strengths



To be summed, weighted by $\Sigma r|W - W_{\rm p}|^2$

Borderies, Goldreich & Tremaine (1984) – planetary rings:

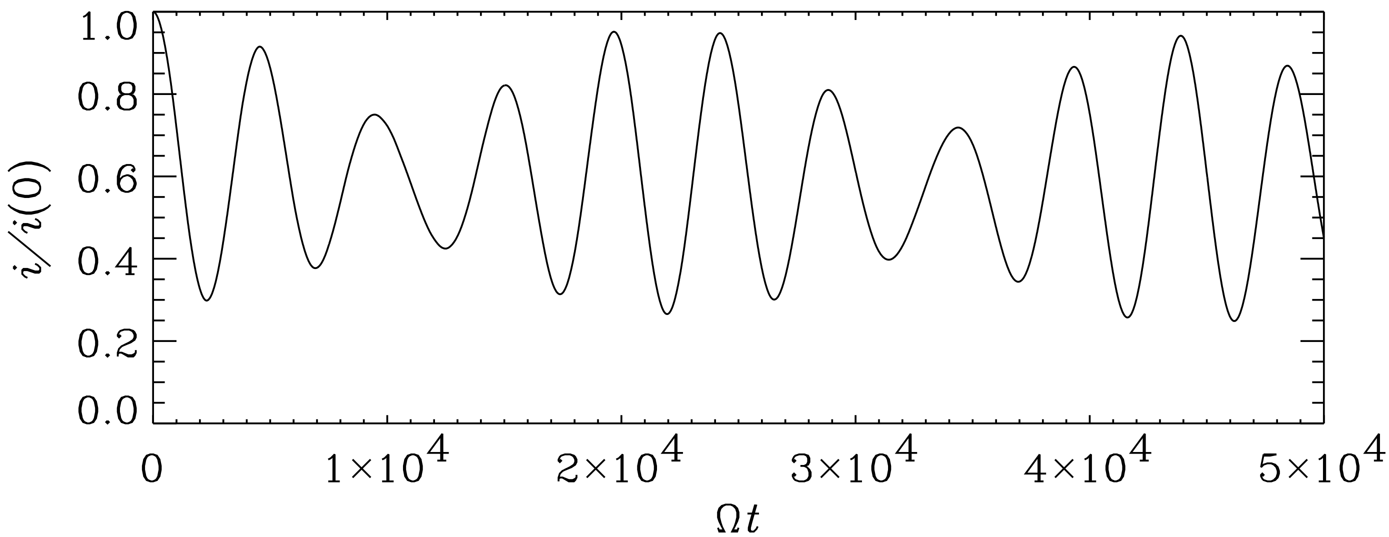
- neglect warping of rings
- inclination always grows

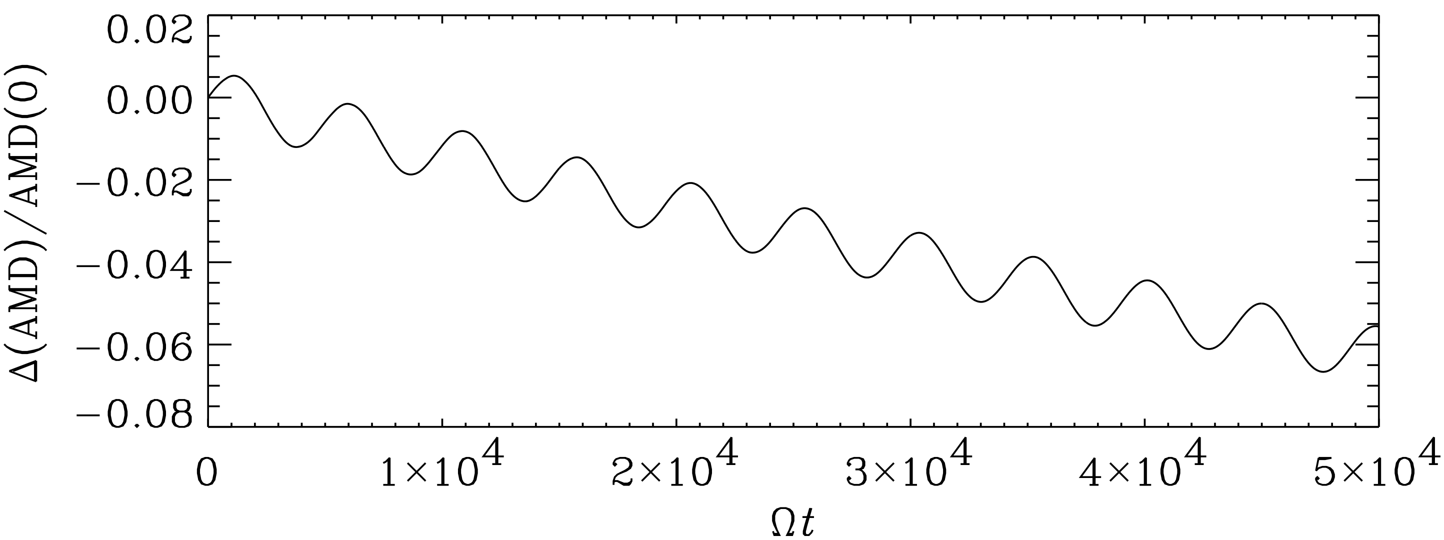
Lubow & Ogilvie (2001) – protoplanetary systems:

- compute global modes including warping and MMRs
- viscous damping typically prevails ($\alpha \gtrsim 10^{-3}$)

Typical time-dependent behaviour

Incline mobile planet with respect to flat disc





Inappropriate to measure $\mathrm{d}i/\mathrm{d}t$; monitor AMD instead

Non-mobile planet: different (erroneous) results

Similar behaviour expected for e

KEY POINTS / TOPICS FOR DISCUSSION

- e and i are shared properties of the planet–disc system
- AMD (including wave contribution) is a positive-definite measure of the bending disturbance
- AMD is conserved in secular exchanges but grows or decays as a result of competition between MMRs and disc-based dissipation
- (B)GT calculations of $\mathrm{d}e/\mathrm{d}t$, $\mathrm{d}i/\mathrm{d}t$ need to be revisited:
  - damping / excitation in the disc
  - modified weighting of resonant torques
- eccentricity dynamics of the disc is very subtle:
  - 2D / 3D
  - viscous / turbulent
  - NSG / SG
  - bias of polar grids?
- high resolution required to see ECR saturation and eccentricity growth